MATHS

SIMILAR TRIANGLES

★ INTRODUCTION

In earlier classes, you have learnt about congruence of two geometric figures, and also some basic theorems and results on the congruence of triangle. Two geometric figures having same shape and size are congruent to each other but two geometric figures having same shape are called similar. Two congruent geometric figures are always similar but the converse may or may not be true.

All regular polygons of same number of sides such as equilateral triangle, squares, etc, are similar. All circles are similar.

In some cases, we can easily notice that two geometric figures are not similar. For example, a triangle and a rectangle can never be similar. In case, we are given two triangles, they may appear to be similar but actually they may not be similar. So, we need some criteria to determine the similarity of two geometric figures. In particular, we shall discuss similar triangles.

★ HISTORICAL FACTS

EUCLID was a very great Greek mathematician born about 2400 years ago. He is called the father of geometry

because he was the first to establish a school of mathematics in Alexandria. He wrote a book on geometry called "The Elements" which has 13 volumes and has been used as a text book for over 2000 years. This book was further systematized by the great mathematician of Greece tike Thales, Pythagoras, Pluto and Aristotle.

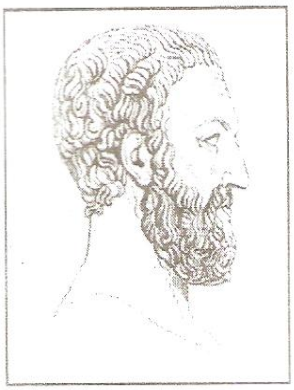
Abraham Lincoln, as a young lawyer was of the view that this Greek book was a splendid sharpener of human mind and improver his power of logic and language.

A king once asked Euclid, "Isn't there an easier way to understand geometry"

Euclid replied : "There is no royal-road way to geometry. Every one has to think for himself when studying."

Euclid : Father of Geometry
(about 300 B.C. Greece)

THALES (640-546 B.C.) a Greek mathematician was the first who initiated and formulated the theoretical study of geometry to make astronomy a more exact science. He is said to have introduced geometry in Greece. He is believed to have found the heights of the pyramids in Egypt, using shadows and the principle of similar triangles. The use of similar triangles has made possible the measurements of heights and distances. He proved the well-known and very useful theorem credited after his name : Thales Theorem.



Thales (640-546 B.C.)

★ CONGRUENT FIGURES

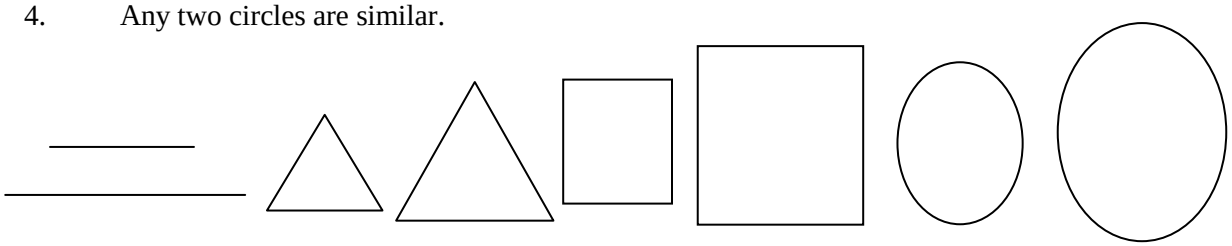
Two geometrical figures are said to be congruent, provided they must have same shape and same size. Congruent figures are alike in every respect.

EX. 1. Two squares of the same length.
2. Two circle of the same radii.
3. Two rectangles of the same dimensions.
4. Two wings of a fan.
5. Two equilateral triangles of same length.

★ SIMILAR FIGURES

Two figures are said to be similar, if they have the same shape. Similar figures may differ in size. Thus, two congruent figures are always similar, but two similar figures need not be congruent.

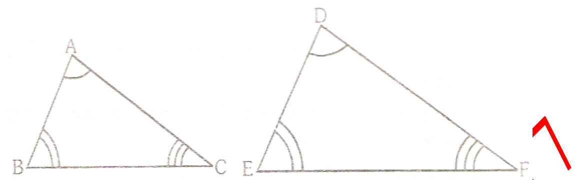
EX. 1. Any two line segments are similar.
2. Any two equilateral triangles are similar
3. Any two squares are similar.
4. Any two circles are similar.



We use the symbol '~' to indicate similarity of figures.

★ SIMILAR TRIANGLES

ΔABC and ΔDEF are said to be similar, if their corresponding angles are equal and the corresponding sides are proportional.



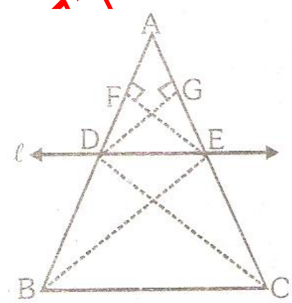
i.e., when $\angle A = \angle D,\ \angle B = \angle E,\ \angle C = \angle F$

and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$.

And, we write $\Delta ABC \sim \Delta DEF$.

The sign '~' is read as 'is similar to'.

THEOREM-1 (Thales Theorem or Basic Proportionality Theorem) : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.



Given : A ΔABC in which line ℓ parallel to BC ($DE \parallel BC$) intersecting AB at D and AC at E.

To prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

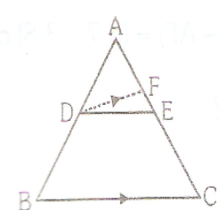
Construction : Join D to C and E to B. Through E drawn EF perpendicular to AB i.e., $EF \perp AB$ and through D draw $DG \perp AC$.

Proof :

	STATEMENT	REASON
1.	Area of (ΔADE) $= \frac{1}{2}(AD \times EF)$ Area of (ΔBDE) $= \frac{1}{2}(BD \times EF)$	Area of $\Delta = \frac{1}{2}$ base × altitude
2.	$\dfrac{Area(\Delta ADE)}{Area(\Delta BDE)} = \dfrac{\frac{1}{2} AD \times EF}{\frac{1}{2} BD \times EF} = \dfrac{AD}{DB}$	By 1.
3.	$\dfrac{Area(\Delta ADE)}{Area(\Delta BDE)} = \dfrac{\frac{1}{2} AE \times DG}{\frac{1}{2} EC \times DG} = \dfrac{AE}{EC}$	Similarly
4.	Area (ΔBDE) = Area (ΔCDE)	Δs BDE and CDE are on the same base BC and between the same parallel lines DE and BC.
5.	$\dfrac{Area(\Delta ADE)}{Area(\Delta BDE)} = \dfrac{AE}{EC}$	By 3. & 4.
6.	$\dfrac{AD}{DB} = \dfrac{AE}{EC}$	By 1. & 5.

Hence proved.

THEOREM-2 (Converse of Basic Proportionality Theorem) : If a line divided any two sides of a triangle proportionally, the line is parallel to the third side.



Given : A ΔABC and DE is a line meeting AB and AC at D and E respectively such that $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

To prove : $DE \parallel BC$

Proof :

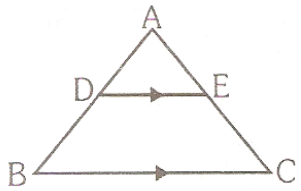
	STATEMENT	REASON
1.	If possible, let DE be not parallel to BC. Then, draw DF $\parallel$ BC	
2.	$\frac{AD}{DB}=\frac{AE}{FC}$	By Basic Proportionality Theorem.
3.	$\frac{AD}{DB}=\frac{AE}{EC}$	Given
4.	$\therefore \frac{AF}{FC}=\frac{AE}{EC}$	From 2 and 3.
	$\Rightarrow \frac{AF}{FC}+1=\frac{AE}{EC}+1$	Adding 1 on both sides.
	$\Rightarrow \frac{AF+FC}{FC}=\frac{AE+EC}{EC}$	By adding.
	$\Rightarrow \frac{AC}{FC}=\frac{AC}{EC}$	AF + FC = AC and AE + EC = AC.
	$\Rightarrow FC=EC \Rightarrow$ E and F coincide.	
	But, DF $\parallel$ BC. Hence DE $\parallel$ BC.	

Hence, proved.

Ex.1 In the adjoining figure. DE $\parallel$ BC.

(i) If AD = 3.4 cm, AB = 8.5 cm and AC = 13.5 cm, find AE

(ii) If $\frac{AD}{DB}=\frac{3}{5}$ and AC = 9.6 cm, find AE.



Sol. (i) Since DE $\parallel$ BC, we have $\frac{AD}{AB}=\frac{AE}{AC}$

$\therefore \quad \frac{3.4}{8.5}=\frac{AE}{13.5} \Rightarrow \frac{3.4\times 13.5}{8.5}=5.4$

Hence, AE = 5.4 cm.

(i) Since DE $\parallel$ BC, we have $\frac{AD}{DB}=\frac{AE}{EC}$

$\therefore \quad \frac{AE}{AC}=\frac{3}{5}\left[\because \frac{AD}{DB}=\frac{3}{5}(Given)\right]$

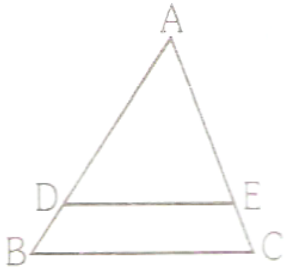
Let AE = x cm. Then, EC = (AC – AE) = (9.6 – x) cm.

$\therefore \quad \frac{x}{9.6-x}=\frac{3}{5} \Rightarrow 5x=3(9.6-x)$

$\Rightarrow \quad 5x = 28.8 - 3x \Rightarrow 8x = 28.8 \Rightarrow x = 3.6.$

$\therefore \quad$ AE = 3.6 cm.

Ex.2 In the adjoining figure, AD = 5.6 cm, AB = 8.4 cm, AE = 3.8 cm and AC = 5.7 cm. Show that DE $\parallel$ BC.



Sol. We have, AD = 5.6 cm, DB = (AB – AD) = (8.4 – 5.6) cm = 2.8 cm.

AE = 3.8 cm, EC = (AC – AE) = (5.7 – 3.8) cm = 1.9 cm.

$\therefore \quad \frac{AD}{DB}=\frac{5.6}{2.8}=\frac{2}{1}$ and $\frac{AE}{EC}\frac{3.8}{1.9}=\frac{2}{1}$

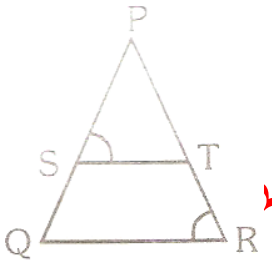
Thus, $\frac{AD}{DB}=\frac{AE}{EC}$

$\therefore \quad$ DE divides AB and AC proportionally.

Hence, DE $\parallel$ BC

Ex.3 In fig, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.

[NCERT]



Sol. It is given that $\frac{PS}{SQ} = \frac{PT}{TR}$

So, $ST \parallel QR$ [Theorem]

Therefore, $\angle PST = \angle PQR$ [Corresponding angles] – (1)

Also, it is given that

$\angle PST = \angle PRQ$ (2)

So, $\angle PRQ = \angle PQR$ [Form 1 and 2]

Therefore $PQ = PR$ [Sides opposite the equal angles]

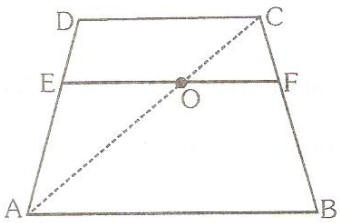
i.e., PQR is an isosceles triangle.

Ex.4 Prove that any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally (i.e., In the same ratio).

OR

ABCD is a trapezium with $DE \parallel AB$. E and F are points on AD and BC respectively such that $EF \parallel AB$. Show that $\frac{AE}{ED} = \frac{BF}{FC}$

[NCERT]



Sol. We are given trapezium ABCD.

$CD \parallel BA$

$EF \parallel AB$ and CD both

We join AC.

It Mets EF at O.

In ΔACD, $OE \parallel CD$

$\Rightarrow \quad \frac{AO}{OC} = \frac{AE}{ED}$ …(i)

(Basic Proportionality Theorem)

In ΔCAB, $OF \parallel AB$

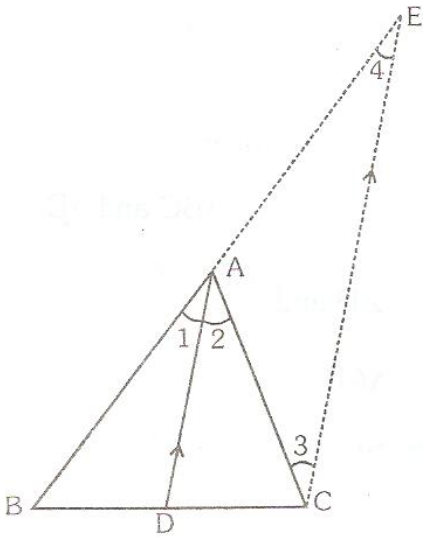
$\Rightarrow \quad \frac{CO}{OA} = \frac{CF}{FB}$ [B.P.T.]

$\Rightarrow \quad \frac{AO}{OC} = \frac{BF}{FC}$ …(ii)

From (i) and (ii)

$\frac{AE}{ED} = \frac{BF}{FC}$

Hence, proved.



Ex.5 Prove that the internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.

(Internal Angle Bisector Theorem)

Sol. **Given :** A Δ ABC in which AD is the internal bisector of $\angle A$.

To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$

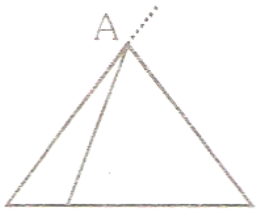
Construction : Draw $CE \parallel DA$, meeting BA produced at E.

Proof :

	STATEMENT	REASON
1.	$\angle 1 = \angle 2$	AD is the bisector of $\angle A$
2.	$\angle 2 = \angle 3$	Alt. $\angle$s are equal, as CE $\parallel$ DA and AC is the transversal
3.	$\angle 1 = \angle 4$	Corres. $\angle$s are equal, as CE $\parallel$ DA and BE is the transversal
4.	$\angle 3 = \angle 4$	From 1, 2 and 3.
5.	AE = AC	Sides opposite to equal angles are equal
6.	In ΔBCE, DA $\parallel$ CE $\Rightarrow \frac{BD}{DC} = \frac{BA}{AE}$ $\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$	By B. P. T. Using 5

Hence, proved.

Remark : The external bisector of an angle divides the opposite side externally in the ratio of the sides containing the angle. i.e., if in a ΔABC, AD is the bisector of the exterior of angle $\angle$A and intersect BC produced in D, $\frac{BD}{CD} = \frac{AB}{AC}$.



★ AXIOMS OF SIMILARITY OF TRIANGLES

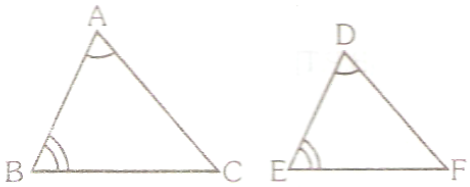
1. AA (Angle-Angle) Axiom of Similarity :

If two triangles have two pairs of corresponding angles equal, then the triangles are similar. In the given figure,

ΔABC and ΔDEF are such that

$\angle A = \angle D$ and $\angle B = \angle E$.

$\therefore$ $\Delta ABC \sim \Delta DEF$



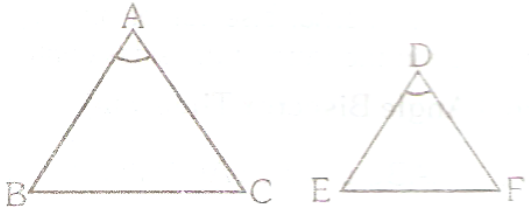
2. SAS (Side-Angle-Side) Axiom of Similarity :

If two triangles have a pair of corresponding angles equal and the sides including them proportional, then the triangles are similar.

In the given fig, ΔABC and ΔDEF are such that

$\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$

$\therefore$ $\Delta ABC \sim \Delta DEF$

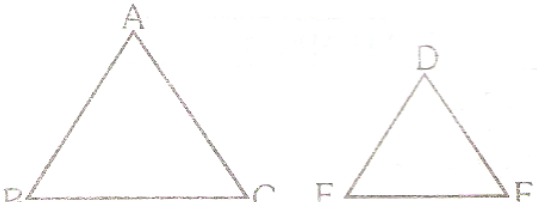


3. SSS (Side- Side- Side) Axiom of Similarity :

If two triangles have three pair of corresponding sides proportional, then the triangles are similar.

If in ΔABC and ΔDEF we have :

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$, then $\Delta ABC \sim \Delta DEF$.



Ex.6. In figure, find $\angle$L.

Sol. In ΔABC and ΔLMN,

$\frac{AB}{LM} = \frac{4.4}{11} = \frac{2}{5}$

$\frac{BC}{MN} = \frac{4}{10} = \frac{2}{5}$ and $\frac{CA}{NL} = \frac{3.6}{9} = \frac{2}{5}$

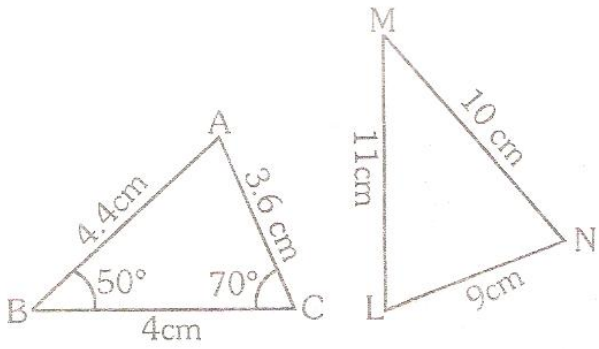
$\Rightarrow$ $\frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{NL}$

$\Rightarrow$ $\Delta ABC \sim \Delta LMN$ (SSS Similarity)

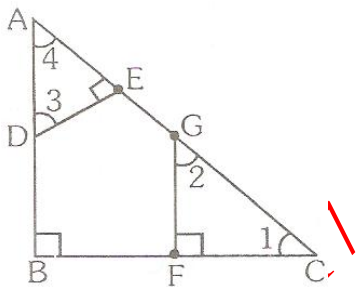
$\Rightarrow$ $\angle L = \angle A = 180^0 - \angle B - \angle C$

$= 180^0 - 50^0 - 70^0 = 60^0$

$\therefore$ $\angle L = 60^0$



Ex.7 In the figure, $AB \perp BC$, $DE \perp AC$, and $GF \perp BC$, Prove that $\Delta ADE \sim \Delta GCF$.



Sol. $\angle 1 + \angle 4 = \angle 1 + \angle 2$ (each side = 90^0)

$\Rightarrow \quad \angle 4 = \angle 2$

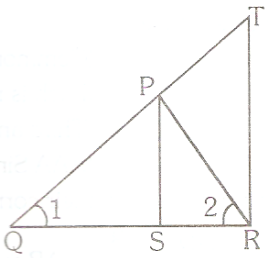
$\Rightarrow \quad \angle A = \angle G \qquad \ldots(i)$

Also $\quad \angle E = \angle F \qquad \ldots(ii)$ (each equal to 90^0)

From (i) and (ii), we get AA similarity for triangle ADE and GCF.

$\Rightarrow \quad \Delta ADE \sim \Delta GCF$

Ex.8 In fig, $\frac{QT}{PR} = \frac{QR}{QS}$ and $\angle 1 = \angle 2$. Prove that $\Delta PQS \sim \Delta TQR$.



Sol. $\angle 1 = \angle 2$ (Given)

$\Rightarrow \quad PR = PQ \qquad \ldots(i)$

(Sides opposite to equal angles in ΔQRP)

Also $\quad \frac{QT}{PR} = \frac{QR}{QS}$ (Given) $\qquad \ldots(i)$

From (i) and (ii), we have

$$\frac{QT}{PR} = \frac{QR}{QS} \Rightarrow \frac{QP}{QT} = \frac{QS}{QR} \qquad \ldots(iii)$$

Now, in triangles PQR and TQR, we have

$\angle PQS = \angle TQR$ (each = $\angle 1$)

and $\quad \frac{PQ}{TQ} = \frac{QS}{QR}$ (from (3))

$\Rightarrow \quad \Delta PQS \sim \Delta TQR$ (SAS Similarity)

Ex.9 In fig, CD and GH are respectively, the medians of ΔABC and ΔFEG, If $\Delta ABC \sim \Delta FEG$, prove that

(i) $\Delta ADC \sim \Delta FHG$

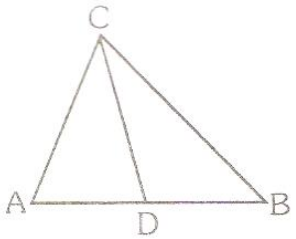
(ii) $\frac{CD}{GH} = \frac{AB}{FE}$ **(NCERT)**

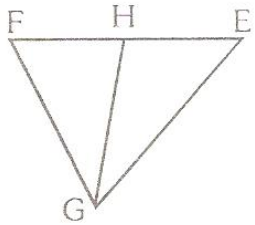
Sol. $\Delta ABC \sim \Delta FEG$ (given)

$\Rightarrow \quad \angle A = \angle F, \qquad \ldots(i)$ ($\because$ the corresponding angles of the similar triangles are equal)

Also, $\quad \frac{AC}{FG} = \frac{AB}{FE}$ (Corresponding sides are proportional)

$\Rightarrow \quad \frac{AC}{FG} = \frac{2AB}{2FH}$ $\left(\begin{matrix} D\,is\,mid-point\,of\,AB \\ H\,is\,mid-point\,of\,FE \end{matrix}\right)$





$\Rightarrow \quad \frac{AC}{AD} = \frac{FG}{FH} \qquad \ldots(ii)$

Now, in triangles ADC and FHG, we have

$\angle A = \angle F$ and $\frac{AC}{AD} = \frac{FG}{FH}$ (By (i) and (ii))

$\Rightarrow \quad \Delta ADC \sim \Delta FHG$ (SAS similarity)

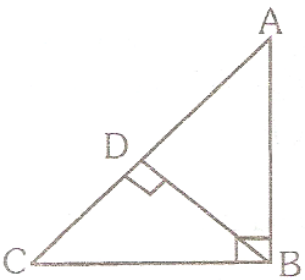
(ii) $\Delta ADC \sim \Delta FHG$

$\Rightarrow \quad \frac{CD}{GH} = \frac{AD}{FH}$ (Corresponding sides proportional)

$\Rightarrow \quad \frac{CD}{GH} = \frac{2 \times AD}{2 \times FH} \quad \Rightarrow \quad \frac{CD}{GH} = \frac{AB}{FE}$

Ex,10 ABC is a right triangle, right angled at B. If BD is the length of perpendicular drawn from B to AC. Prove that :

(i) $\Delta ADB \sim \Delta ABC$ and hence $AB^2 = AD \times AC$ (ii) $\Delta BDC \sim \Delta ABC$ and hence $BC^2 = CD \times AC$



(iii) $\Delta ADB \sim \Delta BDC$ and hence $BD^2 = AD \times DC$ (iv) $\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{BD^2}$

Sol. **Given :** ABC is right angled triangle at B and $BD \perp AC$

To prove :

(i) $\Delta ADB \sim \Delta ABC$ and hence $AB^2 = AD \times AC$

(ii) $\Delta BDC \sim \Delta ABC$ and hence $BC^2 = CD \times AC$

(iii) $\Delta ADB \sim \Delta BDC$ and hence $BD^2 = AD \times DC$

(iv) $\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{BD^2}$

Proof : (i) In two triangles ADB and ABC, we have :

$\angle BAD = \angle BAC$ (Common)

$\angle ADB = \angle ABC$ (Each is right angle)

$\angle ABD = \angle ACB$ (Third angle)

$\angle ADB = \angle ABC$ (AAA Similarity)

Triangle ADB and ABC are similar and so their corresponding sides must be proportion.

$\frac{AD}{AB} = \frac{DB}{DC} = \frac{AB}{AC} \Rightarrow \frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AB \times AB = AC \times AD \Rightarrow AB^2 = AD \times AC$. This proves (a).

(ii) Again consider two triangles BDC and ABC, we have

$\angle BCD = \angle ACB$ (Common)

$\angle BDC = \angle ABC$ (Each is right angle)

$\angle DBC = \angle BAC$ (Third angle

$\therefore$ Triangle are similar and their corresponding sides must be proportional.

i.e., $\angle BAD = \angle BAC$ $\frac{BD}{AB} = \frac{DC}{BC} = \frac{BC}{AC}$

(iii) In two triangle ADB and BDC, we have :

$\Rightarrow$ $\frac{DC}{BC} = \frac{BC}{AC} \Rightarrow BC \times BC = DC \times AC \Rightarrow BC^2 = CD \times AC$ This proves (ii)

$\angle BDA = \angle BDC = 90^0$

$\angle 3 = \angle 2 = 90^0 \angle 1$ [$\because \angle 1 + \angle 2 = 90^0$, $\angle 1 + \angle 3 = 90^0$]

$\angle 1 = \angle 4 = 90^0 \angle 2$ [$\because \angle 1 + \angle 2 = 90^0$, $\angle 2 + \angle 4 = 90^0$]

$\Delta ADB \sim \Delta BDC$ (AAA criterion of similarity)

$\Rightarrow$ Their corresponding sides must be proportional.

$\frac{AD}{BD} = \frac{DB}{DC} = \frac{AB}{BC} \Rightarrow \frac{AD}{BD} = \frac{DB}{DC} \Rightarrow BD \times BD = AD \times AC$

$\therefore$ BD is the mean proportional of AD and DC

(iv) From (i), we have : AB2 = AD × AC

(ii), we have : BC2 = CD × AC

(iii), we have : BD2 = ADDC

Consider $\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{AD \times AC} + \frac{1}{CD \times AC} + \frac{1}{AC}\left[\frac{1}{AD} + \frac{1}{DC}\right]$

$\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{AC}\left[\frac{DC}{AD} + \frac{AD}{DC}\right] = \frac{1}{AC}\left[\frac{AD + DC}{AD \times DC}\right] = \frac{1}{AC}\left[\frac{AC}{AD \times DC}\right]$

$= \frac{1}{AD \times DC} = \frac{1}{BD^2}$ (from (iii))

$\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{BD^2}$

Thus we have proved the following :

If a perpendicular is drawn from the vertex containing the right angle of a right triangle to the hypotenuse then:

(a) Thu triangle on each side of the perpendicular are similar to each other and also similar to the original triangle.

i.e., $\Delta ADB \sim \Delta BDC$, $\Delta ADB \sim \Delta ABC$, $\Delta BDC \sim \Delta ABC$

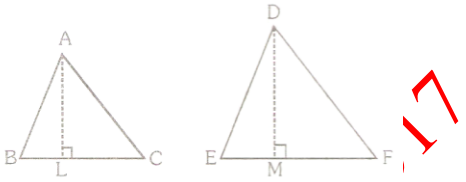
(b) **The square of the perpendicular is equal to the product of the length of two parts into which the hypotenuse is divided by the perpendicular i.e., BD2 = AD × 0DC.**

★ RESULTS ON AREA OF SIMILAR TRIANGLES

Theorem-3 : The areas of two similar triangles are proportional to the squares on their corresponding sides.

Given : $\Delta ABC \sim \Delta DEF$

To prove : $\frac{Area\,of\ \Delta ABC}{Area\,of\ \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$



Construction : Draw $AL \perp BC$ and $DM \perp EF$.

Proof:

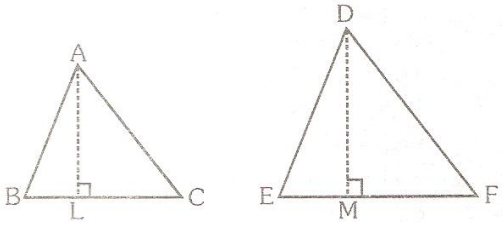
	STATEMENT	REASON
1.	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM}$ $\Rightarrow \frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{BC}{EF} \times \frac{AL}{DM}$	Area of $\Delta = \frac{1}{2} \times \text{Base} \times \text{Height}$
2.	In Δ ALB and Δ DME, we have (i) $\angle$ ALB = $\angle$ DME (ii) $\angle$ ABL = $\angle$ DEM $\therefore \Delta ALB \sim \Delta DME$ $\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$	Each equal to 90^0 $\Delta ABC \sim \Delta DEF \Rightarrow \angle B = \angle E$ AA=axiom
3.	Δ ABC ~ Δ DEF $\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	Corresponding sides of similar Δ s are proportional. Given.
4.	$\frac{AL}{DM} = \frac{BC}{EF}$	Corresponding sides of similar Δ s are proportional. From 2 and 3.
5.	Substituting $\frac{AL}{DM} = \frac{BC}{EF}$ in 1, we get : $\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{BC^2}{EF^2}$	
6.	Combining 3 and 5, we get : $\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$	

Corollary-1 : The areas of two similar triangles are proportional to the squares on their corresponding altitude.

Given : $\Delta ABC \sim \Delta DEF$, $AL \perp BC$ and $DM \perp EF$.

To prove : $\frac{Area\,of\ \Delta ABC}{Area\,of\ \Delta DEF} = \frac{AL^2}{DM^2}$

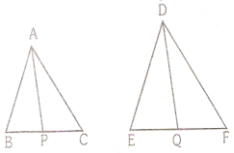
Proof :



	STATEMENT	REASON
1.	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF}=\frac{\frac{1}{2}\times BC\times AL}{\frac{1}{2}\times EF\times DM}$ $\Rightarrow \frac{Area\,\Delta ABC}{Area\,\Delta DEF}=\frac{BC}{EF}=\frac{AL}{DM}$	Area of $\Delta=\frac{1}{2}\times$ Base $\times$ Height
2.	In Δ ALB and Δ DME, we have (i) $\angle$ ALB = $\angle$ DME (ii) $\angle$ ABL = $\angle$ DEM $\Rightarrow \Delta$ ALB ~ Δ DME $\Rightarrow \frac{AL}{DM}=\frac{AB}{DE}$	Each equal to 90^0 Δ ABC ~ Δ DEF $\Rightarrow \angle$ B = $\angle$ E AA=axiom
3.	Δ ABC ~ Δ DEF $\Rightarrow \frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$ $\frac{BC}{EF}=\frac{AL}{DM}$	Corresponding sides of similar Δ s are proportional. Given. Corresponding sides of similar Δ s are proportional.
4.	Substituting $\frac{BC}{EF}=\frac{AL}{DM}$ in 1, we get :	From 2 and 3. Hence proved.
5.	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF}=\frac{AL^2}{DM^2}$	

Corollary-2 : The areas of two similar triangles proportional to the squares on their corresponding medians.

Given : Δ ABC ~ Δ DEF and AP, PQ are their medians. **To prove :** $\frac{Area\,of\,\Delta ABC}{Area\,of\,\Delta DEF}=\frac{AP^2}{DQ^2}$



Proof :

STATEMENT	REASON

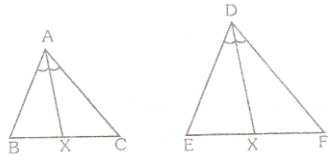
1.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AB^2}{DE^2}$ ……I.	Given Area of two similar Δs are proportional to the squares on their corresponding sides.
2.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{2BP}{2EQ} = \frac{BP}{EQ}$ ……II.	Corresponding sides of similar Δs are proportional
3.	$\frac{AB}{DE} = \frac{BP}{EQ}$ and $\angle A = \angle D$ $\Rightarrow \Delta APB \sim \Delta DQE$ $\Rightarrow \frac{BP}{EQ} = \frac{AP}{DQ}$ …….III $\Rightarrow \frac{AB}{DE} = \frac{AP}{DQ}$ $\Rightarrow \frac{AB^2}{DE^2} = \frac{AP^2}{DQ^2}$ ……..IV	From II and the fact the $\Delta ABC \sim \Delta DEF$ By SAS-similarity axiom From II and III.
4.	$\Rightarrow \frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AP^2}{DQ^2}$	From I and IV.

Hence, proved

Corollary-3 : The areas of two similar triangles proportional to the squares on their corresponding angle bisector segments.

Given : $\Delta ABC \sim \Delta DEF$ and AX, DY are their

To prove : $\frac{Area\,of\,\Delta ABC}{Area\,of\,\Delta DEF} = \frac{AX^2}{DY^2}$

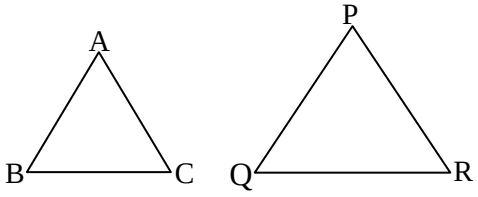


Proof :

	STATEMENT	**REASON**
1.	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AB^2}{DE^2}$	Area of two similar Δs are proportional to the squares on their corresponding sides.
2.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \angle A = \angle D$ $\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle D$ $\Rightarrow \angle BAX = \angle EDY$	Given $\Rightarrow \angle BAX = \frac{1}{2}\angle A$ and $\angle EDY = \frac{1}{2}\angle D$
3.	**In** ΔABX and ΔEDY, we have $\angle BAX = \angle EDY$ $\angle B = \angle E$ $\therefore \Delta ABX \sim \Delta DEY$ $\Rightarrow \frac{AB}{DE} = \frac{AX}{DY} \Rightarrow \frac{AB^2}{DE^2} = \frac{AX^2}{DY^2}$	Given From 2. $\Delta ABC \sim \Delta DEF$ By AA similarity axiom
4.	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AX^2}{DY^2}$	From 1 and 3.

Ex.11 It is given that $\Delta ABC \sim \Delta PQR$, area ($\Delta ABC$) = 36 cm^2 and area ($\Delta PQR$) = 25 cm^2. If QR = 6 cm, find length of BC.

Sol. We know that the areas of similar triangles are proportional to the squares of their corresponding sides.



$$\therefore \quad \frac{Area\,of\,(\Delta ABC)}{Area\,of\,(\Delta PQR)} = \frac{BC^2}{QR^2}$$

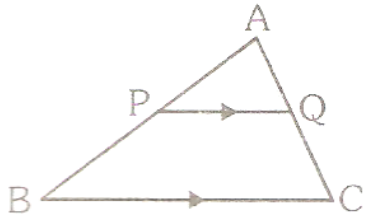
Let BC = x cm. Then.

$$\frac{36}{25} = \frac{x^2}{6^2} \Leftrightarrow \frac{36}{25} = \frac{x^2}{36} \Leftrightarrow x^2 = \frac{36\times 36}{25} \Leftrightarrow x = \left(\frac{6\times 6}{5}\right) = \frac{36}{5} = 7.2$$

Hence BC = 7.2 cn

Ex.12 P and Q are points on the sides AB and AC respectively of Δ ABC such that PQ ‖ BC and divides Δ ABC into parts, equal in area. Find PB : AB.

Sol. Area (Δ APQ) = Area (trap. PBCQ) [Given]



$\Rightarrow$ Area (Δ APQ) = [Area (Δ ABC) – Area (Δ APQ)

$\Rightarrow$ 2 Area (Δ APQ) = Area (Δ ABC)

$$\Rightarrow \quad \frac{Area\,of\,(\Delta APQ)}{Area\,of\,(\Delta ABC)} = \frac{1}{2} \quad \ldots(i)$$

Now, in Δ APQ and Δ ABC, we have

∠ PAQ = ∠ BAC [Common ∠ A]

∠ APQ = ∠ ABC [PQ ‖ BC, corresponding ∠ s are equal]

$\therefore$ Δ APQ ~ Δ ABC.

We known that the areas of similar Δ s are proportional to the squares of their corresponding sides.

$$\therefore \quad \frac{Area\,of\,(\Delta APQ)}{Area\,of\,(\Delta ABC)} = \frac{AP^2}{AB^2} \Rightarrow \frac{AP^2}{AB^2} = \frac{1}{2} \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad \frac{AP}{AB} = \frac{1}{\sqrt{2}} \text{ i.e., AB} = \sqrt{2}\,.\,\text{AP}$$

$\Rightarrow$ AB = $\sqrt{2}$ (AB – PB) $\Rightarrow \sqrt{2}$ PB = ($\sqrt{2}$ – 1) AB

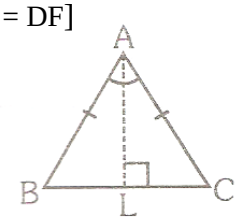
$$\Rightarrow \quad \frac{PB}{AB} = \frac{(\sqrt{2}-1)}{\sqrt{2}}\,.$$

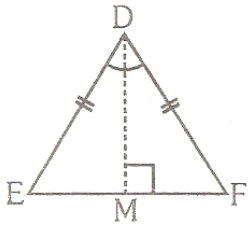
$\therefore$ PB : AB = ($\sqrt{2}$ – 1) : $\sqrt{2}$

Ex.13 Two isosceles triangles have equal vertical angles and there areas are in the ratio 16 : 25. Find the ratio of their corresponding heights.

Sol. Let Δ ABC and Δ DEF be the given triangles in which AB = AC, DE = DF, ∠ A = ∠ D and $\frac{Area\,of\,(\Delta ABC)}{Area\,of\,(\Delta DEF)} = \frac{16}{25}$. Draw AL ⊥ BC and DM ⊥ EF

Now, $\frac{AB}{AC} = 1$ and $\frac{DE}{DF} = 1$ [$\because$ AB = AC and DE = DF]





$$\Rightarrow \quad \frac{AB}{DE} = \frac{AC}{DF}\,.$$

$\therefore$ In Δ ABC and Δ DEF, we have

$$\frac{AB}{DE} = \frac{AC}{DF} \text{ and } \angle A = \angle D$$

$\Rightarrow$ Δ ABC ~ Δ DEF [By SAS similarity axiom]

But, the ratio of the areas of two similar Δ s is the same as the ratio of the square of their corresponding heights.

$$\frac{Area\,of\,(\Delta ABC)}{Area\,of\,(\Delta DEF)} = \frac{AL^2}{DM^2} \quad \Rightarrow \quad \frac{16}{25} = \left(\frac{AL^2}{DM^2}\right) \quad \Rightarrow \quad \frac{AL}{DM} = \frac{4}{5}$$

$\therefore$ AL : DM = 4 : 5, i.e., the ratio of their corresponding heights = 4 : 5,

Ex.14 If the areas of two similar triangles are equal, prove that they are congruent.

Sol. Let Δ ABC ~ Δ DEF and area (Δ ABC) = area (Δ DEF).

Since the ratio of the areas of two similar Δ s is equal to the ratio of the squares on their corresponding sides, we have

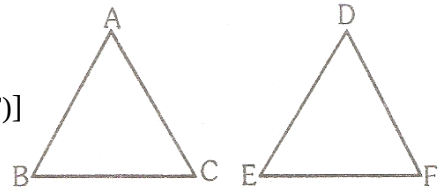
$$\frac{Area\,of\,(\Delta ABC)}{Area\,of\,(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$$

$\Rightarrow \quad \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2} = 1$ [$\because$ Area (Δ ABC) = Area (Δ DEF)]

$\Rightarrow$ $AB^2 = DE^2$, $AC^2 = DF^2$ and $BC^2 = EF^2$

$\Rightarrow$ AB = DE, AC = DF and BC = EF

$\therefore$ $\Delta ABC \cong \Delta DEF$ [By SSS congruence]



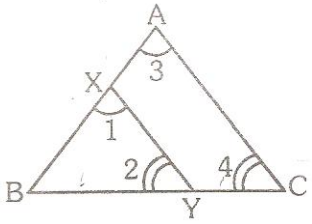
Ex.15 In fig, the line segment XY is parallel to side AC of Δ ABC and it divides the triangle into two parts of equal areas. Find the ratio $\frac{AX}{AB}$. **[NCERT]**

Sol, We are given that XY $\parallel$ AC.

$\Rightarrow$ $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ [Corresponding angles]

$\Rightarrow$ $\Delta BXY \sim \Delta BAC$ [AA similarity]

$\Rightarrow \quad \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{(BY)^2}{(BA)^2}$ [By theorem] …(i)



Also, we are given that

$$ar(\Delta BXY) = \frac{1}{2} \times ar(\Delta BAC) \Rightarrow \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{1}{2} \quad \text{…(ii)}$$

From (i) and (ii), we have $\left(\frac{BX}{BA}\right)^2 = \frac{1}{2} \Rightarrow \frac{BX}{BA} = \frac{1}{\sqrt{2}}$ …(iii)

Now, $\frac{AX}{AB} = \frac{AB - BX}{AB} = 1 - \frac{BX}{AB} = 1 - \frac{BX}{BA} = 1 - \frac{1}{\sqrt{2}}$ [By (iii)]

$= \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$

Hence, $= \frac{AX}{AB} = \frac{2-\sqrt{2}}{2}$

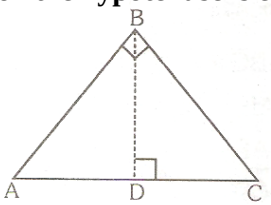
Theorem-4 [Pythagoras Theorem] : In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Given : A Δ ABC in which $\angle B = 90^0$.

To prove : $AC^2 = BA^2 + BC^2$.

Construction : From B, Draw BD $\perp$ AC.

Proof :



STATEMENT	REASON

1.	In Δ ADB and Δ ABC, we have : $\angle$BAD = $\angle$CAB = $\angle$A $\angle$ADB = $\angle$ABC $\therefore \Delta ADB \sim \Delta ABC$ $\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ $\Rightarrow AB^2 = AD \times AC$ ..(i)	Common Each = 90^0 By AA axiom of similarity Corr. sides of similar Δs are proportional
2.	In ΔCDB and ΔCBA, we have : $\Delta CDB = \Delta CBA$ $\angle$BCD = $\angle$ACB $\therefore \Delta CDB \sim \Delta CBA$ $\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$ $\Rightarrow BC^2 = DC \times AC$ ..(ii)	Each = 90^0 Common By AA axiom of similarity Corr. sides of similar Δs are proportional
3.	Adding (i) and (ii), we get $AB^2 + BC^2 = AD \times AC + DC \times AC$ = (AD + DC) $\times AC = AC^2$	$\because AD + DC = AC$

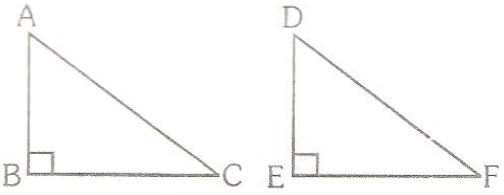
Hence, $AB^2 + BC^2 = AC^2$

Theorem-5 [Converse of pythagoras Theorem] : In a triangle if the square of one sides is equal to the sum of the squares of the squares of the other two sides, then the triangle is right angled.

Given : A Δ ABC in which $AB^2 + BC^2 = AC^2$

To prove : $\angle B = 90^0$.

Construction : Draw a Δ DEF in which CE = AB, EF = BC and $\angle E = 90^0$



Proof :

	STATEMENT	**REASON**
1.	In Δ DEF, we have : $\angle = 90^0$ $\therefore DE^2 + EF^2 = DF^2$ $\Rightarrow AB^2 + BC^2 = DF^2$ $\Rightarrow AC^2 = DF^2$	By Pythagoras Theorem $\because$ DE = AB and EF = BC $\because$ $AB^2 + BC^2 = AC^2$ (Given)
2.	In Δ ABC and Δ DEF, we have : AB = DE BC = DF $\therefore \Delta ABC \cong \Delta DEF$ $\Rightarrow \angle B = \angle E$ $\Rightarrow \angle E = 90^0$	By construction By construction Proved above By SSS congruence c.p.c.t $\because \angle E = 90^0$

Hence, $\angle B = 90^0$

Ex.16 If ABC is an equilateral triangle of side a, prove that its altitude = $\frac{\sqrt{3}}{2}$ a.

Sol. Δ ABC is an equilateral triangle.

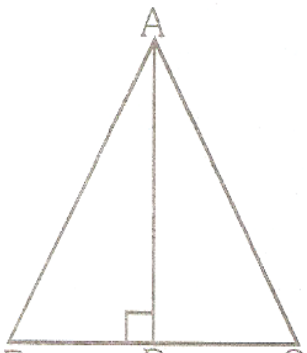
We are given that AB = BC = CA = a. AD is the altitude, i.e., AD $\perp$ BC.

Now, in right angled triangles triangles ABD and ACD, we have

AB = AC [Given]

and AD = AD [Common side]

$\Rightarrow$ Δ ABD = Δ ACD [By RHS congruence]



$\Rightarrow$ BD = CD $\Rightarrow$ BD = DC = $\frac{1}{2}$ BC = $\frac{a}{2}$

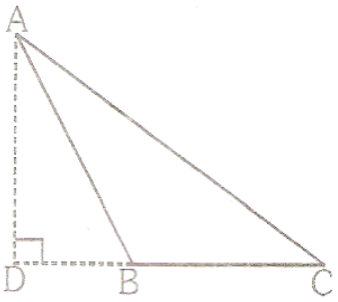
From right triangle ABD,

$AB^2 = AD^2 + BD^2 \quad \Rightarrow a^2 - AD^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow \quad AD^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$

$\Rightarrow \quad AD = \frac{\sqrt{3}}{2}a.$

Ex.17 In a ΔABC, obtuse angled at B, if AD is perpendicular to CB produced, prove that :
$AC^2 = AB^2 + BC^2 + 2BC \times BD$



Sol. In ΔADB, $\angle D = 90^0$.

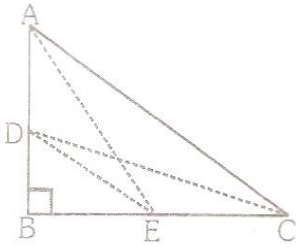
$\therefore \quad AD^2 + DB^2 = AB^2 \quad \ldots(i) \quad$ [By Pythagoras Theorem]

In ΔADC, $\angle D = 90^0$.

$\therefore \quad AC^2 = AD^2 + DC^2 \quad$ [By Pythagoras Theorem]

$= AD^2 + (DB + BC)^2$
$= AD^2 + DB^2 + BC^2 + 2DB \times BC$
$= AB^2 + BC^2 + 2BC \times BD \quad$ [Using (i)]
Hence, $AC^2 = AB^2 + BC^2 + 2BC \times BD$.

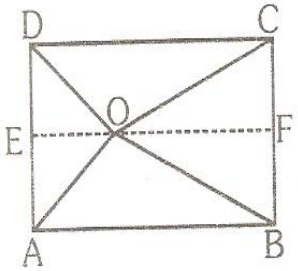
Ex.18 In the given figure, $\angle B = 90^0$. D and E are any points on AB and BC respectively. Prove that :
$AE^2 + CD^2 = AC^2 + DE^2$.



Sol. In ΔABE, $\angle B = 90^0$
$\therefore \quad AE^2 = AB^2 + BE^2 \ldots(i)$
In ΔDBC, $\angle B = 90^0$.
$\therefore \quad CD^2 = BD^2 + BC^2 \ldots(ii)$
Adding (i) and (ii), we get :
$AE^2 + CD^2 = (AB^2 + BC^2) + (BE^2 + BD^2)$
$= AC^2 + DE^2 \quad$ [By Pythagoras Theorem]
Hence, $AE^2 + CD^2 = AC^2 + DE^2$.

Ex.19 A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D. Prove that : $OA^2 + OC^2 = OB^2 + OD^2$

Sol. Through O, draw EOF $\parallel$ AB. Then, ABFE is a rectangle.



In right triangles OEA and OFC, we have :
$OA^2 = OE^2 + AE^2$
$OC^2 = OF^2 + OF^2$
$\therefore \quad OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2$
Again, in right triangles OFB and OED, we have :
$OB^2 = OF^2 + BF^2$
$OD^2 = OE^2 + DE^2$
$\therefore \quad OB^2 + OD^2 = OF^2 + OE^2 + BF^2 + DE^2$
$= OE^2 + OF^2 + AE^2 + CF^2 \quad \ldots(i) \quad [\because BF = AE \ \& \ DE = CF]$
From (i) and (ii), we get
$OA^2 + OC^2 = OB^2 + OD^2$.

Ex.20 In the given figure, ΔABC is right-angled at C.
Let BC = a, CA = b, AB = c and CD = p, where CD $\perp$ AB.

Prove that : (i) cp = ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Sol. (i) Area of $\Delta ABC = \frac{1}{2}\ BC \times CD\ \frac{1}{2}\ cp.$

Also, area of $\Delta ABC = \frac{1}{2}\ BC \times AC\ \frac{1}{2}\ ab.$

$\therefore\ \frac{1}{2}\ cp = \frac{1}{2}\ ab. \Rightarrow cp = ab$

(ii) $cp = ab \Rightarrow p = \frac{ab}{c}$

$\Rightarrow \quad p^2 = \frac{a^2b^2}{c^2}$

$\Rightarrow \quad \frac{1}{p^2} = \frac{c^2}{a^2b^2} = \frac{a^2+b^2}{a^2b^2} \qquad [\because\ c^2 = a^2 + b^2\ ]$

$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{a^2} = \frac{1}{b^2}$

Ex.21 Prove that in any triangle, the sum of the square of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side. **(Appollonius Theorem)**

Sol. Given : A Δ ABC in which AD is a median.

To prove : $AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$ or $AB^2 + AC^2 = 2(AD^2 + BD^2)$

Construction : Draw AE $\perp$ BC.

Proof : $\because$ AD id median

$\therefore$ BD = DC

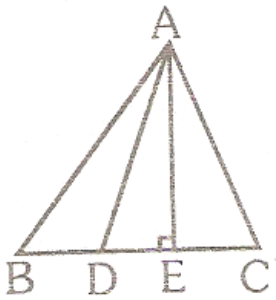
Now, $AB^2 + AC^2 = (AE^2 + BE^2) + (AE^2 + CE^2) = 2AE^2 + BE^2 + CE^2$

$= 2[AD^2 - DE^2] + BE^2 + CE^2$

$= 2AD^2 - 2DE^2 + (BD + DE)^2 + (DC - DE)^2$

$= 2AD^2 - 2DE^2 + (BD + DE)^2 + (DC - DE)^2$

$= 2(AD^2 + BD^2) = = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$



Hence, Proved.

★ **SYNOPSIS**

- **SIMILAR TRIANGLES.** Two triangles are said to be similar if

  (i) Their corresponding angles are equal and (ii) Their corresponding sides are proportional.

- All congruent triangles are similar but the similar triangles need not be congruent.
- Two polygons of the same numbers of sides are similar, if

  (i) their corresponding angels are equal and

  (ii) their corresponding sides are in the same ratio.
- **BASIC PROPORTIONALITY THEOREM.** In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the third side.
- **CONVERSE OF BASIC PROPORTIONALITY THEOREM.** If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.

- **AAA-SIMILARITY.** If in two triangles, corresponding angles are equal, i.e., the two corresponding angles are equal, then the triangles are similar.
- **SSS-SIMILARITY.** If the corresponding sides of two triangles are proportional, then they are similar.
- **SSS-SIMILARITY.** If in triangles one pair of corresponding sides proportional and the included angles are equal then the two triangles are similar.
- The ratio of the areas of similar triangles is equal to the ratio of the squares of their to the sum of the squares.
- **PYTHAGORAS THEOREM.** In a right triangle, if the square of one side is equal to the sum of the squares of the other two sides.
- **CONVERSE OF PYTHAGORAS THEOREM.** In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is a right angle.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. Triangle ABC is such that AB = 3 cm, BC = 2 cm and CA = 2.5 cm. Triangle DEF is similar to ΔABC. If EP 4 cm, then the perimeter of ΔDEF is :
(A) 7.5 cm (B) 15 cm (C) 22.5 cm (D) 30 cm

2. In ΔABC, AB = 3 cm, AC = 4 cm and AD is the bisector of $\angle A$. Then, BD : DC is :
(A) 9 : 16 (B) 16 : 9 (C) 3 : 4 (D) 4 : 3

3. In a equilateral triangle ABC, if $AD \perp BC$, then :
(A) $2AB^2 = 3AD^2$ (B) $4AB^2 = 3AD^2$ (C) $3AB^2 = 4AD^2$ (D) $3AB^2 = 2AD^2$

4. ABC is a triangles and DE is drawn parallel to BC cutting the other sides at D and E. If AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm, then AE is equal to :
(A) 1.4 cm (B) 1.8 cm (C) 1.2 cm (D) 1.05 cm

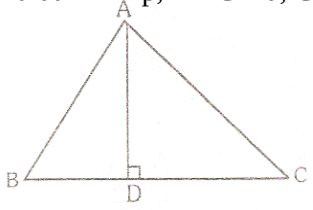
5. The line segments joining the mid points of the sides of a triangle from four triangles each of which is :
(A) similar to the original triangle (B) congruent to the original triangle.
(C) an equilateral triangle (D) an isosceles triangle.

6. In ΔABC and ΔDEF, $\angle A = 50^0, \angle B = 70^0, \angle C = 60^0, \angle D = 60^0, \angle E = 70^0, \angle F = 50^0$, then ΔABC is similar to :
(A) ΔDEF (B) ΔEDF (C) ΔDFE (D) ΔFED

7. D, E, F are the mid points of the sides BC, CA and AB respectively of ΔABC. Then ΔDEF is congruent to triangle
(A) ABC (B) AEF (C) BFD, CDE (D) AFE, BFD, CDE

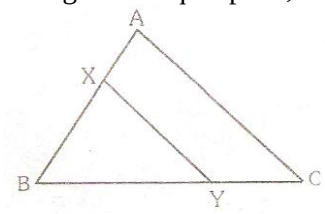
8. If in the triangles ABC and DEF, angle A is equal to angle E, both are equal to 40^0, AB : ED = AC : EF and angle F is 65^0, then angel B is :-
(A) 35^0 (B) 65^0 (C) 75^0 (D) 85^0

9. In a right angled ΔABC, right angled at A, if $AD \perp BC$ such that AD = p, if BC = a, CA = b and AB = c, then :
(A) $p^2 = b^2 + c^2$ (B) $\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{c^2}$



(C) $\frac{p}{a} = \frac{p}{b}$ (D) $p^2 = b^2\, c^2$

10. In the adjoining figure, XY is parallel to AC. If XY divides the triangle into equal parts, then the value of $\frac{AX}{AB}$ =



(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

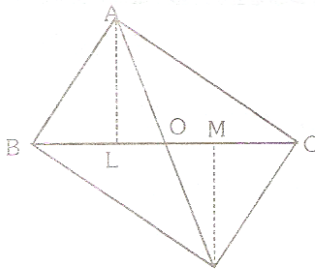
(C) $\frac{\sqrt{2}+1}{\sqrt{2}}$ (D) $\frac{\sqrt{2}-1}{\sqrt{2}}$

11. The ratio of the corresponding sides of two similar triangles is 1 : 3. The ratio of their corresponding heights is :
(A) 1 : 3 (B) 3 : 1 (C) 1 : 9 (D) 9 : 1

12. The areas of two similar triangles are 49 cm^2 and 64 cm^2 respectively. The ratio of their corresponding sides is :
(A) 49 : 64 (B) 7 : 8 (C) 64 : 49 (D) None of these

13. The areas of two similar triangles are 12 cm^2 and 48 cm^2. If the height of the similar one is 2.1 cm, then the corresponding height of the bigger one is :
(A) 4.41 cm (B) 8.4 cm (C) 4.2 cm (D) 0.525 cm

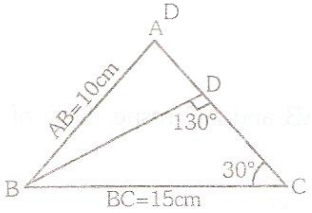
14. In the adjoining figure, ABC and DBC are two triangles on the same base BC, AL ⊥ BC and DM ⊥ BC. Then, $\frac{area(\Delta ABC)}{area(\Delta DBC)}$ is equal to ;



(A) $\frac{AO}{OD}$ (B) $\frac{AO^2}{OD^2}$

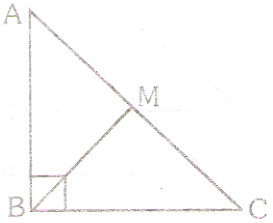
(C) $\frac{AO}{AD}$ (D) $\frac{OD^2}{AO^2}$

15. In the adjoining figure, AD : DC = 2 : 3, then ∠ABC is equal to :
(A) 30^0 (B) 40^0 (C) 45^0 (D) 110^0



16. In ΔABC, D and E are points on AB and AC respectively such that DE ∥ BC. If AE = 2 cm, EC = 3 cm and BC = 10 cm, then DE is equal to ;
(A) 5 cm (B) 4 cm (C) 15 cm (D) $\frac{20}{3}$ cm

17. In the given figure, ∠ABC = 90^0 and BM is a median, AB = 8 cm and BC = 6 cm. Then, length BM is equal to :



(A) 3 cm (B) 4 cm (C) 5 cm (D) 7 cm

18. If D, E, F are respectively the mid points of the sides BC, CA and AB of ΔABC and the area of ΔABC is 24 sq. cm, then the area of ΔDFE is :-
(A) 24 cm^2 (B) 12 cm^2 (C) 8 cm^2 (D) 6 cm^2

19. In a right angled triangle, if the square of the hypotenuse is twice the product of the other two sides, then one of the angles of the triangle is :-
(A) 15^0 (B) 30^0 (C) 45^0 (D) 60^0

20. Consider the following statements :
1. If three sides of a triangles are equal to three sides of another triangle, then the triangles are congruent.
2. If three angles of a triangles are respectively equal to three angles of another triangle, then the two triangles are congruent.

Of these statements,
(A) 1 is correct and 2 is false (B) both 1 and 2 are false
(C) both 1 and 2 are correct (D) 1 is false and 2 is correct

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	C	A	A	D	D	C	B	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	C	A	B	B	C	D	C	A

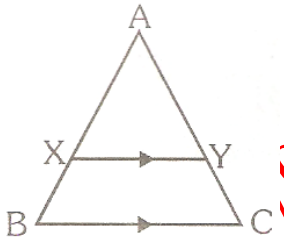
EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. In the given figure, XY ∥ BC.
Given that AX = 3 cm, XB = 1.5 cm and BC = 6 cm.
Calculate :

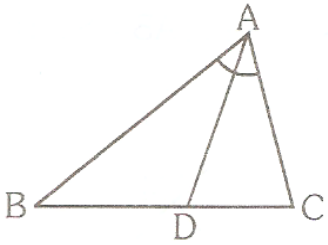
(i) $\frac{AY}{YC}$ (ii) XY



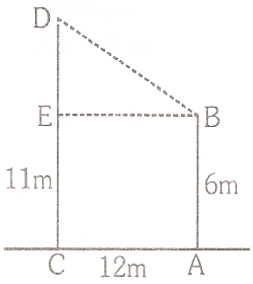
2. D and E are points on the sides AB and AC respectively of Δ ABC. For each of the following cases, state whether DE ∥ BC :

(i) AD = 5.7 cm, BD = 9.5 cm, AE = 3.6 cm, and EC = 6 cm
(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 9.6 cm, and EC = 2.4 cm.
(iii) AB = 11.7 cm, BD = 5.2 cm, AE = 4.4 cm, and AC = 9.9 cm.
(iv) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm, and AE = 2.8 cm.

3. In Δ ABC, AD is the bisector of ∠ A. If BC = 10 cm, BD = 6 cm and AC = 6 cm, find AB.



4. AB and CD are two vertical poles height 6 m and 11 m respectively. If the distance between their feet is 12 m, find the distance between their tops.



5. Δ ABC and Δ PQR are similar triangles such that are (Δ ABC) = 49 cm^2 and area (Δ PQR) = 25 cm^2. If AB = 5.6 cm, find the length of PQ.

6. Δ ABC and Δ PQR are similar triangles such that are (Δ ABC) = 28 cm^2 and area (Δ PQR) = 63 cm^2. If PR = 8.4 cm, find the length of AC.

7. Δ ABC ~ Δ DEF. If BC = 4 cm, EF = 5 cm and area (Δ ABC) = 32 cm^2, determine the area of Δ DEF.

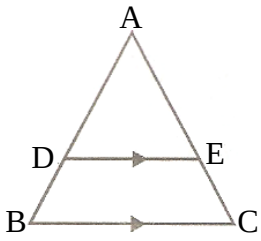
8. The areas of two similar triangles are 48 cm^2 and 75 cm^2 respectively. If the altitude of the first triangle be 3.6 cm, find the corresponding altitude of the other.

9. A rectangular field is 40 m long and 30 m broad. Find the length of its diagonal.

10. A man goes 15 m due west and then 8 m due north. How far is he from the starting point?

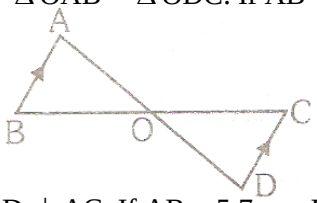
11. A ladder 17 m long reaches the window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

SHORT ANSWER TYPE QUESTIONS

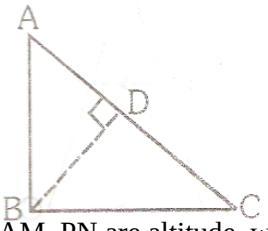


1. In the given fig, DE ‖ BC.
(i) If AD = 3.6 cm, AB = 9 cm and AE = 2.4 cm, find EC.
(ii) If $\frac{AD}{DB} = \frac{3}{5}$ and AC = 5.6 cm, find AE.
(iii) If AD = x cm, DB = (x – 2) cm, AE = (x + 2) cm and EC = (x – 1) cm, find the value of x.

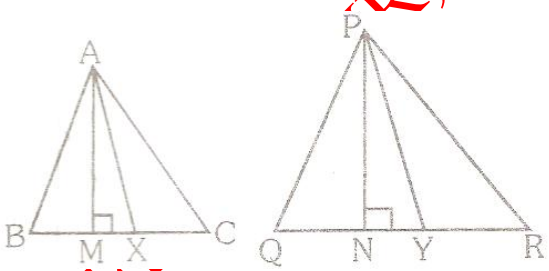
2. In the given figure, BA ‖ DC. Show that $\Delta OAB \sim \Delta ODC$. If AB = 4 cm, CD = 3 cm, OC = 5.7 cm and OD = 3.6 cm, find OA and OB.



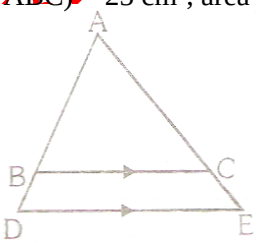
3. In the given figure, $\angle ABC = 90^0$ and BD ⊥ AC. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.



4. In the given figure, $\Delta ABC \sim \Delta PQR$ and AM, PN are altitude, whereas AX and PY are medians. Prove that $\frac{AM}{PN} = \frac{AX}{PY}$



5. In the given figure, BC ‖ DE, area (ΔABC) = 25 cm^2, area (trap. BCED) = 24 cm^2 and DE = 1 4 cm. Calculate the length of BC.

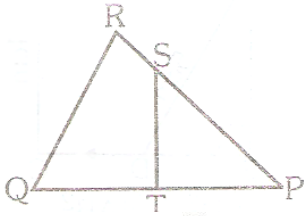


6. In ΔABC, $\angle C = 90^0$. If BC = a, AC = b and AB = c, find :
(i) c when a = 8 cm and b = 6 cm.
(ii) a when c = 25 cm and b = 7 cm.
(iii) b when c = 13 cm and a = 5 cm.

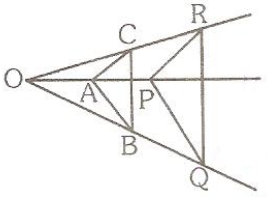
7. The sides of a right triangle containing the right angle are (5x) cm and (3x – 1) cm. If the area of triangle be 60 cm^2, calculate the length of the sides of the triangle.

8. Find the altitude of an equilateral triangle of side $5\sqrt{3}$ cm.

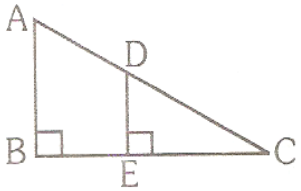
9. In the adjoining figure (not drawn to scale), PS – 4 cm, SR = 2 cm, PT = 3 cm and QT = 5 cm.
(i) Show that $\Delta PQR \sim \Delta PST$. (ii) Calculate ST, if QR = 5.8 cm.



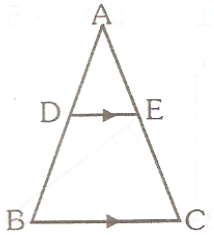
10. In the given figure, AB ‖ PQ and AC ‖ PR. Prove that BC ‖ QR.



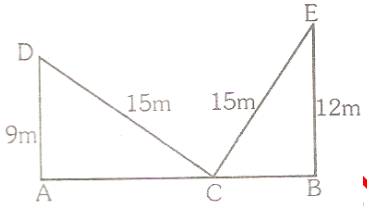
11. In the given figure, AB and DE are perpendicular to BC. If AB = 9 cm, DE = 3 cm and AC = 24 cm, calculate AD.



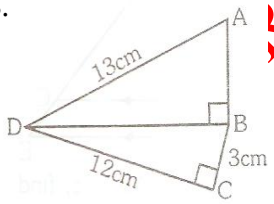
12. In the given figure, DE $\|$ BC. If DE = 4 cm, BC = 6 cm and area (Δ ADE) = 20 cm^2, find the area of Δ ABC.



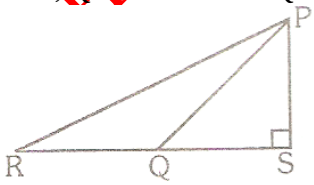
13. A ladder 15 m long reaches a window which is 9 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned is turned to the other side of the street to reach a window 12 m high. Find the width of the street.



14. In the given figure, ABCD is a quadrilateral in which BC = 3 cm, AD = 13 cm, DC = 12 cm and $\angle$ ABD = $\angle$ BCD = 90^0. Calculate the length of AB.

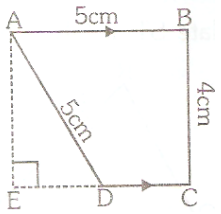


15. In the given figure, $\angle$ PSR = 90^0, PQ = 10 cm, QS = 6 cm and RQ = 9 cm, calculate the length of PR.



16. In a rhombus PQRS, side PQ = 17 cm and diagonal PR = 16 cm. Calculate the area of the rhombus.

17. From the given figure, find the area of trapezium ABCD.

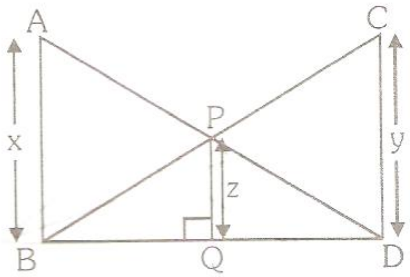


18. In a rhombus ABCD, prove that $AC^2 + BD^2 = 4AB^2$.

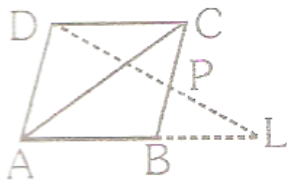
19. A ladder 13 m long rests against a vertical wall. If the foot of the ladder is 5 m from the foot of the wall, find the distance of the other end of the ladder from the ground.

LONG ANSWER TYPE QUESTIONS

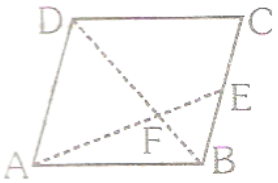
1. In the given figure, it is given that $\angle$ ABD = $\angle$ CDB = $\angle$ PQB = 90^0. If AB = x units, CD = y units and PQ = z units, prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$



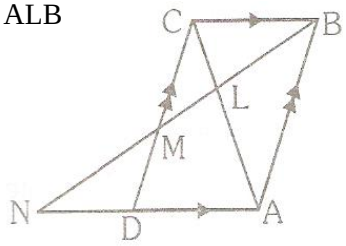
2. In the adjoining figures, ABCD is a parallelogram, P is a point on side BC and DP when produced meets AB produced at L. Prove that : (i) DP : PL = DC : BL (ii) DL : DP = AL : DC.



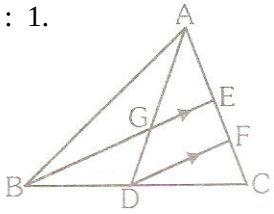
3. In the given figure, ABCD is a parallelogram, E is a point on BC and the diagonal BD intersects AE at F. Prove that DF × FE = FB × FA.



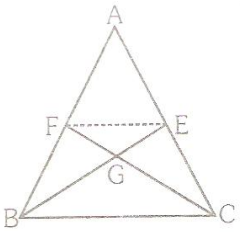
4. In the adjoining figure, ABCD is a parallelogram in which AB = 16 cm BC = 10 cm and L is a point on AC such that CL : LA = 2 : 3. If BL produced meets CD at M and AD produced at N, prove that :
(i) ΔCLB ~ ΔALN (ii) ΔCLM ~ ΔALB



5. In the given figure, medians AD and BE of ΔABC meet at G and DF ‖ BE. Prove that
(i) EF = FC (ii) AG : GD = 2 : 1.

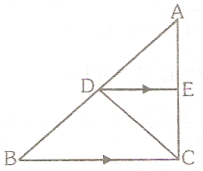


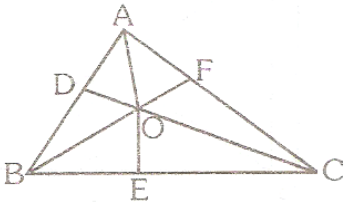
6. In the given figure, the medians BE and CF of ΔABC meet at G. Prove that :
(i) ΔGEF ~ ΔGBC and therefore, BG = 2 GE. (ii) AB × AF = AE × AC.



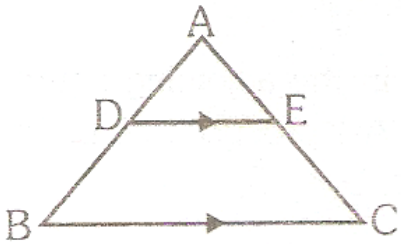
7. In the given figure, DE ‖ BC and BD – DC.
(i) Prove that DE bisects ∠ADC.
(ii) If AD = 4.5 cm, AE = 3.9 cm and DC = 7.5 cm, find CE.
(iii) Find the ratio AD : DB.

8. O is point inside a ΔABC. The bisectors of ∠AOB. ∠BOC and ∠COA meet the sides AB, BC and in points D, E and F respectively. Prove that AD-BE-CF = DB.EC.FA





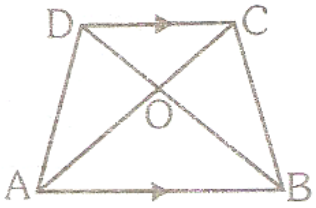
9. In the figure, DE ‖ BC.



(i) Prove that ΔADE and ΔABC are similar.

(ii) Given that AD = $\frac{1}{2}$ BD. Calculate DE, if BC = 4.5 cm.

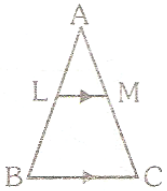
10. In the adjoining figure, ABCD is a trapezium in which AB ‖ DC and AB = 2 DC. Determine the ratio of areas of ΔAOB and ΔCOD



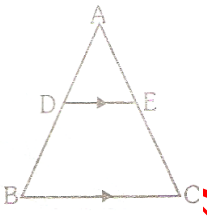
11. In the adjoining figure, LM is parallel to BC. AB = 6 cm, AL = 2 cm and AC = 9 cm. Calculate :

(i) the length of CM.

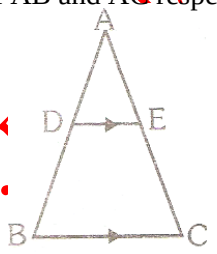
(ii) the value of $\frac{Area(\Delta ALM)}{Area(trap.LBCM)}$



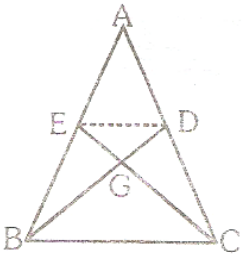
12. In the given figure, DE ‖ BC. and DE : BC = 3 : 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.



13. In ΔABC, D and E are mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC.

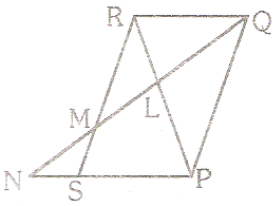


14. In a ΔPQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$. Prove that (i) $\Delta PQL - \Delta RPM$ (ii) QL.RM = PL.PM (iii) PQ^2 = QL.QR



15. In the adjoining figures, the medians BD and CE of a ΔABC meet at G.
Prove that:
(i) $\Delta EGD \sim \Delta CGB$
(ii) BG = 2 GD from (i) above.

16. In the adjoining figure, PQRS is a rarallelog'am with PQ = 15 cm and RQ = 10 cm. L is a point on RP such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N. Find the length of PN and RM.

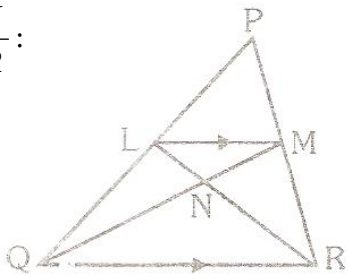


17. In ΔPQR, LM ‖ QR and PM : MR = 3 : 4. Calculate:

(i) $\frac{PL}{PQ}$ and then $\frac{LM}{QR}$:

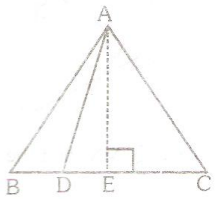
(ii) $\frac{Area(\Delta ALM)}{Area(\Delta MNR)}$

(iii) $\frac{Area(\Delta LQM)}{Area(\Delta LQN)}$



18. In ΔABC, $\angle B = 90^0$ and D is the mid point of BC.
Prove that :
(i) $AC^2 = AD^2 + 3CD^2$
(ii) $BC^2 = 4(AD^2 - AB^2)$

19. In ΔABC, if AB = AC and D is a point on BC. Prove that $BC^2 - AD^2 = BD \times CD$.



SIMILAR TRIANGLE | ANSWER KEY | EXERCISE (X)-CBSE

VELY SHORT ANSWER TYPE QUESTIONS

1. (i) $\frac{1}{2}$ (ii) 4 cm **2.** (i) Yes, (ii) No, (iii) No, (iv) Yes **3.** 9 cm **4.** 13 m **5.** PQ = 4 cm **6.** AC = 5.6 cm **7.** 50 cm^2 **8.** 4.5 cm **9.** 50 m **10.** 17 m **11.** 8m

SHORT ANSWER TYPE QUESTIONS

1. (i) 3, 6 cm, (ii) 2.1 cm, (iii) x = 4 **2.** OA = 4.8 cm, OB = 7.6 cm **3.** 8.1 cm **5.** 10 cm **6.** (i) 10 cm, (ii) 24 cm, (iii) 12 cm **7.** 15 cm,8 cm, 17cm **8.** 7.5 cm **9.** 2.9 cm **11.** 16 cm **12.** 45 cm^2 **13.** 21m **14.** 4 cm **15.** 17 cm **16.** 240 cm^2 **17.** 14 cm^2 **19.** 12 m

LONG ANSWER TYPE QUESTIONS

7. (ii) 6.5 cm, (ii) 3 : 8 **9.** DE = 1.5 cm **10.** 4 : 1 **11.** (i) 6 cm, (ii) $\frac{1}{8}$ **12.** 9 : 16 **13.** 1 : 4

16. PN = 15 cm, RM = 10 cm **17.** (i) $\frac{PL}{PQ} = \frac{LM}{QR} = \frac{3}{7}$ (ii) 3 : 7 (iii) 10 : 7

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PERVIOUS YEARS BOARD QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. ΔABC and ΔDEF are similar, BC = 3 cm, EF = 4 cm and area of ΔABC = 54 cm^2. Determine the area of ΔDEF. **Delhi-1996**

2. In ΔABC, CE $\perp$ AB, BD $\perp$ AC and & BD intersect at P, considering triangles BEP and CPD. Prove that BP $\times$ PD = EP $\times$ PC. **Delhi-1996C**

3. A right triangle has hypotenuse of length q cm and one side of length p cm. if (q – p) = 2, express the length of third side of the right triangle in terms of q. **Al-1996C**

4. In the given figure, ABC is a triangle in which AB = AC. D and E are points on the sides AB and AC respectively, such that AD = AE. Show that the points B, C, E and D are concyclic. **Al-1996C**

5. In a ΔABC, AB = AC and D is a point on side AC, such that BC2 = AC $\times$ CD. Prove that BD = BC. **Al-1997**

6. ΔABC is right angled at B. On side AC, a point D is taken such that AD = DC and AB = BD. Find the measure of $\angle CAB$ **Delhi-1998**

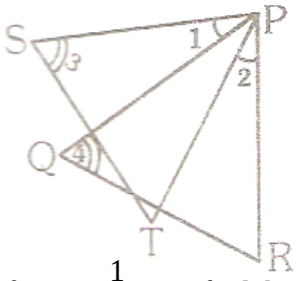
7. In a ΔABC, P and Q are points on the sides AB and AC respectively such that PQ is parallel to BC. Prove that median AD, drawn from A to BC, bisects PQ. **Al-1998**

8. Two poles of height 7 m and 12 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tips. **Al-1998C**

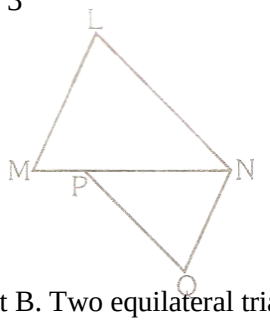
9. In a ΔABC, D and E are points on AB & AC respectively such that DE is parallel to BC and AD : DB = 2 : 3. Determine Area (ΔADE) : Area (ΔABC). **Foreign-1999**

10. In the given figure, $\angle A = \angle B$ and D & E are points on AC and BC respectively such that AD = BE, show that DE $\parallel$ AB. **Delhi-1999**

11. In figure, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Show that PT. QR = PR. ST. **Foreign-2000**



12. In figure, LM ‖ NQ and LN ‖ PQ. If MP = $\frac{1}{3}$ MN, find the ratio of the areas of ΔLMN and ΔQNP. **Foreign-2000**



13. ABC is an isosceles triangle right angled at B. Two equilateral triangles BDC and AEC are constructed with side BC and AC. Prove that area of ΔBCD = $\frac{1}{2}$ area of ΔACE. **Delhi-2001**

14. The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. If the altitude of the first triangle 6.3 cm, find the corresponding altitude of the other. **Al-2001**

15. L and M are the mid-points of AB and BC respectively of ΔABC, right-angled at B. prove that $4LC^2 = AB^2 + 4BC^2$. **Al-2001;Foreign-2001**

16. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm. find the corresponding median of the other. **Al-2001**

17. In an equilateral triangle ABC, AD is the altitude drawn from A on side BC. Prove that $3AB^2 = 4AD^2$. **Delhi-2002**

18. (i) Prove that the equilateral triangle described on the two sides of a right angled triangle are together equal to the equilateral triangle on the hypotenuse in terms of their areas. **Al-2002**

(ii) P is a point in the interior of ΔABC, X, Y and Z are point on lines PA.PB and PC respectively such that XY ‖ AB and XZ ‖ BC. Prove that YZ ‖ BC. **Al-2002 : Delhi-2003 [NCERT]**

(iii) D and E are points on the sides AB and AC respectively of ΔABC such that DE is parallel to BC and AD : DB = 4 : 5. CD and BE intersect each other at F. Find the ratio of the areas of ΔDEF and ΔBCE **Al-2000 : Al-2003**

(iv) P. Q are respectively points on sides AB and AC of triangle ABC. If AP = 2 cm. PB = 4 cm. AQ = 3 cm and QC = 6 cm. prove that BC = 3PQ. **Foreign-2003**

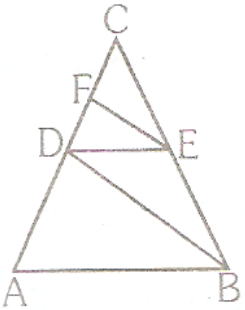
19. D is a point on the side BC of ΔABC such that $\angle$ADC = $\angle$BAC. Prove that $\frac{CA}{CD} = \frac{CB}{CA}$. **Delhi-2002:[NCERT]**

20. ABCD is a trapezium in which AB ‖ DC. The diagonals AC and BD intersect at O. Prove that $\frac{AO}{OC} = \frac{BO}{DO}$ **Al-2004:[NCERT]**

21. In a ΔABC, AD $\perp$ BC and $\frac{BD}{AD} = \frac{AD}{DC}$. Prove that ABC is a right triangle, right angled at A. **Foreign-2004**

22. In a right angled triangle ABC, $\angle A = 90^0$ and AD $\perp$ BC. Prove that $AD^2 = BD \times CD$. **Delhi-2004C, 2006**

23. In fig., AB ‖ DE and BD ‖ EF. Prove that DC2 = CF × AC. **Al-2004C : Delhi-2007**

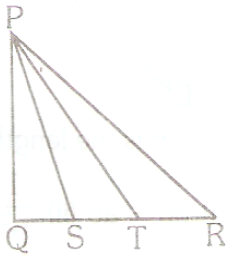


24. If one diagonal of a trapezium divides the other diagonal in the ratio of 1 : 2. prove that one of the parallel sides is double the other. **Foreign-2005**

25. In ΔABC, $AD \perp BC$, prove that $AB^2 + CD^2 = AC^2 + DB^2$. **Delhi-2005C, Al-2006 [NCERT]**

26. Prove that the sum of the squares of the sides of a rhombus is equal to sum of the squares of its diagonals. **Al-2005C [NCERT]**

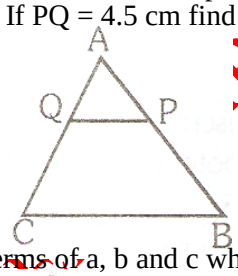
27. In figure, S and T trisect the side QR of a right triangle PQR. Prove that $8PT^2 = 3PR^2 + 5PS^2$.



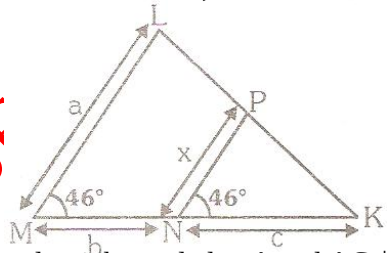
OR

If BL and CM are medians of a triangle ABC right-angled at A, then prove that $4(BL^2 + CM^2) = 5BC^2$. **Al-2006 C; Foreign-2009**

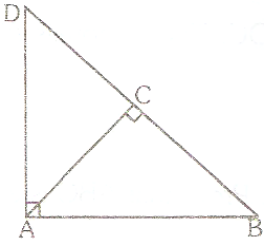
28. In the fig, P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm. If PQ = 4.5 cm find BC. **Delhi-2008**



29. In fig. $\angle M = \angle N = 46^0$ Express x in terms of a, b and c where a, b and c are lengths of LM, MN and NK respectively. **Delhi-2009**



30. In figure, ΔABC is a right triangle, right-angled at A and $AC \perp BD$. Prove that $AB^2 = BC.\ BD$. **Al-2009**



31. In a ΔABC, $DE \parallel BC$. If $DE = \frac{2}{3} BC$ and area of $\Delta ABC = 81\ cm^2$, find the area of ΔADE. **Foregin-2009**

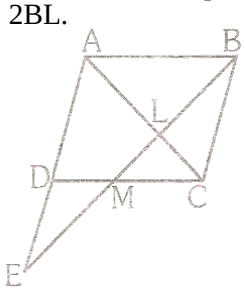
SHORT ANSWER TYPE QUESTIONS

1. P and Q are points on the sides CA and CB respectively of a ΔABC right-angled at C. prove that $AQ^2 + BP^2 = AB^2 + PQ^2$. **Delhi-1996, 2007**

2. ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE. **Al -1997**

3. In Δ ABC, if AD is the median, show that $AB^2 + AC^2 = 2[AD^2 + BD^2]$. **Delhi-1997, 98**

4. In the given figure, M is the mid-point of the side CD of parallelogram ABCD. BM. When joined meets AC is L and AD produced in E. Prove that EL = 2BL. **Al-1998; Delhi-1999, Al-2009**



5. ABC is a right triangle, right-angled at C. if p is the length of the perpendicular from C to AB and a, b, c have the usual meaning, then prove that (i) pc = ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} = \frac{1}{b^2}$ **Delhi-1998, 98 C**

6. In an equilateral triangle PQR, the side QR is trisected at S. Prove that $9PS^2 = 7PQ^2$. **Al-1998, 98C [NCERT]**

7. If the diagonals of a quadrilateral divide each other proportionally, prove that it is trapezium. **Foreign-1999**

8. In an isosceles triangle ABC with AB = AC, BD is a perpendicular from B to the side AC. Prove that $BD^2 - CD^2 = 2CD.AD$. **Foreign-1999**

9. ABC and DBC are two triangles on the same base BC. If AD intersect BC at O. Prove that $\frac{ar.\Delta ABC}{ar.\Delta DBC} = \frac{AO}{DO}$ **Al-1999C; Delhi-2005**

10. In Δ ABC, $\angle$ A is acute. BD and CE are perpendiculars on AC and AB respectively. Prove that $AB \times AE = AC \times AD$. **Al-2003**

11. Points P and Q are on sides AB and AC of a triangle ABC in such a way that PQ is parallel to side BC. Prove that the median AD drawn from vertex A to side BC bisects the segment PQ. **Foreign -2003**

12. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

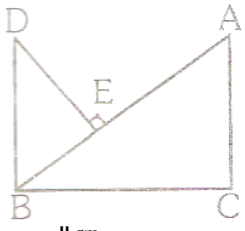
OR

Two Δs ABC and DBC are on the same base BC and on the same side of BC in which $\angle A = \angle D = 90^0$. If CA and BD meet each other at E, show that AE.ED. **Delhi-2008**

13. D and E are points on the sides CA and CB respectively of Δ ABC right-angled at C. prove that $AE^2 + BD^2 = AB^2 + DE^2$.

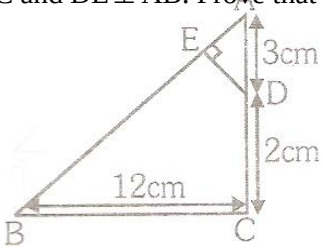
OR

In fig. DB $\perp$ BC, DE $\perp$ AB and AC $\perp$ BC. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$. **Al-2008**

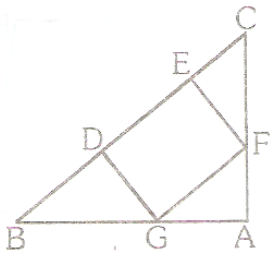


14. E is a point on the side AD produced of a $\|^{gm}$ ABCD and BE intersects CD at F. Show that Δ ABC ~ Δ CFB. **Foreign-2008**

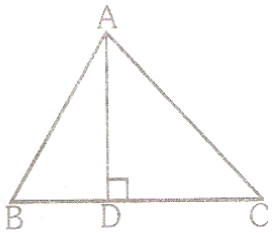
15. In fig, Δ ABC is right angled at C and DE $\perp$ AB. Prove that Δ ABC ~ Δ ADE and hence find the lengths of AE and DE.



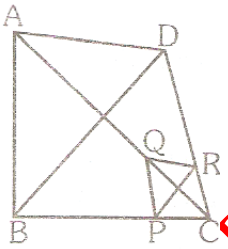
In fig, DEFG is a square and $\angle BAC = 90^0$. Show that $DE^2 = BD \times EC$ **Delhi-2009**



16. In fig, $AD \perp BC$ and $BD = \frac{1}{3} CD$. Prove that $2CA^2 = 2AB^2 + BC^2$. **Al-2009**



17. In fig, two triangles ABC and DBC lie on the same side of base BC. P is a point on BC such that $PQ \parallel BA$ and $PR \parallel BD$. Prove that $QR \parallel AD$. **Foreign-2009**



LONG ANSWER TYPE QUESTIONS

1. In a right triangle ABC, right-angled at C, P and Q are points on the sides CA and CB respectively which divide these sides in the ratio 1 : 2. Prove that **Al-1996C**

(i) $9AQ^2 = 9AC^2 + 4BC^2$ (ii) $0BP^2 = 9BC^2 + 4AC^2$ (iii) $9(AQ^2 + BP^2) = 13AB^2$.

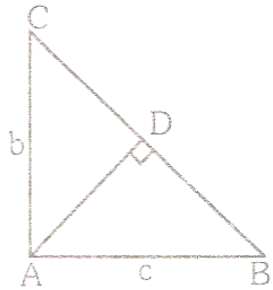
2. The ratio of the areas of similar triangles is equal to the ratio of the square on the corresponding sides, prove. Using the above theorem, prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal. **Delhi-1997C; 2005C; Foreign-2003**

3. Perpendiculars OD, OE and OF are drawn to sides BC, CA and AB respectively from a point O in the interior of a ΔABC. Prove that :

(i) $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$.

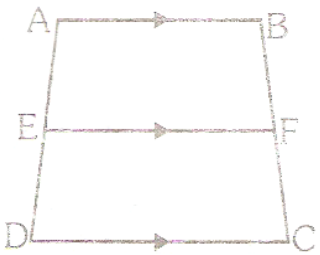
(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$. **Delhi-1997C, [NCERT]**

4. In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides. Prove. Using the above theorem, determine the length of AD in terms of b and C. **Al-1997 C**

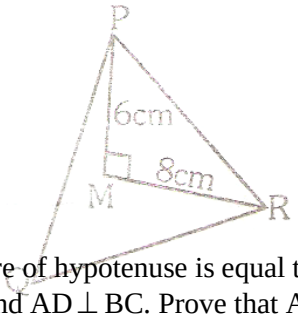


5. If a line is drawn parallel to one side of a triangle, other two sides are divided in the same ratio, Prove. Using this result to prove the following : In the given figure, if ABCD is a trapezium in which $AB \parallel DC \parallel EF$, then

$\frac{AE}{ED} = \frac{BF}{FC}$. **Foreign-1998**



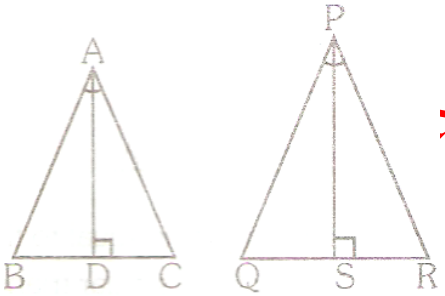
6. State and prove Pythagoras. Use the theorem and calculate are (Δ PMR) from the given figure.
Delhi-1998C, 2006



7. In a right-angled triangle, the square of hypotenuse is equal to the sum of the squares of the two sides. Given that $\angle$B of ΔABC is an acute angle and AD $\perp$ BC. Prove that $AC^2 = AB^2 + BC^2 - 2BC.$ BD. **Delhi-1999**

8. In a right triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using above, solve the following : In quadrilateral ABCD, find the length of CA, if CD $\perp$ DB, CD = 6 m, DB = 12 m and AB = 11 m. **Delhi-2000**

9. Prove that the ratio of the areas of two similar triangles is equal to the squares of their corresponding sides. Using the above, do the following



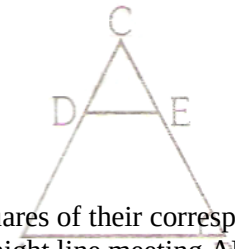
In fig, ΔABC and ΔPQR are isosceles triangles in which $\angle$A = $\angle$P. If $\frac{area(\Delta ABC)}{area(\Delta PQR)} = \frac{9}{16}$, find $\frac{AD}{PS}$. **Al-2000**

10. In a right-angled triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using the above result, find the length of the second diagonal of a rhombus whose side is 5 cm and one of the diagonals is 6 cm. **Al-2001**

11. In a triangle, if the square on one side is equal to the sum of the squares on the other two sides prove that the angle opposite the first side is a right angel.
Using the above theorem and prove that following : In triangle ABC, AD $\perp$ BC and BD = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$. **Al-2003**

12. In a right triangle, prove that the square on hypotenuse is equal to sum of the squares on the other two sides.Using the above result, prove that following : PQR is a right triangle right angled at Q. If S bisects QR, show that $PR^2 = 4PS^2 - 3PQ^2$. **Delhi-2004C**

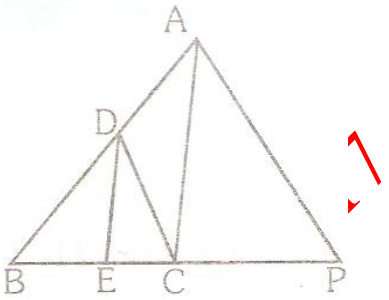
13. If a line is drawn parallel to one side of a trial prove that the other two sides are divided in the same ratio. Using the above result, prove from fig. that AD = BE if $\angle$A = $\angle$B and DE $\parallel$ AB. **Al-2004C**



14. Prove that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides. Apply the above theorem on the following : ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 cm, prove that area of ΔAPQ is one-sixteenth of the area of ΔABC. **Delhi-2005**

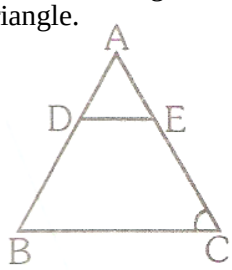
15. If a line is drawn parallel to one side of a triangle, prove that the other two sides are divided in the same ratio. Use the above to prove the following : In the given figure DE ∥ AC and DC ∥ AP.

Prove that $\frac{BE}{EC} = \frac{BC}{CP}$. **Al-2005**



16. In a triangle if the square on one side is equal to the sum of squares on the other two sides, prove that the angle opposite to the first side is a right angle. Using the above theorem to prove the following :
In a quadrilateral ABCD, $\angle B = 90^0$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90^0$. **Al-2205**

17. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following : In figure, DE ∥ AC and BD = CE. Prove that ABC is an isosceles triangle. **Delhi-2007, 2009**



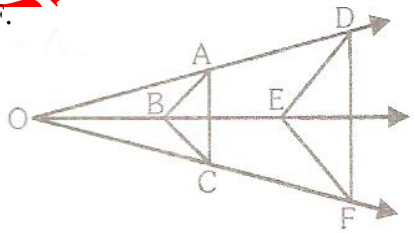
18. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above for the following : If the areas of two similar triangles are equal, prove that they are congruent. **Al-2007**

19. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above result, prove the following :

In a ΔABC, XY is parallel to BC and it divides ΔABC into two parts of equal area. Prove that $\frac{BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$ **Delhi-2008**

20. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above, do the following :
The diagonals of a trapezium ABCD, with AB ∥ DC, intersect each other at the point O. If AB = 2 CD, find the ratio of the area of ΔAOB to the area of ΔCOD. **Al -2008**

21. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following : In the fig, AB ∥ DE and BC ∥ EF. Prove that AC ∥ DF. **Foreign-2008**



2. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above, do the following : In a trapezium ABCD, AC and BD are intersecting at O, AB ∥ DC and AB = 2 CD. If area of $\Delta AOB = 84\ cm^2$, find the area of ΔCOD. **Delhi-2009**

VERY SHORT ANSWER TYPE QUESTIONS

2. 96 cm^2 **3.** $2\sqrt{(q-1)}$ **6.** 60^0 **8.** 13 m **9.** 4.25 **12.** 9 : 4 **14.** 4.9 cm **16.** 8.8 cm **18.** (iii) 16 : 81

28. 13.5 cm **19.** $\left(\frac{ac}{b+c}\right)$ **31.** 36 cm^2

SHORT ANSWER TYPE QUESTIONS

2. $2\sqrt{5}$ cm **15.** AE = $\frac{15}{13}$, DE = $\frac{36}{13}$

LONG ANSWER TYPE QUESTIONS

4. $\frac{bc}{\sqrt{b^2+c^2}}$ **6.** 24 cm^2 **8.** 13 cm **9.** 3 : 4 **10.** 8 cm **21.** 4 : 1 **23.** 21 cm^2

EXERCISE – 1 (FOR OLYMPIADS)

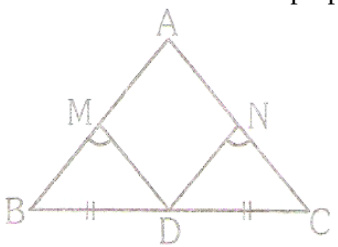
CHOOSE THE CORRECT ONE

1. In a triangle ABC, if AB, BC and AC are the three sides of the triangle, then which of the statements is necessarily true?
(A) AB + BC < AC (B) AB + BC > AC (C) AB + BC = AC (D) $AB^2 + BC^2 = AC^2$

2. The sides of a triangle are 12 cm, 8 cm and 6 cm respectively, the triangle is :
(A) acute (B) obtuse (C) right (D) can't be determined

3. In an equilateral triangle, the incentre, circumcentre, orthocenter and centroid are:
(A) concylic (B) coincident (C) collinear (D) none of these

4. In the adjoining figure D is the midpoint of a Δ ABC. DM and DN are the perpendiculars on AB and AC respectively and DM = DN, then the Δ ABC is :

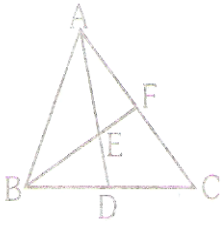


(A) right angled
(B) isosceles
(C) equilateral
(D) scalene

5. Triangle ABC is such that AB = 9 cm, BC = 6 cm, AC = 7.5 cm, Triangle DEF is similar to Δ ABC, If EF = 12 cm then DE is :
(A) 6 cm (B) 16 cm (C) 18 cm (D) 15 cm

6. In Δ ABC, AB = 5 cm, AC = 7 cm. If AD is the angle bisector of < A. Then BD : CD is :
(A) 25 : 49 (B) 49 : 25 (C) 6 : 1 (D) 5 : 7

7. In a Δ ABC, D is the mid-point of BC and E is mid-point of AD, BF passes through E. What is the ratio of AF : FC



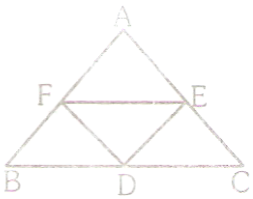
(A) 1 : 1
(B) 1 : 2
(C) 1 : 3
(D) 2 : 3

8. In a Δ ABC, AB = AC and AD $\perp$ BC, then :
(A) AB < AD (B) AB > AD (C) AB = AD (D) AB $\leq$ AD

9. The difference between altitude and base of a right angled triangle is 17 cm and its hypotenuse is 25 cm. What is the sum of the base and altitude of the triangle is ?
(A) 24 cm (B) 31 cm (C) 36 cm (D) can't be determined

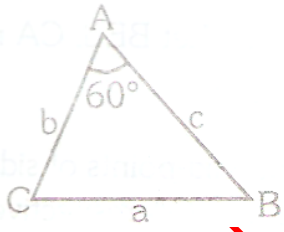
10. If AB, BC and AC be the three sides of a triangle ABC, which one of the following is true ?
(A) AB – BC = AC (B) (AB – BC) > AC (C) (AB – BA) < AC (D) $AB^2 - CB^2 = AC^2$

11. In the adjoining figure D, E and F are the mid-points of the sides BC, AC and AB respectively. Δ DEF is congruent to triangle :



(A) ABC
(B) AEF
(C) CDE , BFD
(D) AFE , BFD and CDE

12. In the adjoining figure $\angle BAC = 60^0$ and BC = a, AC = b and AB = c, then :



(A) $a^2 = b^2 + c^2$
(B) $a^2 = b^2 + c^2 - bc$
(C) $a^2 = b^2 + c^2 + bc$
(D) $a^2 = b^2 + 2bc$

13. If the medians of a triangle are equal, then the triangle is:
(A) right angled (B) isosceles (C) equilateral (D) scalene

14. The incentre of a triangle is determined by the:
(A) Medians (B) angle bisectors
(C) perpendicular bisectors (D) altitudes

15. The point of intersection of the angle bisectors of a triangle is :
(A) orthocenter (B) centroid (C) incentre (D) circumcentre

16. A triangle PQR is formed by joining the mid-points of the sides of a triangle ABC, 'O' is the circumcentre of Δ ABC, then for Δ PQR, the point 'O' is :
(A) incentre (B) circumcentre (C) orthocenter (D) centroid

17. If AD, BE, CF are the altitudes of Δ ABC whose orthocenter is H, then C is the orthocenter of :
(A) Δ ABH (B) Δ BDH (C) Δ ABD (D) Δ BEA

18. In an equilateral Δ ABC, if a, b and c denote the lengths of perpendiculars from A, B and C respectively on the opposite sides, then:
(A) a > b > c (B) a > b < c (C) a = b = c (D) a = c ≠ b

19. Any two of the four triangles formed by joining the midpoints of the sides of a given triangle are:
(A) congruent (B) equal in area but not congruentAB > AD
(C) unequal in area and not congruent (D) none of these

20. The internal bisectors of $\angle B$ and $\angle C$ of Δ ABC meet at O. If $\angle A = 80^0$ then $\angle BOC$ is :
(A) 50^0 (B) 160^0 (C) 100^0 (D) 130^0

21. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is :
(A) centroid (B) incentre (C) circumcentre (D) orthocenter

22. Incentre of a triangle lies in the interior of :
(A) an isosceles triangle only (B) a right angled triangle only
(C) any equilateral triangle only (D) any triangle

23. In a triangle PQR, PQ = 20 cm and PR = 6 cm, the side QR is :
(A) equal to 14 cm (B) less than 14 cm (C) greater than 14 cm (D) none of these

24. If ABC is a right angled triangle at B and M, N are the mid-points of AB and BC, than 4 ($AN^2 + CM^2$) is equal to-
(A) $4AC^2$ (B) $6\ AC^2$ (C) $5\ AC^2$ (D) $\frac{5}{4}\ AC^2$

25. ABC is a right angle triangle at A and AD is perpendicular to the hypotence. Then $\frac{BD}{CD}$ is equal to :
(A) $\left(\frac{AB}{AC}\right)^2$ (B) $\left(\frac{AB}{AD}\right)^2$ (C) $\frac{AB}{AC}$ (D) $\frac{AB}{AD}$

26. Let ABC be an equilateral triangle. Let BE ⊥ CA meeting CA at E, then ($AB^2 + BC^2 + CA^2$) is equal to :
(A) $2BE^2$ (B) $3\ BE^2$ (C) $4\ BE^2$ (D) $6\ BE^2$

27. If D, E and F are respectively the mid-points of sides of BC, CA and AB of a Δ ABC. If EF = 3 cm, FD = 4 cm, and AB = 10 cm, then DE, BC and CA respectively will be equal to :

(A) 6, 8 and 20 cm (B) 4, 6 and 8 cm (C) 5, 6 and 8 cm (D) $\frac{10}{3}$, 9 and 12 cm

28. In the right angle triangle $\angle C = 90^0$. AE and BD are two medians of a triangle ABC meeting at F. The ratio of the area of ΔABF and the quadrilateral FDCE is :

(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 2 : 3

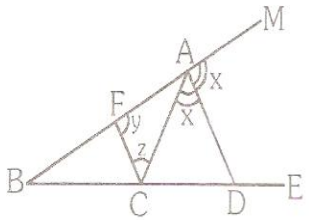
29. The bisector of the exterior $\angle A$ of ΔABC intersects the side BC produced to D. Here CF is parallel to AD.

(A) $\frac{AB}{AC} = \frac{BD}{CD}$

(B) $\frac{AB}{AC} = \frac{CD}{BD}$

(C) $\frac{AB}{AC} = \frac{BC}{CD}$

(D) None of these



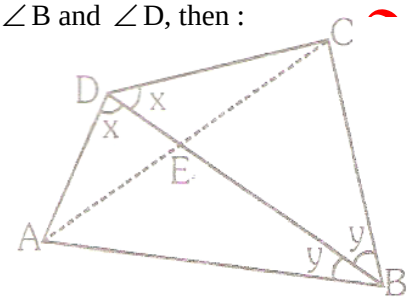
30. The diagonal BD of a quadrilateral ABCD bisects $\angle B$ and $\angle D$, then :

(A) $\frac{AB}{CD} = \frac{AD}{BC}$

(B) $\frac{AB}{BC} = \frac{AD}{CD}$

(C) $AB = AD \times BC$

(D) None of these



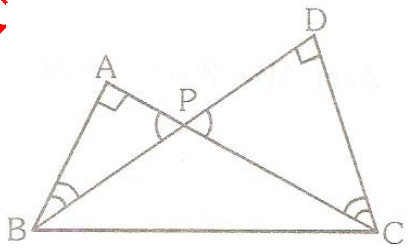
31. Two right triangles ABC and DBC are drawn on the same hypotenuse BC on the same side of BC. If AC and DB intersects at P, then

(A) $\frac{AP}{PC} = \frac{BP}{DP}$

(B) $AP \times DP = PC \times BP$

(C) $AP \times PC \times = BP \times DP$

(D) $AP \times BP \times = PC \times PD$



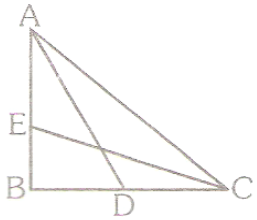
32. In figure, ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE:

(A) $2\sqrt{5}$ cm

(B) 2.5 cm

(C) 5 cm

(D) $4\sqrt{2}$ cm



33. In a ΔABC, AB = 10 cm, BC = 12 cm and AC = 14 cm. Find the length of median AD. If G is the centroid, find length of GA :

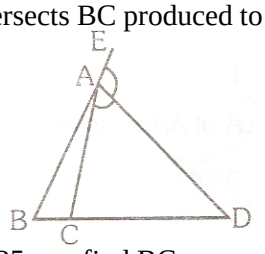
(A) $\frac{5}{3}\sqrt{7}, \frac{5}{9}\sqrt{7}$ (B) $5\sqrt{7}, 4\sqrt{7}$ (C) $\frac{10}{\sqrt{3}}, \frac{8}{3}\sqrt{7}$ (D) $4\sqrt{7}, \frac{8}{3}\sqrt{7}$

34. The three sides of a triangles are given. Which one of the following is not a right triangle ?

(A) 20, 21, 29 (B) 16, 63, 65

(C) 56, 90, 106 (D) 36, 35, 74

35. In the figure AD is the external bisector of $\angle EAC$, intersects BC produced to D. If AB = 12 cm, AC = 8 cm and BC = 4 cm, find CD.

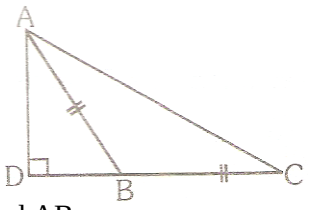


(A) 10 cm
(B) 6 cm
(C) 8 cm
(D)9 cm

36. In ΔABC, AB2 + AC2 = 2500 cm2 and median AD = 25 cm, find BC.

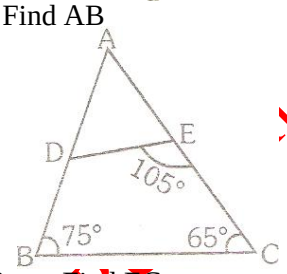
(A) 25 cm (B) 40 cm (C) 50 cm (D) 48 cm

37. In the given figure, AB = BC and $\angle BAC = 150$. AB = 10 cm. Find the area of ΔABC.



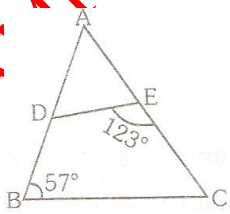
(A) 50 cm^2
(B) 40 cm^2
(C) 25 cm^2
(D) 32 cm^2

38. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and if AE = 10 cm. Find AB



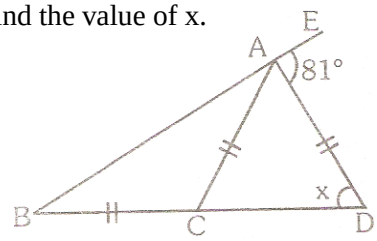
(A) 16 cm
(B) 12 cm
(C) 15 cm
(D) 18 cm

39. In the figure AD = 12 cm. AB = 20 cm and AE = 10 cm. Find EC.



(A) 14 cm
(B) 10 cm
(C) 8 cm
(D) 15 cm

40. In the given fig, BC = AC = AD, $\angle EAD = 81^0$. Find the value of x.



(A) 45^0
(B) 54^0
(C) 63^0
(D) 36^0

41. What is the ratio of inradius to the circumradius of a right angled triangle?

(A) 1 : 2 (B) $1 : \sqrt{2}$ (C) 2 : 5 (D) Can't be determined

ANSWER KEY															
Ans.	B	B	B	B	C	D	B	B	B	C	D	B	C	B	C
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	A	C	A	D	B	D	C	C	A	C	C	A	A	B
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**				
Ans.	C	A	D	D	C	C	C	C	A	B	D				

CO-ORDINATE GEOMETRY

★ INTRODUCTION

In the previous class, you have learnt to locate the position of a point in a coordinate plane of two dimensions, in terms of two coordinates. You have learnt that a linear equation in two variables, of the form ax + c = 0 (either a $\neq 0$ or b $\neq 0$) can be represented graphically as a straight line in the coordinate plane of x and y coordinates. In chapter 4, you have learnt that graph of a equation $ax^2 + bx + c = 0$, a $\neq 0$ is an upward parabola if a > 0 and a downward parabola if a < 0.

In this chapter, you will learn to find the distance between two given points in terms of their coordinates and also, the coordinates of the point which divides the line segment joining the two given points internally in the given ratio.

★ HISTORICAL FACTS

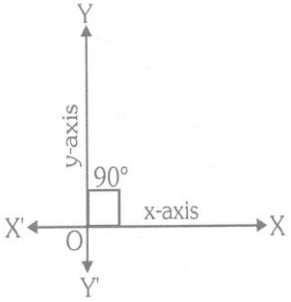
Rene Descartes (1596 - 1650), The 17th century French-Mathematician, was a thinker and a philosopher. He is called the father of Co-ordinate Geometry because he unified Algebra and Geometry which were earlier two distinct branches of Mathematics. Descartes explained that two numbers called co-ordinates are used to locate the position of a point in a plane.

He was the first Mathematician who unified Algebra and Geometry, so Analytical Geometry is also called Algebraic Geometry. Cartesian plane and Cartesian Product of sets have been named after the great Mathematician.

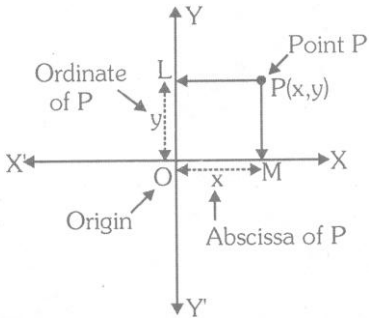
★ RECALL

Cartesian Co-ordinate system:

Let X'OX and Y'OY be two perpendicular straight lines intersecting each other at the point O. Then :

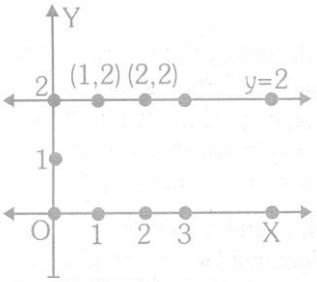


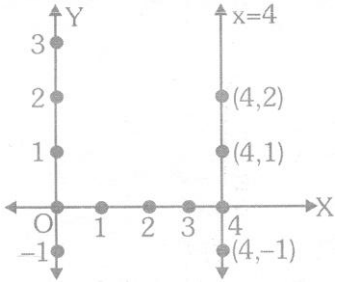
1. X'OX is called the x-axis or the axis of x.
2. Y'OY is called the y-axis or the axis of y.
3. The x-co-ordinate along OX is positive and along OX' negative, y-co-ordinate along OY (upward) is positive and along OY' (downward) is negative.
4. Both X'OX and Y'OY taken together in this order are called the rectangular axes because the angle between them is a right angle.
5. O is called the origin i.e., it is point of intersection of the axes of co-ordinates.



Co-ordinates of a point :

6. Abscissa of a point in the plane is its perpendicular distance with proper sign from y-axis.
7. Ordinate of a point in the plane is its perpendicular distance with proper sign from y-axis.
8. The y-co-ordinate any point on x-axis is zero.
9. The x-co-ordinate any point on x-axis is zero.
10. Any point in the xy-plane, whose y-co-ordinate is zero, lies on x-axis.
11. Any point in the xy-plane, whose x-co-ordinate is zero, lies on x-axis.
12. The origin has coordinates (0, 0).
13. The ordinates of all points on a horizontal line which is parallel to x-axis are equal i.e. y = constant = 2.
14. The abscissa of all points on a vertical line which is a line parallel to y-axis are equal i.e. x-constant = 4





Four Quadrants of a Coordinate plane :

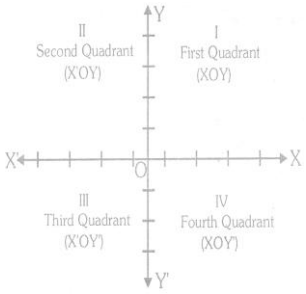
The rectangular axes X'OX and Y'OY divide the plane into four quadrants as below :

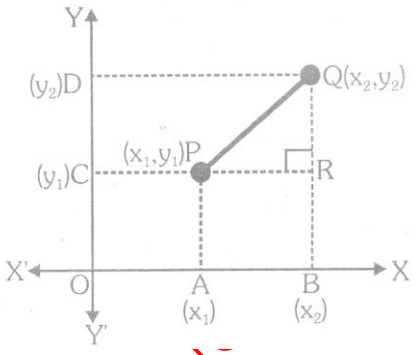
15. Any point in the I quadrant has (+ ve abscissa, + ve ordinate).
16. Any point in the II quadrant has (- ve abscissa, + ve ordinate).
17. Any point in the III quadrant has (- ve abscissa, - ve ordinate).
18. Any point in the IV quadrant has (+ ve abscissa, - ve ordinate).

★ DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) in a rectangular coordinate system is equal to $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Proof : X,OX and Y'OY are the rectangular coordinate axes. P(x_1, y_1) and Q (x_2, y_2) are the given points. We draw PA and QB perpendiculars on the x-axis : PC and QD perpendicular on the y-axis,





Now, CP (produced) meets BQ in R and PR ⊥ BQ.
We find PR = AB = OB – OA = $(x_2 - x_1)$
and QR = BQ – BR = BQ – AP = CD – OC = $(y_2 - y_1)$
In right Δ PRQ, using Pythagoras theorem, we have

$$PQ^2 = PR^2 + QR^2 = (x_2-x_1)^2 + (y_1-y_2)^2 \Rightarrow PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

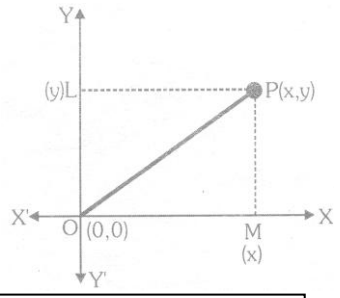
We have taken the positive square roof value because distance between two points is a non-negative quantity.

Distance of a Point from Origin:

The distance of a point (x, y) form origin is $\sqrt{x^2+y^2}$ **.**

Proof : Let us take a point P (x,y) in the given plane of axes X'OX and Y'OY as shown in the fig. Here, the point P (x, y) is in the first quadrant but it can be taken anywhere in all the four quadrants.
We have to find the distance OP, i.e., the distance of the point P from the origin O.

Form the point P, draw PM ⊥ OX and PL ⊥ OY. Then we have
OM = x
MP = OL = y
OP2 = OM2 + MO2 = x2 + y2
Therefore, OP = $\sqrt{x^2+y^2}$

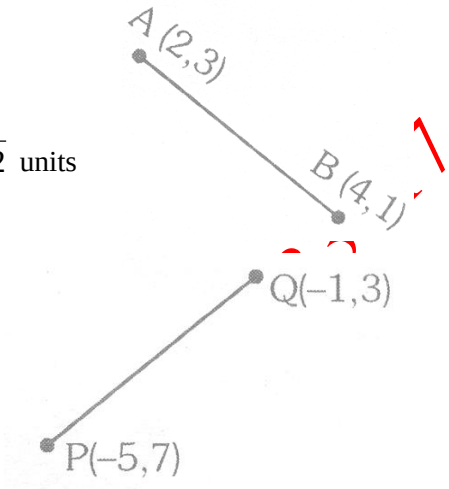


Test For Geometrical Figures :

(a)	For an isosclels	:	Prove that two sides are equal.
(b)	For an equilateral triangle	:	Prove that three sides are equal.
(c)	For a riht-angled triangle	:	Prove that the sum of the squares of two sides is equal to the square of the third side.
(d)	For a square	:	Prove that all sides are equal and diagonals are equal.
(e)	For a rhombus	:	Prove that all sides are equal and diagonals are not equal.
(f)	For a rectangle	:	Prove that the opposites sides are equal and diagonals are also equal.
(g)	For a parallelogram	:	Prove that the opposite sides are equal in length and diagonals are not equal

Ex.1 Find the distance between the following pairs of points:
(a) (2,3), (4,1) (b) (-5, 7), (-1,3) (c) (a,b), (-1, - b) **[NCERT]**

Sol. (a) The given points are : A (2,3), B (4,1).



Required distance $= AB = BA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(b) Distance between P(- 5, 7) and Q(-1, 3) is given by

PQ = QP $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1+5)^2 + (3-7)^2} = \sqrt{16+6} = \sqrt{32}$

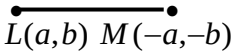
Required distance = PQ = QP = $4\sqrt{2}$ units

(c) Distance LM between L (a,b) and M (-a, -b) is given by

LM $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 = 4b^2}$

$= \sqrt{4(a^2 + b^2} = 2\sqrt{a^2 + b^2}$ units



Ex.2 Find the points on x-axis which are at a distance of 5 units from the point A(-1, 4).

Sol. Let the point on x-axis be :P(x, 0).

Distance = PA = 5 units $\Rightarrow$ $PA^2 = 25$

$\Rightarrow$ $(x+1)^2 + (0-4)^2 = 25$ $\Rightarrow$ $x^2 + 2x + 1 + 16 = 25$

$\Rightarrow$ $x^2 + 2x + 17 = 25$ $\Rightarrow$ $x^2 + 2x - 8 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 + 4(8)}}{2} = \frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2}$$

$$= \frac{-2+6}{2}, \frac{-2-6}{2} = \frac{4}{2}, -\frac{8}{2} = 2, -4$$

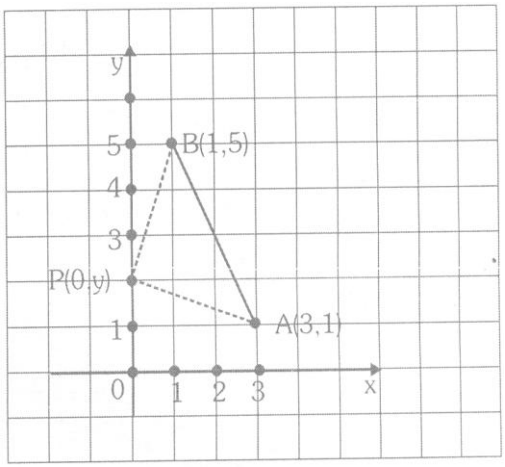
Required point on x-axis are (2, 0) and (-4, 0)

Verification: PA $= \sqrt{(2+1)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

PA $= \sqrt{(-4+1)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Ex.3 What point on y-axis is equidistant from the points (3, 1) and (1, 5)

Sol. Since the required point P(say) is on the y-axis, its abscissa (x-co-ordinate) will be zero. Let the ordinate (y-co-ordinate) of the point be y.



Therefore co-ordinates of the point P are : (0, y)

i.e., P (0, y)

Let A and B denote the points (3, 1) and (1, 5) respectively.
PA = PB …(given)

Squaring we get :

$PA^2 = PB^2$

$\Rightarrow$ $(0-3)^2 + (y-1)^2 = (0-1)^2 + (y-5)^2$

$\Rightarrow \quad 9+y^2 1-2y=1+y^2+25-10y$

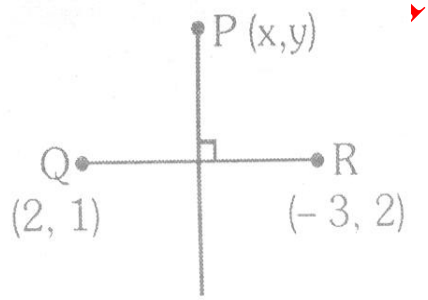
$\Rightarrow \quad y^2-2y+10-y^2-10y+26 \Rightarrow -2y+10y=26-10 \Rightarrow 8y=16 \Rightarrow y=2$

The required point on y-axis equidistant from A(3, 1) and B(1, 5) is P(0, 2).

Ex.4 If Q(2, 1) and R(-3, 2) and P(x, y) lies on the right bisector of QR then show that 5x – y +

Sol Let P(x, y) be a point on the right bisector of QR :

Q(2, 1) and R(-3, 2) are equidistant from P(x, y), then we must have :



PQ = PR

$\Rightarrow \quad PQ^2=PR^2$

$\Rightarrow \quad (x-2)^2+(y-1)^2=(x+3)^2+(y-2)^2$

$\Rightarrow \quad (x^2-4x+4)+(y^2-2y+1)=(x^2+6x+9)+(y^2-4x+4)$

$\Rightarrow \quad -4x-2y+5=6x-4y+13$

$\Rightarrow \quad 10x-2y-8=0$

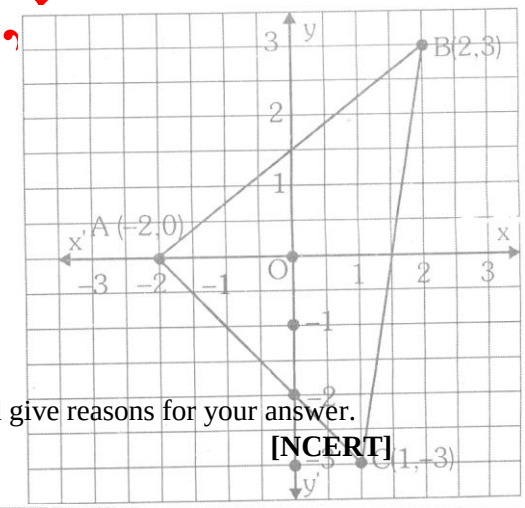
$\Rightarrow \quad 2(5x-y+4)=0$

$\Rightarrow \quad 5x-y+4=0$

Ex.5 The vertices of a triangle are (-2, 0), (2, 3) and (1, -3). Is the triangle equilateral : isosceles or scalene ?

Sol. We denote the given point (-2, 0), (2, 3) and (1, -3) by A, B and C respectively then :

$A(-2,0), B(2,3), C(1,-3)$



$AB=\sqrt{(2+2)^2+(3-0)^2}=\sqrt{(4)^2+(3)^2}=5$

$BA=\sqrt{(1-2)^2+(-3-3)^2}=\sqrt{(-1)^2+(-6)^2}=\sqrt{37}$

$BC=\sqrt{(-2-1)^2+(0+3)^2}=\sqrt{(-3)^2+(3)^2}=3\sqrt{2}$

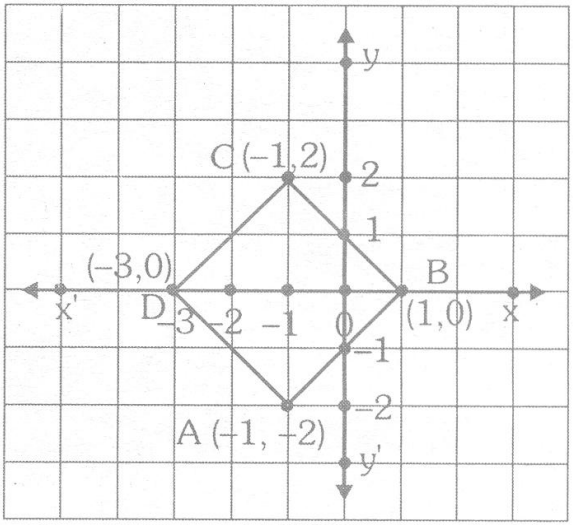
Thus we have $AB \neq BC \neq CA$

$\Rightarrow$ ABC is a scalene triangle

Ex.6 Name the quadrilateral formed, if any, by the following points, and give reasons for your answer.

$(-1,-2),(1,0),(-1,2),(-3,0)$ **[NCERT]**

Sol. $A(-1,-2), B(1,0) C(-1,2), D(-3,0)$



Determine distances : AB, BC, CD, DA, AC and BD.

AB = $\sqrt{(1+1)^2+(0+2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

BC = $\sqrt{(-1-1)^2+(2-0)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

CD = $\sqrt{(-3+1)^2+(0-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

DA = $\sqrt{(-1+3)^2+(-2-0)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

AB = BC = CD = DA

The sides of the quadrilateral are equal ……(1)

AC = $\sqrt{(-1+1)^2+(2+2)^2} = \sqrt{0+16} = 4$

BD = $\sqrt{(-3+1)^2+(0-0)^2} = \sqrt{16+0} = 4$

Diagonal AC = Diagonal BD……(2)

From (1) and (2) we conclude that ABCD is a square.

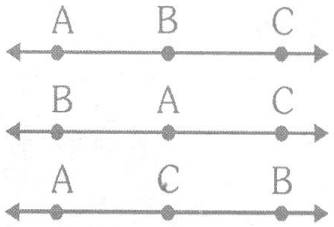
★ COLLINEARITY OF THREE POINTS

Let A, B and C three given points. Point A, B and C will be collinear, If the sum of lengths of any two line-
segments is equal to the length of the third line-segment.
In the adjoining fig. there are three point A, B and C.
Three point A, B and C are collinear if and only if

(i) AB + BC = AC

or (ii) AB + AC = BC

or (iii) AC + BC = AB



Ex.7 Determine whether the points (1, 5) (2, 3) and (-2, -11) are collinear. **[NCERT]**

Sol. The given points are : A(1, 5), B(2, 3) and C(-2, -11).
Let us calculate the distance : AB, BC and CA by using distance formula.

AB = $\sqrt{(2-1)^2+(3-5)^2} = \sqrt{(1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

BC = $\sqrt{(-2-2)^2+(-11-3)^2} = \sqrt{(-4)^2+(-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

CA = $\sqrt{(-2-1)^2+(-11-5)^2} = \sqrt{(-3)^2+(-16)^2} = \sqrt{9+296} = \sqrt{265}$

From the above we see that : AB + BC ≠ CA
Hence the above stated points A(1, 5) B(2, 3) and C(-2, -11) are not collinear.

★ SECTION FORMULA

Coordinates of the point, dividing the line-segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are given by $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

Proof. Let P (x, y) be the point dividing the line-segment joining A(x_1, y_1) internally in the ratio $m_1 : m_2$. We draw the perpendiculars AL, BM and PQ on the x-axis from the points A, B and P respectively. L, M and Q are the points on the x-axis where three perpendiculars meet the x-axis.

We draw AC ⊥ PQ and PD ⊥ BM. Here AC ∥ x-axis and PD ∥ x-axis.

$\Rightarrow$ AC ∥ PD (∵ AC and PD both ∥ x-axis)

$\Rightarrow$ ∠PAC = ∠BPD

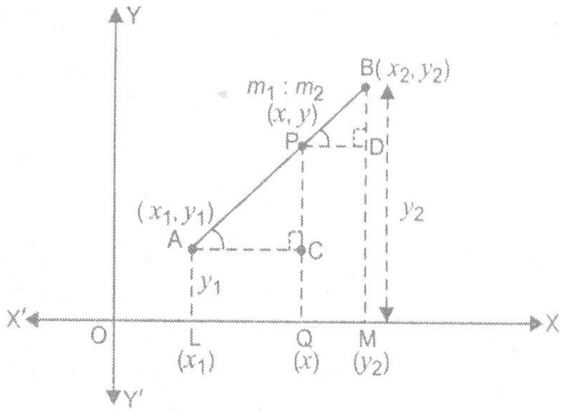
Thus, in ΔACP and ΔPDB, we have

∠PAC = ∠BPD

and ∠ACP = ∠PDB = 90^0

Then by AA similarity criterion,

ΔACP ~ ΔPDB



$\Rightarrow \quad \frac{AC}{PD} = \frac{PC}{BD} = \frac{AP}{PB} = \frac{m_1}{m_2}$

$\Rightarrow \quad \frac{AC}{PD} = \frac{m_1}{m_2}$ ……..(1) and $\frac{PC}{BD} = \frac{m_1}{m_2}$ ……(2)

AC = LQ = OQ – OL = $(x - x_1)$
PD = QM = OM – OQ = $(x_2 - x)$
Putting in (1), we get

$$\frac{(x-x_1)}{(x_2-x)} = \frac{m_1}{m_2} \quad \Rightarrow \quad m_2x - m_2x_1 = m_1x_2 - m_1x$$

$$\Rightarrow \quad m_1x + m_2x = m_1x_2 - m_2x_1 \quad \Rightarrow \quad (m_1+m_2)x = m_1x_2 - m_1x_1$$

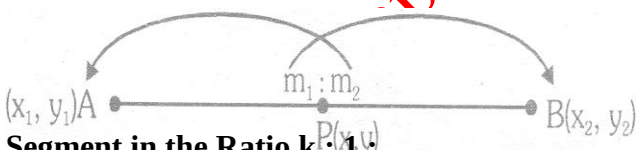
$$\Rightarrow \quad x = \frac{m_1x_2 + m_2x_1}{m_1+m_2}$$

Now, PC = PQ – CQ = PQ – AL = $(y - y_1)$
BD = BM – DM = BM – PQ = $(y_2 - y)$
Putting in (2), we get

$$\frac{y-y_1}{y_2-y} = \frac{m_1}{m_2} \quad \Rightarrow \quad y = \frac{m_1y_2 + m_2y_1}{m_1+m_2}$$

Therefore, the coordinates of the point P are $\left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$

Remark : To remember the section formula, the diagram given below is helpful:



Point Dividing a Line Segment in the Ratio k : 1 :

If P $\left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$ divides the line-segment, joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ we can express it as below :

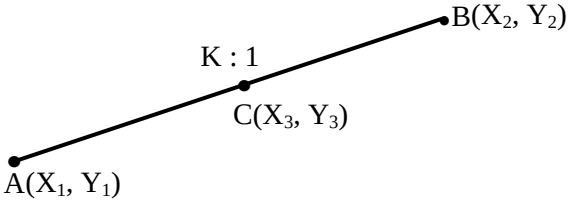
P $\left(\frac{\frac{m_1}{m_2}x_2 + x_1}{\frac{m_1}{m_2}+1}, \frac{\frac{m_1}{m_2}y_2 + y_1}{\frac{m_1}{m_2}+1}\right)$ (By dividing the numerator and the denominator by m_2)

Putting $\frac{m_1}{m_2} = k$, the ratio becomes k : 1 and the coordinates of P are expressed in the form

P $\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 y_1}{k+1}\right)$

Therefore, the coordinates of the point P, which divides the line-segment joining A (x_1, y_1) and $B(x_2, y_2)$ internally in the ratio k : 1, are given by $\left(\frac{kx_2 + x1}{k+1}, \frac{ky_2 + y1}{k+1}\right)$

Collinearity of three points :
Three given points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are said to be collinear if one of them must divide the line segment joining the other two points in the same ratio.



Remark : Three points are called non-collinear if one of them divides the line segment joining the other two points in different ratios

Mid-point Formula :

Coordinates of the mid-point of the line-segment joining (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

The mid-point M (x,y) of the line-segment joining A (x_1, y_1) and B(x_2, y_2) divides the line-segment AB in the ratio 1 : 1. Putting $m_1 - m_2 = 1$in the section formula, we get the coordinates of the mid-point as $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

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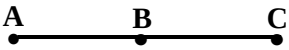
COMPETITION WINDOW

SECTION FORMULA FOR EXTERNAL DIVISION

The co-ordinates of line point which divides the line segment joining the point (x_1, y_1) and (x_2, y_2) externally in the ratio m : n are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

e.g., In the following case, C divides AB externally in ratio AC : BC

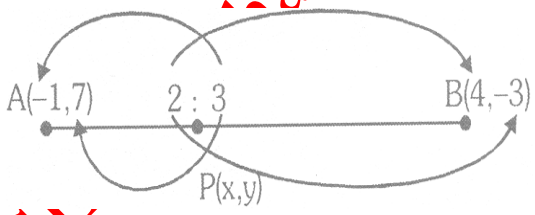


Let A = (1, 0), B = (4, 0) and 5 : 2 be the ratio in which C divides AB externally. Then co-ordinates of C are :

$$\left(\frac{5\times4-2\times1}{5-2}, \frac{5\times0-2\times0}{5-2}\right) = [6,0]$$

Ex.8 Find the co-ordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2 : 3, **[NCERT]**

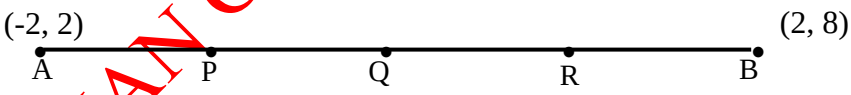
Sol. Let P(x, y) divides the line segment AB joining A(-1, 7) and B(4, -3) in the ratio 2 : 3. Ten by using section formula line the co-ordinates of P are given by :



$$\left(\frac{2\times4\times3\times(-1)}{2+3}, \frac{2\times(-3)+3\times7}{2+3}\right) = P\left(\frac{8-3}{5}, \frac{-6+21}{5}\right) = P\left(\frac{5}{5}, \frac{15}{5}\right) = P(1, 3)$$

Hence the required point of division which divides the line segment joining A(-1, 7) and (4, -3) in the ratio 2 : 3 is P(1, 3).

Sol.



It is given that AB is divided into four equal parts : AP = PQ = QR = RB

Q is the mid-point of AB, then co-ordinates of Q are : $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = \left(\frac{0}{2}, \frac{10}{2}\right) = (0,5)$

P is the mid-point of AQ, then co-ordinates of P are : $\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \left(\frac{-2}{2}, \frac{7}{2}\right) = \left(-1, \frac{7}{2}\right)$

Also, R is the mid-point of QB, then co-ordinates of R are : $\left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(\frac{2}{2}, \frac{13}{2}\right) = \left(-1, \frac{13}{2}\right)$

Hence, required co-ordinates of the points are :

$$P(-1\frac{7}{2}), Q(0, 5), R(1, \frac{13}{2})$$

Ex.10 If the point C(-1, 2) divides the lines segment AB in the ratio 3 : 4, where the co-ordinates of A are (2, 5), find the coordinates of B.

Sol. Let C (-1, 2) divides the line joining A (2, 5) and B (x, y) in the ratio 3 : 4. Then.

$$C\left(\frac{3x+8}{7}, \frac{3y+20}{7}\right) = C(-1, 2)$$

$$\Rightarrow \quad \frac{3x+8}{7} = -1 \quad \& \quad \frac{3y+20}{7} = 2$$

$$\Rightarrow \quad 3x+8 = -7 \quad \& \quad 3y+20 = 14$$

$$\Rightarrow \quad x = -5 \quad \& \quad y = -2$$

The coordinates of B are : B (-5, -2)

Ex.11 Find the ratio in which the line segment joining the points (1, -7) and (6, 4) is divided by x-axis

Sol. Let C (x, 0) divides AB in the ratio k : 1.

By section formula, the coordinates of C are given by :

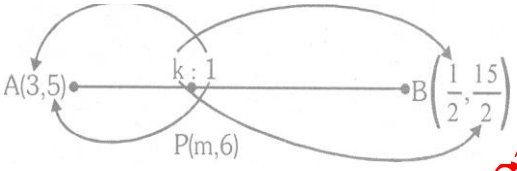
$$C\left(\frac{6k+1}{k+1}, \frac{4k-7}{k+1}\right) \quad \Rightarrow \quad \frac{4k-7}{k+1} = 0$$

$$\Rightarrow \quad 4k-7 = 0 \Rightarrow k = \frac{7}{4}$$

i,e., the x-axis divides AB in the ratio 7 : 4.

Ex.12 Find the value of m for which coordinates (3,5), (m,6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear.

Sol. Let P (m, 6) divides the line segment AB joining A (3,5) $B\left(\frac{1}{2}, \frac{15}{2}\right)$ in the ratio k : 1.



Applying section formula, we get the co-ordinates of P : $\left(\frac{\frac{1}{2}k+3\times1}{k+1}, \frac{\frac{15}{2}k+5\times1}{k+1}\right) = \left(\frac{k+6}{2(k+1)}, \frac{15k+10}{2(k+1)}\right)$

But P (m, 6) = $P\left(\frac{k+6}{2(k+1)}, \frac{15k+10}{2(k+1)}\right)$ $\Rightarrow$ $m = \frac{k+6}{2(k+)}$ and also $\frac{15k+10}{2(k+1)} = 6$

$$\Rightarrow \quad \frac{15k+10}{2(k+1)} = 6 \quad \Rightarrow \quad 15k+10 = 12(k+1)$$

$$\Rightarrow \quad 15k+10 = 12k+12 \quad \Rightarrow \quad 15k-12k = 12-10$$

$$\Rightarrow \quad 3k = 2 \quad \Rightarrow \quad k = \frac{2}{3}$$

Putting $k = \frac{2}{3}$ in the equation $m = \frac{k+6}{2(k+)}$ we get :

$$m = \frac{\left(\frac{2}{3}+6\right)}{2\left(\frac{2}{3}+1\right)} = \frac{\left(\frac{2+18}{3}\right)}{2\left(\frac{2+3}{3}\right)} = \frac{20}{3}\times\frac{3}{10} = \frac{20}{10} \qquad \left(\because k = \frac{2}{3}\right)$$

$$m = \frac{10\times2}{10} = 2$$

Required value of m is 2 $\Rightarrow$ m = 2

Ex.13 The two opposite vertices of a square are (-1, 2) and (3, 2). Find the co-ordinates of the other two vertices.

Sol. Let ABCD be a square and two opposite vertices of it are A(-1, 2) and C(3, 2) ABCD is a square.

$\Rightarrow$ AB = BC

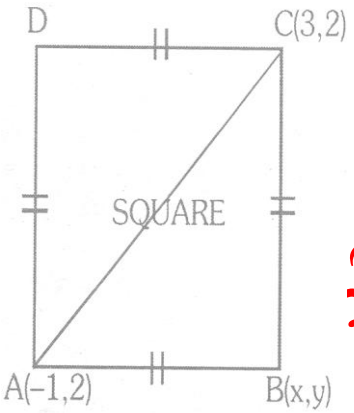
$\Rightarrow$ $AB^2 = BC^2$

$\Rightarrow$ $(x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow$ $x^2+2x+1=x^2-6x+9$

$\Rightarrow$ $2x+6x=9-1=8$

$\Rightarrow$ $8x=8\Rightarrow x=1$



ABCD is right Δ at B, then

$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow$ $(3+1)^2+(2-2)^2=(x+)^2+(y-2)^2+(x-3)^2=(y-2)^2$

$\Rightarrow$ $16=2(y-2)^2+(1+1)^2+(1-3)^2$

$\Rightarrow$ $16=2(y-2)^2+4+4\Rightarrow 2(y-2)^2=16-8=8$

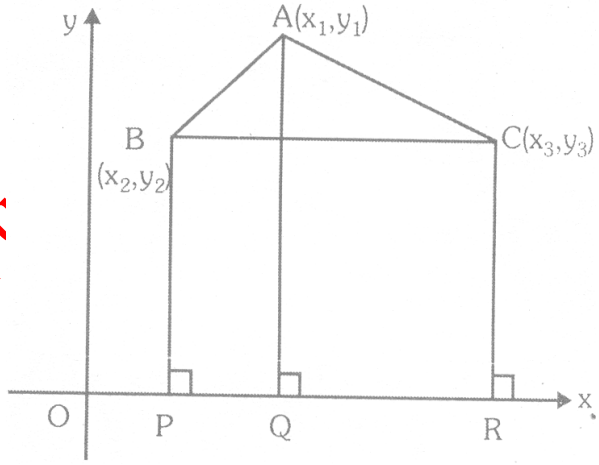
$\Rightarrow$ $(y-2)^2=4 \quad\Rightarrow\quad y-2=\pm 2\Rightarrow$ y = 4 and 0

i.e., when x = 1 then y = 4 and 0

Co-ordinates of the opposite vertices are : B(1, 0) or D(1, 4)

★ AREA OF A TRIANGLE

In your pervious classes, you have learnt to find the area of a triangle in terms of its base and corresponding altitude as below:



Area of triangle = $\frac{1}{2}$ × base × altitude.

In case, we know the lengths of the three sides of a triangle, then the area of the triangle can be obtained by using the Heron's formula.

In this section, we will find the area of a triangle when the coordinates of its three vertices are given. The lengths of the three sides can be obtained by using distance formula but we will not prefer the use of Heron's formula.

Some times, the lengths of the sides are obtained as irrational numbers and the application of Heron's formula becomes tedious. Let us develop some easier way to find the area of a triangle when the coordinates of its vertices are given.

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the given three points. Through A draw AQ $\perp$ OX, through B draw BP $\perp$ OX and through C draw CR $\perp$ OX.

Form the fig. $AQ = y_1$, $BP = y_2$ and $CR = y_3$, $OP = x_2$, $OQ = x_1$ and $OR = x_3$

$\Rightarrow$ $PQ = x_1 - x_2$; $QR = x_3 - x_1$ and $PR = x_3 - x_2$

Area of trapezium = $\frac{1}{2}$ (sum of parallel side) × distance between parallel lines

ar. (Δ ABC) = ar.(Trap.ABCD) + ar. (Trap.AQRC) – ar. (Trap. BPRC)

$= \frac{1}{2}(BP + AQ)\times PQ + \frac{1}{2}(AQ + CR)\times QR - \frac{1}{2}(PB + CR)\times PR$

$=\frac{1}{2}(y_2+y_1)(x_1-x_2)+\frac{1}{2}(y_2+y_3)(x_3-x_1)-\frac{1}{2}(y_2+y_3)(x_3-x_2)$

$=\frac{1}{2}\left|x_1(y_2+y_1-y_1-y_3)+x_2(y_2+y_3-y_2-y_1)+x_3(y_1+y_3-y_2-y_3)\right|$

$=\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

Area of $\Delta ABC = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

Condition of collinearity of three points :

The given points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ will be collinear if the area of the triangle formed by them must be zero because triangle can not be formed.

$\Rightarrow$ area of $\Delta ABC = 0$

$$\Rightarrow \quad \frac{1}{2}\left| x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right| = 0$$

$$\Rightarrow \quad \boxed{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0}$$

is the required condition for three points to be collinear.

Ex.14 The co-ordinates of the ΔABC are A(4, 1), D(3, 2) and C(0, K). Given that the area of ΔABC is 12 unit^2. Find the value of k.

Sol. Area of ΔABC formed by the given-points A(4, 1), B(-3, 2) and C(0, k) is

$$= \frac{1}{2}\left|4(2-k)+(-3)(k-1)+0(1-2)\right|$$

$$= \frac{1}{2}\left|18-4k-3k+3\right| = \frac{1}{2}(11-7k)$$

But area of $\Delta ABC = 12$ unit^2 ……..(given)

$$\frac{1}{2}\left| = \frac{1}{2}(11-7k)\right| = 24 \quad \Rightarrow \quad \left|(11-7k)\right| = 24$$

$$\pm(11-7k) = 24 \quad \Rightarrow \quad 11-7k = 24 \text{ or } -(11-7k) = 24$$

$$-7k = 24-11 = 13 \quad \Rightarrow \quad k = -\frac{13}{7}$$

or $\quad -(11-7k) = 24 \quad \Rightarrow \quad -11+7k = 24$

$$\Rightarrow \quad 7k = 24+11 = 35 \quad \Rightarrow \quad k = \frac{35}{7} = 5$$

Ex.15 Find the area of the quadrilateral whose vertices taken in order are (-4, -2), (-3, -5), (3, -2), and (2, 3). **[NCERT]**

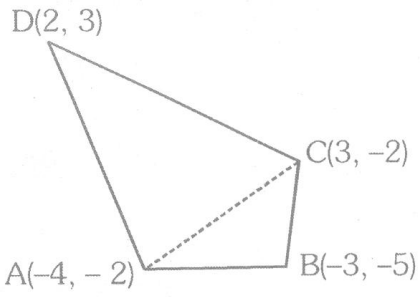
Sol. Join A and C

The given points are A(-4, -2), B(-3, -5), C(-4, -2) and D(2, 3)

Area of ΔABC

$$= \frac{1}{2}\left|(-4)(-5+2)-3(-2+2)+3(-2+5)\right|$$

$$= \frac{1}{2}\left|20-8-6+15\right| = \frac{21}{2} = 10.5 \text{ sq. units}$$



Area of ΔACD

$$= \frac{1}{2}\left|(-4)(-2-3)+3(-2+2)+3(-2+5)\right|$$

$$= \frac{1}{2}\left|20+15\right| = \frac{35}{2} = 17.5 \text{ sq. units}$$

Area of quadrilateral ABCD = ar. (ΔABC) + ar. (ΔACD) = (10.5 + 17.5) sq. units = 28 sq. units

Ex.16 Find the value of p for which the points (-1, 3), (2, p), (5, -1) are collinear.

Sol. The given points A (1, 3), B(2, p), C(5, -1) are collinear.

$\Rightarrow$ Area of ΔABC formed by these points should be zero.

$\Rightarrow$ The area of $\Delta ABC = 0$

$\Rightarrow \quad \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right| = 0$

$\Rightarrow \quad -1(p+1) + 2(-1-3) + 5(3-p) = 0$

$\Rightarrow \quad -p - 1 - 8 + 15 - 5p = 0$

$\Rightarrow \quad -6p + 15 - 9 = 0 \Rightarrow 6p = -6 \Rightarrow p = 1$

Hence the value of p is 1.

COMPETITION WINDOW

AREA OF A QUADRILATERAL

If $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) are vertices of a quadrilateral, its area

$$\frac{1}{2}\left| (x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4) \right|$$

AREA OF A POLYEON

If $(x_1, y_1), (x_2, y_2), (x_3, y_3) \ldots\ldots (x_n, y_n)$ are vertices of a polygon of n sides, its area

$$\frac{1}{2}\left| (x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + ... + (x_ny_1 - x_1y_n) \right|$$

Remark : (i) If the area of a quadrilateral joining the four points is zero, the four points are collinear.
(ii) If two opposite vertex of a square are A(x_1, y_1) and C(x_2, y_2) then it's area is

$$\frac{1}{2}[(x_2 - x_1)^2 + (y_2 - y_1)^2]$$

TRY OUT THE FOLLOWING

(i) Find the area of the quadrilateral formed by joining the four points (1, 1), (3, 4), (5, -2) & (4, -7).
(ii) Find the area of the pentagon whose vertices are A(1, 1), B(7, 21), C(7, -3) D(4, -7) and E(0, -3).
(iii) If the Co-ordinates of two opposite vertex of a square are (a, b) and (b, a), find the area of the square.

ANSWERS

(i) $\frac{41}{2}$ sq. nuits (ii) $\frac{137}{2}$ sq. nuits (i) $(a-b)^2$ sq. nuits

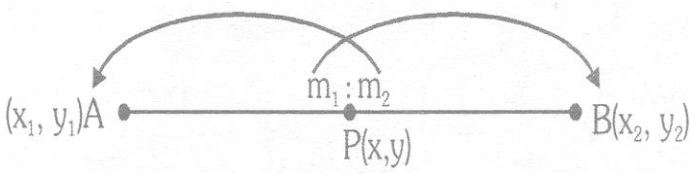
★ SYNOPSIS

▸▸ ***Distance Formula : The distance between two points (x_1, y_2) and (x_2, y_2) in a rectangular coordinate system is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. The distance of a point (x, y) from origin is $\sqrt{x^2 + y^2}$***

Test For Geometrical Figures :

(a)	For an isosceles triangle	:	Prove that at least two sides are equal
(b)	For an equilateral triangle	:	Prove that three sides are equal
(c)	For a right-angled triangle	:	Prove that the sum of the squares of two sides is equal to the square of the third side.
(d)	For a square	:	Prove that all sides are equal and diagonals are equal.
(e)	For a rhombus	:	Prove that all sides are equal and diagonals are not equal.
(f)	For a rectangle	:	Prove that the opposite sides are equal and diagonals are also equal.
(g)	For a parallelogram	:	Prove that the opposite sides are equal in length and diagonals are not equal.

- **Collinearity of three points :** Let A, B and C there given points. Point A, B and C will be collinear if the sum, of lengths of any two line-segment is equal to the length of the third line-segment.
  In the adjoining fig. there are three point A, B and C.
  Three points A, B and C are collinear if and only if
  (i) AB + BC = AC or (ii) AB + AC = BC or (iii) AC + BC = AB
- **Section Formula : Coordinates of the point, dividing the line-segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio : m_1 m_2 are given by** $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$



Mid-point Formula : Coordinates of the mid-point of the line-segment joining (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

- Area of triangle : $\boxed{\text{Area of } \Delta ABC = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + y_3(y_1 - y_2)\right|}$

Condition of collinearity of three points : The given points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ will be collinear if the area of the triangle formed by them must be zero because triangle can not be formed.

$\Rightarrow$ area of $\Delta ABC = 0$

$\Rightarrow$ $\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + y_3(y_1 - y_2)\right| = 0 \Rightarrow \boxed{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0}$

is the required condition for three points to be collinear.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

1. The distance between the points (a, b) and (-a, - b) is :
(A) a^2+b^2 (B) $\sqrt{a^2+b^2}$ (C) 0 (D) $2\sqrt{a^2+b^2}$

2. The distance between points (a + b, b + c) and (a – b, c - b) is :
(A) $2\sqrt{a^2+b^2}$ (B) $2\sqrt{a^2+c^2}$ (C) $2.\sqrt{2b}$ (D) $\sqrt{a^2-c^2}$

3. The distance between points A(1, 3) and B(x, 7) is 5. The value of x > 0 is :
(A) 4 (B) 2 (C) 1 (D) 3.

4. The distance between the points (a cos 20^0 + b sin 20^0,0) and (a sin 20^0 - b cos 20^0) is :
(A) (a + b) (B) (a – b) (C) $\sqrt{a^2-b^2}$ (D) $\sqrt{a^2+b^2}$

5. Mid-point of the line-segment joining the points (-5, 4) and (9, - 8) is :
(A) (–7, 6) (B) (2, – 2) (C) (7, – 6) (D) (–2, 2).

6. The co-ordinates of the points which divides the join of (–2, 2) and (–5, 7) in the ratio 2 : 1 is :
(A) (4, – 4) (B) (–3, 1) (C) (–4, 4) (D) (1, –3).

7. The co-ordinates of the points on x-axis which is equidistant from the points (5, 4) and (–2, 3) are :
(A) (2, 0) (B) (3, 0) (C) (0, 2) (D) (0, 3).

8. The co-ordinates of the points on y-axis which is equidistant from the points (3, 1) and (1, 5) are :
(A) (0, 4) (B) (0, 2) (C) (4, 0) (D) (2, 0).

9. The coordinates of the centre of a circle are (– 6, 1.5). If the ends of a diameter are (– 3, y) and (x, - 2) then:
(A) x = - 9, y = 5 (B) x = 5, y = - 9 (C) x = 9, y = 5 (D) None of these

10. The points (- 2, 2), (8, - 2) and (-4, - 3) are the vertices of a :
(A) equilateral Δ (B) isosceles Δ (C) right Δ (D) None of these

11. The points (1, 7), (4, 2) (- 1, 1) (- 4, 4) are the vertices of a :
(A) parallelogram (B) rhombus (C) rectangle (D) square.

12. The line segment joining (2, - 3) and (5, 6) is divided by x-axis in the ratio:
(A) 2 : 1 (B) 3 : 1 (C) 1 : 2 (D) 1 : 3.

13. The line segment joining the points (3, 5) and (- 4, 2) is divided by y-axis in the ratio:
(A) 5 : 3 (B) 3 : 5 (C) 4 : 3 (D) 3 : 4.

14. If (3, 2), (4, k) and (5, 3) are collinear then k is equal to :
(A) $\frac{2}{3}$ (B) $\frac{2}{5}$ (C) $\frac{5}{2}$ (D) $\frac{3}{5}$

15. If the points (p, 0), (0, q) and (1, 1) are collinear then $\frac{1}{p}+\frac{1}{q}$ is equal to :

(A) – 1 (B) 1 (C) 2 (D) 0

16. Two vertices of a triangle are (-2, -3) and (4, -1) and centroid is at the origin. The coordinates of the third vertex of the triangle are :

(A) (– 2, 3) (B) (–3, – 2) (C) (–2, 4) (D) (4, –2)

17. A (5, 1), B(1, 5) and C(–3, –1) are the vertices of Δ ABC. The length of its median AD is :

(A) $\sqrt{34}$ (B) $\sqrt{35}$ (C) $\sqrt{37}$ (D) 6

18. Three consecutive vertices of a parallelogram are (1, –2), (3, 6) and (5, 10). The coordinates of the fourth vertex are :

(A) (–3, 2) (B) (2,– 3) (C) (3, 2) (D) (– 2, –3)

19. The vertices of a parallelogram are (3, –2), (4, 0), (6,– 3) and (5,– 5). The diagonals intersect at the point M. The coordinates of the point M are :

(A) $\left(\frac{9}{2},\frac{5}{2}\right)$ (B) $\left(\frac{7}{2},\frac{5}{2}\right)$ (C) $\left(\frac{7}{2},\frac{3}{2}\right)$ (D) None of these

20. If two vertices of a parallelogram are (3, 2) and (–1, 0) and the diagonals intersect at (2, –5), then the other two vertex are :

(A) (1,– 10), (5, –12) (B) (1,– 12), (5, –10) (C) (2, –10) (D) (1, – 10), (2, – 12)

OBJECTIVE			ANSWER KEY			EXERCISE-4				
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	D	C	A	C	B	C	A	B	A	C
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	D	A	D	C	C	C	C	C	A	B

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

SHORT ANSWER TYPE QUESTIONS

1. Find the distance between the points A and B in the following :

(i) $A(a+b,b-a),B(a-b,a+b)$ (ii) $A(1,-1),B(-\frac{1}{2},\frac{1}{2})$

2. Find the distance between the points A and B in the following :

(i) $A(8-2),B(3-6)$ (ii) $A(a+b,a-b),B(a-b,-a-b)$

3. A point P lies on the x-axis and has abscissa 5 and a point Q lies on y-axis and has ordinate – 12. Find the distance PQ.

4. Find a relation between x and y such that the point (x, y) is equidistant from (7, 1) and (3, 5).

5. Using distance formula, show that the points A, B and C are collinear.

(i) $A(-1,-1), B(2,3), C(8,11)$ (ii) $A(-4,-2), B(-1,1), C(1, 3)$

6. Find a point on the x-axis which is equidistant from the points (5, 4) and (-2, 3).

7. Find a point on the x-axis which is equidistant from the points (-3, 4) and (2, 3).

8. Find the value of k, if the point (2, 3) is equidistant from the points A(k, 1) and B(7, k).

9. Find the value of k for which the distance between the point A(3k, 4) and B(2, k) is $5\sqrt{2}$ units.

10. Find the co-ordinates of the point which divides the line segment joining the points (1, -3) and (-3, 9) in the ratio 1 : 3 internally.

11. Find the mid-point of AB where A and B are the points (-5, 11) and (7, 3) respectively.

12. The mid-point of a line segment is (5, 8). If one end points is (3, 5), find the second end point.

13. The vertices of a triangle are A(3, 4), (7, 2) and C(-2, -5). Find the length of the median through the vertex A.

14. The co-ordinates of A and B are (1, 2) and (2, 3) respectively. Find the co-ordinates of R on line segment AB so that $\frac{AR}{RB} = \frac{4}{3}$.

15. Find the co-ordinates of the centre of a circle, the co-ordinates of the end points of a diameter being (-3, 8) and (5, 6)

16. Find the co-ordinates of the point where the diagonals of the parallelogram formed by joining the points (-2, -1), (1, 0), (4, 3) and (1, 2) meet.

17. Find the ratio in which the line segment joining the points (3, 5) and (-4, 2) is divided by y-axis.

18. In what ratio in does the point $\left(\frac{1}{2}, \frac{-3}{2}\right)$ divide the line segment joining the points (3, 5) and (-7, 9) ?

19. By using section formula, show that the points (-1, 2), (2, 5) and (5, 8) are collinear.

20. Find the distance of the point (1, 2) from the mid-point of the line segment joining the points (6, 8) and (2, 4).

21. Show that the mid-point of the line segment joining the points (5, 7) and (3, 9) is also the mid-point of the line segment joining the points (8, 6) and (0, 10).

22. Find the area of the triangle whose vertices are (3, 2) (-2, -3) and (2, 3).

23. For what value of m, the points (3, 5), (m, 6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear ?

LONG ANSWER TYPE QUESTION

1. Prove that the points (1, 4), (3, 6) and (9, -2) are the vertices of an isosceles triangle.

2. Find the co-ordinates of the point equidistant from three given points A(5, 1), B(-3, -7) and C(7, -1).

3. Show that the points (7, 10), (-2, 5) and (3, - 4) are the vertices of an isosceles right triangle.

4. Prove that the points (0, 1), (1, 4), (4, 3) and (3, 0) are the vertices of a square.

5. Prove that the points (-4,- 1), (-2,- 4), (4, 0) and (2, 3) are the vertices of a rectangle.

6. If two vertices of an equilateral triangle are (0, 0) and $(3, \sqrt{3})$, find the third vertex of the triangle.

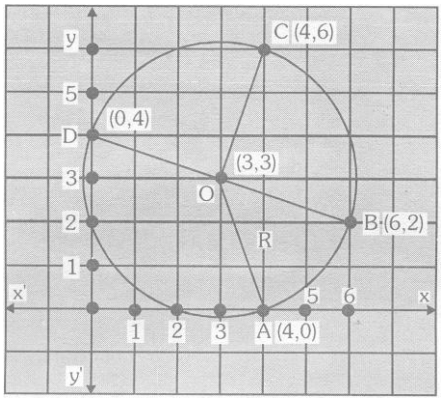
7. A (3, 4) and C (1, -1) are the two opposite angular points of a square ABCD. Find the co-ordinates two vertices

8. Find the co-ordinates of the point equidistant from the point A(-2, -3), B(-1, 0) and C(7, -6).

9. Show that (3, 3) is the centre of the circle passing through the points (4, 6), (0, 4), (6, 2) and (4, 0). What radius of the circle.

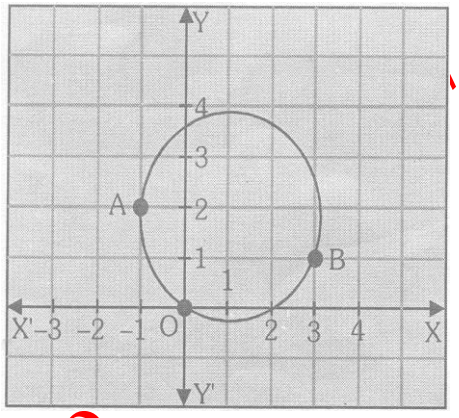
10. If A (2, -1), B(3, 4), C(-2, 3) and D (-3, -2) be four points in a co-ordinates plane, show that ABCD is a rhombus but not a square. Find the area of the rhombus.

11. In figure, find the co-ordinates of the centre of the circle which is drawn through the points A, B and O.



12. The line segment joining the points (3, -1) and (1, 2) is trisected at the points P and Q. If the co-ordinates of P and Q are (p, -2) and $\left(\frac{5}{2}, q\right)$ respectively, find the values of P and Q.

13. What will be the value of y if the point $\left(\frac{23}{5}, y\right)$, divides the line segment joining the points (5, 7) and (4, 5) in the ratio 2 : 3 internally.



14. Find the co-ordinates of the points which divide the line segment joining the points (-4, 0) and (0, 6) in 4 equal parts.

15. If the points (10, 5) (8, 4) and (6, 6) are the mid-points of the sides of a triangle, find its vertices.

16. Find the area of the quadrilateral ABCD formed by the points A (–2, – 2), B (5, 1), C (2, 4) and (–1, 5)..

17. Find the point on the x-axis which is equidistant from the points (–2, 5) and (2, – 3). Hence, find the area of the triangle formed by these points.

18. A (4, 3), B (6, 5) and C (5, – 2) are the vertices of ΔABC.

(i) Find the co-ordinates of the centroid G of ΔABC. Find the area of ΔABC and compare it with area of ΔABC.

(ii) If D is the mid-point of BC, find the co-ordinates of D. Find the co-ordinates of a point P on AD such that AP : PD = 2 : 3. Find the area of ΔABC and compare it with area of ΔABC.

19. ABCDE is a polygon whose vertices are A(–1, 0), B(4, 0), C(4, 4), D(0, 7) and E(–6, 2). Find the area of the polygon.

20. Name the quadrilateral formed by joining the points (1, 2), (5, 4), (3, 8) and (–1, 6) in order. Find also the area of the region formed by joining the mid-points of the sides of this quadrilateral.

CO-ORDINATE GEOMETRY | **ANSWER KEY** | EXERCISE-2 (X)-

SHORT ANSWER TYPE QUESTION :

1. (i) $2\sqrt{a^2+b^2}$ units, **(ii)** $\frac{3\sqrt{2}}{2}$ units **2. (i)** $\sqrt{41}$ units, **(ii)** $2\sqrt{a^2+b^2}$ units **3.** 13 units

4. x – y = 2 **6.** (2, 0) **7.** (0, 6) **8.** k = 13 **9.** k = –1 or k = 3 **10.** (0, 0) **11.** (1, 7) **12.** (7, 11)

13. $\frac{\sqrt{122}}{2}$ units, **14.** $\left(\frac{11}{7}, \frac{18}{7}\right)$ **15.** (1, 7) **16.** (1, 1) **17.** 3 : 4 **18.** 1 : 3 **20.** 5 units **22.** 5 sq. unit

23. m = 2

LONG ANSWER TYPE QUESTIONS :

1. (2, – 4) **2.** $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$ 3. $\left(\frac{9}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{5}{2}\right)$ **4.** (3, – 3) 5. $\sqrt{10}$ units **6.** 24 sq. units

7. $\left(\frac{15}{14}, \frac{25}{14}\right)$ **8.** $p = \frac{7}{3}, q = 0$ **9.** $\frac{31}{5}$ **10.** $\left(-3, \frac{3}{2}\right), (-2, 3), \left(-1, \frac{9}{2}\right)$ **11.** (8, 7), (12, 3), (4, 5) **16.** 26 sq. units

17. (–2, 0), 10 sq. units

18. **(i)** G (5, 2) ; ar (ΔGBC) = 2 sq. units ; ar (ΔGBC) : ar (ΔABC) = 1 : 3

(ii) $D\left(\frac{11}{2}, \frac{3}{2}\right); P\left(\frac{23}{5}, \frac{12}{5}\right)$; ar (ΔPBC) = $\frac{18}{5}$ sq. units ; ar (ΔPBC) : ar (ΔABC) = 3 : 5

19. 44 sq. units **20.** Square ; 10 sq. units.

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

1. Show that the point A(5, 6), B(1, 5), C(2, 1) and D(6, 2) are the vertices of a square. **[Delhi-2004]**

2. Determine the ratio in which the point P(m, 6) divide the join of A(–4, 3) and B(2, 8). Also find the value of m.

OR

A(3, 2) and B(–2, 1) are two vertices of a triangle ABC, whose centroid G has coordinates $\left(\frac{5}{3},-\frac{1}{3}\right)$. Find the coordinates of the third vertex C of the triangle. **[Delhi-2004]**

3. Show that the points A(2, –2), B(14, 10), C(11, 13) and D(–1, 1) are the vertices of a rectangle. **[Al-2004]**

4. Prove that the coordinates of the centroid of a Δ ABC, with vertices. A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are given by $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+_3}{2}\right)$ **[Al-2004]**

5. Determine the ratio in which the point (–6, a) divide the join of A(–3, –1) and B(–8, 9). Also find the value of a. **[Al-2004]**

6. Find the point on the x-axis which is equidistant from the points (–2, 5) and (2, –3) **[Al-2004]**

7. Prove that the points A(0, 1), B(1, 4), C(4, 3) and (3, 0) are the vertices of a square. **[Foreign-2004]**

8. Determine the ratio in which the point (a, –2) divide the join of A(–4, 3) and B(2, –4). Also find the value of a. **[Foreign-2004]**

9. Determine the ratio in which the point P(k, 2) divide the join of A(–3, 5) and B(5, 1). Also find the value of k. **[Foreign-2004]**

10. Determine the ratio in which the point P(b, 1) divide the join of A(7, –2) and B(–5, 6). Also find the value of b. **[Foreign-2004]**

11. The coordinates of the mid-point of the line joining the point (3p, 4) and (–2, 2q) are (5, p). Find the coordinates of p and q. **[Delhi-2004C]**

12. Two vertices of a triangle are (1, 2) and (3, 5). If the centroid of the triangle is at the origin, find the coordinates of the third vertex.

OR

If 'a' is the length of one of the sides of an equilateral triangle ABC, base BC lies on x-axis and vertex B is at the origin, find the coordinates of the vertices of the triangle ABC. **[Delhi-2004C]**

13. Find the ratio in which the one-segment joining the points (6, 4) and (1, –7) is divided by x-axis . **[Al-2004C]**

OR

The coordinates of two vertices A and B of a tnangle An are (1, 4) and (5, 3) respectively. If the coordinates of the centroid of Δ ABC are (3, 3), find the coordinates of the third vertex C. **[Al-2004C]**

14. Find the value of m for which the points with coordinates (3, 5), (m, 6) and $\left(\frac{1}{2},\frac{15}{2}\right)$ are collinear. **[Al-2004C]**

15. Find the value of x such that PQ = QR where the coordinates of P,Q and R are (6,–1);(1,3) and (x,8)respectively.

OR

Find a point on x-axis which is equidistant from the points (7, 6) and (–3, 4). **[Delhi-2004]**

16. The line-segment joining the points (3, –4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, 2) and $\left(\frac{5}{3},q\right)$ respectively, find the values of p and q. **[Delhi-2005]**

17. Prove that the points (0, 0), (5, 5) are vertices of a right isosceles triangle.

OR

If the point P(x, y) is equidistant from the point A(5, 1) and B(–1, 5), prove that 3x = 2y. **[Al-2005]**

18. The line joining the points (2, 1) and (5, –8) is trisected at the points p and Q. If point P lies on the line 2x – y + k = 0, find the value of k. **[Al-2005]**

19. Show that the points (0, –1); (2, 1); (0, 3) and (–2, 1) are the vertices of a square.

OR

Find the value of K such that the point (0, 2) is equidistant from the points (3, K) and (K, 5). **[Foreign-2005]**

20. The base BC of an equilateral Δ ABC lies on y-axis. The coordinates of point C are (0, –3). If the origin is the mid-point of the base BC, find the coordinates of the points A and B. **[Foreign-2005]**

21. Find the coordinates of the point equidistant from the points A(1, 2), B(3, – 4) and C(5, –6).

OR

Prove that the points A(–4, –1), B(–2, –4), C(4, 0) and (2, 3) are the vertices of a rectangle. **[Delhi-2005C]**

22. Find the coordinates of the points which divide the line-segment joining the points (–4, 0) and (0, 6) in three equal parts. **[Delhi-2005C]**

23. Two vertices of Δ ABC are given by A(2, 3) and B(–2, 1) and its centroid is G $\left(1, \frac{2}{3}\right)$. Find the coordinates of the third vertex C of the Δ ABC. **[Al-2005]**

24. Show that the points A(1, 2), B(5, 4), C(3, 8) and D(–1, 6) are the vertices of a square.

OR

Find the co-ordinates of the point equidistant from three given points A(5, 1), B(–3, –7) and C(7,–1) **[Delhi-2006]**

25. Find the value of p for which the points (–1, 3), (2, p) and (5, –1) are collinear. **[Delhi-2006]**

26. If the points (10, 5), (8,4) and (6,6) are the mid. Points of the sides of a triangle, find its vertices. **[Foreign-2006]**

27. In what ratio is the line segment joining the points (–2, –3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

OR

If A (5, –1), B (–3, –2) and C (–1, 8) are the vertices of triangle ABC, find the length of median through A and the coordinates of the centroid. **[Delhi-2006C]**

28. If (–2, –1); (a, 0); (4, b) and C (1, 2) are the vertices of a parallelogram, find the values of a and b. **[Al-2006C]**

29. Show that the points (7, 10), (–2, 5) and (3, –4) are the vertices of an isosceles right triangle. **[Delhi-2007]**

30. In what ratio does the lines x – y – 2 = 0 divides the line segment joining (3, –1) and (8, 9) ? **[Delhi-2007]**

31. Three consecutive vertices of a parallelogram are (–2, 1); (1, 0) and (4, 3). Find the coordinates of the fourth vertex. **[Al-2007]**

32. If the point C(–1, 2) divides the line segment AB in the ratio 3 : 4 where the coordinates of A are (2, 5), find the coordinates of B. **[Al-2007]**

33. For what value of p, are the points (2, 1), (p, –1) and (–1, 3) collinear? **[Delhi-2008]**

34. Determine the ratio in which the line 3x + 4y – 9 = 0 divides joining the points (1, 3) and (2, 7). **[Delhi-2008]**

35. If the distances of P(x, y) from the points A(3, 6) and B(–3, 4) are equal, prove that 3x + y = 5. **[Delhi-2008]**

36. For what value of p, the points (–5, 1), (1, p) and (4, –2) are collinear? **[Delhi-2008]**

37. For what value of k, are the points (1, 1), (3, k) and (–1, 4) are collinear? **[Delhi-2008]**

OR

Find the area of the Δ ABC with vertices A(–5, 7), B(–4, –5) and C(4, 5) **[Al-2008]**

38. If the point P(x, y) is equidistant from the points A(3, 6) and B(–3, 4) prove that 3x + y – 5 = 0. **[Al-2008]**

39. The point R divides the line segment AB, where A(–4, 0) and B(0, 6) such that AR = $\frac{2}{3}$ AB. Find the co-ordinates of R. **[Al-2008]**

40. The co-ordinates of A and B are (1, 2) and (2, 3) respectively. If P lies on AB find co-ordinates of P such that $\frac{AP}{PB}=\frac{3}{4}$. **[Al-2008]**

41. If A(4, – 8), B(3, 6) and C(5, – 4) are the vertices of a Δ ABC, D is the mid point of BC and P is a point on AD joining such that $\frac{AP}{PD}=2$, find the co-ordinates of P. **[Al-2008]**

42. Find the value of k if the points (k, 3), (6, – 2) and (– 3, 4) are collinear. **[Foreign-2008]**

43. If P divides the join of A(–2, –2) and B(2, –4) such that $\frac{AP}{AB}=\frac{3}{7}$, find the co-ordinates of P. **[Foreign-2008]**

44. The mid points of the sides of a triangle are (3, 4), (4, 6) and (5, 7) . Find the co-ordinates of the vertices the triangle. **[Foreign-2008]**

45. Show that A(–3, 2), B(–5, –5), C(2, –3) and D(4, 4) are the vertices of a rhombus. **[Foreign-2008]**

46. Find the ratio in which the line 3x + y – 9 = 0 divides the line-segment joining the points (1, 3) and (2, 7). **[Foreign-2008]**

47. Find the distance between the points $\left(\frac{-8}{5},2\right)$ *and* $\left(\frac{2}{5},2\right)$. **[Delhi-2009]**

48. Find the point on y-axis which is equidistant from the points (5, –2) and (–3, 2).

OR

The line segment joining the points A(2, 1) and B(5, –8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by 2x – y + k = 0, find the value of k. **[Delhi-2009]**

49. If P (x, y) is any point on the line joining the points A(a, 0) and B(0, b), then show that $\frac{x}{a}+\frac{y}{b}=1$. **[Delhi-2009]**

50. Find the point on x-axis which is equidistant from the points (2, – 5) and (– 2, 9) **[Delhi-2009]**

OR

The line segment joining the points P(3, 3), Q(6, – 6) is trisected at the points A and B such that A is nearer to P. If A also lies on the line given by 2x + y + k = 0, find the value of k.

51. If the points A P(4, 3) and B(x, 5) are on the circle with the centre O (2, 3), find the value of x. **[Al-2009]**

52. Find the ratio in which the point (2, y) divides the line segment joining the points A(–2, 2) and B(3, 7). Also find the value of y. **[Al-2009]**

53. Find the area of the quadrilateral ABCD whose vertices are A(–4, –2), B(–3, –5), C(3, –2) and D(2,3). **[Al-2009]**

54. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, – 1), (2, 1) and (0, 3). **[Al-2009]**

55. If the mid-point of the line segment joining the points P(6, b – 2) and Q(2, – 3), find the value of b. **[Foreign-2009]**

56. Show that the points (–2, 5), (3, –4) and (7, 10) are the vertices of a right angled isosceles triangle.

OR

The centre of a circle is $(2\alpha-1,7)$ and it passes through the point (–3, – 1). If the diameter of the circle is 20 units, then find the value(s) of α. **[Foreign-2009]**

57. If C is a point lying on the line segment AB joining A(1, 1) and B(2, – 3) such that 3AC = CB, then find the co-ordinates of C. **[Foreign-2009]**

58. Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear. **[Foreign-2009]**

59. If the points (–2, 1), (a, b) and (4, – 1) are collinear and a – b = 1, then find the values of a and b. **[Foreign-2009]**

60. Find the value of K, if the points A(7, – 2), B(5, 1) and C(3, 2K) are collinear. **[Al-2010]**

61. Find the value of K, if the points A(8, 1), B(3, – 4) and C(2, K) are collinear. **[Al-2010]**

62. Point P divides the line segment joining the points A(–1, 3) and B(9, 8) such that $\frac{AP}{PB} = \frac{K}{1}$. If P lies on the line x – y + 2 = 0, find the value of K. **[Al-2010]**

63. If the points (p, q), (m, n) and (p – m, q – n) are collinear, show that pn = qm. **[Al-2010]**

CO-ORDINATE GEOMETRY CBSE | ANSWER KEY | EXERCISE-2 (X)-

SHORT ANSWER TYPE

2. 3 : 2, – 2/5 or (4, –4) **5.** 3 : 2, 5 **6.** (–2, 0) **8.** 2/7 **9.** 3 **11.** p = 4, q = 2

12. (–4, –7) or A $(a/2, \sqrt{3a}/2)$, B(0, 0), C(a, 0) **13.** 4 : 7 or (3, 2) **14.** 2 **15.** 5 or – 3 or (3, 0)

16. $p = \frac{7}{3}$, q = 0 **18.** k = – 8 **19.** k = 1 **20.** $(\pm 3\sqrt{3}, 0)$ and (0, 3) **21.** (11, 2) **22.** $\left(\frac{-8}{3}, 2\right), \left(\frac{-4}{3}, 4\right)$

23. (3, –2) **24.** (2, –4) **25.** p = 1 **26.** (4, 5), (8, 7), (12, 3) **27.** 2 : 3, (0, 1) or $\sqrt{65}, \left(\frac{1}{3}, \frac{5}{3}\right)$

28. a = 1 , b = 3 **30.** 2 : 3 **31.** (1 , 2) **32.** (–5, –2) **33.** p = 5 **34.** 6 : 25 **36.** – 1

37. – 2 or 53 sq. units **39.** $\left(-1, \frac{9}{2}\right)$ **40.** $\left(\frac{11}{7}, \frac{18}{7}\right)$ **41.** (4, – 2) **42.** $k = -\frac{3}{2}$ **43.** $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

44. (4, 5), (2, 3), (6, 9) **46.** 3 : 4 **47.** 2 **48.** (0, – 2) or – 8 **50.** (–7, 0) or – 8 **51.** 2 **52.** 4 : 1, 6

53. 28 sq. units **54.** 1 sq. units **55.** – 8 **56.** – 4 or 2 **57.** $\left(\frac{5}{4}, 0\right)$ **58.** x + 3y = 7 **59.** a = 1, b = 0

60. 2 **61.** – 5 **62.** $\frac{2}{3}$

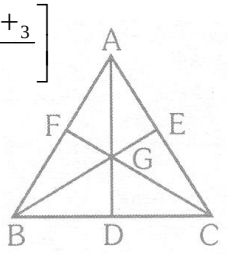
COMPETITION WINDOW

PROPERTIES OF TRIANGLES

Let A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) be the vertices of any Δ ABC, then

1. **Centroid :** It is the point of intersection of the cedians. It divides the median in the ratio of 2 : 1.

Co-ordinates of centroid : G $\left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + _3}{3}\right]$



2. **Incentre :** It is the point of intersection of internal bisectors of the angle. Also it is the centre of the circle touching all the sides of a triangle.

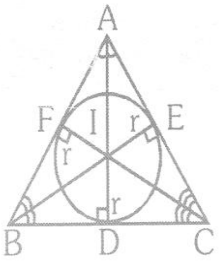
Co-ordinates of incentre $\left[\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right]$. Where a, b, c are the lengths of the sides of triangle.

The radius of incircle.

$$r = \frac{\Delta}{s}$$

Where Δ is the area of Triangle and

$$S = \frac{a+b+c}{2}$$

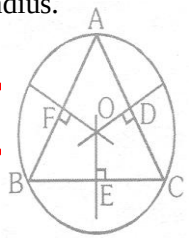


Remark : An angle bisector of a triangle divides the opposite side in the ratio of remaining sides.

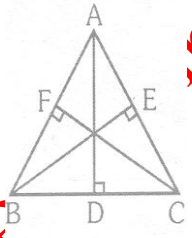
E.g. AD divides BC in the ratio $\frac{BD}{DC} = \frac{AB}{AC}$.

3. **Circumcentre :** It is the point of intersection of perpendicular bisectors of the sides of a triangle. It is also the centre of the circle passing through the vertices of a triangle.

If O is the circumcentre of a triangle ABC, then OA = OB = OC = circumradius.



4. **Orthocentre :** It is the point of intersection of altitudes of a triangle.



Remark :

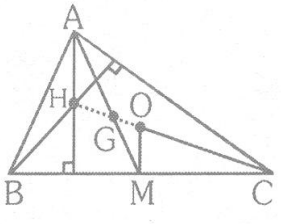
(i) In an equilateral triangle, the centroid, incentre, orthocenter and circumcentre coincide.

(ii) In an isosceles triangle, the centroid, incentre, orthocenter and circumcentre are collinear.

(iii) In a right angled triangle, the circumcentre is the mid point of hypotenuse and the orthocenter is the point where right angle is formed.

(iv) **Euler line :** The circumcentre O, the centroid G and the orthocenter H of a triangle are collinear, the line on which they lie is called Euler line. Also G divides HO in the ratio 2 : 1.

$$\frac{OG}{GH} = \frac{1}{2}$$



TRY OUT THE FOLLOWING

1. Two vertices of a triangle are (– 1, 6) are (5, 2). If its centroid is (0, – 3), find the third vertex.

2. If $\left(\frac{3}{2},0\right), \left(\frac{3}{2},6\right)$ and (–1, 6) are mid-points of the sides of a triangle, then find

(i) Centroid of the triangle (ii) In centre of the triangle

3. If a triangle has it's orthocenter at (1, 1) and circumcentre at $\left(\frac{3}{2},\frac{3}{4}\right)$ then, find the centroid.

ANSWERS

1. (–4, –15) **2.** (i) (2/3, 4) (ii) (1, 2) **3.** (4/3, 5/6)

COMPETITION WINDOW

CONDITIONS FOR A TRIANGLE TO BE ACUTE, OBTUSE OR RIGHT ANGLED

For an acute angled triangle, $a^2+b^2>c^2, b^2+c^2>a^2$ *and* $a^2+b^2>c^2$.

For an obtuse angled triangle, $a^2+b^2>c^2$ (if $\angle$C is obtuse)
$a^2+c^2>b^2$ (if $\angle$B is obtuse)
$b^2+c^2>a^2$ (if $\angle$A is obtuse)

For a right angled triangle, $a^2+b^2=c^2$ (if $\angle C = 90^0$)
$b^2+c^2=a^2$ (if $\angle A = 90^0$)
$a^2+c^2=b^2$ (if $\angle B = 90^0$)

Where a, b and c have their usual meanings.

COMPETITION WINDOW

CONCURRENCY OF LINES

Three lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ are said to be concurrent (lines passing through the same point) if $a_1(b_2c_3-b_3c_2)+a_2(b_3c_1-b_1c_3)+a_3(b_1c_2-b_2c_1)=0$

TRY OUT THE FOLLOWING

1. Prove that the lines 3x + y – 14 = 0, x – 2y = 0 and 3x – 8y + 4 = 0 are concurrent.
2. Show that the lines x – y – 6 = 0, 4x – 3y – 20 = 0 and 6x + 5y + 8 = 0 are concurrent. Also find their common point of intersection.
3. Find the value of λ, if the lines 3x – 4y – 13 = 0, 8x – 11y – 33 = 0 and 2x – 3y + λ = 0 are concurrent.

ANSWER

2. (2, – 4) **3.** $\lambda = -7$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

CHOOSE THE CORRECT ONE

1. The circumcentre of the triangle formed by the lines xy + 2x + 2y + 4 = 0 and x + y + 2 = 0 is :
(A) (– 1, – 2) (B) (– 1, – 1) (C) (– 2, – 2) (D) (0, 0)

2. The vertices of a triangle (a, b – c), (b, c – a) and (c, a – b), then it's centroid lies on :
(A) y-axis (B) x-axis (C) x = 0 (D) None of these

3. The points (1, 2), (3, 8) and (x, 20) are collinear if x =
(A) 4 (B) 5 (C) 6 (D) 7

4. For the triangle whose sides are along the lines x = 0 , y = 0 and $\frac{x}{6}+\frac{y}{8}=1$, the incentre is :
(A) (3, 4) (B) (2, 2) (C) (2, 3) (D) (3, 2)

5. For the triangle whose sides are along the lines y = 15 , 3x – 4y = 0, 5x + 12y = 0, the incentre is :
(A) (1, 8) (B) (– 1, 8) (C) (8, 1) (D) None of these

6. The points D(2, 1), E(–1, –2) and F(3, 3) are the mid points of sides BC, CA and AB respectively of a ΔABC. The vertices A, B and C are :
(A) (0, 0), (6, 6), (– 2, – 4) (B) (0, 1), (6, 6), (2, 4)
(C) (1, 0), (3, 3), (– 2, – 4) (D) None of these

7. The number of integral values of m, for which x-coordinate of the point of intersection of the lines 3x + 4y = 9 and y = mx + 1 is also an integer, is :

(A) 2 (B) 0 (C) 4 (D) 1

8. The radius of the circle inscribed in the triangle formed by lines x = 0, y = 0, 4x + 3y – 24 = 0 is :

(A) 12 (B) 2 (C) $2\sqrt{2}$ (D) 6

9. In a Δ ABC, if A is the point (1, 2) and equations of the median through B and C are respectively x + y = 5 and x = 4, then B is :

(A) (1, 4) (B) (7, – 2) (C) (4, 1) (D) (– 2, 7)

10. The straight line 3x + y = 9 divides the segment joining the points (1, 3) and (2, 7) in the ratio :

(A) 4 : 3 (B) 3 : 4 (C) 4 : 5 (D) 5 : 6

11. Two opposite vertices of a rectangle are (1, 3) and (5, 1). If the equation of a diagonal this rectangle is y = 2x + c, then the value of c is :

(A) – 4 (B) 1 (C) – 9 (D) None of these

12. The radius of the circle passing through the point (6, 2) two of whose diameters are x + y = 6 and x + 2y = 4 is :

(A) 10 (B) $2\sqrt{5}$ (C) 6 (D) 4

13. The straight lines x + y = 0, 3x + y – 4 = 0, x + 3y – 4 = 0 form a triangle which is :

(A) Isosceles (B) Equilateral (C) Right angled (D) None of these

14. The lines segment joining the points (1, 2) and (–2, 1) is divided by the line 3x + 4y = 7 in the ratio :

(A) 3 : 4 (B) 4 : 3 (C) 9 : 4 (D) 4 : 9

15. If a, b, c are in A. P. then the straight line ax + by + c = 0 will always pass through a fixed point whose co-ordinates are :

(A) (1, – 2) (B) (– 1, 2) (C) (1, 2) (D) (– 1, – 2)

16. The lines 8x + 4y = 1, 8x + 4y = 5, 4x + 8y = 3, 4x + 8y = 7 from a :

(A) Rhombus (B) Rectangle (C) Square (D) None of these

17. The incentre of the triangle formed by the lines y = 15, 12y = 5x and 3x + 4y = 0 is :

(A) (8, 1) (B) (– 1, 8) (C) (1, 8) (D) None of these

18. The area of triangle formed by the lines y = x, y = 2x and y = 3x + 4 is :

(A) 4 (B) 7 (C) 9 (D) 8

19. The triangle formed by the lines x + y = 1, 2x + 3y – 6 = 0 and 4x – y + 4 = 0 lies in the :

(A) First quadrant (B) Second quadrant (C) Third quadrant (D) Fourth quadrant

20. A line is drawn through the points (3, 4) and (5, 6). If the line is extended to a point whose ordinate is – 1, then the abscissa of that point is ;

(A) 0 (B) – 2 (C) 1 (D) 2

21. The area of the triangle whose sides are along the lines x = 0, y = 0 and 4x + 5y = 20 is :

(A) 20 (B) 10 (C) $\frac{1}{10}$ (D) $\frac{1}{20}$

22. If a, b, c are all distend, then the equations (b – c) x + (c – a) y + a – b = 0 and $(b^3 - c^3)$ x + $(c^3 - a^3)$ y + $a^3 - b^3$ = 0 represent the same line if :

(A) a + b + c $\neq$ 0 (B) a + b c = 0

(C) a + b = 0 or b + c = 0 (D) None of these

23. The area of the quadrilateral with vertices at (4, 3), (2, – 1), (– 1, 2), (– 3, – 2) is :

(A) 18 (B) 36 (C) 54 (D) None of these

24. If α, β, γ are the real roots of the equation $x^3 - 3px^2 - 1 = 0$, then the centroid of the triangle with vertices $\left(\alpha, \frac{1}{\alpha}\right), \left(\beta, \frac{1}{\beta}\right)$ and $\left(\gamma, \frac{1}{\gamma}\right)$ is at the point :

(A) (p, q) (B) (p/3, q/3) (C) (p + q, p – q) (D) (3p, 3q)

25. The co-ordinates of A, B C are (6, 3), (– 3, 5), (4, – 2) respectively and P is any point (x , y). the ratio of the areas of Δ ABC and Δ ABC is :

(A) $\left|\frac{x-y-2}{7}\right|$ (B) $\left|\frac{x+y-2}{7}\right|$ (C) $\left|\frac{x+y+2}{7}\right|$ (D) None of these

26. The area of a triangle is 5 square units. Two of its vertices are (2, 1) and (3, – 2). The third vertex lie on y = x + 3, the third vertex is :

(A) $\left(\frac{7}{2}, \frac{13}{2}\right) or \left(\frac{-3}{2}, \frac{3}{2}\right)$ (B) $\left(\frac{7}{2}, \frac{-13}{2}\right) or \left(\frac{-3}{2}, \frac{3}{2}\right)$

(C) $\left(\frac{7}{2},\frac{13}{2}\right) or \left(\frac{3}{2},\frac{3}{2}\right)$ (D) None of these

27. The point of intersection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$, lies on the line :

(A) x – y = 0 (B) $x+y=\frac{2ab}{a+b}$ (C) $x-y\frac{2ab}{a+b}$ (D) Both (A) and (B)

28. The point A divides the join of the points (–5, 1) and (3, 5) in the ratio k : 1 and co-ordinates of points B and C are (1, 5) and (7, – 2) respectively. If the area of Δ ABC be 2 units, then k equals :

(A) 7, 9 (B) 6, 7 (C) $7,\frac{31}{9}$ (D) $9,\frac{31}{9}$

29. Q, R and S are the points on the line joining the points P(a, x) and T(b, y) such that PQ = QR = RS = ST, then $\left(\frac{5a+3b}{8},\frac{5x+3y}{8}\right)$ is the mid point of the segment :

(A) PQ (B) QR (C) RS (D) ST

30. The triangle with vertices A(2, 7), B(4, y) and C(–2, 6) is right angled at A if :

(A) y = – 1 (B) y = 0 (C) y = 1 (D) None of these

31. The co-ordinates with of the point which divides the line segment joining (– 3, – 4) and (– 8, 7) externally in the ratio 7 : 5 are :

(A) $\left(\frac{41}{2},\frac{69}{2}\right)$ (B) $\left(\frac{-41}{2},\frac{-69}{2}\right)$ (C) $\left(\frac{-41}{2},\frac{69}{2}\right)$ (D) None of these

32. The distance of the centroid from the orign of the friangle formed by the points (1, 1), (0, – 7) and (– 4, 0) is :

(A) $\sqrt{2}$ (B) $\sqrt{4}$ (C) $\sqrt{3}$ (D) $\sqrt{5}$

33. If A(4, – 3), B(3, – 2) and C(2, 8) are vertices of a triangle, then the distance of it's centroid from the y-axis is :

(A) $\frac{1}{2}$ (B) 1 (C) 3 (D) $\frac{1}{2}$

34. If (5, – 4) and (– 3, 2) are two opposite vertices of a square, then it's area is :

(A) 50 (B) 75 (C) 25 (D) 100

35. A(6, 3), B(–3, 5), C(4, – 2) and (x, 3x) are four points. If the areas of Δ DBC and Δ ABC are in the ratio 1 : 2, then x is equal to :

(A) $\frac{11}{8}$ (B) 3 (C) $\frac{8}{11}$ (D) None of these

36. An equilateral triangle whose circumcentre is (–2, 5), one side is on y-axis, then length of side of the triangle is :

(A) 6 (B) $2\sqrt{3}$ (C) $4\sqrt{3}$ (D) 4

37. A(3, 4), and B(5, – 2) are two given points. If PA = PB and area of Δ PAB = 10. then P is :

(A) (7, 1) (B) (7, 2) (C) (– 7, 2) (D) (– 7, – 1)

38. The distance between foot of perpendiculars drawn from a point (– 3, 4) on both axes is :

39. Point P divides the line segment joining A(– 5, 1) and B(3, 5) internally in the ratio λ : 1. If Q = (1, 5), R = (7, – 2) and area of Δ PQR = 2, then λ equals :

(A) 23 (B) $\frac{29}{5}$ (C) $\frac{31}{9}$ (D) None of these

40. The area of an equilateral triangle whose two vertices are (1, 0) and (3, 0) and third vertex lying in the first quadrant is :

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) None of these

41. ABC is an isosceles triangle. If the co-ordinates of the base are B(1, 3) and C(–2, 7), the co-ordinates of vertex A is

(A) $\left(\frac{-1}{2}, 5\right)$ (B) (1, 6) (C) $\left(\frac{5}{6}, 6\right)$ (D) None of these

42. The area of the quadrilateral formed by the points $(a^2 + 2ab, b^2)$, $(a^2 + b^2, 2ab)$, $(a^2, b^2 + 2ab)$ and (a2 + b2 – 2ab, 4ab) is :
(A) Zero (B) $(a + b)^2$ (C) $a^2 + b^2$ (D) $(a - b)^2$

OBJECTIVE						ANSWER KEY								EXERCISE-4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	B	B	D	C	C	A	A	B	B	B	A	B	A	D	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	A	C	A	B	B	B	B	A	A	B	A	D	C	B	A
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**			
Ans.	C	D	C	A	A	C	B	A	C	C	C	A			

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

CHOOSE THE COREECT ONE

1. The lines ax + 2y + 1 = 0, bx + 3y + 1 = 0 and cx + 4y + 1 = 0 are concurrent, then :
(A) a, b, c are in A.P. (B) a, b, c are in G.p. (C) a, b, c are in H.P. (D) None of these

2. If the lines x + 2ay + a = 0, x + 3by + b = 0 and x 4cy + c = 0 are concurrent, then a, b, c are in ($abc \neq 0$) :
(A) A.P. (B) G.P. (C) H.P. (D) None of these

3. If (0, β) lies on or inside the triangle formed by the lines 3x + y + 2 = 0, 3y – 2x – 5 = 0 and 4y + x – 14 = 0 then :
(A) $\frac{5}{2} \leq \beta \leq \frac{7}{3}$ (B) $\frac{5}{3} \leq \beta \leq \frac{7}{2}$ (C) $\frac{7}{3} \leq \beta \leq \frac{5}{2}$ (D) None of these

4. If a, x_1, x_2 are in G.P. with common ratio r1 and b, y_1, y_2 are in G.P. with common ratio s where s – r = 2, then the area of the triangle with vertices (a, b), (x_1, y_1) and (x_2, y_2) is :
(A) $|ab(r^2 - 1)|$ (B) ab $(r^2 - s^2)$ (C) ab $(s^2 - 1)$ (D) abrs

5. If the circumcentre of a triangle lies at the origin and the centroid is the middle point of the line joining the points $(a^2 + 1, a^2 + {}^1)$ and (2a, – 2a), then the co-ordinates of the orthocenter are :

(A) $\left[\frac{(a+1)^2}{4}, \frac{(a-1)^2}{4}\right]$ (B) $\left[\frac{3}{4}(a+1)^2, \frac{3}{4}(a-1)^2\right]$

(C) $(3(a + 1)^2, 3 (a - 1)^2)$ (D) None of these

6. If every point on the line $(a_1 - a_2)$ x + $(b_1 - b_2)$ y = c is equidistant from the points (a_1, b_1) and (a_2, b_2) then 2c =
(A) $a_1^2 - b_1^2 + a_2^2 + b_2^2$ (B) $a_1^2 + b_1^2 + a_2^2 + b_2^2$ (C) $a_1^2 - b_1^2 - a_2^2 - b_2^2$ (D) None of these

7. A rectangle has two opposite vertices at the points (1, 2) and (5, 5). If the other vertices lie on the line x = 3, the co-ordinates of the vertex nearer the axis of x are :
(A) 3, 1 (B) (3, 2) (C) (3, 4) (D) (3, 6)

8. If the area of the triangle formed by the pair of lines $8x^2 - 6y^2 + y^2 = 0$ and the line 2x + 3y = a is 7, then a is equal
(A) 14 (B) $14\sqrt{2}$ (C) 28 (D) None of these

9. If the centroid of the triangle formed by the pair of lines $2y^2 + 5xy - 3x^2 = 0$ and x + y = k is $\left(\frac{1}{18}, \frac{11}{18}\right)$, then the value of k is :
(A) – 1 (B) 0 (C) 1 (D) None of these

10. If x_1, x_2, x_3 are the abscissa of the points A_1, A_2, A_3 respectively where the lines $y = m_1 x$, $y = m_2 x$, $y = m_3 x$ meet the line $2x - y + 3 = 0$ such that m_1, m_2, m_3, are in A.P., then x_1, x_2, x_3 are in :

(A) A.P. (B) G.P. (C) H.P. (D) None of these

11. The area of the triangle with vertices $\left(1, \frac{\pi}{8}\right), \left(1, \frac{5\pi}{8}\right)$ and $\left(\sqrt{2}\, \frac{3\pi}{8}\right)$ is :

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

12. An equilateral triangle whose orthocenter is $(3, -2)$, one side is on x-axis then vertex of triangle which is not on x-axis is :

(A) $(3, -6)$ (B) $(1, -2)$ (C) $(9, -2)$ (D) $(3, -3)$

13. If O is the origin and the co-ordinates of A and B are (x_1, y_1) and (x_2, y_2) respectively then $OA \times OB \cos \angle AOB$ is equal to :

(A) $x_1y_1 + x_2y_2$ (B) $x_1x_2 + y_1y_2$ (C) $x_1y_2 + x_2y_1$ (D) $x_1x_2 - y_1y_2$

14. If the vertices of a triangle have integral co-ordinates, then the triangle is :

(A) Isosceles (B) Never equilateral (C) Equilateral (D) None of these

15. The circumcentre of the triangle formed by the points $(a\cos\alpha, a\sin\alpha)$, $(a\cos\beta, \sin\beta)$, $(a\cos\gamma, \sin\gamma)$ is

(A) (0, 0) (B) $\left[\left(\frac{a}{3}\right)(\cos\alpha + \cos\beta + \cos\gamma), \left(\frac{a}{3}\right)(\sin\alpha + \sin\beta + \sin\gamma)\right]$

(C) (a, 0) (D) None of these

16. The x co-ordinates of the incentre of the triangle where the mid point of the sides are (0, 1), (1, 1) and (1, 0) is

(A) $2+\sqrt{2}$ (B) $1+\sqrt{2}$ (C) $2-\sqrt{2}$ (D) $1+\sqrt{2}$

17. OPQR is a square and M and N are the mid points of the sides PQ and QR respectively, then ratio of area of square and the triangle OMN is :

(A) 4 : 1 (B) 2 : 1 (C) 8 : 3 (D) 4 : 3

18. The point with co-ordinates (2a, 3a), (3b, 2b) and (c, c) are collinear :

(A) For no value of a, b, c (B) For all value of a, b, c

(C) If a, $\frac{c}{5}$, b are in H.P. (D) If a, $\frac{2c}{5}$, b are in H.P.

19. If co-ordinates of orthocenter and centroid of a triangle are (4, – 1) and (2, 1), then co-ordinates of a point which is equidistant from the vertices of the triangle is :

(A) (2, 2) (B) (3, 2) (C) (2, 3) (D) None of these

20. If the line $y = mx$ meets the lines $x + 2y - 1 = 0$ and $2x - y + 3 = 0$ at the number of points having integral to :

(A) – 2 (B) 2 (C) – 1 (D) 1

21. A triangle is formed by the point O(0, 0), A(0, 21) and B(21, 0). The number of points having integral co-ordinates (both x and y) and strictly inside the triangle is :

(A) 190 (B) 305 (C) 181 (D) 206

22. The straight lines $5x + 4y = 0$, $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ are :

(A) Concurrent (B) The sides of an equilateral triangle

(C) The sides of a right angled triangle (D) None of these

23. A(a, b), B(x_1, y_1) and C(x_2, y_2) are the vertices of a triangle. If a, x_1, x_2 are in G.P. with common ratio r and b, y_1, y_2 are in G.P. with common ratio s, then area of ΔABC is :

(A) ab (r – 1) (s – 1) (s – r) (B) $\frac{1}{2}$ ab (r + 1) (s + 1) (s – r)

(C) $\frac{1}{2}$ (r – 1) (s – 1) (s – r) (D) ab (r + 1) (s + 1) (s – r)

24. If a, b, c are in G.P., then the line $a^2x + b^2y + ac = 0$, will always pass through the fixed point.
(A) (0, 1) (B) (1, 0) (C) (0, – 1) (D) (1, – 1)

25. The lies $\ell x + my + n = 0$, $mx + ny + \ell = 0$ are concurrent if :

(A) $\ell + m n = 0$ (B) $\ell + m - n = 0$

(C) $\ell - m + n = 0$ (D) $\ell^2 + m^2 + n^2 \neq \ell m + mn + n\ell$

26. The sides of a triangle are 3x + 4y, 4x + 3y and 5x + 5y units where x > 0, y > 0. The triangle
(A) Right angled (B) Acute angled (C) Obtuse angled (D) Isosceles

27. The lines x + 2y – 3 = 0, 2x + y – 3 = 0 and the line ℓ are concurrent. If the line ℓ passes through the origin, then its equation is :

(A) x – y = 0 (B) x + y = 0 (C) x + 2y = 0 (D) 2x + y = 0

28. Angles of the triangle formed by the lines $x^2 - y^2 = 0$, x = 7 are :
(A) $45^0, 90^0, 45^0$ (B) $30^0, 60^0, 90^0$ (C) $60^0, 60^0, 60^0$ (D) None of these

29. If the orthocenter and centroid of a triangle are (– 3, 5) and (3, 3) then it's circumcentre is :
(A) (6, 2) (B) (3, – 1) (C) (– 3, 5) (D) (– 3, 1)

30. A triangle with vertices (4, 0), (– 1, – 1), (3, 5) is : **[AIEEE-2002]**

(A) Isosceles and right angled (B) Isosceles but not right angled

(C) right angled but not isosceles (D) Neither right angled nor isosceles

31. The centroid of a triangle is (2, 3) and two of it's vertices are (5, 6) and (– 3, 4). The third vertex of the triangle is : **[AIEEE-2002]**

(A) (2, 1) (B) (2, – 1) (C) (1, 2) (D) (1, – 2)

32. If a vertex of a triangle is (1, 1) and the mid-points of two sides through this vertex are (– 1, 2) and (3, 2), then the centroid of the triangle is : **[AIEEE-2005]**

(A) $\left(-1\frac{7}{3}\right)$ (B) $\left(\frac{-1}{3}, \frac{7}{3}\right)$ (C) $\left(1, \frac{7}{3}\right)$ (D) $\left(\frac{1}{3}, \frac{7}{3}\right)$

33. If non zero numbers a, b, c are in H.P. then the straight the $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point.
That point is : **[AIEEE-2005]**

(A) (1, – 2) (B) (1, – 1/2) (C) (– 1, 2) (D) (– 1, – 2)

34. The line parallel to x-axis passing through the intersection of the lines ax + 2by + 3b = 0 and bx – 2ay – 3a = 0 where (a, b) $\neq$ (0, 0) is : **[AIEEE-2005]**

(A) Above x-axis at a distance 3/2 from it (B) Above x-axis at a distance 2/3 from it

(C) Below x-axis at a distance 3/2 from it (D) Below x-axis at a distance 2/3 from it

35. Let A(h, k), B(1, 1) and C(2, 1) be the vertex of a right angled triangle with AC it's hypotenuse. If the area of the triangle is 1, then the set of value which 'k' can take is given by : **[AIEEE-2007]**

(A) {1, 3} (B) {0, 2} (C) {– 1, 3} (D) {– 3, – 2}

36. The orthocenter of the right triangle with vertices $\left[2,\frac{(\sqrt{3}-1)}{2}\right],\left(\frac{1}{2},-\frac{1}{2}\right)$ and $\left(2,-\frac{1}{2}\right)$ is : **[II T-1993]**

(A) $\left[\frac{3}{2},\frac{\sqrt{3}-3}{6}\right]$ (B) $\left[2,-\frac{1}{2}\right]$ (C) $\left[\frac{5}{4},-\frac{\sqrt{3}-2}{4}\right]$ (D) $\left[\frac{1}{2},\frac{1}{2}\right]$

37. The orthocenter of the triangle formed by the lines xy = 0 and x + y = 1 is : **[II T-1995]**

(A) $\left(\frac{1}{2},\frac{1}{2}\right)$ (B) $\left(\frac{1}{3},\frac{1}{3}\right)$ (C) (0, 0) (D) $\left(\frac{1}{4},\frac{1}{4}\right)$

38. If the vertices P,Q,R of a triangle PQR are rational points, which of the following points of the triangle PQR is (are) not always rational points(s) ? **[II T-1998]**

(A) Centroid (B) Incentre (C) Circumcentre (D) None of these

39. If P(1, 2), Q(4, 6), R(5, 7) and S(a, b) are the vertex of a parallelogram PQRS, then : **[II T-1998]**

(A) a = 2, b = 4 (B) a = 3, b = 4 (C) a = 2, b = 3 (D) a = 3, b = 5

40. The incentre of the triangle with vertex $(1,\sqrt{3})$, (0, 0) and (2, 0) is : **[II T-2000, AIEEE-2002]**

(A) $\left(1,\frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{2}{3},\frac{1}{\sqrt{3}}\right)$ (C) $\left(\frac{2}{3},\frac{\sqrt{3}}{2}\right)$ (D) $\left(1,\frac{1}{\sqrt{3}}\right)$

41. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are : **[II T-2007]**

(A) $\left(\frac{4}{3},3\right)$ (B) $\left(3,\frac{2}{3}\right)$ (C) $\left(3,\frac{4}{3}\right)$ (D) $\left(\frac{4}{3},\frac{2}{3}\right)$

OBJECTIVE **ANSWER KEY**

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	C	B	A	D	C	A	C	C	C	B	A	B	B	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	C	D	D	C	A	A	C	C	A	C	A	A	A	A
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**				
Ans.	B	C	A	C	C	B	C	B	C	D	C				

PROBABILITY

★ INTRODUCTION

In our day-to-day conveersation, we generally use the phrases like :

(i) **Probably,** Satya will visit my house today

(ii) **Most probably,** Megha is preparing for CAT.

(iii) Khusboo is **quite sure** to be on the top.

(iv) **Chances** are high that Regi will head the organization,

The words' probably', 'most probably', 'quite sure', 'chances' etc involve an element of uncertainty.

Probability – Probability is the mathematical measurement of uncertainty.

Probability Theory – it is that branch of mathematics in which the degree of uncertainty (or certainty of occurrence of event) is measured numerically.

★ SOME BASIC CONCEPTS/TERMS

1. **Experiment :** An action or operation which can produce some well defined result is known as **experiment.**

2. **Deterministic experiment :** If we perform an experiment and repeat it under identical conditions, we get almost the same result every time, such an experiment is called a **deterministic experiment.**

3. **Random experiment :** An experiment is said to be a random experiment if it satisfies the following two conditions :

(i) It has more than one possible outcomes.

(ii) It is not possible to predict the outcome (result) in advance.

Ex. (i) Tossing a pair of fair coins. (ii) Rolling an unbiased die.

4. **Outcomes :** The possible results of a random experiment are called **outcomes.**

5. **Trial :** When an experiment is repeated under similar conditions and it does not give the same result each time but may result in any one of the several possible outcomes, the result is called a **trial.**

Ex. If a coin is tossed 100 times, then one toss of the coin is called a trial.

6. **Event :** The collection of all or some outcomes of a random experiment is called an **event.**

Ex. Suppose we toss a pair of coins simultaneously and let E be the event of getting exactly one head. Then, the event E contains HT and TH.

Ex. Suppose we roll a die and let E be the event of getting an even number. Then the event E contains 2, 4 and 6.

7. **Elementary or Simple Event :** An outcome of a trial is called an **elementary event.**

NOTE : An elementary event has only one element.

Ex. Let a pair of coins is tossed simultaneously. Then, possible outcomes of this experiment are.

HH : Getting H on first H on second (= E_1) [H = Head, T = Tail and E = event]

HT : Getting H on first T on second (= E_2)

TH : Getting T on first H on second (= E_3)

and TT : Getting T on first T on second (= E_4)

Here, E_1, E_2, E_3 and E_4 are the element events associated with the random experiment of tossing of two coins.

8. **Compound event or composite event or mixed event :** An event associated to a random experiment and obtained by combining two or more simple events associated to the same random experiment, is called a **compound event .**

OR

A compound event is an aggregate of some simple (elementary) event and is decomposable into simple events.

Ex. If we throw a die, then the event E of getting an odd number is a compound event because the event E contains three elements 1, 3 and 5, which is a compound of three simple events E_1, E_2, and E_3 containing 1, 3 and 5 respectively.

9. **Equally likely events :** The out comes of an experiment are said to be equally likely events if the chances of their happenings are neither less nor greater then other.

In other words, a given number o events are said to be equally likely if none of them is experiment to occur in preference to the others.

Ex. In tossing a coin, getting head (H) and tail (T) are equally likely events.

★ EXPERIMENTAL (OR EMPIRICAL) PROBABILITY

The experiment or empirical probability P(E) of an event is defined as

$$P(E) = \frac{Number\ of\ trials\ in\ which\ the\ event\ happened}{Total\ number\ of\ trials}$$

i.e., P (E) = $\frac{m}{n}$

NOTE : (i) These probabilities are based on results of an actual experiment.

(ii) These probabilities are only 'estimates', i.e., we may get different probabilities for the same event in various experiments.

★ THEORETICAL (OR CLASSICAL) PROBABILITY

The theoretical or classical probability of an event E, written as P(E), is defined as

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

Where the outcomes of the experiment are equally likely.

Ex. A die is thrown once (i) What is the probability of getting a number greater than 4? (ii) What is the probability of getting a number less than or equal to 4?

Sol. The possible outcomes are 1, 2, 3, 4, 5 and 6.
Let E = the event of getting a number greater than 4
and F = The event of getting a number less than or equal to 4
(i) The outcomes favorable to E are 5 and 6.
∴ the number of outcomes favorable to E is 2.

The therefore, P(E) = P(number greater than 4) = $\frac{4}{6} = \frac{2}{3}$

NOTE : Events E and F are not elementary events because event E has 2 outcomes and the event F has 4 outcomes,

<u>ANIMPORTANT REMARK</u>

In the experiment or empirical approach to probability, the probability of events are based on the results of actual experiment and adequate recordings of the happening of the events, while in theoretical approach to probability, we try to find (predict) the probabilities of the events without actually performing the experiment.

★ SOME SPECIAL EVENTS

➤➤ **IMPOSSIBLE EVENT (OR NULL EVENT) :** An event is said to be an impossible event when none of the outcomes is favorable to the event.
The probability of an impossible event = 0 .

Ex. What is the probability, of getting
Sol. The possible outcomes are 1, 2, 3, 4, 5, 6.
Let E = the event of getting a number 8 in a single throw of a die.
Clearly, the number of outcomes favorable to E is 0 and the total number of possible outcomes is 6.

Therefore, P(E) = $\frac{0}{6} = 0$.

Here, E is an impossible event.

⏩ **SURE (OR CERTAIN) EVENT :** An event is said to be a sure (or certain) event when all possible outcomes are favorable to the event.

The probability of a sure event is 1.

Ex. What is the probability of getting a number less than 7 in a single of a die?

Sol. The possible outcomes are : 1, 2, 3, 4, 5, 6.

Let F = the event of getting a number 7 in a single throw of a dice. Clearly, the number of outcomes favorable to F are 1, 2, 3, 4, 5, 6. i.e., the number of outcomes favorable to F is 6.

Therefore, $P(E) = \frac{6}{6} = 1$.

Here, F is an impossible event.

⏩ **COMPLEMENT OF AN EVENT :** Corresponding to every event E associated with random experiment, there is an event 'not E', which occurs only when E does not occur.

The event $\overline{E}$, representing 'not E', is called the complement of the event E.

E and $\overline{E}$, are also called complementary events.

In general, $P(E) + P(\overline{E}) = 1$

i.e., $P(\overline{E}) = 1 - P(E)$ or $P(\text{not E}) = 1 - P(E)$

⏩ **AN IMPORTANTR RESULT :** The probability of an event always lies between 0 and 1.

i.e., $0 \le P(E) \le 1$

PROOF Let m be the number of favorable outcomes of an event E and n be the total number of outcomes.

Then, $0 \le m \le n$ [m cannot be negative integer and m cannot be greater than n]

$$\Rightarrow \quad 0 \le \frac{m}{n} \le \frac{n}{n} \Rightarrow 0 \le \frac{m}{n} \le 1 \Rightarrow a \le P(E) \le 1$$

Thus, the probability of an event always lies between 0 and 1.

NOTES : (i) $0 \le P(E) \le 1$

(ii) Let $E_1, E_2, E_3, \ldots, E_n$ be the n elementary events associated with a random experiment having exactly n outcomes. Then,

$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_n) = 1$

Ex. A bag contains 3 red balls, 4 white balls and 5 green balls. A ball is drawn at random.

Let R = the event of getting a red ball,

W = the event of getting a with ball,

and G = the event of getting a green ball,

Here, total number of balls (outcomes) = 3 + 4 + 5 = 12.

Then, $P(R) = \frac{3}{12}$ [Number of favorable outcomes = 3]

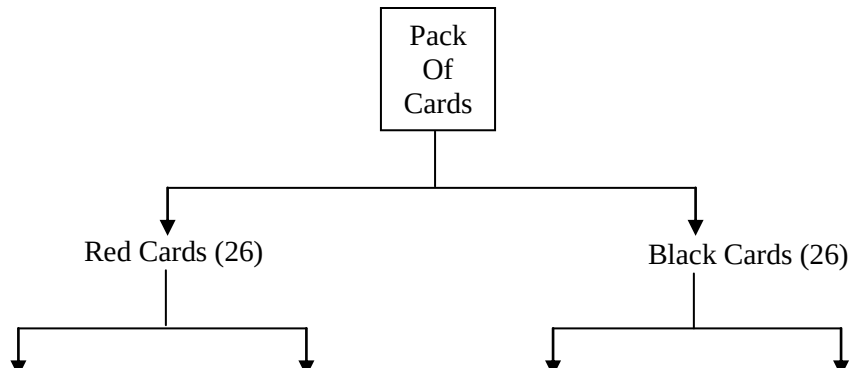
$P(W) = \frac{4}{12}$ [Number of favorable outcomes = 4]

And, $P(G) = \frac{5}{12}$ [Number of favorable outcomes = 5]

Clearly, $\frac{3}{12} + \frac{4}{12} + \frac{5}{12} + \frac{12}{12} = 1.$

i.e., $P(R) + P(W) + P(G) = 1.$

DESIGNATION OF PLAYING CARDS



(i) A beck (pack) of cards contains 52 cards, out of which there are 26 red cards and 26 black cards.
(ii) There are four suits each containing 13 cards.
(iii) The cards in each suit are ace (A). king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.
(iv) Kings, queens and jacks are called **face cards** (4 + 4 + 4 = 12).
(v) Kings, queens jacks and are called **honour cards** (4 + 4 + 4 + 4= 16).

JUST FOR YOU

1. $P(E) = \frac{number\ of\ outcomes\ favourable\ to\ E}{number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$, where outcomes of the experiment are equally likely.
2. Probability of an impossible event = 0.
3. Probability of a sure event = 1.
4. P(E) + P(E) = 1, where E and $\overline{E}$ are complementary events.
5. $0 \le P(E) \le 1$
6. The sum of the probabilities of all the elementary events of an experiment is 1.

Ex.1 Complete the following statements :

(i) Probability of an event E + probability of the event 'not E' = ……

(ii) The Probability of an event that is certain to happen is…… Such an event is called……..

(iii) The Probability of an event is greater than or equal to…… and less than or equal to……..

(iv) P(E) = $\frac{\text{.....}}{Total\ number\ of\ trials}$

Sol. (i) 1 (ii) 1, sure or certain event (iii) 0, 1 (iv) number of trials in which event happened.

Ex.2 Which of the following experiments have equally likely outcomes ? Explain.

(i) A driver attempts to starts a car. The car starts or does not start.

(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

(iii) A coin is tossed. It turns to be a head or a tail.

(iv) A monitor is nominated by the class teacher of a class. It is a boy or a girl.

Sol. (iii) because when a coin is tossed either a head or a tail turns.

Ex.3 Mach the following.

(i) P($\overline{E}$) =	**(a) 0**
(ii) Probability of an impossible event	**(b) 0.5**
(iii) Probability of an event cannot be more than	**(c) 1 – P(E)**
(iv) A cards is drawn form a pack of 52 cards, then the probability of getting a red card is equal to	**(d) 1**

Sol. (i) (c), (ii) (a), (iii) (d), (iv) (b)

Ex.4 If P(E) = 0.05, what is the probability of 'not E'?

Sol. We have P(E) = 0.05

$\therefore$ P(not E) = 1 – P(E) = 1 – 0.05 = 0.95

Therefore, P(not E) = 0.95.

Ex.5 Find the probability of getting a head when a coin is tossed once.

Sol. When a coin is tossed, then all possible outcomes are H and T

Total number of possible outcomes = 2

Let E be the event of getting a head

$\therefore$ number of favorable outcome = 1

Hence, required probability = P(E) = $\frac{Number of\ favourable\ outcomes}{Total\ number of\ possible\ outcomes} = \frac{1}{2}$

Ex.6 Two unbiased coins are tossed simultaneously. Find the probability of getting
(i) one head (ii) one tail (iii) two heads
(iv) at least one head (ii) at most one tail (iii) no head.

Sol. If two unbiased coins are tossed simultaneously, then all possible outcomes are :
HH, HT, TH, TT.
Total number of possible outcomes = 4.

(i) Let A_1 = the event of getting one head.
Then, favorable outcomes are HT, TH.
Number of favorable outcomes = 2.

Hence, required probability = P (getting one head) = P(A_1) = $\frac{2}{4} = \frac{1}{2}$

(ii) Let A_2 = the event of getting one tail.
Then, favorable outcomes are TH, HT.
Number of favorable outcome = 2.

Hence, required probability = P (getting one tail) = P(A_2) = $\frac{2}{4} = \frac{1}{2}$

(iii) Let A_3 = the event of getting two tail.
Then, favorable outcomes is HH
Number of favorable outcome = 1

Hence, required probability = P (getting two heads) = P(A_3) = $\frac{1}{4}$

(iv) Let A_4 = the event of getting at least one head.
Then, favorable outcomes are HT, TH, HH
Number of favorable outcome = 3

Hence, required probability = P (getting at least one head) = P(A_4) = $\frac{3}{4}$

(v) Let A_5 = the event of getting atmost one head.
Then, favorable outcomes are TT, HT, TH.
Number of favorable outcome = 3

Hence, required probability = P (getting atmost one head) = P(A_5) = $\frac{3}{4}$

(vi) Let A_6 = the event of getting no head.
Then, favorable outcomes are TT
Number of favorable outcome = 1

Hence, required probability = P (getting one head) = P(A_6) = $\frac{1}{4}$

Ex.7 Three unbiased coins are tossed together. Find the probability of getting
(i) one head (ii) two heads (iii) all heads (iv) at least two heads

Sol. If three unbiased coins are tossed together, then all possible outcomes are :
HHH, HHT, THH, HTT, THT, TTH, TTT
Total number of possible outcomes = 8

(i) Let A_1 = the event of getting one head.
Then, favorable outcomes are HHT, THT, THH.
Number of favorable outcomes = 3.

Hence, required probability = P (getting one head) = $P(A_1) = \frac{3}{8}$

(ii) Let A_2 = the event of getting two head.
Then, favorable outcomes are HHT, HTH, THH.
Number of favorable outcomes = 3.

Hence, required probability = P (getting two heads) = $P(A_2) = \frac{3}{8}$

(iii) Let A_3 = event of getting all heads.
Then, favorable outcomes are HHH
Number of favorable outcomes = 1

Hence, required probability = P (getting all head) = $P(A_3) = \frac{1}{8}$

(iv) Let A_4 = event of getting at least two heads.
Then, favorable outcomes are HHT, HTH, THH, HHH
Number of favorable outcomes = 4

Hence, required probability = P (getting at least two heads) = $P(A_4) = \frac{4}{8} = \frac{1}{2}$

Ex.8 A die is thrown once. Find the probability of getting
(i) a prime number (ii) a number lying between 2 and 6 (iii) an odd number.

Sol. If a die is thrown, then all possible outcomes are 1, 2, 3, 4, 5, 6.
Total number of possible outcomes = 6.

(i) Let A_1 = event of getting a prime number.
Then, the favorable outcomes are 2, 3, 5.
Number of favorable outcomes = 3.

Hence, required probability = P (getting a prime number) = $P(A_1) = \frac{3}{6} = \frac{1}{2}$

(ii) Let A_2 = event of getting a number lying between 2 and 6.
Then, the favorable outcomes are 3, 4, 5.
Number of favorable outcomes = 3.

Hence, required probability = P (getting a number lying between 2 and 6) = $P(A_2) = \frac{3}{6} = \frac{1}{2}$

(iii) Let A_3 = event of getting an odd number.
Then, the favorable outcomes are 1, 3, 5.
Number of favorable outcomes = 3.

Hence, required probability = P (getting an odd number) = $P(A_3) = \frac{3}{6} = \frac{1}{2}$

Ex.9 A die is thrown twice. What is the probability that
(i) 5 will not come up either time? (ii) 5 will come up at least once?

Sol. If a die is thrown twice, then all the possible outcomes are :
(1, 1), (1, 2), (1,3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2,3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3,3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4,3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5,3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6,3), (6, 4), (6, 5), (6, 6).
Total number of getting 5 not either time

(i) Let A_1 = event of getting 5 not either time.
Then, the favorable outcomes are: (1, 1), (1, 2), (1,3), (1, 4), (1, 6),
(2, 1), (2, 2), (2,3), (2, 4), (2, 6),
(3, 1), (3, 2), (3,3), (3, 4), (3, 6),

(4, 1), (4, 2), (4,3), (4, 4), (4, 6),
(6, 1), (6, 2), (6,3), (6, 4), (6, 6).

Number of favorable outcomes = 25

Hence, required probability = P (5 will not come up either time) = $P(A_1) = \frac{25}{36}$

(ii) Let A_2 = event of getting 5 at least once.
Then, the favorable outcomes are:
(1, 5), (2, 5), (3,5), (4, 5), (5, 1), (5, 2), (5, 3), (5,4), (5, 5), (5, 6), (6, 5).
Number of favorable outcomes = 11

Hence, required probability = P (5 will not come up at least once) = $P(A_2) = \frac{11}{36}$

Ex.10 A pair of dice is thrown simultaneously. Find the probability of getting
(i) a doublet
(ii) sum of the numbers on two dice is always 7
(iii) an even number on the first die and a multiple of 3 on the other.

Sol. If a pair of dice is thrown simultaneously, then all the possible outcomes are :
(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),(6,1), (6,2), (6,3), (6,4), (6,5), (6,6).
Total number of possible outcome = 36

(i) Let A_1 = event of getting a doublet.
Then, the favorable outcomes are (1, 1), (2, 2), (3,3), (4, 4), (5, 5), (6, 6).
Number of favorable outcomes = 6

Hence, required probability = P (getting a doublet) = $P(A_1) = \frac{6}{36} = \frac{1}{6}$

(ii) Let A_2 = event of getting a sum of numbers on two dice is always 7
Then, the favorable outcomes are (1, 6), (2, 5), (3, 4), (5, 2), (6, 1).
Number of favorable outcomes = 6

Hence, required probability = P (getting a sum of the numbers on two dice is always 7) = $P(A_2) = \frac{3}{36} = \frac{1}{6}$

(iii) Let A_3 = event of getting an even umber on the first die and a multiple of 3 on the other.
Then, the favorable outcomes are (2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6).
Number of favorable outcomes = 6
Hence, required probability = P (getting an even umber on the first die and a multiple of 3 on the other)
= $P(A_3) = \frac{6}{36} = \frac{1}{6}$

Ex.11 Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is
(a) 8 (b) 13 (iii) less than or equal to 12? (NCERT)

Sol. If two dice, one blue and one grey, are thrown at the same time, then all possible outcomes are :
(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),(6,1), (6,2), (6,3), (6,4), (6,5), (6,6).

Total number of possible outcome = 36

(i) Let A_1 = event of getting a sum two numbers appearing on the top o the dice is 8.
Then, the favorable outcomes are (2, 6), (2, 5), (4, 4), (5, 3), (6, 2).
Number of favorable outcomes = 5.

Hence, required probability = $P(A_1) = \frac{5}{36}$

(ii) Let A_2 = event of getting a sum two numbers appearing on the top o the dice is 13.
Then, the favorable outcomes = 0.

Hence, required probability = $P(A_2) = \frac{0}{36} = 0$

(iii) Let A_3 = event of getting a sum two numbers appearing on the top o the dice is less than or equal to 12.

Then, the favorable outcomes = all the possible outcomes = 36.

Hence, required probability = $P(A_3) = \frac{36}{36} = 1$.

Ex.12 A cards is drawn at random from a well shuffled deck of 52 cards. Find the probability that the card drawn is –

(i) a red card (ii) a non-ace (iii) a king or a jack (iv) neither a king nor a queen.

Sol. If a card is drawn at random from a well shuffled deck of 52 cards, then total number of possible outcomes = 52

(i) Let A_1 = event of getting a red card.
Then, the favorable outcomes = 26.

Hence, required probability = P (getting a red card) = $P(A_1) = \frac{26}{52} = \frac{1}{2}$

(ii) Let A_2 = event of getting a non-ace
Then, the favorable outcomes = 48. [$\because$ there are 4 aces in a pack of playing cards]

Hence, required probability = P (getting a non-ace) = $P(A_2) = \frac{48}{52} = \frac{12}{13}$

(iii) Let A_3 = event of getting a king or a jack.
There are 4 king cards and 4 jack cards.
Hence, required probability = $P(A_3)$ = P (getting a king or a jack) = P (getting a king) + P (getting a jack)

$$= \frac{4}{52} + \frac{4}{52} + \frac{8}{52} + \frac{2}{13}$$

(iv) Let A_4 = event of getting neither a king nor a queen.
There are 4 king cards and 4 queen cards.
Hence, required probability = $P(A_4)$ = P (getting neither a king nor a queen)
= 1 – P (getting a king or a queen)
= 1 – P (getting a king) + P (getting a queen)]

$$= 1 - \left(\frac{4}{52} + \frac{4}{52}\right) = 1 - \frac{8}{52} = \frac{44}{52} = \frac{11}{13}$$

ALITER: Let A_4 : event of getting neither king nor queen.
$\therefore$ no, of favorable outcomes.
i.e., neither king nor queen cards = 52 – 8 = 44

Hence, $P(A_4) = \frac{44}{52} = \frac{11}{13}$

Ex.13 All the three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting

(i) a black face card (ii) a queen (iii) a black card

Sol. If all the three face cards of spades are removed from a well-shuffled pack of 52 cards, then there are 49 cards left in the pack.

(i) Let A_1 = event of getting a black face card.
There are 3 black face cards left. (face cards of club)

Hence, required probability = $P(A_1)$ = P (getting a black face card) = $\frac{3}{49}$

(ii) Let A_2 = event of getting a queen.
There are three queens left.

Hence, required probability = $P(A_2)$ = P (getting a queen) = $\frac{3}{49}$

(iii) Let A_3 = event of getting a black card.
There are 23 black cards left.

Hence, required probability = $P(A_3)$ = P (getting a black card) = $\frac{23}{49}$

Ex.14 **Five cards, the-ten, jack, queen, king and ace of diamonds, are well-shuffled with their faces downwards. One card is then picked up at random.**

(i) **What is the probability that the card is the queen?**

(ii) **If the queen is drawn and put aside, what is the probability that the second card picked up is**

(a) an ace ? **(b) a queen ?**

Sol. There are five cards as the ten, jack, queen, king and ace of diamond.

(i) Let A = event of getting a queen

There is only one queen out of the five cards.

Hence, required probability = P(A) = P (getting a queen) = $\frac{1}{5}$

(ii) When a queen is drawn and put aside four cards, the ten, jack, king and ace are left. Therefore.

(a) required probability = P (getting an ace) = $\frac{1}{4}$

(b) required probability = P (getting a queen) = $\frac{0}{4}$ = 0.

Ex.15 **A box contains 5 red. 4 green and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is**

(i) white **(ii) neither red nor white**

Sol. Total number of balls in the box = 5 + 4 + 7 = 16 .

Let A_1 = event of getting a red ball

A_2 = event of getting a white ball.

(i) There are 7 white balls in the box.

Hence, required probability = $P(A_2)$ = P (getting a white ball) = $\frac{7}{16}$

(ii) There are 7 white and 5 red balls in the box.

Hence, required probability = P (getting neither red nor white ball)

= 1 – P (getting either red or white ball)

= 1 – P (getting a red) + P (getting a white ball)]

$= 1 - \left(\frac{5}{16} + \frac{7}{16}\right) = 1 - \frac{12}{16} = \frac{4}{16} = \frac{1}{4}$

ALITER P(getting neither red nor white ball) = P (getting a green ball) = $\frac{4}{16} = \frac{1}{4}$

Ex.16 **A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of red ball, determine the number of blue balls in the bag.**

Sol. There are 5 red balls in a bag.

Let number of blue balls be x.

Let A_1 = event of getting a red ball

and A_2 = event of getting a blue ball.

$P(A_1)$ = P (getting a red ball) = $\frac{5}{x+5}$

$P(A_2)$ = P (getting a blue ball) = $\frac{x}{x+5}$

$\because 2P(A_1) = P(A_2) \Rightarrow \frac{2\times 5}{x+5} = \frac{x}{x+5} \Rightarrow 10 = x \Rightarrow x = 10$

Hence, required number of blue balls = 10.

Ex.17 **A box contains 5 red marbles, 8 white marbles and 4 green marbles one marble is taken out of the box at random. What is the probability that the marble taken out will be**

(i) red **(ii) white** **(iii) not green?**

Sol. Total number of marbles in the box = 5 + 8 + 4 = 17.

Let A_1 = event of getting a red marble

A_2 = event of getting a white marble

and A_3 = event of getting a green marble.

(i) There are 5 red marbles in the box.

Hence, required probability = $P(A_1)$ = P (getting a red marble) $= \frac{5}{17}$

(ii) There are 8 white marbles in the box.

Hence, required probability = $P(A_2)$ = P (getting a white marble) $= \frac{8}{17}$

(iii) There are 4 green marbles in the box.

$\therefore$ $P(A_3)$ = P (getting a green marble) = $\frac{4}{17}$

Hence, required probability = P (not getting a green marble)

= 1 – P (getting a green marble) = 1 – $P(A_3)$ $= 1 - \frac{4}{17} = \frac{13}{17}$.

Ex.18 A box contains 19 balls bearing numbers 1, 2, 3,……, 19 respectively. A ball is drawn at random from the box. Find the probability that the number on the ball is –
(i) a prime number (ii) even number
(iii) divisible by 3 or 5 (iv) neither divisible by 5 nor by 10.

Sol. Total number of balls in the box = 19

$\therefore$ number of all possible outcomes = 19

(i) Let A_1 = event of getting a prime number.
Then, the favorable outcomes are 2, 3, 5, 7, 11, 13, 17, 19.
Number of favorable outcomes = 8.

Hence, required probability = P (getting a prime number) = $P(A_1)$ $= \frac{8}{19}$.

(ii) Let A_2 = event of getting an even number.
Then, the favorable outcomes are 2, 4, 6, 8, 10, 12, 14, 16, 18.
Number of favorable outcomes = 9.

Hence, required probability = P (getting an even number) = $P(A_2)$ $= \frac{9}{19}$.

(iii) Let A_3 = event of getting a number divisible by 3 or 5.
Then, the favorable outcomes are 3, 5, 6, 9, 10, 12, 15, 18.
Number of favorable outcomes = 8.

Hence, required probability = P (getting a number divisible by 3 or 5) = $P(A_3)$ $= \frac{8}{19}$.

(iv) Let A_4 = event of getting a number divisible by 5 or 10.

Then, the favorable outcomes are 5, 10, 15. Number of favorable outcomes = 3.

$\therefore$ P (getting a number divisible by 5 or 10) = $P(A_4)$ = $\frac{3}{19}$

Hence, required probability = P (getting a number neither divisible by 5 nor by 10)

= 1 – P (getting a number neither divisible by 5 nor by 10) $= 1 - \frac{3}{19} = \frac{16}{19}$

Ex.19 Seventeen cards numbered 1, 2, 3, 4,……, 16, 17 are put in a box and mixed thoroughly. One person drawn a card from the box. Find the probability that the number on the card is
(i) odd (ii) a prime (iii) divisible by 3 (iv) divisible by 2 and 3 both

Sol. There are seventeen cards in the box.
$\therefore$ number of all possible outcomes = 17.

(i) Let A_1 = event of getting an odd number.
Then, the favorable outcomes are 1, 3, 5, 7, 9, 11, 13, 15, 17.
Number of favorable outcomes = 9.

Hence, required probability = P (getting an odd number) = $P(A_1) = \frac{9}{17}$.

(ii) Let A_2 = event of getting a prime number.
Then, the favorable outcomes are 2, 3, 5, 7, 11, 13, 17.
Number of favorable outcomes = 7.

Hence, required probability = P (getting a prime number) = $P(A_2) = \frac{7}{17}$.

(iii) Let A_3 = event of getting a number divisible by 3.
Then, the favorable outcomes are 3, 6, 9, 12, 15.
Number of favorable outcomes = 5.

Hence, required probability = $P(A_3) = \frac{5}{17}$.

(iv) Let A_4 = event of getting a number divisible by 2 and 3 both.
Then, the favorable outcomes are 6, 12.
Number of favorable outcomes = 2.

Hence, required probability = $P(A_4) = \frac{2}{17}$.

Ex.20 Find the probability that a number selected at random from the numbers 1 to 15 not a prime number when each of the given numbers is equally likely to be selected.

Sol. The total given numbers = 25
Then, the favorable outcomes (prime numbers) are 2, 3, 5, 7, 11, 13, 17, 19, 23.
Number of favorable outcomes = 9.
Let A = event of getting a non-prime number.
$\therefore$ number of non-prime number = 25 – 9 = 16

$\therefore$ required probability = P (getting a non-prime numbers) = $P(A) = \frac{16}{25}$.

Ex.21 A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears
(i) a two digit number (ii) a perfect square number (iii) a number divisible by 5.

Sol. The total of discs = 90. Number of possible outcomes = 90.
(i) Let A_1 = event of getting a two digit number.
There are 9 single-digit numbers and 81 two-digit numbers.
Then, the number of favorable outcomes = 81.

Hence, required probability = $P(A_1)$ = P (getting a two-digit number) = $\frac{81}{90} = \frac{9}{10}$.

(ii) Let A_2 = event of getting a perfect square number.
Then, the number of favorable outcomes are 1, 4, 9, 16, 25, 36, 49, 64, 81.
Number of favorable outcomes = 9.

Hence, required probability = $P(A_2)$ = P (getting a perfect square number) = $\frac{9}{90} = \frac{1}{10}$.

(iii) Let A_3 = event of getting a number divisible by 5.
Then, the number of favorable outcomes are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90.
Number of favorable outcomes = 18.

Hence, required probability = $P(A_3)$ = P (getting a number divisible by 5) = $\frac{18}{90} = \frac{1}{5}$.

Ex.22 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not is defective. One pen is taken out at random from this from this lot. Determine the probability that the pen taken out is a good one.

Sol. There are 12 defective pens and 132 good pens.
$\therefore$ Total number of possible outcomes = 12 + 132 = 144.
Let A = event of getting a good pen
Then, the number of favorable outcomes = 132

Hence, required probability = P(A) = P (getting a good pen) $= \frac{132}{144} = \frac{11}{12}$.

Ex.23 A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it t her. What is the probability that (i) she will buy it (ii) she will not buy it?

Sol. There are 144 ball pens.
$\therefore$ total number of possible outcomes = 144.

(i) Let A_1 = event of buying a good pen.
There are 20 ball pens which are defective out of 144 ball pens.
$\therefore$ number of good ball pens = 144 – 20 = 124

Hence, required probability = $P(A_1)$ = P (buying a good pen) $= \frac{124}{144} = \frac{31}{36}$.

(ii) Let A_2 = event of not buying a good pen i.e., buying a defective pen.
Then, the number of favorable outcomes = 20.

Hence, required probability = $P(A_2)$ = P (not buying a good pen) $= \frac{20}{144} = \frac{5}{36}$.

Ex.24 Savita and Hamida are friend. What is the probability that both will have
(i) different birthdays (ii) the same birthday (ignoring leap year). (NCERT)

Sol. There are 365 days in a year.
$\therefore$ the total number of possible outcomes = 365.

(i) Let A_1 = event that Hamida's birthday is different from Savita's birthday Then, the number of favorable outcomes for her birthday = 365 – 1 = 364.

Hence, required probability = $P(A_1)$ = P (Hamida's birthday is different from Savita's birthday) $= \frac{364}{365}$.

(ii) Let A_2 = the event the Savita and Hamida have the same birthday
Hence, required probability = $P(A_2)$ = P (Savita and Hamida have the same birthday)

= 1 – P (both have different birthday) $= \frac{364}{365} = \frac{1}{365}$.

Ex.25 A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

Sol. There are 24 marbles in a jar.
$\therefore$ the total number of possible outcomes = 24.
Let number of green marbles be x.

(ii) Let A_1 = event of getting a green marble.

$\therefore$ required probability = $P(A_1)$ = P (getting a green marble) = $\frac{x}{24}$

But it is given that the probability of green marble is $\frac{2}{3}$

$\therefore \quad \frac{2}{3} = \frac{x}{24} \Rightarrow x = \frac{2 \times 24}{3} \Rightarrow x = 16$

So, number of green marbles = 16.
Hence, number of blue marble in jar = 24 – 16 = 8.

Ex.36 What is the probability that an ordinary year has 53 Sundays?

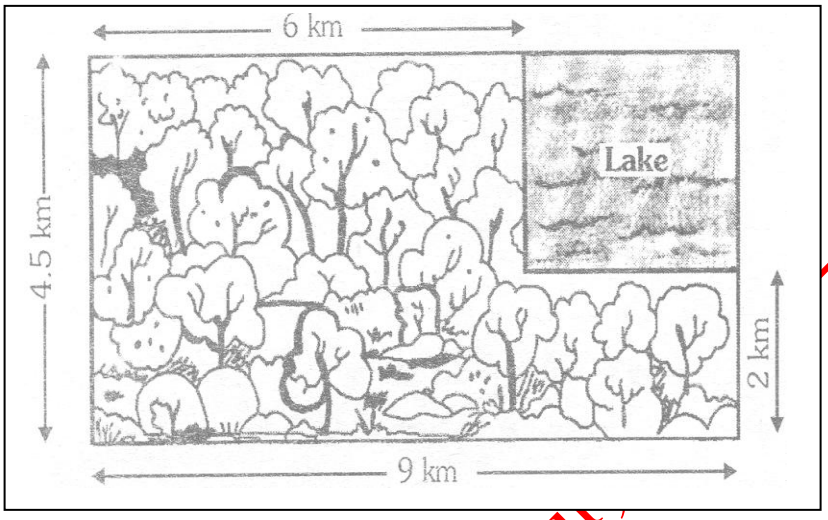
Sol. There are 365 day i.e., 52 weeks and 1 day in an ordinary year.
This 1 day can be anyone of the 7 days of the week.

$\therefore$ P (this day is sunday) = $\frac{1}{7}$

Also, 52 weeks have 52 Sundays.

Hence, required probability = P (an ordinary year has 53 sundays) $= \frac{1}{7}$.

Ex.27 **A missing hellcopter is reported to have crashed somewhere in the rectangular region in figure. What is the probability that it crashed inside the lake shown in the figure? (NCERT)**



Sol. The helicopter is equally likely to crash anywhere in the region.

Area of entire rectangular region, where the helicopter can crash = (4.5×9) km^2 = 40.5 km^2.

Area of the lake = (2.5×3) km^2 = 7.5 km^2.

Let A = the event that the helicopter crashed inside the lake.

Then, number of favorable outcomes = 7.5 km^2

Hence, required probability = P(A) = P (helicopter crashed in the lake) $= \frac{7.5}{40.5} = \frac{75}{405} = \frac{5}{27}$.

Ex.28 **A piggy bank contains hundred 50p coins, fifty Re. 1 coins, twenty Rs. 2 coins and ten Rs. 5 coins. If it is likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin**
(i) will be a 50p coin (ii) will not be a Rs. 5 coin?

Sol. Number of 50p coins = 100

Number of Rs. 1 coins = 50

Number of Rs. 2 coins = 20

Number of Rs. 5 coins = 10

$\therefore$ total number of coins = 180.

$\therefore$ the total number of possible outcomes = 180.

(i) Let A_1 = event of getting 50p coin.

Then, the number of favorable outcomes = 100.

Hence, required probability = P(A_1) = P (getting a 50p coin) $= \frac{100}{180} = \frac{5}{9}$.

(ii) Let A_2 = event of getting not a Rs. 5 coin.

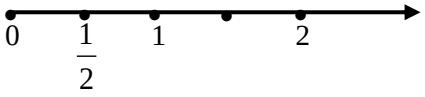
Hence, required probability = P(A_2) = P (not getting Rs. 5 coin)

= 1 – P (getting a Rs. 5 coin)

$= 1 - = \frac{10}{180} = \frac{170}{180} = \frac{17}{18}$.

Ex.29 **In a musical chair game, the person playing the music has been advised to stop playing the music at any time within two minutes after she/he starts playing. What is the probability that the music will stop within the first half minute after starting?**

Sol. The possible outcomes are all the numbers between 0 and 2.
This is the portion of the number line from 0 to 2.



Let A = the event that the music is stopped within the first half-minute.

Then, the favorable outcomes are points on the number line from 0 to $\frac{1}{2}$

The distance from 0 to 2 is 2, while the distance from 0 to $\frac{1}{2}$ is $\frac{1}{2}$

Since all the outcomes are equally likely, therefore, the total distance = 2 and favorable to A = $\frac{1}{2}$.

Hence, required probability = P(A) = P (the music is stopped within the first half minute) $= \frac{\frac{1}{2}}{2} = \frac{1}{4}$

Ex.30 **There are 40 students in class X of a school of whom 25 are girls and 15 are days. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical. Then she puts cards in a bag and stir them thoroughly. She, then draws one card from the bag. What is the probability that the name written on the card is the name of :**
(i) a girl **(ii) a boy?** **(NCERT)**

Sol. There are 40 students out of which 25 are girls and 15 are days.
$\therefore$ number of all possible outcomes = 40.

(i) Let A_1 = event that the name written on the card is the name of a gils.
Then, the number of favorable outcomes = 25.
Hence, required probability = $P(A_1) = \frac{25}{40} = \frac{3}{8}$.

(ii) Let A_2 = event that the name written on the card is the name of a bot.
Then, the number of favorable outcomes = 15.
Hence, required probability = $P(A_2) = \frac{15}{40} = \frac{3}{8}$.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

1. If A be the event such that $P(A) = \frac{2}{5}$, then P(not A) is equal to

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{5}$ (d) None of these

An unbiased die is thrown (Q. NO. 2 to 6)

2. The probability of getting a prime number is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

3. The probability of getting a multiple of 3is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{3}{6}$ (d) $\frac{4}{6}$

4. The probability of getting a number greater than 1is

(a) $\frac{1}{6}$ (b) $\frac{2}{6}$ (c) $\frac{4}{6}$ (d) $\frac{5}{6}$

5. The probability of getting a number between 1 and 6 is

(a) $\frac{1}{6}$ (b) $\frac{2}{6}$ (c) $\frac{4}{3}$ (d) $\frac{2}{3}$

6. The probability of getting an odd number is

(a) $\frac{1}{6}$ (b) $\frac{2}{6}$ (c) $\frac{4}{6}$ (d) None of these

Two unbiased coins are tossed simultaneously (Q. No. 7 to 10)

7. The probability of getting one head is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) None of these

8. The probability of getting two heads

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) None of these

9. The probability of getting no head is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) None of these

10. The probability of getting at least one head is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) None of these

One card is drawn from a pack of 52 cards (Q. No. 11 to 14)

11. The probability of getting a jack card is

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{4}{13}$

12. The probability of getting a face card is

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{4}{13}$

13. The probability of getting a '10' of black suit is

(a) $\frac{1}{26}$ (b) $\frac{1}{13}$ (c) $\frac{2}{26}$ (d) None of these

14. The probability of getting a red and a king card is

(a) $\frac{5}{26}$ (b) $\frac{1}{13}$ (c) $\frac{7}{26}$ (d) None of these

15. A bag contains 4 red balls and 3 green balls. A ball is drawn at random. The probability a green ball is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

16. P(E) + P($\overline{E}$) is equal to
The probability of getting a jack card is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) None of these

Which one of the following cannot be the probability of an event (Q. No. 17 to 18)

17. (a) $\frac{1}{3}$ (b) $\frac{11}{36}$ (c) $\frac{-2}{3}$ (d) 1

18. (a) $\frac{2}{7}$ (b) 0 (c) $\frac{13}{29}$ (d) $\frac{5}{2}$

Choose the correct alternative for each of the following and justify your answer (Q. No. 19 to 22)

19. Probability of an impossible event is equal to

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) None of these

20. If $P(E_1)=\frac{1}{6}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{1}{6}$, where E_1, E_2, E_3 and E_4 are elementary events of a random experiment, then $P(E_4)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) None of these

21. Cards each marked with one of the numbers 4, 5, 6,…., 20 are placed in box and mixed thoroughly. One card is drawn at random from the box. Then, the probability of getting an even prime number is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) None of these

22. A bag contains 5 red and 4 black balls. A ball is drawn at random from the bag. Then, the probability of getting a black ball is

(a) $\frac{1}{5}$ (b) $\frac{4}{9}$ (c) $\frac{1}{5}$ (d) $\frac{1}{4}$

OBJECTIVE **ANSWER KEY** EXERCISE-4

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	C	B	D	D	D	A	B	A	C	A	C	A	D	C
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**								
Ans.	C	C	D	B	C	A	B								

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

SUBJCTIVE TYPE QUESTIONS

1. (a) Two dice are thrown at the same time. Complete the following table

Event : Sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

Number in first throw

Number in second throw	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(b) A die is numbered in such a way that its faces show the numbers 1, 2, 3, 4, 5, 6. It is thrown two times and the total score in two throws is noted.

Complete the following table which gives a few values of the total score on the two throws.

(c) Justify the statement : "Tossing a coin a fair of deciding which team should get the batting first at the beginning of a cricket game"

2. Which of the following experiment have not equally likely outcomes? Explain.

(i) A trial is made to answer a true-false question. The answer is right or wrong.

(ii) A baby is born. It is boy or a girl.

(iii) Kushagra appears in an interview. He is selected or not selected.

(iv) A die is thrown. It turns to be an even or an odd number.

3. Match the following :

A black die and a white die are thrown at the same time.

(i)	The probability of getting a total of 9.	(a) $\frac{1}{6}$
(ii)	The probability of getting a total of 10.	(b) $\frac{5}{36}$
(iii)	The probability of getting a total of more then 9.	(c) $\frac{1}{12}$
(iv)	The probability of getting the sum of the two numbers is 8.	(d) $\frac{1}{9}$

4. (a) The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow?

(b) If The probability of winning a game is 0.6, what is the probability of losing it?

5. Find the probability of getting a tail when a coin is tossed once.

6. (a) Two unbiased coins are tossed simultaneously. Find the probability of getting

(i) exactly one head (ii) exactly one tail (iii) two tails

(iv) at least one tail (v) atmost one tail (vi) no tail.

(b) Harpreet tosses two different coins simultaneously. What is the probability that she gets at least one head?

7. Three unbiased coins are tossed together. Find the probability of getting

(a) (i) one tail (ii) two tails (iii) all tails (iv) at least two tails

(b) (i) at most two tails (ii) at most two heads.

8. (a) A die is thrown once. Find the probability of getting

(i) a multiple of 2

(ii) a number lying between 1 and 5

(iii) an odd number.

(b) A child has a die whose six faces show the letters as given below

A	B	C	D	E	A

The die is thrown once. What is the probability of getting (i) A (ii) D?

9. A die is thrown twice. What is the probability that

(i) 3 will not come up either time?

(ii) 6 will come up at least once?

10. A pair of dice is thrown simultaneously. Find the probability of getting

(i) a multiple of 3 on both dice

(ii) sum of the numbers on two dice is always less than 7.

(iii) an odd number on the first die and a prime number on the other.

11. Two dice, one blue and green are thrown at the same time. What is the probability that sum of the two umbers appearing on the top of the dice is

(i) 9 (ii) greater than 10 (iii) less than or equal to 11

12. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(a) (i) a king of red colour (ii) a face card (iii) a red face card (iv) a jack of hearts

(b) (i) a spade (ii) the queen of diamonds (iii) neither a red card nor a queen.

(c) (i) a non-face card (ii) a black king or a red queen

13. (a) From a pack of 52 playing cards jacks, queens, kings and aces of red colour are removed. From a remaining, a card is drawn at random. Find the probability that the card drawn is

(i) a black queen (ii) a red card (iii) a ten (iv) a picture card [jacks, queens and kings are picture cards]

(b) All cards of ace, jack and queen are removed from a deck of playing cards. One card is drawn at random from the remaining cards. Find the probability that the card drawn is

(i) a face card

(ii) not a face card

14. Five cards – the ten, jack, queen, king and ace of diamonds are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is jack?

(ii) If the king is drawn and put aside, what is the probability that the second card picked up is

(a) a queen (b) a ten?

15. (a) A bag contains 7 red, 5 white and 3 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is

(b) (i) A bag contains 5 white balls, 7 red balls, 4 black and 2 blue balls. One ball is drawn at random from the bag. What is the probability that the ball drawn is

(1) white or blue (2) red or black (3) not white (4) neither white nor black

(ii) A bag contains 6 red balls, 8 white balls, 5 green balls and 3 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is

(1) white (2) red or black (3) not green (4) neither white nor black

(iii) A bag contains a red balls, a blue balls, a yellow balls, all the balls being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability that the she takes out the

(1) yellow ball (2) red ball (3) blue ball?

(iv) A box contains 7 red balls, 8 green balls, 5 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is

(1) white (2) neither red nor white

(c) Poonam buys a fish from a shop for her aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish 8 female fish. What is the probability that the fish taken out is a male fish?

16. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?

If 6 more black balls are put in the box, the probability of drawing a black ball is now doubled of what it was before, find x.

17. (a) A box contain 3 blue, 2 white and 4 red marbles. If a marble is drawn at random from the box. What is the probability of that it will be

(i) white (ii) blue (iii) red?

(b) A bag contain 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the

probability of getting
(i) a white ball or a green ball (ii) neither a green ball nor a red ball

18. (a) A box contains 20 balls bearing numbers 1, 2, 3,….., 20 respectively. A ball is drawn at random from the box what is the probability that the number on the ball is

(b) Find the probability that a number selected at random from the numbers 1, 2, 3, 4, 5,….., 34, 35 is a
(i) prime number (ii) multiple of 7 (iii) multiple of 3 or 5

(c) Cards bearing numbers 3 to 19 are put in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the card drawn is
(i) even (ii) a prime (iii) divisible by 2 and 3 both.

19. Fifteen cards numbered 1, 2, 3, 4,…..,14, 15 are put in a box and mixed thoroughly. A man drawn a card at random from the box. Find the probability that the number on the card is

(i) an odd number (ii) a multiple of 4 (iii) divisible by 5 (iv) divisible by 2 and 3 both.
(v) less than or equal to 10.

20. (a) There are 30 cards numbered from 1 to 30. One card is drawn at random. Find the probability that the number of the selected card is not divisible by 3.

(b) A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, and these are equally likely outcomes. What is the probability that it will point at
(i) 8 (ii) an odd number (iii) a number greater than 2 (iv) a number less than 9.

21. A box contains 50 discs which are numbered from 1 to 50. If one disc is drawn at random from the box, find the probability that it bears
(i) a two digit number less than (ii) a prime number (iii) a number divisible by 3

22. (i) A lot of 20 bulbs contains 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

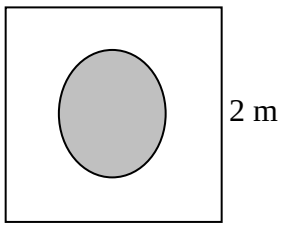
23. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujata, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that
(i) it is acceptable to Jimmy? (ii) it is acceptable to Sujata.

24. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

25. The probability of selecting a green marble at random from a jar that contains only green, white and yellow marbles is $\frac{1}{4}$. The probability of selecting a white marble at random from the same jar is $\frac{1}{3}$. If this jar contains 10 yellow marbles, what is the total number of marbles in the jar.

26. What is the probability that a leap year has 53 Sundays?

27. Suppose you drop a die at random on the rectangular region shown in figure. What is the probability that it will lend inside the circle with diameter 1 m?



28. A purse contains 10 five hundred rupee note, 20 hundred rupee notes, 30 fifty rupee note and 40 ten rupee note. If it is likely that one of the notes will fall out when the purse turns upside. What is the probability that the note
(i) will be a fifty rupee note (ii) will not be a five hundred rupee note.

29. In a musical chair game, the person playing the music has been advised to stop playing the music at any time within three minutes after she starts playing. What is the probability that the music will stop within the first half minute after starting?

30. There are 44 students in class X of a school of whom 32 are boys and 12 are girls. The class teacher has to selected one student as a class representative. He writes the name of each student on a separate card, the cards being identical. Then he puts cards in a bag and stir them thoroughly. He then drawn one card from the bag. What is the probability that the name written on the card is the name of
(i) a girl? (ii) a boy?

- **Subjective type Question**

1. (a)

Sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(b)

+	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2	3	4	4	5	5	8
3	4	5	5	6	6	9
3	4	5	5	6	6	9
6	7	8	8	9	9	12

2. (iii), because selection depends on number of factors, (constraints) **3.** (i)- (d), (ii)- (c), (iii)- (a), (iv)- (b)

4. (a) 0.15, (b) 0.4 **5.** $\frac{1}{2}$ **6.** (a) (i) $\frac{1}{2}$, (ii) $\frac{1}{2}$, (iii) $\frac{1}{4}$, (iv) $\frac{3}{4}$, (v) $\frac{3}{4}$, (vi) $\frac{1}{4}$, (b) $\frac{3}{4}$

7. (a) (i) $\frac{3}{8}$, (ii) $\frac{3}{8}$, (iii) $\frac{1}{8}$, (iv) $\frac{1}{2}$: (b) (i) $\frac{7}{8}$, (ii) $\frac{7}{8}$ **8.** (a) (i) $\frac{1}{2}$, (ii) $\frac{1}{2}$, (iii) $\frac{1}{2}$; (b) (i) $\frac{1}{3}$, (i) $\frac{1}{6}$

9. (i) $\frac{25}{36}$, (ii) $\frac{11}{36}$, **10.** (i) $\frac{1}{9}$, (ii) $\frac{5}{12}$, (iii) $\frac{1}{4}$, **11.** (i) $\frac{1}{9}$, (ii) $\frac{1}{12}$, (iii) $\frac{25}{36}$,

13. (a) (i) $\frac{1}{22}$, (ii) $\frac{9}{22}$, (iii) $\frac{1}{11}$, (iv) $\frac{3}{22}$; (b) (i) $\frac{1}{10}$, (ii) $\frac{9}{10}$, **14.** (i) $\frac{1}{5}$, (ii) $\frac{1}{4}$, (b) $\frac{1}{4}$,

15. (a) (i) $\frac{4}{5}$, (ii) $\frac{4}{5}$, (iii) $\frac{7}{15}$;

(b) (i) (1) $\frac{7}{18}$, (2) $\frac{11}{18}$, (3) $\frac{13}{18}$, (4) $\frac{1}{2}$; (ii) (1) $\frac{4}{11}$, (2) $\frac{9}{22}$, (3) $\frac{17}{22}$, (4) $\frac{1}{2}$; (iii) (1) $\frac{1}{3}$, (2) $\frac{1}{3}$, (3) $\frac{1}{3}$;

(iv) (1) $\frac{1}{4}$, (2) $\frac{2}{5}$, (c) $\frac{5}{13}$

16. $\frac{x}{12}, x = 3$ **17.** (a) (i) $\frac{2}{9}$, (ii) $\frac{1}{3}$, (iii) $\frac{4}{9}$; (b) (i) $\frac{3}{4}$, (ii) $\frac{7}{20}$

18. (a) (i) $\frac{1}{2}$, (ii) $\frac{13}{20}$, (iii) $\frac{2}{5}$, (iv) $\frac{9}{10}$; (b) (i) $\frac{11}{35}$, (ii) $\frac{1}{7}$ (iii) $\frac{16}{35}$; (c) (i) $\frac{8}{17}$, (ii) $\frac{7}{17}$, (iii) $\frac{3}{17}$,

19. (i) $\frac{8}{15}$, (ii) $\frac{1}{5}$, (iii) $\frac{1}{5}$, (iv) $\frac{2}{15}$, (v) $\frac{2}{3}$, **20.** (a) $\frac{2}{3}$; (b) (i) $\frac{1}{8}$, (ii) $\frac{1}{2}$, (iii) $\frac{3}{4}$, (iv) 1

21. (i) $\frac{2}{5}$, (ii) $\frac{3}{10}$, (iii) $\frac{8}{25}$ **22.** (i) $\frac{1}{5}$, (ii) $\frac{15}{19}$ **23.** (i) 0.88 (ii) 0.96 **24.** (i) 0.008 **25.** 24 **26.** $\frac{2}{7}$

27. $\frac{\pi}{24}$ **28.** (i) $\frac{3}{10}$, (ii) $\frac{9}{10}$ **29.** $\frac{1}{6}$ **30.** (i) $\frac{3}{11}$, (ii) $\frac{8}{11}$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

1. Mark Question :

1. From a well shuffled pack of cards, a card is drawn at random. Find the probability of getting a black queen. **[Delhi- 2008]**

2. A bag contains 4 red and 6 black balls. A ball is taken out of the bag at random. From the probability of getting a black ball. **[Al-2008]**

3. A die is thrown once. Find the probability of getting a number less than 3. **[Foreign-2008]**

4. Cards bearing numbers 3 to 20 are placed in a bag and mixed thoroughly. A card is taken out from the bag at random. What is the probability that the number on the card taken out is an even number? **[Delhi-2008 C]**

5. Two friends were born in the year 2000. What is the probability that they have the same birthday? **[Al-2008 C]**

OR

Two coins are tossed simultaneously. Find the probability of getting exactly one head. **[Al-2008]**

2 Mark Question :

1. A die is thrown once. Find the probability of getting **[Delhi-2008]**

(i) A prime number

(ii) A number divisible by 2.

2. Cards, carked with numbers 5 to 50, are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken card is :

(i) A prime number less than 10.

(ii) A number which is a perfect square

3. A pair of dice is thrown once. Find the probability of getting the same number on each dice. **[Foreign-2008]**

4. A bag contains 5 red, 4 blue and 3 green balls. A ball is taken out of the bag at random. Find the probability that the selected ball is (i) of red colour (ii) not of green colour.

OR

A card is drawn at random from a well-shuffled deck of playing cards. Find the probability of drawing a

(i) face card (ii) card which is neither a king nor a red card. **[Delhi-2008 C]**

5. Two dice are thrown simultaneously. Find the probability that the sum of the two numbers appearing on the top is less than or equal to 10.

OR

The king, queen and jack of diamonds are removed from a pack of 52 cards and than the pack is well shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (i) diamonds (ii) a jack. **[Al-2008 C]**

3 Marks Question :

1. A bag contains 5 black, 7 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) red (ii) black or white (iii) not black. **[Delhi-2004]**

2. A bag contains 7 black, 8 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) red (ii) black or white (iii) not black. **[Delhi-2004]**

3. A bag contains 6 black, 7 red and 2 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) red (ii) black or white (iii) not black. **[Delhi-2004]**

4. A bag contains 4 red, 5 black and 6 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) white (ii) red (iii) not black (iv) red or white. **[Al-2004]**

5. A bag contains 4 red, 5 black and 6 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) white (ii) red (iii) not black (iv) red or white. **[Al-2004]**

6. A bag contains 3 red, 5 black and 7 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) white (ii) red (iii) not black (iv) red or white. **[Al-2004]**

7. A bag contains 6 red, 5 black and 4 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) white (ii) red (iii) not black (iv) red or white. **[Al-2004]**

8. 15 cards, numbered 1, 2, 3,…., 15 are put in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the card drawn bears (i) an even number (ii) a number divisible by 2 or 3. **[Al-2004]**

9. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither an ace nor a king. **[Foreign-2004]**

10. Out of 400 bulbs in a box, 15 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective.

OR

Find the probability of getting 53 Fridays in a leap year. **[Al-2004 C]**

11. A bag contains 8 red, 6 white and 4 black balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) red or white (ii) not black (iii) neither white nor black **[Al-2005]**

12. A bag contains 5 white balls, 7 red balls, 4 black balls, and 2 blue balls. Out ball is drawn at random from the bag. What is the probability that the ball drawn is :
(i) white or blue (ii) red or black (iii) not white (iv) neither white nor black **[Delhi-2006]**

13. A card is drawn at random from a well shuffled deck of playing cards. Find the probability that the card drawn is :
(i) a king or a jack (ii) a non ace (iii) a red card (iv) neither a king nor a queen **[Delhi-2006]**

14. A card is drawn at random from a well shuffled deck of playing cards. Find the probability that the card drawn is :
(i) a card of spade or an ace (ii) a red king (iii) neither a king nor a queen (iv) either a king or queen **[Delhi-2006]**

15. A box contains 19 balls bearing numbers 1, 2, 3, ….., 19. A ball is drawn at random from the box. What is the probability that the number of the ball is (i) a prime number (ii) divisible by 3 or 5 (iii) neither divisible by 5 nor by 10 (iv) an even number. **[Delhi-2006 C]**

16. Find the probability that a number selected at random from the numbers, …., 35 is a (i) prime number (ii) multiple of 7 (iii) multiple of 3 or 5. **[Delhi-2006 C]**

17. From a pack of 52 playing cards, jacks, queens, kings and aces of red colour are removed. From the remaining cards, a card is drawn at random. Find the probability that the card drawn is : (i) a black queen (ii) a red card (iii) a black jack (iv) a picture card (jacks, queens and kings are picture cards.) **[Al-2006 C]**

18. Cards marked with numbers 3, 4, 5, …, 50 are placed in a box and mixed thoroughly. One cards is drawn at random from the box. Find the probability that number on the drawn card is (i) divisible by 7 (ii) a number which is a perfect square. **[Delhi-2007]**

19. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue from the bag is thrice that of a red ball, find the number of blue balls in the bag. **[Delhi-2007]**

20. A box contains 5 red balls, 4 green balls and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is : (i) white (ii) neither red nor white. **[Al-2006]**

21. All the three face cards of spades are removed from a deck of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting a card of (i) a black face card (ii) a queen (iii) a black card. **[Al-2009]**

22. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (i) heart (ii) queen (iii) clubs. **[Delhi-2009]**

23. Two dice are thrown simultaneously. What is the probability that
(i) 5 will not come up on either of them?
(ii) 5 will come up on at least one?
(iii) 5 will come up at both dice? **[Al-2009]**

OR

A box has cards numbered 14 to 99. Cards are mixed thoroughly and a card is drawn from the bag at random. Find the probability that the number on the card, drawn from the box is :
(i) an odd number
(ii) a perfect square number
(iii) a number divisible by 7. **[Foreign-2009]**

1 Mark :

1. $\frac{1}{26}$ **2.** $\frac{3}{5}$ **3.** $\frac{1}{3}$ 4. $\frac{1}{2}$ **5.** $\frac{1}{366}$ OR $\frac{1}{2}$

2 Marks :

1. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ **2.** (i) $\frac{1}{23}$ (ii) $\frac{5}{46}$ 3. $\frac{1}{6}$ **4.** (i) $\frac{5}{12}$ (ii) $\frac{3}{4}$ OR (i) $\frac{3}{13}$ (ii) $\frac{6}{13}$ **5.** $\frac{11}{12}$ OR (i) $\frac{10}{49}$, (ii) $\frac{3}{49}$

3 Marks :

1. (i) $\frac{7}{15}$ (ii) $\frac{8}{15}$ (iii) $\frac{2}{3}$ 2. (i) $\frac{1}{3}$ (ii) $\frac{2}{3}$ (iii) $\frac{8}{15}$ 3. (i) $\frac{7}{15}$ (ii) $\frac{8}{15}$ (iii) $\frac{3}{5}$ 4. (i) $\frac{2}{5}$ (ii) $\frac{4}{15}$ (iii) $\frac{2}{3}$ (iv) $\frac{2}{3}$

5. (i) $\frac{2}{5}$ (ii) $\frac{4}{15}$ (iii) $\frac{2}{3}$ (iv) $\frac{2}{3}$ 6. (i) $\frac{7}{15}$ (ii) $\frac{1}{5}$ (iii) $\frac{2}{3}$ (iv) $\frac{2}{3}$ 7. (i) $\frac{4}{15}$ (ii) $\frac{2}{5}$ (iii) $\frac{2}{3}$ (iv) $\frac{2}{3}$ 8. (i) $\frac{7}{15}$ (ii) $\frac{2}{3}$

9. $\frac{11}{13}$ 10. $\frac{77}{80}$ OR $\frac{2}{7}$ 11. (i) $\frac{7}{9}$ (ii) $\frac{7}{9}$ (iii) $\frac{4}{9}$ 12. (i) $\frac{7}{18}$ (ii) $\frac{11}{18}$ (iii) $\frac{13}{18}$ (iv) $\frac{1}{2}$ 13. (i) $\frac{2}{13}$ (ii) $\frac{12}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{11}{13}$

14. (i) $\frac{4}{13}$ (ii) $\frac{1}{26}$ (iii) $\frac{11}{13}$ (iv) $\frac{2}{13}$ 15. (i) $\frac{8}{19}$ (ii) $\frac{8}{19}$ (iii) $\frac{16}{19}$ (iv) $\frac{9}{19}$ 16. (i) $\frac{11}{35}$ (ii) $\frac{1}{7}$ (iii) $\frac{16}{35}$ 17. (i) $\frac{1}{22}$ (ii)

$\frac{9}{22}$ (iii) $\frac{1}{22}$ (iv) $\frac{3}{22}$ 18. (i) $\frac{1}{8}$ (ii) $\frac{5}{48}$ 19. 15 20. (i) $\frac{7}{16}$ (ii) $\frac{1}{4}$ 21. (i) $\frac{3}{49}$ (ii) $\frac{3}{49}$ (iii) $\frac{23}{49}$ 22. (i) $\frac{13}{49}$ (ii) $\frac{3}{49}$ (iii) $\frac{23}{49}$ **23.** (i) $\frac{25}{36}$ (ii) $\frac{11}{36}$ (iii) $\frac{1}{36}$ OR (i) $\frac{1}{2}$ (ii) $\frac{4}{43}$ (iii) $\frac{13}{86}$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

1. Find the probability of getting a head in a throw of a coin.

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) None of these

Directions (for Q. No. 2-5) : Two fair coins are tossed simultaneously. Find the probability of

2. Getting only one head

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

3. Getting two heads

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{8}$

4. Getting at least two heads

(A) $\frac{7}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{4}{5}$

5. Getting at least two heads

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 1

Directions (for Q. No. 6-12) : Three fair coins are tossed simultaneously. Find the probability of

6. Getting one head

(A) 0 (B) $\frac{3}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{8}$

7. Getting one tail

(A) 1 (B) $\frac{1}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{8}$

8. Getting at least one heads

(A) $\frac{7}{8}$ (B) $\frac{1}{8}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$

9. Getting two heads

(A) $\frac{3}{5}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{2}{5}$

10. Getting two heads

(A) $\frac{3}{8}$ (B) $\frac{7}{8}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

11. Getting at least one head and one tail

(A) $\frac{2}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{10}$ (D) $\frac{4}{3}$

12. Getting more heads than the number of tails

(A) 2 (B) $\frac{7}{8}$ (C) $\frac{5}{8}$ (D) $\frac{1}{2}$

13. Getting a number less than 7 but greater than 0

(A) 0 (B) $\frac{3}{4}$ (C) 1 (D) $\frac{7}{8}$

14. Getting a multiple of 3.

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{5}{6}$ (D) None of these

15. Getting a prime number

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{5}{7}$ (D) $\frac{5}{8}$

16. Getting an even number

(A) $\frac{1}{2}$ (B) $\frac{4}{5}$ (C) $\frac{5}{7}$ (D) $\frac{5}{8}$

Directions (for Q. No. 17 and 18) : A coin is tossed successively three times. Find the probability of

17. Getting exactly one head or two heads.

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{8}$

18. Getting no heads.

(A) 0 (B) 1 (C) $\frac{1}{8}$ (D) $\frac{7}{8}$

Directions (for Q. No. 19-27) : Two dice are rolled simultaneously. Find the probability of

19. Getting a total of 9

(A) $\frac{1}{3}$ (B) $\frac{1}{9}$ (C) $\frac{8}{9}$ (D) $\frac{9}{10}$

20. Getting a sum greater than 9

(A) $\frac{10}{11}$ (B) $\frac{5}{6}$ (C) $\frac{1}{6}$ (D) $\frac{8}{9}$

21. Getting a total of 9 or 11

(A) $\frac{2}{99}$ (B) $\frac{20}{99}$ (C) $\frac{1}{6}$ (D) $\frac{1}{10}$

22. Getting a doublet

(A) $\frac{1}{12}$ (B) 0 (C) $\frac{5}{8}$ (D) $\frac{1}{6}$

23. Getting a doublet of even number

(A) $\frac{5}{8}$ (B) $\frac{1}{12}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$

24. Getting a multiple of two on one die and a multiple of three on the other.

(A) $\frac{15}{36}$ (B) $\frac{25}{36}$ (C) $\frac{11}{36}$ (D) $\frac{5}{6}$

25. Getting the sum of numbers on the two faces divisible by 3 or 4.n even number

(A) $\frac{4}{9}$ (B) $\frac{1}{7}$ (C) $\frac{5}{9}$ (D) $\frac{7}{12}$

26. Getting the sum as a prime number

(A) $\frac{3}{5}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

27. Getting atleast one 5.

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{5}{36}$ (D) $\frac{11}{36}$

Directions (for Q. No. 28-35) : One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to drawn. Find the probability that

28. The card drawn is black

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{8}{13}$ (D) can't be determined

29. The card drawn is a queen

(A) $\frac{1}{12}$ (B) $\frac{1}{13}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

30. The card drawn is black and a queen

(A) $\frac{1}{13}$ (B) $\frac{1}{52}$ (C) $\frac{1}{26}$ (D) $\frac{5}{26}$

31. The card drawn is either black or a queen

(A) $\frac{15}{26}$ (B) $\frac{13}{17}$ (C) $\frac{7}{13}$ (D) $\frac{5}{26}$

32. The card drawn is either king or a queen

(A) $\frac{5}{26}$ (B) $\frac{1}{13}$ (C) $\frac{2}{13}$ (D) $\frac{12}{13}$

33. The card drawn is either a heart, a king or a queen

(A) $\frac{17}{52}$ (B) $\frac{21}{52}$ (C) $\frac{19}{52}$ (D) $\frac{9}{26}$

34. The card drawn is neither a spade nor a king

(A) 0 (B) $\frac{9}{13}$ (C) $\frac{1}{2}$ (D) $\frac{4}{13}$

35. The card drawn is neither a ace nor a king

(A) $\frac{11}{13}$ (B) $\frac{1}{2}$ (C) $\frac{2}{13}$ (D) $\frac{11}{26}$

36. From a well shuffled pack of 52 cards, three cards are drawn at random. Find the probability of drawing an ace, a king and a jack.

(A) $\frac{16}{5525}$ (B) $\frac{16}{625}$ (C) $\frac{16}{3125}$ (D) None of these

37. Four cards are drawn at random from a pack of 52 cards. Find the probability of getting all the four cards of same number.

(A) $\frac{17}{1625}$ (B) $\frac{1}{20825}$ (C) $\frac{7}{25850}$ (D) None of these

38. From a well shuffled pack of 52 cards, four cards are accidently dropped. Find the probability that one card is missing from each suit.

(A) $\frac{17}{20825}$ (B) $\frac{2197}{20825}$ (C) $\frac{197}{1665}$ (D) None of these

39. Four cards are drawn at random from a pack of 52 cards. Find the probability of getting all the four cards different number.

(A) $\frac{141}{4165}$ (B) $\frac{117}{833}$ (C) $\frac{264}{4165}$ (D) None of these

Directions (for Q. No. 40-13) : Four dice are thrown simultaneously. Find the probability that

40. All of them show the same face.

(A) $\frac{1}{216}$ (B) $\frac{15}{16}$ (C) $\frac{15}{36}$ (D) $\frac{1}{2}$

41. All of them show the different face.

(A) $\frac{3}{28}$ (B) $\frac{5}{18}$ (C) $\frac{15}{36}$ (D) $\frac{11}{36}$

42. Two of them show the same face and remaining two show the different faces.

(A) $\frac{4}{9}$ (B) $\frac{5}{9}$ (C) $\frac{11}{18}$ (D) $\frac{7}{9}$

43. At least two of them show the same face.

(A) $\frac{37}{72}$ (B) $\frac{11}{66}$ (C) $\frac{47}{72}$ (D) $\frac{25}{36}$

44. What is the probability that the number selected from the numbers 1, 2, 3, ….., 20, is a prime number when each of the given numbers is equally likely to be selected ?

(A) $\frac{7}{10}$ (B) $\frac{2}{15}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

45. Tickets numbered from 1 to 18 are mixed up together and then a ticket is drawn at random. Find the probability that the ticket has a number which is a multiple of 2 or 3.

(A) $\frac{1}{3}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$

46. In a lottery of 100 tickets numbered 1 to 100, two tickets are drawn simultaneously. Find the probability that both the tickets drawn have prime numbers.

47. In the previous question, find the probability that none of the tickets drawn has a prime number.

(A) $\frac{29}{66}$ (B) $\frac{17}{33}$ (C) $\frac{37}{66}$ (D) $\frac{17}{50}$

48. Find the probability that a leap year selected at random will contain 53 Sundays.

(A) $\frac{5}{7}$ (B) $\frac{3}{4}$ (C) $\frac{4}{7}$ (D) $\frac{2}{7}$

Directions (for Q. No. 49-53) : A bag contains 8 red and 4 green balls. Find the probability that

49. The ball drawn is red when one ball is selected at random.

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{5}{6}$

50. All the 4 balls drawn are red when 4 balls drawn at random.

(A) $\frac{17}{32}$ (B) $\frac{14}{99}$ (C) $\frac{7}{12}$ (D) None of these

51. All the 4 balls drawn are green when 4 balls drawn at random.

(A) $\frac{1}{495}$ (B) $\frac{7}{99}$ (C) $\frac{5}{12}$ (D) None of these

52. Two balls are red and one ball is green when three balls are drawn at random.

(A) $\frac{56}{99}$ (B) $\frac{112}{495}$ (C) $\frac{78}{495}$ (D) None of these

53. Three balls are drawn and none of them is red.

(A) $\frac{68}{99}$ (B) $\frac{7}{99}$ (C) $\frac{4}{495}$ (D) None of these

54. The odds in favor of an event are 2:7. find the probability of occurrence of this event.

(A) $\frac{2}{9}$ (B) $\frac{5}{12}$ (C) $\frac{7}{12}$ (D) None of these

55. The odds against of an event are 5:7. find the probability of occurrence of this event.

(A) $\frac{3}{8}$ (B) $\frac{7}{12}$ (C) $\frac{2}{7}$ (D) None of these

56. If there are two children in a family, find the probability that there is atleast one girl in the family.

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) None of these

57. From a group of 3 men and 2 women, two persons are selected at random. Find the probability that at least one woman is selected.

(A) $\frac{1}{5}$ (B) $\frac{7}{10}$ (C) $\frac{2}{5}$ (D) None of these

58. A box contains 5 defective and 15 non-defective bulbs. Two bulbs are chosen at random. Find the probability that both the bulbs are non-defective

(A) $\frac{5}{19}$ (B) $\frac{3}{20}$ (C) $\frac{21}{38}$ (D) None of these

59. In the previous question, find the probability that at least 3 bulbs are defective when 4 bulbs are selected at random.

(A) $\frac{31}{969}$ (B) $\frac{7}{20}$ (C) $\frac{1}{20}$ (D) None of these

OBJECTIVE **ANSWER KEY**

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	A	B	B	C	D	D	A	B	C	D	D	C	B	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	A	B	C	B	C	C	D	B	C	C	B	D	A	B	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	C	C	C	B	A	A	B	B	C	A	B	B	C	C	C
Que.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	
Ans.	A	C	D	A	B	A	B	C	A	B	C	B	C	A	

QUADRATIC EQUATIONS

★ INTRODUCTION

When a polynomial f(x) is equated to zero, we get an equation which is known as a polynomial equation. If f(x) is a linear polynomial than f(x) = 0 is called a linear equation. For example, $3x - 2 = 0$, $4t + \frac{3}{5} = 0$ etc. are linear equations. If f(x) is quadratic polynomial i.e., $f(x) = ax^2 + bc + c$, $a \neq 0$, then $f(x) = 0$ i.e., $ax_2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation. Such equations arise in many real life situations. In this chapter, we will learn about quadratic and various ways of finding their zeros or roots. In the end of the chapter, we will also discuss some applications of quadratic equations in daily life situations.

★ HISTORICAL FACTS

On clay tables dated between 1800 BC and 1600 BC, the ancient Babylonians left the earliest evidence of the discovery of quadratic equations, and also gave early methods for solving them.
Indian mathematician Baudhayana who wrote a Sulba Sutra in ancient India circa 8th century BC first used quadratic equations of the form : $ax^2 = c$ and $ax^2 + bx = c$ and also gave methods for solving them.
Babylonian mathematicians from circa 400 BC and Chinese mathematicians from circa 200 BC used the method of completing the square to solve quadratic equations with positive roots, but did not have a general formula.
Euclid, a Greek mathematician, produced a more abstract geometrical method around 300 BC.
The first mathematician to have found negative solutions with the general algebraic formula was Brahmagupta (India, 7th century). He gave the first explicit (although still not completely general) solutions of the quadratic equations $ax^2 + bx = c$ as follows :
"To the absolute number multiplied by four times the [coefficient of the] square, add the square of the [coefficient of the] middle term; the square root of the same, less the [coefficient of the] middle term, being divided by twice the [coefficient of the] square is the value."

This is equivalent to : $x = \frac{\sqrt{4ac + b^2} - b}{2a}$

Muhammad ibn Musa al-Kwarizmi (Persia, 9th century) developed a set of formulae that worked for positive solutions.
Bhaskara II (1114-1185), an Indian mathematician-astronomer, solved quadratic equations with more than one unknown and is considered the originator of the equation.
Shridhara (India, 9th century) was one of the first mathematicians to give a general rule for solving a quadratic equation.

★ QUADRATIC EQUATIONS

A polynomial equations of degree two is called a quadratic equation.
Ex. $2x^2 - 3x + 1 = 0$, $4x - 3x^2 = 0$ and $1 - x^2 = 0$
General form of quadratic equations : $ax^2 + bx + c = 0$, where a,b,c, are real numbers and $a \neq 0$.
Moreover, it is general form of a quadratic equation in standard form.
Types of Quadratic Equations : A quadratic equation can be of the following types :
(i) $b = 0$, $c \neq 0$ i.e., of the type $ax^2 + c = 0$ **(Pure quadratic equation)**
(ii) $b \neq 0$,$c = 0$ i.e., of the type $ax^2 + bx = 0$
(iii) $b = 0$, $c = 0$ i.e., of the type $ax^2 = 0$
(iv) $b \neq 0$,$c \neq 0$ i.e., of the type $ax^2 + bx + c = 0$ **(Mixed or complete quadratic equation)**
Roots of quadratic equation : $x = \alpha$ is said to be root of the quadratic equation ax2 + bx + c = 0, $a \neq 0$ iff $x = \alpha$ satisfies the quadratic equation i.e. in other words the value of $a\alpha^2 + b\alpha + c$ is zero.
Solving a quadratic equation : The determination of all the roots of a quadratic equation is called solving the quadratic equation.

Ex.1 Check whether the following are quadratic equations :

(i) $(x+1)^2 = (x-3)$ (ii) $(x-2)(x+1) = (x-1)(x+3)$ (iii) $(x-3)(2x+1) = x(x+5)$

Sol. (i) Here, the given equation is $(x+1)^2 = 2(x-3)$

$\Rightarrow$ $x^2+2x+1 = 2x-6 \Rightarrow x^2+2x-2x+1+6 = 0$

$\Rightarrow$ $x^2+7 = 0 \Rightarrow x^2+0.x+7 = 0$, which is of the form $ax^2+bx+c = 0$

Hence, $(x+1)^2 = 2(x-3)$ is a quadratic equation.

(ii) Here, the given equation is $(x-2)(x+3) = (x-1)(x+3)$

$\Rightarrow$ $x^2+x-2x-2 = x^2+3x-x-3 \Rightarrow x^2-x^2-x-2x-2+3 = 0 \Rightarrow -3+1 = 0$,

which is not of the form $ax^2+bx+c = 0$

Hence, $(x-2)(x+1) = (x-1)$ is not a quadratic equation.

(iii) Here, the given equation is $(x-3)(2x+1)\,x(x+5)$

$\Rightarrow$ $2x^2+x-6x-3 = x^2+5x \Rightarrow 2x^2-x^2-5x-5x-3 = 0 \Rightarrow x^2-10x-3 = 0$,

which is of the form $ax^2+bx+c = 0$

Hence, $(x-3)(2x+1) = x(x+5)$ is a quadratic equation.

Ex.2 In each of the following, determine whether the given values are the solution of the given equation or not :

(i) $\frac{2}{x^2}-\frac{5}{x}+2 = 0; x = 5, x = \frac{1}{2}$ (ii) $a^2x^2-3abx+2b^2 = 0; x = \frac{a}{b}, x = \frac{b}{a}$

Sol. (i) Putting x = 5 and $x = \frac{1}{2}$ in the given equation.

$\frac{2}{(5)^2}-\frac{5}{5}+2$ and $\frac{2}{\left(\frac{1}{2}\right)^2}-\frac{5}{\left(\frac{1}{2}\right)}+2$

$\Rightarrow$ $\frac{2}{25}-1+2$ and $\frac{2}{\frac{1}{4}}-\frac{5}{\frac{1}{2}}+2$

$\Rightarrow$ $\frac{2}{25}+1$ and $8-10+2 \Rightarrow \frac{27}{25}$ and 0

i.e., x = 5 does not satisfy but $x = \frac{1}{2}$ satisfies the given equation.

Hence, x = 5 is not a solution but $x = \frac{1}{2}$ is a solution of $\frac{2}{x^2}-\frac{5}{x}+2 = 0$.

(ii) Putting $x = \frac{a}{b}$ and $x = \frac{b}{a}$ in the given equation.

$a^2\left(\frac{a}{b}\right)^2-3ab\left(\frac{a}{b}\right)+2b^2$ and $a^2\left(\frac{b}{a}\right)^2-3ab\left(\frac{b}{a}\right)+2b^2$

$\Rightarrow$ $\frac{a^2}{b^2}+2b^2-3a^2$ and 0

i.e., $x = \frac{a}{b}$ does not satisfy but $x = \frac{b}{a}$ satisfies the given equation.

Hence, $x = \frac{b}{a}$ is a solution but $x = \frac{a}{b}$ is not a solution of $a^2x^2-3abx+2b^2 = 0$.

Ex.3 Find the values of p and q for which $x = \frac{3}{4}$ and x = – 2 are the roots of the equation $px^2+qx-6 = 0$.

Sol. Since $x = \frac{3}{4}$ and x = – 2 are the roots of the equation $px^2+qx-6 = 0$.

$\therefore$ $p\left(\frac{3}{4}\right)^2+q\left(\frac{3}{4}\right)-6 = 0$ and $p(-2)^2+q(-2)-6 = 0$

$\Rightarrow \quad p \times \frac{9}{16} + q \times \frac{3}{4} - 6 = 0$ and $4p - 2q - 6 = 0$

$\Rightarrow \quad \frac{9p + 12q - 96}{16} = 0$ and $4p - 2q - 6 = 0$

$\Rightarrow \quad 9p + 12q - 96 = 0$ and $4p - 2q - 6 = 0$

$\Rightarrow \quad 3p + 4q - 32 = 0$ …(i)

and $2p - q - 3 = 0$ …(ii)

Multiplying (2) by 4, we get $8p - 4q - 12 = 0$ …(iii)

Adding (1) and (3), we get $p = 4$

Putting the value of p in equation (2), we get

$23 \times 4 - q - 3 = 0 \Rightarrow q = 5$

Hence, $p = 4$, $q = 5$.

★ METHODS OF SOLVING QUADRATIC EQUATIONS

Solution by factorization method

Algorithm :

Step-I : Factorize the constant term of the given quadratic equation.

Step-II : Express the coefficient of middle term as the sum or difference of the factors obtained in step-I. Clearly, the product of these two factors will be equal to the product of the coefficient of x^2 and constant term.

Step-III : Split the middle term in two parts obtained in stop-II

Step-IV : Factorize the quadratic equation obtained in step-III by grouping method.

Ex.4 Solve the following quadratic equation by factorization method $x^2 - 2ax + a^2 - b^2 = 0$

Sol. Factors of the constant term $a^2 - b^2$ are$(a - b)$&$(a + b)$ also coefficient of the middle term $= -2a = -[(a - b)+(a + b)]$

$\Rightarrow \quad x^2 - 2ax + a^2 - b^2 = 0$

$\Rightarrow \quad x^2 - \{(a - b) + (a + b)\} x + (a + b)(a - b) = 0$

$\Rightarrow \quad x^2 - (a - b) x - (a + b) x + (a - b)(a + b) = 0$

$\Rightarrow \quad x[x - (a - b)] - (a + b)[x - (a - b)] = 0$

$\Rightarrow \quad [x - (a - b)][x - (a + b)] = 0$

$x - (a - b) = 0$ or $x - (a + b) = 0$

$x = a - b$, $x = a + b$

Ex.5 Solve the quadratic equation $5x^2 = -16x - 12$ by factorization method.

Sol. $5x^2 = -16x - 12$

$5x^2 + 16x + 12 = 0$

$5x^2 + 10x + 6x + 12 = 0$

$5x(x + 2) + 6(x + 2) = 0$

$(x + 2)(5x + 6) = 0$

$x + 2 = 0 \Rightarrow x = -2$

$5x + 6 = 0 \Rightarrow x = \frac{-6}{5}$

Solution by factorization method

Algorithm :

Stop-I : Obtain the quadratic equation. Let the quadratic equation be $ax^2 + bx + c = 0$, $a \neq 0$.

Stop-II : Make the coefficient of x^2 unite by dividing throughout by it, if it is not unity that is obtain

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Stop-III : Shift the coefficient term $\frac{c}{a}$ on R.H.S. to get $x^2+\frac{b}{a}=-\frac{c}{a}$

Stop-IV : Add square of half of the coefficient of x. i.e., $\left(\frac{b}{2a}\right)^2$ on both sides to obtain.

$$x^2+2\left(\frac{b}{2a}\right)x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$$

Stop-V : Write L.H.S. as the perfect square and simplify R.H.S. to get $\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$

Stop-VI : Take square root of both sides to get $x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$

Stop-VII : Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on R.H.S. i.e., $x=-\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$

Ex.6 Solve : $9x^2 - 15x + 6 = 0$

Sol. Here, $9x^2 - 15x + 6 = 0$

$\Rightarrow$ $x^2-\frac{15}{9}x+\frac{6}{9}=0$ [Dividing throughout by 9]

$\Rightarrow$ $x^2-\frac{5}{3}x+\frac{2}{3}=0$

$\Rightarrow$ $x^2-\frac{5}{3}x-\frac{2}{3}=0$ [Shifting the constant term on RHS]

$\Rightarrow$ $x^2-2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2=\left(\frac{5}{6}\right)^2+\frac{2}{3}$ [Adding square of half of coefficient x on both sides]

$\Rightarrow$ $\left(x-\frac{5}{6}\right)^2=\frac{25}{36}+\frac{2}{3}\Rightarrow\left(x-\frac{5}{6}\right)^2=\frac{25-24}{36}\Rightarrow\left(x-\frac{5}{6}\right)^2=\frac{1}{36}$

$\Rightarrow$ $x-\frac{5}{6}=\pm\frac{1}{6}$ [Taking square root of both sides]

$\Rightarrow$ $x=\frac{5}{6}=\pm\frac{1}{6}$ $\Rightarrow$ $x=\frac{5}{6}+\frac{1}{6}=1$ or, $x=\frac{5}{6}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$

$\Rightarrow$ $x=1$ or, $x=\frac{2}{3}$

Ex.7 Solve the equation $x^2-(\sqrt{3}+1)x+\sqrt{3}=0$ by the method of completing the square.

Sol. We have,

$x^2-(\sqrt{3}+1)x+\sqrt{3}=0$

$\Rightarrow$ $x^2-(\sqrt{3}+1)x=-\sqrt{3}$

$\Rightarrow$ $x^2-2\left(\frac{\sqrt{3}+1}{2}\right)x+\left(\frac{\sqrt{3}+1}{2}\right)^2=-\sqrt{3}+\left(\frac{\sqrt{3}+1}{2}\right)^2$

$\Rightarrow$ $\left(x-\frac{\sqrt{3}+1}{2}\right)^2=\frac{-4\sqrt{3}+(\sqrt{3}+1)^2}{4}$

$\Rightarrow \quad \left(x-\frac{\sqrt{3}+1}{2}\right)^2=\left(\frac{\sqrt{3}-1}{2}\right)^2 \qquad \Rightarrow x-\frac{\sqrt{3}+1}{2}=\pm\frac{\sqrt{3}-1}{2}$

$\Rightarrow \quad x-\frac{\sqrt{3}+1}{2}\pm\frac{\sqrt{3}-1}{2} \qquad \Rightarrow x=\sqrt{3},\ 1$

Hence, the roots are $\sqrt{3}$ and 1.

Solution by Quadratic Formula "Sreedharacharya's Rule"

Consider quadratic equation $ax^2+bx+c=0, a\neq 0$ then $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\therefore$ The roots of x are

$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$

$\Rightarrow \quad x=\frac{-b+\sqrt{D}}{2a}$ or, $x=\frac{-b-\sqrt{D}}{2a}$, where $D = b^2 - 4ac$

Thus, if $D=b^2-4ac\geq 0$, then the quadratic equation ax2 + bx + c = 0 has real roots α and β given by

$\alpha=\frac{-b+\sqrt{D}}{2a}$ and $\beta=\frac{-b-\sqrt{D}}{2a}$

Discriminant : If $ax^2+bx+c=0, a\neq 0 (a,b,c\in R)$ is a quadratic equation, then the expression $b^2 - 4ac$ is known as its discriminant and is generally denoted by D or Δ.

Ex.8 Solve the quadratic equation $x^2 - 6x + 4 = 0$ by using quadratic formula (Sreedharacharya's Rule).

Sol. On comparing the given equation $x^2 - 6x + 4 = 0$ with the standard quadratic equation $ax^2 + bx + c = 0$, we get a = 1, b = – 6, c = 4

Hence the required roots are

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}=\frac{6\pm\sqrt{36-16}}{2}=\frac{6\pm\sqrt{20}}{2}=\frac{6\pm\sqrt{4\times 5}}{2}$$

$$=\frac{2(3\pm\sqrt{5})}{2}=3\pm\sqrt{5}$$

COMPETITION WINDOW

SOLUTIONS OF EQUATIONS REDUCIBLE TO QUADRATIC FORM

Equations which are not quadratic at a glance but can be reduced to quadratic equations by suitable transformations Some of the common types are :

Type-I : $ax^4 + bx^2 + c = 0$

This can be reduced to a quadratic equation by substituting $x^2 = y$ i.e., $ay^2 + by + c = 0$

e.g. Solve $2x^4 - 5x^2 + 3 = 0$

Putting $x^2 = y$, we get $2y^2 - 5y + 3 = 0$

$\Rightarrow \quad (2y-3)(y-1)=0 \Rightarrow y=\frac{3}{2}$ or 1

$\Rightarrow \quad (2y-3)(y-1)=0 \Rightarrow y=\frac{3}{2}$ or 1

Type-II : $aip(x)\}^2 + b.p(x) + c = 0$ where p(x) is an expression in 'x'

Put $p(x) = y$, $\{p(x)\}^2 = y^2$ to get the quadratic equation $ay^2 + by + c = 0$.

e.g. Solve $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0$, $x \in R$

Putting $x^2 + 3x = y$, we get y2 – y – 6 = 0

Solving, we get y = 3 or – 2

$\Rightarrow \quad x^2+3x=3$ or $x^2+3x=-2$

$\Rightarrow \quad x\dfrac{-3\pm\sqrt{21}}{2}$ or $x=-2$ or -1.

Type-III : $ap(x)+\dfrac{b}{p(x)}=c$, where p(x) is an expression in x.

Put p(x) = y to obtain the quadratic equation $ay^2 - cy + b = 0$.

e.g. Solve $\dfrac{x}{x+1}+\dfrac{x+1}{x}=\dfrac{34}{15}$

Putting $\dfrac{x}{x+1}=y$, we get, $y=\dfrac{1}{y}=\dfrac{34}{15}$

$\Rightarrow \quad 15y^2-34y+15=0 \Rightarrow y=\dfrac{5}{3}$ or $\dfrac{3}{5}$

$\Rightarrow \quad \dfrac{x}{x+1}=\dfrac{5}{3}$ or $\dfrac{x}{x+1}=\dfrac{3}{5} \Rightarrow x=\dfrac{-5}{2}$ or $\dfrac{3}{2}$

Type-IV : (i) $a\left[x^2+\dfrac{1}{x^2}\right]+b\left[x+\dfrac{1}{x}\right]+c=0$ **(ii)** $a\left[x^2+\dfrac{1}{x^2}\right]+b\left[x-\dfrac{1}{x}\right]+c=0$

If the coefficient of b in the given equation contains $x+\dfrac{1}{x}$, then replace $x^2+\dfrac{1}{x^2}$ by $\left(x^2+\dfrac{1}{x^2}\right)^2-2$ and put $x+\dfrac{1}{x}=y$. In case the coefficient of b is $x-\dfrac{1}{x}$, then replace $x^2+\dfrac{1}{x^2}$ by $\left(x-\dfrac{1}{x}\right)^2+2$ and put $x-\dfrac{1}{x}=y$.

e.g. Solve $9\left[x^2+\dfrac{1}{x^2}\right]-9\left[x+\dfrac{1}{x}\right]-52=0$

Putting $x+\dfrac{1}{x}=y$, we get : $9(y^2-2)-9y-52=0$

$\Rightarrow \quad y=\dfrac{10}{3}$ or $y=-\dfrac{7}{3} \quad \Rightarrow x+\dfrac{1}{x}=\dfrac{10}{3}$ or $x+\dfrac{1}{x}=-\dfrac{7}{3}$

$\Rightarrow \quad x=\dfrac{1}{3}$ or 3 or $x=\dfrac{-7\pm\sqrt{13}}{6}$

Type-V : (x + a) (x + b) (x + c) (x + d) + k = 0, such that a + b = c + d.

Rewrite the equation in the form

{(x + a) (x + b)} . {(x + c) (x + d)} + k = 0

Put $x^2 + x(a + b) = x^2 + x(c + d) = y$ to obtain a quadratic equation in y i.e. (y + ab) (y + cd) = k.

e.g. Solve (x + 1) (x + 2) (x + 3) (x + 4) = 120

$\because \quad$ 1 + 4 = 2 + 3, we write the equation in the following form :

{(x + 1) (x + 4)} . {(x + 2) (x + 3)} = 120

$\Rightarrow \quad (x^2 + 5x + 4)(x^2 + 5x + 6) = 120$

Putting $x^2 + 5x = y$, we get (y + 4) (y + 6) = 120

$\Rightarrow \quad y = -16$ or 6

$\Rightarrow \quad x^2 + 5x = -16$ or $x^2 + 5x = 6$

$\Rightarrow \quad x = -6$ or 1 ($x^2 + 5x + 16$ has no real solution)

Type-VI : $\sqrt{ax+b}=(cx+d)$

Square both sides to obtain $(ax + b) = (cx + d)^2$

or $c^2x^2 + (2cd - a)x + d^2 - b = 0$

Reject those values of x, which do not satisfy both $ax+b\geq 0$ and $cx+d\geq 0$

e.g. Solve : $\sqrt{2x+9}+x=13$

$\Rightarrow$ $(2x + 9) = (13 - x)^2$ (on squaring both sides)

$\Rightarrow$ $x^2 - 28x + 160 = 0$

$\Rightarrow$ $x = 20$ or 8

$x = 20$ does not satisfy $2x+9 \geq 0$. So, $x = 8$ is the only root.

Type-VII : $\sqrt{ax^2+bx+c} = dx+e$

Square both sides to obtain the quadratic equation $x^2 (a - d^2) + x (b - 2de) + (c - e^2) = 0$. solve it and reject those value of x which do not satisfy $ax2 + bx + c \geq 0$ and $dx + e \geq 0$.

e.g. Solve : $\sqrt{3x^2+x+5} = x-3$

$\Rightarrow$ $3x^2 + x + 5 = (x - 3)^2$ (On squaring both sides)

$\Rightarrow$ $2x^2+7x-4=0 \Rightarrow x=\frac{1}{2}$ or -4

No value of x satisfy $3x^2+x+5 \geq 0$ and $x-3 \geq 0$

Type-VIII : $\sqrt{ax+b} \pm \sqrt{cx+d} = e$

Square both sides and simplify in such a manner that the expression involving radical sing on one side and all other terms are on the other side. square both sides of the equation thus obtained and simplify it to obtain a quadratic in x. Reject these values which do not satisfy $ax + b \geq 0$ and $cx + d \geq 0$.

e.g. Solve : $\sqrt{4-x}+\sqrt{x+9}=5$

$\Rightarrow$ $\sqrt{4-x} = 5-\sqrt{x+9}$

$\Rightarrow$ $x+15 = 5\ \sqrt{x+9}$ (on squaring both sides)

$\Rightarrow$ $(x+15)^2 = 25\ \sqrt{x+9}$ (on squaring both sides)

$\Rightarrow$ $x = 0$ or -5

Clearly, $x = 0$ and $x = -5$ satisfy $4 - x \geq 0$ and $x + 9 \geq 0$.

Hence, the roots are 0 and -5

★ NATURE OF THE ROOTS OF THE QUADRATIC EQUATION

Let the quadratic equation be $ax2 + bx + c = 0$. …(i)

Where $a \neq b$ and $a, b, c \in R$.

The roots of the given equation are given by $x=\frac{-b\pm\sqrt{D}}{2a}$.

i.e., of α and β are two roots of the quadratic equation (i). Them.

$$\alpha=\frac{-b+\sqrt{D}}{2a} \text{ and } \beta=\frac{-b-\sqrt{D}}{2a}$$

Now, the following cases are possible.

Case-I : When $D > 0$.

Roots are real and unequal (distinct).

The roots are given by $\alpha=\frac{-b+\sqrt{D}}{2a}$ and $\beta=\frac{-b-\sqrt{D}}{2a}$

Remark : Consider a quadratic equation $ax2 + bx+ c = 0$. where $a,b,c \in Q$, $a \neq b$ and $D > 0$ them :

(i) If D is a perfect square, then roots are rational and unequal.

(ii) If D is not a perfect square, then roots are irrational and unequal. If one root is of the form $p+\sqrt{q}$ (where p is rational and $\sqrt{q}$ is a surd) then the other root will be $p-\sqrt{q}$.

Case-II : When D = 0.

Roots are real and equal and each root $\alpha = \frac{-b}{2a} = \beta$

Case-III : When D < 0.

No real roots exist. Both the roots are imaginary.

Remark : If D < 0, the roots are of the form a $\pm$ ib (a,b $\in$ R&I = $\sqrt{-1}$). If one root is a + ib, then other root will be a – ib.

e.b. $x^2 - 3x + 12 = 0$ has D = – 39 < 0

$\therefore$ It's roots are, $\alpha = \frac{-b+\sqrt{D}}{2a}$ and $\beta = \frac{-b-\sqrt{D}}{2a}$

or $\alpha = \frac{3+\sqrt{-39}}{2}$ and $\beta = \frac{3-\sqrt{-39}}{2}$

or $\alpha = \frac{3}{2}+\frac{i\sqrt{39}}{2}$ and $\beta = \frac{3}{2}-\frac{i\sqrt{39}}{2}$

COMPETITION WINDOW

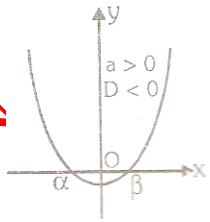
GEOMETRICAL REPRESENTATION OF QUADRATIC EXPRESSION

Consider the quadratic expression, $y = ax^2 + bx + c$, $a \neq 0$ & a,b,c $\in$ R then :

(i) The graph between x,y is always a parabola. If a > 0, then the shape of the parabola is concave upwards & if a < 0 then the shape of the parabola is concave downwards.

(ii) The graph of $y = ax^2 + bx + c$ can be divided into 6 categories which are as follows :

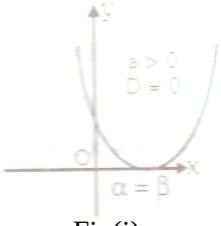
(Let the roots of the equation $ax^2 + bx + c = 0$ be α and β)



Fig(i)
Roots are real and distinct

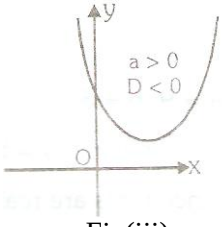
$ax^2+bx+c>0 \forall x \in (-\infty,\alpha)\cup(\beta,\infty)$

$ax^2+bx+c<0 \forall x \in (\alpha,\beta)$



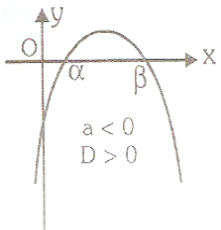
Fig(i)
Roots are coincident

$ax^2+bx+c>0 \forall x \in R-(\alpha)$

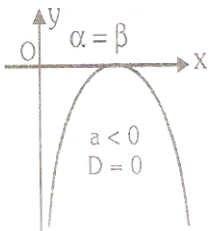


Fig(iii)
Roots are complex conjugates

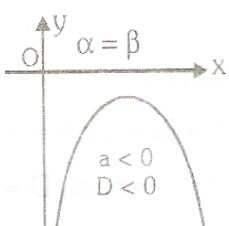
$ax^2+bx+c>0 \forall x \in R$



Fig(iv)



Fig(v)



Fig(vi)

$ax^2+bx+c<0\forall x\in(\alpha,\beta)$ $ax^2+bx+c>0\forall x\in R-(\alpha)$ $ax^2+bx+c>0\forall x\in R$

$ax^2+bx+c>0\forall x\in(-\infty,\alpha)\cup(\beta,\infty)$

Remark : (i) The quadratic expression $ax^2 + bx + c > 0 \ \forall \ x \in R \Rightarrow a > 0, D < 0$ (fig (iii))

(ii) The quadratic expression $ax^2 + bx + c > 0 \ \forall \ x \in R \Rightarrow a > 0, D < 0$ (fig (vi))

Ex.9 Find the nature of the roots of the following equations. If the real roots exist, find them.

(i) $2x^2 - 6x + 3 = 0$ (ii) $2x^2 - 3x + 5 = 0$

Sol. (i) The given equation $2x^2 - 6x + 3 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 2, b = -6$ and $c = 3$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (-6)^2 - 4.2.3 = 36 - 24 = 12 > 0$

$\because$ $D > 0$, roots are real and unequal.

Now, by quadratic formula, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Hence the roots are $x = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

(ii) Here, the given equation is $2x^2 - 3x + 5 = 0$;

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 2, b = -3$ and $c = 5$.

$\therefore$ Discriminant, $D = b^2 - 4ac = 9 - 4 \times 2 \times 5 = 9 - 40 = -31$

$\because$ $D < 0$, the equation has no real roots.

Ex.10 Find the value of k for each of the following quadratic equations, so that they have real and equal roots :

(i) $9x^2 + 18kx + 16 = 0$ (ii) $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

Sol. (i) The given equation $9x^2 + 8kx + 16 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 9, b = 8k$ and $c = 16$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (8k)^2 - 4 \times 9 \times 16 = 64k^2 - 576$

Since roots are real and equal, so

$D = 0 \Rightarrow 64k^2 - 576 = 0 \Rightarrow 64k^2 = 576$

$\Rightarrow$ $k^2 = \frac{576}{64} = - \Rightarrow k = \pm 3$

Hence, $k = 3, -3$

(ii) The given equation $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = (k + 1), b - 2 (k - 1)$ and $c = 1$

$\therefore$ Discriminant, $D = b^2 - 4ac = 4 (k - 1)^2 - 4 (k + 1) \times 1$

$= 4(k^2 - 2k + 1) - 4k - 4$

$\Rightarrow$ $4k^2 - 8k + 4 - 4k - 4 = 4k^2 - 12k$

Since roots are real and equal, so

$D = 0 \Rightarrow 4k^2 - 12k = 0 \Rightarrow 4k (k - 3) = 0$

$\Rightarrow$ either $k = 0$ or $k - 3 = 0 \Rightarrow k = 0$ or $k = 3$

Hence, k = 0, 3.

Ex.11 Find the set of value of k for which the equations $kx^2 + 2x + 1$ has distinct real roots.

Sol. The given equation is $kx^2 + 2x + 1 = 0$

$\therefore$ $D = (4 - 4 \times k \times 1) = 4c - 4k$

For distinct and real roots, we must have, $D > 0$.

Now, $D = (4 - 4k) > 0 \Leftrightarrow 4 > 4k \Leftrightarrow 4k < 4 \Leftrightarrow k < 1$.

$\therefore$ Required set = $\{k \in R : k < 1\}$

Ex.12 Find the of k for which the equations $5x^2 - kx + 4 = 0$ has real roots.

Sol. The given equation is $5x^2 - kx + 4 = 0$

$\therefore$ $D = k^2 - 4 \times 5 \times 4 = k^2 - 80$

For real roots, we must have, $D \geq 0$.

Now, $D > 0 \Leftrightarrow k^2 - 80 \geq 0 \Leftrightarrow k^2 \geq 80 \Leftrightarrow k \geq \sqrt{80}$ or $k \leq -\sqrt{80} \Leftrightarrow k \geq 4\sqrt{5}$ or $k \leq -4\sqrt{5}$.

COMPETITION WINDOW

ROOTS UNDER PARTICUALR CASES

(A) Let the quadratic equation $ax^2 + bx + c = 0$ has real roots and

(i) If $b = 0 \Leftrightarrow$ roots are of equal magnitude but of opposite sign.

(ii) If $c = 0 \Leftrightarrow$ one roots is zero and the other is $-\frac{b}{a}$

(iii) If $a = c \Leftrightarrow$ roots are of opposite sign.

(iv) If $a > 0$ $c < 0$; $a < 0$ $c > 0$ $\Leftrightarrow$ roots are of opposite sign.

(v) If $a > 0, b > 0, c < 0$; $a < 0, b < 0, c < 0$ $\Leftrightarrow$ both roots are negative $(\alpha + \beta < 0 \,\&\, \alpha\beta > 0)$

(vi) If $a > 0, b < 0, c > 0$; $a < 0, b > 0, c < 0$ $\Leftrightarrow$ both roots are positive $(\alpha + \beta < 0 \,\&\, \alpha\beta > 0)$

(vii) If $a + b + c = 0 \Leftrightarrow$ One of the roots is 1 and the other roots is $\frac{c}{a}$.

(viii) If $a = 1$ $b, c \in Z$ and the roots are rational numbers, then these roots must be integers.

(ix) If $a, b, c \in Q$ and D is a perfect square $\Leftrightarrow$ roots are rational.

(x) (A) If $a, b, c \in Q$ and D is positive but not a perfect square $\Leftrightarrow$ roots are irrational.

(B) If $ax^2 + bx + c = 0$ is satisfied by more then two values, it is an identity and $a = b = c = 0$ and vice versa

(C) The quadratic equation whose roots are reciprocal of the roots of $ax^2 + bx + c = 0$ is $cx^2 + bx + a = 0$ (i.e. the coefficients are writer in reverse order).

★ **SUM & PRODUCT OF THE ROOTS**

Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.

Then $\alpha = \frac{-b + \sqrt{+b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{+b^2 - 4ac}}{2a}$

$\therefore$ The sum of roots $\alpha + \beta = -\frac{b}{a} = -\frac{Ceoff.of\ x}{Ceoff.of\ x^2}$

and product of roots = $\alpha.\beta = \frac{c}{a} = -\frac{\text{costant term}}{\text{coefficient of } x^2}$

★ FORMATION OF QUADRATIC EQUATION

Consider the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.
Let α and β be the roots of the quadratic equation

$\therefore \quad \alpha + \beta = -\frac{b}{a}$ and $\alpha.\beta = \frac{c}{a}$

Hence the quadratic equation whose roots are α and β is given by

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e. x^2 – (sum of the roots) x + product of the roots = 0

Ex.13 Form the quadratic equation in each of the following cases when the roots are :

(i) $2+\sqrt{5}2-\sqrt{5}$ (ii) a and $\frac{1}{a}$

Sol. (i) Here roots are $\alpha = 2+\sqrt{5}$ and $\beta = 2-\sqrt{5}$

$\therefore$ Sum of roots $= \alpha + \beta(2+\sqrt{5}) + (2-\sqrt{5})$

$\therefore \quad \alpha + \beta = 4$

and product of the roots $= \alpha.\beta = (2+\sqrt{5}).(2-\sqrt{5}) = 4-5 = -1$

$\therefore \quad \alpha\beta = -1$

$\therefore$ Required equation is
x^2 – (sum of the roots) x + product of the roots = 0

or $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

or $x^2 - (4)x + (-1) = 0$

$\therefore \quad x^2 - 4x - 1 = 0$

(ii) Here roots are a and $\frac{1}{a}$

$\therefore \quad \alpha + \beta = a + \frac{1}{a}$ and $\alpha.\beta = a \times \frac{1}{a} = 1$

Here the required equation is $x^2 - \left(a + \frac{1}{a}\right)x + 1 = 0$

COMPETITION WINDOW

CONDITION FOR TWO QUADRATIC EQUATION TO HAVE A COMMON ROOT

Suppose that the quadratic equation $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$ (where a, a' ≠ 0 and ab' – a'b ≠ 0) have a common root. Let this common root be α Than $a\alpha^2 b\alpha + c = 0$ and $a'\alpha^2 b'\alpha + c' = 0$
Solving the above equations, we get,

$$\frac{\alpha^2}{bc'-b'c} = \frac{\alpha}{a'c-ac'} = \frac{1}{ab'-a'b}$$

$\Rightarrow \quad \alpha^2 = \frac{bc'-b'c}{ab'-a'b}$ and $\alpha = \frac{a'c-ac'}{ab'-a'b}$

Eliminating α, we get : $\frac{(a'c-ac')^2}{(ab-a'b)^2} = \frac{bc'-b'c}{ab'-a'b}$

$\Rightarrow \quad (a'c-ac')^2 = (bc'-b'c)(ab'-a'b)$

This is the required condition for two quadratic equations to have a common root. To obtain the common root, make coefficient of x^2 in both the equation same and subtract one equation from the other to obtain a linear equation in x. Solve it for x to obtain the common root.

Ex. For which value of k will the equations $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ have one common root.

Sol. Let the common root be α than, $\alpha^2 - k\alpha - 21 = 0$ and $\alpha^2 - 3k\alpha - 35 = 0$.

Solving by Cramer rule, we have : $\frac{\alpha^2}{-35k-63k} = \frac{\alpha}{-21-35k} = \frac{1}{-3k+k}$

$\therefore \quad \alpha = \frac{-98k}{-56} = \frac{-7k}{4}$ and $\frac{7k}{4} = \frac{28}{k} \Rightarrow 7k^2 = 28 \times 4 \Rightarrow k = \pm 4$

CONDITION FOR TWO QUADRATIC EQUATION TO HAVE THE SAME ROOT

Two quadratic equations $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$ have the same roots if and only if

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

★ APPLICATIONS OF QUADRATIC EQUATIONS

Algorithm :

The method of problem solving consists of the following three steps :-

Stop-I : Translating the word problem in to symbolic language (mathematical statement) which means identifying relationships existing in the problem & then forming the quadratic equation.

Stop-II : Solving the quadratic equation thus formed

Stop-III: Interpreting the solution of the equation which means translating the result of mathematical statement into verbal language.

Type-I : Problems Based On Numbers.

Ex.14 The difference of two numbers is 3 and their product is 504. Find the numbers.

Sol. Let the required numbers be x and (x – 3). Then,

$x(x-3) = 504$

$\Rightarrow \quad x^2 - 3x - 504 = 0 \Rightarrow x^2 - 24x + 21x - 504 = 0$

$\Rightarrow \quad x(x-24) + 21(x-24) = 0 \Rightarrow (x-24)(x+21) = 0$

$\Rightarrow \quad x - 24 = 0$ or $x + 21 = 0 \Rightarrow x = 24$ or $x = -21$

If x = – 21, then the numbers are – 21 and – 24.

Again, if x = 24, then the numbers are 24 and 21.

Hence, the numbers are – 21, – 24 or 24, 21

Ex.15 The sum of the square of two consecutive odd positive integers is 290. find the integers.

Sol. Let the two consecutive odd positive integers be x and (x + 2). Then.

$x^2 + (x+2)^2 = 290$

$\Rightarrow \quad x^2 + x^2 + 4x = 290 \Rightarrow x^2 + 2x - 143 = 0$

$\Rightarrow \quad x^2 + 13x - 11x - 143 = 0 \Rightarrow x(x+13) - 11(x+13) = 0$

$\Rightarrow \quad (x+13)(x-11) = 0 \Rightarrow x = -13$ or $x = 11$

If x = – 21, then the numbers are – 21 and – 24.

But – 13, is not an odd positive integer.

Hence, the required integers are 11 and 13.

Type-II : Problems Based On Ages :

Ex.16 Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two fifth of Varun's age. Find their present ages.

Sol. Let the present ages of Varun and Swati be x years and y years respectively.

Seven years ago,

Varun's age = (x – 7) years and Swati's age = (y – 7) years.

$\therefore \quad (x-7) = 5(y-7)^2 \Rightarrow x - 7 = 5(y^2 - 14y + 49)$

$\Rightarrow \quad x = 5y^2 - 70y + 245 + 7 \Rightarrow x = 5y^2 - 70y + 252 \quad \ldots(i)$

Three years hence,

Varun's age = (x + 3) years and Swati's age = (y + 3) years.

$\therefore \quad (y+3) = \frac{2}{5}(x+3) \Rightarrow 5y + 15 = 2x + 6 \Rightarrow x = \frac{5y+9}{2} \quad \ldots(ii)$

From (i) and (ii) we get $5y^2 - 70y + 252 = \frac{5y+9}{2}$

$\Rightarrow \quad 10y^2 - 140y + 504 = 5y + 9 \Rightarrow 10y^2 - 145y + 495 = 0 \Rightarrow 2y^2 - 29y + 99 = 0$

$\Rightarrow \quad 2y^2 - 18y - 11y + 99 = 0 \Rightarrow 2y(y-9) - 11(y-9) = 0$

$\Rightarrow$ (y – 9) (2y – 11) = 0 $\Rightarrow$ y = 9 or $y = \frac{11}{2}$

$\because$ $y = \frac{11}{2}$ is not possible $\left[\because \frac{11}{2} < 7\right]$

So, y = 9.

$\therefore$ $\frac{5\times 9+9}{2} = 27$ [From (ii)]

Hence, the Varun's present age is 27 years and Swati's present age is 9 years..

Type-III : Problems Based On Geometrical Concepts :

Ex.17 The length of the hypotenuse of a right triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

Sol. Let ΔABC be a right triangle, right angled at B.

Let AB = x. Then

AC = (2x + 1) and BC = (2x + 1) – 2 = 2x – 1

$\Rightarrow$ ΔABC, $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow$ $(2x + 1)^2 = x^2 + (2x – 1) \Rightarrow 4x^2 + 4x + 1 = x^2 + 4x^2 – 4x + 1$

$\Rightarrow$ $x^2 = 8x \Rightarrow x = 8$ cm,

$\therefore$ BC = 2x – 1 = 2×8 + 1 = 15 cm

AC = 2x + 1 = 2×8 + 1 = 17 cm

Hence, the sides of the given triangle are 8cm, 15 cm and 17 cm.

Type-IV : Problems Based On Perimeter/Age :

Ex.18 Is it possible to design a rectangular park of perimeter 80 cm and area 400 m^2? If so, find its length and breadth.

Sol. Let the length and breadth of the rectangular park be ℓ and b respectively. Then,

2 (ℓ + b) = 80

ℓ + b = 40 $\Rightarrow$ ℓ = (40 – b)

And area of the park = 400 m^2

$\therefore$ ℓ b = 400

$\Rightarrow$ (40 – b) b = 400 $\Rightarrow 40b – b^2 = 400$

$\Rightarrow$ $b^2 – 40b + 400 = 0 \Rightarrow b^2 – 20b + 400 = 0$

$\Rightarrow$ b (b – 20) – 20 (b – 20) = 0 $\Rightarrow$ (b – 20) (b – 20) = 0

$\Rightarrow$ $(b – 20)^2 = 0 \Rightarrow b – 20 = 0 \Rightarrow b = 20$ m

$\therefore$ $\ell = 40 – b = 40 – 20 = 20$ m

Hence, length and breadth of the park are 20 m and 20 m respectively.

Thus, it is possible to design a rectangular park of perimeter 80 m and area 400 m^2

Type-V : Problems Based On Time and Distance :

Ex.19 A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less tor the same journey. Find the speed of the train.

Sol. Let the speed of the train be x km/h. Then,

Time taken to cover the distance of 360 km = $\frac{360}{x}$ hours.

If the speed of the train increased by 5 km/h. Them,

Time taken to cover the same distance = $\left(\frac{360}{x+5}\right)$h

According to the question, $\frac{360}{x} - \frac{360}{x+5} = 1$

$\Rightarrow$ $\frac{360(x+5)-360x}{x(x+5)} = 0 \Rightarrow 360x + 1800 - 360x = x^2 + 5x$

$\Rightarrow \quad x^2+5x-1800=0 \Rightarrow x^2+45x-40x-1800=0$

$\Rightarrow \quad x(x+45)-40(x+45)=0 \Rightarrow (x+45)(x-40)=0$

$\Rightarrow \quad x=-45 \text{ or } x=40$

But the speed can not be negative.

Hence, the speed of the train is 40 km/h.

Type-VI : Problems Based On Time and Work :

Ex.20 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank respectively. Find the time in which each tap can separately fill the tank.

Sol. Let the tap of larger diameter takes x hours to fill the tank. Then, the tap of smaller diameter takes (x + 10) hours to fill the tank.

$\therefore$ The portion of tank filled by the larger tap in one hour $=\frac{1}{x}$, the portion of tank filled by the smaller tap in one hour $=\frac{1}{x+10}$

And the portion of tank filled by both the smaller and the larger tap in one hour $=\frac{1}{9\frac{3}{8}}=\frac{8}{75}$

$\therefore \quad \frac{1}{x}+\frac{1}{x+10}=\frac{8}{75}$

$\Rightarrow \quad \frac{x+10+x}{x(x+10)}=\frac{8}{75} \Rightarrow \frac{2x+10}{x^2+10x}=\frac{8}{75}$

$\Rightarrow \quad 15x + 750 = 8x^2 + 80x \Rightarrow 8x^2 - 70x - 750 = 0$

$\Rightarrow \quad 4x^2 - 35x - 375 = 0 \Rightarrow 4x^2 - 60x + 25x - 375 = 0$

$\Rightarrow \quad 4x (x - 15) + 25 (x - 15) = 0 \Rightarrow (x - 15) (4x + 25) = 0$

$\Rightarrow \quad x = 15 \text{ or } x=\frac{-25}{4}$

But the speed can not be negative.

Hence, the larger tap takes 15 hours and the smaller tap takes 25 hours.

Type-VI : Miscellaneous Problems :

Ex.21 300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students.

Sol. Let the number of students be x. Then,

The number of apples received by each student = $\frac{300}{x}$

if there is 10 more students, i.e., (x + 10) students. Then,

The number of apples received by each student = $\frac{300}{x+10}$

According to the question, $\frac{300}{x}-\frac{300}{x+10}=1$

$\Rightarrow \quad \frac{300x+3000-300x}{x(x+10)}=1 \Rightarrow 3000=x^2+10x$

$\Rightarrow \quad x^2+10x-3000=0 \Rightarrow x^2+60x-50x-3000=0$

$\Rightarrow \quad x(x+60)-50(x+60)=0 \Rightarrow (x+60)(x-50)=0$

$\Rightarrow \quad x=-60 \text{ or } x=50$

But the number of students can not be negative.
Hence, the number of students is 50.

SYNOPSIS

Quadratic Equation : A quadratic equation in one variable x is of the form $ax^2 + bx + c = 0$, $a \neq 0$ where a, b and c are real numbers.

Roots of the quadratic equation : A real number α is said to be a root of the quadratic equation or a zero of the quadratic polynomial if and only if α satisfies the equation i.e., which make LHS = RHS.

Sreedharacharya formula : $ax^2 + bx + c = 0$, $a \neq 0$, $b^2 - 4ac \geq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \alpha \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Nature of roots : A quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has :

(i) No real roots if $D < 0$. (ii) Two distinct real roots if $D > 0$. (iii) Two equal real roots if $D = 0$.

Relation between roots of equation : $ax^2 + bx + c = 0$, $a \neq 0$

Sum of roots = $\alpha + \beta = \frac{-b}{a}$, Product of roots = $\alpha\beta = \frac{c}{a}$

Formation of quadratic equation when roots are given : $ax^2 + bx + c = 0$ $[x^2 - (\alpha + \beta)x + \alpha\beta]$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOSSE THE CORRECT ONE

1. Which of the following quadratic expression can be expressed as a product of real linear factors?

(A) $x^2 - 2x + 3$ (B) $3x^2 - \sqrt{2x} - \sqrt{3}$ (C) $\sqrt{2x^2} - \sqrt{5x} + 3$ (D) None of these

2. Two candidates attempt to solve a quadratic equation of the form $x^2 + px + q = 0$. One starts with a wrong value of p and finds the roots to be 2 and 6. The other starts with a wrong value of q and finds the roots to be 2 and – 9. Find the correct roots of the equation :

(A) 3, 4 (B) – 3, – 4 (C) 3, – 4 (D) – 3, 4

3. Solve for x : $15x^2 - 7x - 36 = 0$

(A) $\frac{5}{9}, -\frac{4}{3}$ (B) $\frac{9}{5}, -\frac{4}{3}$ (C) $\frac{5}{9}, -\frac{3}{4}$ (D) None of these

4. Solve for y : $7y^2 - 6y - 13\sqrt{7} = 0$

(A) $\sqrt{7}, 2\sqrt{7}$ (B) $3, \frac{2}{\sqrt{7}}$ (C) $\frac{13}{\sqrt{7}}, -\sqrt{7}$ (D) None of these

5. Solve for x : $6x^2 + 40x = 31$

(A) $\frac{3}{8}, \frac{2}{5}$ (B) $\frac{3}{8}, \frac{3}{2}$ (C) $0, \frac{8}{3}$ (D) $\frac{8}{3}, \frac{5}{2}$

6. Determine k such that the quadratic equation $x^2 + 7(3 + 2k) - 2x(1 + 3k) = 0$ has equal roots :

(A) 2, 7 (B) 7, 5 (C) $2, -\frac{10}{9}$ (D) None of these

7. Discriminant of the roots of the equation $-3x^2 + 2x - 8 = 0$ is

(A) – 92 (B) – 29 (C) 39 (D) 49

8. The nature of the roots of the equation $x^2 - 5x + 7 = 0$ is

(A) No real roots (B) 1 real root (C) Can't be determined (D) None of these

9. The roots of $a^2x^2 + abx = b^2$, $a \neq 0$ are :

(A) Equal (B) Non-real (C) Unequal (D) None of these

10. The equation $x^2 - px + q = 0$ p, q $\in$ R has no real roots if :

(A) $p^2 > 4q$ (B) $p^2 < 4q$ (C) $p^2 = 4q$ (D) None of these

11. Determine the value of k for which the quadratic equation $4x^2 - 3kx + 1 = 0$ has equal roots :

(A) $\pm\left[\frac{2}{3}\right]$ (B) $\pm\left[\frac{4}{3}\right]$ (C) ± 4 (D) ± 6

12. Find the value of k such that the sum of the square of the roots of the quadratic equation $x^2 - 8x + k = 0$ is 40 :

(A) 12 (B) 2 (C) 5 (D) 8

13. Find the value of p for which the quadratic equation $x^2 + p(4x + p - 1) + 2 = 0$ has equal roots :

(A) $-1, \frac{2}{3}$ (B) 3, 5 (C) $1, -\frac{4}{3}$ (D) $\frac{4}{3}, 2$

14. The length of a hypotenuse of a right triangle exceeds the length of its base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle (in cm) :

(A) 6, 8 10 (B) 7, 24, 25 (C) 8, 15, 17 (D) 7, 40, 41

15. A two digit number is such that the product of it's digits is 12. When 9 is added to the number, the digits interchange their places, find the number :

(A) 62 (B) 34 (C) 26 (D) 43

16. A plane left 40 minutes late due to bad weather and in order to reach it's destination, 1600 km away in time, it had to increase it's speed by 400 km/h from it's usual speed. Find the usual speed of the plane :

(A) 600 km/h (B) 750 km/h (C) 800 km/h (D) None of these

17. The sum of the squares of two consecutive positive odd numbers is 290. Find the sum of the numbers :

18. A shopkeeper buys a number of books for Rs. 80. If he had bought 4 more for the same amount, each book would have cost Re. 1 less. How many books did he buy?

(A) 8 (B) 36 (C) 24 (D) 28

19. The squares have sides x cm and (x + 4) cm. The sum of their areas is 656 cm^2. find the sides of the square.

(A) 8 cm, 12 cm (B) 12 cm, 15 cm (C) 6 cm, 10 cm (D) 16 cm, 20 cm

20. The real values of a for which the quadratic equation $2x^2 - (a^3 + 8a - 1)\,x + a^2 - 4a = 0$ possesses roots of opposite signs are given by :

(A) $a > 6$ (B) $a > 9$ (C) $0 < a < 4$ (D) $a < 0$

OBJECTIVE				ANSPER KEY						EXERCISE -I					
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	B	B	B	C	D	C	A	A	C	B	B	A	A	C	B
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	C	B	B	D	C										

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. State which of the following equations are quadratic equation :

(i) $3x + \frac{1}{x} - 8 = 0$ (ii) $18x^2 - 6x = 0$ (iii) $x^2 - 5x = 7 - 6x^3$ (iv) $x^2 = 25$ (v) $6x^5 + 3x^2 - 7 = 0$

(vi) $x + \frac{1}{x^2} = 3$ (vii) $5x^2 + 6x = 7$ (viii) $5x^3 - 2x - 3 = 0$ (ix) $\frac{3x}{4} - \frac{5x^2}{8} = \frac{7}{8}$

(x) $\sqrt{x} + \frac{1}{\sqrt{x}} = 4$ (xi) $(x+1)(x+3) = 0$ (xii) $(2x+1)(3x+2) = 6(x-1)(x-2)$

(xiii) $16x^2 - 3 = (2x+5)(5x-3)$ (xiv) $(x-2)^2 + 1 = 2x - 3$ (xv) $x(x+1) + 8 = (x+2)(x-2)$

(xvi) $x(2x-3) = x^2 + 1$ (xvii) $(x+2)^3 = x^3 - 4$ (xviii) $x^2 + \frac{2}{x^2} = 3$

2. Represent each of the following situations in the form of a quadratic equation :

(i) The sum of the squares of two consecutive positive integers is 545. We need to find the integers.

(ii) The hypotenuse of a right triangle is 25 cm. The difference between the length of the other two sides of the triangle is 5 cm. We need to find the lengths of these sides.

(iii) One year ago, the father was 8 times as old as his son. Now his age is square of the son's age. We need to find their present ages.

(iv) Ravi and Raj together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out the number of toys produced on that day.

(v) A cottage industry product a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys product in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

3. In each of the following determine whether the given values are the solutions of the given equation or not :

(a) (i) $x^2 - 7x + 12 = 0$; $x = 3, x = 4$

(ii) $x^2 - \sqrt{2}x - 4 = 0$; $x = -\sqrt{2}, x = -2\sqrt{2}$

(iii) $10x - \frac{1}{x} = 3$; $x \neq 0, x = \frac{1}{2}, x = \frac{-1}{2}$

(iv) $\frac{a}{(x-b)}+\frac{b}{(x-a)}=2;\ (x\neq a,b)\ ;\ x=(a+b), x=\frac{a+b}{2}$

(b) (i) $x^2-(\sqrt{2}+\sqrt{3})x+\sqrt{6}=0;\ x=\sqrt{2}, x=\sqrt{3}$

(ii) $\frac{x}{a}+\frac{b}{x}=\frac{a+b}{a};\ (x\neq 0), x=a, x=b$

4. In each of the following find the value of k for which the given value is a solution of the given equation :

(i) $(x+3)(2x-3k)=0:\ x=6$ (ii) $3\sqrt{7}x^2-4x+k=0:\ x=\frac{\sqrt{7}}{3}$

5. Find the value of p and q for which $x=\frac{2}{3}$ and x = – 3 are the roots of the equation $px^2 + 7x + q = 0$.

SHORT ANSWER TYPE QUESTIONS

Find the solutions of the following quadratic equations by factorization method and check the solutions (1-24) :

1. $27x^2 - 12 = 0$

2. $3\left(\frac{x}{2}+1\right)^2=27$

3. $16(x-4)^2 = 9(x+3)^2$

4. $x^2 - 300 = 0$

5. $x^2 + (a-b)x = ab$

6. $(3x + a)(3x + b) = ab$

7. $x^2-(1+\sqrt{2})x+\sqrt{2}=0$

8. $3\sqrt{7}x^2+4x-\sqrt{7}=0$

9. $\sqrt{3}y^2+11y+6\sqrt{3}=0$

10. $abx^2 - (a^2 + b^2)x + ab = 0$

11. $x^2-\frac{x}{12}-\frac{1}{12}=0$

12. $x^2+\left(a+\frac{1}{a}\right)x+1=0$

13. $x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$

14. $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

15. $\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0$

16. $\frac{5}{x-5}+\frac{4}{x-3}=\frac{3}{x-3}$

17. $\frac{5}{x-5}+\frac{2}{x-2}=\frac{3}{x-3}+\frac{4}{x-4}$

18. $\frac{x}{x+1}+\frac{x+1}{x}=\frac{34}{15}$

19. $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$

20. $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

21. $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5$

22. $\frac{4x-3}{2x+1}-10\left(\frac{2x+1}{4x-3}\right)=3$

23. $2\left(\frac{x+2}{2x-3}\right)-9\left(\frac{2x-3}{x+2}\right)=3$

24. $\frac{x}{2x+1}+\frac{2x+1}{x}=\frac{58}{21}$

Find the roots of each of the following quadratic equations by the method of completing the squares (25 - 29)

25. $x^2 - 6x + 4 = 0$

26. $2x^2 - 5x + 3 = 0$

27. $\sqrt{5}x^2+9x+4\sqrt{5}=0$

28. $(5z + 2a)(3z + 4b) = 8ab$

29. $2\sqrt{2}x^2+\sqrt{15}x+\sqrt{2}=0$

30. Find the solutions of $3x^2-2\sqrt{6}x+2=0$ by the method of completing the squares when
(i) x is a rational number (ii) x is a real number

31. Find the solutions of $15x^2 + 3 = 17x$, when (i) x is a rational number (ii) x is a real number.

32. Find the solutions of $5x^2 - 6x - 2 = 17x$, when (i) x is a rational number (ii) x is a real number.

Find the roots of each of the following quadratic equations by using the quadratic formula (33 - 50) :

33. $4x^2 + 3x + 5 = 0$

34. $x^2 - 16x + 64 = 0$

35. $3x^2 - 5x + 2 = 0$

36. $2x^2 2\sqrt{2}x+1=0$

37. $3x^2 2\sqrt{5}x-5=0$

38. $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

39. $x+\frac{1}{x}=3, x\neq 0$

40. $\frac{x-2}{4}=\frac{x+2}{x}, x\neq 0$

41. $y-\frac{15}{4y}+1=0, y\neq 0$

42. $\frac{x-3}{x+3}-\frac{x+3}{x-3}=6\frac{6}{7}, x\neq -3,3$

43. $\frac{x+2}{x-2}+\frac{x-2}{x+2}=\frac{5}{2}, x\neq 2,-2$

44. $\frac{x}{x-1}+\frac{x-1}{x}=4, x\neq 0,1$

45. $\frac{1}{x}-\frac{1}{x-2}=3, x\neq 0,2$

46. $\left(\frac{2x-3}{x-1}\right)-4\left(\frac{x-1}{2x-3}\right)=3, x\neq 1,\frac{3}{2}$

47. $(x^2 - 2x)^2 - 4(x^2 - 2x) + 3 = 0$

48. $(y^2 - 4y)^2 + 11(y^2 + 4y) + 28 = 0$

49. $2\left(\frac{x}{x+1}\right)^2-5\left(\frac{x}{x+1}\right)+2=0, x\neq -1$

50. $(x^2 + 3x + 2)^2 - 8(x^2 + 3x) - 4 = 0$

51. Find the nature of the roots of the following equations. If the real roots exist, find them :

(a) (i) $6x^2 + x - 2 = 0$ (ii) $2x^2+5\sqrt{3}x+6=0$ (iii) $2x^2 - 6x + 3 = 0$ (iv) $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

(b) Find the discriminant of roots of equation $3x^2-2x+\frac{1}{3}=0$ and hence find the nature of its roots. Find them, if they are real.

(c) What is the nature of roots of the quadratic equation $4x^2 - 12x - 9 = 0$?

52. Find the value of k for each of the following quadratic equations, so that they have two real and equal roots :

(a) (i) $2x^2 + kx + 3 = 0$ (ii) $kx^2 - 2\sqrt{5}x + 4 = 0$ (iii) $4x^2 - 2(k + 1)x + (k + 4) = 0$
(iv) $(k - 3)x^2 + 4(k - 3)x + 4 = 0$

(b) (i) $x^2 - 2(k + 1)x + k^2 = 0$ (ii) $(k + 4)x^2 + (x + 1)x + 1 = 0$ (iii) $kx^2 - 2\sqrt{5}x + 4 = 0$
(iv) $2kx^2 - 40x + 25 = 0$

(c) (i) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$ (ii) $x^2 - kx + 4 = 0$ (iii) $2x^2 - (k - 2)x + 1 = 0$

53. Determine the value(s) of p for which the quadratic equation $2x^2 + px + 8 = 0$ has (i) real roots.

54. Show that the equation $x^2 + px - 1 = 0$ has (i) real and distinct roots for all real values of p.

55. (a) If -2 is a root of the quadratic equation $x^2 + px + 2 = 0$ and the quadratic equation $2x^2 + px + k = 0$ has equal roots, find the value of k.

(b) If -2 is a root of the quadratic equation $x^2 + px + 2 = 0$ and the quation $2x^2 + px + q = 0$ has equal roots, find the value of p and q.

56. If the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$ or ad = ac.

57. Prove that both the roots of the equation $(x + a)(x + b) + (x + b)(x + c) + (x + c)(x + a) = 0$ are always real and can not be equal unless a = b = c.

58. (a) If the root of the equation $x^2 + 2cx + ab = 0$ are real and unequal, prove that the equation $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ has no real roots.

(b) Prove that the equation $x^2 (a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$ has no real roots, if $ad \neq bc$.

(c) If p,q,r and s are real number such that $pr = 2(q + s)$ than show that atleast one of the equation $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ has real roots.

SBJECTIVE — ANSPER KEY — EXERCISE -2 (x)-CBSE

• VERY SHORT ANSWER TYPE QUESTIONS

1. Equations in questions No. (i), (ii), (iv), (vii), (ix), (xiii), (xiv), (xvi) and (xvii) are quadratic equations.

2. (i) $x^2 + x - 272 = 0$, where x is the smaller integer. (ii) $x^2 + 5x - 300 = 0$, where x is the length of one side.
(iii) $x^2 - 8x + 7 = 0$, where x (in years) is the present age of son.
(iv) $x^2 - 45x + 324 = 0$, where x is the number of marbles with Ravi.
(v) $x^2 - 55x + 750 = 0$, where x (in km/h) is the speed of the train.

3. (a) (i) Both are solution (ii) $x = -\sqrt{2}$ is a solution but $x = -2\sqrt{2}$ is not a solution.
(iii) $x = \frac{1}{2}$ is a solution but $x = \frac{-1}{2}$ is not a solution. (iv) Both are solution
(b) (i) Both are solution (ii) Both are solution

4. (i) k = 4, (ii) $k = -\sqrt{7}$ **5.** p = 3, q = – 6

• SHORT ANSWER TYPE QUESTIONS

1. $\frac{2}{3}, -\frac{2}{3}$ **2.** 4, -8 **3.** 1, 25 **4.** $10\sqrt{3}, -10\sqrt{3}$ **5.** b, - a **6.** $0, -\frac{(a+b)}{3}$ **7.** $\sqrt{2}$ **8.** $-\frac{\sqrt{7}}{3}, \frac{\sqrt{7}}{7}$ **9.** $-\frac{2}{\sqrt{3}}, -3\sqrt{3}$
10. $\frac{a}{b}, \frac{b}{a}$ **11.** $\frac{1}{3}, -\frac{1}{4}$ **12.** $a, \frac{1}{a}$ **13.** $\frac{-a}{a+b}, -\frac{(a+b)}{a}$ **14.** –a, –b **15.** – 1 **16.** 12, – 2 **17.** $0, \frac{7}{2}$ **18.** $-\frac{5}{2}, \frac{3}{2}$
19. 5, – 1 **20.** $6, \frac{40}{13}$ **21.** $-10, -\frac{1}{5}$ **22.** $-\frac{4}{3}, \frac{1}{8}$ **23.** $\frac{11}{5}, \frac{5}{8}$ **24.** $3, -\frac{7}{11}$ **25.** $3 \pm \sqrt{5}$ **26.** $1, \frac{3}{2}$ **27.** $-\sqrt{5}, \frac{-4}{\sqrt{5}}$
28. $0, -\frac{6a + 20b}{15}$ **29.** No solution **30.** (i) No solution (ii) $\sqrt{\frac{2}{3}}$ **31.** (i) No solution (ii) $\frac{17 + \sqrt{109}}{30}, \frac{17 - \sqrt{109}}{30}$
32. (i) No solution (ii) $\frac{3 \pm \sqrt{19}}{5}$ **33.** No solution **34.** 8 **35.** $1, \frac{2}{3}$ **36.** $\frac{1}{\sqrt{2}}$ **37.** $\frac{\sqrt{5}}{3}, -\sqrt{5}$ **38.** $\frac{-2b}{a}, \frac{-2b}{3a}$
39. $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$ **40.** $3 + \sqrt{17}, 3 - \sqrt{17}$ **41.** $\frac{3}{2}, -\frac{5}{2}$ **42.** $-4\frac{9}{4}$ **43.** 6, – 6 **44.** $\frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}$ **45.** $\frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}$
46. $\frac{1}{2}, \frac{4}{3}$ **47.** –1, 3, $1 + \sqrt{2}, 1 - \sqrt{2}$ **48.** – 2, – 1, $\frac{-3 \pm \sqrt{21}}{2}, \frac{-3 \pm \sqrt{5}}{2}$ **49.** – 2, 1 **50.** 1, 0, – 3 – 4
51. (a) (i) $\frac{1}{2}, \frac{-2}{3}$ (ii) $\frac{-\sqrt{3}}{2}, -2\sqrt{3}$ (iii) $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$ (iv) $\frac{-2b}{a}, \frac{-2b}{3a}$ (b) $\frac{1}{3}, \frac{1}{3}$ (c) Roots are real and unequal

52. (a) (i) $k = \pm 2\sqrt{6}$ (ii) $k = \frac{5}{4}$ (iii) = – 3, 5 (iv) k = 4 ; (b) (i) $k = \frac{-1}{2}$ (ii) k = 5, – 3 (iii) $k = \frac{5}{4}$ (iv) k = 8

(c) (i) k = 14 (ii) k = ± 4 (iii) $k = 2 \pm \sqrt{2}$ **53.** (i) p = ± 8 (ii) $p \leq -8$ or $p \geq 8$, $p \in R$ **55.** $k = \frac{9}{4}$,(b) p = 3, $q = \frac{9}{8}$

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

APPLICATIONS TO WORD PROBLEMS

1. Find the numbers whose sum is 40 and product 375.

2. The difference between two integers is 4. Their product is 221. Find the numbers.

3. The sum of a natural number and its reciprocal is $\frac{65}{8}$. Find the natural numbers.

4. Divide 27 into two parts such that the sum of their reciprocals is $\frac{3}{20}$.

5. The sum of two numbers is 12 and the sum of their squares is 74. Find the natural numbers.

6. Find two consecutive natural numbers, the sum of whose squares is 145.

7. Find two consecutive positive even integers, whose product is 224.

8. The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.

9. Find two consecutive multiples of 3 whose product is 270.

10. A number consists of two digits whose product is 18. If 27 is added to the number, the digits interchange their places. Find the number

11. A two-digit number contains the smaller of the two digits in the unit place. The product of the digits is 40 and the difference between the digits is 3. Find the number.

12. The sum of numerator and denominator of a certain fraction is 10. If 1 is subtracted from both the numerator and denominator, the fraction is decreased by $\frac{2}{21}$. Find the fraction.

13. Two years ago, a man's age was three times the square of his son's age. In three years time, his age will be four times his son's age. Find their present ages.

14. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. Find the time taken by the first pipe alone to fill the tank.

15. A booster pump can be used for filling as well as for emptying a tank. The capacity of the tank is 2400 m^3. The emptying capacity of the tank is 10 m^3 per minute higher than its filling capacity and the pump needs 8 minutes lesser to empty the tank than it needs to fill it. What is the filling capacity of the pump?

16. Albert goes to his friend's house which is 15 km away from his house. He covers half of the distance at a speed of x km/hr and the remaining at (x + 2) km/hr. If he takes 2 hrs 30 min. to cover the whole distance, find x.

17. (i) A train covers a distance of 780 km at x km/hr. Had the speed been (x – 5) km/hr, the time taken to cover the same distance would have been increased by 1 hour. Write down an equation in x and solve it to evaluate x.

(ii) A train covers a distance of 600 km at x km/hr. Had the speed been (x + 20) km/hr, the time taken to cover the same distance would have been reduced by 5 hour. Write down an equation in x and solve it to evaluate x.

18. By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car.

19. The distance by road between two towns A and B, is 216 km, and by rail it is 208 km. A car travels at a speed of x km/hr and the train travels at a speed which is 16 km/hr faster than the car.

(i) Write down the time taken by the car to reach town B from A, in terms of x.

(ii) Write down the time taken by the train to reach town B from A, in terms of x.

(iii) If the train takes 2 hours less than the car to reach town B, obtain an equation in x and solve it .

(iv) Hence, find the speed of the train .

20. Car A travels x km for every litre of petrol, while car B travels (x + 5) km for every litre of petrol.

(i) Write down the number of litres used by car A and B in covering a distance of 400 km.

(ii) If car A used 4 litres of petrol more than car B in covering 400 km, write an equation in x and solve it to determine the number of litres of petrol used by car B for the journey.

21. The speed of a boat in still water is x km/hr and the speed of the stream is 3 km/hr.

(i) Write the speed of the boat upstream, in terms of x.

(ii) Write the speed of the boat downstream, in terms of x.

(iii) If the boat goes 15 km upstream and 22 km downstream in 5 hours, write an equation in x to represent the statement.

(iv) Solve the equation to evaluate x.

22. The hypotenuse of right triangle is 20 m. If the difference between the lengths of other sides be 4 m. find the other sides.

23. The length of the sides of a right triangle are (2x – 1) m, and (4x + 1) m, where $x > 0$. Find :

(i) The value of x. (ii) The area of the triangle.

24. Two squares have sides x cm and (x + 5) cm. The sum of their areas is 697 sq. cm.

(i) Express this as an algebraic equation in x.

(ii) Solve this equation to find the sides of the squares .

25. The length of a rectangle is 8 metres more than its breadth and its area is 425 m^2.

(i) Taking x metres as the breadth of the rectangle, write an equation in x that represents the above statement.

(ii) Solve the above equation and find the dimensions of the rectangle.

26. The ratio between the length and the breadth of a rectangular field is 3 : 2. If only the length is increased by 5 metres, the new area of the field will be 2600 sq. metres. What is the breadth of the rectangular field?

27. The perimeter of a rectangular plot of land is 114 metres and its area is 810 square metres.

(i) Take the length of plot as x metres. Use the perimeter 114 m to write the value of the breadth in terms of x.

(ii) Use the values of length, breadth and area to write an equation in x.

(iii) Solve the equation to find the length and breadth of the plot.

28. Write a rectangular garden 10 m wide and 20 m long, we wish to pave a walk around the borders of uniform width so as to leave an area of 96 m^2 for flowers. Haw wide should the walk be ?

29. The area of right-angle triangle is 96 m^2. If the base is three times its altitude, find the base.

30. The length of the parallel sides of trapezium are (x + 8) cm and (2x +3) cm, and the distance between them is (x + 4) cm. If its area is 590 cm^2, find the value of x.

31. A man buys an article for Rs. x and sells it for Rs. 56 at a gain of x%. Find the value of x.

32. Rohit is on tour and he has Rs. 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by Rs. 3. For how many days Rohit is on tour?

33. Rs. 6400 were divided equally among x persons. Had this money been divided equally among (x + 14) persons, each would have got Rs. 28 less. Find the value of x.

34. Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member increased Rs. 10. Find how many students went for the picnic.

35. A shopkeeper buys x books for Rs. 720.

(i) Write the cost of 1 book in terms of x.

(ii) If the cost of per book were Rs. 5 less, the number of books that could be bought for Rs. 720 would be 2 more. Write down the equation in x for the above situation and solve it to find x.

36. A piece of cloth costs Rs. 35. If the length of the piece would have been 4 m longer and each metre costs Rs. 1 less, the cost would have remained unchanged. How long is the piece?

37. A fruit seller-bought x apples for Rs. 1200.

(i) Write the cost price of each apple in terms of x.

(ii) If 10 of the apple were rotten and he sold each of the rest at Rs. 3 more than the cost price of each, write the selling price of (x – 10) apples.

(iii) If he made a profit of Rs. 60 in this transaction, from an equation in x and solve it to evaluate x.

38. Vibha and Sanya distribute Rs. 100 each in charity. Vibha distributes money to 5 more people than Sanya and Sanya gives each Re 1 more than Vibha. How many people are recipients of the charity?

SBJECTIVE **ANSWER** **EXERCISE -3 (x)-CBSE**

Applications To Word Problems

1. 15, 25 **2.** 13, 17 or 13, – 17 **3.** 8 **4.** 15, 12 **5.** 5, 7 **6.** 8, 9 **7.** 14, 16 **8.** 27, 29, 31 **9.** 15, 18

10. 36 **11.** 85 **12.** $\frac{3}{7}$ **13.** 29 years, 5 years **14.** 15 hours **15.** 50 m^3/ min **16.** x = 4

17. (i) $x^2 - 5x - 3900 = 0$, x = 65 (ii) $x^2 + 20x - 2400 = 0$, x = 40 **18.** 30 km/hr

19. (i) $\frac{216}{x}$ hrs (ii) $\frac{208}{(x+16)}$ hrs (iii) $x^2 + 12x - 1728 = 0$, x = 36 (iv) 52 km/hr

20. (i) $\left(\frac{400}{x}\right)$ litres and $\left(\frac{400}{x+5}\right)$ litres (ii) $\frac{400}{x+5} - \frac{400}{(x+5)} = 4, x = 20$. Car B consumed 16 litres.

21. (i) (x – 3) km/hr (ii) x + 3 km/hr (iii) $\frac{15}{(x+3)} + \frac{22}{(x+3)} = 5$ (iv) x = 8 **22.** 16 m, 12,

23. (i) x = 3 (ii) 30 m^2 **24.** (i) $x^2 + 5x - 336 = 0$ (ii) 16 cm, 21 cm 25. (i) $x^2 + 8x - 425 = 0$ (ii) 17 m, 25 m

26. 40 m **27.** (i) Breadth = (57 – x) m (ii) $x^2 - 57x + 810 = 0$ (iii) ℓ = 30 m, b = 27 m **28.** 2 m **29.** 24 m

30. x = 16 **31.** x = 40 **32.** 20 days **33.** x = 50 **34.** 16 **35.** (i) Rs. $\left(\frac{720}{x}\right)$ (ii) $x^2 + 2x - 228 = 0$ x = 16 **36.** 10 m

37. (i) Rs. $\left(\frac{1200}{x}\right)$ (ii) Rs. $(x - 10)\left(\frac{1200}{x} + 3\right)$ (iii) $x^2 - 30x - 4000 = 0$, x = 80 **38.** 45

EXERCISE – 4 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD QUESTIONS

SHORT ANSWER TYPE QUESTIONS

1. Find the values of k so that (x – 1) is a factor of $k^2x^2 - 2kx\ 3$. **[CBSE-Delhi-2003]**

2. Solve using the quadratic formula : $x^2 - 4x + 1 = 0$ **[ICSE-2003]**

3. Solve for x : $4x^2 - 2(a^2 + b^2)\ x + a^2b^2 = 0$ **[CBSE-Delhi-2004]**

4. Solve for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$ **[CBSE-Delhi-2004]**

5. Solve for x : $9x^2 - 9\ (a + b)\ x = [2a^2 + 5ab + 2b^2] = 0$ **[CBSE-Delhi-2004]**

6. Using quadratic formula, solve the following quadratic equation for x : $p^2x^2 + (p^2 - q^2)\ x - q^2 = 0$ **[CBSE-Al-2004]**

7. Using quadratic formula, solve the following quadratic equation for x : $x^2 - 2x + (a^2 - b^2) = 0$ **[CBSE-Al-2004]**

8. Using quadratic formula, solve the following quadratic equation for x : $x^2 - 4x + 4a^2 - b^2 = 0$ **[CBSE-Al-2004]**

9. Solve for x : $9x^2 - 6a^2x + (a^4 - b^4) = 0$ **[CBSE-Foreign-2004]**

10. Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$ **[CBSE-Foreign-2004]**

11. Solve for x : $16x^2 - 8a^2x + (a^4 - b^4) = 0$ **[CBSE-Foreign-2004]**

12. Solve for x : $36x^2 - 12ax + (a^2 - b^2) = 0$ **[CBSE-Delhi-2004C]**

13. Solve the equation $3x^2 - x - 7 = 0$ and give your answer correct to two decimal places. **[ICSE-2004]**

14. Solve for x : $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ **[CBSE-Foreign-2005]**

OR

Solve for x : $x^2 - 2\ (a^2 + b^2)\ x + (a^2 - b^2)^2 = 0$ **[CBSE-Delhi-2006C]**

15. Solve $x^2 - 5x - 10 = 0$ and give your answer correct to two decimal places **[ICSE-2005]**

16. Using quadratic formula, solve for x : $9x^2 - 3\ (a + b)\ x + ab = 0$.

OR

The sum of the square of two consecutive natural numbers is 421. Find the numbers. **[CBSE-Dehli-2005C]**

17. Using quadratic formula, solve the following for x : $9x^2 - 3\ (a^2 + b^2)\ x + a^2\ b^2 = 0$

OR

The sum of the square of three consecutive positive integers is 50. Find the integers. **[CBSE-Al-2005C]**

18. Rewrite the following as a quadratic equation in x and then solve for x : $\frac{4}{x} - 3 = \frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$

[CBSE-Al-2006C]

19. Solve $2x - \frac{1}{x} = 7$ and give your answer correct to 2 decimal places. **[ICSE-2006]**

20. Solve $x^2 - 3x - 9 = 0$ and give your answer correct to 2 decimal places. **[ICSE-2007]**

21. Find the roots of the following equation : $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$; $x \neq -4, 7$ **[CSBE-Delhi-2008]**

22. Is x = – 2 a solution of the equation $x^2 - 2x + 8 = 0$? **[CSBE-Al-2008]**

23. Is x = – 3 a solution of the equation $2x^2 + 5x + 3 = 0$? **[CSBE-Al-2008]**

24. Is x = – 4 a solution of the equation $2x^2 + 5x - 12 = 0$? **[CSBE-Al-2008]**

25. Show that x = – 3 is a solution of $x^2 + 6x + 9 = 0$. **[CSBE-Foreign-2008]**

26. Show that x = – 3 is a solution of $2x^2 + 6x - 3 = 0$. **[CSBE-Foreign-2008]**

27. Show that x = – 2 is a solution of $3x^2 + 13x + 14 = 0$. **[CSBE-Foreign-2008]**

28. Find the discriminant of the equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$. **[CSBE-Al-2009]**

29. The sum of two numbers is 8. Determine the numbers if the sum of their reciprocals is $\frac{8}{15}$. **[CSBE-Al-2009]**

30. Write the nature of roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$. **[CSBE-Foreign-2009]**

LONG ANSWER TYPE QUESTIONS

1. An aeroplane traveled a distance of 400 km at an average speed of x km/hr. On the return journey, the speed was increased by 40 km/hr. Write down an expression for the time taken for (i) the onward journey, (ii) the return journey. If the return journey took 30 minutes less than the onward journey, write an equation in x and find the value of x. **[ICSE-2002]**

2. In an auditorium, seats were arranged in rows and columns. The number of rows was equal to number of seats in each row. When the number of rows was doubled and the number of seats in each row was reduced by 10, the total number of seats increased by 300. Find (i) the number of rows in the original arrangement, (ii) the number of seats in the auditorium after rearrangement. **[ICSE-2003]**

3. Solve for x : $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5$; given that $x\neq -3,\ x\neq \frac{1}{2}$ **[CSBE-Delhi-2004]**

4. Solve for x : $2\left(\frac{x-1}{x+3}\right)-7\left(\frac{x+3}{x-1}\right)=5$; given that $x\neq -3,\ x\neq 1$ **[CSBE-Delhi-2004]**

5. Solve for x : $2\left(\frac{2x+3}{2x+1}\right)-10\left(\frac{2x+1}{2x-3}\right)=3$; given that $x\neq 3,\ x\neq \frac{-3}{2}$ **[CSBE-Delhi-2004]**

6. Solve for x : $2\left(\frac{4x-3}{2x+1}\right)-9\left(\frac{2x-3}{x+2}\right)=3$; given that $x\neq \frac{-1}{2};\ x\neq \frac{3}{4}$

OR

300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students. **[CSBE-Al-2004]**

7. Solve for x : $2\left(\frac{x+2}{2x-3}\right)-9\left(\frac{2x-3}{x+2}\right)=3$; given that $x\neq \frac{3}{2};\ x\neq -2$

OR

An aeroplane takes one hour less for a journey of 1200 km if its speed is increased by 100 km/hour from its usual speed. Find the its usual speed. **[CSBE-Foreign-2004]**

8. A two digit number is four times the sum of its digits and is also equal to twice the product of its digits. Find the number **[CSBE-Delhi-2004C]**

9. A two digit number is seven times the sum of its digits and is also equal to 12 less than three times the product of its digits. Find the number **[CSBE-Delhi-2004C]**

10. A two digit number is 5times the sum of its digits and is also equal to 5 more than twice the product of its digits. Find the number **[CSBE-Delhi-2004C]**

11. The sum of two number a and b is 15, and the sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$. Find the number **[CSBE-Delhi-2005]**

12. The sum of two number is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the number **[CSBE-Delhi-2005]**

13. The sum of two number is 18. The sum of their reciprocals is $\frac{1}{4}$. Find the number **[CSBE-Delhi-2005]**

14. A two digit number is such that the product of its digits is 15. If 18 is added to the number, the digits interchange their places. Find the number **[CSBE-Al-2005]**

15. A two digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number **[CSBE-Al-2005]**

16. A two digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number **[CSBE-Al-2005]**

17. The sum of the square of two natural number is 34. If the first number is one less than twice the second number, find the number **[CBSE-Foreign-2005]**

18. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hour from its usual speed. Find the usual speed of the train. **[CSBE-Delhi-2005C, 2006]**

19. Solve for $x\ \frac{x+1}{x-1}+\frac{x-2}{x+2}=3:(x\neq 1,-2)$ **[CSBE-Al-2005C]**

OR

Aeroplane left 30 minutes later than its scheduled time and in order to reach destination 1500 km away in time, it has to increase its speed by 250 km/h from its usual speed. Determine its usual speed.

20. Solve for x : $\frac{1}{a+b+x}+\frac{1}{a}+\frac{1}{x}:a\neq 0,b\neq 0,x\neq 0$

OR

Solve for x : $abx^2 + (b^2 - ac)\ x - bc = 0$ **[CSBE-Delhi-2005]**

21. Solve for x : $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

OR

Solve for x : $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}\ (x\neq 2,4)$ **[CSBE-Al-2005]**

22. By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car. **[ICSE-2005]**

23. Solve for x : $12abx^2 - (9a^2 - 8b^2)\ x - 6ab = 0$

OR

A two digit number is such that the product of its digits is 35. When 18 is added to number, the digits interchange their places. Find the number. **[CBSE-Dehli-2006]**

24. Using quadratic formula, solve the equation : $a^2b^2x^2 - (4b^4 - 3a^4)\ x - 12a^2b^2 = 0$

OR

The sum of two natural numbers is 8. Determine the numbers if the sum of their reciprocals is $\frac{8}{15}$.

[CBSE-Al-2006]

25. Solve for x : $(a + b)^2 x^2 + 8\ (a^2 - b^2)\ x + 16\ (a - b)^2 = 0$

OR

Two number differ by 3 and their product is 504. Find the number. **[CBSE-Foreign-2006]**

26. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speeds of the two trains. **[CBSE-Foreign-2006]**

27. Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two-fifth of Varun's age. Find their present ages. **[CBSE-Delhi-2006C]**

28. A 2-digit number is such that product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

OR

A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for the journey. Find the original speed of the train **[CBSE-Al-2006C]**

29. A shopkeeper buys x books for Rs. 720. (i) Write the cost of 1 book in terms of x, (ii) If the cost per book were Rs. 5 less, the number of books that could be bought for Rs. 720 would be 2 more.
Write down the equation in x for the above situation and solve it to find x. **[ICSE-2006]**

30. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

OR

By increasing the list price of a book by Rs. 10 a person can buy 10 less books for Rs. 1200. Find the original list price of the book. **[CBSE-Delhi-2007]**

31. The numerator of a fraction is one less than its denominator. If three is added to each of the numerator and denominator, the fraction is increased by $\frac{3}{28}$. Find the fraction.

OR

The difference of squares of two natural numbers is 45. The square of the smaller umber is four times the larger number. Find the numbers. **[CBSE-Al-2007]**

32. Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member increased by Rs. 10. Find how many students went for the picnic. **[ICSE-2008]**

33. In a class test, the sum of the marks obtained by P in mathematics and science is 28. Had he got 3 more marks in mathematics and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

OR

The sum of the areas of two squares is 640 m^2. If the difference in their perimeters be 64 m, find the sides of the two squares. **[CBSE-Delhi-2008]**

34. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **[CBSE-Al-2008]**

35. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to it's hole at the base of pillar. Seeing the snake the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?

OR

Two difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the two numbers.

[CBSE-Foreing-2008]

36. The sum of the squares of two consecutive odd numbers is 394. Find the numbers. **[CBSE-Delhi-2009]**

37. Solve the following equation for x : $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

OR

If (– 5) is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k. **[CBSE-Al-2009]**

38. A trader bought a number of articles for Rs. 900. Five articles were found damaged. He sold each of the remaining articles at Rs. 2 more than what he paid for it. He got a profit of Rs. 80 on the whole transaction. Find the number of articles he bought.

OR

Two years ago a man's age was three times the square of his son's age. Three years hence his age will be four times his son's age. Find their present ages. **[CBSE-Foreing-2009]**

39. A girl is twice as old as her sister. Four years hence. The product of their ages (in yeras) will be 160. Find their present ages. **[CBSE-Al-2010]**

SBJECTIVE	ANSPER KEY	EXERCISE -4 (x)-CBSE

- **SHORT ANSWER TYPE QUESTION**

1. (–1, 3) **2.** $[2+\sqrt{3}, 2-\sqrt{3}]$ **3.** $x = \frac{a^2}{2}, \frac{b^2}{2}$ **4.** $x = \frac{(a^2+b^2)}{2}.\frac{(a^2-b^2)}{2}$ **5.** $x = \frac{(2a+b)}{3}.\frac{(a+2b)}{3}$ **6.** $\frac{q^2}{p^2}, 1$

7. a + b, a – b **8.** 2a + b, 2a – b **9.** $\frac{(a^2+b^2)}{3}, \frac{(a^2-b^2)}{3}$ **10.** $\frac{(a+b)}{3}\frac{(a-b)}{3}$ **11.** $\frac{(a^2+b^2)}{4}, \frac{(a^2-b^2)}{4}$

12. $\frac{(a+b)}{6}\frac{(a-b)}{6}$ **13.** 1.70, – 1.37 **14.** $\frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$ or $(a+b)^2, (a-b)^2$ **15.** 6.53, – 1.53 **16.** $\frac{a}{3}.\frac{b}{3}$ or 14, 15

17. $\frac{a^2}{3}.\frac{b^2}{3}$ or 3, 4, 5 **18.** x = – 2, 1 **19.** 3.64 – 0.14 **20.** 4.85, – 1.85 **21.** 2, 1 **22.** No **23.** No

24. Yes 28.64 29. 3and 5

- **LONG ANSWER TYPE QUESTION**

1. (i) $\left(\frac{400}{x}\right)$ hrs. (ii) $\left(\frac{400}{x+40}\right)$ hrs; x = 160 km/hr 2. (i) 30 (ii) 1200 3. x = – 10 1/5 4. $x = -\frac{23}{5}, -1$

6. $x = -\frac{4}{3}, \frac{1}{8}$ or 50 7. $x = \frac{5}{8}, \frac{11}{5}$ or 300 km/hr 8. 36 9. 84 10. 45 11. 5,10 12. 4,12 13. 6,12 14. 35 15. 45

16. 27 17. 5 and 3 18. 25 km/hr 19. x = – 5, 2 or 750 km/hr 20. x = – a, – b or $x = \frac{c}{b}, \frac{-b}{a}$

21. $x = \frac{1}{b^2}, -\frac{1}{a^2}$ or $x = \frac{5}{2}$,5 22. 30 km/hr 23. $x = \frac{-2b}{3a}, \frac{3a}{4b}$ or 57 24. $x = \frac{-3a^2}{b^2}, \frac{4b^2}{a^2}$ or 3 and 5

25. $x = \frac{-4(a-b)}{a+b}$ or 21, 24 or – 21, – 24 26. 40 km/hr, 50 km/hr 27. 9 years, 27 years 28. 92 or 45 km/hr

29. (i) Rs. $\left(\frac{720}{x}\right)$ (ii) x2 + 2x – 288 = 0, x = 16 **30.** 10 and 5 or Rs. 30 **31.** 3/4 or 9 and 6 **32.** 16

33. Marks in maths : 12(9), Marks in science : 16(19) **34.** 6 km/hr or 25 hrs and 15 hrs

35. 12 m or (7 and 3) or (– 3 and – 7) **36.** 13 and 15 **37.** $\frac{2a+b}{3}, \frac{a+2b}{3}$ or p = 7 and $k = \frac{7}{4}$

38. 75 or son's age = 5 years and man's age = 29 years. **29.** 6 years and 12 years

EXERCISE – 5 (FOR OLYMPIADS)

CHOOSE THE CORRECT ONE

1. The roots of the equation (x – a) (x – b) (x – c) + (x – b) (x – c) +(x – c) (x – a) = 0 are :

(A) Real (B) Not real (C) Imaginary (D) Rational

2. The integral values of k for which the equation (k – 2) x^2 + 8x + k + 4 = 0 has both the roots real, distinct and negative is :

(A) 0 (B) 2 (C) 3 (D) – 4

3. If the roots of the equation $\frac{x^2-bx}{ac-c} = \frac{m-1}{m+1}$ are equal and of oppositve sign, then the value of m will be :

(A) $\frac{a-b}{a+b}$ (B) $\frac{b-a}{a+b}$ (C) $\frac{a+b}{a-b}$ (D) $\frac{b+a}{b-a}$

4. If α, β are the roots of the equation x^2 + 2x + 4 = 0, then $\frac{1}{\alpha^3}\frac{1}{\beta^3}$ is equal to :

(A) $-\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 32 (D) $\frac{1}{32}$

5. If $x = \sqrt{6+\sqrt{6+\sqrt{6+...to\infty}}}$, then :

(A) x is an irrational number (B) 2 < x < 3

(C) x = 3 (D) None of these

6. If α, β are the roots of the equation $x^2 + 7x + 12 = 0$, then the equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ is :

(A) $x^2 + 50x + 49 = 0$ (B) $x^2 - 50x + 49 = 0$ (C) $x^2 - 50x - 49 = 0$ (D) $x^2 + 12x + 7 = 0$

7. The values of k (k > 0) for which the equation $x^2 + kx + 64 =$ and $x^2 - 8x + k = 0$ both will have real roots is :

(A) 8 (B) 16 (C) – 64 (D) None of these

8. If α, β are the roots of the equation $x^2 + bx - c = 0$, then the equation whose roots are b and c is :

(A) $x^2 + \alpha x - \beta = 0$ (B) $x^2 - [(\alpha + \beta) + \alpha\beta]x - \alpha(\alpha + \beta) = 0$

(C) $x^2 + (\alpha\beta + \alpha + \beta)x + \alpha\beta(\alpha + \beta) = 0$ (D) $x^2 + (\alpha\beta + \alpha + \beta)x - \alpha\beta(\alpha + \beta) = 0$

9. Solve for y : $9y^4 - 29y^2 + 20 = 0$

(A) $\pm 2, \pm\frac{2}{3}$ (B) $\pm 3, \pm\frac{3}{\sqrt{5}}$ (C) $\pm 1, \pm\frac{2\sqrt{5}}{3}$ (D) None of these

10. Solve for x : $x^6 - 26x^3 - 27 = 0$

(A) – 1, 3 (B) 1, 3 (C) 1, – 3 (D) – 1, – 3

11. Solve : $\sqrt{2x+9} + x = 3$:

(A) 4, 16 (B) 8, 20 (C) 2, 8 (D) None of these

12. Solve : $\sqrt{2x+9} - \sqrt{x-4} = 3$

(A) 4, 16 (B) z8, 20 (C) 2, 8 (D) None of these

13. Solve for x : $2\left[x^2 + \frac{1}{x^2}\right] - 9\left[x + \frac{1}{x}\right] + 14 = 0$:

(A) $\frac{1}{2}$, 1, 2 (B) 2, 4, $\frac{1}{3}$ (C) $\frac{1}{3}$, 4, 1 (D) None of these

14. Solve x : $6\left[x^2 + \frac{1}{x^2}\right] - 25\left(x + \frac{1}{x}\right) + 12 = 0$:

(A) $-\frac{1}{3}, -\frac{1}{2}$, 2, 3 (B) $\frac{1}{3}, \frac{1}{2}$, 2, 3 (C) $\frac{1}{3}, \frac{1}{2}$, – 2 , – 3 (D) None of these

15. Solve for x : $\sqrt{x^2 + x - 6} - x + 2 = \sqrt{x^2 - 7x + 10}$, $x \in R$:

(A) 2, 6, $-\frac{10}{3}$ (B) 2, 6 (C) – 2 , – 6 (D) None of these

16. Solve for x : $3^{x+2} + 3^{-x} = 10$

(A) – 3, – 2 (B) – 2, 0 (C)2 , 3 (D) None of these

17. Solve for x : (x + 1) (x + 2) (x + 3) (x + 4) = 24 (x ∈ R) :

(A) 0, – 5 (B) 0, 5 (C) 0, – 2 (D) 0, 2

18. The sum of all the real roots of the equation $|x-2|^2 + |x-2|^2 - 2 = 0$ is :

(A) 2 (B) 3 (C) 4 (D) None of these

19. If a, b ∈ {1, 2, 3, 4}, then the number of quadratic equation of the form ax2 + bx + 1 = 0, having real roots is :

(A) 6 (B) 7 (C) 8 (D) None of these

20. The number of real solutions of $x - \frac{1}{x^2 - 4} = 2 - \frac{1}{x^2 - 4}$ is :

(A) 0 (B) 1 (C) 2 (D) Infinite

21. If $(2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1} = \frac{2}{2-\sqrt{3}}$, then x is equal to :

(A) 0 (B) 1 (C) 2 (D) Both (A) and (C)

22. The quadratic equation $3x^2 + 2(a^2 + 1)\,x + a^2 - 3a + 2 = 0$ possesses roots of opposite sign then a lies in :

(A) $(-\infty, 0)$ (B) $(-\infty, 1)$ (C) (1, 2) (D) (4, 9)

23. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has :

(A) No solution (B) One solution (C) Two solution (D) More than two solution

24. The number of real solutions of the equation $2|x|^2 - 5|x| + 2 = 0$ is :

(A) 0 (B) 4 (C) 2 (D) None of these

25. The number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$:

(A) 0 (B) 2 (C) 3 (D) 6

26. The number of real solutions of the equation $2^{3x^2-7x+4} = 1$ is :

(A) 0 (B) 4 (C) 2 (D) Infinitely many

27. If the equation $(3x)^2 + (27 \times 3^{1/k} - 15)\,x + 4 = 0$ has equal roots, then k =

(A) – 2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 0

28. If $x = \sqrt{2+\sqrt{2+\sqrt{2+\ldots+\infty}}}$, then x is :

(A) 1 (B) 2 (C) 3 (D) None of these

29. Equation $ax^2 + 2x + 1$ has one double root if :

(A) a = 0 (B) a = – 1 (C) a = 1 (D) a = 2

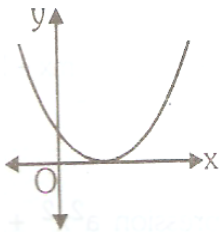
30. Solve for x : (x + 2) (x – 5) (x – 6) (x + 1) = 144 :

(A) –1, –2, –3 (B) 7, – 3, 2 (C) 2, – 3, 5 (D) None of these

31. If f(x) = $\frac{2x+5}{x^2+x+5}$, then find f(f(– 1))

(A) $\frac{149}{155}$ (B) $\frac{155}{147}$ (C) $\frac{155}{149}$ (D) $\frac{147}{155}$

32. What does the following graph represent?



(A) Quadratic polynomial has just one root.

(B) Quadratic polynomial has equal one roots.

(C) Quadratic polynomial has no root.

(D) Quadratic polynomial has equal roots and constant term is non-zero.

33. Consider a polynomial $ax^2 + bx + c$ such that zero is one of it's roots then :

(A) $c = 0, x = \frac{-b}{a}$ satisfies the polynomial equation

(B) $c \neq 0, x = \frac{-b}{a}$ satisfies the polynomial equation

(C) , $x = \frac{-b}{a}$ satisfies the polynomial equation

(D) Polynomial has equal roots.

34. For a parabola opening upwards and above x-axis, quadratic will have :

(A) Equal roots and a = 0 (B) Unequal roots and $a \neq 0$

(C) No roots, a > 0 (D) No roots, a < 0

35. The equation $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$ has :

(A) An extraneous root between – 5 and – 1.

(B) An extraneous root between – 10 and – 6.

(C) Two extraneous roots.

(D) A real root between 20 and 25.

36. Consider a quadratic polynomial $f(x) = ax^2 - x + c$ such that $ac > 1$ and it's graph lies below x-axis then :

(A) $a < 0, c > 0$ (B) $a < 0, c < 0$ (C) $a > 0, c > 0$ (D) $a > 0, c < 0$

37. If α, β are the roots of a quadratic equation $x^2 - 3x + 5 = 0$, then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is :

(A) $x^2 + 4x + 1 = 0$ (B) $x^2 - 4x + 4 = 0$ (C) $x^2 - 4x - 1 = 0$ (D) $x^2 + 2x + 3 = 0$

OBJECTIVE ANSPER KEY EXERCISE -5

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	A	B	C	B	B	C	C	A	B	B	A	A	B
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	B	A	C	B	A	D	C	A	B	A	C	B	B	C	B
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**								
Ans.	C	D	A	C	B	B	B								

EXERCISE – 6 (FOR IIT-JEE/AIEEE)

CHOOSE THE CORRECT ONE

Based on Graph of Quadratic Expression

1. If the expression $\left[mx - 1 + \frac{1}{x}\right]$ is non negative for all positive real x, then the minimum value of m must be :

(A) $\frac{-1}{2}$ (B) 0 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

2. The expression $a^2x^2 + bx + 1$ will be positive for all $x \in R$ if :

(A) $b^2 > 4a^2$ (B) $b^2 < 4a^2$ (C) $4b^2 > a^2$ (D) $4b^2 < 4a^2$

3. If x be real, then $3x2 + 14x + 11 > 0$ when :

(A) $x < -\frac{3}{2}$ (B) $x > -\frac{3}{4}$ (C) $x > -2$ (D) Never

4. For what value of a the curve $y = x2 + ax + 25$ touches the x-axis :

(A) 0 (B) ± 5 (C) ± 10 (D) None of these

5. The integer k for which the inequality $x^2 - 2(4k - 1)x + 15k2 - 2k - 7 > 0$ is valid for any x is :

(A) 2 (B) 3 (C) 4 (D) 6

6. The value fo the expression $x^2 - 2bx + c$ will be positive for all real x if :

(A) $b^2 - 4c > 0$ (B) $b^2 - 4c < 0$ (C) $c^2 < b$ (D) $b^2 < c$

7. If the roots fo the quadratic equation $ax^2 + bx + c = 0$ are imaginary then for all values of a, b, c and x R the expression $a^2x^2 + abx + ac$ is :

(A) Positive (B) Non-negative (C) Negative (D) May be positive, zero or negative

Based on Maximum & Minimum Value of the Expression :

8. The range of $y = \frac{x+2}{2x^2+3x+6}$, if x is real, is :

(A) $-\frac{1}{13} \le y \le \frac{1}{3}$ (B) $\frac{1}{13} \le y \le \frac{1}{3}$ (C) $-\frac{1}{13} \le y \le \frac{1}{13}$ (D) None of these

9. If $x \in R$ and $k = \frac{(x^2 - x + 1)}{(x^2 + x + 1)}$, then :

(A) $x \le 0$ (B) $\frac{1}{3} \le k \le 3$ (C) $k \ge 5$ (D) None of these

10. For all real values of x, the maximum value of the expression $\frac{x}{x^2-5x+9}$ is :

(A) 1 (B) 45 (C) 90 (D) None of these

11. If x be real then the maximum and minimum value of the expression $\frac{x^2-3x+4}{x^2+3x+4}$ are

(A) 2, 1 (B) $7, \frac{1}{7}$ (C) $5, \frac{1}{5}$ (D) None of these

12. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is : **[AIEEE-2006]**

(A) $\frac{17}{7}$ (B) $\frac{1}{4}$ (C) 41 (D) None of these

Based on the Concept of Common Roots :

13. The value of k, so that the equation $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have one root in common is :

(A) – 2, – 3 (B) $-3, -\frac{27}{7}$ (C) – 5, – 6 (D) None of these

14. If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \neq 0$, then the common factor is :

(A) (x – 3) (B) (x – 6) (C) (x – 8) (D) None of these

15. The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is :

(A) 0, 2 (B) 0, – 2 (C) 2, – 2 (D) None of these

16. If the equation $x^2 + bx + c = 0$ and $x^2 + cx + b = 0$, $(b \neq c)$ have a common root then :

(A) b + c = 0 (B) b + c = 1 (C) b + c + 1 = 0 (D) None of these

17. If both the roots of the equation $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then 2r – p is equal to :

(A) 1 (B) – 1 (C) 2 (D) 0

18. If every pair from among the equation $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$ and $x^2 rx + pq = 0$ has a common root, then the sum of three common roots is :

(A) 2 (p + q + r) (B) p + q + r (C) – (p + q + r) (D) pqr

19. If $x^2 - ax - 21 = 0$ and $x^2 - 3ax + 35 = 0$; $a > 0$ have a common root, then a is equal to :

(A) 1 (B) 2 (C) 4 (D) 5

20. The values of a for which the quadratic equation $(1 - 2a)x^2 - 6ax - 1 = 0$ and $ax^2 - x + 1 = 0$ have at least one root in common are :

(A) $\frac{1}{2}, \frac{2}{9}$ (B) $0, \frac{1}{2}$ (C) $\frac{2}{9}$ (D) $0, \frac{1}{2}, \frac{2}{9}$

21. If the quadratic equation $2x^2 + ax + b = 0$ and $2x^2 + bx + a = 0$ $(a \neq 0)$ and $ax^2 - x + 1 = 0$ have a common root, the value of a + b is :

(A) – 3 (B) – 2 (C) – 1 (D) 0

22. If the equation $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have a common root and $b \neq c$, then their other roots will satisfy the equation :

(A) $x^2 - (b + c)x + bc = 0$ (B) $x^2 - ax + bc = 0$

(C) $x^2 + ax + bc = 0$ (D) None of these

23. If both the roots of the equation $x^2 + mx + 1 = 0$ and $(b - c)x^2 + (c - a)x + (a - b) = 0$ are common then :

(A) m = – 2 (B) m = – 1 (C) m = 0 (D) m = 1

24. The quadratic equation $x^2 - 6x + a = 0$ and $x^2 - cx + ab = 0$ have one common root. The other roots of first and second equation are integers in the ratio 4 : 3. Then common root is : **[AIEEE-2008]**

(A) 1 (B) 4 (C) 3 (D) 2

Miscellaneous :

25. Solve $x^2 - 5x + 4 = 0$:

(A) $1 < x < 4$ (B) $-4 < x < 1$ (C) $x < 1$ and $x > 4$ (D) None of these

26. Solve $-x^2 + 6x - 8 = 0$:

(A) $-2 < x < 4$ (B) $-4 < x < -2$ (C) $2 < x < 4$ (D) None of these

27. For all $x \in R$, $x^2 + 2ax + 10 - 3a > 0$ then the interval in which 'a' lies is : **[IIT Screening-2004]**

(A) $(-\infty,-5)$ (B) $(-5, 2)$ (C) $(5, \infty)$ (D) $(2, 5)$

28. The solution set contained in R of the inequation : $3^x + 3^{1-x} - 4 < 0$ is : **[EAMCET-2003]**

(A) (1, 3) (B) (0, 1) (C) (1, 2) (D) (0, 2)

29. The number of real solution of the equation $x^2 - 3|x| + 2 = 0$ is : **[AIEEE-2003]**

(A) 3 (B) 2 (C) 4 (D) 1

30. Product of real roots the equation $t^2x^2 + |x| + 9 = 0$: **[AIEEE-2002]**

(A) Is always positive (B) Is always negative (C) Does not exist (D) None of these

31. For the equation 3x2 + px + 3 = 0, p > 0. If one of the roots is square of the other, then p =

(A) $\frac{1}{2}3$ (B) 1 (C) 3 (D) $\frac{2}{3}$ **[IIT Screening-2000]**

32. The roots of the equation $|x^2 - x - 6| = x + 2$ are :

(A) – 2, 1, 4 (B) 0, 2, 4 (C) 0, 1, 4 (D) – 2, 2, 4

33. If α, β are the roots of $x^2 + x + 1 = 0$, the equation whose roots are $(\alpha^{19}, \beta^7$ is : **[IIT 1994]**

(A) $x^2 - x - 1 = 0$ (B) $x^2 - x + 1 = 0$ (C) $x^2 + x - 1 = 0$ (D) $x^2 + x + 1 = 0$

34. The equation of the smallest degree with real coefficients having 1 + 1 as one of the roots is :

(A) $x^2 + x + 1 = 0$ (B) $x^2 - 2x + 2 = 0$ (C) $x^2 + 2x + 2 = 0$ (D) $x^2 + 2x - 2 = 0$

[Kerala Engineering -2002]

35. If a, b, c, d are positive reals such that a + b + c + d = 2 and M = (a + b) (c + d), then :

(A) $0 < M \leq 1$ (B) $1 \leq M \leq 2$ (C) $2 \leq M \leq 3$ (D) $3 \leq M \leq 4$

[IIT Screening-2000]

36. Let a, b, c be real numbers such that 4a + 2b + c = 0 and ab > 0; then the quadratic equation $ax^2 + bx + c = 0$ has :

(A) Real roots (B) Non-real roots **[IIT 1990]**

(C) Purely imaginary roots (D) Only one real roots

37. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then the biquadratic P(x) Q(x) = 0 has :

(A) All the four roots real (B) No real roots **[IIT 1989]**

(C) Atleast imaginary roots (D) Two equal roots

38. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has : **[IIT 1989]**

(A) Two roots (B) Infinitely many roots (C) Only one roots (D) No root

39. Number of values of x satisfying the equation $(15+4\sqrt{14})^t + (15-4\sqrt{14})^t = 30$, where $t = x^2 - 2|x|$:

(A) 0 (B) 2 (C) 4 (D) 6

40. The of values of x which satisfy the expression : $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{16})^{x^2-3} = 10$

(A) $\pm 2, \pm\sqrt{3}$ (B) $\pm\sqrt{2}, \pm 4$ (C) $\pm 2, \pm\sqrt{2}$ (D) $\pm\sqrt{2}, \pm\sqrt{3}$

41. If α and β are the roots of the equation $ax^2 + bx + c$, where (a,b,c) > 0, then α and β are :

(A) Rational numbers (B) Real and negative (C) Negative real parts (D) None of these

42. The number of quadratic equation which remain unchanged by squaring their roots, is :

(A) 0 (B) 2 (C) 4 (D) Infinitely many

43. If the equation $(\lambda^2+5\lambda+6)x^2+(\lambda^2-3\lambda+2)x+(\lambda^2-4)=0$ has more than two roots, then the value of λ is

(A) 2 (B) 3 (C) 1 (D) – 2

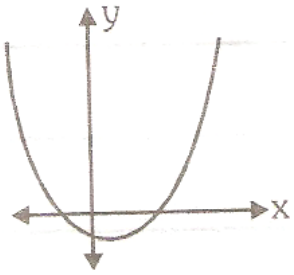
44. Find all the integral values of a for which the quadratic equation (x – a) (x – 10) + 1 = 0 has integral roots :

(A) 12, 8 (B) 4, 6 (C) 2, 0 (D) None of these

45. If one root of the quadratic equation $px^2 + qx + r = 0$ ($p \neq 0$) is a surd $\frac{\sqrt{a}}{\sqrt{a}+\sqrt{a-b}}$ where p,q,r,a,b are all rationals then the other root is :

(A) $\frac{\sqrt{a}}{\sqrt{a}+\sqrt{a-b}}$ (B) $a+\frac{\sqrt{a(a-b)}}{b}$ (C) $\frac{a+\sqrt{a(a-b)}}{b}$ (D) $\frac{\sqrt{a}-\sqrt{a-b}}{\sqrt{b}}$

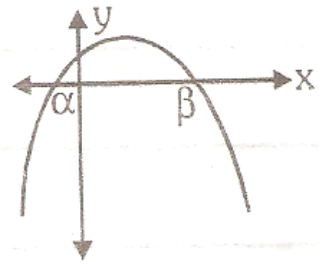
46. Graph of $y = ax^2 + bx + c$ is given adjacently. What conclusions can be drawn from the graph :



(i) a > 0 (ii) b < 0 (iii) c < 0 (iv) $b^2 - 4ac > 0$

(A) (i) and (iv) (B) (ii) and (iii) (C) (i), (ii) & (iv) (D) (i), (ii), (iii) & (iv)

47. The adjacently figure shows the graph of $y = ax^2 + bc + c$. Then which of the following is correct :



(i) a > 0 (ii) b > 0 (iii) c > 0 (iv) $b^2 < 4ac$

(A) (i) and (iv) (B) (ii) and (iii) (C) (iii) & (iv) (D) None of these

OBJECTIVE — ANSWER KEY — EXERCISE -6

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	B	B	C	B	D	A	A	B	A	B	C	B	C	B
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	D	B	C	C	B	A	A	D	C	C	B	B	C	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	C	D	D	B	A	A	C	D	C	C	C	C	A	A	C
Que.	**46**	**47**													
Ans.	D	B													

AREAS RELATED TO CIRCLES

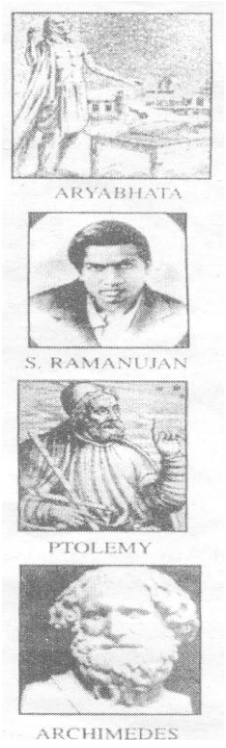
★ INTRODUCTION

In earlier classes, we have studied methods of finding perimeters and area of simple plane figures such as rectangles, squares, parallelograms, triangle and circles. In our daily life, we come across many objects which are related to circular shape in some form or the other. For example, cycle wheels, wheel arrow, drain cover, bangles, flower beds, circular paths etc. That is why the problem of finding perimeters and areas related to circular figures is of great practical importance. In this chapter, we shall discuss problems on finding the area of some combinations of plane figures involving circles or parts of circles. Let us first recall the concepts related to their perimeter and area of a circle.

★ HISTORICAL FACTS

Mensuration is that branch of mathematics which studies the method of measurements. Measurement is a very important human activity. We measure the length of a cloth for stitching. The area of a wall for painting, the perimeter of a plot for fencing. We do many other measurements of similar nature in our daily life. All these measurements, we shall study in this chapter called Mensuration.

π (pi) occupies the most significant place in measurement of surface area as well as volume of various solid and plane figures. The value of π is not exactly known. The story of the accuracy by which the value of π was estimated is an interesting one.



Mathematically $\pi = \dfrac{Cirumfereane\ of\ \ a\ circle}{Diameter\ of\ \ the\ circle}$

$\pi = \dfrac{(10+4)\ x\ 8+62000}{20{,}000} = \dfrac{62832}{20{,}000} = 3.1416$

According to **S. Ramanujan,** the value of π

$$\pi = \frac{355}{113}$$

According to **Archimedes ;** the value of π is given below :

$$\pi = \frac{Cirumference\ of\ \ a\ circle}{Diameter\ of\ \ the\ circle} = 3\frac{10}{71} < \pi < 3\frac{10}{70}$$

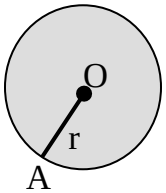
Ptolemy, $\pi = 3\dfrac{17}{120}$

The **Egyptians,** $\pi = \left(\dfrac{16}{9}\right)^2 = 3.160$

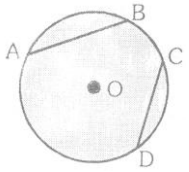
Note: π (pi) is an irrational number. It cannot be expressed as the ratio of whole numbers. However, the ratio 22 : 7 is often used as approximation for it.

★ RECALL

(A) Circle: Circle is the locus of a point which moves in such a manner that its distance from a fixed point O remains constant (the same). The fixed point is **called the centre** O and the constant distance OA is **called its radius.**

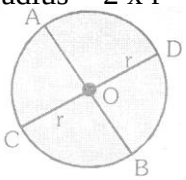


(B) **Chord :** A line segment joining two points on a circle is called a chord of the circle. If fig. AB and CD are two chords of the circle.



(C) **Diameter :** A chord passing through the centre of the circle is called the diameter. In fig, AOB and COD are diameter of the circle i.e., the diameter is the largest chord of the circle.

Length of diameter = Twice the radius = 2 x r = AOB = COD



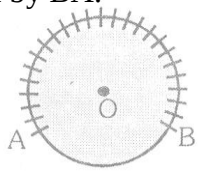
(D) **Circumference :** The perimeter of the circle or the length of boundary of the circle is called its circumference i.e. the distance covered by traveling once around a circle is called the perimeter or circumference. The circumference of a circle is given by $2\pi r$. It is well-known fact that the ratio of the circumference of a circle to its diameter bears a constant ratio.

$$\pi = \frac{Circumference\ of\ a\ circle}{Diameter\ of\ the\ circle}$$

$\Rightarrow$ Circumference = π x diameter = π x 2r = $2\pi r$ where r is the radius of the circle.

(E) **Arc.** Any part of a circle is called an arc of the circle. Two points A and B on a circle divides it into two arcs. In general one arc is greater than other. The smaller arc is called minor arc and greater arc is called major arc.

In the given fig, AB is an arc of a circle with centre O, denoted by $\overset{\frown}{AB}$. The remaining part of the circle shown by the dotted lines is represented by BA.

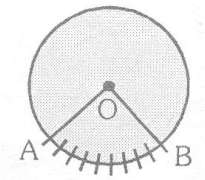


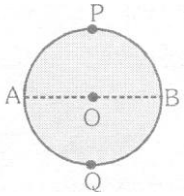
(F) **Central Angle :** Angle subtended by an arc at the centre of a circle is called its central angle. In fig. the centre of the circle is O.

Central angle made by $\overset{\frown}{AB}$ at the centre O = $\angle AOB = \theta$

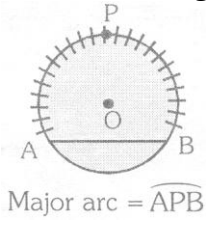
If $\theta^o < 180^o$ then the arc $\overset{\frown}{AB}$ is called the minor arc and the arc $\overset{\frown}{AB}$ is called major arc.

(G) **Semi-circle :** A diameter divides a circle into two congruent arcs. Each of these two arc is called a semicircle. In the given fig. of circle with centre O, and are semicircles. Is the half of the circle. $\overset{\frown}{APB}$ $\overset{\frown}{BQA}$

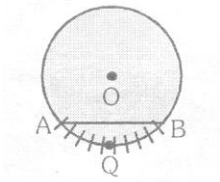




(H) **Major arc :** An arc whose length is more than the length of the semi-circle is called a major arc:

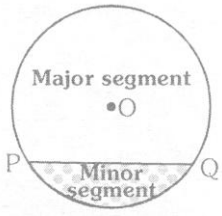


(I) **Minor arc :** An arc whose length is less than the length of semi-circle is called a minor arc.



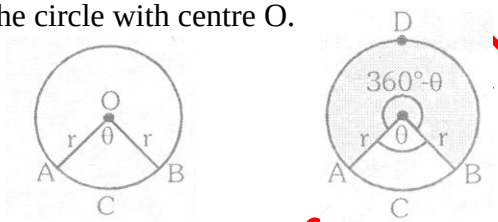
Minor arc = $\overset{\frown}{BQA}$

(J) **Segment :** A segment of a circle is the region bounded by an arc and its chord, including the arc and the chord.



The shaded segment containing the minor arc is called a minor segment, while the unshaded segment containing the major arc is called the major segment.

(K) **Sector of a circle :** A sector of a circle is a region enclosed by an arc and its two bounding radii. In the fig OACBO is a sector of the circle with centre O.



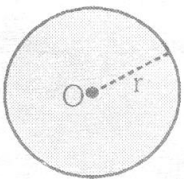
If arc AB is a minor arc then OACBO is a called the minor segment of the circle. The remaining part OADBO of the circle is called the major sector of the circle.

★ **FORMULA**

1. **For a circle of radius = r units, we have**

(a) Circumference of the circle = $(2\pi r)$ units = (πd) units,
Where d is the diameter.

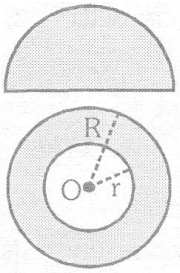
(b) Area of the circle = (πr^2) sq. units.



II. **For a semi-circle of radius = r units, we have**

(a) Area of the semi-circle = $(\frac{1}{2}\pi r^2)$ sq. units

(b) Perimeter of the semi-circle = $(\pi r + 2r)$ units.



III. **Area of a Circular Ring :**

If R and r be the outer and inner radii of a ring, then
Area of the ring = $\pi (R^2 - r^2)$ sq. units

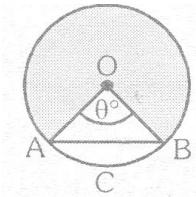
IV **Results on Sectors and Segments :**

Suppose an arc ACB makes an angle θ at the centre O of a circle of radius = r units. Then :

(a) Length of arc ACB = $\left(\frac{2\pi r\theta}{360}\right)$ units

(b) Area of sector OACBO = $\left(\frac{\pi r^2\theta}{360}\right)$ sq. units

$= \frac{1}{2} x\, r\, x \left(\frac{2\pi r\theta}{360}\right)$ sq. units $= \left(\frac{1}{2} x\ radius\ x\ arc\ length\right)$ sq. units



(c) Perimeter of sector OACBO = length of arc ACB + OA + OB = $\left(\frac{2\pi r\theta}{360} + 2r\right)$ units.

(d) Area of segment ACBA = (Area of sector OACBO) – (Area of ΔOAB) = $\left(\frac{\pi r^2\theta}{360} - \frac{1}{2}r^2\sin\theta\right)$ sq. units.

(e) Perimeter of segment ACBA = arc ACB + chord AB) units.

(f) Area of Major segment BDAB = (Area of circle) – (Area of segment ACBA).

V. Rotations Made By a Wheel:

(a) Distance moved by a wheel in 1 revolution = Circumference of the wheel

(b) Number of rotations made by a wheel in unit time = $\frac{Distance\ moved\ by\ it\ in\ unit\ time}{Circuference\ of\ the\ wheel}$

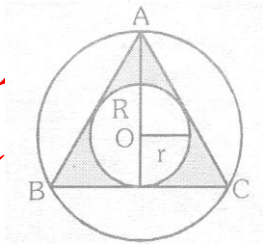
VI. Facts About Clocks:

(a) Angle described by minute hand in 60 minutes = 360^o

(b) Angle described by minute hand in 5 minutes = $\left(\frac{360}{60}\ x\ 5\right)^o = 30^o$

(c) Angle described by hour hand in 12 hours = 360^o.
Angle described by hour hand in 1 hour = 30^o.

VII. In an equilateral triangle of side a units, we have:



(a) Height of the triangle, h = $\frac{\sqrt{3}}{2}$ a units.

(b) Area of the triangle = $\left(\frac{\sqrt{3}}{4}a^2\right)$ sq. units.

(c) Radius of incircle, r = $\frac{1}{3}$h = $\left(\frac{1}{3}.\frac{\sqrt{3}}{2}a\right) = \left(\frac{a}{2\sqrt{3}}\right)$ units.

(d) Radius of circumcircle, R = $\frac{2}{3}h = \left(\frac{2}{3}.\frac{\sqrt{3}}{2}a\right) = \left(\frac{a}{\sqrt{3}}\right)$ units.

Thus, r = $\frac{a}{2\sqrt{3}}$ and R = $\frac{a}{\sqrt{3}}$

Ex.1 Calculate the circumference and area of a circle of radius 5.6 cm.

Sol. We have :

$Circum\ ference\ of\ the\ circle = 2\pi r = \left(2\ x\frac{22}{7}x5.6\right)$ cm = 35.2 cm.

Area of the circle = $\pi r^2 = \left(\frac{22}{7}x5.6x5.6\right)$ cm^2 = 98.56 cm^2.

Ex.2 The circumference of a circle is 123.2 cm. Calculate :
(i) the radius of the circle in cm,
(ii) the area of the circle, correct to nearest cm^2.

Sol. (i) Let the radius of the circle be r cm.
Then, its circumference = (2π r) cm.

$\therefore$ 2π r = 123.2 $\Rightarrow$ 2 x $\frac{22}{7}$ x r = 123.2 $\Rightarrow$ r = (123.2 x $\frac{7}{44}$) = 19.6 cm.

$\therefore$ Radius of the circle = 19.6 cm.

(ii) Area of the circle = π r^2 = ($\frac{22}{7}$ x 19.6 x 19.6) cm^2= 1207.36 cm^2.

$\therefore$ Area of the circle, correct to nearest cm^2 =m 1207 cm^2.

Ex.3 The area of a circle is 301.84 cm^2. Calculate:
(i) the radius of the circle in cm.
(ii) the circumference of the circle, correct to nearest cm

Sol. (i) Let the radius of the circle be r cm.
Then, its area = πr^2 cm^2 = 301.84

$\Rightarrow \quad \frac{22}{7} \times r^2 = 301.84$

$\Rightarrow \quad r^2 = \left(301.84 \times \frac{7}{22}\right) = 96.04 \Rightarrow r = \sqrt{96.04} = 9.8$ cm.

$\therefore$ Radius of the circle = 9.8 cm.

(ii) Circumference of the circle = $2\pi r = \left(2 \times \frac{22}{7} \times 9.8\right)$ cm = 61.6 cm.

$\therefore$ Circumference of the circle, correct to nearest cm = 62 cm.

Ex.4 The perimeter of a semi-circular protractor is 32.4 cm. Calculate :
(i) the radius of the protractor in cm,
(ii) the area of the protractor in cm^2.

Sol. (i) Let the radius of the protractor be r cm.
Then, its perimeter = $(\pi r + 2r)$ cm.

$\therefore \quad \pi r + 2r = 32.4 \Rightarrow (\pi + 2)r = 32.4$

$\Rightarrow \left(\frac{22}{7} + 2\right) r = 32.4 \Rightarrow \frac{36}{7} r = 32.4 \Rightarrow r = \left(32.4 \times \frac{7}{36}\right)$ cm = 6.3 cm.

Radius of the protractor = 6.3 cm.

(ii) Area of the protractor = $\frac{1}{2}\pi r^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 6.3 \times 6.3\right)$ cm^2 = 62.37 cm^2.

$\therefore$ Area of the protractor = 62.37 cm^2.

Ex.5 The area enclosed by the circumferences of two concentric circles is 346.5 cm^2. If the circumference of the inner circle is 88 cm, calculate the radius of the outer circle.

Sol. Let the radius of inner circle be r cm.
The, its circumference = $(2\pi r)$ cm.

$\therefore 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = \left(88 \times \frac{7}{44}\right) = 14$ cm.

$\therefore$ Radius of the inner circle is, r = 14 cm.
Let the radius of the outer circle be R cm.
Than, area of the ring = $(\pi R^2 - \pi r^2)$ cm^2

$= \pi (R^2 - r^2)$ cm^2 $= \frac{22}{7} \times [R^2 - (14)^2]$ cm^2

$= \left(\frac{22}{7} R^2 - 616\right)$ cm^2

$\therefore \quad \frac{22}{7} R^2 - 616 = 346.5 \Rightarrow \frac{22}{7} R^2 = 962.5$

$\Rightarrow \quad R^2 = \left(962.5 \times \frac{7}{22}\right) = 306.25 \Rightarrow R = \sqrt{306.25} = 17.5$ cm.

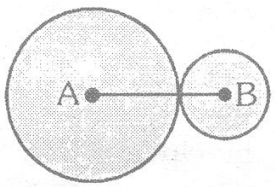
Hence, the radius of the outer circle is 17.5 cm.

Ex.6 Two circles touch externally. The sum of their areas is 130π sq. cm and distance between their centres is 14 cm. Determine the radii of the circles.

Sol. Let the radii of the given circles be R cm and r cm respectively. As the circles touch externally, distance between their centres = (R + r) cm.

$\therefore \quad R + r = 14 \qquad \ldots(i)$

Sum of their areas = $(\pi R^2 + \pi r^2)$ cm^2 = $\pi (R^2 + r^2)$ cm^2.



$\therefore \quad \pi(R^2 + r^2) = 130\pi$

$\Rightarrow \quad R^2 + r^2 = 130 \quad \ldots.(ii)$

We have the identity, $(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

$(14)^2 + (R - r)^2 = 2 \times 130$ [From (i) and (ii)]

$(R - r)^2 = 64$

$R - r = 8 \quad \ldots.(iii)$

On solving (i) and (iii), we get R = 11 and r = 3.

Hence, the radii of the given circles are 11 cm and 3 cm.

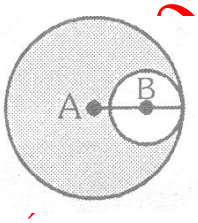
Ex.7 Two circles touch internally. The sum of their areas is 116π sq. cm and the distance between their centres is 6 cm. Find the radii of the given circles.

Sol. Let the radii of the given circles be R cm and r cm respectively. As the circles touch internally, distance between their centres = (R – r) cm.

$\therefore \quad R - r = 6 \quad \ldots.(i)$

Sum of their areas = $(\pi R^2 + \pi r^2)\ cm^2 = \pi(R^2 + r^2)\ cm^2$

$\therefore \ \pi(R^2 + r^2) = 116\ \pi \Rightarrow R^2 + r^2 = 116 \quad \ldots(iii)$



We have the identity, $(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

$\Rightarrow \quad (R + r)^2 + 6^2 = 2 \times 116$ [Using (i) and (ii)]

$\Rightarrow \quad (R + r)^2 = 196$

$\Rightarrow \quad R + r = \sqrt{196} = 14 \quad \ldots.(iii)$

On solving (i) and (iii), we get R = 10 and r = 4.

Hence, the radii of the given circles are 10 cm and 4 cm.

Ex.8 The wheel of a cart is making 5 revolutions per second. If the diameter of the wheel is 84 cm, find its speed in km/hr. Give your answer, correct to nearest km.

Sol. Radius of the wheel = 42 cm.

Circumference of the wheel = $2\pi r = \left(2 \times \frac{22}{7} \times 42\right)$ cm = 264 cm.

Distance moved by the wheel in 1 revolution = 264 cm.

Distance moved by the wheel in 5 revolutions = (264 x 5) cm = 1320 cm.

$\therefore$ Distance moved by the wheel in 1 second = 1320 cm.

Distance moved by the wheel in 1 hour = (1320 x 60 x 60) cm.

$= \left(\frac{1320 \times 60 \times 60}{100 \times 1000}\right)$ km

$\therefore$ Speed of the cart = $\left(\frac{1320 \times 60 \times 60}{100 \times 1000}\right)$ km/hr = 47.52 km/hr.

Hence, the speed of the cart, correct to nearest km/hr is 48 km/hr.

Ex.9 The diameter of the driving wheel of a bus is 140 cm. How many revolutions must the wheel make in order to keep a speed of 66 km/hr?

Sol. Distance to be covered in 1 min. = $\left(\frac{66 \times 1000}{60}\right)$ m = 1100 m.

Radius of the wheel = $\left(\frac{140}{2}\right)$ cm = 70 cm = 0.70 m.

Circumference of the wheel = $2\pi r = \left(2 \times \frac{22}{7} \times 0.70\right)$ m = 4.4 m.

$\therefore$ Number of revolutions per minute = $\left(\frac{1100}{4.4}\right) = 250$.

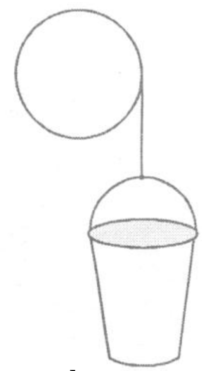
Hence, the wheel must make 250 revolutions per minute.

Ex.10 A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the bucket ascends in 1 min. 28 seconds with a uniform speed of 1.1 m / sec, calculate the number of complete revolutions the wheel makes in raising the bucket.

Sol. Time taken by bucket to ascend = 1 min. 28 sec. = 88 sec. Speed = 1.1 m/sec.

Length of the rope = Distance covered by bucket to ascend

= (1.1m x 88) m = (1.1 x 88 x 100) cm = 9680 cm.



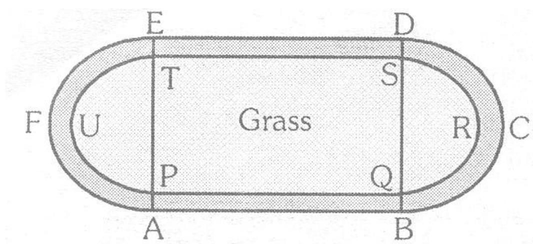
Radius of the wheel = $\frac{77}{2}$ cm.

Circumference of the wheel = $2\pi r = 2x\left(\frac{22}{7}x\frac{77}{2}\right)$ cm = 242 cm.

` ∴ Number of revolutions = $\frac{Length\ of\ the\ rope}{Circumference\ of\ the\ wheel} = \left(\frac{9680}{242}\right)$ = 40.

Hence, the wheel makes 40 revolutions to raise the bucket.

Ex.11 The figure shows a running track surrounding a grass enclosure PQRSTU. The enclosure consists of a rectangle PQST with a semi-circular region at each end. Given, PQ = 200 m and PT = 70 m.



(i) Calculate the area of the grassed enclosure in m^2.

(ii) Given that the track is of constant width 7 m, calculate the outer perimeter ABCDEF to the track.

Sol. (i) Diameter of each semi-circular region of grassed enclosure = PT = 70 m,

∴ Radius of each one of them = 35 m.

Area of grassed enclosure

= (Area of rect. PQST) + 2 x $\frac{1}{2}\pi r^2 = \left[(200x70)+\frac{22}{7}x35x35\right]$ m^2 = 17850 m^2.

(ii) Diameter of each outer semi-circle of the track = AE = (PT + 7 + 7) m = 84 m.

∴ Radius of each one of them = 42 m.

Outer perimeter ABCDEF = (AB + DE + semi-circle BCD + semi-circle EFA)

= (2PQ + 2 x circumference of semi-circle with radius 42 m)

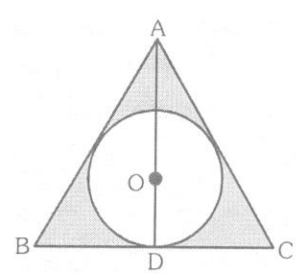
= (2 x 200 + 2 x π x 42) m = $\left[2\ x\ 200+2\ x\frac{22}{7}x\ 42\right]$ m = 664 m.

Ex.12 In an equilateral triangle of side 24 cm, a circle is inscribed, touching its sides. Find the area of the remaining portion of the triangle. Take $\sqrt{3}$ = 1.73 and π = 3.14.

Sol. Let Δ ABC be the given equilateral triangle in which a circle is inscribed.

Side of the triangle, a = 24 cm.

Height of the triangle, h = $\left(\frac{\sqrt{3}}{2}xa\right)cm = \left(\frac{\sqrt{3}}{2}x\ 24\right)cm = 12\sqrt{3}\,cm.$



Radius of the incircle, r = $\frac{1}{3}h = \left(\frac{1}{3}x12\sqrt{3}\right)cm = 4\sqrt{3}\,cm.$

∴ Required Area = Area of the shaded region

= (Area of Δ ABC) – (Area of incircle)

$= \left(\frac{\sqrt{3}}{4} x\ 24\ x\ 24 - \pi\, x\, 4\sqrt{3}\, x\, 4\sqrt{3}\right) \text{cm}^2$

$= (144\sqrt{3} - 3.14 \text{ x } 48)\ \text{cm}^2 = (144 \text{ x } 1.73 - 3.14 \text{ x } 48)\ \text{cm}^2$

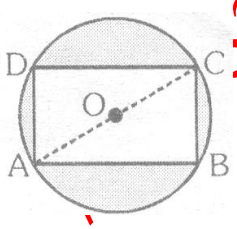
$= [48 \text{ x } (3 \text{ x } 1.73 - 3.14)]\ \text{cm}^2 = (48 \text{ x } 2.05)\ \text{cm}^2 = 98.4\ \text{cm}^2$

Ex.13 In the given figure, a circle circumscribes a rectangle with sides 12 cm and 9 cm. Calculate :

(i) the circumference of the circle to nearest cm,

(ii) the area of the shaded region, correct to 2 places of decimal, in cm^2.

Take $\pi = 3.14$.



Sol. Let ABCD be the rectangle with AB = 12 cm and BC = 9 cm.

$\therefore$ AC = $\sqrt{AB^2 + BC^2} = \sqrt{(12)^2 + 9^2} = \sqrt{225} = 15\,\text{cm}$.

Let O be the mid-point of AC.

Then, O is the centre and OA, the radius of the circum-circle.

$\therefore$ Radius, OA = $\frac{1}{2}$AC = $\left(\frac{1}{2} x 15\right)$ cm = 7.5 cm.

$\therefore$ (i) Circumference of the circle = 2π r = (2 x 3.14 x 7.5) cm = 47.1 cm.

Hence, the circumference of the circle, correct to nearest cm is 47 cm.

(ii) Area of shaded region = (Area of the circle) – (Area of the rectangle)

$= \left[\left(3.14 x \frac{15}{2} x \frac{15}{2}\right) - (12 x 9)\right] cm^2$

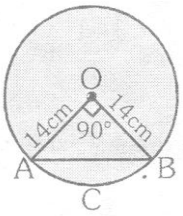
= (176.625 – 108) cm^2 = 68.625 cm^2 = 68.63 cm^2.

Ex.14 A chord of a circle of radius 14 cm makes a right angle at the centre. Calculate :

(i) the area of the minor segment of the circle, (ii) the area of the major segment of the circle.

Sol. Let AB be the chord of a circle with centre O and radius 14 cm such that $\angle AOB = 90^o$.

Thus, r = 14 cm and $\theta = 90^o$.



(i) Area of sector OACB = $\frac{\pi r^2 \theta}{360} = \left(\frac{22}{7} x 14 x \frac{90}{360}\right) \text{cm}^2 = 154\ \text{cm}^2$.

Area of ΔOAB = $\frac{1}{2} r^2 \sin\theta = \left(\frac{1}{2} x 14 x 14 x \sin 90^o\right)\ \text{cm}^2 = 98\ \text{cm}^2$.

$\therefore$ Area of minor segment ACBA = (Area of sector OACB) – (Area of ΔOAB) = (154 – 98) cm^2 = 56 cm^2.

(ii) Area of major segment BDAB

= (Area of the circle) – (Area of minor segment ACBA)

$= \left[\left(\frac{22}{7} x 14 x 14\right) - 56\right]\ \text{cm}^2$ = (616 – 56) cm^2 = 560 cm^2.

Ex.15 The minute hand of a clock is 10.5 cm long. Find the area swept by it in 15 minutes.

Sol. Angle described by minute hand in 60 minutes = 360^o.

Angle described by minute hand in 15 minutes = $\left(\frac{360}{60} x 15\right)^o = 90^o$.

Thus, required area is the area of a sector of a circle with central angle, $\theta = 90^o$. and radius, r = 10.5 cm.

Required area = $\left(\frac{\pi r^2}{360}\right) = \left(\frac{22}{7} x 10.5 x 10.5 x \frac{90}{360}\right)\ \text{cm}^2 = 86.63\ \text{cm}^2$.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. If the radii of two circles are 7 cm and 24 cm, then the radius of circle having area equal to the sum of the areas of the two circles, is

(A) 31 cm (B) 25 cm

(C) 17 cm (D) 28 cm

2. The cost of fencing a circular field at the rate of Rs. 24 per metre is RS. 5280. Then the cost of ploughing the field, at the rate of 50 paise/m^2, is

(A) Rs. 2875 (B) Rs. 3850

(C) Rs. 1925 (D) Rs. 1825

3. The inner circumference of a circular track is 220 m, and the track is 14 m wide. The cost of leveling the track, at 50 paise/m^2, is

(A) Rs. 1848 (B) Rs. 1663.2

(C) Rs. 1478.4 (D) None of these

4. The area of a sector, of a circle with radius 7 cm and angle of the sector is 60^o. is

(A) $\frac{144}{3}$ cm^2 (B) $\frac{154}{21} cm^2$

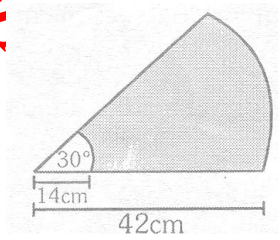
(C) $\frac{150}{7}$ cm^2 (D) $\frac{77}{3}$ cm^2

5. In a circle of radius 21 cm, an are subtends an angle of the centre. The area of the segment formed by the corresponding chord of the arc is

(A) 40.63 cm^2 (B) 421.73 cm^2

(C) 429.43 cm^2 (D) 40.27 cm^2

6. AB and CD are respectively arcs of two concentric circles of radii 42 cm and 14 cm and centre O as shown in the adjoining figure. If $\angle AOB = 30^o$, then the area of the shaded region is



(A) $\frac{1232}{3} cm^2$ (B) $\frac{1220}{3} cm^2$

(C) 411cm^2 (D) None of these

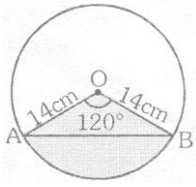
7. In the given figure, the shaded area is

(A) 205.03 cm^2

(B) 205.04 cm^2

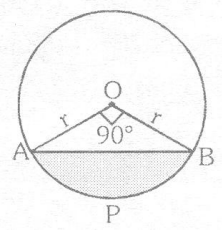
(C) 205.33 cm^2

(D) 205.35 cm^2



8. In the given figure, the area of the segment APB is

(A) $\frac{1}{4}\pi r^2$



(B) $\frac{1}{4}(\pi-2)r^2$

(C) $\frac{1}{4}(\pi-1)r^2$

(D) None of these

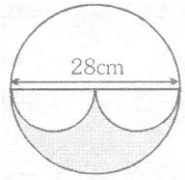
9. In the given figure, the area of shaded region is

(A) 462 cm^2

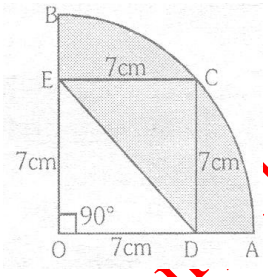
(B) 308 cm^2

(C) 616 cm^2

(D) 154 cm^2



10. In the given figure, ODCE is a square then the area of shaded region is



(A) 52.5 cm^2 (B) 24.5 cm^2 (C) 49 cm^2 (D) None of these

OBJECTIVE				ANSWER KEY					EXERCISE	
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	A	D	D	A	C	B	D	A

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

Remark : Take $\pi=\frac{22}{7}$, unless mentioned otherwise.

1. A sheet is 11 cm long and 2 cm wide. Circular pieces 0.5 cm in diameter are cut from it to prepare discs. Calculate the number of discs that can be prepared.

2. Find the circumference and area of a circle of radius 17.5 cm.

3. Find the circumference and area of a circle of diameter 91 cm.

4. Find the circumference and area of a circle of radius 15 cm. (Take $\pi=3.14$)

5. The circumference of a circle is 123.2 cm. Taking $\pi=\frac{22}{7}$, calculate:

(i) the radius of the circle in cm;

(ii) the area of the circle in cm^2, correct to the nearest cm^2;

(iii) the effect on the area of the circle if the radius is doubled

6. Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 m^2.

7. The area of a circle is 394.24 cm^2. Calculate : (i) the radius of the circle, (ii) the circumference of the circle.

8. Find the perimeter and area of a semi-circular of a plate of radius 25 cm (Take $\pi=3.14$).

9. The perimeter of a semi-circular metallic plate is 86.4 cm. Calculate the radius and area of the plate.

10. The circumference of a circle exceeds its diameter by 180 cm. Calculate
(i) the radius (ii) the circumference and (iii) the area of the circle.

11. A copper wire when bent in the form of a square encloses an area of 272.25 cm^2. If the same wire is bent into the form of a circle, what will be the area enclosed by the wire?

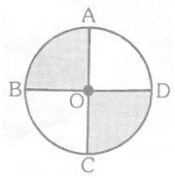
12. A copper wire when bent in the form of a equilateral triangle has an area of $121\sqrt{3}$ cm^2. If the same wire is bent into the form of a circle, find the area enclosed by the wire.

13. The circumference of a circle field is 528 m.

14. The cost of leveling a circular field at Rs2 per sq. metre is Rs 33957. Calculate:
(i) the area of the field; (ii) the radius of then field; (iii) the circumference of the field;
(iv) the cost of fencing it at Rs 2.75 per metre.

15. The cost of fencing a circular field at Rs 9.50 per metre is Rs 2926. Find the cost of ploughing the field at Rs 1.50 per sq. metre.

16. AC and BD are two perpendicular diameters of a circle ABCD. Given that the area of the shaded portion is 308 cm^2, calculate : (i) the length of AC; and (ii) the circumference of the circle.

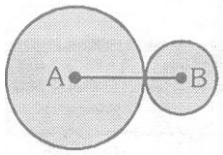


17. The sum of the radii of two circles is 140 cm and the difference of their circumference is 88 cm. Find the radii of the two circles.

18. The sum of the radii of two circles is 84 cm and the difference of their areas is 5544 cm^2. Calculate the radii of the two circles .

19. Two circles touch externally. The sum of their areas is 117π cm^2 and the distance between their centres is 15 cm. Find the radii of the two circles.

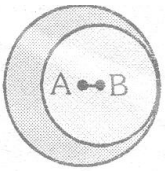
20. Two circles touch internally. The sum of their areas is and the distance between then centres is 4 cm. Find the radii of the circles.



21. Find the area of a ring whose outer and inner radii are 19 cm and 16 cm respectively.

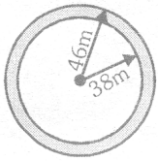
22. A path of width 8 m runs around a circular park whose radius is 38 m. Find the area of the path.

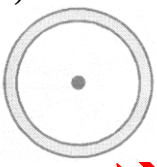
23. The areas of two concentric circles are 962.5 cm^2 and 1386 cm^2 respectively. Find the width of the ring.

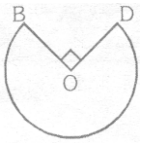


24. The area enclosed between two concentric circles is 770 cm^2. If the radius of the outer circle is 21 cm, calculate the radius of the inner circle.

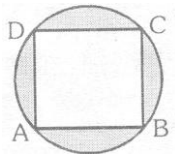
25. In the given figure, the area enclosed between two concentric circles is 808.5 cm^2. The circumference of the outer circle is 242 cm. Calculate : (i) the radius of the inner circle, (ii) the width of the ring.







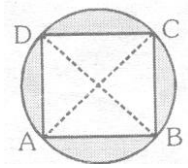
26. Find the area of a circle circumscribing an equilateral triangle of side 15 cm. [Take π = 3.14].



27. Find the area of a circle inscribed in an equilateral triangle of side 18 cm. [Take π = 3.14].

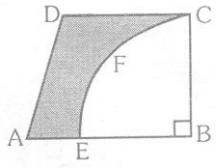
28. The shape of the top of a table in a restaurant is that of a segment of a circle with centre O and $\angle BOD = 90^o$. BO = OD = 60 cm. Find: (i) the area of the top of the table; (ii) the perimeter of the table. [Take π = 3.14].

29. In the given figure, ABCD is a square of side 5 cm inscribed in a circle. Find:
(i) the radius of the circle, (ii) the area of the shaded region. [Take π = 3.14]

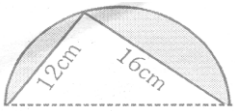


30. In the given figure, ABCD is a rectangle inscribed in a circle. If two adjacent sides of the rectangle be 8 cm and 6 cm, calculate : (i) the radius of the circle; and (ii) the area of the shaded region. [Take π = 3.14].

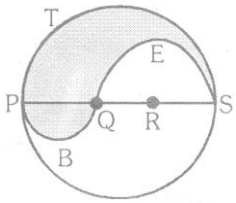
31. In the given figure, ABCD is a piece of cardboard in the shape of a trapezium in which AB || DC, $\angle ABC = 90^o$. From this piece, quarter circle BEFC is removed. Given DC = BC = 4.2 cm and AE = 2 cm.
Calculate the area of the remaining piece of the cardboard.



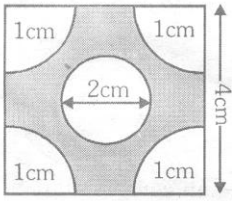
32. Find the perimeter and area of the shaded region in the given figure. (Take π = 3.142).



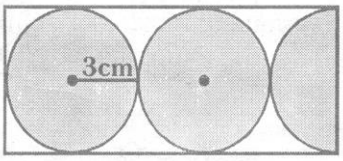
33. In the given figure, PQRS is a diameter of circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters. If PS = 12 cm, find the perimeter and the area of the shaded region. [Take π = 3.14].



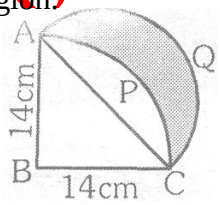
34. Find the perimeter and area of the shaded region shown in the figure. The four corners are circle quadrants and at the centre, there is a circle. [Take π = 3.14].



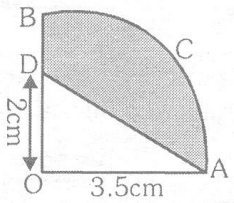
35. In the given figure, find the area of the unshaded portion within portion within the rectangle. [Take π = 3.14].



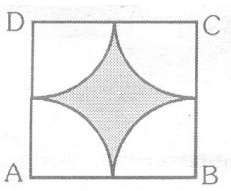
36. In the given figure, ABCP is a quadrant of circle of radius 14 cm. With AC as diameter, a semi-circle is drawn. Find the area of the shaded region.



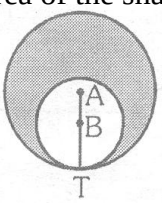
37. In the given, OACB is a quadrant of a circle. The radius OA = 3.5 cm, OD = 2 cm. Calculate the area of the shaded portion.



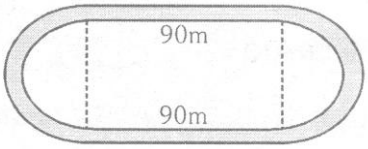
38. In the given figure, ABCD is a square of side 14 cm and A, B, C, D are centres of circular arcs, each of radius 7 cm. Find the area of the shaded region.



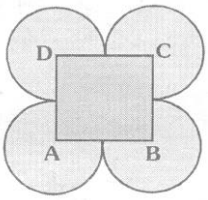
39. In the given figure, two circles with centres A and B touch each other at the point T. If AT = 14 cm and AB = 3.5 cm, find the area of the shaded region.



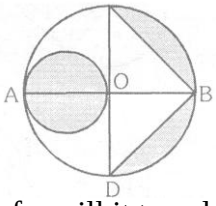
40. In the adjoining figure, the inside perimeter of a running track with semi-circular ends and straight parallel sides is 312 m. The lengths of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.



41. In the given figure, ABCD is a square of side 7 cm and A, B, C, D are centres of equal circles which touch externally in pairs. Find the area of the shaded region.



42. In the given figure, AB is the diameter of a circle with centre O and OA = 7 cm. Find the area of the shaded region.



43. The diameter of a wheel is 1.26 m. How far will it travel in 500 revolutions?

44. The wheel of the engine of a train $4\frac{2}{7}$ m in circumference makes 7 revolutions in 3 seconds. Find the speed of the train in km per hour.

45. A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 30 revolutions?

46. A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.

AREAS RELATED TO CIRCLES **ANSWER KEY** EXERCISE – 2 (X)-CBSE

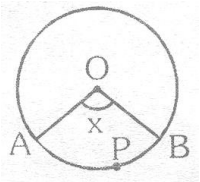
1. 88 **2.** 110 cm, 962.5 cm^2 **3.** 286 cm, 6506.5 cm^2 **4.** 94.2 cm, 706.5 cm^2
5. (i) 19.6 cm (ii)mm 1207 cm^2 (iii) area becomes four times. **6.** 56 m **7.** (i) 1.2 cm (ii) 70.4cm
8. 128.5 cm, 981.25 cm^2 **9.** 168. cm, 443.52 cm^2 **10.** (i) 42 cm (ii) 264 cm (iii) 5544 cm^2 **11.** 346.5 cm^2
12. 346.5 cm^2 **13.** (i) 84 m (ii) 22176 m^2 (iii) Rs 33264 **14.** (i) 16978.5 m^2 (ii) 73.5 m (iii) 462 m (iv) Rs 1270.50 **15.** Rs 11319 **16.** (i) 28 cm (ii) 88 cm **17.** 77 cm, 63cm **18.** 52.5 cm; 31.5 cm
19. 9 cm, 6 cm **20.** 11 cm, 7 cm **21.** 330 cm^2 **22.** 2112 m^2 **23.** 3.5 m^2 **24.** 14 cm
25. 35 cm, 3.5 cm **26.** 235.5 cm^2 **27.** 84.78 cm^2 **28.** (i) 8478 cm^2 (ii) 402.60 cm
29. (i) $\frac{5}{2}\sqrt{2}$ cm (ii) 14.25 cm^2 **30.** (i) 5 cm (ii) 30.5 cm^2 **31.** 7.28 cm^2 **32.** 59.4 cm, 61.1 cm^2
33. 37.68 cm, 37.68 cm^2 **34.** 20.56 cm, 9.72 cm^2 **35.** 19.35 cm^2 **36.** 98 cm^2 **37.** 6.125 cm^2
38. 42. cm^2 **39.** 269.5 cm^2 **40.** 636.57 m^2 **41.** 164.5 cm^2 **42.** 66.5 cm^2 **43.** 1980 m
44. 36 km/hr **45.** 50 **46.** 14 m

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

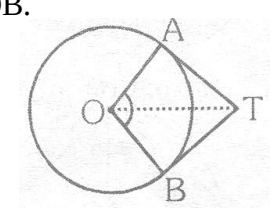
VERY SHORT ANSWER TYPE QUESTIONS

1. In the fig. O is the centre of a circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle. Find x.

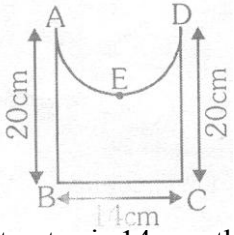


[Delhi-2008]

2. In fig., if $\angle ATO = 40^o$, find $\angle$ AOB. **[AI-2008]**



3. Find the perimeter of the given figure, where AED is a semi circle and ABCD is a rectangle. **[AI-2008]**

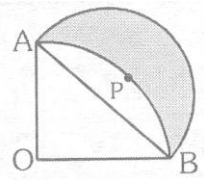


4. If the diameter of a semicircular protractor is 14 cm, then find it's perimeter. **[AI-2009]**

5. The length of the minute hand of a wall clock is 7 cm. How much area does it sweep in 20 minutes? **[Foreign-2009]**

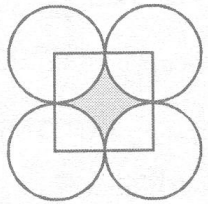
SHORT ANSWER TYPE QUESTIONS

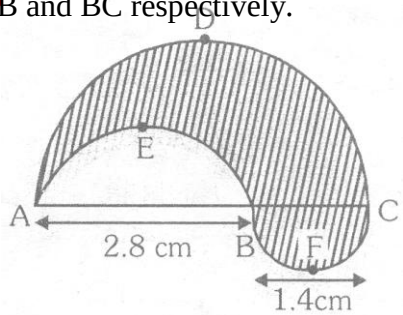
1. In fig. AOBPA is quadrant of a circle of radius 14 cm. A semicircle with AB as diameter is draw. Find the area of the shaded region.



2. Four circles are described about the four corners of a square of a square so that each touches two of the others as shown in fig. Find the area of the shaded region. Each side of the square is 14 cm. (Take π = 22/7) **[Delhi-2007]**

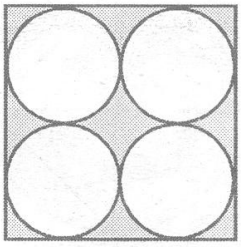
3. In the fig., find the perimeter of shaded region where ADC, AEB and BFC are semicircles on diameters AC, AB and BC respectively.



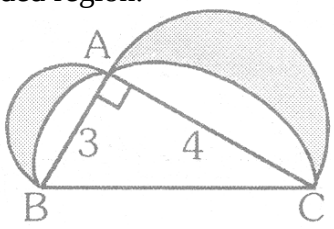


OR

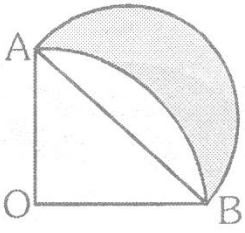
Find the area of the shaded region in the fig., where ABCD is a square of side 14 cm. **[Delhi-2008]**



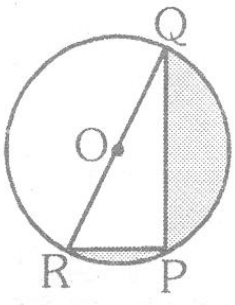
4. In fig., ABC is a right-angled triangle, right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region. **[AI-2008]**



5. In the fig., ABC is a quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of the shaded region. **[Foreign-2008]**

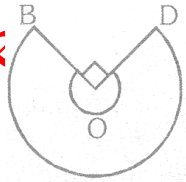


6. In fig., PQ = 24 cm, PR = 7 cm and O is the centre of the circle. Find the area of shaded region. (Take π = 3.14) **[Delhi-2009]**



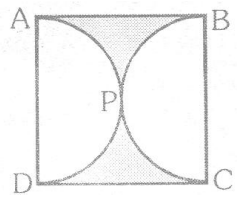
7. In figure, the shape of the top of a table in a restaurant is that of a sector of a circle with centre O and $\angle BOD = 90^o$. If BO = OD = 60 cm, find.

(i) the area of the top of the table (ii) The perimeter of the table top. (Take π = 3.14)



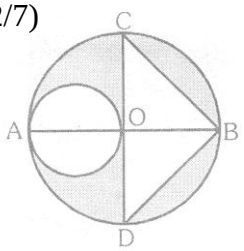
OR

In fig., ABCD is a square of side 14 cm and APD and BPC are semicircles. Find the area of shaded region.

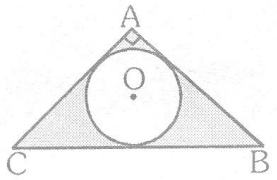


(Take π = 22/7) **[Foreign-2009]**

8. In fig., AB and CD are two perpendicular diameters of a circle with centre O. If OA = m7 cm. find the area of the shaded region. (Take π = 22/7) **[AI-2010]**



LONG ANSWER TYPE QUESTIONS



1. In fig., ABC is a right triangle right angled at A. Find the area of shaded region if AB = 6 cm, BC = 10 cm and O is the centre of the incircle of ΔABC. (Take π = 3.14) **[Delhi-2009]**

2. The area of an equilateral triangle is $49\sqrt{3}\,cm^2$. Taking each angular point as centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles. (Take $\sqrt{3}$ = 1.73)

- **VERY SHORT ANSWER TYPE QUESTIONS**

1. 100^o **2.** 100^o **3.** $(7\pi + 54)$ cm **4.** 36 cm **5.** $\frac{154}{3}\ cm^2$

- **SHORT ANSWER TYPE QUESTIONS**

1. 98 cm^2 **2.** 42 cm^2 **3.** 13.2 cm or 42 cm^2 **4.** 6 sq. units **5.** 98 cm^2 **6.** 161.3 cm^2

- **LONG ANSWER TYPE QUESTIONS**

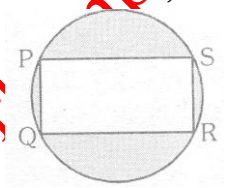
1. 11.44 cm^2 **2.** 7.77 cm^2

EXERCISE – 4 (FOR OLYMPIADS)

CHOOSE THE CORRECT ONE

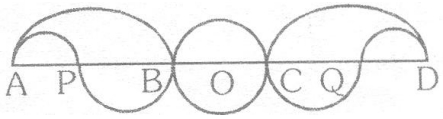
1. In the adjoining figure PQRS is a rectangle 8 cm x 6 cm, inscribed in the circle. The area of the shaded portion will be :

(A) 48 cm^2
(B) 42.50 cm^2
(C) 32.50 cm^2
(D) 30.5 cm^2



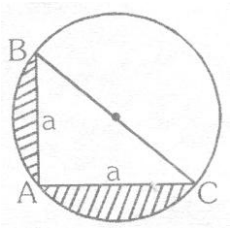
2. In the adjoining figure AB = CD = 2BC = 2BP = 2 CQ. In the middle, a circle with radius 1 cm is drawn. In the rest figure all are the semicircular arcs. What is the perimeter of the whole figure?

(A) 4 π
(B) 8 π
(C) 10 π
(D) None of these



3. If BC passes through centre of the circle, then the area of the shaded region in the given figure is :

(A) $\frac{a^2}{2}(3-\pi)$
(B) $a^2\left(\frac{\pi}{2}-1\right)$
(C) $2a^2(\pi - 1)$
(D) $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$



4. Two circles of unit radii, are so drawn that the centre of each lies on the circumference of the other. The area of the region common to both the circles, is :

(A) $\frac{(4\pi - 3\sqrt{3})}{12}$ (B) $\frac{(4\pi - 6\sqrt{3})}{12}$

(C) $\frac{(4\pi - 3\sqrt{3})}{6}$ (D) $\frac{(4\pi - 6\sqrt{3})}{6}$

5. The area of the largest possible square inscribed in a circle of unit radius (in square unit) is :

(A) 3 (B) 4 (C) $2\sqrt{3\pi}$ (D) 2

6. The area of the largest triangle that can be inscribed in a semicircle of radius r is:

(A) r^2 cm^2 (B) $\left(\frac{r}{3}\right)^2 cm^2$ (C) $r\sqrt{2}$ cm^2 (D) $3\sqrt{3r}$ cm^2

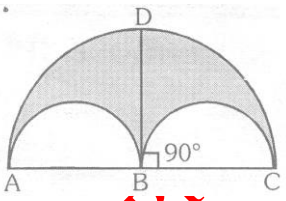
7. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is :

(A) $6\sqrt{3r}$ (B) 6r (C) 3r (D) 12r

8. If a regular circumscribes a circle of radius r, then its perimeter is :

(A) $4\sqrt{3}r$ (B) $6\sqrt{3}r$ (C) 6r (D) $12\sqrt{3}r$

9. In the adjoining figure there are three semicircles in which BC = 6 cm and BD = $6\sqrt{3}$ cm. What is the area of the shaded region (in cm):

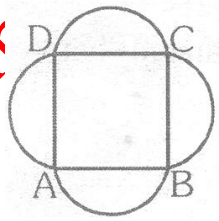


(A) 12π
(B) 9π
(C) 27π
(D) 28π

10. ABCD is a square of side a cm. AB, BC, CD and AD all are the chords of circles with equal radii each. It the chords subtends an angle of 120^o at their respective centres, find the total area of the given figure, where arcs are part of the circles:

(A) $\left[a^2 + 4\left(\frac{\pi a^2}{9} - \frac{a^2}{3\sqrt{2}}\right)\right]$

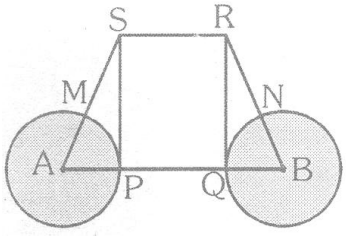
(B) $\left[a^2 + 4\left(\frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}}\right)\right]$



(C) $[9a^2 - 4\pi + 3\sqrt{3a^2}]$

(D) None of these

11. In the adjoining figure PQRS is a square and MS = RN and A, P, Q and B lie on the same line. Find the ratio of the area of two circles to the area of the square. Given that AP = Ms.



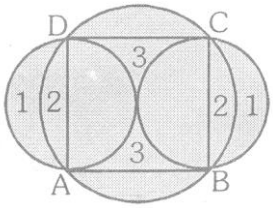
(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{3\pi}{2}$

(D) $\frac{6}{\pi}$

Direction for questions number (12 to 14) : In the adjoining figure ABCD is a square. A circle ABCD is passing through all the four vertices of the square. There are two more circles on the sides AD and BC touching each other inside the square, AD and BC are the respective diameters of the two smaller circles. Area of the square is 16 cm^2.



12. What is the area of region 1?

(A) 2.4 cm^2 (B) $\left(2-\frac{\pi}{4}\right)cm^2$

(C) 8 cm^2 (D) $(4\pi - 2)$ cm^2

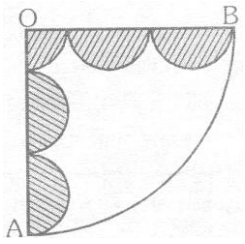
13. What is the area of region 2?

(A) 3 $(\pi - 2)$ cm^2 (B) $(\pi - 3)$ cm^2 (C) $(2\pi - 3)$ cm^2 (D) 4 $(\pi - 2)$ cm^2

14. What is the area of region 3?

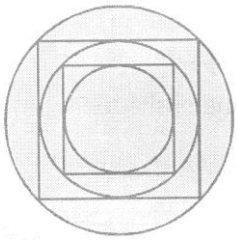
(A) $(4 - 4\pi)$ cm^2 (B) 4 $(4 - \pi)$ cm^2 (C) $(4\pi - 2)$ cm^2 (D) $(3\pi + 2)$ cm^2

15. A circular paper is folded along its diameter, then again it is folded to form a quadrant. Then it is cut as shown in the figure, after it the paper was reopened in the original circular shape. Find the ratio of the original paper to that of the remaining paper? (The shaded portion is cut off from the quadrant. The radius of quadrant OAB is 5 cm and radius of each semicircle is 1 cm) :



(A) 25 : 16 (B) 25 : 9 (C) 20 : 9 (D) None of these

Directions for questions number 16-18 : A square is inscribed in a circle then another circle is inscribed in the square. Another square is then inscribed in the circle. Finally a circle is inscribed in the innermost square. Thus there are 3 circles and 2 squares as shown in the fig. The radius of the outer-most circle is R.



16. What is the radius of the inner-most circle?

(A) $\frac{R}{2}$ (B) $\frac{R}{\sqrt{2}}$ (C) $\sqrt{2}R$ (D) None of these

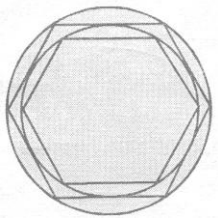
17. What is the sum of areas of all the squares shown in the figure?

(A) $3R^2$ (B) $3\sqrt{2}R^2$ (C) $\frac{3}{\sqrt{2}}R^2$ (D) None of these

18. What is the ratio of sum of circumferences of all the circles to the sum of perimeters of all the squares?

(A) $(2+\sqrt{3})\pi R$ (B) $(3+\sqrt{2})\pi R$ (C) $3\sqrt{3}\pi R$ (D) None of these

Directions for questions number 19-21 : A regular hexagon is inscribed in a circle of radius R. Another circle is inscribed in the hexagon. Now another hexagon is inscribed in the second (smaller) circle.



19. What is the sum of perimeters of both the hexagons?

(A) $(2+\sqrt{3})R$ (B) $3(2+\sqrt{3})R$ (C) $3(3+\sqrt{2})R$ (D) None of these

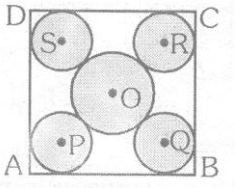
20. What is the ratio of area of inner circle to the outer circle?

(A) 3 : 4 (B) 9 : 16 (C) 3 : 8 (D) None of these

21. If there are some more circles and hexagons inscribed in the similar way as given above, then the ratio of each side of outermost hexagon (largest one) to that of the fourth (smaller one) hexagon is (fourth hexagon means the hexagon which is inside the third hexagon from the outerside.):

(A) $9 : 3\sqrt{2}$ (B) $16 : 9$ (C) $8 : 3\sqrt{3}$ (D) None of these

22. In the adjoining diagram ABCD is a square with side 'a' cm. In the diagram the area of the larger circle with centre 'O' is equal to the sum of the areas of all the rest four circles with equal radii, whose centres are P, Q, R, and S. What is the ratio between the side of square and radius of a smaller circle?



(A) $(2\sqrt{2} + 3)$ (B) $(2 + 3\sqrt{2})$ (C) $(4 + 3\sqrt{2})$ (D) Can't be determined.

23. There are two concentric circles whose areas are in the ratio of 9 : 16 and the difference between their diameters is 4 cm. What is the area of the outer circle?

(A) 32 cm^2 (B) 64π cm^2 (C) 36 cm^2 (D) 48 cm^2

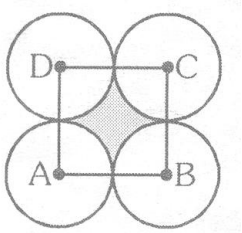
24. ABCD is a square, 4 equal circles are just touching each other whose centres are the vertices A, B, C, D of the square. What is the ratio of shaded to the unshaded area within square?

(A) $\frac{8}{11}$

(B) $\frac{3}{11}$

(C) $\frac{5}{11}$

(D) $\frac{6}{11}$



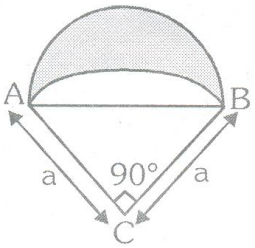
25. In the adjoining figure ACB is a quadrant with radius 'a'. A semicircle is drawn outside the quadrant taking AB as a diameter. Find the area of shaded region :

(A) $\frac{1}{4}(\pi - 2a^2)$

(B) $\left(\frac{1}{4}\right)(\pi a^2 - a^2)$

(C) $\frac{a^2}{2}$

(D) Can't be determined



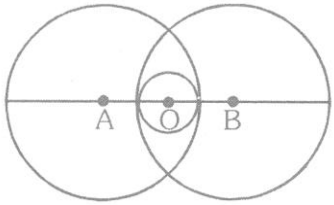
26. There are two circles intersecting each other. Another smaller circle with centre O, is lying between the common region of two larger circles. Centre of the circle (i.e., A, O and B) are lying on a straight line. AB = 16 cm and the radii of the larger circles are 10 cm each. What is the area of the smaller circle?

(A) 4π cm^2

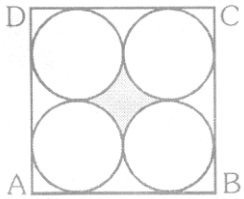
(B) 2π cm^2

(C) $\frac{4}{\pi}$ cm^2

(D) $\frac{\pi}{4}$ cm^2

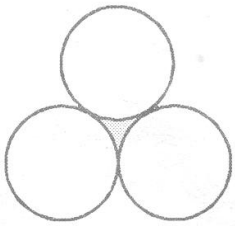


27. ABCD is a square, inside which 4 circles with radius 1 cm, each are touching each other. What is the area of the shaded region?



(A) $(2\pi - 3)$ cm^2
(B) $(4 - \pi)$ cm^2
(C) $(16 - 4\pi)$ cm^2
(D) None of these

28. Three circles of equal radii touch each other as shown in figure. The radius of each circle is 1 cm. What is the area of shaded region?



(A) $\left(\frac{2\sqrt{3}-\pi}{2}\right)$ cm^2
(B) $\left(\frac{3\sqrt{2}-\pi}{3}\right)$ cm^2
(C) $\frac{2\sqrt{3}}{\pi}$ cm^2
(D) None of these

OBJECTIVE					ANSWER KEY							EXERCISE – 4			
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	D	C	D	C	D	A	B	A	C	B	B	C	D	B	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**		
Ans.	A	A	D	B	A	C	B	B	B	C	A	B	A		

ARITHMETIC PROGRESSIONS

★ INTRODUCTION

Consider the following arrangement of numbers :

(i) 1, 3, 5, 7, …….. (ii) 3, 6, 12, 24, …….. (iii) 1, 4, 9, 16, ……..

In each of the above arrangements, we observe some patterns. In (i) we find that the succeeding terms are obtained by adding a fixed number [i.e. 2], in (ii) by multiplying with a fixed number [i.e. 2], in (iii) we find that they are squares of natural numbers.

In this chapter, we shall discuss one of these patterns in which succeeding terms are obtained by adding a fixed number to the preceding terms. We shall also see how to find their n^{th} terms and the sum of n consecutive terms, and use this knowledge in solving some daily life problems.

★ HISTORICAL FACTS

Gauss was a very talented and gifted mathematician of 19^{th} century who developed the formula :

$1 + 2 + 3 + 4 + \ldots + (n-1) + n = \frac{n(n+1)}{2}$ for the sum of first n natural numbers at the age of 10. He did this in the following way :

$S = 1 + 2 + 3 \ldots + (n-2) + (n-1) + n$

$S = \underline{n + (n-1) + (n-2) + \ldots + 3 + 2 + 1}$

$2S = (n+1) + (n+1) + (n+1) + \ldots + (n+1) + (n+1) + (n+1)$

$= (n+1)(1 + 1 + 1 + \ldots \text{ up to n times})$

$2S = (n+1)\,n \Rightarrow S = \frac{n(n+1)}{2}$

Even when he was a little child of three he could read and make mathematical calculation himself. Gauss proved the fundamental theorem of Algebra when he was 20 years old. His contribution to mathematics has been immense because his formulae were used in applied field of Astronomy, Differential Geometry and Electricity widely all over the world by scientists.

★ SEQUENCE

In our daily life, we come across the arrangement of numbers or objects in an order such as arrangement of students in a row as per their roll numbers, arrangement of books in the library, etc.

An arrangement of numbers depends on the given rule :

Given Rule	Arrangement of numbers
Write 3 and then add 4 successively	3. 7, 11, 15, 19,….
Write 3 and then multiply 4 successively	3, 12, 48, 192,…..
Write 4 and then subtract 3 successively	4, 1, - 2, -5,…..
Write alternately 5 and – 5	5, - 5, 5, -5,…

Thus, a sequence is an ordered arrangement of numbers according to a given rule.

Terms of a Sequence : The individual numbers that form a sequence are the terms of a sequence.

For example : 2, 4, 6, 8, 10,…… forming a sequence are called the first, second third, fourth and fifth,…. terms of the sequence.

The terms of a sequence in successive order is denoted by ‘T’$_n$ or ‘a’$_n$ The nth term ‘T’n is called the general terms of the sequence.

★ SERIES

The sum of terms of a sequence is called the series of the corresponding sequence. $T_1 + T_2 + T_3 + \ldots$ is an infinite series, where as $T_1 + T_2 + T_3 + \ldots + T_{n-1} + T_n$ is a finite series of n terms.

$S_n = T_1 + T_2 + T_3 + \ldots + T_{n-2} + T_{n-1} + T_n$

$S_{n-1} = T_1 + T_2 + T_3 + \ldots + T_{n-2} + T_{n-1}$

$S_n - S_{n-1} = T_n$

OR $\boxed{T_n = S_n - S_{n-1}}$

Ex.1 Write the first five terms of the sequence, whose nth term is $a_n = \{1 + (-1)^n\}n$.

Sol. $a_n = \{1 + (-1)^n\}n$

Substituting n = 1, 2, 3, 4 and 5, we get

$a_1 = \{1 + (-1)^1\}\ 1 = 0$; $a_2 = \{1 + (-1)^2\}\ 2 = 4$;

$a_3 = \{1 + (-1)^3\}\ 3 = 0$; $a_4 = \{1 + (-1)^4\}\ 4 = 8$;

$a_5 = \{1 + (-1)^5\}\ 5 = 0$

Thus, the required terms are : 0, 4, 0, 8 and 0.

Ex.2 Find the 20th term of the sequence whose nth term is, $a_n = \dfrac{n(n-2)}{n+3}$

Sol. $a_n = \dfrac{n(n-2)}{n+3}$. Putting n = 20, we obtain $a_{20} = \dfrac{20(20-2)}{20+3}$

Thus, $a_{20} = \dfrac{360}{23}$

Ex3. The Fibonacci sequence is defined by $a_1 = 1 = a_2$; $a_n = a_{n-1} + a_{n-2}$ for n > 2. Find $\dfrac{a_{n+1}}{a_n}$, for n = 1,2,3,4,5,

Sol. We have $a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$

Substituting n = 3, 4, 5 and 6, we get,

$a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

and $a_6 = a_5 + a_4 = 5 + 3 = 8$

Now, we have to find $\dfrac{a_{n+1}}{a_n}$ for n = 1, 2, 3, 4 and 5

For, n = 1, $\dfrac{a_2}{a_1} = \dfrac{1}{1} = 1$

n = 2, $\dfrac{a_3}{a_2} = \dfrac{2}{1} = 2$

n = 3, $\dfrac{a_4}{a_3} = \dfrac{3}{2}$

n = 4, $\dfrac{a_5}{a_4} = \dfrac{5}{3}$

n = 5, $\dfrac{a_6}{a_5} = \dfrac{8}{5}$

Hence, the required values are 1, 2, $\dfrac{3}{2}, \dfrac{5}{3}$ and $\dfrac{8}{5}$

COMPETITION WINDOW

SERIES OF NATURAL NUMBERS

1. The sum of first n natural numbers i.e. 1 + 2 + 3 +……+ n is usually written as $\sum n$.

$$\boxed{\sum n = \frac{n(n+1)}{2}}$$

2. The sum of squares of first n natural numbers i.e. $1^2 + 2^2 + 3^2 + \ldots + n^2$ is usually written as $\sum n^2$.

$$\boxed{\sum n^2 = \frac{n(n+1)(2n+1)}{6}}$$

3. The sum of cubes of first n natural numbers i.e. $1^3 + 2^3 + 3^3 + \ldots + n^3$ is usually written as $\sum n^3$.

$$\boxed{\sum n^3 = \left(\frac{n(n+1)^2}{2}\right) = \left(\sum n\right)^2}$$

★ PROGRESSON

It is not always possible to write each and every sequence of some rule.
For example of prime numbers 2, 3, 5, 7, 11,… cannot be expressed explicitly by stating a rule and we do not have any expression for writing the general term of this sequence.
The sequence that follows a certain pattern is called a progression. Thus, the sequence 2, 3, 5, 7, 11,… is not a progression. In a progression, we can always write the nth term.

Consider the following collection of numbers : (i) 1, 3, 5, 7… (ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$.

From the above collection of numbers, we observe that
(i) Each term is greater than the previous by 2.
(ii) In each term the numerator is 1 and the denominator is obtained by adding 1 to the preceding denominator.

Thus, we observe that the collection of numbers given in (i) and (ii) follow a certain pattern and as such are all progressions.

★ ARITHMETIC PROGRESSIONS

An arithmetic progression is that list of numbers in which the first term is given and each term, other than the first term is obtained by adding a fixed number 'd' to the preceding term.
The fixed term **'d'** is known as the **common difference** of the arithmetic progression. It's value can be positive, negative or zero. The **first term** is denoted by **'a'** or **'a_1'** and the **last term** by 'ℓ'.
Ex. Consider a sequence 6, 10, 14, 18, 22, …

Hence, $a_1 = 6, a_2 = 10, a_3 = 14, a_4 = 18, a_5 = 22$
$a_2 - a_1 = 10 - 6 = 4$
$a_3 - a_2 = 14 - 10 = 4$
$a_4 - a_3 = 18 - 14 = 4$

Therefore, the sequence is an arithmetic progression in which the first term a = 6 and the common difference d = 4.

Symbolical form : Let us denote the first term of an AP by a_1, second term by a_2,….nth term by a_n and the common difference by d. Then the AP becomes $a_1, a_2, a_3, \ldots\ldots a_n$.

So, $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$.

General form : In general form, an arithmetic progression with first term 'a' and common difference 'd' can be represented as follows :

a, a + d, a + 2d, a + 3d, a + 4d,….

Finite AP : An AP in which there are only a finite number of terms is called a finite AP. It may be noted that each such AP has a last term.

Ex. (a) The heights (in cm) of some students of school standing in a queue in the morning assembly are 147, 148, 149,…., 157.

(b) The minimum temperatures (in degree Celsius) recorded for a week in the month of January in a city arranged in ascending order are – 3.1, - 3.0, - 2.9, - 2.8, - 2.7, - 2.6, - 2.5

Infinite AP : An AP in which the number of terms is not finite is called infinite AP. It is note worthy that such APs do not have a last term.

Ex. (a) 1, 2, 3, 4,……

(b) 100, 70, 40, 10,….

Least Information Required : To know about an AP, the minimum information we need to know is to know both – the first term a and the common difference d.

For instance if the first term a is 6 and the common difference d is 3, then AP is 6, 9, 12, 15,…

Similarly, when

a = - 7, d = - 2, the AP is – 7, - 11, -13,….

a = 1.0, d = 0.1, the AP is 1.0, 1.1, 1.2, 1.3,…

So if we know what a and d are we can list the AP.

Ex.4 In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs.150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs. 10000 is deposited at compound interest 8% per annum. **[NCERT]**

Sol. (i) Taxi fare for 1 km = Rs. 15 = a_1

Taxi fare for 2 kms = Rs. 15 + 8 = Rs. 23 = a_2

Taxi fare for 3 kms = Rs. 23 + 8 = Rs. 31 = a_3

Taxi fare for 4 kms = Rs. 31 + 8 = Rs. 39 = a_4 and so on.

$a_2 - a_1$ = Rs. 23 – 15 = Rs. 8

$a_3 - a_2$ = Rs. 31 – 23 = Rs. 8

$a_4 - a_3$ = Rs. 39 – 31 = Rs. 8

i.e., $a_{k+1} - a_k$ is the same every time.

So, this list of numbers form an arithmetic progression with the first term a = Rs 15 and the common difference d = Rs. 8

(ii) Amount of air present in the cylinder = x units (say) = a_1

Amount of air present in the cylinder after one time removal of air by the vacuum pump $= x - \frac{x}{4} = \frac{3x}{4}$ units a_2

Amount of air present in the cylinder after two time removal of air by the vacuum pump

$= -\frac{3x}{4} - \frac{1}{4}\left(\frac{3x}{4}\right) = \frac{3x}{4} - \frac{3x}{16} = \frac{9x}{16}$ units $= \left(\frac{3}{4}\right)^2$ x units = a_3

Amount of air present in the cylinder after three times removal of air by the vacuum pump $= \left(\frac{3}{4}\right)^2 x - \frac{1}{4}\left(\frac{3}{4}\right)^2 x \Rightarrow \left(1-\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 x \Rightarrow \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)^2 x = \left(\frac{3}{4}\right)^3$ x units = a_4 and so on.

$a_2 - a_1 = \frac{3x}{4} - x = \frac{x}{4}$ units

$a_3 - a_2 = \left(\frac{3}{4}\right)^2 x - \frac{3}{4}x = -\frac{3}{16}$ x units

As $a_2 - a_1 \neq a_3 - a_2$, this list of numbers does not form an AP.

(iii) Cost of digging the well after 1 metre of digging = Rs. 150 = a_1
Cost of digging the well after 2 metres of digging = Rs. 150 + 50 = Rs 200 = a_2
Cost of digging the well after 3 metres of digging = Rs. 200 + 50 = Rs 250 = a_3
Cost of digging the well after 4 metres of digging = Rs. 250 + 50 = Rs 300 = a_4
and so on.
$a_2 - a_1$ = Rs 200 – 150 = 50
$a_3 - a_2$ = Rs 250 – 200 = 50
$a_4 - a_3$ = Rs 300 – 250 = 50
i.e., $a_{k-1} - a_k$ is the same every time. So this list of numbers forms an AP with the first term a = Rs. 150 and the common difference d = Rs.50

(iv) Amount of money after 1 year = Rs. $10000\left(1+\frac{8}{100}\right) = a_1$

Amount of money after 2 year = Rs. $10000\left(1+\frac{8}{100}\right)^2 = a_2$

Amount of money after 3 year = Rs. $10000\left(1+\frac{8}{100}\right)^3 = a_3$

Amount of money after 4 years = Rs. $10000\left(1+\frac{8}{100}\right)^4 = a_4$

$a_2 - a_1$ = Rs. $10000\left(1+\frac{8}{100}\right)^2$ - Rs. $10000\left(1+\frac{8}{100}\right)$

= Rs. $10000\left(1+\frac{8}{100}\right)\left(1+\frac{8}{100}-1\right) \Rightarrow$ Rs. $10000\left(1+\frac{8}{100}\right)\left(\frac{8}{100}\right)$

$a_3 - a_2$ = Rs. $10000\left(1+\frac{8}{100}\right)^3$ - Rs. $10000\left(1+\frac{8}{100}\right)^2 \Rightarrow$ Rs. $10000\left(1+\frac{8}{100}\right)^2\left(1+\frac{8}{100}-1\right) =$ Rs. $10000\left(1+\frac{8}{100}\right)^2\left(\frac{8}{100}\right)$

As $a_2 - a_1 \neq a_3 - a_2$, this list of numbers does not form an AP.

Ex.5 Write first four terms of the AP, when the first term a and the common difference d are given as follows
(i) a = 4, d = 5 (ii) a = - 1.25, d = - 0.25

Sol. (i) a = 4, d = 5

First term, a = 4
Second term = 4 + d = 4 + 5 = 9
Third term = 9 + d = 9 + 5 = 14
Fourth term = 14 + d = 14 + 15 = 19

Hence, first four terms of the given AP are 4, 9, 14, 19.

(ii) a = – 1.25, d = – 0.25

First term = a = – 1.25

Second term = - 1.25 + d = - 1.25 + (- 0.25) = - 1.50

Third term = - 1.50 + d = - 1.50 + (-0.25) = - 1.75

Fourth term = - 1.75 + d = - 1.75 + (-0.25) = - 2.00

Hence, first four terms of the given AP are – 1.25, - 1.50, - 1.75, - 2.00

Ex.6 For the AP $\frac{3}{2},\frac{1}{2},-\frac{1}{2},-\frac{3}{2},\ldots..$ write the first term a and the common difference d. Also write the next two terms after the given last term - $\frac{3}{2}$.

Sol. We have $a_1 = \frac{3}{2}, a_2 = \frac{1}{2}, a_3 = -\frac{1}{2}, a_4 = -\frac{3}{2}$ and so on.

Thus, $a = \frac{3}{2}$

$a_2 = a_1 = \left(\frac{1}{2}\right) - \left(\frac{3}{2}\right) = -1,$

$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right) = -1,$

$a_4 - a_3 = \left(-\frac{3}{2}\right) - \left(-\frac{1}{2}\right) = -1,$ and so on.

$\Rightarrow$ d = - 1

Now, we find the successor of - $\frac{3}{2}$.

$a_5 = \left(-\frac{3}{2}\right) + d = \left(-\frac{3}{2}\right) + (-1) = -\frac{5}{2}$

Then $a_6 = a_5 + d = \left(-\frac{5}{2}\right) + (-1) = -\frac{7}{2}$

Hence, the next two terms after the given term $-\frac{3}{2} are -\frac{5}{2}, -\frac{7}{2}$.

COMPETITION WINDOW

GEOMETRIC PROGRESSION

1. A sequence of non-zero numbers $a_1, a_2, a_3, \ldots, a_n$ is said to be a geometric sequence or G.P.

iff $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots..$

i.e. iff $\frac{a_{n+1}}{a_n}$ = a constant for all n.

This constant is called the common ratio of the G.P. and is usually denoted by 'r'. e.g., 3, 9, 27, 81,….

A general G.P. is a, ar, ar^2,…

When the terms of a geometric sequence are added, we get a geometric series.

HARMONIC PROGRESSION

A sequence of non-zero numbers $a_1, a_2, \ldots a_n$ is said to be a harmonic sequence or H.P.

iff $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots \frac{1}{a_n}$ are in A.P.

e.g., (i) 12, 6, 4, 3….. (ii) 10, 30, - 30, - 10, - 6,…..

A general H.P. is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} + \ldots$.

Where is the first term and d is the common difference of the A.P.

★ GENERAL TERM OF AN ARITHEMETIC PROGRESSION

The formula for writing general term or the nth term of an arithmetic progression is

$$a_n = a + (n - 1)d$$

Where, a is the first term of arithmetic progression,

and d is the common difference of arithmetic progression.

★ r^{th} TERM OF FINITE ARITHMETIC PROGRESSION FROM THE END

Let there be an arithmetic progression with first term a and common difference d. If there are n terms in the arithmetic progression, then

$$r^{th} \text{ term from the end} = a + (n - r)d$$

Also, if ℓ is the last term of the arithmetic progression then r^{th} term from the end is the r^{th} term of an arithmetic progression whose first term is ℓ and common difference is – d.

$$\text{rth term form the end} = \ell + (r - 1)(- d)$$

Ex.7 Find the 30th term of the AP : 10, 7, 4…. **[NCERT]**

Sol. The given A.P. is 10, 7, 4,…..

Here, a = 10, d = 7 – 10 = - 3 and n = 30

we have $a_n = a + (n - 1)d$

So, $a_{30} = 10 + (30 - 1)(-3)$

$\Rightarrow$ $a_{30} = 10 - 87 \Rightarrow a_{30} = - 77$

$\therefore$ The 30^{th} term of the given AP is – 77.

Ex.8 The 6^{th} term of an arithmetic progression is – 10 and the 10^{th} term is – 26. Determine the 15^{th} term of the AP.

Sol. Let first term and the common difference of the AP be a and d respectively.

6^{th} term = - 10 (Given)

$\Rightarrow$ $a + 5(6 - 1)d = - 10$ [$\because$ $a_n = a + (n - 1)d$]

$\Rightarrow$ $a + 5d = - 10$ ….(i)

10^{th} term = - 26 (Given)

$\Rightarrow$ $a + (10 - 1)d = - 26$

$\Rightarrow$ $a + 9d = - 26$ ….(ii)

Solving (i) and (ii) we get

$a = 10, d = - 4$

Therefore, 15^{th} term of the AP

= $a + (15 - 1)d$ [$\because$ $a_n = a + (n - 1)d$]

= a + 14d

= 10 + 14 (-4)

= 10 – 56 = - 46

Hence, the 15^{th} term of AP is – 46.

Ex.9 Find the 6th term from the end of the AP 17, 14, 11,…., - 40.

Sol. The given AP 17, 14, 11,…., - 40

Here, a = 17, d = 14 – 17 = - 3, ℓ = - 40

Let there be n terms in the given AP.

Then, nth term = - 40

$\Rightarrow$ a + (n – 1)d = - 40 [$\because a_n = a + (n-1)\,d$]

$\Rightarrow$ 17 + (n – 1) (-3) = - 40

$\Rightarrow$ (n – 1) (-3) = - 40 – 17

$\Rightarrow$ (n – 1) (-3) = - 57

$\Rightarrow$ $n - 1 = \frac{-57}{-3}$

$\Rightarrow$ n – 1 = 19

$\Rightarrow$ n = 19 + 1

$\Rightarrow$ n = 20

Hence, there are 20 terms in the given AP, Now, 6th term from the end

= a + (20 – 6)d [$\because$ rth term from the end = a + (n – r)d]

= a + 14d

= 17 + 14(-3)

= 17 – 42 = - 25

Hence, the 6th term from the end of the given AP is – 25.

Ex.10 is 200 any term of the sequence 3, 7, 11, 15,…?

Sol. The given sequence is 3, 7, 11, 15,…

$a_2 - a_1 = 7 - 3 = 4$

$a_3 - a_2 = 11 - 7 = 4$

$a_4 - a_3 = 15 - 11 = 4$

As $a_{k+1} - a_k$ is the same for k = 1, 2, 3, etc., the given sequence form an AP.

Here, a = 3, d = 4

Let 200 be the nth term of the given sequence. Then,

$a_n = 200$

$\Rightarrow$ a + (n – 1)d = 200 $\Rightarrow$ 3 + (n – 1) 4 = 200

$\Rightarrow (n-1) = \frac{197}{4}$ $\Rightarrow n = \frac{197}{4} + 1 \Rightarrow n = \frac{201}{4}$.

But n should be a positive integer. So, 200 is not term of the given sequence.

COMPETITION WINDOW

GENERAL TERM OF A.G.P.

The nth terms of a G.P. is a, ar^2,….ar^{n-1} is $\boxed{T_n = ar^{n-1}}$

rth TERM FROM THE END OF FINITE G.P.

Let a be the first term and r be the common ratio of a finite G.P. consisting of n terms, then

$$\boxed{\text{rth term from the end} = ar^{n-r}}$$

Also, if ℓ is the last term of the G.P. then

$$\boxed{\text{rth term from the end} = \ell\left[\frac{1}{r}\right]^{n-1}}$$

GENERAL TERM OF A H.P.

To find the nth term of an H.P., find the nth term of the corresponding A.P. obtained by the reciprocals of the terms of the given H.P. Now the reciprocal of the nth term of an A.P., will be the nth term of the H.P.

Try out the Following:

1. Find the 9^{th} term and the general term of the progression $\frac{1}{4}, \frac{-1}{2}, 1, -2....$

2. Which term of the G.P. 5, 10, 20, 40, ….is 5120?

3. The fourth, seventh and the last term of a G.P. are 10, 80 and 2560 respectively Find the first term and the number of terms in the G.P.

4. Find the 9th term of progression $\frac{1}{7}+\frac{1}{14}+\frac{1}{21}+\frac{1}{28}+....$

5. If the pth term of a H.P. is qr and it's qth term is pr, then find it's rth term.

6. Find the 6th term of the series $2+1\frac{3}{4}+1\frac{5}{9},....$

ANSWER KEY

1. 64; $(-1)^{n-1}\,2^{n-3}$ **2.** 11^{th} term **3.** $\frac{10}{8}$, 12 terms **4.** $\frac{1}{63}$ **5.** pq **6.** $\frac{7}{6}$

★ **SELECTION OF TERMS IN AN AP**

Sometimes we require certain number of terms in AP. The following ways of selecting terms are generally very convenient.

Number of terms	Terms	Common difference
3	a-d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + 3d, a + 5d	2d

It should be noted that in case of an odd number of terms, the middle term is a and the common difference is d while in case of an even number of terms the middle terms are a – d, a + d and the common difference is 2d.

Remark–1 : If the sum of terms is not given, then select terms as a, a + d, a + 2d,….

Remark–2 : If three numbers a, b, c in order are in AP. Then

b – a = Common difference = c – b

$\Rightarrow$ b – a = c – b

$\Rightarrow$ 2b = a + c

Thus, a, b, c are in AP if and only if $\boxed{2b = a + c}$

Remark–3 : If a, b, c are in AP, then b is known as the arithmetic mean (AM) between a and c.

Remark–4 : If a, x, b are in AP Then,

$$2x = a + b \Rightarrow x = \frac{a+b}{2}$$

Thus, AM between a and b is $\frac{a+b}{2}$.

Ex.11 The sum of three numbers in AP is -3, and their product is 8. Find the numbers.

Sol. Let the numbers be (a – d), a, (a + d). Then,

Sum = - 3 $\Rightarrow$ (a – d) + a + (a + d) = - 3 $\Rightarrow$ 3a = - 3 $\Rightarrow$ a = - 1

Now, product = 8

$\Rightarrow$ (a – d) (a) (a +d) = 8

$\Rightarrow$ $a(a^2 - d^2) = 8$

$\Rightarrow$ $(-1)(1 - d^2) = 8$ [$\because$ a = - 1]

$\Rightarrow$ $d^2 = 9 \Rightarrow d = \pm 3$

If d = 3, the numbers are – 4, -1, 2. If d = - 3, the numbers are 2, - 1, - 4

Thus, the numbers are – 4, - 1, 2 or 2, - 1, - 4

Ex.12 Find four numbers in AP, whose sum is 20 and the sum of whose squares is 120.

Sol. Let the numbers be (a – 3d), (a – d), (a + d), (a + 3d). Then,

Sum = 20

$\Rightarrow$ (a – 3d) + (a –d) + (a + d) + (a + 3d) = 20 $\Rightarrow$ 4a = 20 $\Rightarrow$ a = 5

Now sum of the squares = 120

$\Rightarrow$ $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$

$\Rightarrow$ $4a^2 + 20d^2 = 120$

$\Rightarrow$ $a^2 + 5d^2 = 30 \Rightarrow 25 + 5d^2 = 30$

$\Rightarrow$ $25 + 5d^2 = 30 \Rightarrow 5d^2 = 5 \Rightarrow d = \pm 1$

If d = 1, then the numbers are 2, 4, 6, 8. If d = - 1, then the numbers are 8, 6, 4, 2.

Thus, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2.

Ex.13 If 2x, x + 10, 3x + 2 are in AP. Find the value of x.

Sol. Since, 2x, x + 10, 3x + 2 are in AP.

$\therefore$ 2 (x + 10) = 2x + (3x + 2)

$\Rightarrow$ 2x + 20 = 5x + 2

$\Rightarrow$ 3x = 18

$\Rightarrow$ x = 6.

COMPETITION WINDOW

SELECTION OF TERMS IN G.P.

Sometimes it is required to select a finite number of terms in G.P. It is always convenient if we select the terms in the following manner :

No. of Terms	Terms	Common Ratio
3	$\frac{a}{r}$, a, ar	R
4	$\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3	r^2
5	$\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2	r

If the product of the numbers is not given, then the numbers are taken as a, ar, ar^2, ar^3,…

TRY OUT THE FOLLOWING

1. If the sum of three numbers in G.P. is 38 and their product is 1728, find them.
2. Find the three numbers in G.P. is whose sum is 13 and the sum of whose squares is 91.
3. Find four numbers in G.P. whose sum is 85 and product is 4096.
4. Three numbers are in G.P. whose sum is 70. If the extremes be each multiplied by 4 and the means by 5, they will be in A.P. Find the numbers.
5. Find four numbers in G.P. in which the third term is greater than the first by 9 and the second term is greater than the fourth by 18.
6. The product of first three terms of a G.P. is 1000. If 6 is added to it's second term and 7 added to its third term, the terms become in A.P. Find the G.P.

ANSWERS

1. 8, 12, 18, or 18, 12, 8 **2.** 1, 3, 9 or 9, 3, 1 **3.** 1, 4, 16, 64 or 64, 16, 4, 1
4. 10, 20, 40, or 40, 20, 10 **5.** 3, - 6, 12, -24 **6.** 5, 10, 20,… or 20, 10, 5…

★ SUM TO N TERMS OF AN ARITHMETIC PROGRESSION

The sum S_n of n terms of an arithmetic progression with first term 'a' and common difference 'd' is

$$\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$$

OR $\boxed{S_n = \frac{n}{2}[a + \ell]}$

Where ℓ = last term.

Remark–1 : In the formula $S_n = \frac{n}{2}[2a + (n-1)\,d]$, there are four quantities viz. S_n, a, n and d. If any three of these are known, the fourth can be determined. Sometimes, two of these quantities are given.
In such a case, remaining two quantities are provided by some other relation.

Remark–2 : If the sum S_n of n terms of a sequence is given, then n^{th} term a^n of the sequence can be determined by using the following formula :
$\mathbf{a_n = S_n - S_{n-1}}$
i.e., the nth term of an AP is the difference of the sum to first n terms and the sum to first (n – 1) terms of it.

Ex.14 Find the sum of the AP: $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,...., to 11 terms. **[NCERT]**

Sol. Here, $a = \frac{1}{15}$

$d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$

$n = 11$

We know that

$\Rightarrow \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right] \Rightarrow S_{11} = \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$

$\Rightarrow \quad S_{11} = \frac{11}{2}\left[\frac{3}{10}\right] \Rightarrow S_{11} = \frac{33}{20}$

So, the sum of the first 11 terms of the given AP is $\frac{33}{20}$.

Ex.15 Find the sum : 34 + 32 + 30 + …. + 10

Sol. 34 + 32 + 30 + … + 10

This is an AP

Here, a = 34

d = 32 – 34 = - 2

ℓ = 10

Let the number of terms of the AP be n.

We know that

$a_n = a + (n-1)d$

$\Rightarrow \quad 10 = 34 + (n-1)(-2) \Rightarrow (n-1)(-2) = -24$

$\Rightarrow \quad n - 1 = \frac{-24}{-2} = 12 \Rightarrow n = 13$

Again, we know that

$S_n = \frac{n}{2}(a+\ell) \Rightarrow S_{13} = \frac{13}{2}(34+10)$

$\Rightarrow \quad S_{13} = 286$

Hence, the required sum is 286.

Ex.16 Find the sum of all natural numbers between 100 and 200 which are divisible by 4.

Sol. All natural numbers between 100 and 200 which are divisible by 4 are

104, 108, 112, 116,….196

Here, $a_1 = 104$

$a_2 = 108$

$a_3 = 112$

$a_4 = 116$

⋮ ⋮

$\therefore \quad a_2 - a_1 = 108 - 104 = 4$

$a_3 - a_2 = 112 - 108 = 4$

$a_4 - a_3 = 116 - 112 = 4$

⋮ ⋮ ⋮

$\therefore \quad a_2 - a_1 = a_3 - a_2 = a_4 - a_3 =$ …..(= 4 each)

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, a = 104

d = 4

ℓ = 196

Let the number of terms be n. Then

$\ell = a + (n-1)d$

$\Rightarrow$ $196 = 104 + (n - 1)4$

$\Rightarrow$ $196 - 104 = (n - 1)4$

$\Rightarrow$ $92 = (n - 1)\,4$

$\Rightarrow$ $(n - 1)\,4 = 92$

$\Rightarrow$ $n - 1 = \frac{92}{4}$

$\Rightarrow$ $n - 1 = 23$

$\Rightarrow$ $n = 23 + 1 \Rightarrow n = 24$

Again, we know that

$$S_n = \frac{n}{2}(a + \ell)$$

$\Rightarrow$ $S_{24} = \left(\frac{24}{2}\right)(104 + 196)$

$=$ $(12)\,(300) = 3600$

Hence, the required sum is 3600.

Ex.17 Find the number of terms of the AP 54, 51, 48,…so that their sum is 513.

Sol. The given AP is 54, 51, 48,….

Here, $a = 54$, $d = 51 - 54 = -3$

Let the sum of n terms of this AP be 513.

We know that

$$S_n = \frac{n}{2}\,[2a + (n - 1)d]$$

$\Rightarrow$ $513 = \frac{n}{2}[2\,(54) + (n - 1)\,(-3)]$ $\Rightarrow$ $513 = \frac{n}{2}[108 - 3n + 3]$

$\Rightarrow$ $513 = \frac{n}{2}[111 - 3n]$ $\Rightarrow$ $1026 = n\,[111 - 3n]$

$\Rightarrow$ $1026 = 111n - 3n^2$ $\Rightarrow$ $3n^2 - 111n + 1026 = 0$

$\Rightarrow$ $n^2 - 37n + 342 = 0$ [Dividing throughout by 3]

$\Rightarrow$ $n^2 - 18n - 19n + 342 = 0$

$\Rightarrow$ $n(n - 18) - 19(n - 18) = 0$

$\Rightarrow$ $(n - 18)\,(n - 19) = 0$

$\Rightarrow$ $n - 18 = 0$ or $n - 19 = 0$

$\Rightarrow$ $n = 18, 19$

Hence, the sum of 18 terms or 19 terms of the given AP is 513.

Note : Actually 19^{th} term

$=$ a_{19}

$=$ $a + (19 - 1)\,d$ [$\because a_n = a + (n - 1)\,d$]

$=$ $a + 18d$

$=$ $54 + 18\,(-3)$

$=$ $54 - 54 = 0$

Ex.18 Find the AP whose sum to n terms is $2n^2 + n$.

Sol. Here, $S_n = 2n^2 + n$ (Given)

Put n = 1, 2, 3, 4,…., in succession, we get

$S_1 = 2(1)^2 + 1 = 2 + 1 = 3$

$S_2 = 2(2)^2 + 2 = 8 + 2 = 10$

$S_3 = 2(3)^2 + 3 = 18 + 3 = 21$

$S_4 = 2(4)^2 + 4 = 32 + 4 = 36$

and so on.

$\therefore \quad a_1 = S_1 = 3$

$a_2 = S_2 - S_1 = 10 - 3 = 7$

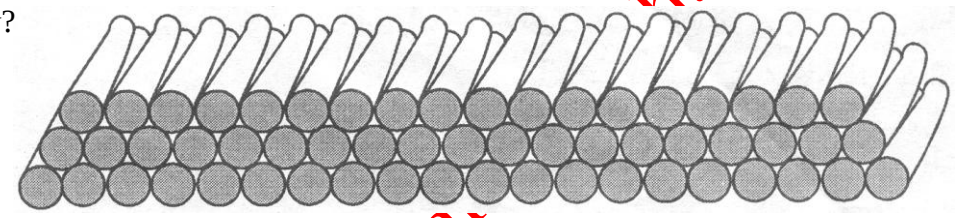
$a_3 = S_3 - S_2 = 21 - 10 = 11$

$a_4 = S_4 - S_3 = 36 - 21 = 15$

and so on.

Hence, the required AP is 3, 7, 11, 15,…

Ex.19 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?



[NCERT]

Sol. The number of logs in the bottom row, next row, row next to it and so on form the sequence 20, 19, 18, 17, …….

$a_2 - a_1 = 19 - 20 = -1$

$a_3 - a_2 = 18 - 19 = -1$

$a_4 - a_3 = 17 - 18 = -1$

i.e., $a_{k+1} - a_k$ is the same every time.

So, the above sequence forms an AP.

Here, a = 20

$d = -1$

$S_n = 200$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 200 = \frac{n}{2}[2(20) + (n-1)(-1)] \quad \Rightarrow \quad 200 = \frac{n}{2}[40 - n + 1]$$

$$\Rightarrow \quad 200 = \frac{n}{2}[41 - n] \quad \Rightarrow \quad 400 = n[41 - n]$$

$$\Rightarrow \quad n[41 - n] = 400 \quad \Rightarrow \quad 41n - n^2 = 400$$

$$\Rightarrow \quad n^2 - 41n + 400 = 0 \quad \Rightarrow \quad n^2 - 25n - 16n + 400 = 0$$

$$\Rightarrow \quad n(n - 25n) - 16(n - 25) = 0 \quad \Rightarrow \quad (n - 25)(n - 16) = 0$$

$$\Rightarrow \quad n - 25 = 0 \text{ or } n - 16 = 0 \quad \Rightarrow \quad n = 25 \text{ or } n = 16$$

$$\Rightarrow \quad n = 25, 16$$

Hence, the number of rows is either 25 or 16.

Now, number of logs in row
= Number of logs in 25th row
= a_{25}
= $a + (25 - 1)d$ $[\because a_n = a + (n - 1)d]$
= $a + 24d$
= $20 + 24\ (-1)$ $\Rightarrow 20 - 24 = -4$
Which is not possible.
Therefore, n = 16 and
Number of log in top row
= Number of logs in 16th row
= a_{16}
= $a + (16 - 1)\ d$ $[\because a_n = a + (n - 1)d]$
= $a + 15\ d$
= $20 + 15\ (-1)$
= $20 - 15 = 5$
Hence, the 200 logs are placed in 16 rows and there are 5 logs in the top row.

COMPETITION WINDOW

SUM OF n TERMS OF A.G.P.

If S_n is the sum of first n terms of the G.P. a, ar, ar^2,…..,

i.e., $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$, then $\boxed{S^n = \frac{a(1-r^n)}{1-r} \text{ or } \frac{a(r^n-1)}{r-1}}$, $r \neq 1$

Also, $\boxed{S^n = \frac{a-r\ell}{1-r} \text{ or } \frac{r\ell-a}{r-1}}$, $r \neq 1$, where ℓ is the last term i.e. the nth term .

For r = 1, Sn = na

SUM OF AN INFINITE G.P.

The sum of an infinite G.P. with first term a and common ratio r, where $-1 < r < 1$, is

$$\boxed{S_\infty = \frac{a}{1-r}}$$

If $r \geq 1$, then the sum of an infinite G.P. tends to infinity.

SUM OF n TERMS OF A H. P.

There is no specific formula to find the sum of n terms of H.P. To solve the questions of this progression, first of all convert it in A.P. then use the properties of A.P.

TRY OUT THE FOLLOWING

1. Find the sum of seven terms of the G.P. 3, 6, 12, ….

2. Find the sum to 7 terms of the sequence $\left[\frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}\right], \left[\frac{1}{5^4}+\frac{2}{5^5}+\frac{3}{5^6}\right], \left[\frac{1}{5^7}+\frac{2}{5^8}+\frac{3}{5^9}\right]$.

3. Find the sum of the series $2 + 6 + 18 + \ldots + 4374$.

4. How many terms of the sequence 1, 4, 16, 64…. will make the sum 5461?

5. Find the sum to infinite of the G.P. $\frac{-5}{4}, \frac{5}{16}, \frac{-5}{64}$,….

6. The first term of a G.P. is 2 and the sum of infinity is 6. Find the common ratio

7. If each term of an infinite G.P. is twice the sum of the terms following it, then find the common ratio of the G.P.

ANSWERS

1. 381 **2.** $\frac{19}{62}\left[1-\frac{1}{5^{21}}\right]$ **3.** 6560 **4.** – 1 **5.** $\frac{2}{3}$ **6.** $\frac{1}{3}$

★ **PROPERTIES OF ARITHMETICAL PROGRESSIONS**

1. If a constant is added to or subtracted from each term of an A.P., then the resulting sequence is also an A.P. with the same common difference.

2. If each term of a given A.P. is multiplied or divided by a non-zero constant K, then the resulting sequence is also an A.P. with common difference Kd or d/K, where d is the common difference of the given A.P.

3. In a finite A.P., the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of first and last term.

4. Three numbers a, b, c, are in A.P. iff 2b = a + c.

5. A sequence is an A.P. iff it's nth term is a linear expression in n i.e., $a_n = A_n + B$ are constants. In such a case, the coefficient of n is the common difference of the A.P.

6. A sequence is an A.P. iff the sum of it's first n terms is of the form $An^2 + Bn$, where A, B are constants. independent of n. In such a case, the common difference is 2 A.

7. If the terms of an A.P. are chosen at regular intervals, then they form an A.P.

COMPETITION WINDOW

ARITHMETIC MEANS

1. If there numbers a, b, c are in A.P. then b is called the arithmetic mean (A.M.) between a and c.

2. The arithmetic mean between two numbers a and b is $\frac{a+b}{2}$

3. $A_1, A_2 \ldots, A_n$ are said to be n A.M.s between two numbers a and b. iff a, $A_1, A_2, \ldots, A_n$, b are in A.p.
Let d be the common difference of the A.P.
Clearly, b = (n + 2)th term of the A.P.

$\Rightarrow$ b = a + (n + 1) d

$\Rightarrow$ $d = \frac{b-a}{n+1}$

Hence, $A_1 = a + d = a + \frac{b-a}{n+1}$, $A_2 = a + 2d = a + \frac{2(b-a)}{n+1}$

……

……

$A_n = a + nd = a + \frac{n(b-a)}{n+1}$

4. The sum of n A.M.'s between two numbers a and b is n times the single A.M. between then i.e., $n\left[\frac{a+b}{2}\right]$

GEOMETRIC MEANS

1. If three non-zero numbers a, b, c are in G.P. then b is called the geometric mean (G.M.) between a and b.

2. The geometric mean between two positive numbers a and b is $\sqrt{ab}$

HARMONIC MEAN

1. If three non-zero numbers a, b, c are in H.P., then is called the harmonic mean (H.M.) between a and b,

2. The harmonic mean between numbers a and b is $\frac{2ab}{a+b}$

Remark : If A, G, H denote respectively, the A.M., the G.M. and the H.M. between two distinct positive numbers, then

(i) A, G, H are in G.P. (ii) A > G > H

★ **SYNOPSIS**

1. **Sequence :** A sequence is an ordered arrangement of numbers according to a given rule.
2. **Terms :** The numbers in a sequence are called its terms.
3. **Series :** The sum of terms of a sequence is called the series of the corresponding sequence.
4. **Progression :** A progression is a sequence whose terms obey a certain pattern.
5. **Arithmetic Progression :** Arithmetic progression is a sequence if the difference of a term a and its predecessor is always constant.
6. **Common Difference :** The difference between two successive terms of an A.P. is called common difference.
7. **General Term :** General term or nth term or last term of an A.P. is $T_n = \ell = a_n = a + (n-1)\,d$, where 'a' is the first term and 'd' the common difference.
8. **Sum of n terms of an A.P. :** $S_n = \frac{n}{2}\{2a + (n-1)d\} = \frac{n}{2}\{a + \ell\}$

   Where ℓ = last term = a + (n – 1) d

   nth term, $a_n = S_n - S_{n-1}$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. If the sum of first n terms of an AP be $3n^2 - n$ and it's common difference is 6, then its first term is :
(A) 2 (B) 3 (C) 1 (D) 4

2. If 7^{th} and 13^{th} terms of an A.P. be 34 and 64 respectively, then it's 18^{th} term is :
(A) 87 (B) 88 (C) 89 (D) 90

3. The sum of all 2-digit odd numbers is':
(A) 2475 (B) 2530 (C) 4905 (D) 5049

4. The fourth term of an A.P. is 4. Then the sum of the first 7 terms is :
(A) 4 (B) 28 (C) 16 (D) 40

5. In an A.P. $s_1 = 6$, $s_7 = 105$, then $s_n : s_{n-3}$ is same as :
(A) (n + 3) : (n – 3) (B) (n + 3) : n (C) n : (n – 3) (D) None of these

6. In an A.P. $s_3 = 6$, $s_6 = 3$, then it's common difference is equal to :
(A) 3 (B) – 1 (C) 1 (D) None of these

7. The number of terms common to the two A.P. s
2 + 5 + 8 + 11 + ….+ 98 and 3 + 8 + 13 + 18 + … + 198
(A) 33 (B) 40 (C) 7 (D) None of these

8. (p + q)th and (p – q)th terms of an A.P. are respectively m and n, The P^{th} term is :

(A) $\frac{1}{2}(m+n)$ (B) $\sqrt{mn}$ (C) m + n (D) mn

9. The first, second and last terms of an A.P. are a, b and 2a. The number of terms in the A.P. is:

(a) $\frac{b}{b-a}$ (B) $\frac{b}{b+a}$ (C) $\frac{a}{b-a}$ (D) $\frac{a}{a+b}$

10. Let s_1, s_2, s_3 be the sums of n terms of three series in A.P., the first term of each being 1 and the common differences 1, 2, 3 respectively. If $s_1 + s_3 = \lambda s_2$, then the value of λ is :

(A) 1 (B) 2 (C) 3 (D) None of these

11. Sum of first 5 terms of an A.P. is one fourth of the sum of next five terms. If the first term = 2, then the common difference of the A.P. is :

(A) 6 (B) – 6 (C) 3 (D) None of these

12. If x, y, z are in A.P., then the value of (x + y – z) (y + z –x) is equal to :

(A) $8yz - 3y^2 - 4z^2$ (B) $8yz - 3z^2 - 4y^2$ (C) $8yz + 3y^2 - 4z^2$ (D) $8yz - 3y^2 + 4z^2$

13. The number of numbers between 105 and 1000 which are divisible by 7 is :

(A) 142 (B) 128 (C) 127 (D) None of these

14. If the numbers a, b, c, d, e form an A.P. then the value of a – 4b + 6c – 4d + e is equal to :

(A) 1 (B) 2 (C) 0 (D) None of these

15. If s_n denotes the sum of first n terms of an A.P., whose common difference is d, then $s_n - 2s_{n-1} + s_{n-2}$ (n > 2) is equal to :

(A) 2d (B) – d (C) d (D) None of these

16. The sum of all 2-digited numbers which leave remainder 1 when divided by 3 is:

(A) 1616 (B) 1602 (C) 1605 (D) None of these

17. The first term of an A.P. of consecutive integers is $p^2 + 1$. The sum of 2p + 1 terms of this series can be expressed as :

(A) $(p+1)^2$ (B) $(2p+1)(p+1)^2$ (C) $(p+1)^3$ (D) p3 + $(p+1)^3$

18. If the sum of n terms of an AP is $2n^2$ 5n, then its nth term is –

(A) 4n – 3 (B) 3n – 4 (C) 4n + 3 (D) 3n + 4

19. If the last term of an AP is 119 and the 8th term from the end is 91 then the common difference of the AP is –

(A) 2 (B) 4 (C) 3 (D) – 3

20. If $(a_n) = \{2.5, 2.51, 2.52, \ldots\}$ and $\{b_n\} = \{3.72, 3.73, 3.74, \ldots\}$ be two AP's then $a_{100005} - b_{100005}$ =

(A) – 1.22 (B) 1.22 (C) 1.2 (D) – 1.02

OBJECTIVE			ANSWER KEY									EXERCISE – 1			
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	C	A	B	A	B	C	A	A	B	B	A	C	C	C
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	C	D	C	B	A										

SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE

1. Write the first five terms of each of the sequences, whose nth terms are:

(i) $a_n = \frac{(-1)^n(2n+1)}{6}$ (ii) $a_n = \frac{1}{n} + (-1)^n$ (iii) $a_n = n\left[\frac{n^2+5}{4}\right]$ (iv) $a_n = (-1)^{n-1}\, 5^{n+1}$

2. Find the indicated terms in each of the following sequences whose nth terms are:

(i) $a_n = 2^n + n^3$; a^3 (ii) $a_n = \frac{n^2 - n + 1}{n}$, a_{10} (iii) $a_n = (-1)^{n-1}n^3$; a_9 (iv) $a_n = (n-1)(2-n)(3+n)$; a_{20}

3. Write the first five terms of each of the following sequences and obtain the corresponding series.

(i) $a_1 = 1$, $a_n = a_{n-1} + 2$, $n \geq 2$ (ii) $a_1 = 4$, $a_{n+1} = 2na_n$

4. Write the first term a and the common difference d of the AP : -5, -1, 3, 7….

5. Write the first term a and the common difference d of the AP : -1.1, -3.1, - 5.1, - 7.1……

6. Write the arithmetic progression when first term a = - 1 and common difference d = $\frac{1}{2}$.

7. Write the arithmetic progression when first term a = - 1.5 and common difference d = - 0.5.

8. Find the common difference and write the next four terms of the AP : $1, \frac{1}{4}, \frac{3}{2}, \ldots$..

9. Write the sequence with nth term, $a_n = 3 + 4n$.

10. Find out whether the sequence 3, 3, 3, 3, … is an AP. If it is, find out the common difference.

11. Find the common difference and write the next two terms of the AP : 1.8, 2.0, 2.2, 2.4…

12. Find out whether the sequence $1^2, 3^2, 5^2\ 7^2, \ldots$ is an AP. If it is, find out the common difference.

13. Find the common difference and write the next two terms of the AP $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$.

14. Find the 18th term of the AP. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}$ ….

15. Which term of the AP : 84, 80, 76,… is 0?

16. Is 302 a term of the AP : 3, 8, 13,…?

17. How many terms are there in the AP : 7, 13, 19,…..205 ?

18. Find the sum of the arithmetic progression : -26, -24, -22,…. to 36 terms.

19. Show that the sequence defined by $a_n = 2n^2 + 3$ is not an A.P.

20. Find the 14th term of the A.P. 9, 5, 1, - 3…

21. Find the nth term of the sequence m – 1, m – 3, m – 5,…

22. Is – 150 a term of the A.P. 11, 8, 5, 2….

SHORT ANSWER TYPE QUESTIONS

1. Show that the sequence defined by $a_n = 5n - 7$ is an AP. Find its common difference.

2. Prove that no matter what the real numbers a and b are, the sequence with nth term a + nb is always an AP. What is the common difference?

3. The first term of an AP is 5, the common difference is 3 and the last term is 80, find the number of terms.

4. If the nth term of the AP. 9, 7, 5… is same as the nth term of the AP. 15, 12, 9,…., find n.

5. Find the 12th term from the end of the arithmetic progression : 3, 5, 7, 9,…..201?

6. Find n if the given value of x is the nth term of the AP : $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots$; x = 550.

7. Which term of the AP : 3, 10, 17,…. will be 84 more than its 13th term?

8. Find the 8th term from the end of the AP : 7, 10, 13,….184

9. Find the value of x for which (8x + 4), (6x – 2) and (2x + 7) are in AP.

10. If the sum of a certain number of terms starting from first of an AP 25, 22, 19,…, is 116. Find the last term.

11. How many terms of he sequence 18, 16, 14,…. Should be taken so that their sum is zero?

12. Find the sum of first n odd natural numbers.

13. Find the sum of all even integers between 101 and 999.

14. Find the sum : $7 + 10\frac{1}{2} + 14 + \ldots + 84$.

15. Find the sum of the first 15 terms of the sequence having nth term as : $a_n = 3 + 4n$.

16. The 6th and 17th terms of an AP. are 19 and 41 respectively, find the 40th term.

17. In a certain AP, the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

18. Find the second term and nth term of an AP whose 6th term is 12 and the 8th term is 22.

19. An AP consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32nd term.

20. Find $a_{30} - a_{20}$ for the AP : a, a + d, a + 2d, a + 3d,….

21. Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

22. Find the term of the arithmetic progression 9, 12, 15, 18,…. Which is 39 more than its 36th term.

23. The sum of three terms of AP. Is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.

24. The sum of three numbers in AP. is 12 and the sum of their cubes is 288. Find the numbers.

25. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.

26. How many terms are there in the AP whose first and fifth terms are – 14 and 2 respectively and the sum of the terms is 40?

27. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

28. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

29. In an AP if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms?

30. Find the sum of the first 25 terms of an AP whose nth terms is given by $a_n = 2 - 3n$.

31. If x, x + 10 and 3x + 2 are in A.P., find the value of x.

32. If x + 1, 3x and 4x + 2 are in A.P., find the fifth term of A.P.

33. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P., then prove that $2b^2 = a^2 + c^2$.

34. If a, $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ and b are in A.P. and $a \neq b$, then find the value of n.

35. Find the nth term and 100^{th} term of the sequence 7 + 3 – 1 – 5….

36. Which term of the A.P., 3, 8, 13, 18,…., is 78?

37. Which term of the A.P., 21, 18, 15,… is – 81?

38. Which term of the A.P., 121, 117, 113,…., is it's first negative term?

39. Which term of the sequence 114, 109, 104,… is the first negative term?

40. How many terms are there in the A.P. $-1, \frac{-5}{6}, -\frac{2}{3}, \frac{-1}{2}, \ldots\ldots, \frac{10}{3}$?

41. How many two digit numbers are divisible by 3?

42. How many three digit numbers are divisible by 5?

43. If the 5th and 21st terms of an A.P. are 14 and – 14 respectively, then which term of the A.P. is zero?

44. In an A.P., the fourth term exceeds four times the 12th term by one and the third term exceeds twice the tenth term by five, find the A.P.

45. Determine the A.P. whose fourth term is 15 and the difference of 6th term from 10th term is 16.

46. The 4th term of an A.P. is zero, prove that its 25th term is triple its 11^{th} term.

47. If the pth term of an A.P. is $\frac{1}{q}$, and qth term of an A.P. is $\frac{1}{q}$, then show that its (pq)th term is 1.

48 . The sum of three numbers in A.P. is 21 and their product is 231. Find the numbers.

49. The sum of three numbers in A.P. is 3 and their product is – 35. Find the numbers.

50. Divided 20 into four parts which are in A.P. such that the product of the first and fourth and the product of the second and third is in the ratio 2 : 3.

51. If the sum of first n terms of an A.P. is given by $s_n = 5n^2 + 3n$, find the nth term of the A.P.

52 The sum of the first 9 terms of an A.P. is 81 and the sum of it's first 20 terms is 400. Find the first term, the common difference and the sum up to 15^{th} term

53. The sum of n terms of two arithmetic progressions are in the ratio (3n + 8) : (7n + 15).Find the ratio of their (i) 12th terms (ii) 15th terms.

54. If $s_n = n^2p$ and $s_m = m^2p$, $m \neq n$ is an A.P. prove that $s_p = p^3$.

55. The first, second and the last terms of an A.P. are m, n and 2m respectively. Show that it's sum is $\frac{3mn}{2(n-m)}$

56. If the mth term of an A.P. is 20 and nth term is 10, then show that sum of it's first (m + n) terms is $\frac{m+n}{2}\left[30+\frac{10}{m-n}\right]$.

57. If s_1, s_2, s_3 are the sum of n terms of three arithmetic progressions, the first term of each being unity and the respective common difference being 1, 2, 3; prove that $s_1 + s_3 = 2s_2$.

58. Two A.P.'s have the same common difference. If the first term of the two A.P's are 3 and 8 respectively, find the difference between their sum to first 30 terms.

59. If in an A.P., the sum of 12 terms is equal to 18 and the sum of 18 terms is equal to 12, then prove that the sum of 30 terms is – 30.

60. Find the sum of all two digit natural numbers which are divisible by 4.

61. Find the sum of all 3-digit natural numbers which are divisible by 13.

62. Find the sum of all 2-digit numbers which when divided by 5 leave remainder 1.

63. Find the sum of all multiples of 9 lying between 300 and 700.

LONG ANSWER TYPE QUESTION

1. If pth, qth and rth terms of an AP are a, b, c respectively, then show that :
(i) a(q – r) + b (r – p) + c (p – q) = 0, (ii) (a – b)r + (b – c)p + (c – a)q = 0

2. In a garden bed, there are 23 rose plants in the first row, twenty one in the second row, nineteen in the third row and so on. There are five plants in the last row. How many rows are there of rose plants?

3. If the mth term of an AP is $\frac{1}{n}$ and the nth term is $\frac{1}{m}$, show that the sum of mn terms is $\frac{1}{2}$(mn + 1).

4. If the sum of m terms of an AP is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.

5. The sum of the first p,q,r terms of an AP. are a,b,c respectively. Show that:
$$\frac{a}{q}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

6. A manufacturer of radio sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the product increases uniformly by a fixed number every year, find (i) the production in the first year
(ii) the total product in 7 years and (ii) the product in the 10th year.

7. A man is employed to count Rs. 10710. He counts at the rate of Rs. 180 per minute for half an hour. After this he counts at the rate of Rs 3 less every minute thatn the preceding minute. Find the time taken by him to count the entire amount.

8. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid, find the value of the first installment.

9. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are 2.5 metre apart, what is the length of the wood required for the rungs?

10. The digits of a positive integer, having three digits, are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

11. Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by $\frac{1}{2}$ km each succeeding hour. After how many hours will the second car overtake the first car if both cars go non – stop?

12. A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will it take him to clear the loan?

13. 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more workers dropped the third day and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.

14. Along a road lie an odd number of stones place at intervals of 10 metres. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried the job with one of the end stones by carrying them in succession. In carrying all the stones he covered a distance of 3 km. Find the number of stones.

15. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120°, and the common difference is 5°. Find the number of sides of the polygon.

16. A man saved Rs. 16500 in ten years after the first he saved Rs. 100 more than he did in the preceding year. How much did he save in the first year?

17. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid, find the value of the first installment.

18. A piece of equipment cost a certain factory Rs. 600.000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year and so on. What will be it's value at the end of 10 years, all percentages applying to the original cost?

ARITHMETIC PROGRESSIONS	ANSWER KEY	EXERCISE – 2 (X)–CBSE

• VERY SHORT ANSWER TYPE QUESTIONS

1. (i) $\frac{-1}{2}, \frac{5}{6}, \frac{-7}{6}, \frac{3}{2}$ *and* $\frac{-11}{6}$ (ii) $0, \frac{3}{2}, \frac{-2}{3}, \frac{5}{4}$ *and* $\frac{-4}{5}$ (iii) $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21$ *and* $\frac{75}{2}$ (iv) 25, - 125, 625, - 3125 and 15625

2. (i) 35 (ii) $\frac{91}{10}$ (iii) 729 (iv) – 7866 **3.** (i) 1, 3, 5, 7, 9 ; 1 + 3 + 5 + 7 +9 (ii) 4, 8, 32, 192, 1536 : 4 + 8 32 + 192 + 1536

4. a = - 5, d = 4 **5.** a = -1.1, d = -2 **6.** $-1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \ldots\ldots$ **7.** –1.5, -2, -2.5, -3,….

8. $\frac{5}{4}; a_4 = \frac{11}{4}, a_5 = \frac{16}{4}, a_6 = \frac{21}{4}, a_7 = \frac{26}{4}$ **9.** 7, 11, 15, 19,…. **10.** Yes, d = 0 **11.** d = 0.2, a_5 = 2.6, a_6 = 2.8

12. No **13.** $d = \frac{1}{4}; a_5 = 1, a_6 = \frac{5}{4}, a_7 = \frac{26}{4}$ **14.** $35\sqrt{2}$ **15.** 22^{nd} term **16.** No **17.** 34

18. 324 **20.** – 43 **21.** m – 2n + 1 **22.** No

• SHORT ANSWER TYPE QUESTIONS

1. 5 **2.** b **3.** 26 **4.** 7 **5.** 179 **6.** 100 **7.** 25th **8.** 163 **9.** $\frac{15}{2}$ **10.** 4 **11.** 19 **12.** n^2 **13.** 246950

14. $\frac{2093}{2}$ **15.** 525 **16.** 87 **18.** a_2 = - 8, a_n = 5n – 18 **19.** 69 **20.** 10d **21.** 100 **22.** 49th **23.** 1, 7, 13

24. 2, 4, 6 or 6, 4, 2 **26.** 10. **27.** 38,6973 **29.** 1150 **30.** – 925 **31.** x = 9 **32.** 24 **34.** 1
35. 11 – 4n, - 389 **36.** 16th **37.** 15th, 8^{th} **38.** 32nd **39.** 24th **40.** 27 **41.** 30 **42.** 180
43. 3rd **44.** 27, 25, 23, 21 **45.** 3, 7, 11, 15, 19,…. **48.** 3, 7, 11 or 11, 7, 3 **49.** – 5, 1, 7 or 7, 1, - 5
50. 2, 4, 6 and 8 **51.** 10 n – 2 **52.** a = 1, d = 2, s_{15} = 225 **53.** (i) 7 : 16 (ii) 95 : 218 **58.** 150 **60.** 1188
61. 37674 **62.** 963 **63.** 21978

• LONG ANSWER TYPE QUESTIONS

2. 10 rows **6. (i)** 550 **(ii)** 4375 **(iii)** 775 **7.** 89 minutes **8.** Rs. 51 **9.** 3.5 minters **10.** 852
11. 9 hours **12.** 20 months **13.** 25 days **14.** 25 stones **15.** 9 sides **16.** Rs. 1200 **17.** Rs. 51 **18.**10500

EXERCISE – 3 (FOR SCHOOL/BOARDS EXAMS)

PERVIOUS YEARS BOARD (CBSE) QUESTIONS

1. The nth term (t_n) of an Arithmetic progression is given by $t_n = 7n + 1$. Find the sum of the first 30 terms of Arithmetic progression. **[Foreign – 2004]**

2. The 10th term of an Arithmetic progression (A.P.) is 57 and its 15th term is 87. Find the Arithmetic Progression. **[Foreign – 2004]**

3. If the sum of first n terms of an A.P. is given by $S_n = 3n^2 + 2n$, find the nth term of the A.P.

OR

If m times the mth terms of an A.P. is equal to n times its nth term, find its (m + n)th term.

[Delhi-2004C]

4. How many terms of the A.P.3,5,7,… must be taken so that the sum is 120? **[Delhi-2004C]**

5. If the sum of first n terms of an A.P. is given by $S_n = 4n^2 – 3n$, find the nth term of the A.P.

[Delhi-2004C]

6. If the sum of first of an A.P. is given by $S_n = 2n^2 + 5n$, find the nth term of the A.P. **[Delhi-2004C]**

7. Find the sum of first 15 terms of an A.P. whose nth term is 9 – 5n.

OR

If the sum to first n terms of an A.P. is given by $S_n = 5n^2 + 3n$, find the nth term of the A.P.

[AI-2004C]

8. Find 10^{th} term from end of an A.P. 4, 9, 14, . . 254 **[Delhi-2005]**

9. Find the number of terms of the A.P. 54, 51, 48,….so that their sum is 513.

OR

If the nth term of an A.P. is (2n + 1), find the sum of first n terms of the A.P. **[Delhi-2005]**

10. Find the sum of all two digits odd positive numbers. **[AI-2005]**

11. The 8th term of an Arithmetic Progression is zero. Prove that its 38th term is triple of it 18th term.

[AI-2005]

12. Find the sum of all two digit positive numbers divisible by 3. **[Foreign-2005]**

13 If fifth term of the A.P. is zero, show that its 33rd term is four times its 12th term **[Foreign-2005]**

14. Which term of the A.P. 5, 9, 13,... is 81? Also find the sum 5 + 9 + 13 +… + 81 **[Delhi-2005C]**

15. The sum of first n terms of an A.P. is given by $(n^2 + 3n)$. Find the 20th term of the progression.

[Delhi-2005C]

16. Find the sum of the first 51 terms of the A.P. whose 2nd term is 2 and 4th term is 8. **[AI-2005C]**

17. The sum of the first n terms of an A.P. is given by $S_n = 3n^2 – n$. Determine the A.P. and its 25^{th} term.

OR

The sum of three numbers in A.P. is 27 and their product is 405. Find the numbers. **[AI-2005C]**

18. The 6^{th} term of an Arithmetic progression (A.P.) is -10 and the 10th term is – 26 Determine the 15th term of the A.P. **[Delhi-2006]**

19. Find the sum of all the natural numbers less than 100 which are divisible by 6. **[AI-2006]**

20. How many terms are there in A.P. whose first term and 6th term are – 12 and 8 respectively and sum of all its terms is 120?

21 Using A.P., find the sum of all 3-digit natural numbers which are multiples of 7. **[Delhi-2006C]**

22. In an A.P. the sum of first n terms is $\frac{5n^2}{2}+\frac{3n}{2}$ Find its 20th term. **[AI-2006C]**

23. Find the sum of first 25 terms of an A.P. whose nth term is 1 – 4n. **[Delhi-2007]**

24. Which term of the A.P. 3, 15, 27, 39, …will be 132 more than its 54th term? **[Delhi-2007]**

25. In an A.P., the sum of its first n terms is $n^2 + 2n$. Find its 18th term. **[AI-2007]**

26. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively. Find the sum of all terms of this A.P. **[AI-2007]**

27. The nth term of an A.P. is 7 – 4n. Find its common difference. **[Delhi-2008]**

28. The sum of n terms of an A.P. is $5n^2 – 3n$. Find the A.P. Hence, find its 10th term. **[Delhi-2008]**

29. The nth term of an A.P. is 6n + 2. Find it's common difference. **[Delhi-2008]**

30. Find the 10th term from the end of the A.P. 8, 10, 12,…., 126 **[Delhi-2008]**

31. Write the next term of the A.P. $\sqrt{8}, \sqrt{18}, \sqrt{32}$,…. **[AI-2008]**

32. The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P. **[AI-2008]**

33. The first term of an A.P. is p and it's common difference is q. Find it's 10th terms is. **[AI-2008]**

34. For what value of n are the terms of two A.P.'s 63, 65, 67,….., and 3, 10, 17 ,…. equal?

OR

If m times the mth tem of an A.P. is equal to n times it's nth term, find the (m + n)th term of the A.P. **[Foreign-2008]**

35. In an A.P. the first term is 8, nth term is 33 and sum to first n terms is 123. Find n and d, the common difference. **[Foregin-2008]**

36. In an A.P., the term is 22, nth term is – 11, and sum to first n terms is 66. Find n and d, the common difference. **[Foreign-2008]**

37. In an A.P., the first term is 22, nth term is – 11, and sum to first n terms is 66. Find n and d, the common difference. **[Foreign-2008]**

38. For what value of p, are 2p – 1, 7 and 3p three consecutive terms of an A.P.? **[Delhi-2009]**

39. If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 – 4n$, then find its nth term. **[Delhi-2009]**

40. The sum of 4th and 8th terms of an A.P. is 24 and sum of 6th and 10th terms is 44. Find A.P. **[Delhi-2009]**

41. If S_n the sum of first n terms an A.P. is given by $S_n = 5n^2 + 3n$, then find its nth term. **[Delhi-2009]**

42. The sum of 5th 9th terms of an A.P., is 72 and the sum of 7th and 12th terms is 97. Find the A.P. **[Delhi-2009]**

43. If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a. **[AI-2009]**

44. Which term of the A.P. 3, 15, 27, 39,… will be 120 more than it's 21str term? **[AI-2009]**

45. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P. **[AI-2009]**

46. Which term of the A.P. 4, 12, 20, 28,…. will be 120 more than it's 21st term? **[AI-2009]**

47. For what value of k, are the numbers x, 2x + k and 3x + 6 three consecutive terms of an A.P.? **[Foreign-2009]**

48. The 17th term of an A.P. exceeds its 10th term by 7. Find the common difference. **[Foreign-2009]**

49. If 9th term of an A.P. is zero, prove that its 29th term is double of it's 19th term. **[Foreign-2009]**

50. If 5th term of an A.P. is zero, prove that it's 23rd term is three times it's 11th term. **[Foreign-2009]**

51. If the 7th term of an A.P. is zero, prove that it's 27th term is five times it's 11th term. **[Foreign-2009]**

SUBJECTIVE **ANSWER KEY** **EXERCISE-4 (X)-CBSE**

1. 3285 **2.** 3, 9, 15, 21… **3.** 6n – 1 or 0 **4.** 10 terms **5.** 8n – 7 **6.** 4n + 3
7. – 465 or 10n – 2 **8.** 209 **9.** 18 or 19 OR n (n + 2) **10.** 2,475 **12.** 1,665 **14.** 860
15. 42 **16.** 3,774 **17.** 146 OR (3, 9,15) or (15, 9,3) **18.** – 46 **19.** 816 **20.** 12
21. 70, 336 **22.** 99 **23.** – 1275 **24.** 65th term **25.** 38 **26.** 5412 **27.** – 4
28. 2, 12, 22..; $a_{10} = 92$ **29.** 6 **30.** 108 **31.** $5\sqrt{2}$ **32.** – 13, - 8, - 3
33. p + 9q **34.** n = 13 or $a_{m+n} = 0$ **35.** n = 6, d = 5 **36.** n = 15, d = - 3
37. n = 12, d = - 3 **38.** p = 3 **39.** 6n – 7 **40.** – 13, – 8, – 3 **41.** 10n – 2
42. 6, 11, 16, 21,… **43.** a = $\frac{7}{5}$ **44.** 31st term **45.** a = 2, $a_{13} = 26$ **46.** 36th term
47. k = 3 **48.** d = 1

EXERCISE – 4 (FOR OLYMPIADS)

CHOSSE THE CORRECT ONE

1. For a series whose nth term is $\frac{n}{x}+y$, the sum of r terms is :

(A) $\frac{r(r+1)}{2x}+ry$ (B) $\frac{r(r-1)}{2x}$ (C) $\frac{r(r-1)}{2x}-ry$ (D) $\frac{r(r+1)}{2y}-rx$

2. If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P., then $\left[\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right]\left[\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right]$ is equal to :

(A) $\frac{4}{ac}-\frac{3}{b^2}$ (B) $\frac{b^2-ac}{a^2b^2c^2}$ (C) $\frac{4}{ac}-\frac{1}{b^2}$ (D) None of these

3. The sum of first 24 terms of an A.P. $a_1, a_2, a_3,\ldots$; if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, is equal to :

(A) 90 (B) 180 (C) 900 (D) 1800

4. A student read common difference of an AP is – 2 instead of 2 and got the sum of first five terms as – 5. The actual sum of first five terms is :

(A) 25 (B) – 25 (C) – 35 (D) 35

5. The sum of n terms of two A.P's are in the ratio of (7n + 1) : (4n + 27). The ratio of their 11th terms is –

(A) 2 : 3 (B) 4 : 3 (C) 5 : 4 (D) 5 : 6

6. If $1^2 + 2^2 + 3^2 + \ldots n^2 = 1015$, then the value of n is :

(A) 13 (B) 14 (C) 15 (D) None of these

7. The sum of the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\ldots$ upto 9 terms is:

(A) $-\frac{5}{6}$ (B) $-\frac{1}{2}$ (C) 1 (D) $-\frac{3}{2}$

8. The sum of first n odd natural numbers is:

(A) n^2 (B) $2n$ (C) $\frac{n(n-1)}{2}$ (D) $\frac{n(n+1)}{2}$

9. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference will be:

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

10. If A.M between two numbers is 5 and their G.M. is 4, then their H.M. is:

(A) $\frac{16}{5}$ (B) $\frac{14}{5}$ (C) $\frac{11}{5}$ (D) None of these

11. If A is the single A.M. between two numbers a and b and S is the sum of n A.M.'s between them, then $\frac{S}{A}$ depends upon:

(A) n, a, b (B) n, a (C) n, b (D) n

12. If the A.M. between the roots of a quadratic equation is 8 and the G.M. is 5, then the equation is:

(A) $x^2 + 10x - 25 = 0$ (B) $x^2 - 8x + 5 = 0$
(C) $x^2 - 16x + 25 = 0$ (D) $x^2 - 16x - 25 = 0$

13. If c is the harmonic mean between a and b, then $\frac{c}{a} + \frac{c}{b}$ is equal to:

(A) 2 (B) $\frac{a+b}{ab}$ (C) $\frac{ab}{a+b}$ (D) None of these

14. If a,b,c,d,e,f are in A.P. then e-c is equal to :

(A) 2(c – a) (B) 2(f – d) (C) 2(d – c) (D) d – c

15. 20th term of the series : $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$ is :

(A) $\frac{441}{4}$ (B) $\frac{443}{2}$ (C) $\frac{445}{2}$ (D) $\frac{439}{2}$

16. If the value of $1^3 + 2^3 + 3^3 + \ldots + n^3 = 2025$, then the value of $1 + 2 + 3 + \ldots + n$ is:

(A) 675 (B) 81 (C) 45 (D) 285

17. If the value of $1 + 2 + 3 + \ldots + n$ is 55, then the value of $1^3 + 2^3 + 3^3 + \ldots + n^3$ is:

(A) 165 (B) 385 (C) 3025 (D) 555

18. The nth term of the series $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \ldots$ is :

(A) $\frac{n-1}{2}$ (B) $\frac{n^2+1}{2}$ (C) $\frac{n+1}{2}$ (D) $\frac{n^2-1}{2}$

19. $1^2 + 1 + 2^2 + 2 + 3^2 + 3 + \ldots + n^2 + n$ is equal to :

(A) $\frac{n(n+1)}{2}$ (B) $\left[\frac{n(n+1)}{2}\right]^2$ (C) $\frac{n(n+1)(n+2)}{3}$ (D) $\frac{n(n+)(n+2)(n+3)}{4}$

20. The next term of the sequence $\frac{1}{4}, \frac{1}{36}, \frac{1}{144} \ldots$ is :

(A) $\frac{1}{576}$ (B) $\frac{1}{400}$ (C) $\frac{1}{1296}$ (D) None of these

21. If the sum of first n natural numbers is one-fifth of the sum of their squares, then n equals:

(A) 5 (B) 6 (C) 7 (D) 8

22. The nth term of the series $1 + 3 + 6 + 10 + 15 + \ldots$ is :

(A) $\frac{n(n+1)}{2}$ (B) $n^2 - n + 1$ (C) n (n + 1) (D) None of these

23. The sum of the series $1^2 + 1 + 2^2 + 2 + 3^2 + 3 + \ldots$ + up to n terms is :

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n+1)(n+2)}{3}$ (C) $\left[\frac{n(n+1)}{2}\right]^2$ (D) None of these

24. The nth term of the sequence 1, $\sqrt{2}, 3^{\frac{1}{3}}, 2^{\frac{1}{2}}, \ldots$ is :

(A) $n^{\frac{1}{n}}$ (B) n^n (C) $\left(\frac{1}{n}\right)^n$ (D) None of these

25. The sum of n terms of the series $(1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + ,,,$ is :

(A) $\frac{n(n+1)}{2}$ (B) $\frac{-n(n+1)}{2}$ (C) $-n(2n+1)$ (D) None of these

26. If A_1 and A_2 be the two A.M.s between two numbers p and q, then $(2A_1 - A_2)(2A_2 - A_1)$ is equal to :

(A) p + q (B) p – q (C) pq (D) None of these

27. If $\frac{1}{a}, \frac{a^n + b^n}{a^{n+1} + b^{n+1}}, \frac{1}{b}$ are in A.P., then n is equal to :

(A) 0 (B) – 1 (C) $\frac{1}{2}$ (D) None of these

28. If $S_n = nP + \frac{1}{2}n(n-1)Q$ where S_n denotes the sum of the first n terms of an A.P., then the common difference of the A.P. is

(A) P + Q (B) 2P + 3Q (C) 2Q (D) Q

29. If a,b,c are positive reals, then least value of $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ is :

(A) 1 (B) 6 (C) 9 (D) None of these

30. The sum of first four terms of an A.P. is 56 and sum of last four terms is 112. If the first term is 11, then the number of terms is :

(A) 10 (B) 12 (C) 11 (D) None of these

31. For an A.P., $S_{2n} = 3S_n$. The value of $\frac{S_{3n}}{S_n}$ is equal to :

(A) 4 (B) 6 (C) 8 (D) 10

32. The ratio of the 7th to the (n – 1)th mean between 1 and 31, when n arithmetic means are inserted between then, is 5 : 9. The value of n is :

(A) 12 (B) 13 (C) 14 (D) 15

33. The first, second and last terms of an A.P. are a,b, and 2a respectively, the sum of the series is :

(A) $\frac{3ab}{2(b+a)}$ (B) $\frac{3ab}{2(b-a)}$ (C) $\frac{3ab}{2(a-b)}$ (D) None of these

34. Sum of first m terms of an A.P. is 0. If a be the first term of the A.P., then the sum of next n terms is :

(A) $\frac{-a(m+n)m}{m-1}$ (B) $\frac{-a(m+n)n}{m-1}$ (C) $\frac{-a(m+n)n}{n-1}$ (D) $\frac{-a(m+n)m}{n-1}$

35. If A_1 and A_2 be the two A.M.s between two numbers a and b, then $A_2 - A_1$ is equal to

(A) a + b (B) b – a (C) $\frac{b-a}{3}$ (D) None of these

36. The sum of terms equidistant from the beginning and end in an A.P. is equal to :

(A) Last term (B) First term
(C) Sum of the first and the last term (D) None of these

37. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then bc^2, ca^2, ab^2 are in:

(A) A.P. (B) G.P. (C) H.P. (D) None of these

38. If $a_1, a_2, a_3, \ldots a_n$ are in A.P. and $a_1 > 0$ for all I, then : $\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} =$

(A) $\frac{n}{\sqrt{a_1} + \sqrt{a_n}}$ (B) $\frac{n}{\sqrt{a_n} - \sqrt{a_1}}$ (C) $\frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$ (D) None of these

39. If a,b,c are in A.P. and also $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., then:

(A) $a = b \neq c$ (B) $a \neq b = c$ (C) $a = b = c$ (D) $a \neq b \neq c$

40. If a,b,c are in H.P., then $\frac{a-b}{b-c}$ equals :

(A) $\frac{b}{a}$ (B) $\frac{a}{b}$ (C) $\frac{a}{c}$ (D) None of these

41. An, A.P. consists pf n (odd) terms and it's middle term is m. Then the sum of the A.P. is :

(A) 2mn (B) $\frac{1}{2}mn$ (C) mn (D) mn^2

42. If A,G and H denote respectively the A.M., G.M. and H.M. between two positive numbers a and b, then A-G is equal to :

(A) a – b (B) $\frac{2ab}{a+b}$ (C) $\frac{1}{2}(\sqrt{a}-\sqrt{b})^2$ (D) None of these

43. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference is :

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

OBJECTIVE								ANSWER KEY				EXERCISE – 4			
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	A	C	D	B	B	D	A	C	A	D	C	A	C	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	C	C	C	B	C	A	D	A	C	C	B	D	C	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**		
Ans.	B	C	B	B	C	C	A	C	C	C	C	C	C		

EXERCISE – 5 (FOR IIT – JEE/AIEEE)

CHOOSE THE CORRECT ONE

1. The sum of the n terms of the series $\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+\ldots$ is : **[Kerala Engineering-2003]**

(A) $\frac{3^n(2n+1)+1}{2(3^n)}$ (B) $\frac{3^n(2n+1)-1}{2(3^n)}$ (C) $\frac{n3^n-1}{2(3^n)}$ (D) $\frac{3^n-1}{2}$

2. If the third term of a G.P. is p, then the product of it's first 5 terms is : **[Kerala Engineering-2003]**

(A) p^3 (B) p^2 (C) p^{10} (D) p^5

3. If $a_1, a_2, \ldots\ldots a_n$ are n A.M's between a and b, then $2\sum_{i=1}^{n} a_i$ = **[Kerala Engineering-2003]**

(A) ab (B) n (a + b) (C) $\frac{n(a+b)}{ab}$ (D) $\frac{a+b}{n}$

4. $4^{\frac{1}{2}} x 4^{\frac{1}{4}} x 4^{\frac{1}{8}}$ x …… to ∞ is a root of the equation : **[Kerala Engineering-2003]**

(A) $x^2 - 4 = 0$ (B) $x^2 - 4x + 6 = 0$ (C) $x^2 - 5x + 4 = 0$ (D) $x^2 - 3x + 2 = 0$

5. If a,b,c are in A.P., then which one of the following is not true? **[Kerala Engineering-2003]**

(A) a + k, b + k, c + k are in A.P. (B) ka, kb, kc are in A.P.

(C) a^2, b^2, c^2 are in A.P. (D) a + b, c + a, b + c are in A.P.

6. The sum of the series: $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots.+\frac{1}{\sqrt{n^2-1}+\sqrt{n^2}}$ is equal to **[AMU–2002]**

(A) $\frac{2n+1}{\sqrt{n}}$ (B) $\frac{\sqrt{n}+1}{\sqrt{n}+\sqrt{n-1}}$ (C) $\frac{n+\sqrt{n^2-1}}{2\sqrt{n}}$ (D) n – 1

7. Sum of infinite number of terms of a G.P. is 20 and sum of their squares is 100. The common ratio of the G.P. is : **[AIEEE–2002]**

(A) 5 (B) $\frac{3}{5}$ (C) $\frac{8}{5}$ (D) $\frac{1}{5}$

8. $1^3 - 2^3 + 3^3 - 4^3 + \ldots + 9^3 =$ **[AIEEE–2002]**

(A) 425 (B) – 425 (C) 475 (D) – 475

9. If $y = x - x^2 + x^3 - x^4 + \ldots$ to ∞, then the value of x will be $(-1 < x < 1)$:

(A) $y + \frac{1}{y}$ (B) $\frac{y}{1+y}$ (C) $y - \frac{1}{y}$ (D) $\frac{y}{1-y}$

10. The two geometric means between 1 and 64 are: **[Kerala Engineering-2002]**

(A) 1 and 64 (B) 8 and 16 (C) 4 and 16 (D) 3 and 16

11. The sum of infinite terms of the geometric progression $\frac{\sqrt{2+1}}{\sqrt{2-1}}, \frac{1}{2-\sqrt{2}}, \frac{1}{2}, \ldots$ is :

[Kerala Engineering-2002]

(A) $\sqrt{2}\left(\sqrt{2}+1\right)^2$ (B) $\left(\sqrt{2}+1\right)^2$ (C) $5\sqrt{2}$ (D) $3\sqrt{2}+\sqrt{5}$

12. If the nth term of the geometric progression, $5, -\frac{5}{2}, \frac{5}{4}, -\frac{5}{8}, \ldots is \frac{5}{1024}$, then the value of n is

[Kerala Engineering-2002]

(A) 11 (B) 10 (C) 9 (D) 4

13. In a harmonic progression, pth term is q and qth term is p, then the (pq)th term is:

(A) $\frac{p+q}{pq}$ (B) 0 (C) $\frac{pq}{p+q}$ (D) 1

14. Suppose a,b,c are A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$; then the value of a is:

[IIT Screening-2002]

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

15. Let the positive numbers a,b,c,d be in A.P., then abc, abd, acd, bcd are: **[IITScreening-2001]**

(A) Not in A.P./G.P./H.P. (B) In A.P.

(C) In G.P. (D) In H.P.

16. If the sum of the first 2n terms of the A.P. 2, 5, 8,… is equal to the sum of first n term of the A.P.57, 59, 61,… then n equals : **[IIT Scereening-2001]**

(A) 10 (B)12 (C)11 (D) 13

17. If $\frac{3+5+7+\ldots upto n terms}{5+8+11+\ldots upto 10 terms} = 7$, then the value of n is:

(A) 35 (B) 36 (C) 37 (D) 40

18. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in: **[DCE-2000]**

(A) G.P. (B) A.P. (C) H.P. (D) None of these

19. Consider an infinite geometric series with first term a and common ratio r. If it's sum is 4 and the second term is $\frac{3}{4}$, then : **[IIT-Screening-2000]**

(A) $a = \frac{7}{4}, r = \frac{3}{7}$ (B) $a = 2, r = \frac{3}{8}$ (C) $a = \frac{3}{2}, r = \frac{1}{2}$ (D) $a = 3, r = \frac{1}{4}$

20. If 4th term of an H.P. is 5 and 5th term is 4, then it's 20th term is:

(A) Zero (B) $\frac{4}{5}$ (C) 1 (D) $\frac{5}{4}$

21. H.M. between two numbers is 4. The A.M. 'A' and the G.M. 'G' between them satisfy the relation $2A + G^2 = 27$. The numbers are:

(A) 6, 3 (B) 4, 2 (C) 6, 9 (D) 3, 5

22. The sum of an infinite G.P. is 3. The sum of the series formed by squaring its terms is also 3. The series is:

(A) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...$ (B) $\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+...$

(C) $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+...$ (D) $1-\frac{1}{3}+\frac{1}{3^2}+-\frac{1}{3^3}+...$

23. If first three terms of a sequence $\frac{1}{16}, a, b, \frac{1}{6}$ are in G.P. and last three are in H.P., then values of a and b are respectively :

(A) $-\frac{1}{4}, 1$ (B) $\frac{1}{12}, \frac{1}{9}$

(C) Both (A) and (B) are true (D) $\frac{1}{9}, \frac{1}{12}$

24. The sum of few terms of a ratio series is 728, if common ratio is 3 and last term is 486, then first term of the series is :

(A) 1 (B) 2 (C) 3 (D) 4

25. The 6th term of a G.P. is 32 and it's 8th term is 128; the common ratio of the G.P. is :

(A) – 1 (B) 2 (C) 4 (D) – 4

26. Let $a_1, a_2, \ldots\ldots, a_{10}$ be A.P., $h_1, h_2, \ldots, h_{10}$ be in H.P. If $a_1 = h_1 = 2$, $a_{10} = h_{10} = 3$, then $a_4\, h_7$ = **[IIT]**

27. In a G.P., the first term is a, second term is b and the last term is c, then sum of the series is: **[AMU]**

28. If H is the harmonic mean between a and b, then $\frac{H+a}{H-a}+\frac{H+b}{H-b}$ is equal to: **[AMU]**

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) None of these

29. Let A_1, A_2 be two AMs and G_1, G_2 be two GMs between a and b, then $\frac{A_1+A_2}{G_1\ G_2}$ =

(A) $\frac{a+b}{2ab}$ (B) $\frac{2ab}{a+b}$ (C) $\frac{a+b}{ab}$ (D) $\frac{a+b}{\sqrt{ab}}$

30. a,b,c are three unequal numbers such that a,b,c are in A.P. ; b – a, c – b, a are in G.P. then a : b :c : : **[Karnataka-CET]**

(A) 1 : 2 : 4 (B) 2 : 3: 5 (C) 1 : 2: 3 (D) 1 :3 : 5

31. The first two terms of an infinite G.P. are together equal to 5 and every term is 3 times the sum of all the terms that follow it ; the common ratio of the G.P. is : **[AMU]**

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) 3 (D) 4

32. The eighth term of a G.P. is 128 and common ratio is 2. The product of it's first five terms is:

(A) 4^6 (B) 4^3 (C) 4^5 (D) 4^4

33. The sum of infinity of $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+\ldots$ is : **[AMU]**

(A) $\frac{3}{16}$ (B) $\frac{1}{5}$ (C) $\frac{1}{24}$ (D) $\frac{1}{16}$

34. p,q,r are in A.P. and each is numerically less than 1. Let : **[Karnataka-CET]**

$x = 1 + p + p^2 + \ldots\ldots$ to ∞

$y = 1 + q + q^2 + \ldots\ldots$ to ∞

$z = 1 + r + r^2 + \ldots\ldots$ to ∞, then x,y,z are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

35. If the numbers p,q,r are in A.P., then m^{7p}, m^{7q}, m^{7r} (m > 0) are in :

(A) A.P. (B) G.P. (C) H.P. (D) None of these

36. If the pth, qth and rth terms of a G.P. are ℓ, m and n respectively, then $\ell^{q-r} m^{r-p} n^{p-q}$ is :

(A) 1 (B) 0 (C) pqr (D) ℓ mn

37. $\frac{1}{1x2}+\frac{1}{2x3}+\frac{1}{3x4}+....+\frac{1}{n(n+1)}$ equals : **[AMU]**

(A) $\frac{n+1}{n}$ (B) $\frac{n(n+1)}{6}$ (C) $\frac{n}{n+1}$ (D) $\frac{n^2}{n+1}$

38. Sum of n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+....$ is equal to :

(A) $2^n - n - 1$ (B) $1 - 2^{-n}$ (C) $2^n - 1$ (D) $n + 2^{-n} - 1$

39. If $a^x = b^y = c^z$ and a,b,c are in G.P. then x, y,z are in :

(A) A.P. (B) G.P. (C) H.P. (D) None of these

40. If a_n be the nth term of a G.P. of positive numbers and $\sum_{n=1}^{100} a_{2n} = \alpha, \sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the common ratio of the G.P. is : **[IIT]**

(A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$ (C) $\sqrt{\frac{\alpha}{\beta}}$ $\sqrt{\frac{\beta}{\alpha}}$

41. If p,q,r are in A.P. and x,y,z are in G.P, then $x^{q-r} y^{r-p} z^{p-q}$ is equal to :

(A) p + q + r (B) xyz (C) 1 (D) px + qy +rz

42. If a,b,c are in A.P., then 10^{ax+10}, 10^{bx+10}, 10^{cx+10}, $x \neq 0$ are in :

(A) A.P. (B) G.P. only when x > 0

(C) G.P. for all x (D) G.P. only when x < 0

43. If the sum of roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in :

(A) G.P. (B) H.P. (C) A.P. (D) None of these

44. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then integral values of p and q are respectively.

(A) – 2, – 32 (B) – 2, 3 (C) – 6, 3 (D) – 6,– 32

45. If a,b,c,d and x are all real and $(a^2 + b^2 + c^2)\ x^2 - 2(ab + bc + cd)\ x + (b^2 + c^2 + d^2) \leq 0$ then :

(A) a,b,c,d are in G.P. (B) a,b,c,d are in A.P. (C) a,b,c,d are in H.P. (D) None of these

OBJECTIVE — ANSWER KEY — EXERCISE – 4

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	B	C	C	D	B	A	D	C	A	A	D	D	D
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	C	A	B	D	C	A	B	C	B	B	D	B	C	C	C
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	B	C	A	C	B	A	C	D	C	A	C	C	B	A	A

Important Notes

CIRCLES

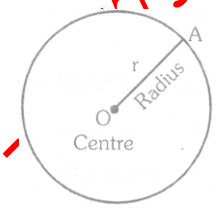
|★ INTRODUCTION

In class IX, we have studied that a circle is a collection of all points in a plane which are at a constant distance from a fixed point. The fixed point is called the centre and the constant distance is known as the radius. We have also studied various terms related to a circle like chord, segment, sector, are etc. Now we shall study properties of a line touching a circle at one point.

★ RECALL

Circle

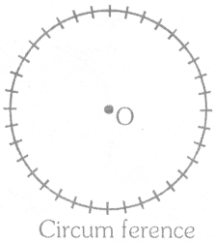
A circle is the locus of a point which moves in such a way that it is always at the constant distance from a fixed point in the plane. **The fixed point** 'O' is called the centre of the circle. **The constant distance** 'OA' between the centre (O) and the moving point (A) is called the **Radius** of the circle.



Circumference

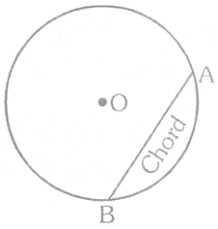
The distance round the circle is called the circumference of the circle.

$2\pi r$ = circumference of the circle
= Perimeter of the circle.
= boundary of the circle
r is the radius of the circle.



Chord

The chord of a circle is a line segment joining any two points on the circumference. AB is the chord of the circle with centre O. In fig. AB is the chord of the circle.

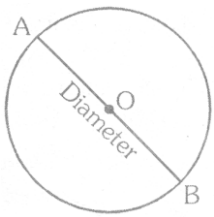


Diameter

A line segment passing through the centre of the circle and having its end points on the circle is called diameter. If r is the radius of the circle then the diameter of the circle is twice the radius i.e., d = 2r

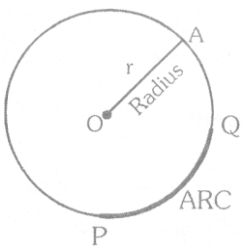
AOB is a diameter of the circle whose centre is O

AOB = OA + OB = r + r + 2r.



Arc of a circle

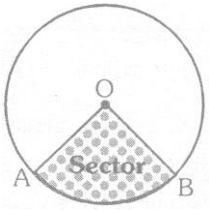
If P and Q be any two points on the circle then the circle is divided into two pieces each of which is an arc. Now we denote the arc from P to Q in counter clock-wise direction by $\overset{\frown}{PQ}$ and the are from Q to P in clock-wise direction by $\overset{\frown}{QP}$.



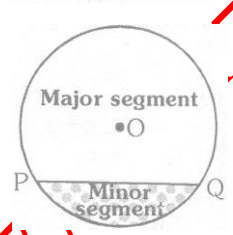
Sector of a circle

The part of a circle bounded by two radii and arc is called sector. In fig, the part of the plane region enclosed0 by $\overset{\frown}{AB}$ and its bounding radii OA and OB is a sector of the circle with centre O.

Segment of a circle

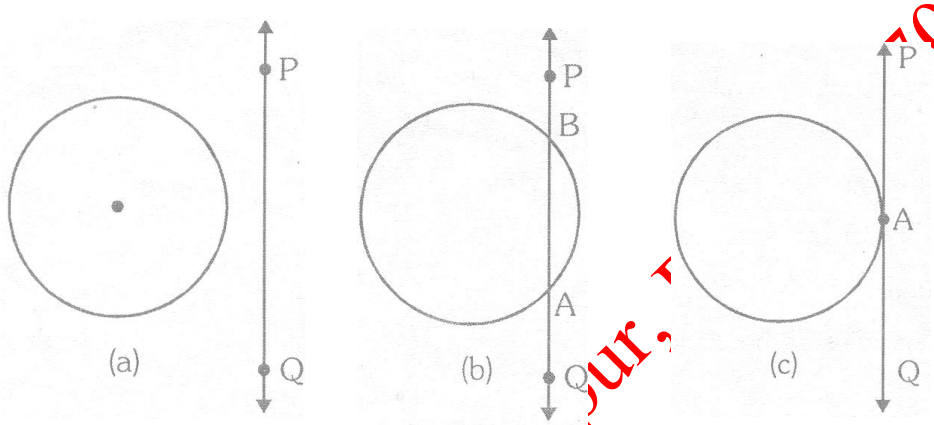


Let PQ be a chord of a circle with centre O and radius r, then PQ divides the region enclosed by the circle into two parts. Each part is called a segment of the circle. The part containing the minor arc is called the **minor segment** and the part containing the major arc is called the **major segment.**



★ INTERSECTION OF A CIRCLE AND A LINE

Consider a circle with centre O and radius r and a line PQ in a plane. We find that there are three different positions a line can take with respect to the circle as given below in fig.



(a) The line PQ does not intersect the circle.
In fig. (a) the line PQ and the circle have no common point. In this case PQ is called a non-intersecting line with respect to the circle.

(b) The line PQ intersect the circle in more than one point. In fig. (b), there are two common points A and B between the line PQ and the circle and we call line PQ as a secant of the circle.

(c) The line intersect the circle in a single point i.e. the line intersect the circle in only one points In fig. (c) you can verify that there is only one point 'A' which is common to the line PQ in the given circle. In this case the line is called a tangent to the circle.

Secant

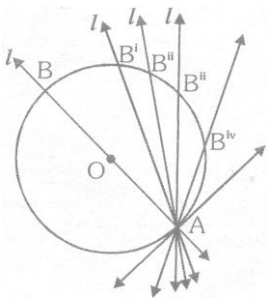
A secant is a straight line that cuts the circumference of the circle at two distinct (different) points i.e., if a circle and a line have two common points then the line is said to be secant to the circle.

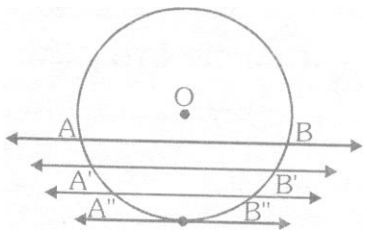
Tangent

A tangent is a straight line that meets the circle at one and only one point. This point 'A' is called point of contact or point of tangency in fig. (c).

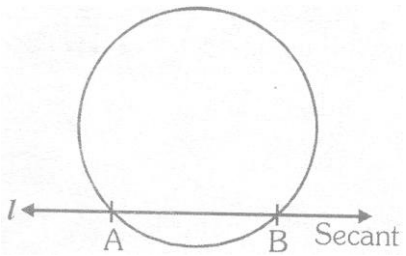
Tangent as a limiting case of a secant

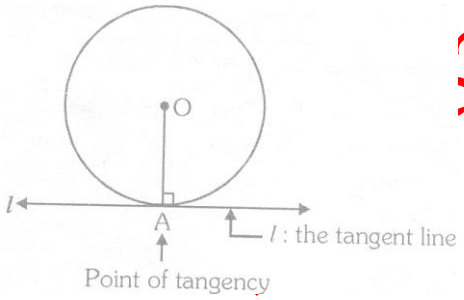
In the fig. the secant ℓ cuts the circle at A and B. If this secant ℓ is turned around the point A, keeping A fixed then B moves on the circumference closer to A. In the limiting position, B coincides with A. The secant ℓ becomes the tangent at A. Tangent to a circle is a secant when the two end points of its corresponding chord coincide.





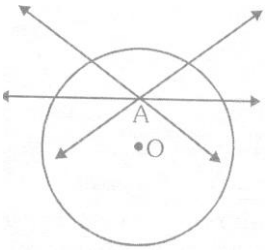
In the fig. ℓ is a secant which cuts the' circle at A and B. If the secant is moved parallel to itself away from the centre, then the points A and B come closer and closer to each other. In the limiting position, they coincide into a single point at A, the secant ℓ becomes the tangent at A. Thus a tangent line is the limiting case of a secant when the two points of intersection of the secant and a circle coincide with the point A. i.e., the common point of the tangent and the circle is called the point of contact and the tangent is said to touch the circle at the common point.





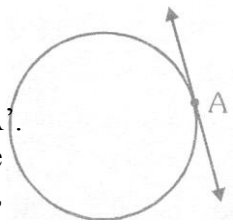
Note: The line containing the radius through the point of contact is called normal to the circle at the point.

★ NUMBER OF TANGENTS TO ACIRCLE FROM A POINT

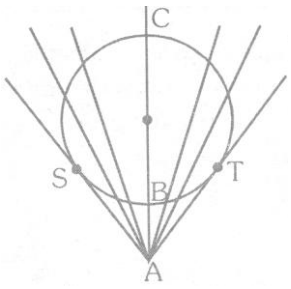


1. If a point A lies inside a circle, no line passing through 'A' can be a tangent to the circle. i.e., No tangent can be drawn from the point A.

2. If A lies on the circle, then one and only one tangent can be drawn to pass through 'A'.
i.e. Exactly one tangent can be drawn through A.



3. If A lies outside the circle then exactly two tangents can be drawn through 'A'.
In the fig., a secant ABC is drawn from a point 'A' outside the circle, if the secant is turned around A in the clockwise direction, in the limiting position, it becomes a tangent at T. Similarly if the secant is turned in the anti-clockwise direction, in the limiting position, it becomes a tangent at S. Thus from a point A outside a circle only two tangents can be drawn. The points S and T where the lines touch the circle are called the points of contact.



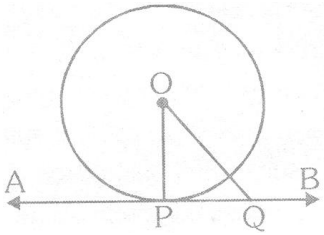
★ PROPERTIES OF TANGENT TO A CIRCLE

Theorem-1 : The tangent at any point of a circle and the radius through the point are perpendicular to each other.

Given : A circle with centre O. AB is a tangent to the circle at a point P and OP is the radius through P.

To prove : OP $\perp$ AB.

Construct : Take a point Q, other than P, on tangent AB. Join OQ.



Proof :

	STATEMENT	REASON
1.	Since Q is a point on tangent AB, other than the point P, so Q will lie outside the circle $\therefore$ OQ will intersect the circle at some point R.	Tangent at P intersects the circle at points P only.
2.	$\therefore$ OR < OQ $\Rightarrow$ OP < OQ	Part is less than the whole. OR = OP = radius.
3.	Thus, OP is shorter than any other line segment joining O to any point of AB.	
4.	OP $\perp$ AB	Of all line segments drawn from O to line AB, the perpendicular is the shortest

Hence, proved.

Remark 1 : A pair of tangents drawn at two points of a circle are either parallel or they intersect each other at a point outside the circle.

Remark 2 : If two tangents drawn to a circle are parallel to each other, then the line-segment joining their points of contact is a diameter of the circle.

Remark 3 : The distance between two parallel tangents to a circle is equal to the diameter of the circle, i.e., twice the radius.

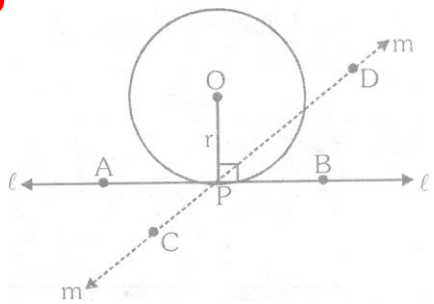
Remark 4 : A pair of tangents drawn to a circle at the end point of a diameter of a circle are parallel to each other.

Remark 5 : A pair of tangents drawn to a circle at the end points of a chord of the circle, other than a diameter, intersect each other at a point outside the circle.

Corollary 1: A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle

Given: O is the centre and r be the radius of the circle. OP is a radius of the circle. Line ℓ is drawn through P so that OP $\perp \ell$

To prove: Line ℓ is tangent to the circle at P.



Construction: Suppose that the line ℓ is not the tangent to the circle at P. Let us draw another straight line m which is tangent to the circle at P. Take two points A and B (other that P) on the line ℓ and two points C and D on m.

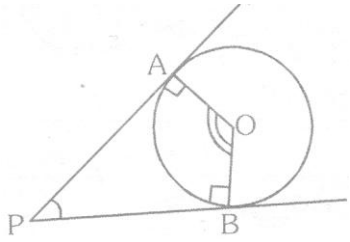
Proof:

	STATEMENT	REASON
1.	OP $\perp \ell$ $\Rightarrow \angle OPB = 90^o$	Given
2.	OP $\perp$ m	By theorem

	$\Rightarrow \angle OPD=90^o$ $\Rightarrow \angle OPD = <OPB$	 Each = 90^o

But a part cannot, be equal to whole. This gives contradiction. Hence, our supposition is wrong.
Therefore, the line ℓ is tangent to the circle at P

Corollary 2: If O be the centre of a circle and tangents drawn to the circle at the points A and B of the circle intersect each other at P, then $\angle AOB + \angle APB = 180^o$.



Proof:

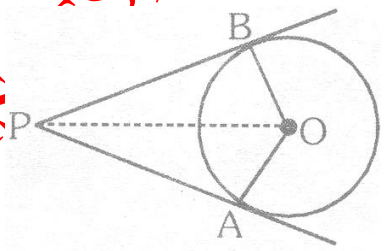
	STATEMENT	REASON
1.	$OP \perp PA$ & $OB \perp PB$ $\Rightarrow \angle OPB = \angle OBP = 90^0$	 By theorem
2.	$\angle AOB + \angle OBP + \angle OAP + \angle APB = 360^o$ $\Rightarrow \angle AOP + 90^o + 90^o + \angle APB = 360^o$ $\Rightarrow \angle AOB + \angle APB + 180^o = 360^o$ $\Rightarrow \angle AOB + \angle APB = 360^o - 180^o$ $\Rightarrow \angle AOB + \angle APB = 180^o$	

Hence, Proved

Theorem-2: If two tangents are drawn to a circle from an exterior point, then
(i) the tangents are equal in length
(ii) the tangents subtend equal angles at the centre
(iii) the tangents are equally inclined to the line joining the point and the centre of the circle.

Given : PA and PB are two tangents drawn to a circle with centre O, from an exterior point P.

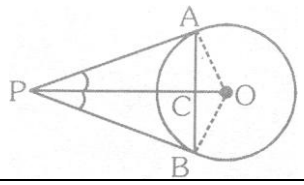
To prove: (i) PA = PB (ii) $\angle AOP = \angle BOP$, (iii) $\angle APO = \angle BPO$.



Proof:

	STATEMENT	REASON
1.	In ΔAOP and ΔBOP: OA = OB $\angle OAP = \angle OBP = 90^o$	Radii of the same circle. Radius through point of contact is perpendicular to the tangent
	OP = OP $\therefore \Delta AOP \cong \Delta BOP$	Common.
2.	Hence, we have (i) PA = PB (ii) $\angle AOP = \angle BOP$ (iii) $\angle APO = \angle BPO$.	 c.p.c.t c.p.c.t c.p.c.t

Corollary 3: If PA and PB are two tangents from a point to a circle with centre 0 touching it at A and B prove that OP is perpendicular bisector of AB.



Proof:

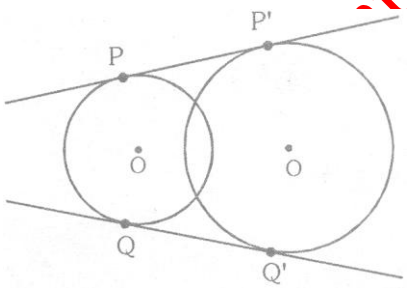
	STATEMENT	REASON
1.	For ΔACP and ΔBCP (i) PA= PB (ii) PC = PC (iii) $\angle ACP = \angle BPC$	 Lengths of two tangents from P are equal Common PO bisector $\angle APB$
2.	$\Delta ACP \cong \Delta BCP$	SAS congruency
3.	AC = BC	c.p.c.t
4.	$\angle ACP = \angle BCP = \frac{1}{2} \times 180^o = 90^o$	

Therefore, OP is perpendicular bisector of AB.
Hence proved.

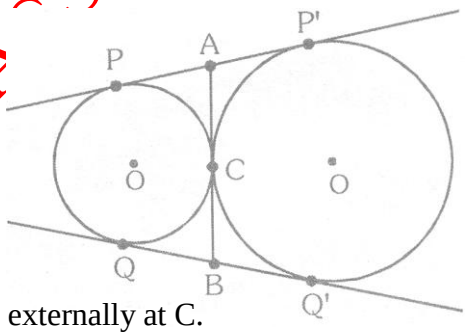
★ **COMMON TANGENTS OF TWO CIRCLES**

Two circles in a plane, either intersect each other in two points or touch each other at a point or they neither intersect nor touch each other.

Common Tangent of two intersecting circles : Two circles intersect each other in two points A and B. Here, PP' and QQ' are the only two common tangents. The case where the two circles are of unequal radii, we find the common tangents PP' and QQ' are not parallel.

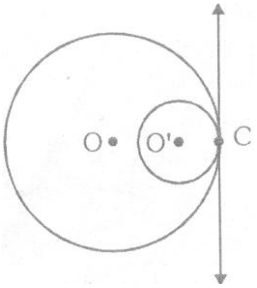


Common tangents of two circles which touch each other externally at a point:



Two circles touch other externally at C.
Here, PP', QQ' and AB are the three common tangents drawn to the circles.

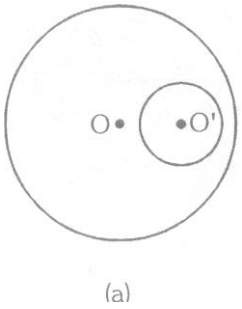
Common tangents of two circles which touch each other internally at a point:



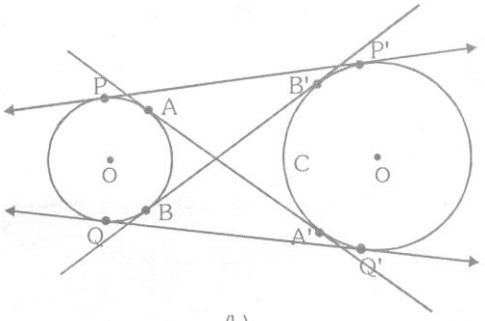
Two circles touch other internally at C. Here, we have only one common tangent of the two circles.

Common tangents of two non-intersecting and non-touching circles:

Here, we observe that in figure (a), there is no common tangent but in figure (b) there are four common tangents PP' QQ', AA and BB'.



(a)



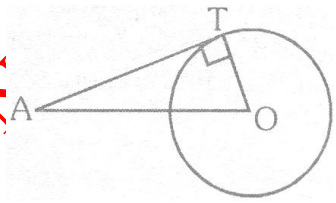
(b)

Ex.1. A point A is 26 cm away from the centre of a circle and the length of tangent drawn from A to the circle is 24 cm. Find the radius of the circle.

Sol. Let O be the centre of the circle and let A be a point outside the circle such that OA = 26 cm.

Let AT be the tangent to the circle.

Then, AT = 24 cm. Join OT.



Since the radius through the point of contact is perpendicular to the tangent, we have $\angle OTA = 90^o$. In right ΔOTA, we have

$$OT^2 = OA^2 - AT^2$$

$$= [(26)^2 - (24)^2] = (26 + 24)(26 - 24) = 100.$$

$\Rightarrow$ $OT = \sqrt{100} = 10$ cm.

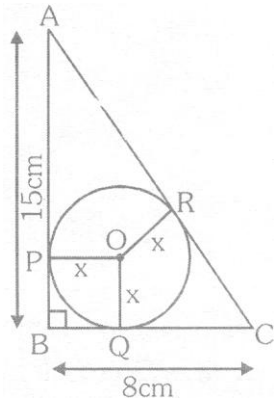
Hence, the radius of the circle is 10 cm.

Ex.2. In the given figure, ΔABC is right-angled at B, in which AB = 15 cm and BC = 8 cm. A circle with centre O has been inscribed in ΔABC. Calculate the value of x, the radius of the inscribed circle.

Sol. Let the inscribed circle touch the sides AB, BC and CA at P, Q and R respectively. Applying Pythagoras theorem on right ΔABC, we have

$$AC^2 = AB^2 + BC^2 = (15)^2 + (8)^2 = (225 + 64) = 289$$

$\Rightarrow$ $AC = \sqrt{289} = 17$ cm.



Clearly, OPBQ is a square.

[$\therefore \angle OPB = 90^o$, $\angle PBQ = 90^o$, $\angle OQB = 90^o$ and OP = OQ = x cm]

$\therefore$ BP = BQ = x cm.

Since the tangents to a circle from an exterior point are equal in length, we have AR = AP and CR = CQ.

Now, AR = AP = (AB – BP) = (15 – x) cm

CR = CQ = (BC – BQ) = (8 – x) cm.

AC = AR + CR $\Rightarrow$ 17 = (15 – x) + (8 – x) $\Rightarrow$ 2x = 6 $\Rightarrow$ x = 3.

Hence, the radius of the inscribed circle is 3 cm.

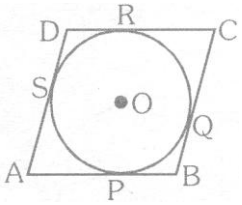
Ex.3 If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

Sol. Let ABCD be a parallelogram whose sides AB, BC, CD and DA touch a circle at the points P, Q, R and S respectively.

Since the lengths of tangents drawn from an external point to a circle are equal, we have

AP = AS, BP = BQ, CR = CQ and DR = DS.

$\therefore$ AB + CD = AP + BP + CR + DR



= AS + BQ + CQ + DS
= (AS + DS) + (BQ + CQ)
= AD + BC

Now, AB + CD = AD + BC

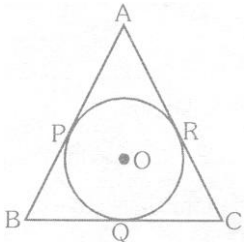
$\Rightarrow$ 2AB = 2BC [$\because$ Opposite sides of a || gm are equal]

$\Rightarrow$ AB = BC

$\therefore$ AB = BC = CD = AD.

Hence, ABCD is a rhombus.

Ex.4 In the given figure, the in circle of ΔABC touches the sides AB, BC and CA at the points P, Q, R respectively. Show that AP + BQ + CR = BP + CQ + AR = $\frac{1}{2}$ (Perimeter of ΔABC



Sol. Since the lengths of two tangents drawn from an external point to a circle are equal, we have

AP = AR, BQ = BP and CR = CQ

$\therefore$ AP + BQ + CR = AR + BP + CQ ....(i)

Perimeter of ΔABC = AB + BC + CA = AP + BP + BQ + CQ + AR + CR

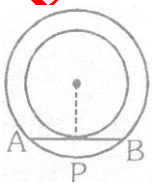
= (AP + BQ + CR) + (BP + CQ + AR)

= 2(AP + BQ + CR) [Using (i)]

$\therefore$ AP + BQ + CR = BP + CQ + AR = $\frac{1}{2}$ (Perimeter of ΔABC).

Ex.5 In two concentric circles, prove that a chord of larger circle which is tangent to smaller circle is bisected at the point of contact.

Sol. Let there be two concentric circles, each with centre O.



Let AB be a chord of larger circle touching the smaller circle at P. Join OP.

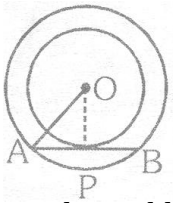
Since OP is a radius of smaller circle and APB is a tangent to it at the point P, so OP $\perp$ AB.

But the perpendicular from the centre to a chord, bisects the chord.

$\therefore$ AP = PB

Hence, AB is bisected at the point P.

Ex.6 Two concentric circles are of radii 13 cm and 5 cm. Find the length of the chord of the outer circle which touches the inner circle.



Sol. Let O be the centre of the concentric circles and let AB be a chord of the outer circle, touching the inner circle at P. Join OA and OP.

Now, the radius through the point of contact is perpendicular to the tangent.

$\therefore$ OP $\perp$ AB.

Since, the perpendicular from the centre to a chord, bisects the chord, AP = PB. Now, in right ΔOPA, we have OA = 13 cm and OP = 5 cm.

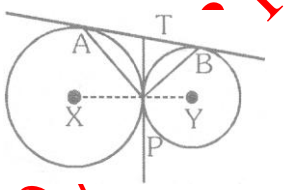
$\therefore$ $OP^2 + AP^2 = OA^2 \Rightarrow AP^2 = OA^2 - OP^2 = (13^2 - 5^2) = (169 - 25) = 144.$

$\Rightarrow AP = \sqrt{144} = 12$ cm.

$\therefore$ AB = 2AP = (2 x 12) cm = 24 cm.

Hence, the length of chord AB = 24 cm.

Ex.7 In the given figure, PT is a common tangent to the circles touching externally at P and AB is another common tangent touching the circles at A and B. Prove that:



(i) T is the mid-point of AB

(ii) $\angle APB = 90^o$

(iii) If X and Y are centres of the two circles,
show that the circle on AB as diameter touches the line XY.

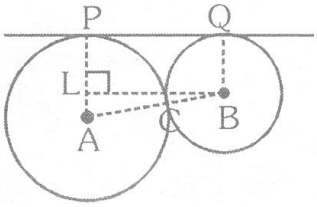
Sol. (i) Since the two tangents to a circle from an external point are equal, we have
TA = TP and TB = TP.
$\therefore$ TA = TB [Each equal to TP]
Hence, T bisects AB, i.e., T is the mid-point of AB.

(ii) TA = TP $\Rightarrow \angle TAP = \angle TPA$
TB = TP $\Rightarrow \angle TBP = \angle TPB$
$\therefore \angle TAP + \angle TBP = \angle TPA + \angle TPB = \angle APB$
$\Rightarrow \angle TAP + \angle TBP = \angle APB = 2\angle APB$
$\Rightarrow 2\angle APB = 180^o$ [$\therefore$ The sum of the $\angle$s of a Δ is 180^o]
$\Rightarrow \angle APB = 90^o$

(iii) Thus, P lies on the semi-circle with AB as diameter.
Hence, the circle on AD as diameter touches the line XY.

Ex.8 Two circles of radii 25 cm and 9cm touch each other externally. Find the length of the direct common tangent.

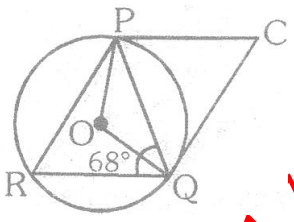
Sol. Let the two circles with centres A and B and radii 25 cm and 9 cm respectively touch each other externally at a point C.



Then, AB = AC + CB = (25 + 9) cm = 34 cm.
[$\therefore$ Radius through point of contact is perpendicular to the tangent]
Draw, BL $\perp$ AP.
Then, PLBQ is a rectangle.
Now, LP = BQ = 9 cm and PQ = BL
$\therefore$ AL = (AP – LP) = (25 – 9) cm = 16 cm.
From right fΔ ALB, we have
$AB^2 = AL^2 + BL^2 \Rightarrow BL^2 = AB^2 - AL^2 = (34)^2 - (16)^2 = (34 + 16)(34 - 16) = 900$
$\Rightarrow BL = \sqrt{900} = 30$ cm.
$\therefore$ PQ = BL = 30 cm.

Hence, the length of direct common tangent is 30 cm.

Ex.9 In the given figure, PQ = QR, $\angle RQP = 68^o$, PC and CQ are tangents to the circle with centre O. Calculate the values of : (i) $\angle QOP$ (ii) $\angle QCP$



Sol. (i) In ΔPQR,

$PQ = QR \Rightarrow \angle PRQ + \angle QPR$ [$\angle$s opp. to equal sides of a Δ are equal]

Also, $\angle QPR + \angle RQP + \angle PRQ = 180^o$ [Sum of the $\angle$s of a Δ is 180^o]

$\Rightarrow \quad 68^o + 2\angle PRQ = 180^o$

$\Rightarrow \quad 2\angle PRQ = (180^o - 68^o) = 112^o$

$\Rightarrow \quad \angle PRQ = 56^o.$

$\therefore \quad \angle QOP = 2\angle PRQ = (2 \times 56^o) = 112^o$. [Angle at the centre is double the angle on the circle]

(ii) Since the radius through the point of contact is perpendicular to the tangent, we have

$\angle OQC = 90^o$ and $\angle OPC = 90^o$.

Now, $\angle OQC + \angle QOP + \angle OPC + \angle QCP = 360^o$ [Sum of the $\angle$s of a quad. Is 360^o)

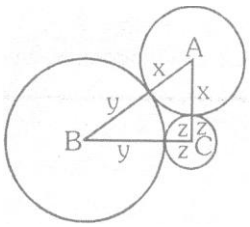
$\Rightarrow 90^o + 112^o + 90^o + \Rightarrow \angle QCP = 360^o.$

$\Rightarrow \angle QCP = (360^o - 292^o) = 68^o.$

Ex.10 With the vertices of ΔABC as centres, three circles are described, each touching the other two externally. If the sides of the triangle are 9 cm, 7 cm and 6 cm, find the radii of the circles.

Sol. Let AB = 9 cm, BC = 7 cm and CA = 6 cm.

Let x, y, z, be the radii of circles with centres A, B, C respectively.



Then, x + y = 9, y + z = 7 and z + x = 6.

Adding, we get $2(x + y + z) = 22 \Rightarrow x + y + z = 11$.

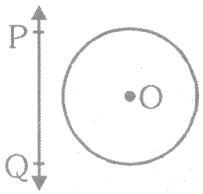
$\therefore x = [(x + y + z) - (y + z)] = (11 - 7)$ cm = 4cm.

Similarly, y = (11 – 6) cm = 5 cm and z = (11 – 9) cm = 2 cm.

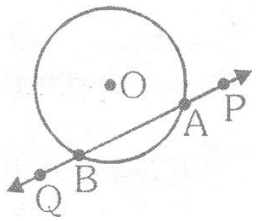
Hence, the radii of circles with centres A, B, C are 4 cm, 5cm and 2 cm respectively.

★ SYNOPSIS

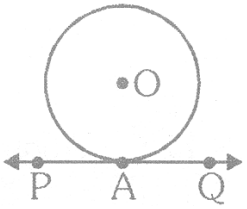
➤ If a circle and a line have no common point, then line is called a non-intersecting line with respect to the circle.



- If a circle and a line have two common points or a line intersect a circle in two distinct points, then line is called secant to the circle.



- If a line and a circle have only one point common, or a line intersect the circle in only one point, then it is called tangent to the circle.



- There is only one tangent at a point of the circle.
- The common point of the tangent and the circle is called the point of contact.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- The line containing the radius through the point of contact of tangent is called the normal to the circle at the point.
- There is no tangent to the circle passing through a point lying inside the circle.
- There are exactly two tangents to a circle through a point lying outside the circle.
- The length of the segment of the tangent from the external point and the point of contact with the circle is called the length of the tangent.
- The lengths of tangents drawn from an external point to a circle are equal.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. A point P is 10 cm from the centre of a circle. The length of the tangent drawn from P to the circle is 8 cm. The radius of the circle is equal to

(A) 4 cm (B) 5 cm (C) 6 cm (D) None of these.

2. A point P is 25 cm from the centre of a circle. The radius of the circle is 7 cm and length of the tangent drawn from P to the circle is x cm. The value of x =

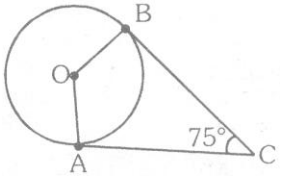
(A) 20 cm (B) 24 cm (C) 18 cm (D) 12 cm.

3. In fig, O is the centre of the circle, CA is tangent at A and CB is tangent at B drawn to the circle. if <ACB = 75^o, then <AOB =

(A) 75^o

(B) 85^o

(C) 95^o



(D) 105^o

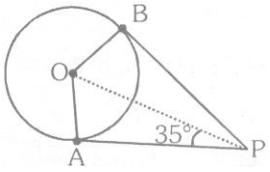
4. In figure 10.75, PA and PB are the two tangents drawn to the circle. 0 is the centre of the circle. A and B are the points of contact of the tangents PA and PB with the circle. If $\angle OPA = 35^o$, then $\angle POB =$

(A) 55^o

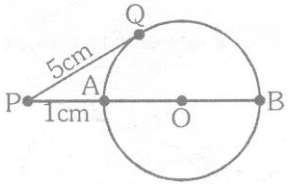
(B) 65^o

(C) 75^o

(D) 85^o



5. In fig, O is the centre of the circle. PQ is tangent to the circle and secant PAB passes through the centre O. If PQ = 5 cm and PA = 1 cm, then the radius of the circle is



(A) 8 cm (B) 12cm (C) 10cm (D) 6cm

6. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q such that OQ = 12cm. Length PQ is

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}\ cm$

7. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25cm. The radius of the circle is

(A) 7 cm (B) 12 cm (C) 15cm (D) 24.5 cm

8. The length of the tangent from a point A at a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is

(A) $\sqrt{7}\ cm$ (B) 7 cm (C) 5 cm (D) 25cm

9. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80^o then $\angle POA$ is equal to

(A) 50^o (B) 60^o (C) 70^o (D) 80^o

10. If TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110^o$, then, $\angle PTQ$ is equal to

(A) 60^o (B) 70^o (C) 80^o (D) 90^o

11. PQ is a tangent to a circle with centre O at the point P. If ΔOPQ is an isosceles triangle, then $\angle OQP$ is equal to

(A) 30^o` (B) 45^o (C) 60^o (D) 90^o

12. Two circle touch each other externally at C and AB is a common tangent to the circles. Then, $\angle ACB =$

(A) 60^o (B) 45^o (C) 30^o (D) 90^o

13. ABC is a right angled triangle, right angled at B such that BC = 6 cm and AB = 8 cm. A circle with centre O is inscribed in ΔABC. The radius of the circle is

(A) 1 cm (B) 2 cm (C) 3 cm (D) 4 cm

14. PQ is a tangent drawn from a point P to a circle with centre O and QOP is a diameter of the circle such that $\angle POR = 120^o$, then $\angle OPQ$ is

(A) 60^o (B) 45^0 (C) 30^o (D) 90^o

15. If four sides of a quadrilateral ABCD are tangential to a circle, then

(A) AC + AD = BD + CD (B) AB + CD = BC + AD

(C) AB + CD = AC + BC (D) AC + AD = BC + DB

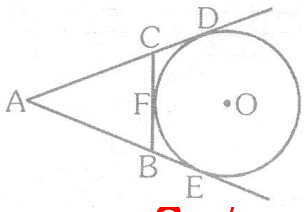
16. The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is

(A) $\sqrt{7}$ cm (B) $2\sqrt{7}$ cm (C) 10 cm (D) 5 cm

17. AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that CD = 4 cm, then AB is equal to

(A) 4 cm (B) 6 cm (C) 8 cm (D) 12 cm

18. In the adjoining figure, if AD, AE and BC are tangents to the circle at D, E and F respectively. Then,



(A) AD = AB + BC + CA (B) 2AD = AB + BC + CA

(C) 3AD = AB + BC + CA (D) 4AD = AB + BC + CA

OBJECTIVE				ANSWER KEY					EXERCSE
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	C	B	D	A	B	D	A	C	A	B
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans.	B	D	B	C	B	B	C	B		

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

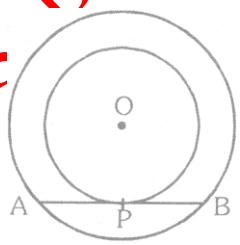
SUBJECTIVE TYPE QUESTIONS

SHORT ANSWER TYPE QUESTIONS

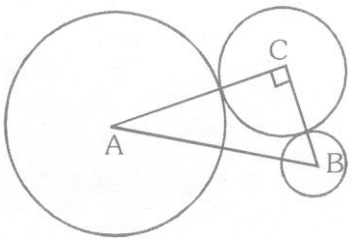
1. Find the length of the tangent drawn to a circle of radius 8 cm, from a point which is at a distance of 10 cm from the centre of the circle.

2. A point P is 7 cm away from the centre of the circle and the length of tangent drawn from P to the circle is 15 cm. Find the radius of the circle.

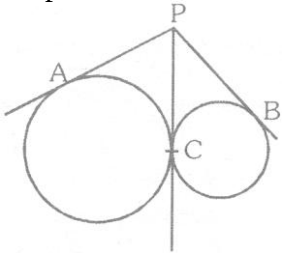
3. There are two concentric circles, each with centre O and of radii 10 cm and 26 cm respectively. Find the length of the chord AB of the outer circle which touches the inner circle at P.



4. A and B are centres of circles of radii 9 cm and 2 cm such that AB = 17 cm and C is the centre of the circle of radius r cm which touches the above circles externally. If $\angle ACB = 90^o$, write an equation in r and solve it.



5. Two circles touch each other externally at a point C and P is a point on the common tangent at C. If PA and PB are tangents to the two circles, prove that PA = PB.



6. Two circles touch each other internally. Prove that the tangents drawn to the two circles from any point on the common tangent are equal in length.

7. Two circles of radii 18 cm and 8 cm touch externally. Find the length of a direct common tangent to the two circles.

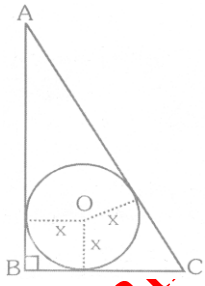
8. Two circles of radii 8 cm and 3 cm have their centres 13 cm apart. Find the length of a direct common tangent to the two circles.

9. Two circles of radii 8 cm and 3 cm have a direct common tangent of length 10 cm. Find the distance between their centres, up to two places of decimal.

10. With the vertices of ΔPQR as centres, three circles are described, each touching the other two externally. If the sides of the triangle are 7 cm, 8 cm and 11 cm, find the radii of the three circles.

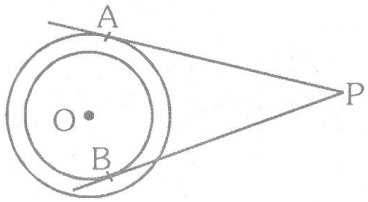
LONG ANSWER TYPE QUESTIONS

1. ΔABC is right-angled triangle with AB = 12 cm and AC = 13 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of x, the radius of the inscribed circle.

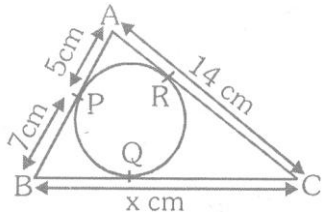


2. PQR is a right-angled triangle with PQ = 3 cm and QR = 4 cm. A circle which touches all the sides of the triangle is inscribed in the triangle. Calculate the radius of the circle.

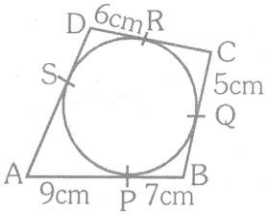
3. In the given figure, O is the centre of each one of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to outer and inner circle respectively. If PA = 10 cm, find the length of PB, up to two places of decimal.



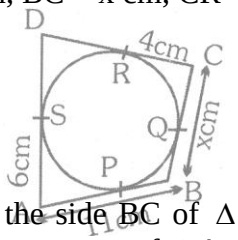
4. In the given figure, ΔABC is circumscribed. The circle touches the sides AB, BC and CA at P, Q, R respectively. If AP = 5 cm, BP = 7 cm, AC = 14 cm and BC = x cm, find the value of x.



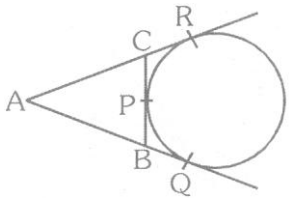
5. In the given figure, quadrilateral ABCD is circumscribed. The circle touches the sides AB, BC, CD and DA at P, Q, R, S respectively. If AP = 9 cm, BP = 7 cm, CQ = 5 cm and DR = 6 cm, find the perimeter of quad. ABCD.



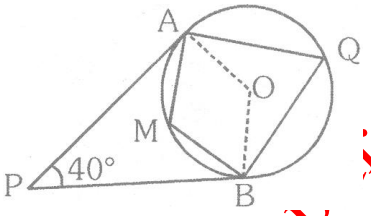
6. In the given figure, the circle touches the sides AB, BC, CD and DA of a quadrilateral ABCD at the points P, Q, R,S respectively. If AB = 11 cm, BC = x cm, CR = 4 cm and AS = 6 cm, find the value of x.



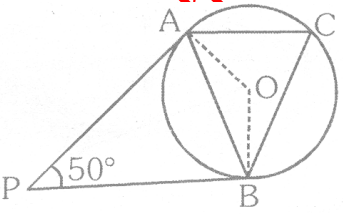
7. In the given figure, a circle touches the side BC of ΔABC at P and AB and AC produced at Q and R respectively. If AQ = 15 cm, find the perimeter of ΔABC.



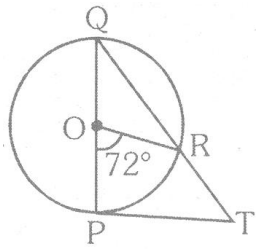
8. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 40^o$, find $\angle AQB$ and $\angle AMB$.



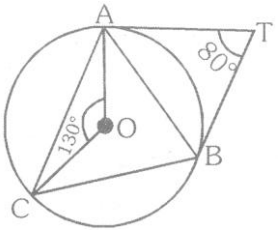
9. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 50^o$, find:
(i) $\angle AOB$ (ii) $\angle OAB$ (iii) $\angle ACB$



10. In the given figure PQ is a diameter of a circle with centre O and PT is a tangent at. QT meets the circle at R. If $\angle POR = 72^o$, find $\angle PTR$.

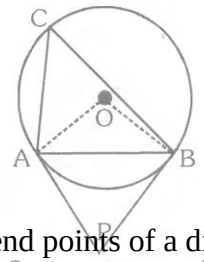


11. In the given figure, O is the centre of the circumcircle of ΔABC. Tangents at A and B intersect at T. If $\angle ATB = 80^o$ and $\angle AOC = 130^o$, Calculate $\angle CAB$.

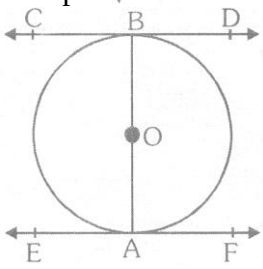


12. In the given figure, PA and PB are tangents to a circle with centre O and ΔABC has been inscribed in the

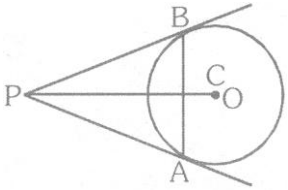
circle such that AB = AC. If $\angle BAC = 72^o$, calculate (a) $\angle AOB$ (B0 $\angle APB$.



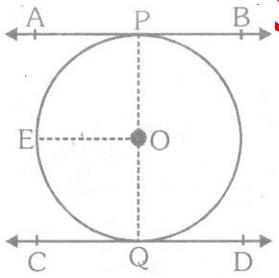
13. Show that the tangent lines at the end points of a diameter of a circle are parallel.



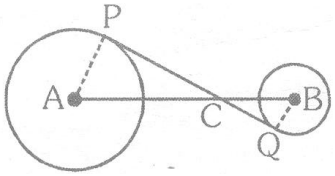
14. Prove that the tangents at the extremities of any chord make equal angles with the chord.



15. Show that the line segment joining the points of contact of two parallel tangents passes through the centre.

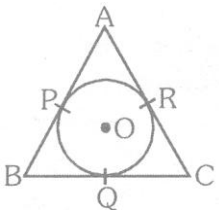


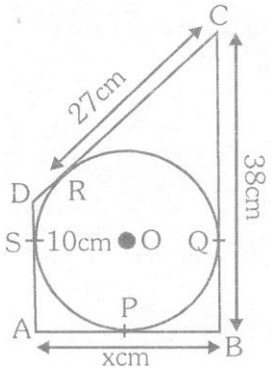
16. In the given figure, PQ is a transverse common tangent to two circles with centres A and B and of radil 5 cm and 3 cm respectively If PQ intersects AB at C such that CP = 12 cm, calculate AB.

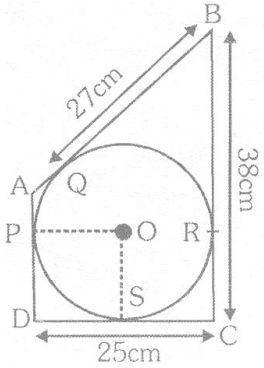


17. ΔABC is an isosceles triangle in which AB = AC, circumscribed about a circle. Prove that the base is bisected by the point of contact.

18. In the given figure quadrilateral ABCD is circumscribed and AD $\perp$ AB. If the radius of incircle is 10 cm, find the value of x.







19. In the given figure, a circle is inscribed in quad. ABCD. If BC = 38 cm, BQ = 27 cm, DC = 25 cm and AD ⊥ DC, find the radius of the circle.

CIRCLE **ANSWER KEY** **EXERCISE – 2 (X) – CBSE**

SHORT ANSWER TYPE QUESTIONS:

1. 6 cm **2.** 8 cm **3.** 48 cm **4.** $r^2 + 11r - 102 = 0$, r = 6 **7.** 24 cm **8.** 12 cm

9. 11.8 cm **10.** 5 cm, 2 cm, 6 cm

LONG ANSER TYPE QUESTIONS:

1. 2 cm **2.** 1 cm **3.** 10.95 cm **4.** 16 cm **5.** 54 cm **6.** x = 9 **7.** 30 cm

8. $\angle AQB = 70^o$, $\angle AMB = 110^o$ **9.** (i) 130^o, (ii) 25^o, (iii) 65^o **10.** 54^o **11.** 65^o

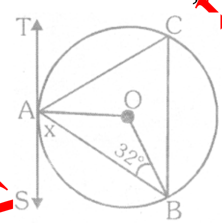
12. (a) 108^o, (b) 72^o **16.** 20.8 cm **18.** 21 cm **19.** 14 cm

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

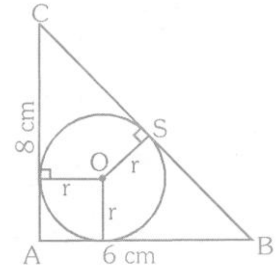
PREVIOUS YEARS BOARD QUESTIONS

VERY SHORT ANSWER QUESTIONS

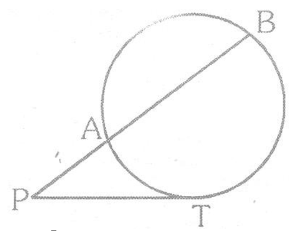
1. In the given figure, TAS is a tangent to the circle, with centre O, at the point A. If $\angle OBA = 32^o$, find the value of x. **[Delhi-1996C]**



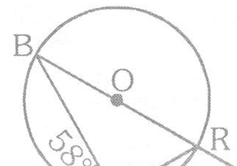
2. In the given figure, ABC is a right angled triangle right angled at A, with AB = 6cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of r, the radius of the inscribed circle. **[AI-1998]**



3. In the given figure, PT is tangent to the circle at T. If PA = 4 cm and AB = 5 cm, find PT. **[Delhi-19980]**

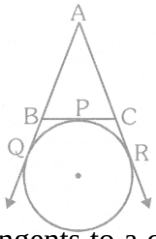


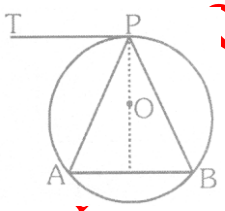
4. In the figure, O is the centre of the circle, PQ is tangent to the circle at A. If $\angle PAB = 58^o$, find $\angle ABQ$ and $\angle AQB$.



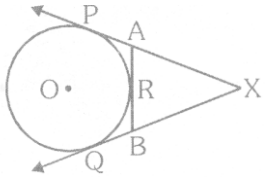
5. In figure, a circle touches the side BC of ΔABC at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of ΔABC.

6. A tangent PT is drawn parallel to a chord AB as shown in figure. Prove that APB is an isosceles triangle. **[Foreign – 2000]**

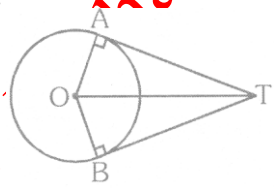




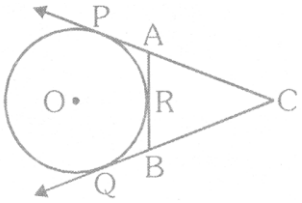
7. In figure, XP and XQ are two tangents to a circle with centre O from a point X outside the circle. ARB is tangent to circle at R. Prove that XA + AR = XB + BR. **[Delhi-2003]**



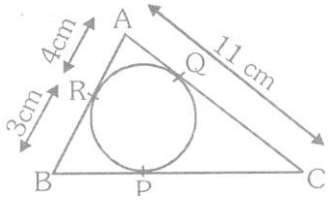
8. In fig, if $\angle ATO = 40^o$, find $\angle AOB$. **[AI-2008]**



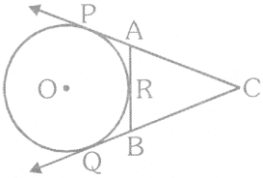
9. In fig., CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then find the length of BR. **[Delhi-2009]**



10. In fig., ΔABC is circumscribing a circle. Find the length of BC. **[AI-2009]**



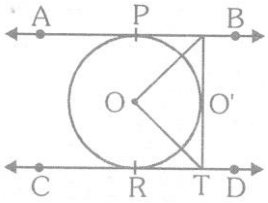
11. In fig., CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC. **[AI-2010]**



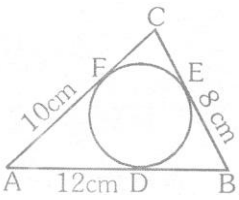
SHORT ANSWER TYPE QUESTIONS

1. If ΔABC is isosceles with AB = AC, prove that the tangent at A to the circumcircle of ΔABC is parallel to BC. **[AI-1998C]**

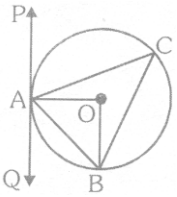
2. In figure, AB and CD are two parallel tangents to a circle with centre O. ST is tangent segment between the two parallel tangents touching the circle at Q. Show that $\angle SOT = 90^o$. **[AI-2000]**



3. A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm as shown in figure. Find AD, BE and CF. **[Delhi-2001]**



4. PAQ is a tangent to the circle with centre O at a point A as shown in figure. If $\angle OBA = 35^o$, find the value of $\angle BAQ$ and $\angle ACB$.

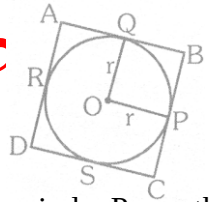


5. AB is diameter and AC is a chord of a circle such that $\angle BAC = 30^o$. If then tangent at C intersects AB produced in D, prove that BC = BD. **[Delhi-2003]**

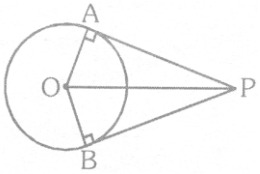
6. ABC is an isosceles triangle in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact.

OR

In the fig., a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^o$. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius (r) of the circle.

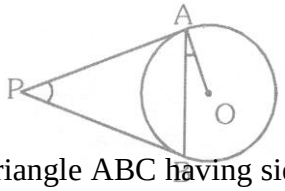


7. In fig., OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle. **[AI-2008]**



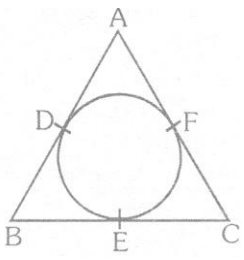
8. Prove that a parallelogram circumscribing a circle is a rhombus. **[Foreign-2008]**

9. Two tangents PA and PB are drawn to a circle with centre O from and external point P. Prove that $\angle APB = 2\angle OAB$.

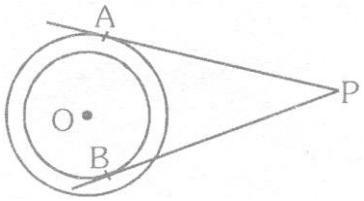


10. In fig., a circle is inscribed in a triangle ABC having side BC = 8 cm, AC = 10 cm and AB = 12 cm. Find AD, BE and CF

[Foreign–2009]

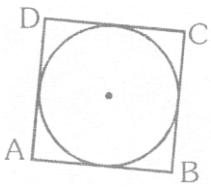


11. In fig., there are two concentric circles with centre O and of radii 5 cm and 3 cm. From an external Point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP. **[AI – 2010]**

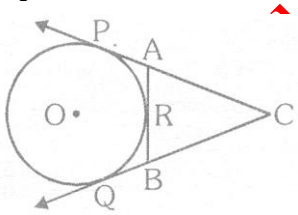


LONG ANSWER TYPE QUESTIONS

1. Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above, prove the following :
A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.
[Delhi-2008, AI-2009]

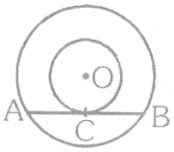


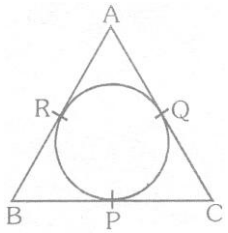
2. Prove that the lengths of the tangents drawn from an external point to a circle are equal. Using the above, do the following:
In the fig., TP and TQ are tangents from T to the circle with centre O and R is any point on the circle.
If AB is a tangent to the circle at R, prove that TA + AR = TB + BR. **[AI-2008]**



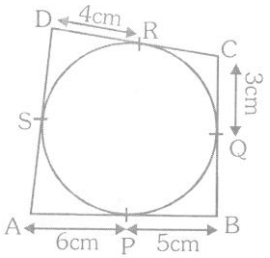
3. Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above do the following :
ABC is an isosceles triangle in which AB = AC, circumscribe about a circle as shown in the fig. Prove that the base is bisected by the point of contact. **[Foreign-2008]**

4. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following:
In fig., O is the centre of the two concentric circles. AB is a chord of the larger circle touching the small circle at C. Prove that AC = BC.
[AI-2009]





5. Prove that the length of the tangents drawn from an external point to a circle are equal. Using the above, do the following :
In fig, quadrilateral ABCD is circumscribing a circle. Find the perimeter of the quadrilateral ABCD.
[Foreign-2009]



VERY SHORT ANSWER TYPE QUESTIONS:

1. $x = 58^o$ **2.** r = 2 cm **3.** PT = 6 cm **4.** $32^o, 26^o$ **5.** 10 cm **8.** 100^o
9. 4 cm **10.** 10cm **11.** 7 cm

SHORT ANSWER TYPE QUESTIONS:

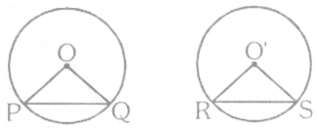
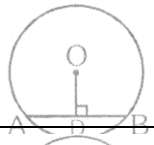
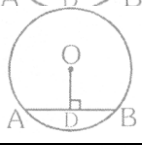
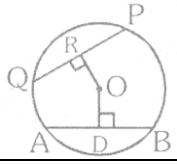
3. 7 cm, 5cm, 3 cm **4.** 55^o and 55^o **6.** 11 cm **10.** AD = 7 cm, BE = 5 cm and CF = 3 cm
11. $4\sqrt{10}$ cm

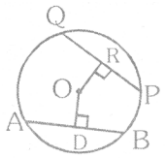
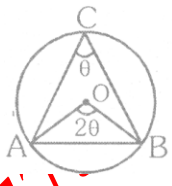
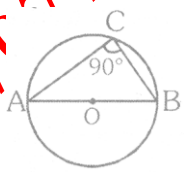
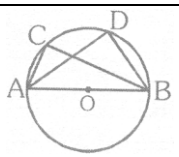
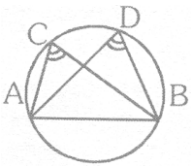
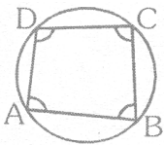
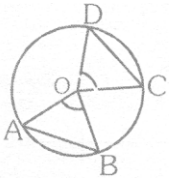
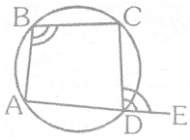
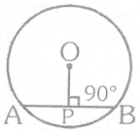
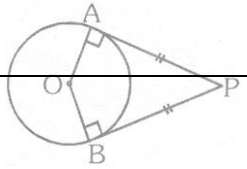
LONG ANSWER TYPE QUESTIONS:

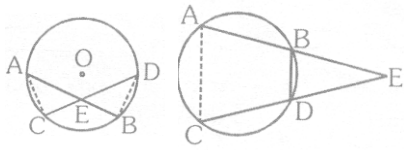
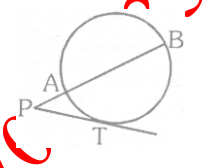
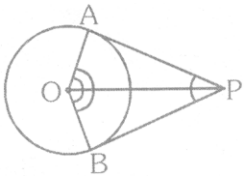
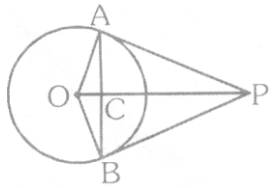
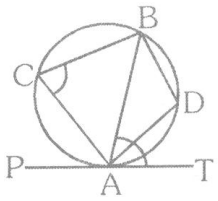
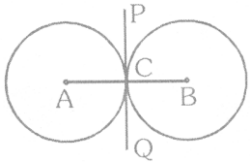
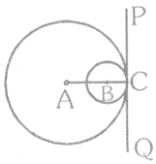
5. 36 cm

COMPETITION WINDOW

SOME IMPORTANT THEOREMS:

S. No.	Theorem	Diagram
1.	In a circle (or in congruent circles) equal chords are made by equal arcs. {OP = OQ} = {O'R = O'S} and $\overset{\frown}{PQ} = \overset{\frown}{RS}$ $\therefore$ PQ = RS	
2.	Equal arcs (or chords) subtend equal angles at the centre i.e., if $\overset{\frown}{PQ} = \overset{\frown}{AB}$ (or PQ = AB) $\therefore$ $\angle POQ = \angle AOB$	
3.	The perpendicular from the centre of a circle to a chord bisects the chord i.e., if OD $\perp$ AB $\therefore$ AB = 2AD = 2BD	
4.	The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord. $\because$ AD = DB $\therefore$ OD $\perp$ AB	
5.	Perpendicular bisector of a chord passes through the centre. i.e., if OD $\perp$ AB and AD = DB $\therefore$ O is the centre of the circle.	
6.	Equal chords of a circle (or of congruent circles) are equidistant from the centre. $\therefore$ AB = PQ	

	$\therefore$ OD = OR	
7.	Chords which are equidistant from the centre in a circle (or in congruent, circles) are equal. $\therefore$ OD = OR $\therefore$ AB = PQ	
8.	The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. m $\angle$AOB = 2m $\angle$ACB.	
9.	Angle in a semicircle is a right angle.	
10.	Angle in the same segment of a circle are equal i.e., $\angle$ACB = $\angle$ADB	
11.	If line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle. $\angle$ACB = $\angle$ADB $\therefore$ Points A, C, D, B are co cyclic i.e., lie on the circle	
12.	The sum of pair of opposite angles of a cyclic quadrilateral is 180^o $\angle$DAB + $\angle$BCD = 180^o and $\angle$ABC + $\angle$CDA = 180^o (converse of this theorem is also true)	
13.	Equal chords (or equal arcs) of a circle (or congruent circles subtend equal angles at the centre. AB = CD (or $\overset{\frown}{AB}$ = $\overset{\frown}{CD}$) $\therefore$ $\angle$AOB = $\angle$COD	
14.	If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. m $\angle$CDE = m $\angle$ABC	
15.	A tangent at any point of a circle is perpendicular to the radius through the point of contact. (converse of this theorem is also true)	
16.	The lengths of two tangents drawn from an external point to a circle are equal. i.e., AP = BP	

17.	If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E) then AE x BE = CE x DE	
18.	If PB be a secant which intersects the circle at A and B and PT be a tangent at T then PA . PB = $(PT)^2$	
19.	From an external point from which the tangents are drawn to the circle with centre O, then (a) They subtend equal angles at the centre. (b) They are equally inclined to the line segment joining the centre of that point. $\angle AOP = \angle BOP$ and $\angle APO = \angle BPO$	
20.	If P is an external point from which the tangents to the circle with centre O touch it at A and B then OP is the perpendicular bisector of AB. OP $\perp$ AB and AC = BC	
21.	**Alternate Segment Theorem :** If from the point of contact of a tangent, a chord is drawn then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram. $\angle BAT = \angle BCA$ and $\angle BAP = \angle BDA$	
22.	The point of contact of two tangents lies on the straight line joining the two centres. (a) When two circles touch externally then the distance between their centres is equal to sum of their radii i.e. AB = AC + BC (a) When two circles touch internally then the distance between their centres is equal to the difference between their radii i.e. AB = AC – BC	 

EXERCISE – 4 (FOR OLMPLADS]

CHOOSE THE CORRECT ONE

1. If the diagonals of cyclic quadrilateral are equal, then the quadrilateral is

(A) rhombus (B) square (C) rectangle (D) none of these

2. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is a

(A) rectangle (B) square (C) parallelogram (D) cyclic quadrilateral

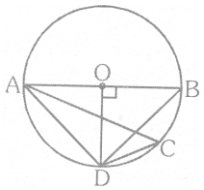
3. In the given figure, AB is the diameter of the circle. Find the value of $\angle ACD$:

(A) 30^o

(B) 60^o

(C) 45^o

(D) 25^o



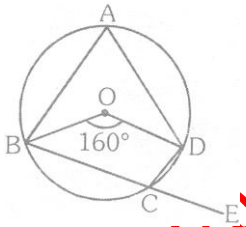
4. Find the value of $\angle DCE$:

(A) 100^o

(B) 80^o

(C) 90^o

(D) 75^o



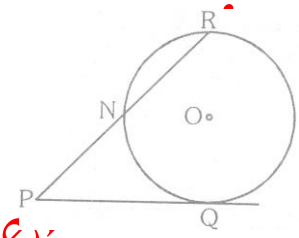
5. In the given figure, PQ is the tangent of the circle. Line segment PR intersects the circle at Nand R.PQ = 15 cm, PR = 25 cm, find PN:

(A) 15 cm

(B) 10 cm

(C) 9 cm

(D) 6 cm



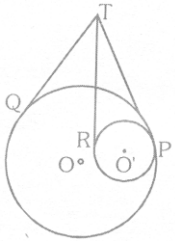
6. In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find TQ : TR

(A) 8 : 7

(B) 7 : 8

(C) 5 : 4

(D) 1 : 1



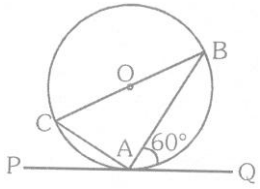
7. In the given figure, PAQ is the tangent. BC is the diameter of the circle. m $\angle BAQ = 60^o$, find m $\angle ABC$:

(A) 25^o

(B) 30^o

(C) 45^o

(D) 60^o



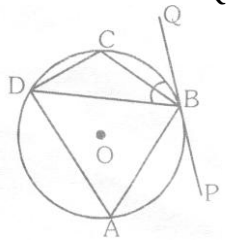
8. ABCD is a cyclic quadrilateral PQ is a tangent at B. If $\angle DBQ = 65^o$, then $\angle BCD$ is :

(A) 35^o

(B) 85^o

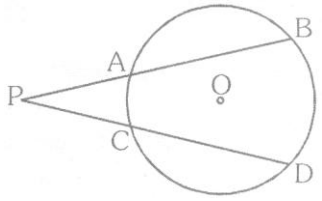
(C) 115^o

(D) 90^o

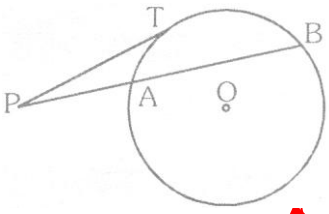


9. In the given figure, AP = 2 cm, BP = 6 cm and CP = 3 cm. Find DP:
(A) 6 cm
(B) 4 cm
(C) 2 cm
(D) 3 cm

10. In the given figure, AP = 3 cm, BA = 5 cm and CP = 2 cm. Find CD :
(A) 12 cm
(B) 10 cm
(C) 9 cm
(D) 6 cm



11. In the figure, tangent PT = 5 cm, PA = 4 cm, find AB :
(A) $\frac{7}{4}$ cm
(B) $\frac{11}{4}$ cm
(C) $\frac{9}{4}$ cm
(D) can't be determined

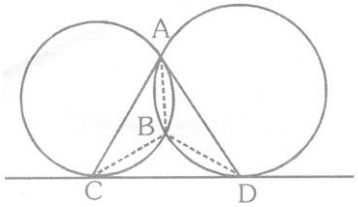


12. Two circles of radii 13 cm and 5 cm touch internally each other. Find the distance between their centres :
(A) 18 cm (B) 12 cm (C) 9 cm (D) 8 cm

13. Three circles touch each other externally. The distance between their centre is 5 cm. 6 cm and 7 cm. Find the radii of the circles :
(A) 2 cm, 3 cm, 4 cm (B) 3 cm, 4 cm, 1 cm
(C) 1 cm, 2.5 cm, 3.5 cm (D) 1 cm, 2 cm, 4 cm

14. If AB is a chord of a circle, P and Q are two points on the circle different from A and B, then:
(A) the angle subtended by AB at P and Q are either equal or supplementary .
(B) the sum of the angles subtended by AB at P and Q is always equal two right angles.
(C) the angles subtended at and Q by AB are always equal.
(D) the sum of the angles subtended at P and Q is equal to four right angles.

15. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B, then: $\angle$ CAD + $\angle$ CBD = ?
(A) 120^o
(B) 90^o
(C) 360^o
(D) 180^o



16. In a circle of radius 5 cm, AB and AC are the two chords such that AB = AC = 6 cm. Find the length of the chord BC.
(A) 4.8 cm (B) 10.8 cm (C) 9.6 cm (D) none of these

17. In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is :
(A) 23 cm (B) 30 cm (C) 15m cm (D) none of these

18. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :
(A) $\sqrt{3}:2$ (B) $\sqrt{3}:1$ (C) $\sqrt{5}:1$ (D) none of these

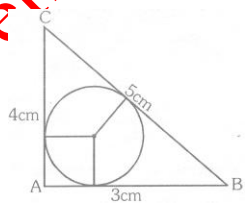
19. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the other circle which is outside the inner circle, is of length :
(A) $2\sqrt{2}\,cm$ (B $3\sqrt{2}\,cm$ (C) $2\sqrt{3}\,cm$ (D) $4\sqrt{2}\,cm$

20. Through any given set of four points P, Q, R, S it is possible to draw:
(A) atmost one circle (B) exactly one circle (C) exactly two circles (D) exactly three circles

21. The distance between the centers of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is:
(A) 4 cm (B) 6 cm (C) 8 cm (D) 10 cm

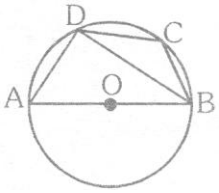
22. The number of common tangents that can be drawn to two given circles is at the most :
(A) 1 (B) 2 (C) 3 (D) 4

23. ABC is a right angled triangle AB = 3 cm, BC = 5 cm and AC = 4 cm, then the inradius of the circle is :
(A) 1 cm
(B) 1.25 cm
(C) 1.5 cm
(D) none of these

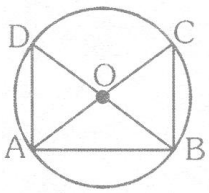


24. A circle has two parallel chords of lengths 6 cm and 8 cm. If the chords are 1 cm apart and the centre is on the same side of the chords, then a diameter of the circle is of length:
(A) 5 cm (B) 6 cm (C) 8 cm (D) 10 cm

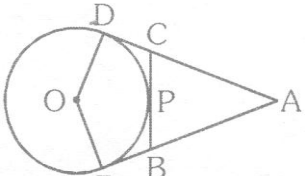
25. In the adjoining figure AB is a diameter of the circle and $\angle BCD = 130^o$. What is the value of $\angle ABD$?
(A) 30^o
(B) 50^o
(C0 40^o
(D) None of these



26. In the given figure O is the centre of the circle and $\angle BAC = 25^o$. then the value of $\angle ADB$ is :
(A) 40^o
(B) 55^o
(C) 50^o
(D) 65^o

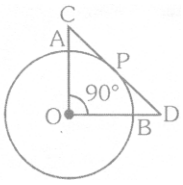


27. In the given circle O is the centre of the circle and AD, AE are the two tangents. BC is also a tangent, then:



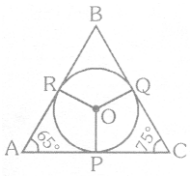
(A) AC + AB = BC
(B) 3AE = AB + BC + AC
(C) AB + BC + AC = 4AE
(D) 2AE = AB + BC + AC

28. In a circle O is the centre and ∠COD is right angle. AC = BD and CD is the tangent at P. What is the value of AC + CP, if the radius of the circle is 1 metre?



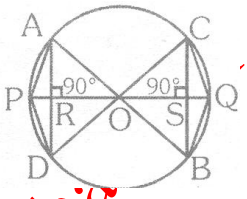
(A) 105 cm
(B) 141.4 cm
(C) 138.6 cm
(D) Can't be determined

29. In a triangle ABC, O is the centre of incircle PQR, ∠BAC = 65°, ∠BCA = 75°, find ∠ROQ :



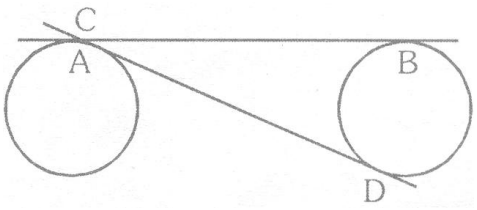
(A) 80°
(B) 120°
(C) 140°
(D) Can't be determined

30. In the adjoining figure O is the centre of the circle. ∠AOD = 120°. If the radius of the circle be 'r', then find the sum of the areas of quadrilaterals AODP and OBQC:



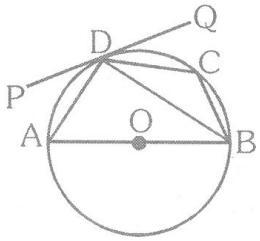
(A) $\frac{\sqrt{3}}{2}r^2$
(B) $3\sqrt{3}r^2$
(C) $\sqrt{3}r^2$
(D) None of these

31. There are two circles each with radius 5 cm. Tangent AB is 26 cm. The length of tangent CD is :



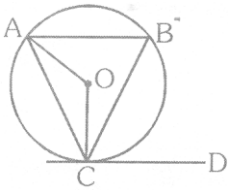
(A) 15 cm
(B) 21 cm
(C) 24 cm
(D) Can't be determined

32. In the adjoining figure O is the centre of the circle and AB is the diameter. Tangent PQ touches the circle at D. ∠*BDQ* 48°. Find the value of ∠DBA : ∠DCB :



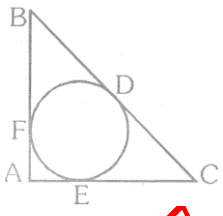
(A) $\frac{22}{7}$
(B) $\frac{7}{22}$
(C) $\frac{7}{12}$
(D) Can't be determined

33. In the given diagram O is the centre of the circle and CD is a tangent, ∠CAB and ∠ACD are supplementary to each other ∠OAC 30°. Find the value of ∠OCB :



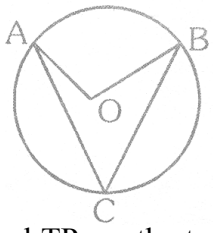
(A) 30°
(B) 20°
(C) 60°
(D) None of these

34. In the given diagram an incircle DEF is circumscribed by the right angled triangle in which AF = 6 cm and EC = 15 cm. Find the difference between CD and BD:



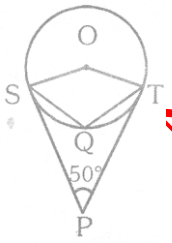
(A) 1 cm
(B) 3 cm
(C) 4 cm
(D) Can't be determined

35. In the adjoining figure 'O' is the centre of circle, $\angle CAO = 25^o$ and $\angle CBO = 35^o$. What is the value of $\angle AOB$?



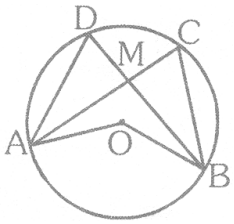
(A) 55^o
(B) 110^o
(C) 120^o
(D) Data insufficient

36. In the given figure 'O' is the centre of the circle SP and TP are the two tangents at S and T respectively. $\angle SPT$ is 50^o, the value of $\angle SQT$ is :



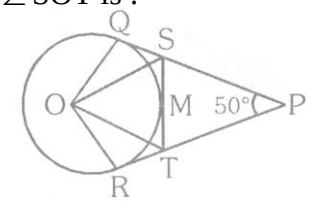
(A) 125^o
(B) 65^o
(C) 115^o
(D) None of these

37. In the given figure of circle, 'O' is the centre of the circle $\angle AOB = 130^o$. What is the value of $\angle DMC$?



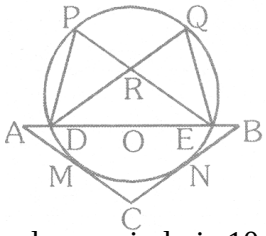
(A) 65^o
(B) 125^o
(C) 85^o
(D) Can't be determined

38. In the adjoining figure 'O' is the centre of the circle of the circle and PQ, PR and ST are the three tangents. $\angle QPR = 50^o$, then the value of $\angle SOT$ is :



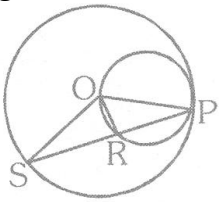
(A) 30^o
(B) 75^o
(C) 65^o
(D) Can't be determined

39. ABC is an isosceles triangle and AC, BC are the tangents at M and N respectively. DE is the diameter of the circle. $\angle ADP = \angle BEQ = 100^o$. What is value of $\angle PRD$?



(A) 60^o
(B) 50^o
(C) 20^o
(D) Can't be determined

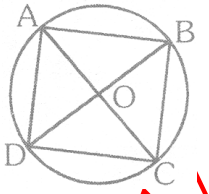
40. In the adjoining figure the diameter of the larger circle is 10 cm and the smaller circle touches internally the lager circle at P and passes through O, the centre of the larger circle. Chord SP cuts the smaller circle at R and OR is equal to 4 cm. What is the length of the chord SP?



(A) 9 cm
(B) 12 cm
(C) 6 cm

(D) $8\sqrt{2}$ cm

41. In the given figure ABCD is a cyclic quadrilateral DO = 8 cm and CO = 4 cm. AC is the angle bisector of $\angle$BAD. The length of AD is equal to the length of AB. DB intersects diagonal AC at O, then what is the length of the diagonal AC?'



(A) 20 cm
(B) 24 cm
(C) 16 cm
(D) None of these

OBJECTIVE					ANSEWER KEY						EXERCISE – 4				
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	C	D	C	B	C	D	B	C	B	B	C	D	A	A	D
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	B	B	D	A	C	B	A	D	C	D	D	B	C	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**				
Ans.	C	B	A	A	C	C	D	C	C	C	A				

Important Notes

CONSTRUCTIONS

★ **INTRODUCTION**

In class IX, we have discussed a number of constructions with the help of ruler and compass e.g. bisecting a line segment, bisecting an angle, perpendicular bisector of line segment, some more constructions of triangles etc. with their justifications. In this chapter we will discuss more constructions by using the knowledge of the earlier construction.

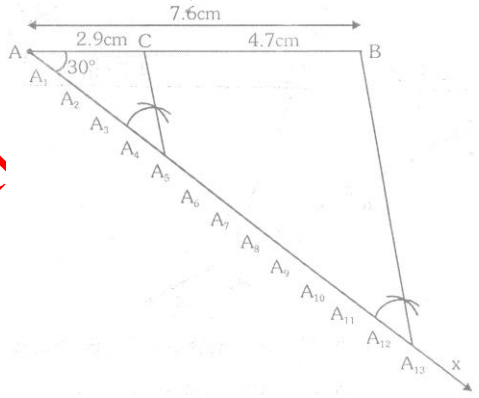
★ **DIVISION OF A LINE SEGMENT**

Let us divide the given line segment in the given ratio say 5 : 8. This can be done in the following two ways:

(i) Use of Basic Proportionality Theorem.

(ii) Constructing a triangle similar to a given triangle.

Construction – 1: Draw a segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts . (NCERT)



Steps of Constructions:

Step 1 : Draw any ray AX making an angle of 30^o with AB.

Step 2 : Locate 13 points : A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12} and A_{13} So that:

$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = \ldots\ldots = A_{11}\ A_{12} = A_{12}\ A_{13}$

Step 3 : Join B with A_{13}.

Step 4 : Through the point A_5, draw a line $A_5C \| A_{13}B$ such that $\angle AA_5C$ = corr. $\angle AA_{13}B$ intersecting AB at a point C.

Then AC : CB = 5 : 8.

Let us see how this method gives us the required division.

Since A_5C is parallel to A_{13} B.

Therefore $\dfrac{AA_5}{A_5A_{13}} = \dfrac{AC}{CB}$ (Basic Proportionality Theorem)

By construction, $\dfrac{AA_5}{A_5A_{13}} = \dfrac{5}{8}$

Therefore $\dfrac{AC}{CB} = \dfrac{5}{8}$

This given that C divides AB in the ratio 5 : 8.

By measurement, we find, AC = 2.9 cm, CB = 4.7 cm.

By Calculation: $AC = \dfrac{7.6\,x\,5}{13} = \dfrac{38}{13} = 2.9$

$BC = \dfrac{7.6\,x\,8}{13} = \dfrac{60.8}{13} = 4.67 = 4.7\,\text{cm}.$

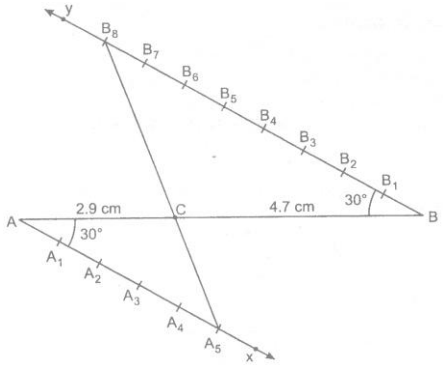
Alternative Solutions

Step 1 : Draw a line segment AB = 7.6 cm and to be divided in the ratio 5 : 8.

Step 2 : Draw any ray AX making an angle of 30^o with AB.

Step 3 : Draw a ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$. i.e. $\angle ABY$ = corr. $\angle BAX$.

Step 4 : Locate the points A_1, A_2, A_3, A_4, A_5, on AX and B_1, B_2, B_3, B_4, B_5, B_6, B_7, and B_8 on BY such that :



$AA_1 = A_1A_2 = \ldots\ldots = A_4A_5 = BB_1 = B_1B_2 = \ldots\ldots\ldots = B_6B_7 = B_7B_8$.

Step 5 : Join A_5B_8. Let it intersect AB at a point C. then AC : CB = 5 : 8.

Here ΔAA_5C is similar to ΔBB_8C

Then $\dfrac{AA_5}{BB_8} = \dfrac{AC}{BC}$

Since by construction, $\dfrac{AA_5}{BB_8} = \dfrac{5}{8}$ Therefore $\dfrac{AC}{CB} = \dfrac{5}{8}$

By measurement : AC = 2.9 cm, BC = 4.7 cm.

Constructions – 2 : Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle. (NCERT)

Sol. First all we are to construct a triangle ABC with given sides, AB = 6 cm, BC = 7 cm, CA = 5 cm. Given a triangle ABC, we are required to construct a triangle whose sides are $\frac{7}{5}$ of the corresponding sides of ΔABC.

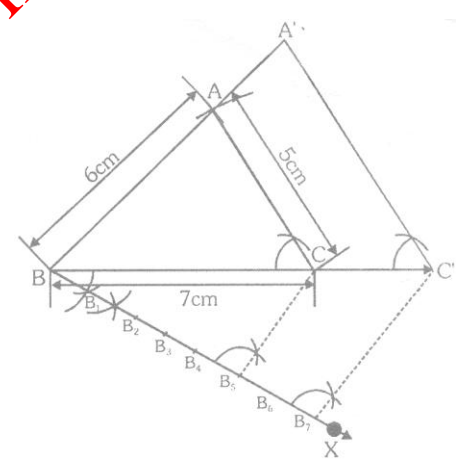
Steps of Construction :

Step 1 : Draw any ray BX making an angle of 30^o with the base BC of ΔABC on the opposite side of the vertex A.

Step 2 : Locate seven points B_1, B_2, B_3, B_4, B_5, B_6 and B_7 on BX so that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

[Note that the number of points should be greater of m and n in the scale factor $\frac{m}{n}$.]



Step 3 : Join B_5 (the fifth point) to C and draw a line through B_7 parallel to B_5C, intersecting the extended line segment BC at C'.

Step 4 : Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.

Then, A'B'C is the required triangle.

For justification of the construction.

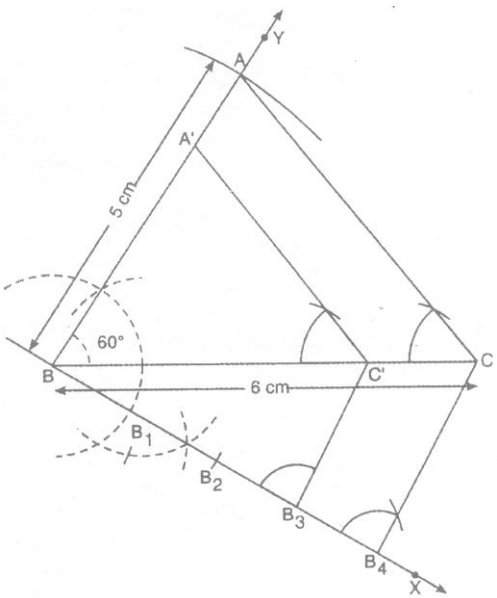
$\Delta ABC \approx \Delta A'BC'$

Therefore, $\dfrac{AB}{A'B} = \dfrac{AC}{A'C} = \dfrac{BC}{BC'}$

But $\dfrac{BC}{BC'} = \dfrac{BB_5}{BB_7} = \dfrac{5}{7}$

Therefore $\dfrac{A'B}{AB} = \dfrac{A'C}{AC} = \dfrac{BC'}{BC} = \dfrac{7}{5}$

Construction – 3 : Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^o$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC. (NCERT)



Sol. Given a triangle ABC, we are required to construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

Step of Constructions :

Step 1 : Draw a line segment BC = 6 cm.

Step 2 : At B construct $\angle CBY = 60^o$ and cut off AB = 5 cm, join AB and AC. ABC is the required Δ.

Step 3 : Draw any ray BX making an acute angle say 30^o with BC on the opposite side of the vertex A, $\angle CBX = 30^o$ downwards.

Step 4 : Locate four (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX, so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

Step 5 : Join B_4C and draw a line through B_3 (the 3rd point) parallel to B_4C to intersect BC at C'.

Step 6 : Draw a line through C' parallel to the line CA to intersect BA at A'.

Then A'BC' is the required triangle whose each side is $\frac{3}{4}$ times the corresponding sides of them ΔABC,

Let us now see how this construction gives the required triangle.

For justification of the construction.

$$\frac{BC'}{C'C} = \frac{3}{1}$$

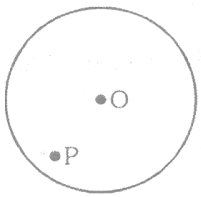
Therefore $$\frac{BC}{BC'} = \frac{BC' + C'C}{BC'} = \frac{BC'}{BC'} + \frac{C'C}{BC'} = 1 + \frac{1}{3} = \frac{4}{3}$$

$\Rightarrow$ BC' = $\frac{3}{4}$ BC, Also C'A' is parallel to CA.

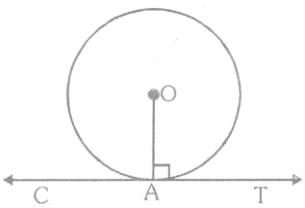
Therefore $\Delta A'BC' \approx \Delta ABC \Rightarrow \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$

★ **CONTRUCTION OF TANGENTS TO A CIRCLE**

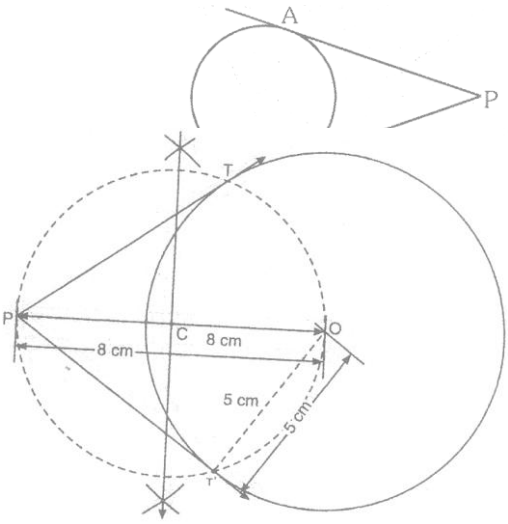
(a) If a point lies inside a circle, we can not draw any tangent to the circle i.e., No tangent is possible in this case



(b) If a point lies on the circle, then there is only one tangent to the circle at this point. The tangent to a circle at any point is perpendicular to the radius passing through the point of contact.



(c) Two tangents are drawn from an external point to circle, they are equal in length.



Construction 4 : Draw a circle of radius 5 cm. From a point 8 cm away from its centre, construct pair of tangents to the circle measure their lengths.

Sol.

Steps of Construction :

Step-1 : Draw a circle with radius 5 cm whose centre is O.

Step-2 : Take a point P at a distance 8 cm from the centre O such that OP = 8cm.

Step-3 : Bisect the line segment OP at the point C such that OC = CP = 4 cm.

Step-4 : Taking C as centre and OC as arc, draw a dotted circle to intersect the given circle at the points T and T'.

Step-5 : Join PT and PT'

PT and PT' are the required pair of tangents to the circle.

By measurement we obtain PT = PT' = 6.2 cm (Answer)

Verification: PT = PT' = $\sqrt{8^2 - 5^2} = \sqrt{64-25} = \sqrt{39} = 6.2$ cm (Answer)

Construction 5. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

(NCERT)

Sol.

Steps of Construction:

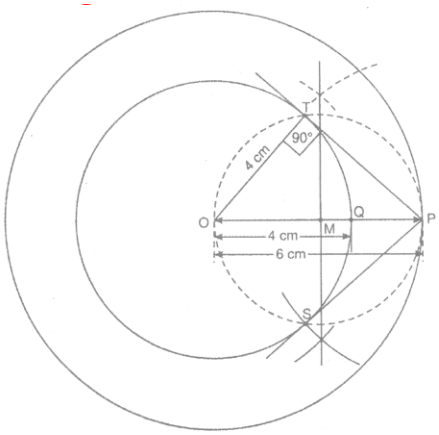
Step 1 : Draw two concentric circles with centre O and radii 4 cm and 6 cm such that OP = 6 cm, OQ = 4 cm.

Step 2 : Join OP and bisect it at M. i.e. M is the mid-point of OP i.e. OM = PM = 3 cm.

Step 3 : Taking M as centre with OM as radius draw a circle intersecting the smaller circle in two points namely T and S.

Step 4 : Join PT and PS.

PT and PS are the required tangents from a point P to the smaller circle, whose radius is 4 cm. By measurement: PT = 4.5 cm.



Verification. OTP is right Δ at T

$OP^2 = OT^2 + PT^2$

$6^2 = 4^2 + PT^2 \Rightarrow PT^2 = 36 - 16 = 20$

$PT = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5} = 2 \times 2.24 = 4.48$ cm

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

1. Draw a line segment of length 7.5 cm and divide it internally in the ratio 3 : 2. Measure the two parts.

2. Divide a line segment 8.8 cm long internally in the ratio 4 : 7 and measure the two parts.

3. Draw a line segment of length 13.5 cm and divide it internally in the ratio 2 : 3 : 4. Measure each part.

4. Construct a triangle with sides AB = 4 cm. BC = 5 cm and AC = 6 cm and then another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

5. Construct a triangle ABC whose sides are 4 cm, 5 cm, 7 cm. Construct another triangle similar to ΔABC and with sides $\frac{2}{3}$rd of the corresponding sides of triangle ABC.

6. Draw a right triangle in which the sides (other than hypotenuse) are of length 5 cm and 12 cm. Then construct another triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the given triangle.

7. Construct an isosceles triangle whose base is 6 cm and altitude 3 cm and then another triangle whose sides are $\frac{4}{5}$ times the corresponding sides of the isosceles triangle.

8. Draw a triangle ABC with sides BC = 8 cm, $\angle B = 30^o$, $\angle A = 45^o$. Then construct a triangle whose sides are $\frac{5}{4}$ times the corresponding sides of ΔABC.

9. Construct a ΔABC, whose perimeter is 10.5 cm and base angles are 60^o and 45^o. Construct another Δ whose sides are $\frac{4}{3}$ of the corresponding sides of the ΔABC.

10. Draw two tangents to a circle of radius 4 cm from a point P at a distance 7 cm from its centre. Also measure the length of the two tangents. Are they equal? Give reasons for your answer.

11. Construct a circle with radius equal to 3 cm. Draw two tangents to it inclined at an angle of 60^o at their point of intersection. Measure their lengths and verify the results by calculation.

12. Draw two tangents to a circle of radius 4 cm inclined at an angle of 45^o to each other.

13. Construct a tangent to a circle of radius 3 cm from a point on the concentric circle of radius 5 cm and measure its length. Also verify the measurement by actual calculation.

14. Draw a circle of radius 2.5 cm. Take two points P and Q on one of its extended diameter each at a distance of 7.5 cm from its centre. Draw tangents to the circle from these two points P and Q.

15. Draw a line segment AB of length 10 cm. Taking A as centre, draw a circle of radius 5 cm and taking B as centre, draw a circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

1. Draw a line segment AB = 7 cm. Divide it internally in the ratio of (i) 3 : 5, (ii) 5 : 3. **[2000 C]**

2. From a point P on the circle of radius 4 cm, draw a tangent to the circle with using the centre. Also write the steps of construction. **[2000]**

3. Draw circle of radius 4.5 cm. Take a point P on it. Construct a tangent at the point P without using the centre of the circle. Write the steps of construction. **[2001]**

4. Divide a line segment of length 5.6 cm internally in the ratio (i) 3 : 2 (ii) 2 : 3. **[2001]**

5. Construct a ΔABC in which base AB = 6 cm, $\angle C = 60^o$ and the median CD = 5 cm. Construct a $\Delta AB'C'$ similar to ΔABC with base AB' = 8 cm. **[2002]**

6. Draw a circle of radius 3.5 cm. From a point P on the circle draw a tangent to the circle without using its centre.. **[2003]**

7. Draw a circle of radius 5 cm. Take a point P on it, without using the centre of the circle, construct a tangent at the point P. Write the steps of construction also. **[2003]**

8. Draw a circle of diameter 12 cm. From a point P, 10 cm away from its centre, construct a pair of tangent to the circle. Measure the lengths of the tangent segments. **[2004 C]**

9. Draw a circle of radius 3.5 cm. Form a point P, outside the circle at a distance of 6 cm from the centre of circle, draw two tangent to the circle. **[2005]**

10. Construct a ΔABC in which AB = 6.5 cm, $\angle B = 60^o$ and BC = 5.5 cm. Also construct a triangle AB'C' similar to ΔABC, whose each side is $\frac{3}{2}$ of the corresponding side of the ΔABC. **[Delhi-2008]**

11. Draw a ΔABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^o$. Construct a $\Delta AB'C'$ similar to ΔABC such that sides of $\Delta AB'C'$ are $\frac{3}{4}$ of the corresponding sides of ΔABC. **[AI-2008]**

12. Draw a right triangle in which the sides containing the right angle are 5 cm and 4 cm. Construct a similar triangle whose sides are $\frac{5}{3}$ times the sides of the above triangle. **[Foreign-2008]**

13. Construct a ΔABC in which BC = 6.5 cm, AB = 4.5 cm and $\angle ABC = 60^o$. Construct a triangle similar to this triangle whose sides are $\frac{3}{4}$ of the corresponding sides of triangle ABC. **[Delhi-2008]**

14. Draw a right triangle in which sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle. **[AI-2009]**

15. Draw a circle of radius 3 cm. From a point P, 6 cm away from it's centre, construct a pair of tangents to the circle. Measure the lengths of the tangents. **[Foreign-2009]**

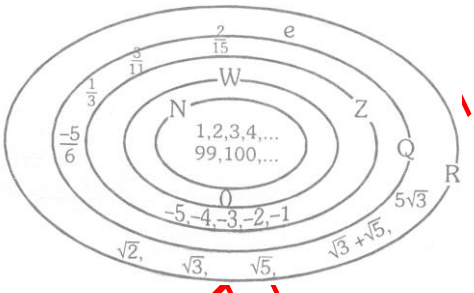
16. Construct a triangle ABC in which AB = 8 cm, BC = 10 cm and AC = 6 cm. Then construct another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of ΔABC. **[AI-2010]**

17. Construct a triangle ABC in which BC = 9 cm, $\angle B = 60^o$ and AB = 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of ΔABC. **[AI-2010]**

18. Construct a triangle ABC in which BC = 8 cm, $\angle B = 60^o$ and $\angle C = 45^o$. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC. **[AI-2010]**

REAL NUMBERS

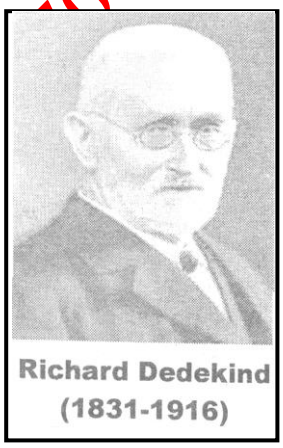
★ INTRODUCTION



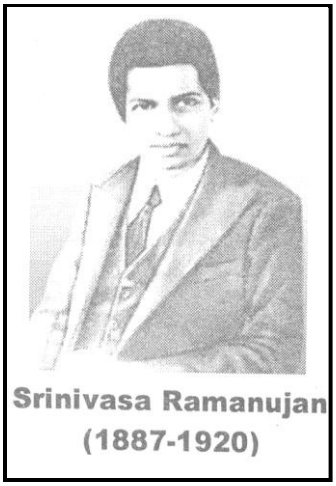
"God gave us the natural number, all else is the work of man". It was exclaimed by Leopold Kronecker (1823-1891). The reputed German Mathematician. This statement reveals in a nut shell the significant role of the universe of numbers played in the evolution of human though.

N	:	The set of natural number,
W	:	The set of whole numbers,
Z	:	The ser of Integers ,
Q	:	The set of rationales,
R	:	The set of Real Numbers.

★ HISTORICAL FACTS


Richard Dedekind
(1831-1916)

Dedekind was the first modern mathematician to publish in 1872 the mathematically rigorous definition of irrational numbers. He gave explanation of their place in the real Numbers System. He was able to demonstrate the completeness of the real number line. He filled in the "holes' in the system of Rational numbers with irrational Numbers. This innovation the made Richard Dedekind an immortal figure in the history of Mathematics.


Srinivasa Ramanujan
(1887-1920)

Srinivasa Ramanujan (1887-1920) was one of the most outstanding mathematician that India produced. He worked on history of Numbers and discovered wonderful properties of numbers. He stated intuitively many complicated result in mathematics. Once a great mathematician Prof. Hardy come of India to see Ramanujan. Prof. Hardy remarked that the he has traveled in a taxi with a rather dull number viz. 1729. Ramanujan jumped up and said, Oh! No. 1729 is very interesting number. It is the smallest number which can be expressed as the sum of two cubes in two different ways.

viz $1729 = 1^3 + 12^3$,

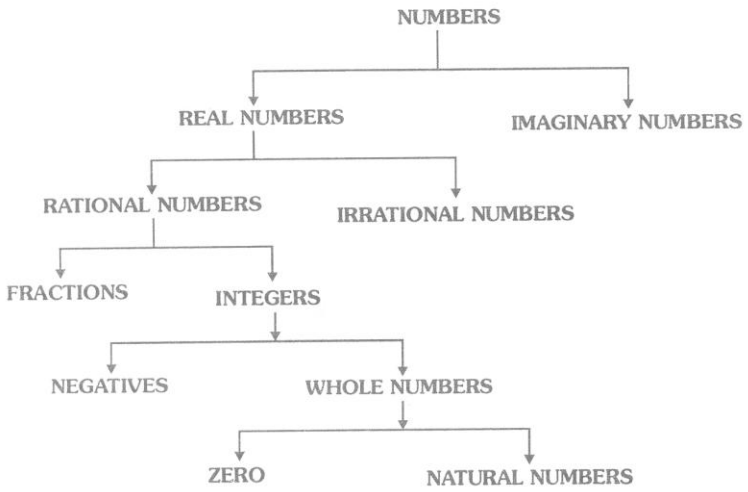
$1729 = 9^3 + 10^3$,

$\Rightarrow$ $1729 = 1^3 + 12^3 = 9^3 + 10^3$

★ RECALL

In our day to life, we deal with different types of numbers which can be broadly classified as follows.

CLASSIFICATION OF NUMBERS



(i) **Natural numbers (N) :** N + {1, 2, 3, 4… ∞ }

Remark :
(i) The set N is infinite i.e. it has unlimited members.
(ii) N has the smallest element namely '1'.
(iii) N has no largest element. i.e., give me any natural number, we can find the bigger number from the given number.
(iv) N does not contain '0' as a member. i.e., '0' is not a member of the set N.

(ii) **Whole numbers (W)** W = {0, 1, 2, 4… ∞ }

Remark :
(i) The set of whole number is infinite (unlimited elements)
(ii) This set has the smallest members as '0'. i.e. '0' the smallest whole number. i.e., set W contain '0' as a member.
(iii) The set of whole numbers has no largest member.
(iv) Emery natural number is a whole number.
(v) Non-zero smallest whole number is '1'.

(iii) **Integers (I or Z) :** I or Z = {- ∞ …-3, -2, -1, 0, +2, +3…+ ∞ }
Positive integers : {1, 2, 3…}, **Negative integers :** {…. -4, -3, -2, -1}

Remark :
(i) This set Z is infinite .
(ii) It has neither the greatest nor the lest element.
(iii) Every natural number is an integer.
(iv) Every whole number is an integer.
(iv) The set of non-negative integer = {0, 1, 2, 3, 4,…..}
(v) The set of non-positive integer = {…….-4, -3, -2, -1, 0}

(iv) **Rational numbers :-** These are real numbers which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Ex. $\frac{2}{3}, \frac{37}{15}, \frac{-17}{19}, -3, 0, 10, 4.33, 7.123123123\ldots\ldots\ldots\ldots$

Remark :

(i) Every integer is a rational number.

(ii) Every terminating decimal is a rational number .

(iii) Every recurring decimal is a rational number.

(iv) A non-terminating repeating decimal is called a recurring decimal.

(v) Between any two rational numbers there are an infinite number of rational numbers. This property is known as the density rational numbers.

(vi) If a and b are two rational numbers then $\frac{1}{2}(a+b)$ lies between a and b.

$$a < \frac{1}{2}(a+b) < b$$

n rational number between two different rational numbers a and b are :

$$a+\frac{(b-a)}{n+1}; a+\frac{2(b-a)}{n+1}; a+\frac{3(b-a)}{n+1}; a+\frac{4(b-a)}{n+1}; \ldots\ldots\ldots a+\frac{n(b-a)}{n+1};$$

(vii) Every rational number can be represented either as a terminating decimal or a non-termination repeating (recurring) decimals.

(viii) Types of rational numbers :- (a) Terminating decimal numbers and
(b) Non-termination repeating (recurring) decimal numbers

(v) Irrational numbers :- A number is called irrational number , if it can not be written in the form $\frac{p}{q}$, where p & q are integers and $q \neq 0$. All Non-terminating & Non-repeating decimal numbers are Irrational numbers .

Ex. $\sqrt{2}, \sqrt{3}, \ 3\sqrt{2}, \ 2+\sqrt{3}, \ \sqrt{2+\sqrt{3}}, \ \ \pi, \ e, \ etc$

(vi) Real numbers :- The totality of rational numbers and irrational numbers is called the set of real numbers i.e. rational numbers and irrational numbers taken together are called real numbers .
Every real number is either a rational number or an irrational number.

★ NATURE OF THE DECIMAL EXPANSION OF RATIONAL NUMBERS

Theorem -1 : Let x be a rational number whose decimal expansion terminates. Then we can express x in the form $\frac{p}{a}$, where p and q are co-primes, and the prime factorisation of q is of the form $2^m \times 5^n$, where m, n are non-negative integers.

Theorem-2 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is the $2^m \times 5^n$, where m, n are non-negative integers. Then , x has a decimal expansion which terminates.

Theorem-3 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^m \times 5^m$, where m, n are non-negative integers. Then , x has a decimal expansion which is non-terminating repeating

Ex. (i) $\frac{189}{125} = \frac{189}{5^3} = \frac{189}{2^0 \times 5^3}$

we observe that prime factorization of the denominators of these rational numbers are of the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, $\frac{189}{125}$ has terminating decimal expansion.

(ii) $\frac{17}{6} = \frac{17}{2 \times 3}$

we observe that the prime factorization of the denominator of these rational numbers are not of the form $2^m \times 5^n$, where m, n are non-negative integers. Hence $\frac{17}{6}$ has non-terminating and repeating decimal expansion

(iii) $\frac{17}{8} = \frac{17}{2^3 \times 5^0}$

So, the denominator 8 of $\frac{17}{8}$ is of the form $2^m \times 5^n$, where m, n are non-negative integers. Hence $\frac{17}{8}$ has terminating decimal expansion.

(iv) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Clearly, 455 is not of the form $2^m \times 5^n$, So, the decimal expansion of $\frac{64}{455}$ is non-terminating repeating.

★ PROOF OF IRRATIONALITY OF $\sqrt{2}, \sqrt{3}, \sqrt{5}, \ldots\ldots\ldots$

Ex.1 Prove that $\sqrt{2}$ is not a rational number or there is no rational whose square is 2. (CBSE (outside Delhi) 2008).

Sol. Let us find the square root of 2 by long division method as shown below.

	1.414215
1	2.000000000000
+1	1
24	100
4	96
281	400
+1	281
2824	11900
+4	11296
28282	60400
+2	56564
282841	383600
+1	282841
2828423	10075900
3	8485269
28284265	159063100
+5	141421325
28284270	17641775

$\sqrt{2} = 1.414215$

Clearly, the decimal representation of $\sqrt{2}$ is neither terminating nor repeating.
We shall prove this by the method of contradiction .

If possible, let us assume that $\sqrt{2}$ is a rational number.

Then $\sqrt{2} = \frac{a}{b}$ where a, b are integers having no common factor other than 1.

$\Rightarrow \left(\sqrt{2}\right)^2 = \left(\frac{a}{b}\right)^2$ (squaring both sides)

$2 = \frac{a^2}{b^2}$

$a^2 = 2b^2$

$\Rightarrow$ 2 divides a^2

$\Rightarrow$ 2 divides a

Therefore let $a = 2c$ for some integer c.

$\Rightarrow$ $a^2 = 4c^2$.

$\Rightarrow$ $2b^2 = 4c^2$

$\Rightarrow \quad b^2 = 2c^2$

$\Rightarrow$ 2 divides b^2

$\Rightarrow$ 2 divides b

Thus, 2 is a common factor of a and b.

But, it contradicts our assumption that a and b have no common factor other than 1.

So, our assumption that $\sqrt{2}$ is a rational, is wrong.

Hence, $\sqrt{2}$ is irrational.

Ex.2 Prove that $\sqrt[3]{3}$ is irrational .

Sol. Let $\sqrt[3]{3}$ be rational $= \frac{p}{q}$, where p and q $\in$ Z and p, q have no common factor except 1 also q > 1.

$\therefore \frac{p}{q} = \sqrt[3]{3}$

Cubing both sides

$\frac{p^3}{q^3} = 3$

Multiply both sides by q^2

$\frac{p^3}{q} = 3q^2$, Clearly L.H.S is rational since p, q have no common factor.

$\therefore$ p^3, q also have no common factor while R.H.S. is an integer .

$\therefore$ L.H.S. $\neq$ R.H.S. which contradicts our assumption that $\sqrt[3]{3}$ is Irrational .

Ex. 3 Prove that $2+\sqrt{3}$ is irrational . **[Sample paper (CBSE) 2008]**

Sol. Let $2+\sqrt{3}$ be a rational number equals to r

$\therefore \quad 2+\sqrt{3} = r$

$\sqrt{3} = r-2$

Here L.H.S. is an irrational number while R.H.S. r – 2 is rational. $\therefore$ S.H.S. $\neq$ R.H.S

Hence it contradicts our assumption that $2+\sqrt{3}$ is rational .

$\therefore 2+\sqrt{3}$ is irrational.

Ex.4 Prove that $\sqrt{2}+\sqrt{3}$ is irrational.

Sol. Let $\sqrt{2}+\sqrt{3}$ be rational number say 'x' $\Rightarrow x = \sqrt{2}+\sqrt{3}$

$x^2 = 2+3+2\sqrt{3}.\sqrt{2} = 5+2\sqrt{6}$

$\Rightarrow \quad x^2 = 5+2\sqrt{6} \Rightarrow \sqrt{6} = \frac{x^2-5}{2}$

As x, 5 and 2 are rationales $\Rightarrow \frac{x^2-5}{2}$ is a rational number .

$\Rightarrow \sqrt{6} = \frac{x^2-5}{2}$ is a rational number

Which is contradiction of the fact that $\sqrt{6}$ is a irrational number.

Hence our supposition is wrong $\Rightarrow \sqrt{2}+\sqrt{3}$ is an irrational number .

★ **EUCLID'S DIVISION LEMMA OR EUCLID'S DIVISION ALGORITHM**

For any two positive integers **a** and **b** there exist unique integers q and r such that

A = bq + r, where 0 ≤ r < b.

Let us consider a = 217, b = 5 and make the division of 217 by 5 as under :

Divined

↓

Divisor → 5) 217(43 ← **Quotient**

20

17

15

2 ← **Remainder**

i.e.

Dividends	=	**Divisor**	×	**Quotient**	+	**Remainder**
(a)	=	**(b)**	×	**(q)**	+	**(r)**

e.g. **(i)** Consider number 23 and 5, then :

$23 = 5 \times 4 + 3$

Comparing with a = bq + r

we get, a = 23, b = 5, q = 4, r = 3 and 0 ≤ r < b (as 0 < 3 < 5)

(ii) Consider positive integers 18 and 4

$18 = 4 \times 4 + 2$

For 18 (= a) and 4 (= b) we have q = 4, r = 2 and 0 ≤ r < b

In the relations a = bq + r, where 0 ≤ r < b is nothing but a statement of the long division of number a by b in which q is the quotient obtained and r is the remainder.

Ex.5 **Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m +1 for some integer m.**

Sol. Let **a** and **b** are two positive integers such that **a** is greater than **b**, then :

a = bq + r ; where q and r are also positive integers and 0 ≤ r < b

Taking b = 3, we get :

a = 3q7 + r ; where 0 ≤ r < 3

⇒ The value of positive integer **a** will be

3q + 0, 3q + 1 or 3q + 2

i.e., 3q, 3q + 1or 3q + 2

Now we have to show that square of positive integers 3q, 3q + 1 and 3q + 2 can be expressed as 3m or 3m +1 for some integer m.

∴ **Square of 3q = $(3q)^2$**

= $9q^2 = 3(3q^2) = 3m$; where m is some integer and $m = 3q^2$

Square of 3q + 1 = (3q + 1)2

= $9q^2 = 6q + 1$

= $3(3q^2 + 2q) + 1 = 3m + 1$ for some integer and m $3q^2 + 2q$.

Square of 3q + 2 = $(3q + 2)^2$

= $9q^2 = 12q + 4$

= $9q^2 = 12q + 3 + 1$

= $3(3q^2 + 4q + 1) + 1 = 3m + 1$ for some integer and $m = 3q^2 + 4q + 1$.

∴ The square of any positive integer is either of the form 3m or 3m +1 for some integer m.

Ex.6 **Show that one and only out of n; + 2 or n + 4 is divisible by 3, where is any positive integer.**

Sol. Consider any two positive integers **a** and **b** such that **a** is grater than **b**, then according to Euclid's division algorithm –

a = bq + r ; where q and r positive integers and $0 \leq r < b$

Let a = n and b = 3, then

a = bq + r $\Rightarrow$ n = 3q r ; where $0 \leq r < 3$.

r = 0 $\Rightarrow$ n = 3q + 0 = 3q

r = 1 $\Rightarrow$ n = 3q + 1

and r = 2 $\Rightarrow$ n = 3q + 2

if n = 3q ; **n is divisible by 3**

If n = 3q +1 ; then n + 2 = 3q + 1 + 2

= 3q +3 ; which is divisible by 3

$\Rightarrow$ **n + 2 is divisible by 3**

If n = 3q + 2 ; then n + 4 = 3q + 2 + 4

= 3q + 6 ; which is divisible by 3

$\Rightarrow$ **n + 4 is divisible by 3**

Hence , if n is any positive integer, then one and only one out n, n + 2 or n + 4 is divisible by 3.

★ **APPLICATION OF EUCLID'S DIVISION LEMMA FOR FINDING H.C.F. OF POSITIVE INTEGERS**

Algorithm :

Consider positive integers 418 and 33

Step. (a) Taking bitter number (418) as a and smaller number (33) as b.

Express the numbers are a = bq + r

418 = 33 × 12 +22

Step. (b) Now taking the divisor 33 and remainder 22, apply the Euclid's division method to get.

33 = 22 × 1 + 11 [Expressing as a = bq + r]

Step. (c) Again with new divisor 22 and new remainder 11, apply the Euclid's division algorithm to get

22 = 11 × 2 + 0

Step. (d) Since, the remainder = 0 so we can not proceed further.

Step. (e) The last divisor is 11 and we say H.C.F. of 418 and 33 = 11

Ex.7 Use Euclid's algorithm to find the HCF of 4052 and 12576.

Sol. Using a = bq + r, where $0 \leq r < b$.

Clearly, 12576 > 4052 [a = 12576, b = 4051]

$\Rightarrow$ 12576 = 4051 × 3 + 420

$\Rightarrow$ 4052 = 420 × 9 + 272

$\Rightarrow$ 402 = 272 × 1 + 148

$\Rightarrow$ 272 = 148 × 1 + 124

$\Rightarrow$ 148 = 124 × 1 + 24

$\Rightarrow$ 124 = 24 × 5 + 4

$\Rightarrow$ 24 = 4 × 6 + 0

The remainder at this stage is 0. So, the divisor at this stage, i.e., 4 is the HCF of 12576 and 4052.

Ex.8 Find the HCF of 1848, 3058 and 1331.

Sol. Two numbers 1848 and 3058 , where 3058 > 1848

$3058 = 1848 \times 1 + 1210$

$1848 = 1210 \times 1\ 638$ [Using Euclid's division algorithm to the given number 1848 and 3058]

$1210 = 638 \times 1\ 572$

$638 = 572 \times 1 + 66$

$527 = 66 \times 8 + 44$

$66 = 44 \times 1 + 22$

$44 = \boxed{22} \times 2 + 0$

Therefore HCF of 1848 and 3058 is 22.

HCF (1848 and 3058) = 22

Let us find the HCF of the numbers 1331 and 22.

$1331 = 22 \times 60 + 11$

$22 = \boxed{11} \times 2 + 10$

$\therefore$ HCF of 1331and 22 is 11

$\Rightarrow$ HCF (22, 1331) = 11

Hence the HCF of the given numbers 1848, 3058 and 1331 is 11.

HCF (1848, 3058, 1331) = 11

Ex.9 Using Euclid's division, find the HCF of 56, 96 and 404 **[Sample paper (CBSE) - 2008]**

Sol. Using Euclid's division algorithm, to 56 and 96.

$96 = 56 \times 1 + 40$

$56 = 40 \times 1 + 16$

$40 = 16 \times 2 + 8$

$16 = \boxed{8} \times 2 + 0$

Now to find HCF of 8 and 404

We apply Euclid's division algorithm to 404 and 8

$404 = 8 \times 50 + 4$

$8 = \boxed{4} \times 2 + 0$

Hence 4 is the HCF of the given numbers 56, 96 and 404.

★ THE FUNDAMENTAL THEOREM OF ARITHMETIC

Statement – "Every composite number can be factorized as a product of prime numbers in a unique way, except for the order in which the prime numbers occur.

e.g. **(i)** $90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

(ii) $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$

(iii) $12600 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$

In general, a composite number is expressed as the product of its prime factors written in ascending order of their values.

COMPETITION WINDOW

NUMBER OF FACTORS OF A NUMBER

To get number of factors (or divisors) of a number N, express N as

$N = a^p . b^q . c^r . d^s$……(a, b, c, d are prime numbers and p, q, r, s are indices)

Then the number of total divisors or factors of N = (p + 1) (q + 1) (r + 1) (s + 1) ……

Eg. $540 = 2^2 \times 3^3 \times 5^1$

$\therefore$ total number of factors of 540 = (2 + 1) (3 + 1) (1 + 1) = 24

SUM OF FACTORS OF A NUMBER

The sum of all factors of $N = \dfrac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)(d^{s+1}-1)}{(a-1)(b-1)(c-1)()d-1}$

Eg. $270 = 2 \times 3^3 \times 5$

$\therefore$ Sum of factors of $270 = \dfrac{(2^{1+1}-1)(3^{3+1}-1)(5^{1+1}-1)}{(2-1)(3-1)()5-1} = \dfrac{3\times 80\times 24}{1\times 2\times 4} = 720$

PRODUCT OF FACTORS

The product of factors of composite number $N = N^{n/2}$, where n is the total number of factors of N.

Eg. $360 = 2^3 \times 3^2 \times 5^1$

$\therefore$ No. of factors of 360 = (3 + 1) (2 + 1) (1 + 1) = 24

Thus, the product of factors = $(360)^{24/2} = (360)^{12}$

NUMBER OF ODD FACTORS OF A NUMBER

To get the number of odd factors of a number N, express N as

$N = (p_1^a \times p_2^b \times p_3^c \times .....) \times (e^x)$

(where p^1, p^2, p^3……are the odd prime factors and e is the even prime factor)

Then the total number of odd factors = (a + 1) (b + 1) (c + 1)…….

Eg. $90 = 2^1 \times 3^2 \times 5^1$

$\therefore$ Total number of odd factors of 90 = (2 + 1)(1 + 1) = 6

NUMBER OF EVEN FACTORS OF A NUMBER

Number of even factors of a number = Total number of factors – Total number of odd factors.

NUMBER OF WAYS TO EXPRESS A NUMBER AS A PRODUCT OF TWO FACTORS

Let n be the number of total factors of a composite number .

Case – 1 : If the composite number is not a perfect square then number of ways of expressing the composite number as a product of two factors $= \dfrac{n}{2}$

Case – 2 : If the composite number is a perfect square then

(a) Number of ways of expressing the composite number as a product of two factors $= \dfrac{n+1}{2}$

(b) Number of ways of expressing the composite number as a product of two distinct factors $= \dfrac{(n-1)}{2}$

★ **USING THE FUNDAMENTAL THEOREM OF ARITHMETIC TO FIND H.C.F. AND L.C.M.**

For any two number a and b.

(a) L.C.M. (Least common multiple) = Product of each prime factor with highest powers

$$\text{L.C.M. (a,b)} = \frac{\text{Product of the numbers or } (a \times b)}{\text{H.C.F. (a, b)}}$$

(b) H.C.F. (Highest common factor) = Product of common prime factor with lowest power.

$$\text{H.C.F. (a, b)} = \frac{\text{Product of the numbers or } (a \times b)}{\text{L.C.M. (a, b)}}$$

Remark : The above relations hold only for two numbers.

COMPETITION WINDOW

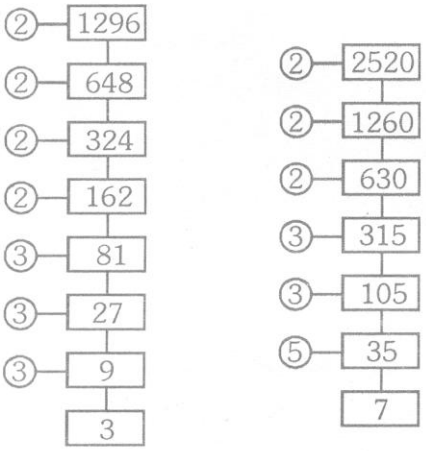
For any three positive integers p, q, r –
HCF (P, q, r) × LCM (p, q, r) ≠ p × q × r
However , the following results hold good for the three positive integers p, q and r :

$$\text{LCM (p, q, r)} = \frac{\text{p.q.r.HCF (p, q, r)}}{\text{HCF (p, q) . HCF (q, r) . HCF (p, r)}} \qquad \text{HCF (p, q, r)} = \frac{\text{p.q.r.LCM (p, q, r)}}{\text{LCM (p, q) . LCM (q, r) . LCM (p, r)}}$$

Ex.10 Find the L.C.M. and H.C.F. of 1296 and 2520 by applying the fundamental theorem of arithmetic method i.e. using the prime factorisation method.

Sol. $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times = 2^4 \times 3^4$

$2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2^3 \times 3^2 \times 5 \times 7$



L.C.M. = $2^4 \times 3^4 \times 5 \times 7 = 45360$

H.C.F. = $2^3 \times 3^2 \times = 72$

Ex.11 Given that H.C.F. (306, 657) = 9. Find L.C.M. (306, 657)

Sol. H.C.F. (306, 657) = 9 means H.C.F. of 306 and 657 = 9

Required L.C.M. (306, 657) means required L.C.M. of 306 and 657.

For any two positive integers ;

$$\text{their L.C.M.} = \frac{\text{Product to the number}}{\text{Their H.C.F.}}$$

$$\text{i.e., L.C.M. (306, 657)} = \frac{306 \times 657}{9} = 22,338$$

Ex.12 Given that L.C.M. (150, 100) = 300, find H.C.F. (150, 100)

Sol. L.C.M. (150, 100) = 300

$\Rightarrow$ L.C.M. of 150 and 100 = 300

Since, the product of number 150 and 100 = 150 × 100

$$\frac{\text{Product of 150 and 100}}{\text{L.C.M.(150. 100)}} \qquad \frac{150 \times 100}{300}$$

And , we know : H.C.F. (150, 100) = = = 50

Ex.13 Explain why 7 × 13 +13 are composite numbers :

Sol. (i) Let $7 \times 11 \times 13 + 13 = (7 \times 11 + 1) \times 13$

$= (77 + 1) \times 13 = 78 \times 13 \Rightarrow 7 \times 11 \times 13 + 13 = 2 \times 3 \times 13 \times 13$

$= 2 \times 3 \times 13^2$ is a composite number as powers of prime occur.

COMPETITION WINDOW

HCF AND LCM OF FRACTIONS

HCF of Fractions : The greatest common fraction is called the CHF of the given fractions.

$$\text{HCf of fractions} = \frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$$

For example : The HCF of $\frac{4}{3}, \frac{4}{9}, \frac{2}{15}, \frac{36}{21} = \frac{HCF \ \ of \ \ 4,4,2,36}{LCM \ \ of \ \ 3,9,15,21} = \frac{2}{315}$

Called the LCM of the fractions.

$$\text{LCM of fractions} = \frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$$

For example : The LCM of $\frac{4}{3}, \frac{4}{9}, \frac{2}{15}, \frac{36}{21} = \frac{LCM \ \ of \ \ 4,4,2,36}{HCF \ \ of \ \ 3,9,15,21} = \frac{36}{3} = 12$

HCF AND LCM OF DECIMALS

HCF

Step-1 : First of all equate the number of places in all the numbers by using zeros, wherever required .
Step-2 : Then considering these number as integers find the HCF of these numbers.
Step-3 : Put the decimal point in the resultant value as many places before the right most digit as that of in the every equated number.

Ex. Find the HCF of 0.0005, 0.005, 0.15, 0.175, 0.5 and 3.5

Sol. $0.0005 \Rightarrow 5$ $0.0050 \Rightarrow 50$ $0.1500 \Rightarrow 15$ $0.1750 \Rightarrow 1750$

$0.5000 \Rightarrow 5000$ $3.5000 \Rightarrow 35000$

Then the HCF of 5, 50, 1500, 1750, 5000, and 35000 is 5. So, the HCF of the given numbers is 0.0005.

LCM

Step-1: First of all equate the number of places in all the given numbers by putting the minimum possible number of zeros at the end of the decimal numbers. wherever even required.
Step-2 : Now consider the equated numbers as integers and then find the LCM of these numbers.
Step-3 : Put the decimal point in the LCM of the number as many places as that of in the equated numbers.

Ex. Find LCM of 1.8, 0.54 and 7.2.

Sol. 1.8 → 1.80, 0.54 → 0.54, 7.2 → 7.20; 180, 54, 720 ; Now the LCM of 180, 54 and 720 is 2160. Therefore the required LCM is 21.60.

★ SYNOPSIS

1. Euclid Division Algorithm : Given any two positive integers a and b, b $\neq$ 1. a > b and a is not divisible by b, there exists two (unique) integers q and r such that

$$a = q + r \text{ , where } r < b$$

2. **Prime Factorization Theorem :** Every composite number can be expressed as a product of prime factors, and the decomposition is unique, apart from the order of factors.

(The fundamental Theorem of Arithmetic)

i.e. given any composite number x, we can find unique prime factors p_1, p_2, p_3……p_n such that
$x = p_1 \times p_2 \times p_3 \times \ldots\ldots \times p_n$

3. **HCF and LCM of two numbers :** Let a, b be given numbers, Let each of these is expressed at a product of prime factors.
   - **(i)** The product of the smaller powers of the common prime numbers is the HCF.
   - **(ii)** The product of the prime numbers is either or both of these expression taken with greater power is the required LCM.
   - **(iii)** HCF (a, b) × LCM (a, b) = a × b

4. Rational Numbers $\frac{p}{q}, q \neq 0$ has a terminating decimal expansion if the prime factors of q are only 2's and 5's or both

5. Let $x = \frac{p}{q}$ be a rational number such the prime factorization of q is of the form $2^n . 5^m$ where n, m are non-negative integers, then x ahs a decimal expansion which terminates.

6. A rational number $\frac{p}{q}, q \neq 0$ has terminating repeating decimal expansion if the prime factors of q are other than 2 and 5 or both .

7. Let $x = \frac{p}{q}$ be a rational number such that the prime factorization of q is not of the form $2^n . 5^m$, where n and m are non negative integers, then x has a decimal expansion which is non-terminating repeating .

8. **Irrational Numbers :** $\sqrt{2}, \sqrt{3}, \sqrt{5}, 3\sqrt{3}, \sqrt{2}+\sqrt{3}, \pi, e$ are all irrational numbers. numbers which are expressed as non-terminating and non-repeated decimals are called the irrational numbers.

9. Real Numbers are a combination of the rational numbers and the irrational numbers .

SOLVED NCERT EXERCISE

EXERCISE : 1.1

1. **Use Euclid's division algorithm to find the HCF of :**
   (i) 135 and 225 **(ii) 196 and 38220** **(iii) 867 and 255.**

Sol. **(i)** **135 and 225,** Start with the lager integer, that is, 225. Apply the division lemma to 225 and 135, to get.

$225 = 135 \times 90$

Since the remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to get

$138 = 90 \times 1 + 45$

We consider the new divisor 90 and the new remainder 45, and apply the division lemma to get

$90 = 45 \times 2 + 0$

The remainder has now become zero, so our procedure stops.

Since the divisor at this stage is 45, the HCF of 225 and 135 is 45.

[Rest Try Yourself]

2. **Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.**

Sol. Let us start with taking a, where a is any positive odd integer. We apply the division algorithm, with a and b = 6. Since $0 \leq r < 6$, the possible remainders are 0, 1, 2, 3, 4, 5. That is , a can be 6q or 6q + 1, or 6q +2 , or 6q + 3, or 6q + 4 or 6q + 5 where q is the quotient, However, since a is odd, we do not consider the cases 6q, 6q + 2 and 6q + 4 (since all the three are divisible by 2). Therefore, any positive odd integer is of the form 6q + 1, or 6q + 3, 6q + 5.

3. **An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march ?**

Sol. Hint : Find HCF of 616 & 32

4. **Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.**

Sol. Let a be any odd positive integer. We apply the division lemma with a and b = 3. Since $0 \leq r < 3$, the possible remainders are 0, 1 and 2. That is, a can be 3q, or 3q +1, or 6q + 2, where q is the quotient .

Now, $(3q)^2 = 6q^2$

which can be written in the form 3m, since 9 is divisible by 3.

Again , $(3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1$

Which can be written in the form 3m + 1 since $9q^2 + 6q$, i.e., $3(3q^2 + 2q)$ is divisible by 3.

Lastly, $(3q + 2)^2 = 9q^2 + 12q + 4 = (9q^2 + 12q + 3) + 1 = 3(3q^2 + 4q + 1) + 1$

which can be written in the form 3m + 1, since $9q^2 + 12q + 3$, i.e., $3(3q^2 + 4q + 1)$ is divisible by 3.

Therefore, the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

5. **Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, + 1 or 9m + 8**

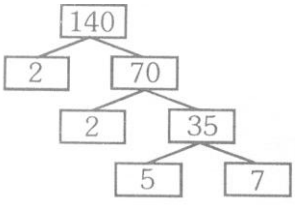
Sol. **Try Yourself**

EXERCISE : 1 . 2

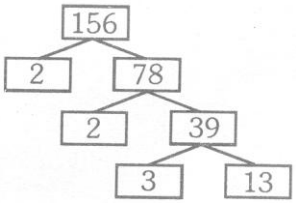
1. **Express each number as product of its prime factors**

(i) 140 **(ii) 156** **(iii) 3825** **(iv) 5005** **(v) 7429**

Sol. (i) 140



(ii) 156



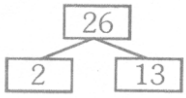
So, $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$ So, $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

[Rest Try Yourself]

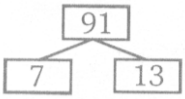
2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Sol.(i) 26 and 91



So, $26 = 2 \times 13$



So, $91 = 7 \times 13$

Therefore, LCM (26, 91) = $2 \times 7 \times 13 = 182$

HCF (26, 91) = 13

Verification LCM × HCF = $182 \times 13 = 2366$ and $26 \times 91 = 2366$

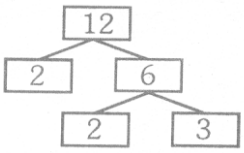
i.e., LCM × HCF product of two numbers.

[Rest Try Yourself]

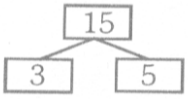
3. Find the LCM and HCF of the following integers by applying the prime factorization method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

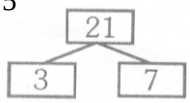
Sol.(i) 12, 15 and 21



So, $12 = 2 \times 2 \times 3 = 2^2 \times 3$



So, $15 = 3 \times 5$



So, $21 = 3 \times 7$

Therefore, HCF (12, 15, 21) = 3 ; LCM = (12, 15, 21) = $2^2 \times 3 \times 5 \times 7 = 420$ **[Rest Try Yourself]**

4. Given that HCF (306, 657) = 9. find LCM (306, 657)

Sol. $\text{LCM}(306, 657) = \dfrac{306 \times 657}{HCF\ (306, 657)} = \dfrac{306 \times 657}{9} = 22338$

5. Check whether 6" can end with the digit 0 for any natural number n.

Sol. If the number 6^n, for any neural number n, end with digit 0, then it would be divisible by 5. That is the prime factorization of 6^n, would contain the prime number 5. This is not possible because $6^n = (2 \times 3)^n = \times 3^n$; so the only prime in the factorization of 6^n are 2 and 3 and the uniqueness of the Fundamental Theorem of Arithmetic guarantees

that there are no other primes in the factorization of 6^n, So, there is no nature number n for which 6^n, ends with the digit zero.

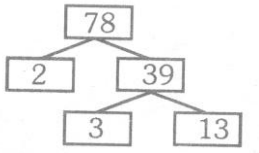
6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol. (i) $7 \times 11 \times 13 + 13 = (7 \times 11 + 1) \times 13$

$= (77 + 1) \times 13$

$= 78 \times 13 = (2 \times 3 \times 13) \times 13$

So, $78 = 2 \times 3 \times 13 = 2 \times 3 \times 13^2$



Since, $7 \times 11 \times 13 + 13$ can be expressed as a product of primes, therefore, it is a composite number.

(ii) **[Try Yourself]**

7. **There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point ?**

Sol. [Hint : Take LCM of 18 and 12]

EXERCISE : 1 . 3

1. **Prove $\sqrt{5}$ is irrational.**

Sol. Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find co prime integers a and b ($\neq 0$) such that

$$\sqrt{5} = \frac{a}{b}$$

$\Rightarrow \quad \sqrt{5}\, b = a$

Squaring on both sides, we get

$5b^2 = a^2$

Therefore, 5 divides a^2.

Therefore, 5, divides a

So, we can write $a = 5c$ for same integer c.

Substituting for a, we get

$5b^2 = 25c^2$

$\Rightarrow \quad b^2 = 5c^2$

This means that 5 divides b^2, and so 5 divides b.

Therefore , a and b have at least 5 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

This contradiction arose because of our incorrect assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is irrational.

2. **prove that $3 + 2\sqrt{5}$ is irrational.**

Sol. Let us assume, to the contrary, $3 + 2\sqrt{5}$ is rational.

That is, we can find co prime integers a and b ($b \neq 0$) such that $3 + 2\sqrt{5} = \frac{a}{b}$

Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \quad \frac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \quad \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2} = \sqrt{5}$

Since a and b are integers, we get $\frac{a}{2b} - \frac{3}{2}$ is rational , and so $\frac{a-3b}{2b} = \sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has risen because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

3. Prove that the following are irrationals :

(i) $\frac{1}{\sqrt{2}}$ **(ii)** $7\sqrt{5}$ **(iii)** $6+\sqrt{2}$

Sol. [Try yourself]

EXERCISE : 1 . 4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

(i) $\frac{13}{3125}$ **(ii)** $\frac{17}{8}$ **(iii)** $\frac{64}{455}$ **(iv)** $\frac{15}{1600}$ **(v)** $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^2}$ **(vii)** $\frac{129}{2^2 5^7 7^5}$ **(viii)** $\frac{16}{15}$ **(ix)** $\frac{35}{50}$ **(x)** $\frac{77}{210}$

Sol. (i) $\frac{13}{3125} = \frac{13}{5^5}$

Hence , q = 5^5 , which is of the form $2^n 5^m$ (n = 0, m = 5). So, the rational number $\frac{13}{3125}$ has a terminating decimal expansion.

(ii) $\frac{17}{8} = \frac{17}{2^3}$

Hence, q = 2^3, which is of the form $2^n 5^m$ (n = 3, m = 0). So, the rational number $\frac{17}{8}$ has a terminating decimal expansion.

(iii) $\frac{64}{455} = \frac{64}{5\times 7\times 13}$

Hence, q=5×7×13, which is not of the form 2n5m. so, the rational number $\frac{64}{455}$ has a non-terminating repeating decimal expansion.

[Rest Try Yourself]

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Sol. (i) $\frac{13}{3125}$

$= \frac{13}{5^5} = \frac{13\times 2^5}{5^5 \times 2^5} = \frac{416}{10^5} = 0.00416$

(ii) $\frac{17}{8} = \frac{17}{2^3}$

$= \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 5^3}{10^3} = \frac{2125}{10^3} = 2.125$

[Rest Try Yourself]

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational, or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q ?

(i) 43.123456789

(ii) 0.120 1200 12000 120000……..

. $34.1\overline{23456789}$

Sol. (i) 43.123456789

Since, the decimal expansion terminates, so the given real number is rational and therefore of the form $\frac{p}{q}$

43.123456789

$= \frac{4312345678\,9}{1000000000}$

$= \frac{4312345678\,9}{10^9}$

$= \frac{4312345678\,9}{(2 \times 5)^9}$

$= \frac{4312345678\,9}{2^9 5^9}$

Hence, $q = 2^9 5^9$

The prime factorization of q is of the form $2^n 5^m$, where n = 9, m = 9

(ii) 0.120 1200 12000 120000……

Since, the decimal expansion is neither terminating nor non-ter4mnating repeating, therefore, the given real number is not rational .

(iii) **Try Yourself**

EXERCISE – 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

Choose The Correct One

1. $\sqrt{2}$ is –

(A) An integer
(B) A rational number
(C) An irrational number
(D) None of these

2. $\frac{1}{\sqrt{3}}$ is –

(A) A rational number (B) An irrational number
(C) a whole number (D) None of these

3. $7\sqrt{3}$ is –

(A) An irrational (B) A natural number
(C) A rational number (D) None of these

4. $5-\sqrt{3}$ is –

(A) An integer (B) A rational number
(C) An irrational number (D) None of these

5. $\pi = \frac{\text{Circumference of the circle}}{\text{Diameter of the circle}}$

(A) A rational number (B) A whole number
(C) A positive integer (D) None of these

6. HCF(p, q) × LCM (p, q) =

(A) p + q (B) $\frac{p}{q}$ (C) p × q (D) p^q

7. HCF(p, q, r) . LCM (p, q, r) =

(A) $\frac{pq}{r}$ (B) $\frac{qr}{p}$ (C) p, q, r (D) None of these

8. If $\sqrt[3]{32} = 2^x$ then x is equal to

(A) 5 (B) 3 (C) $\frac{3}{5}$ (D) $\frac{5}{3}$

9. 0.737373… =

(A) $(0.73)^3$ (B) $\frac{73}{100}$ (C) $\frac{73}{99}$ (D) None of these

10. If p is a positive prime integer, then $\sqrt{p}$ is –

(A) A rational number (B) An irrational number
(C) a positive integer (D) None of these

11. LCM of three numbers 28, 44, 132 is –

(A) 528 (B) 231 (C) 462 (D) 924

12. If a is a positive integer and p be a prime number and p divides a^2, then

(A) a divides p (B) p divides a (C) p^2 divides a (D) None of these

13. Evaluate $\sqrt[3]{\left(\frac{1}{6}\right)^{-2}}$

(A) 4 (B) 16 (C) 32 (D) 64

14. If $a = \frac{a+\sqrt{3}}{2-\sqrt{3}}$, $b = \frac{2-\sqrt{3}}{2+\sqrt{3}}$ then the value of a + b is –

(A) 14 (B) – 14 (C) $8\sqrt{3}$ (D) $-\sqrt{3}$

15. If $x = 0.\overline{16}$, then 3x is –

(A) $0.\overline{48}$ (B) $0.\overline{49}$ (C) $0.\bar{5}$ (D) 0.5

16. Find the value of x then $\left(\frac{3}{5}\right)^{2x-3} = \left(\frac{5}{3}\right)^{x-3}$

(A) x = 2 (B) x = - 2 (C) x = 1 (D) x = - 1

17. $1.\bar{3}$ is equal to –

(A) 3/4 (B) 2/3 (C) 4/3 (D) 2/5

18. The product of $4\sqrt{6}$ and $3\sqrt{24}$ is –

(A) 124 (B) 134 (C) 144 (D) 154

19. If x = $(7 + 4\sqrt{3})$, then the value of $x^2 + \frac{1}{x^2}$ is –

(A) 193 (B) 194 (C) 195 (D) 196

20. If $16 \times 8^{n+2} = 2^m$, then m is equal to –

(A) n + 8 (B) 2n +10 (C) 3n + 2 (D) 3n + 10

ANSWER KEY **EXERCISE - 1**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	B	A	C	D	C	D	D	C	B	D	B	B	B	A
Que.	16	17	18	19	20										
Ans.	A	C	C	B	D										

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions

1. Show that product of two numbers 60 and 84 is equal to the product of their HCF and LCM.

2. The product of two numbers is 396 × 576 and their LCM is 6336. Find their HCF.

3. Without actually performing the long division, state whether the following rational numbers have a terminating decimal expansion or a non-terminating repeating decimal expansion :

(i) $\frac{1}{7}$ (ii) $\frac{1}{11}$ (iii) $\frac{22}{7}$ (iv) $\frac{3}{5}$ (v) $\frac{7}{20}$ (vi) $\frac{2}{13}$ (vii) $\frac{27}{40}$ (viii) $\frac{13}{125}$ (ix) $\frac{23}{7}$ (x) $\frac{42}{100}$

4. Write down the decimal expansions of the following rational numbers :

(i) $\frac{241}{2^3 5^2}$ (ii) $\frac{19}{256}$ (iii) $\frac{25}{1600}$ (iv) $\frac{9}{30}$ (v) $\frac{133}{2^3 5^4}$

5. Show that 5309 and 3072 are prime to each other.

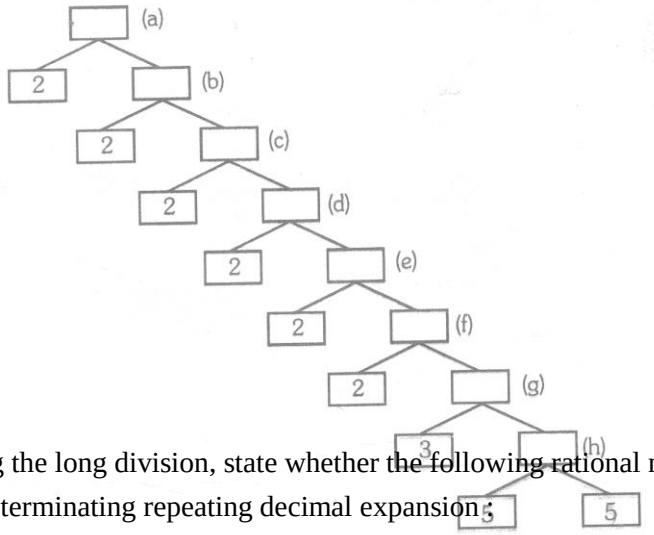
6. The HCF of two numbers is 119 and their LCM is 11781. If one of the numbers is 1071, find the other.

7. The LCM of two numbers is 2079 and their HCF is 27. If one of the numbers is 189, find the other.

8. Find the prime factorization of the following numbers:

(i) 10000 (ii) 2160 (iii) 396 (iv) 4725 (v) 1188

9. Find the missing numbers in the following factorization :



10. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion :

(i) $\frac{11}{125}$ (ii) $\frac{19}{128}$ (iii) $\frac{32}{405}$ (iv) $\frac{15}{3200}$ (v) $\frac{29}{2401}$

11. Write down the decimal expansions of the following rational numbers :

(i) $\frac{5}{8}$ (ii) $\frac{12}{125}$ (iii) $\frac{13}{625}$ (iv) $\frac{7}{64}$ (v) $\frac{7}{8}$

Short Answer Type Questions

12. Use Euclid's algorithm to find the HCF of 4052 and 12576.

13. Find the HCF 84 and 105. using Euclid's algorithm.

14. Find the HCF of 595 and 107, using Euclid's algorithm.

15. Find the HCF of 861 and 1353, using Euclid's algorithm.

16. Find the HCF of 616 and 1300, using Euclid's algorithm.

17. Show that every positive even integer is of the form 2q, and that very positive odd integer is of the form 2q + 1, where q is some integer .

18. Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some inerter.

19. Show that one and only out of n, n + 2 or n + 4 is divisible by 3, where n is any positive integer.

20. Find the greatest length which can be contained exactly in 10 m 5 dm 2cm 4mm and 12m 7dm 5cm 2mm.

21. Find the greatest measure which is exactly contained in 10 liters 857 millilitres and 15 litres 87 millilitres.

22. Consider the number 4^n, where n is a natural number. Check whether there is any value of $n \in N$ for which 4^n ends with the digit zero.

23. Find the LCM and HCF of 6 and 20 by the prime factorization method.

24. Find the HCF of 12576 and 4052 by using the prime factorization method.

25. Find the HCF and LCM of 6, 72 and 120 using the prime factorization method.

26. Find the prime factors of the following numbers:

(i) 1300 (ii) 13645 (iii) 3456

27. Find the LCM and HCF of 18, 24, 60, 150

28. Find the HCF and LCM of 60, 32, 45, 80, 36, 120

29. Split 4536 and 18511 into their prime factors and hence find their LCM and HCF.

30. Prove that $\sqrt{5}$ is irrational.

31. Prove that $\sqrt{7}$ is irrational.

32. Prove that $\frac{1}{\sqrt{3}}$ is irrational.

33. Prove that $3\sqrt{5}$ is irrational.

34. Prove that $3-\sqrt{3}$ is irrational.

35. Prove that $7+\sqrt{2}$ is irrational.

36. Prove that $5-\sqrt{5}$ is irrational.

37. Prove that $3\sqrt{2}$ is irrational.

38. Use Euclid's division lemma to find the HCF of

(i) 13281 and 15844 (ii) 1128 and 1464 (iii) 4059 and 2190

(iv) 10524 and 12752 (v) 10025 and 14035

39. What is the greatest number by which 1037 and 1159 can both be divided exactly ?

40. Find the greatest number which both 2458090 and 867090 will contain an exact number of times.

41. Find the greatest weight which can be contained exactly in 3 kg 7 hg 8 dag 1 g and 9 kg 5 dag 4 g.

42. Find the LCM of the following using prime factorization method. :

(i) 72, 90, 120

(ii) 24, 63, 70

(iii) 455, 117, 338

(iv) 225, 240, 208

(v) 2184, 2730, 3360

43. Prove that $\sqrt{3}$ is irrational.

44. Prove that $2\sqrt{2}$ is irrational.

45. Prove that $\frac{1}{\sqrt{5}}$ is irrational.

46. Prove that $7+\sqrt{3}$ is irrational.

47. Prove that $8-\sqrt{2}$ is irrational.

- **Very Short Answer Type Questions**

2. 36.

3. (i) Non-terminating repeating ; (ii) Non-terminating repeating ; (iii) Non-terminating repeating

(iv) Terminating ; (v) Terminating (vi) Non-terminating repeating (vii) Terminating ; (viii) Terminating

(ix) Non-terminating repeating ; (x) Terminating

4. (i) 1.205 ; (ii) 0.07421875 ; (iii) 0.015625 ; (iv) 0.0266 **6.** 1309 **7.** 297

8. (i) $2^4 \times 5^4$; (ii) $2^4 \times 3^3 \times 5$; (iii) $2^2 \times 3^2 \times 11$; (iv) $3^3 \times 5^2 \times 7$; (v) $2^2 \times 3^3 \times 11$

9. (a) 4800 ; (b) 2400 ; (c) 1200 ; (d) 600 ; (e) 300 ‘ (f) 150 ‘ (g) 75 ‘ (h) 25

10. (i) Terminating ; (ii) Terminating ; (iii) Non-terminating repeating ;(iv) Terminating ;(v)Non-terminating repeating

11. (i) 0.625 ; (ii) 0.96 ; (iii) 0.0208 ; (iv) 0.109375 ; (v) 0.875

- **Short Answer Type Questions**

12. 4 **13.** 21 **14.**119 **15.** 123 **16.** 4 **20.** 4mm **21.** 141 mmlilitres **22.** No **23.** 60, 2

24. 4 **25.** 6, 360 **26.** (i) $2^2 \times 5^2 \times 13$; (ii) $3\times5\times7\times13$; (iii) $2^7 \times 3^3$ **27.** 1800, 6 **28.** 1, 1440

29. 149688, 567 **38.** (i) 233 ; (ii) 24 ; (iii) 3 ; (iv) 4 ; (v) 2005. **39.** 61 **40.** 10 **41.** 1 hg 9 dag 9 g

42. (i) 360 ; (ii) 2520 ; (iii) 106470 ; (iv) 46800 ; (v) 43680

EXERCISE – 3 (FOR SCHOOL / BOARD EXAMS

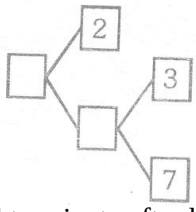
PREVIOUS YEARS BOARD (CBSE) QUESTIONS

Questions Carrying 1 Mark

1. If $\frac{p}{q}$ is a rational number ($q \neq 0$), what is condition of q so that the decimal representation of $\frac{p}{q}$ is terminating ? **[Delhi-2008]**

2. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$. **[AI-2008]**

3. Complete the missing entries in the following factor tree : **Foreign - 2008]**



4. The decimal expansion of the rational number $\frac{43}{2^4 5^3}$, will terminate after how many places of decimals ? **[Delhi - 2009]**

5. Find the [HCF × LCM] for the numbers 100 and 190. **[AI - 2009]**

6. Find the [HCF × LCM] for the numbers 105 and 120. **[AI - 2009]**

7. Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion. **[Foreign - 2009]**

8. The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, write the other number. **[Foreign - 2009]**

Questions Carrying 3 Marks

9. Show that $5-2\sqrt{3}$ is an irrational number **[Delhi - 2008]**

10. Show that $2-\sqrt{3}$ is an irrational number **[Delhi - 2008]**

11. Show that $5+3\sqrt{2}$ is an irrational number **[Delhi - 2008]**

12. Prove that $\sqrt{3}$ is an irrational number . **[Delhi – 2009/AI-2008]**

13. Use Euclid’s Division Lemma to show the square of any positive integer is either of the form 3m or 3m + 1 for some integer m. **[Foreign – 2008 / AI-2008]**

14. Prove that $\sqrt{2}$ is an irrational number. **[Delhi – 2009/AI-2008]**

15. Prove that $\sqrt{5}$ is an irrational number. **[Delhi – 2009/AI-2008]**

16. Prove that $3+\sqrt{2}$ is an irrational number. **[AI-2008]**

17. Prove that $5-2\sqrt{3}$ is an irrational number. **[AI-2008]**

18. Prove that $3+5\sqrt{2}$ is an irrational number. **[AI-2009]**

19. Show that the square of any positive odd integers is of the form 8m + 1, for some integer m. **[Foreign-2009]**

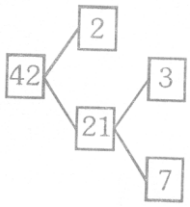
20. Prove that $7+3\sqrt{2}$ is not a rational number. **[ForeignAI-2009]**

1. $q = 2^n \times 5^m$, where n and m are whole numbers.

2. $\sqrt{2} = 1.41.....$, $\sqrt{3} = 1.73.....$

$\therefore$ One rational no. between $\sqrt{2}$ and $\sqrt{3}$ is 1.5.

3.



4. After 4 decimal ; $\frac{43}{2^4 5^3} = \frac{43}{2000} = 0.0215$

5. HCF × LCM = 100× 190 = 19000

6. HCF × LCM = 105 × 120 = 12600

7. $\frac{51}{1500} = \frac{17}{500}; 500 = 2^2 \times 5^3 (2^m \cdot 5^n)$. So, it has terminating expansion.

8. Other number $= \frac{9 \times 360}{45} = 72$

EXERCISE – 4 (FOR OLYMPIADS)

Choose The Correct One

1. The greatest possible number with which when we divide 37 and 58, leaves the respective remainder of 2 and 3, is -

(A) 2 (B) 5 (C) 10 (D) None of these

2. The largest possible number with which when 60 and 98 are divided, leaves the remainder 3 in each case, is –

(A) 38 (B) 18 (C) 19 (D) None of these

3. The largest possible number with which when 38, 66 and 80 are divided the remainders remain the same is –

(A) 14 (B) 7 (C) 28 (D) None of these

4. What is the least possible number which when divided by 24, 32 or 42 in each case it leaves the remainder 5 ?

(A) 557 (B) 677 (C) 777 (D) None of these

5. In Q.N. 4, how many numbers are possible between 666 and 8888 ?

(A) 10 (B) 11 (C) 12 (D) 13

6. What is the least number which when divided by 8, 12 and 16 leaves 3 as the remainder in each case, but when divided by 7 leaves no remainder ?

(A) 147 (B) 145 (C) 197 (D) None of these

7. What is the least possible number which when divided by 18, 35 or 42 leaves 2, 19, 26 as the remainders respectively ?

(A) 514 (B) 614 (C) 314 (D) None of these

8. What is the least possible number which when divided by 2, 3, 4, 5, 6 leaves the remainders 1, 2, 3, 4, 5 respectively ?

(A) 39 (B) 48 (C) 59 (D) None of these

9. In Q.No. 8, what is the least possible 3 digit number which is divisible by 11 ?

(A) 293 (B) 539 (C) 613 (D) None of these

10. How many numbers lie between 11 and 1111 which when divided by 9 leave a remainder of 6 and when divided by 21 leave a remainder of 12 ?

(A) 18 (B) 28 (C) 8 (D) None of these

11. If x divides y (written as x | y) and y | z, (x, y, z $\in$ z) then –

(A) x | z (B) z | y (C) z | x (D) None of these

12. If x | y, where x > 0, y > 0 (x, y $\in$ z) then –

(A) x < y (B) x = y (C) x $\leq$ y (D) x $\geq$ y

13. If a | b, then gcd of a and b is –

(A) a (B) b (C) ab (D) Can't be determined

14. If gcd of b and c is g and d | b & d | c, then –

(A) d = g (B) g | d (C) d | g (D) None of these

15. If x, y $\in$ R and | x | + | y | = 0, then –

(A) x > 0, y < 0 (B) x < 0, y > 0 (C) x = 0, y = 0 (D) None of these

16. If a, b, c $\in$ R and $a^2 + b^2 + c^2 = ab + bc + ca$, then –

(A) a = b = c (B) a = b = c = 0 (C) a, b, c are distinct (D) None of these

17. If x, y $\in$ R and x < y $\Rightarrow$ $x^2 > y^2$ then –

(A) x > 0 (B) y > 0 (C) x < 0 (D) y < 0

18. If x, y $\in$ R and x > y $\Rightarrow$ | x | > | y | , then –(A)(B)(C)(D)

(A) x > 0 (B) y > 0 (C) x < 0 (D) y < 0

19. If x, y $\in$ R and x > y $\Rightarrow$ | x | < | y | , then –

(A) x < 0 (B) x > 0 (C) y > 0 (D) y < 0

20. π and e are –

(A) Natural numbers (B) Integers (C) Rational numbers (D) Irrational numbers.

21. If a, b $\in$ R and a < b, then –

(A) $\frac{1}{a} < \frac{1}{b}$ (B) $\frac{1}{a} > \frac{1}{b}$ (C) $a^2 > b^2$ (D) Nothing can be said

22. If x is a non-zero rational number and xy is irrational , then y must by –

(A) a rational number (B) an irrational number (C) non-zero (D) an integer

23. The arithmetical fraction that exceeds it's square by the greatest quantity is –

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) None of these

24. If x and y are rational numbers such that $\sqrt{xy}$ is irrational , then $\sqrt{x} + \sqrt{y}$ is –

(A) Rational (B) Irrational (C) Non-real (D) None of these

25. If x and y are positive real numbers, then –

(A) $\sqrt{x} + \sqrt{y} \geq \sqrt{x+y}$ (B) $\sqrt{x} + \sqrt{y} < \sqrt{x+y}$

(C) $\sqrt{x} + \sqrt{y} = \sqrt{x+y}$ (D) None of these

26. If $(\sqrt{2} + \sqrt{3})^2 = a + b\sqrt{6}$, where a, b $\in$ Q, then –

(A) a = 5, b = 6 (B) a = 5, b = 2 (C) a = 6, b = 5 (D) None of these

27. If x $\in$ R, then | x | =

(A) x (B) –x (C) max {x, -x} (D) min {x, -x}

28. $\frac{15}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{125}}$ is equal to –

(A) $\sqrt{5}(5 + \sqrt{2})$ (B) $\sqrt{5}(2 + \sqrt{2})$ (C) $\sqrt{5}(\sqrt{2} + 1)$ (D) $\sqrt{5}(3 + \sqrt{2})$

29. $\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}$ is equal to –

(A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{\sqrt{2}}$ (C) $\frac{\sqrt{2}}{\sqrt{3}}$ (D) $\sqrt{6}$

30. The expression $\frac{\sqrt{3}-1}{2\sqrt{2}-\sqrt{3}-1}$ is equal to –

(A) $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ (B) $\sqrt{6}-\sqrt{4}+\sqrt{3}-\sqrt{2}$

(C) $\sqrt{6}-\sqrt{4}-\sqrt{3}+\sqrt{2}$ (D) None of these

31. If x, y, z are real numbers such that $\sqrt{x-1}+\sqrt{y-2}+\sqrt{z-3}=0$ then the values of x, y, z are respectively

(A) 1, 2, 3 (B) 0, 0, 0

(C) 2, 3, 1 (D) None of these

32. If a, b, c $\in$ R and a > b $\Rightarrow$ ac < bc, then –

(A) $c \geq 0$ (B) $c \leq 0$

(C) c > 0 (D) c < 0

33. If a, b, c $\in$ R and ac = bc $\Rightarrow$ a = b, then –

(A) $c \geq 0$ (B) $c \leq 0$

(C) $c = 0$ (D) $c \neq 0$

34. Between any two distinct rational numbers –

(A) There lie infinitely many rational numbers.

(B) There lies only one rational number.

(C) There lie only finitely many numbers.

(D) There lie only rational numbers.

35. The total number of divisors of 10500 except 1 and itself is –

(A) 48 (B) 50

(C) 46 (D) 56

36. The sum is the factors of 19600 is –

(A) 54777 (B) 33667

(C) 5428 (D) None of these

37. The product of divisors of 7056 is –

(A) $(84)^{48}$ (B) $(84)^{44}$

(C) $(84)^{45}$ (D) None of these

38. The number of odd factors (or divisors) of 24 is –

(A) 2 (B) 3 (C) 1 (D) None of these

39. The number of even factors (or divisors) of 24 is –

(A) 6 (B) 4 (C) 8 (D) None of these

40. In how many ways can 576 be expressed as a product of two distinct factors ?

(A) 10 (B) 11 (C) 21 (D) None of these

OBJECTIVE	ANSWER KEY														EXERCISE - 4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**B**	**C**	**A**	**B**	**D**	**A**	**B**	**C**	**B**	**A**	**A**	**C**	**A**	**C**	**C**
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	**A**	**D**	**B**	**A**	**D**	**D**	**B**	**B**	**B**	**A**	**B**	**C**	**D**	**D**	**A**
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**					
Ans.	**A**	**D**	**D**	**A**	**C**	**A**	**C**	**A**	**A**	**A**					

COMPETITION WINDOW

COMPLEX NUMBERS

The idea of complex numbers was introduced, so that all algebraic equations could have solutions. Over the real numbers, the square root on negative number is not defined.

Leonhard Euler for the first time introduced the symbol iota (i) in 1748, (i is the first letter of Latin word 'imaginaries'] for $\sqrt{-1}$ with the property $i^2 = -1$.

$i = \sqrt{-1}$ so $i^2 = -1$.

Imaginary Numbers : Square root of a negative number is called imaginary number, e.g. $\sqrt{-1}, \sqrt{-2}, \sqrt{-9/4}$ etc.

$\sqrt{-2}$ can be written as

$\sqrt{-2} = \sqrt{2} \times \sqrt{-1} = \sqrt{2}i$

Remark :

1. If a, b are positive real numbers, then $\sqrt{-a} \times \sqrt{-b} = -\sqrt{ab}$
2. For any two real number $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ is not valid if a and b both are negative.
3. For any positive real number a, we have $\sqrt{-a} = \sqrt{-1 \times a} = \sqrt{-1} \times = \sqrt{a} = i\sqrt{a}$

E.g

1. $\sqrt{-144} = \sqrt{-1 \times 144} = \sqrt{-1} \times \sqrt{144} = 12i$
2. $\sqrt{-4} \times \sqrt{-\frac{9}{4}} = 2i\left(\frac{3i}{2}\right) = 3i^2 = -3$
3. $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9} = 5i + 6i + 6i = 17i$

Integral powers of i : We have $i = \sqrt{-1}$ so $i^2 = -1,\ i^3 = -i,\ i^4 = 1$

For any n $\in$ N, we have

$i^{4n} = 1$,

$i^{4n+2} = 1$,

$i^{4n+2} = -1$,

$i^{4n+3} = -i$

E.g.

1. $i^{35} = i^3 = -i$
2. $i^{-999} = \frac{1}{i^{999}} = \frac{1}{i^3} = \frac{i}{i^4} = \frac{i}{1} = i$
3. $\left[i^{19} + \left(\frac{1}{i}\right)^{25}\right]^2 = \left[i^{19} + \frac{1}{i^{25}}\right]^2 = \left[i^3 + \frac{1}{i}\right]^2 = \left[-i + \frac{i^3}{i^4}\right]^2 = [-i + i^3]^2 = (-i - i)^2 = 4i^2 = -4$

Complex Numbers : If a, b are two real numbers, then a number of the form a + ib is called a complex number.
e.g. 7 + 2i, -1 + i, 3 -2i etc

If z = a + ib is a complex number, then 'a' is called the real part of z (Re (z)) and 'b' is called the imaginary part of z(Im(z)).

Equality of complex numbers : Two complex numbers $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are equal if $a_1 = a_2$ and $b_1 = b_2$.

Algebra of complex combers : Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ then

(i) $z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2) = (a_1+a_2) + i\,(b_1 + b_2)$

(ii) $z_1 - z_2 = (a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + i\,(b_1 - b_2)$

(iii) $z_1 \cdot z_2 = (a_1 + ib_1) \cdot (a_2 + ib_2) = (a_1a_2 - b_1b_2) + i\,(a_1b_2 + b_1a_2)$

(iv) $$\frac{z_1}{z_2} = \frac{(a_1 + ib_1)}{(a_2 + ib_2)} = \frac{(a_1a_2 + b_1b_2)}{a_2^2 + b_2^2} + i\frac{(b_1a_2 - a_1b_2)}{a_2^2 + b_2^2}$$

Multiplicative Inverse of a complex number : Corresponding to every non-zero complex number $z = a + ib$, there exists a complex number $z^{-1} = x + iy$ such that

$$z \cdot z^{-1} = 1 \quad (z \neq 0)$$

$$z^{-1} = \frac{1}{z} = \frac{1}{a + ib} = \frac{a - ib}{(a + ib)(a - ib)} = \frac{a - ib}{a^2 + b^2}$$

Conjugate of a complex number : Let $z = a + ib$ be a complex number. Then the conjugate of z is dented by $\bar{z}$ and is equal to $a - ib$.

Thus $z = a + ib \Rightarrow \bar{z} = a - ib$

E.g. if $z = 3 + 4i \Rightarrow \bar{z} = 3 - 4i$

Modulus of a complex number : The modulus of a complex number $z = a + ib$ is denoted by $|z|$ and is defined as

$$|z| = \sqrt{a^2 + b^2}$$

$|z|$ is also called the absolute value of z.

EXERCISE – 5 (FOR ITT-JEE/AIEEE)

Choose The Correct One

1. The value of i^{457} is -

(A) 1 (B) – 1 (C) i (D) – i

2. The value $i^{37} + \frac{1}{i^{67}}$ is –

(A) 1 (B) – 1 (C) 2i (D) –2

3. The value of $\left(i^{41} + \frac{1}{i^{257}}\right)^9$ is –

(A) 1 (B) 0 (C) –1 (D) 2

4. The value of $(i^{77} + i^{70} + i^{87} + i^{414})^3$

(A) – 8 (B) – 6 (C) 6 (D) 8

5. The value of the expression $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$ is -

(A) –1 (B) 1 (C) 0 (D) i

6. The standard form of (1 + i) (1 + 2i) is –

(A) 3 + I (B)–3 + i (C) 1 – 3i (D) 1 – + 3i

8. The standard form of $\frac{(1+i)(1+\sqrt{3i})}{(1-i)}$ is –

(A) $-\sqrt{3}+i$ (B) $\sqrt{3}-i$ (C) $1-i\sqrt{3}$ (D) $1+i\sqrt{3}$

9. The standard form of $\frac{3-4i}{(4-2i)(1+i)}$ is –

(A) $\frac{1}{4}+\frac{3}{4}i$ (B) $\frac{1}{4}-\frac{3}{4}i$ (C) $\frac{3}{4}+\frac{1}{4}i$ (D) $\frac{3}{4}-\frac{1}{4}i$

10. If (x + iy) (2 – 3i) = 4 + i , then real values of x and y are –

(A) $x=5,\ y=14$ (B) $x=\frac{13}{5},\ y=\frac{14}{13}$

(C) $x=\frac{5}{13},\ y=\frac{14}{13}$ (D) None of these

11. If $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3ii}=i,$ then real values of x and y are –

(A) $x=3,\ y=-1$ (B) $x=-1,\ y=3$

(C) $=1,\ y=-2$ (D) $x=-1,\ y=-3$

12. The conjugate of 4 – 5i is –

(A) 4 + 5i (B) – 4 – 5i (C) – 4 + 5i (D) 4 – 5i

13. The conjugate of $\frac{1}{3+5i}$ is –

(A) $\frac{1}{34}(3+5i)$ (B) 3 + 5i (C) $\frac{1}{3-5i}$ (D) $\frac{34}{3-5i}$

14. The conjugate of $\frac{(1+i)(2+i)}{3+i}$ is –

(A) $\frac{3}{5}+\frac{4}{5}i$ (B) $\frac{3}{5}-\frac{4}{5}i$ (C) $-\frac{3}{5}-\frac{4}{5}i$ (D) $\frac{3}{5}+\frac{4}{5}i$

15. The multiplicative inverse of 1 – i is –

(A) 1 + i (B) $\frac{1}{1+i}$ (C) $\frac{1}{2}+\frac{1}{2}i$ (D) None of these

16. The multiplicative inverse of $(1+\sqrt{3})^2$ is –

(A) $-\frac{1}{8}-\frac{i\sqrt{3}}{8}$ (B) $(1-i\sqrt{3})^2$ (C) $\frac{1}{8}+\frac{i\sqrt{3}}{8}$ (D) None of these

17. The value of $2x^3 + 2x^2 – 7x + 72$, when $x=\frac{3-5i}{2}$ is –

(A) 4 (B) – 4 (C) 2 (D) 0

18. The value of $x^4 + 4x^3 + 6x^2 + 4x + 9$. when $x=-1+\sqrt{2}$ is –

(A) 12 (B) 10 (C) 14 (D) 8

19. If $a+ib=\frac{c+i}{c-i},$ where c is real, then $a^2 + b^2$ =

(A) i (B) 1 (C) – 1 (D) 0

20. If $(x + iy)^{1/3} = a + ib$, x, y, a b $\in$ R, then $\frac{x}{a} + \frac{y}{b} =$

(A) 4 (B) $4(a^2 + b^2)$ (C) $4(a^2 - b^2)$ (D) $(a^2 - b^2)$

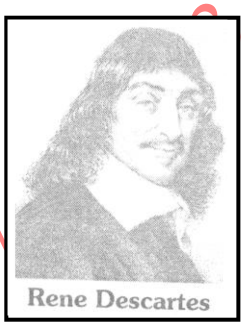
OBJECTIVE	ANSWER KEY														EXERCISE - 5
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**C**	**C**	**B**	**A**	**A**	**B**	**D**	**A**	**B**	**C**	**A**	**A**	**A**	**B**	**C**
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	**A**	**A**	**A**	**B**	**C**										

POLYNOMIALS

★ INTRODUCTION

In class IX, have studied the polynomials in one variable and their degrees. We have also learnt about the values the zeros of a polynomial. In the this chapter, we wil discuss more about the zeros of a polynomial and the relationship between the zeros and the coefficients of a polynomial with particular reference to quadratic polynomials. In addition, statement and simple problems on division algorithm for polynomials with real coefficients will be discussed.

★ HISTORICAL FACTS


Rene Descartes

Determining the roots of polynomials, or 'solving algebraic equations", is among the oldest problems in mathematics. However, elegant and practical notation we use today only developed beginning in the 15th century. Before that, equations were written out in words. For example, an algebra problem from the Chinese Arithmetic in Nine Sections, begins "Three sheaf of good crop, two sheaf of mediocre crop, and one sheaf of bad crop are sold for 29 dou". We would write 3x + 2y + z = 29.

The earliest known use of the equal sign is in Robert Recorder's The Whetstone of Witte, 1557. The signs + for addition, - for subtraction, and the use of letter for and unknown appear in Michael Stifel's Arithmetical Integra, 1544. **Rene Descartes**, in La geometric, 1637, introduced the concept of the graph of polynomial equation. He popularized the use of letters from the beginning of the alphabet to denote constants and letters from the end of the alphabet to denote variables, as can be seen in the general formula for a polynomial, where the a's denote constants and x denotes a variable. Descartes introduced the use of superscripts to denote exponents as well.

★ RECALL

(i) **Polynomials :** An algebraic expression of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x^1 + a_0 x^0$ where $a_n \neq 0$ and $a_0, a_1, a_2, ...a_n$ are real numbers and each power of x is a positive integer, is called a polynomial.

Hence, a_n, a_{n-1}, a_{n-2}, are coefficients of $x^n, x^{n-1}...............x^0$ and $a_n x^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, ...$ are terms of the polynomial . Here the term $a_n x^n$ is called the ***leading term*** and its coefficient a_n, the ***leading coefficient***, For example : $p(u) = \frac{1}{2}u^3 - 3u^2 + 2u - 4$ is a polynomial in variable u.

$\frac{1}{2}u^3, -3^2, 2u, -4$ are know as terms of polynomial and $\frac{1}{2}, -3, 2, -4$ are their respective coefficients.

$6x^{-2}$	This is NOT a polynomial term	Because the variable has a negative exponent
$\frac{1}{x^2}$	This is NOT a polynomial term	Because the variable is in the denominator
sqrt (x)	This is NOT a polynomial term	Because the variable is inside a radical
$4x^2$	This IS a polynomial term	Because it obeys all the rules

(ii) **Types of Polynomials :** Generally we divide the polynomials in three categories .

Types of Polynomials

- **Based on number of distinct variables**
- **Based on degree**
- **Based on number of terms**

Polynomials classified by number of distinct variables

Number of distinct variables	Name	Example
1	Univariate	x + 9
2	Bivariate	x + y + 9
3	Trivariate	x + y + z + 9

Generally, a polynomial in more than one variable is called a **multivariate polynomial.** A second major way of classifying polynomials is by their degree. Recall that the degree of a term is the sum of the exponents on variables, and that the degree of a polynomial is the largest degree of any one term.

Polynomials classified by degree

Degree	Name	Example
$-\infty$	Zero	0
0	(non-zero) constant	1
1	Linear	x + 1
2	quadratic	$x^2 + 1$
3	cubic	$x^3 + 2$
4	quadratic (or biquadratic)	$x^4 + 3$
5	quintic	$x^5 + 4$
6	sextic (or hexic)	$x^6 + 5$
7	septic (or heptic)	$x^7 + 6$
8	octic	$x^8 + 7$
9	nonic	$x^9 + 8$
10	decic	$x^{10} + 9$

Usually, a polynomial of degree n, for n grater than 3, is called a polynomial of degree n, although the phrases quartic polynomial and quintic polynomial are sometimes used.

The polynomial 0, which may be considered to have no terms at all, is called the **zero polynomial.** Unlike other constant polynomials, its degree is not zero. Rather the degree of the zero polynomial is either left explicitly undefined , or defined to be negative (either -1 or $-\infty$)

Polynomials classified by number of non-zero terms

Number of non-zero terms	Name	Example
0	zero polynomial	0
1	monomial	x^2
2	binomial	$x^2 + 1$
3	trinomial	$x^2 + x + 1$

If a polynomial has only one variable, then the terms are usually written either from highest degree to lowest degree ("descending powers") or from lowest degree to highest degree ("ascending powers").

(iii) **Value of a Polynomial :** If p(x) is a polynomial in variable x and α is any real number, then the value obtained by replacing x by α in p(x) is called value of p(x) at x = α and is denoted by p(x).

For example : Find the value of $p(x) = x^3 - 6x^2 + 11x - 6$at = -2

$\Rightarrow p(-2) = (-2)^3 - 6(-2)^2 + 11(-2) - 6 = -8 - 24 - 22 - 6 \Rightarrow p(-2) = -60$

(iv) **Zero of a Polynomial :** A real number α is zero of the polynomial p(x) if p(α) = 0.

For example : consider $p(x) = x^3 - 6x^2 + 11x - 6$

$$p(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$$
$$p(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$$
$$p(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$$

Thus, 1, 2 and 3 are called the zero of polynomial p(x).

★ GEOMETRICAL MEANING OF THE ZEROS OF A POLYNOMIAL

Geometrically the zeros of a polynomials f(x) are the x-co-ordinates of the points where the graph y = f(x) intersects x-axis. To understand it, we will see the geometrical representations of linear and quadratic polynomials.

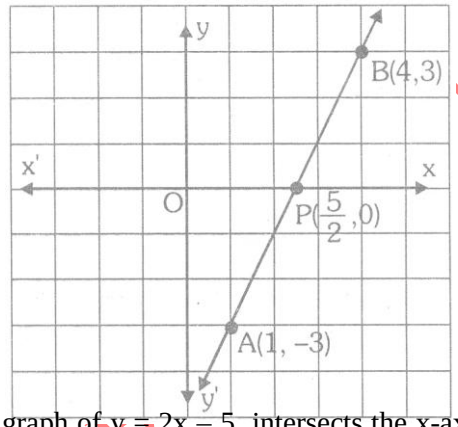
Geometrical Representation of the zero of a Linear Polynomial

Consider a linear polynomial, y = 2y – 5.

The following table lists the values of y corresponding to different values of x.

x	1	4
y	- 3	32

On plotting the points A(1, -3) and B(4, 3) and joining them, a straight line is obtained.



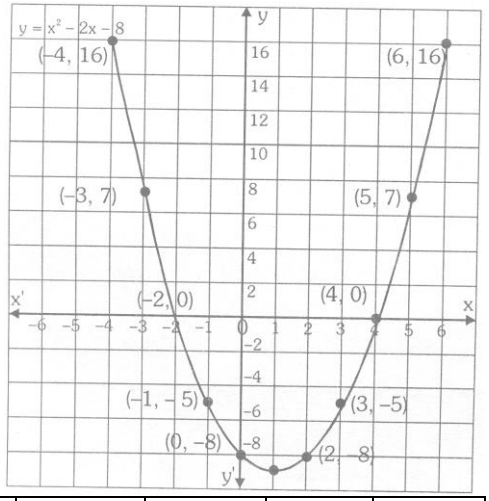
From, graph we observer that the graph of y = 2x – 5 intersects the x-axis at $\left(\frac{5}{2}, 0\right)$ whose x-coordinate is $\frac{5}{2}$. Also, zero of 2x – 5 is $\frac{5}{2}$.

Therefore, we conclude that the linear polynomial as + b has one and only one zero, which is the x-coordinate of the point where the graph of y = ax + b intersects the x-axis

Geometrical Representation of the zero of a quadratic Polynomial :

Consider quadratic polynomial, y = x2 – 2x – 8,

The following table gives the values of y or f(x) for various values of x.



x	– 4	– 3	– 2	– 1	0	1	2	3	4	5	6
$y = x^2 - 2x - 8$	16	7	0	–5	–8	–9	–8	–5	0	7	16

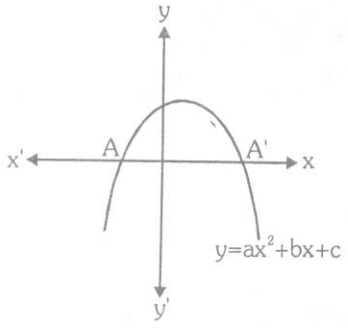
On plotting the points (-4, 16), (-3, 7)(-2, 0), (-1, -5), (0, -8), (1, -9), (2, -8), (3, -5), (4, 0), (5, 7) and (6, 16) on a graph paper and drawing a smooth free hand curve passing through these points, the curve thus obtained represents the graph of the polynomial y = x2 – 2x – 8. This is called a parabola.

It is clear from the table that –2 and 4 are the zeros of the quadratic polynomial x2 – 2x – 8. Also, we observe that –2 and 4 are the x-coordinates of the points where the graph of y = x2 – 2x – 8 intersects the x-axis.

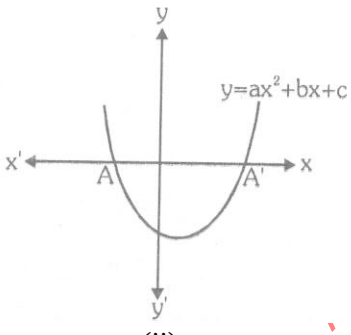
Consider the following cases –

Case-I : Here, the graph cuts x–axis at two distinct points A and A'.

The x-coordinates of A and A' are two zeroes of the quadratic polynomial ax2 + bx + c.

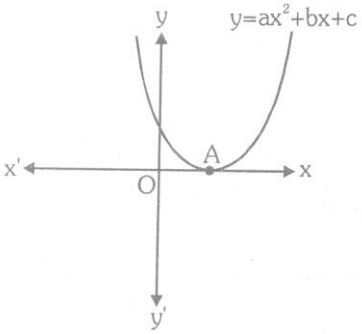


(i)

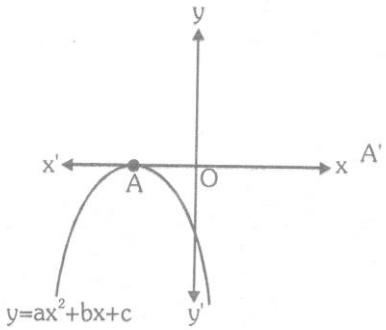


(ii)

Case-II : Here, the graph cuts the x-axis at exactly one point, i.e., at two coincident points. So, the two points A and A' of Case (i) coincide here to become one point A.



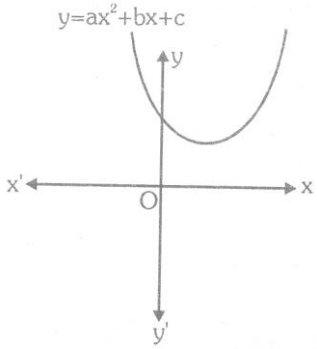
(iii)



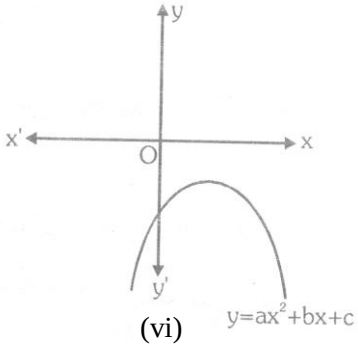
(iv)

The x-coordinate of A is the only zero for the quadratic polynomial ax2 + bx + c in this case.

Case-III : Here, the graph is either completely above the x-axis or completely below the x-axis, So, it does not cut the x-axis at any point.



(v)



(vi)

So, the quadratic polynomial ax2 + bx + c has no zero in this case.

So, you can see geometrically that a quadratic polynomial can have either two distinct zeroes or one zero, or no zero. This also means that a polynomial of degree 2 has atmost two zeroes.

Remark : In general given a polynomial p(x) of degree n, the graph of y = p(x) intersects the x-axis at atmost n points. Therefore, a polynomial p(x) of degree n has atmost n zeros.

★ Relationship Between The Zeros And Coefficients Of A Polynomial

For a linear polynomial ax + b, (a ≠ 0), we have,

$$\frac{\text{(constant term)}}{\text{(coefficient of x)}}$$

zero of a linear polynomial $= -\frac{b}{a} = -$

For a quadratic polynomial ax2 + b + c (a ≠ 0), with α and β as it's zeros, we have

$$\text{Sum of zeros} = \alpha + \beta = -\frac{b}{a} = -\frac{\text{(coefficient of x)}}{\text{(coefficient of } x^2\text{)}}$$

$$\text{Product of zeros} = \alpha\beta = \frac{c}{a} = \frac{\text{(constant term)}}{\text{(coefficient of } x^2\text{)}}$$

If α and β are the zeros of a quadratic polynomial f(x). Then polynomial f(x) is given by

$f(x) = K\{x^2 - (\alpha + \beta)x + \alpha\ \beta\}$

or $f(x) = K\{x^2 - (\text{sum of the zeros})\ x + \text{product of the zeros}\}$

where K is a constant .

COMPETITION WINDOW

RELATIONSHIP BETWEEN THE ZEROS AND COEFFICIENTS OF CUBIC POLYNOMIAL

For a cubic polynomial $ax^3 + bx^2 + cx + d$ ($a \neq 0$), with α, β and λ at its zeros, we have :

Sum of three zeros $= \alpha + \beta + \lambda = \frac{-b}{a}$

Sum of the product of its zeros taken two at a time $= \alpha\beta + \beta\lambda + \lambda\alpha = \frac{c}{a}$

Product of its zeros $= \alpha\beta\lambda = \frac{-d}{a}$

The cubic polynomial whose zeros are α, β and λ is given by

$f(x) = \{x^3 - (\alpha + \beta + \lambda)x^2 + (\alpha\beta + \beta\lambda + \lambda\alpha)x - \alpha\beta\lambda\}$

RELATIONSHIP BETWEEN THE ZEROS AND COEFFICIENTS OF A BI-QUADRATIC POLYNOMIAL

For a bi-quadratic polynomial $ax^4 + bx^3 + cx^2 + dx + e$ ($a \neq 0$), with α, β, λ and δ as its zeros, we have :

Sum of four zeros $= \alpha + \beta + \lambda + \delta = \frac{-b}{a}$

Sum of the product of its zeros taken two at a time $= \alpha\beta + \alpha\lambda + \alpha\delta + \beta\lambda + \beta\delta + \lambda\delta = \frac{c}{a}$

Sum of the product of its zeros taken three at a time $= \alpha\beta\lambda + \alpha\beta\delta + \beta\lambda\delta + \lambda\delta\alpha = \frac{-b}{a}$

Product of all the four zeros $= \alpha\beta\lambda\delta = \frac{e}{a}$

The bi-quadratic polynomial whose zeros are α, β, λ and δ is given by

$f(x) = \{x^4 - (\alpha + \beta + \lambda + \delta)x^3 + (\alpha\beta + \alpha\lambda + \alpha\delta + \beta\lambda + \beta\delta + \lambda\delta)x^2 - (\alpha\beta\lambda + \alpha\beta\delta + \beta\lambda\delta + \lambda\delta\alpha)x + \alpha\beta\lambda\delta\}$

Ex. 1 **Find the zeros of the quadratic polynomial $x^2 + 7x + 12$, and verify the relation between the zeros and its coefficients**

Sol. We have,

$f(x) = x^2 + 7x + 12 = x^2 + 4x + 3x + 12$

$\Rightarrow$ $f(x) = x(x + 4) + 3(x + 4)$

$\Rightarrow$ f(x) = (x + 4)(x + 3)

The zeros of f(x) are given by

f(x) = 0

$\Rightarrow$ $x^2 + 7x + 12 = 0$

$\Rightarrow$ (x + 4)(x + 3) = 0

$\Rightarrow$ x + 4 = 0 or, x + 3 = 0

$\Rightarrow$ x = - 4 or x = - 3

Thus, the zeros of $f(x) = x^2 + 7x + 12$ are $\alpha = -4$ and $\beta = -3$

Now, sum of the zeros $= \alpha + \beta = (-4) + (-3) = -7$

and $-\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^2} = -\dfrac{7}{1} = -7$

$\therefore$ Sum of the zeros = $-\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$

Product of the zeros $= \alpha\beta = (-4) \times (-3) = 12$

and, $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{12}{1} = 12$

$\therefore$ Product of the zeros = $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Ex.2 **Find the zeros of the quadratic polynomial $f(x) = abx^2 + (b^2 + ac)\,x + be$ and verify the relationship between the zeros and its coefficients.**

Sol. $f(x) = abx^2 + (b^2 + ac)\,x + bc = abx^2 + b^2x + acx + bc$.

$= bx\,(ax + b) + c\,(ax + b) = (ax + b)\,(bx + c)$

So, the value of f(x) is zero when ax + b = 0 or bx + c = 0, i.e. $x = \dfrac{-b}{a}$ or $x = \dfrac{-c}{b}$

Therefore, $\dfrac{-b}{a}$ and $\dfrac{-c}{b}$ are the zeros (or roots) of f(x).

Now, sum of zeros $= \left(\dfrac{-b}{a}\right) + \left(\dfrac{-c}{b}\right) = \dfrac{-b^2 - ac}{ab} = \dfrac{-(b^2 + ac)}{ab} = -\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$

Product of zeros $= \left(\dfrac{-b}{a}\right)\left(\dfrac{-c}{b}\right) = \dfrac{bc}{ab} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

★ SYMMETRIC FUNCTIONS OF THE ZEROS

Let α, β be the zeros of a quadratic polynomial, then the expression of the form $\alpha + \beta; (\alpha^2 + \beta^2)$; $\alpha\beta$ are celled the functions of the zeros. By symmetric function we mean that the function remain invariant (unaltered) in values when the roots are changed cyclically. In other words, an expression involving α and β which remains unchanged by interchanging α and β is called symmetric function of α and β.

Some useful relations involving α and β are :-

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

(ii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

(iii) $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

(iv) $\alpha^2+\beta^2=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

(v) $\alpha^3-\beta^3=(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)$

(vi) $\alpha^4-\beta^4=(\alpha^2+\beta^2)(\alpha+\beta)(\alpha-\beta)=[(\alpha+\beta)^2-2\alpha\beta](\alpha+\beta)\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

(vii) $\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2=[(\alpha+\beta)^2-2\alpha\beta]^2-2(\alpha\beta)^2$

(viii)

$\alpha^5+\beta^5=(\alpha^3+\beta^3)(\alpha^2+\beta^2)-\alpha^2\beta^2(\alpha+\beta)=[(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)][(\alpha+\beta)^2-2\alpha\beta]-(\alpha\beta)^2(\alpha+\beta)$

Ex.3 If α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$ then calculate :

(i) $\alpha^2+\beta^2$ **(ii)** $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Sol. Since α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$

$\therefore \quad \alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

(i) We have,

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow \alpha^2+\beta^2=\left(\frac{-b}{a}\right)^2-\frac{2c}{a}=\frac{b^2-2ac}{a^2}$

(ii) We have, $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}=\frac{\left(-\frac{b}{a}\right)^3-3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)}{\frac{c}{a}}\Rightarrow\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{3abc-b^3}{a^2c}$

Ex.4 If α and β are the zeros of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$, find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$

Sol. Since α and β are the zeros of the polynomial $p(s) = 3s^2 - 6s + 4$.

$\therefore \quad \alpha+\beta=\frac{-(-6)}{3}=2$ and $\alpha\beta=\frac{4}{3}$

We have $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{\alpha^2+\beta^2}{\alpha\beta}+2\left(\frac{\beta+\alpha}{\alpha\beta}\right)+3\alpha\beta$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}+\frac{2(\alpha+\beta)}{\alpha\beta}+3\alpha\beta=\frac{(2)^2-2\times\frac{4}{3}}{\frac{4}{3}}+\frac{2\times2}{\frac{4}{3}}+3\times\frac{4}{3}=8$

Ex.5 If α and β are the roots (zeros) of the polynomial $f(x) = x^2 - 3x + k$ such that $\alpha-\beta=1$, find the value of k.

Sol. Since α and β are the roots (zeros) of the polynomial $f(x) = x^2 - 3x + k$.

$\therefore \quad \alpha+\beta=\frac{-(-3)}{1}=3$ and $\alpha\beta = k$.

We have $\alpha-\beta=1\Rightarrow(\alpha-\beta)^2=(1)^2\Rightarrow\alpha^2-2\alpha\beta+\beta^2=1$

$\Rightarrow \quad (\alpha^2+\beta^2)-2\alpha\beta=1\Rightarrow\{(\alpha+\beta)^2-2\alpha\beta\}-2\alpha\beta=1$

$\Rightarrow \quad (\alpha+\beta)^2-4\alpha\beta=1\Rightarrow(3)^2-4\times k=1$

$\Rightarrow \quad 9 - 4k = 1 \Rightarrow 4k = 8 \Rightarrow k = 2$

Hence, the value of k is 2.

Ex.6 **If α, β are the zeros of the polynomial $f(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k for this to be possible .**

Sol. Since α and β are the zeros of the polynomial $f(x) = 2x^2 + 5x + k$

$\therefore \quad \alpha + \beta = \frac{-5}{2}$ and $\alpha\beta = \frac{k}{2}$

Now, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

$\Rightarrow \quad (\alpha^2 + \beta^2 + 2\alpha\beta) - \alpha\beta = \frac{21}{4}$

$\Rightarrow \quad (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$

$\Rightarrow \quad \frac{25}{4} - \frac{k}{2} = \frac{21}{4} \qquad \left[\because \alpha + \beta = -\frac{5}{2} \text{ and } \alpha\beta = \frac{k}{2}\right]$

$\Rightarrow \quad -\frac{k}{2} = -1$

$\Rightarrow \quad k = 2$

Ex.7 **Find a quadratic polynomial each with the given numbers as the sum and product of its zeros prospectively.**

(i) $\frac{1}{4}, -1$ **(ii)** $\sqrt{2}, \frac{1}{3}$

Sol. We know that a quadratic polynomial which the sum and product of its zeros are given i given by –

$f(x) = k\{x^2 - (\text{Sum of the zeros})\, x + \text{Product of the zeros}\}$, where k is a constant.

(i) Required quadratic polynomial f(s) is given by

$$f(x) = k\left(x^2 - \frac{1}{4}x - 1\right)$$

(ii) Required quadratic polynomial f(s) is given by

$$f(x) = k\left(x^2 - \sqrt{2}x + \frac{1}{3}\right)$$

Ex.8 **If α, β are the zeros of the polynomial $ax^2 + bx + c$, find a polynomial whose zeros are $\frac{1}{a\alpha + b}$ and $\frac{1}{a\beta + b}$**

Sol. since α and β are the zeros of the polynomial $ax^2 + bc + c$.

$\therefore \quad \alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Since $\frac{1}{a\alpha + b}$ and $\frac{1}{a\beta + b}$ are the zeros of the require polynomial

$\therefore \quad$ sum of the zeros $= \frac{1}{a\alpha + b} + \frac{1}{a\beta + b} = \frac{a\beta + b + a\alpha + b}{(a\alpha + b)(a\beta + b)}$

$$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2} = \frac{a\times\left(\frac{-b}{a}\right)+2b}{a^2\times\left(\frac{c}{a}\right)+ab\times\left(\frac{-b}{a}\right)+b^2} = \frac{b}{ac}$$

Product of the zeros $= \left(\frac{1}{a\alpha+b}\right)\left(\frac{1}{a\beta+b}\right) = \frac{1}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

$$= \frac{1}{a^2\times\frac{c}{a}+ab\times\left(\frac{-b}{a}\right)+b^2} = \frac{1}{ac}$$

Hence, the required polynomial = x^2 – (sum of zeros) x + product of zeros $= x^2 - \left(\frac{b}{ac}\right)x + \frac{1}{ac}$

★ DIVISION ALGORITHM FOR POLYNOMIALS

If f(x) is a polynomial and g(x) is a non-zero polynomial, then there exist two polynomials q(x) and r(x) such that f(x) = g(x) ×q(x) + r(x) , where r(x) = 0 or degree r(x) < degree g(x). In other words,

Dividend = Divisor × Quotient + Remainder

Remark : If r(x) = 0, then polynomial g(x) is a factor of polynomial f(x).

Ex.9 **divide the polynomial $2x^2 + 3x + 1$ by the polynomial x + 2 and verify the division algorithm .**

Sol. We have

$$\begin{array}{r|l} x+2 & 2x^2+3x+1 \quad (2x-1 \\ & \underline{2x^2+4x} \\ & -x+1 \\ & \underline{-x-2} \\ & 3 \end{array}$$

Clearly, quotient = 2x -1 and remainder = 3

Also, (x + 2) (2x - 1) + 3 = $2x^2 + 4x - x - 2 + 3 = 2x^2 + 3x + 1$

i.e., $2x^2 + 3x + 1 = (x + 2)(2x - 1) + 3$. Thus, Dividend = Divisor × Quotient + Remainder.

Ex.10 **Check whether the polynomial $t^2 - 3$ is a factor of the polynomial $2t^4 + 3t^3 - 2t^2 = 9t - 12$, by dividing the second polynomial by the first polynomial.**

Sol. We have

$$\begin{array}{r|l} t^2-3 & 2t^4+3t^3-2t^2-9t-12 \quad (2t^2+3t+4 \\ & \underline{2t^4 \qquad\quad -6t^2} \\ & 3t^3+4t^2-9t-12 \\ & \underline{3t^3 \qquad\quad -9t} \\ & 4t^2 \qquad -12 \\ & \underline{4t^2 \qquad -12} \\ & 0 \end{array}$$

Since the remainder is zero, therefore, the polynomial $t^2 - 3$ is a factor of the polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

$$\begin{array}{r|l} x^2-2 & 2x^4-3x^3-3x^2+6x-2 \quad (2x^2-3x+1 \\ & \underline{2x^4 \qquad\quad -4x^2} \\ & -3x^3+x^2+6x-2 \\ & \underline{-3x^3 \qquad\quad +6x} \\ & x^2 \qquad -2 \\ & \underline{x^2 \qquad -2} \\ & 0 \end{array}$$

Ex.11 **Find all the zeros of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeros are $\sqrt{2}$ and $\sqrt{2}$.**

Sol. Let p(x) $2x^4 - 3x^3 - 3x^2 + 6x - 2$ be the given polynomial. Since two zeros are $\sqrt{2}$ and $-\sqrt{2}$ so, $(x - \sqrt{2})$ and $(x + \sqrt{2})$ are both factors of the given polynomial p(x).

Also, $(x - \sqrt{2}) = (x^2 - 2)$ is a factor of the polynomial. Now , we divide the given polynomial by $x^2 - 2$.

By division algorithm, we have

$$2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$$

$$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})(x + \sqrt{2})(2x^2 - 2x - x + 1)$$

$$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})(x + \sqrt{2})\{2x(x - 1) - (x - 1)\}$$

$$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})(x + \sqrt{2})(x - 1)(2x - 1)$$

When p(x) = 0, $x = \sqrt{2}, -\sqrt{2}, 1$ and $\frac{1}{2}$

Hence, all the zeros of the polynomial $2x^4 - 3x^3 - 3x^2 + 6x - 2$ are $\sqrt{2}, -\sqrt{2}, 1$ and $\frac{1}{2}$

Ex.12 **On dividing $f(x) = x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x)**

Sol. Here, Divided $= x^3 - 3x^2 + x + 2$,

Quotient $= x - 2$,

Remainder $= -2x + 4$ and Divisor = g(x).

Since **Dividend = Divisor × Quotient + Remainder**

So , $x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$$\Rightarrow \quad g(x) \times (x - 2) = x^3 - 3x^2 + x + 2 + 2x - 4$$

$$\Rightarrow \quad g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2} = \frac{(x - 2)(x^2 - x + 1)}{x - 2} = x^2 - x + 1$$

Hence, $g(x) = x^2 - x + 1$.

★ SYNOPSIS

1. The highest power of the variable (x) in a polynomial p(x) is called a degree of polynomial p(x).

2. A polynomial of degree one is called linear polynomial :

$p(x) = ax + b$, where $a \neq 0$ | a = coefficient of x ; b = constant term

3. A polynomial of a degree two is called quadratic polynomial :

$p(x) = ax^2 + bx + c$, where $a \neq 0$ | a = coefficient of x^2 ; b = coefficient of x ; c = constant term

4. A polynomial of degree three is called a cubic polynomial : $p(x) = ax^3 + bx2 + cx + d$, $a \neq 0$.

5. The zeros of a polynomial p(x) are precisely the x-coordinates of the point where the graph of y = p(x) intersects the x-axis.

6. The graph of the quadratic function $y = ax^2 + bx + c$, $a \neq 0$ is a parabola .

7. The parabola opens upwards if $a > 0$ and opens downwards if $a < 0$.

8. A polynomial of degree n can have at most n zeros. So the quadratic polynomial can have at most two zeros and a cubic polynomial can have at most three zeros.

9. If α, β are the zeros of a quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ then

Sum of its zeros $= \alpha + \beta = -\frac{b}{a}$ and Product of its zeros $= \alpha\beta = \frac{c}{a}$.

10. If α, β, γ are the zeros of a cubic polynomial $ax^3 + bx^2 + cx + d$, $a \neq 0$ then

Sum of its zeros $= \alpha + \beta + \gamma = -\frac{b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

Sum of the products of zeros taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

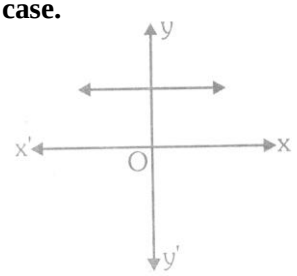
Product of its zeros $= \alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

11. The division algorithm states that given any polynomial p(x) and any non-zero polynomial g(x) then we can find quotient polynomial q(x) and remainder polynomial r(x) such that :
p(x) = g(x). q(x) + r(x) where deg. of r(x) < degree of g(x), deg of r(x) = 0.

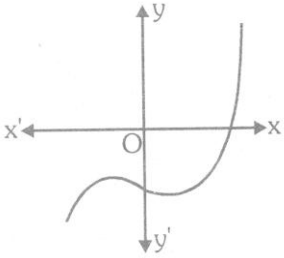
SOLVED NCERT EXERCISE

EXERCISE : 2 . 1

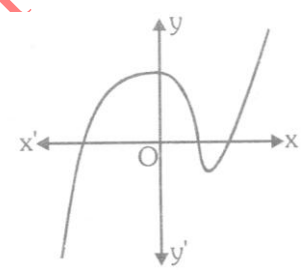
1. The graph of y – p(x) are given in fig below, for some polynomials p(x). Find the number of zeros of p(x), in each case.



(i)



(ii)



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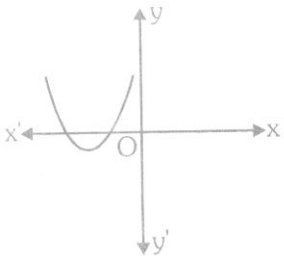

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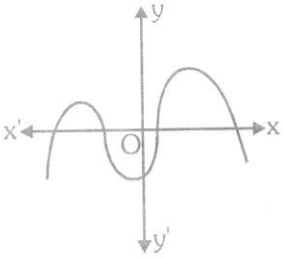
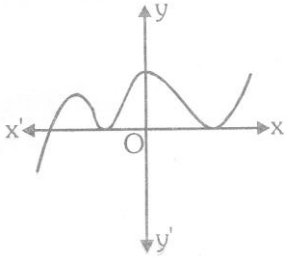


(v)



(vi)

Sol. (i) Graph of y = p(x) does not intersect the x-axis. Hence, polynomial p(x) has no zero.
(ii) Graph of y = p(x) intersects the x-axis at one and only one point.
Hence, polynomial p(x) has **one end only one** real zero.
[Rest Try Yourself]

EXERCISE : 2.2

1. Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients.
(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$ (iii) $6x^2 - 3 - 7x$ (iv) $4u^2 + 8u$
(v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Sol. (i) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x - 4) + 2(x - 4) = (x + 2)(x - 4)$
Zeros are –2 and 4.

$\frac{\text{-Coefficient of } x}{\text{Coefficient of } x^2}$

$\frac{\text{Constant term}}{}$

Sum of the zeros = (–2) + (4) = 2 = $\frac{-(-2)}{1}$ =

Product of the zeros $= (-2)(4) = -(8) = \frac{(-8)}{1}$ =

(ii) $4s^2 - 4s + 1 = (2s - 1)^2$

The two zeros are $\frac{1}{2}, \frac{1}{2}$

Sum of the two zeros $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{\text{-Coefficient of x}}{\text{Coefficient of } x^2}$

Product of two zeros $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

[Rest Try Yourself]

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $\frac{1}{4}, -1$ **(ii)** $\sqrt{2}, \frac{1}{3}$ **(iii)** $0, \sqrt{5}$ **(iv) 1, 1** **(v)** $-\frac{1}{4}, \frac{1}{4}$ **(iv) 4, 1**

Sol. (i) Let the quadratic polynomial be $ax^2 + bx + c$

Then $-\frac{b}{a} = \frac{1}{4}$ and $\frac{c}{a} = -1$

i.e., $\frac{b}{a} = \frac{-1}{4}$ and $\frac{c}{a} = \frac{-1}{1}$

We select a = LCM (4, 1) = 4

Then $\frac{b}{4} = \frac{-1}{4}$ and $\frac{c}{4} = -1 \Rightarrow b = -1$ and $c = -4$.

Substituting $a = 4, b = -1, c = -4$ in $ax^2 + bx + c$, we get the required polynomial $4x^2 - x - 4$

(ii) $-\frac{b}{a} = \sqrt{2}, \quad \frac{c}{a} = \frac{1}{3}$

$\Rightarrow \quad \frac{b}{a} = \frac{-\sqrt{2}}{1}, \quad \frac{c}{a} = \frac{1}{3}$

Select a = LCM (1, 3) = 3.

Then $\frac{b}{3} = -\sqrt{2}$ and $\frac{c}{a} = \frac{1}{3} \Rightarrow b = -3\sqrt{2}$ and c = 1.

Substituting $a = 3, b = -3\sqrt{2}$ and c = 1 in $ax^2 + bx + c$, we get the required polynomial $3x^2 - 3\sqrt{2}x + 1$

[Rest Try Yourself]

EXERCISE : 2.3

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3, g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5, g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6, g(x) = 2 - x^2$.

$x^2 - 2 \overline{)x^3 - 3x^2 + 5x - 3} \; q(x) = (x - 3)$

$x^3 \quad -2x$

$- \quad +$

$-3x^2 + 7x - 3$

Sol. (i)

Hence, Quotient q(x) = x – 3 and Remainder r(x) = 7x – **9**

[Rest Try Yourself]

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

(i) $t^2 - 3, 2t^4 + 3t^3 - 9t - 12$ **(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$**

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Sol. (i)

$t^2 - 3 \overline{)2t^4 + 3t^3 - 2t^2 - 9t - 12}$ q (t) $2t^2 + 3t + 4$

$2t^4 \quad - 6t^2$ (− +)

$3t^2 + 4t^2 - 9t - 12$

$3t^2 \quad - 9t$ (− +)

$4t^2 - 12$

$4t^2 - 12$ (− +)

Remainder = 0

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

[Rest Try Yourself]

3. Obtain all other zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, If two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Two of the zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)$ is a factor of the polynomial .

i.e., $x^2 - \frac{5}{3}$ is a factor.

i.e., $(3x^2 - 5)$ is a factor of the polynomial. Then we apply the division algorithm as below :

$3x^2 - 5 \overline{)3x^4 + 6x^3 - 2x^2 - 10x - 5}$ q (x) $= x^2 + 2x + 1$

$3x^4 \quad - 5x^2$ (− +)

$6x^3 + 3x^2 - 10x - 5$

$6x^3 \quad - 10x$ (− +)

$3x^2 - 5$

$3x^2 - 5$ (− +)

×

The other two zeros will be obtained from the quadratic polynomial $q(x) = x^2 + 2x + 1$

Now $x^2 + 2x + 1 = (x + 1)^2$.

Its zeros are –1 , –1.

Hence, all other zeros are –1 , –1.

4. **On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x – 2 and –2x + 4, respectively. Find g(x).**
[Try Yourself]

5. **Give examples of polynomial p(x), g(x) and r(x), which satisfy the division algorithm and**
(i) deg p(x) = deg q(x) (ii) deg q(x) = deg r(x) (iii) deg r(x) = 0,

Sol. (i) $p(x) = 2x^2 + 2x + 8$, $g(x) = 2x^0 = 2$; $q(x) = x^2 + x + 4$; $r(x) = 0$
(ii) $p(x) = 2x^2 + 2x + 8$, $g(x) = x^2 + x + 9$; $q(x) = 2$; $r(x) = -10$
(iii) $p(x) = x^3 + x + 5$; $g(x) = x^2 + 1$; $g(x) = x$; $r(x) = 5$.

EXERCISE – 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

Choose The Correct One

1. Quadratic polynomial having zeros 1 and –2 is -
(A) $x^2 - x + 2$ (B) $x^2 - x - 2$
(C) $x^2 + x - 2$ (D) None of these

2. If (x – 1) is a factor of $k^2x^3 - 4kx - 1$, then the value of k is –
(A) 1 (B) – 1
(C) 2 (D) – 2

3. For what value of a is the polynomial $2x^4 - ax^3 = 4x^2 + 2x + 1$ divisible by 1 – 2x ?
(A) a = 25 (B) a = 24 (C) a = 23 (D) a = 22

4. If one of the factors of $x^2 + x - 20$ is (x + 5), then other factor is -
(A) (x – 4) (B) (x – 5) (C) (x – 6) (D) (x – 7)

5. If α, β be the zeros of the quadratic polynomial $2x^2 + 5x + 1$, then value of $\alpha + \beta + \alpha\beta =$
(A) – 2 (B) – 1 (C) 1 (D) None of these

6. If α, β be the zeros of the quadratic polynomial $2 - 3x - x^2$, then $\alpha + \beta =$
(A) 2 (B) 3 (C) 1 (D) None of these

7. Quadratic polynomial having sum of it's zeros 5 and product of it's zeros – 14 is-
(A) $x^2 - 5x - 14$ (B) $x^2 - 10x - 14$
(C) $x^2 - 5x + 14$ (D) None of these

8. If x = 2 and x = 3 are zeros of the quadratic polynomial $x^2 + ax + b$, the values of a and b respectively are :
(A) 5, 6 (B) – 5 , – 6 (C) –5, 6 (D) 5, 6

9. If 3 is a zero of the polynomial $f(x) = x^4 - x^3 - 8x^2 + kx + 12$, then the value of k is -
(A) – 2 (B) 2 (C) – 3 (D) $\frac{3}{2}$

10. The sum and product of zeros of the quadratic polynomial are – 5 and 3 respectively the quadratic polynomial is equal to -

(A) $x^2 + 2x + 3$ (B) $x^2 – 5x + 3$ (C) $x^2 + 5x + 3$ (D) $x^2 + 3x – 5$

11. On dividing $x^3 – 3x^2 + x + 2$ by polynomial g(x), the quotient and remainder were x – 2 and 4 – 2x respectively then g(x) :

(A) $x^2 + x + 1$ (B) $x^2 + x – 1$

(C) $x^2 – x – 1$ (D) $x^2 – x + 1$

12. If the polynomial $3x^3 – x3 – 3x + 5$ is divided by another polynomial x – 1 – x2 , the remainder comes out to be 3, then quotient polynomial is –

(A) 2 – x (B) 2x – 1 (C) 3x + 4 (D) x – 2

13. If sum of zeros $= \sqrt{2}$, product of its zeros $= \frac{1}{3}$. The quadratic polynomial is –

(A) $3x^2 - 3\sqrt{2}x + 1$ (B) $\sqrt{2}x^2 + 3x + 1$

(C) $3x^2 - 2\sqrt{3}x + 1$ (D) $\sqrt{2}x^2 + x + 3$

14. If $-\frac{1}{3}$ is the zeros of the cubic polynomial $f(x) = 3x^3 – 5x^2 – 11x – 3$ the other zeros are :

(A) – 3 , –1 (B) 1, 3 (C) 3, –1 (D) –3, 1

15. If α and β are the zeros of the polynomial $f(x) = 6x^2 – 3 – 7x$ then $(\alpha + 1)(\beta + 1)$ is equal to –

(A) $\frac{5}{2}$ (B) $\frac{5}{3}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

16. Let $p(x) = ax^2 + bx + c$ be q quadratic polynomial. It can have at most –

(A) One zero (B) Two zeros

(C) Three zeros (D) None of these

17. The graph of the quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ is always-

(A) Straight line (B) Curve

(C) Parabola (D) None of these

18. If 2 and $-\frac{1}{2}$ as the sum and product of its zeros respectively then the quadratic polynomial f(x) is –

(A) $x^2 – 2x – 4$ (B) $4x^2 – 2x + 1$

(C) $2x^2 + 4x – 1$ (D) $2x^2 – 4x – 1$

19. If α and β are the zeros of the polynomial $f(x) = 16x^2 + 4x – 5$ then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to –

(A) $\frac{2}{5}$ (B) $\frac{5}{2}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

20. If α and β are the zeros of the polynomial $f(x) = 15x^2 – 5x + 6$ then $\left(1 + \frac{1}{\alpha}\right)\left(1 + \frac{1}{\beta}\right)$ is equal to –

(A) $\frac{13}{3}$ (B) $\frac{13}{2}$ (C) $\frac{16}{3}$ (D) $\frac{15}{2}$

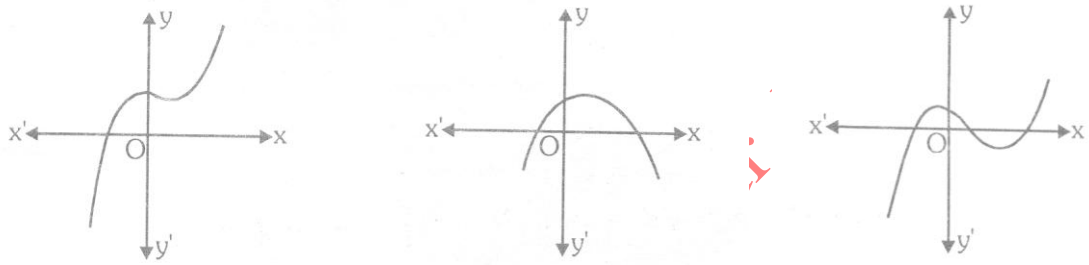
OBJECTIVE					ANSWER KEY										EXERCISE - 1
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	A	A	A	A	D	A	C	B	C	D	D	A	C	B
Que.	16	17	18	19	20										
Ans.	B	C	D	D	A										

EXERCISE – 2 (FOR SCHOOL / BOARD EXAMS)

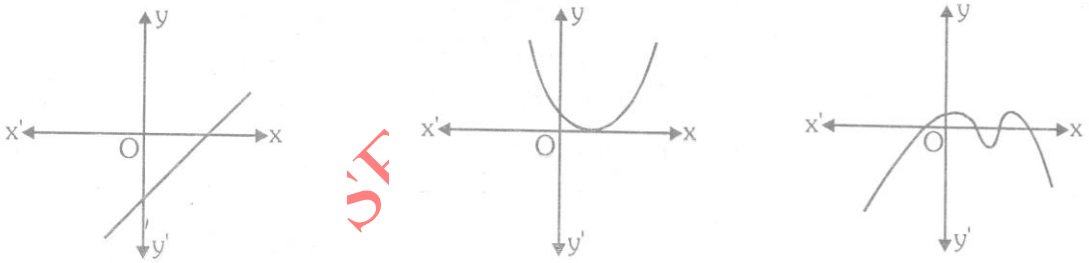
SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions

1. Look at the graph in fig given below. Each is the graph of y = p(x) , where p(x) is a polynomial. For each of the graph, find the number of zeros of p(x).

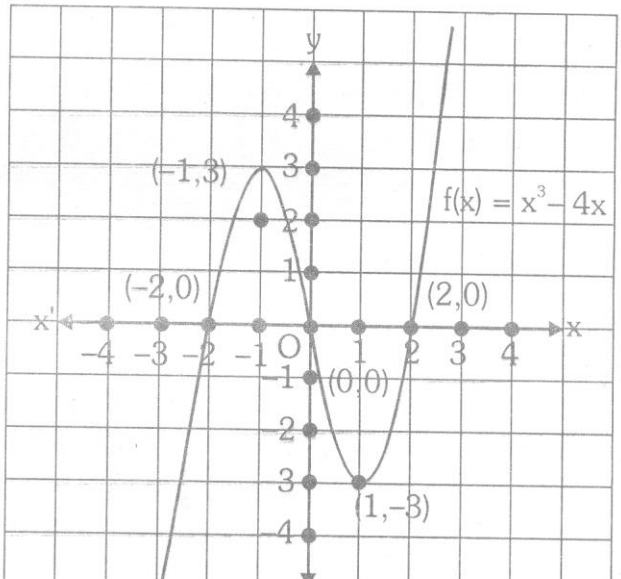


(i) (ii) (iii)



(iv) (v) (vi)

2. Consider the cubic polynomial $f(x) = x^3 - 4x$. Find from the fig, the number of zeros of the above stated polynomial

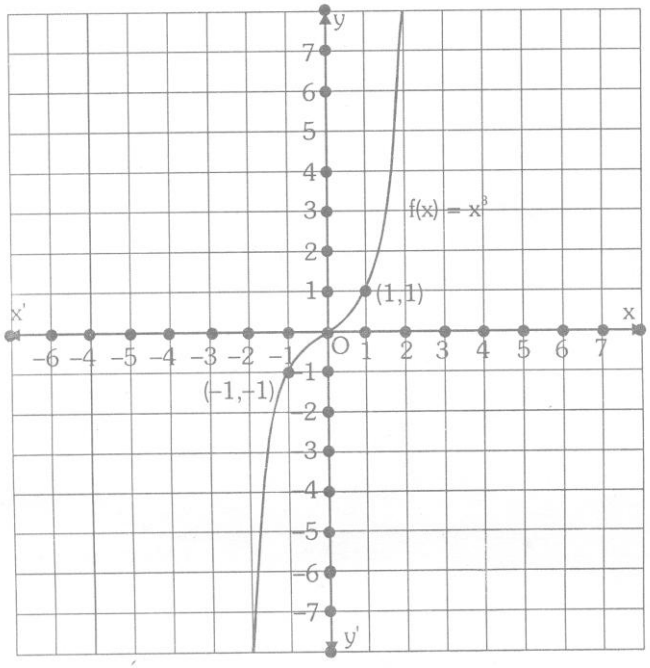


3. Let $f(x) = x^3$

The graph of the polynomial is shown in fig.

(i) Find the number of zeros of polynomial f(x).

(ii) Determine the co-ordinates of the points, at which the graph intersects the x-axis



Short answer Type Questions

1. Find the zeros of the following quadratic polynomial and verify the relationship between the zeros and their coefficients.

(i) $6x^2 - x - 1$ (ii) $25x(x+1)+4$ (iii) $4x^2 + 4x + 1$ (iv) $48y^2 - 13y - 1$ (v) $63 - 2x - x^2$

(vi) $2x^2 - 5x$ (vii) $49x^2 - 81$ (viii) $4x^2 - 4x - 3$

2. Find a quadratic polynomial each with the given numbers as the zeros of the polynomial .

(i) $3+\sqrt{7}, 3-\sqrt{7}$ (ii) $2\sqrt{3}, -2\sqrt{3}$ (iii) $-\frac{3}{7}, -\frac{2}{3}$ (iv) $\sqrt{3}, 3\sqrt{3}$ (v) $2+3\sqrt{2}, 2-3\sqrt{2}$ (vi) $\frac{8}{3}, \frac{5}{2}$

3. Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively .

(i) $4\sqrt{3}, 9$ (ii) $2\sqrt{3}-1, 3-\sqrt{3}$ (iii) $0, -\frac{1}{4}$ (iv) $\frac{-10}{\sqrt{3}}, 7$ (v) $\frac{5}{6}, \frac{25}{9}$ (vi) $\frac{-2\sqrt{5}}{3}, -\frac{5}{3}$ (vii) $-\sqrt{3}, \frac{1}{4}$ (viii) $-\frac{6}{5}, \frac{9}{25}$

(ix) $\sqrt{2}, -12$

4. If α and β are the zeros of the polynomial $f(x)\ 5x^2 + 4x - 9$ then evaluate the following :

(i) $\alpha - \beta$ (ii) $\alpha^2 + \beta^2$ (iii) $\alpha^2 - \beta^2$ (iv) $\alpha^3 + \beta^3$ (v) $\alpha^3 - \beta^3$ (vi) $\alpha^4 - \beta^4$

5. If one of the zeros of the quadratic polynomial $2x^2 + px + 4$ is 2, find the other zero. Also find the value of p.

6. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is the reciprocal of the other, find the value of a .

7. If the product of zeros of the polynomial $ax^2 - 6x - 6$ is 4, find the value of a.

8. Find the zeros of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeros and the coefficients of the polynomial .

9. Determined if 3 is a zero of p(x) $= \sqrt{x^2-4x+3}+\sqrt{x^2-9}-\sqrt{4x^2-14x+6}$

10. If α and β be two zeros of the quadratic polynomial $ax^2 + bx + c$, then valuate :

(i) $\alpha^2+\beta^2$ (ii) $\alpha^3+\beta^3$ (iii) $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$ (iv) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

11. Find the value of k :

(i) If α and β are the zeros of the polynomial $x^2 - 5x + k$ where $\alpha-\beta=1$

(ii) If α and β are the zeros of the polynomial x2 – 8x + k such that $\alpha^2+\beta^2=40$.

(iii) If α and β are the zeros of the polynomial $x^2 - 6x + k$ such that $3\alpha+2\beta=20$.

12. If 2 and 3 are zeros of polynomial $3x^2 - 2kx + 2m$, find the values of k and m.

13. If one zeros of polynomial $3x^2 = 8x + 2x + 1$ is seven times the other, then find the zeros and the value of k.

14. If α and β are the zeros of the polynomial $2x^2 - 4x + 5$. Form the polynomial where zeros are :

(i) $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ (ii) $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ (iii) $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$

15. If α and β are the zeros of the quadratic polynomial $x^2 - 3x + 2$, find a quadratic polynomial whose zeros are :

(i) $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$ (ii) $\frac{\alpha-1}{\alpha+1}$ and $\frac{\beta-1}{\beta+1}$

16. If the sum of the squares of zeros of the polynomial $5x^2 + 3x + k$ is $-\frac{11}{25}$, find the value of k .

17. If one zero of the quadratic polynomial $2x^2 - (3k + 1) x - 9$ is negative of the other, find the value of k .

18. If α and β are the two zeros of the quadratic polynomial $x^2 - 2x + 5$, find a quadratic polynomial whose zeros are $\alpha+\beta$ and $\frac{1}{\alpha}+\frac{1}{\beta}$

19. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$, prove that $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2}{q}+2$

20. Apply the division algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by g(x) as given below :

(i) $p(x) = 3x^3 + 2x^2 + x + 1$; $g(x) = x^3 + 3x + 2$

(ii) $p(x) = x^6 + x^4 - x^2 - 1$; $g(x) = x^3 - x^2 + x - 1$

(iii) $p(x) = 2x^5 + 3x^4 + 4x^2 + 3x + 2$; $g(x) = x^3 + x^2 + x + 1$

(iv) $p(x) = x^3 - 3x^2 - x + 3$; $g(x) = x^2 - 4x + 3$

21. Find the quotient q(x) and remainder r(x) of the following when f(x) is divided by g(x). Verify the division algorithm.

(i) $f(x) = x^6 + 5x^3 + 7x + 3$; $g(x) = x^2 + 2$

(ii) $f(x) = x^4 + 2x^2 + 1$; $g(x) = x^3 + 1$

(iii) $f(x) = 4x^4 - 7x^2 + 18x - 1$; $g(x) = 2x + 1$

(iv) $f(x) = 5x^3 - 70x^2 + 153x - 342$; $g(x) = x^2 - 10x + 6$

22. Check whether g(y) is a factor of f(y) by applying the division algorithm :

(i) $f(y) = 2y^4 + 3y^3 - 2y^2 - 9y - 12$, $g(y) = y^2 - 3$

(ii) $f(y) = 3y^4 + 5y^3 - 7y^2 + 2y + 2$, $g(y) = y^2 + 3y + 1$

(iii) $f(y) = y^5 - 4y^3 + y^2 + 3y + 1$, $g(y) = y^3 - 3y + 1$

23. (a) If 1 is the zero of $f(x) = k^2x^2 - 3kx - 1$ then find the value (s) of k.

(b) If 1 and – 2 are the zeros of $f(x) = x^3 + 10x^2 + ax + b$, then find the values of a and b.

(c) Find p and q such that 3 and – 1 are the zeros of $f(x) = x^4 + px^3 + qx^2 + 12x - 9$.

(d) If 3 is the zero of $f(x) = x^4 - x^3 - 8x^2 + kx + 12$, then find the value of k.

24. (a) Find all the zeros of $3x^3 + 16x^2 + 23x + 6$ if two its zeros are – 3 and – 2 .

(b) Determine all the zeros of $4x^3 + 12x^2 - x - 3$ if two of its zeros are $-\frac{1}{2}$ and $\frac{1}{2}$.

(c) Determine all the zeros of $x^3 + 5x^2 - 2x - 10$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$

(d) Determine all the zeros of $4x^3 + 12x^2 - x - 3$ if one of its zeros is $\frac{5}{2}$

(e) Determine all the zeros of $4x^3 + 5x^2 - 180x - 225$ if one of its zeros is $-\frac{5}{4}$.

25. (a) Find all the zeros of $3x^4 - 10x^3 + 5x^2 + 10x - 8$ if three of its zeros are 1, 2 and – 1 .
(b) Obtain all the zeros of $2x^4 + 5x^3 - 8x^2 - 17x - 6$ if three of its zeros are $-1, -3, 2$.
(c) Determine all the zeros of $x^4 - x3 - 8x^2 + 2x + 12$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

26. (a) Obtain all other zeros of the polynomial $2x^3 - 4x - x^2 + 2$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.
(b) Find all the zeros of $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeros are $2+\sqrt{3}$ and $2-\sqrt{3}$.
(c) Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeros are 2 and – 2 .
(d) Find all the zeros of the polynomial $2x^4$ +7x3 – $19x^2 - 14x + 30$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

27. (a) On dividing $f(x) = 3x^3 + x^2 + 2x + 5$ by a polynomial $g(x) = x^2 + 2x + 1$, the remainder $r(x) = 9x + 10$. Find the quotient polynomial q(x).
(b) On dividing f(x) be a polynomial $x - 1 - x^2$, the quotient q(x) and remainder r(x) are (x – 2) and 3 respectively . Find f(x).
(c) On dividing $x^5 - 4x^3 + x^2 + 3x + 1$ by polynomial g(x), the quotient and remainder are $(x^2 - 1)$ and 2 respectively. Find g(x).
(d) On dividing $f(x) = 2x^5 + 3x^4 + 4x^3 + 4x^2 + 3x + 2$ by a polynomial g(x), where $g(x) = x^3 + x^2 + x + 1$, the quotient obtained as $2x^2 + x + 1$. Find the remainder r(x).

POLYNOMIALS **ANSWER KEY** **EXERCISE – 2 (X) CBSE**

- **Very Short Answer Type Questions**

1. (i) One zero, (ii) Two zero, (iii) One zero, (v) One zero, (vi) Four zeros
2. Three zeros **3.** (i) One zero, (ii) (0, 0)

- **Short Answer Type Questions**

1. (i) $-\frac{1}{3}, \frac{1}{2}$ (ii) $-\frac{1}{5}, \frac{-4}{5}$, (iii) $\frac{-1}{2}, \frac{-1}{2}$, (iv) $\frac{1}{3}, \frac{-1}{16}$, (v) 7, -9 , (vi) $0, \frac{5}{2}$ (vii) $\frac{9}{7}, \frac{-9}{7}$, (viii) $\frac{3}{2}, \frac{-1}{2}$

2. (i) $x^2 - 6x + 2$, (ii) $x^2 - 12$, (iii) $21x^2 + 33x + 6$, (iv) $x^2 - 4\sqrt{3}x + 9$, (v) $x^2 - 4x - 14$, (vi) $6x^2 - 31x + 40$

3. (i) $x^2 - 4\sqrt{3}x + 9$, (ii) $x^2 - (2\sqrt{3} - 1)x + (3 - \sqrt{3})$, (iii) $4x^2 - 1$, (iv) $3x^2 + 10\sqrt{3}x + 21$ (v) $18x^2 - 15x + 50$.
(vi) $3x^2 + 2\sqrt{5}x - 5$, (vii) $4x^2 + 4\sqrt{3}x + 1$ (viii) $25x^2 + 30x + 9$, (ix) $x^2 - \sqrt{2}x - 12$

4. (i) $\frac{14}{5}$, (ii) $\frac{106}{25}$, (iii) $\frac{-56}{25}$, (iv) $\frac{-604}{125}$ (v) $\frac{854}{125}$ (vi) $\frac{-5936}{125}$ **5.** p = - 6 , other zero = 1 **6.** a = 3 **7.** $a = \frac{-31}{2}$

8. 2 and $\frac{-2}{5}$ **9.** Yes **10.** (i) $\frac{b^2 - 2ac}{a^2}$ (ii) $\frac{3abc - b^3}{a^3}$ (iii) $\frac{3abc - b^3}{c^3}$ (iv) $\frac{3abc - b^3}{a^2 c}$ **11.** (i) 6 (ii) 12 (iii) –16

12. $k = \frac{15}{2}$, $m = 9$ **13.** $\frac{1}{3}, \frac{7}{3}, k = \frac{-5}{3}$ **14.** (i) $\frac{1}{5}(5x^2 - 4x + 2)$ (ii) $\frac{1}{25}(25x^2 + 4x + 4)$ (iii) $\frac{1}{5}(5x^2 - 8x + 8)$

15. (i) $20x^2 - 9x + 1$ (ii) $3x^2 - x$ **16.** 2 **17.** $-\frac{1}{3}$ **18.** $5x^2 - 12x + 4$

20. (i) q(x) = 3, r = $2x^2 - 8x - 5$, (ii) $q(x) = x^3 + x^2 + x + 1$, r(x) = 0, (iii) $q(x) = 2x^2 + x + 1$, (x) = x + 1,
(iv) q(x) = x + 1, r(x) = 0

21. (i) $q(x) = x^4 - 2x^2 + 5x + 4$, r(x) = - (3x + 5), (ii) q(x) = x, $r(x) = 2x^2 - x + 1$,
(iii) $q(x) = 2x^3 - x^2 - 3x + \frac{11}{2}$, $r(x) = -\frac{13}{2}$, (iv) q(x) = 5x – 20, r(x) = – 127x – 22

22. (i) g(y) is a factor of f(y), (ii) g(x) is a factor of f(x), (iii) g(t) is not a factor of f(t)
23. (a) $k = \pm 1$, (b) a = 7, b = -18, (c) p = - 8, q = 12, (d) k = 2

24. (a) $-2, -3, -\frac{1}{3}$, (b) $\frac{1}{2}, -\frac{1}{2}, 3$, (c) $\sqrt{2}, -\sqrt{2}, -5$, (d) $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}$, (e) $-\frac{5}{4}, 3\sqrt{5}, -3\sqrt{5}$

25. (a) $1, 2, -1, \frac{4}{3}$ (b) $-1, -3, 2, -\frac{1}{2}$, (c) $\sqrt{2}, -\sqrt{2}, 3, -2$

26. (a) $\frac{1}{2}$, (b) $2 \pm \sqrt{3}, 1, -\frac{1}{2}$, (c) 2, – 2 , 5 and – 6, (d) $\pm \sqrt{2}, \frac{3}{2}$ and – 5

27. (a) q(x) = 3x – 5 , (b) f(x) = $-x^3 + 3x^2 - 3x + 5$, (c) g(x) = $x^3 - 3x + 1$, (d) r(x) = x + 1,

EXERCISE – 3 (FOR SCHOOL / BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

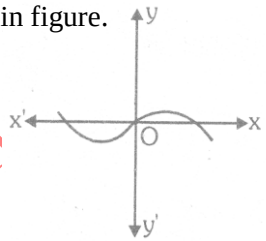
Questions Carrying 1 Mark

1. Write the zeros of the polynomial $x^2 + 2x + 1$. **[Delhi – 2008]**

2. Write the zeros of the polynomial $x^2 - 2x - 6$. **[Delhi – 2008]**

3. Write a quadratic polynomial , the sum and product of whose zeros are 3 are – 2 respectively. **[Delhi – 2008]**

4. Write the number of zeros of the polynomial y = f(x) whose graph is given in figure. **[AI– 2008]**



5. If (x + a) is a factor of $2x^2 + 2ax + 5x + 10$, find a **[Foreign – 2008]**

6. For what value of k, (- 4) is a zero of the polynomial $x^2 - x - (2x + 2)$? **[Delhi – 2009]**

7. For what value of p, (-4) is a zero of the polynomial $x^2 - 2x - (7p + 3)$? **[Delhi – 2009]**

8. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then find the value of a. **[AI– 2009]**

9. Write the polynomial , the product and sum of whose zeros $-\frac{9}{2}$ and $-\frac{3}{2}$ respectively **[Foreign – 2009]**

10. Write the polynomial , the product and sum of whose zeros are $-\frac{13}{5}$ and $-\frac{3}{5}$ respectively **[Foreign – 2009]**

Questions Carrying 2 Marks

11. Find the zeros of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeros and the co-efficient of the polynomial . **[Delhi – 2008]**

12. Find the zeros of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeros and the coefficients of the quadratic polynomial **[Delhi – 2008]**

13. Find the quadratic polynomial sum of whose zeros is 8 and their product is 12. Hence, find the zeros of the polynomial . **[AI– 2008]**

14. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other. Find the value of 'a' **[AI– 2008]**

15. If the product of zeros of the polynomial $ax^2 - 6x - 6$ is 4, find the value of 'a' **[AI– 2008]**

16. Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of it's zeros are 2 and – 2 . **[Foreign – 2008]**

17. Find all the zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of it's zeros are $\sqrt{2}$ and $-\sqrt{2}$ **[Foreign – 2008]**

18. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out of be (ax +b) , find a and b. **[Delhi – 2009]**

19. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be px + q. Find the values of p and q. **[Delhi – 2009]**

20. Find all the zeros of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of it's zeros are $-\sqrt{2}$ and $\sqrt{2}$ **[AI – 2009]**

21. Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of it's zeros are $-\sqrt{3}$ and $\sqrt{3}$. **[AI – 2009]**

22. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by polynomial $2x^2 - 5$, then find the value of the a and b **[Foreign – 2009]**

1. x = - 1 **2.** 3, -2 **3.** $x^2 - 3x - 2$ **4.** 3 **5.** 2 **6.** 9 **7.** 3 **8.** a = 1 **9.** $2x^2 + 3x - 9$ **10.** $5x^2 + 3x - 13$

11. $\left[\frac{-1}{3},\frac{3}{2}\right]$ **12.** $\left[\frac{-2}{2},2\right]$ **13.** $x^2 - 8x + 12$; (6, 2) **14.** 3 **15.** $\frac{-3}{2}$ **16.** 2, -2 , -6 and 5 **17.** $\sqrt{2},-\sqrt{2}-5$ and $\frac{3}{2}$

18. a = 1, b = 2 **19.** p = 2, q = 3 **20.** $-\sqrt{2},\sqrt{2}$ and -3 **21.** $-\sqrt{3},\sqrt{3}$ and $-\frac{1}{2}$ **22.** a = - 20, b = - 25

EXERCISE – 4 (FOR OLYMPIADS)

Choose The Correct One

1. If α, β and γ are the zeros of the polynomial $2x^3 - 6x^2 - 4x + 30$. then the value of $(\alpha\beta + \beta\gamma + \gamma\alpha)$ is
(A) – 2 (B) 2 (C) 5 (D) –30

2. If α, β and γ are the zeros of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=$
(A) $-\frac{b}{a}$ (B) $\frac{c}{d}$ (C) $-\frac{c}{d}$ (D) $-\frac{c}{a}$

3. If α, β and γ are the zeros of the polynomial $f(x) = ax^3 - bx^2 + cx - d$, then $\alpha^2 + \beta^2 + \gamma^2 =$
(A) $\frac{b^2-ac}{a^2}$ (B) $\frac{b^2+2ac}{b^2}$ (C) $\frac{b^2-2ac}{a}$ (D) $\frac{b^2-2ac}{a^2}$

4. If α, β and γ are the zeros of the polynomial $f(x) = x^3 + px^2 - pqrx + r$, then $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}=$
(A) $\frac{r}{p}$ (B) $\frac{p}{r}$ (C) $-\frac{p}{r}$ (D) $-\frac{r}{p}$

5. If the parabola $f(x) = ax^2 + bx + c$ passes through the points (-1, 12), (0, 5) and (2, -3) , the value of a + b + c is –
(A) - 4 (B) -2 (C) Zero (D) 1

6. If a, b are the zeros of $f(x) = x^2 + px + 1$ and c, d are the zeros of $f(x) = x^2 + qx + 1$ the value of $E = (a - c)(b - c)(a + b)(b + d)$ is –
(A) $p^2 - q^2$ (B) $q^2 - p^2$ (C) $q^2 + p^2$ (D) None of these

7. If α, β are zeros of $ax^2 + bx + c$ then zeros of $a^3x^2 + abcx + c^3$ are -
(A) $\alpha\beta, \alpha+\beta$ (B) $\alpha^2\beta, \alpha\beta^2$ (C) $\alpha\beta, \alpha^2\beta^2$ (D) α^3, β^3

8. Let α, β be the zeros of the polynomial $x^2 - px + r$ and $\frac{\alpha}{2}, 2\beta$ be the zeros of $x^2 - qx + r$, Then the value of r is –
(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$ (C) $\frac{2}{9}(q-2)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

9. When $x^{200} + 1$ is divided by $x^2 + 1$, the remainder is equal to –
(A) x + 2 (B) 2x – 1 (C) 2 (D) - 1

10. If $a(p+q)^2 + 2bpq + c = 0$ and also $a(q + r)^2 + 2bqr + c = 0$ then pr is equal to –
(A) $p^2+\frac{a}{c}$ (B) $q^2+\frac{c}{a}$ (C) $p^2+\frac{a}{b}$ (D) $q^2+\frac{a}{c}$

11. If a, b and c are not all equal and α and β be the zeros of the polynomial $ax^2 + bx + c$, then value of $(1+\alpha+\alpha^2)(1+\beta+\beta^2)$ is :
(A) 0 (B) positive (C) negative (D) non-negative

12. Two complex number α and β are such that $\alpha + \beta = 2$ and $\alpha^4 + \beta^4 = 272$, then the polynomial whose zeros are α and β is –
(A) $x^2 - 2x - 16 = 0$ (B) $x^2 - 2x + 12 = 0$ (C) $x^2 - 2x - 8 = 0$ (D) None of theses

13. If 2 and 3 are the zeros of $f(x) = 2x^3 + mx^2 - 13x + n$, then the values of m and n are respectively –
(A) -5 , - 30 (B) -5, 30 (C) 5, 30 (D) 5, -30

14. If α, β are the zeros of the polynomial $6x^2 + 6px + p^2$, then the polynomial whose zeros are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ is –

(A) $3x^2 + 4p^2x + p^4$ (B) $3x^2 + 4p^2x - p^4$

(C) $3x^2 - 4p^2x + p4$ (D) None of theses

15. If c, d are zeros of $x^2 - 10ax - 11b$ and a, b are zeros of $x^2 - 10cx - 11d$, then value of a + b + c + d is –

(A) 1210 (B) -1 (C) 2530 (D) -11

16. If the ratio of the roots of polynomial $x^2 + bx + c$ is the same as that of the ratio of the roots of $x^2 + qx + r$, then –

(A) $br^2 = qc^2$ (B) $cq^2 = rb^2$ (C) $q^2c^2 = b^2r^2$ (D) bq = rc

17. The value of p for which the sum of the squares of the roots of the polynomial $x^2 - (p-2)x - p - 1$ assume the least value is -

(A) -1 (B) 1 (C) 0 (D) 2

18. If the roots of the polynomial $ax^2 + bx + c$ are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$ then the value of $(a + b + c)^2$ is-

(A) $b^2 - 2ac$ (B) $b^2 - 4ac$ (C) $2b^2 - ac$ (D) $4b^2 - 2ac$

19. If α, β and γ are the zeros of the polynomial $x^3 + a_0x^2 + a_1x + a_2$, then $(1-\alpha^2)(1-\beta^2)(1-\gamma)$ is

(A) $(1 - a_1)^2 + (a_0 - a_2)^2$ (B) $(1 + a_1)^2 - (a_0 + a_2)^2$

(C) $(1 + a_1)^2 + (a_0 + a_2)^2$ (D) None of these

20. If α, β, γ are the zeros of the polynomial $x^3 - 3x + 11$, then the polynomial whose zeros are $(\alpha+\beta)(\beta+\gamma)$ and $(\gamma+\beta)$ is –

(A) $x^3 + 3x + 11$ (B) $x^3 - 3x + 11$

(C) $x^3 + 3x - 11$ (D) $x^3 - 3x - 11$

21. If α, β, γ are such that $\alpha+\beta+\gamma = 2$, $\alpha^2+\gamma^2 = 6$, $\alpha^3+\beta^3+\gamma^3 = 8$, then $\alpha^4+\beta^4+\gamma^4$ is equal to –

(A) 10 (B) 12 (C) 18 (D) None of these

22. If α, β are the roots of $ax^2 + bx + c$ and $\alpha + k$, $\beta + k$ are the roots of $px^2 + qx + r$, then k =

(A) $-\frac{1}{2}\left[\frac{a}{b} - \frac{p}{q}\right]$ (B) $\left[\frac{a}{b} - \frac{p}{q}\right]$ (C) $\frac{1}{2}\left[\frac{b}{a} - \frac{q}{p}\right]$ (D) (ab – pq)

23. If α, β are the roots of the polynomial $x^2 - px + q$, then the quadratic polynomial, the roots of which are $(\alpha^2-\beta^2)(\alpha^3-\beta^3)$ and $\alpha^3\beta^2 + \alpha^2\beta^3$:

(A) $px^2 - (5p + 7q)x - (p^6q^6 + 4p^2q^6) = 0$

(B) $x^2 - (p^5 - 5p^3q + 5pq^2)x + (p^6q^2 - 5p^4q^3 + 4p^2q^4) = 0$

(C) $x^2 - (p^3q - 5p^5 + p^4q) - (p^6q^2 - 5p^2q^6) = 0$

(D) All of the above

24. The condition that $x^3 - ax^2 + bx - c = 0$ may have two of the roots equal to each other but of opposite signs is :

(A) ab = c (B) $\frac{2}{3}a = bc$ (C) $a^2b = c$ (D) None of these

25. If the roots of polynomial $x^2 + bx + ac$ are α, β and roots of the polynomial $x^2 + ax + bc$ are α, γ then the values of α, β, γ respectively are –

(A) a,b,c (B) b,c,a (C) c,a,b (D) None of these

26. If one zero of the polynomial $ax^2 + bx + c$ is positive and the other negative then (a, b, c $\in$ R, a $\neq$ 0)

(A) a and b are of opposite signs. (B) a and c are of opposite signs.

(C) b and c are of opposite signs. (D) a,b,c are all of the same sign.

27. If α, β are the zeros of the polynomial $x^2 - px + q$. then $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$ is equal to -

(A) $\frac{p^4}{q^2} + 2 - \frac{4p^2}{q}$ (B) $\frac{p^4}{q^2} - 2 + \frac{4p^2}{q}$ (C) $\frac{p^4}{q^2} + 2q - \frac{4p^2}{q}$ (D) None of these

28. If α, β are the zeros of the polynomial $x^2 - px + 36$ and $\alpha^2 + \beta^2 = 9$, then p =

(A) ± 6 (B) $\pm$ 3 (C) $\pm$ 8 (D) ± 9

29. If α, β are zeros of $ax^2 + bx + c$, $ac \neq 0$, then zeros of $cx^2 + bx + a$ are –

(A) $-\alpha, \ -\beta$ (B) $\alpha, \frac{1}{\beta}$ (C) $\beta, \frac{1}{\alpha}$ (D) $\frac{1}{\alpha}, \frac{1}{\beta}$

30. A real number is said to be algebraic if it satisfies a polynomial equation with integral coefficients. Which of the following numbers is not algebraic :

(A) $\frac{2}{3}$ (B) $\sqrt{2}$ (C) 0 (D) π

31. The bi-quadratic polynomial whose zeros are 1, 2, $\frac{4}{3}$,–1 is :

(A) $3x^4 - 10x^3 + 5x^2 + 10x - 8$ (B) $3x^4 + 10x^3 - 5x^2 + 10x - 8$

(C) $3x^4 + 10x^3 + 5x^2 - 10x - 8$ (D) $3x^4 - 10x^3 - 5x^2 + 10x - 8$

32. The cubic polynomials whose zeros are $4, \frac{3}{2}$ and -2 is :

(A) $2x^3 + 7x^2 + 10x - 24$ (B) $2x^3 + 7x^2 - 10x - 24$

(C) $2x^3 - 7x^2 - 10x + 24$ (D) None of these

33. If the sum of zeros of the polynomial $p(x) = kx^3 - 5x^2 - 11x - 3$ is 2, then k is equal to :

(A) $k = -\frac{5}{2}$ (B) $k = \frac{2}{5}$ (C) k = 10 (D) $k = \frac{5}{2}$

34. If $f(x) = 4x^3 - 6x^2 + 5x - 1$ and α, β and γ are its zeros , then $\alpha\beta\gamma =$

(A) $\frac{3}{2}$ (B) $\frac{5}{4}$ (C) $-\frac{3}{2}$ (D) $\frac{1}{4}$

35. Consider $f(x) = 8x^4 - 2x^2 + 6x - 5$ and $\alpha, \beta, \gamma, \delta$ are it's zeros then $\alpha + \beta + \gamma + \delta =$

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $-\frac{3}{2}$ (D) None of these

36. If $x^2 - ax + b = 0$ and $x^2 = px + q = 0$ have a root in common and the second equation has equal roots, then –

(A) b + q = 2ap (B) $b + q = \frac{ap}{2}$ (C) b + q = ap (D) None of these

OBJECTIVE					ANSWER KEY										EXERCISE - 4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**A**	**C**	**D**	**B**	**C**	**B**	**B**	**D**	**C**	**B**	**D**	**C**	**B**	**C**	**A**
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	**B**	**B**	**B**	**B**	**D**	**C**	**C**	**B**	**A**	**C**	**B**	**A**	**D**	**D**	**D**
Que.	**31**	**32**	**33**	**34**	**35**	**36**									
Ans.	**A**	**C**	**D**	**D**	**D**	**B**									

EXERCISE - 5 (FOR IIT-JEE/AIEEE)

Choose The Correct One

1. If the sum of the two zeros of $x^3 + px^2 + qx + r$ is zero, then pq = **[EAMCET - 2003]**

(A) $-r$ (B) r (C) 2r (D) $-2r$

2. Let $a \neq 0$ and p(x) be a polynomial of degree grater than 2. If p(x) leaves remainders a and – a when divided respectively by x + a and x – a , the remainder when p(x) is divided by $x^2 - a^2$ is **[EAMCET - 2003]**

(A) 2x (B) – 2x (C) x (D) – x

3. If one root of the polynomial $x^2 + px + q$ is square of the other root, then **[IIT-Screening - 2003]**

(A) $p^3 - q(3p - 1) + q^2 = 0$ (B) $p^3 - q(3p + 1) + q^2 = 0$

(C) $p^3 + q(3p - 1) - q^2 = 0$ (D) $p^3 + q(3p + 1) - q^2 = 0$

4. If α, β are the zeros of $x^2 + px + 1$ and γ, δ be those of $x^2 + qx + 1$, then the value of $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) =$ **[DCE-2000]**

(A) $p^2 - q^2$ (B) $q^2 - p^2$ (C) p^2 (D) q^2

5. The quadratic polynomial whose zeros are twice the zeros of $2x^2 - 5x + 2 = 0$ is – **[Kerala Engineering -2003]**

(A) $8x^2 - 10x + 2$ (B) $x^2 - 5x + 4$ (C) $2x^2 - 5x + 2$ (D) $x^2 - 10x + 6$

6. The coefficient of x in $x^2 + px + q$ was taken as 17 in place of 13 and it's zeros were found to be – 2 and – 15. The zeros of the original polynomial are - **[Kerala Engineering -2003]**

(A) 3, 7 (B) – 3 , 7 (C) – 3 , – 7 (D) – 3, – 10

7. If $\alpha + \beta = 4$ and $\alpha^2 + \beta^2 = 44$, then α, β are the zeros of the polynomial . **[Kerala Engineering -2003]**

(A) $2x^2 - 7x + 6$ (B) $3x^2 + 9x + 11$ (C) $9x^2 - 27x + 20$ (D) $3x^2 - 12x + 5$

8. If α, β, γ are the zeros of the polynomial $x^3 + 4x + 1$, then $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1} =$

(A) 2 (B) 3 (C) 4 (D) 5 **[EAMCET-2003]**

9. If α, β are the zeros of the quadratic polynomial $4x^2 - 4x + 1$, then $\alpha^3 + \beta^3$ is –

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) 16 (D) 32

10. The value of 'a', for which one root of the quadratic polynomial $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2$ is twice as large as the other, is - **[AIEEE -2003]**

(A) $-\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) $\frac{1}{3}$

11. Let α, β be the zeros of $x^2 + (2 - \lambda)x - \lambda$. The values of λ for which $\alpha^2 + \beta^2$ is minimum is –

(A) 0 (B) 1 (C) 2 (D) 3 **[AMU-2002]**

12. If 1 + 2i is a zero of the polynomial $x^2 + bx + c$, $b, c \in R$, then (b, c) is given by –

(A) (2. – 5) (B) (- 3, 1) (C) (-2, 5) (D) (3, 1)

13. If 2 + i is a zero of the polynomial $x^3 - 5x^2 + 9x - 5$, the other zeros are –

(A) 1 and 2 – i (B) – 1 and 3 + i (C) 0 and 1 (D) None of these

14. The value of λ for which one zero of $3x^2 - (1 + 4\lambda)x + \lambda^2 + 2$ may be one-third of the other is –

(A) 4 (B) $\frac{33}{8}$ (C) $\frac{17}{4}$ (D) $\frac{31}{8}$

15. If 1 – i is a zero of the polynomial $x^2 + ax + b$, then the values of a and b are respectively .

(A) 2, 1 (B) – 2 , 2 **[Tamil Nadu Engineering 2002]**

(C) 2, 2 (D) 2, - 2

16. If the sum of the zeros of the polynomial $x^2 + px + q$ is equal to the sum of their squares, then –

(A) $P^2 - q^2 = 0$ (B) $p^2 + q^2 = 0$ (C) $p^2 + p = 2q$ (D) None of these

17. Let α, β be the zeros of the polynomial (x – a) (x – b) – c with $c \neq 0$. then the zeros of the polynomial $(x-\alpha)(x-\beta)$ + c are : **[IIT-1992, AIEEE - 2002]**

(A) a, c (B) b, c (C) a, b (D) a + c, b + c

18. If p, q are zeros of $x^2 + px + q$. then **[AIEEE - 2002]**

(A) p = 1 (B) p = 1 or 0 (C) p = - 2 (D) p = - 2 or 0

19. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3, \beta^2 = 5\beta - 3$, then the polynomial whose zeros are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is :

(A) $3x^2 - 25x + 3$ (B) $x^2 - 5x + 3$ **[AIEEE - 2002]**

(C) x2+5x – 3 (D) $3x^2 - 19x + 3$

20. If $\alpha \neq \beta$ and the difference between the roots of the polynomials $x^2 + ax + b$ and $x^2 + bx + a$ is the same, then **[AIEEE - 2002]**

(A) a + b + 4 = 0 (B) a + b – 4 = 0 (C) a – b + 4 = 0 (D) a – b – 4 = 0

21. If the zeros of the polynomial $ax^2 + bx + c$ be in the ratio m : n, then

(A) $b^2 mn = (m^2 + n^2) ac$ (B) $(m + n)^2 ac = b^2 mn$

(C) $b^2 (m^2 + n^2) = mnac$ (D) None of these

COMPREHENSION BASED QUESTIONS

Maximum and Minimum value of a quadratic expression :

At $x = \frac{-b}{2a}$, we get the maximum or minimum value of the quadratic expression, $y = ax^2 + bx + c$

(i) When a > 0, the expression $ax^2 + bx + c$ gives minimum value $= \frac{4ac - b^2}{4a}$

(ii) When a < 0, the expression $ax^2 + bx + c$ gives maximum value $= \frac{4ac - b^2}{4a}$

Based on above information, do the following questions :

22. The minimum value of the expression $4x^2 + 2x + 1$ $(x \in R)$ is -

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

23. If x be real , the maximum value of $7 + 10x - 5x^2$ is –

(A) 12 (B) 15 (C) 16 (D) 18

24. If p and q $(\neq 0)$ are the zeros of the polynomial $x^2 + px + q$, then the least value of $x^2 + px + q$ $(x \in R)$ is –

(A) $-\frac{1}{4}$ (B) $\frac{1}{4}$ (C) $-\frac{9}{4}$ (D) $\frac{9}{4}$

25. If x is real, the minimum value of $x^2 - 8x + 17$ is –

(A) – 1 (B) 0 (C) 1 (D) 2

OBJECTIVE					ANSWER KEY								EXERCISE - 5		
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**B**	**D**	**A**	**B**	**B**	**D**	**D**	**C**	**A**	**B**	**B**	**C**	**A**	**D**	**B**
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**					
Ans.	**C**	**C**	**B**	**D**	**A**	**B**	**C**	**A**	**C**	**C**					

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

★ INTRODUCTION

In class IX, we have read about linear equations in two variables. A linear equation is a rational and integral equating of the first degree.

For example, the equations : $3x + 2y = 7$, $2x - \sqrt{3}y = \sqrt{5}$, $y - 4x = \sqrt{3}$ are linear equations in two variables, since in each case

(i) Neither x nor y is under a radical sign i.e., x and y rational.
(ii) Neither x nor y in the denominator.
(ii) The exponent f x and y in each term is one.

In general, $ax + by + c = 0$: a, b, c $\in$ R ; $a \neq 0$ and $b \neq 0$ is a linear equation in two variables. A linear equation in two variables has an infinite number of solutions. The graph of a linear equation in two variables is always a straight line . In this chapter, we shall study about systems of linear equations in two variables, solution of system of linear equations in two variables and graphical and algebraic methods of solving a system of linear equations in two variables. In the end of the chapter, we shall be discussing some applications of linear equations in two variables in simple problems areas.

★ HISTORICAL FACTS

Diophantus, the last genius of Alexandria and the best algebraic mathematician of the Greek-Roman Era, has made a unique contribution in the development of Algebra and history of mathematics. He was born in the 3rd century and lived for 84 years. Regarding his age it has been told in VINODIKA of Greek collections.
"He spent one-sixth of his life in childhood, his beard grew after one twelfth more, after another one-seventh he married, five years later his son was born, the son lived to half the father's age, and Diophantus died four years after his son."

i.e. $\frac{x}{6}+\frac{x}{12}+\frac{x}{7}+5+\frac{x}{2}+4=x \Rightarrow 9x=756 \Rightarrow x=84$ Years.

He was known as the father of Algebra. Arithimetica is his famous book.

★ RECALL

(i) **Equation :** An statement of equality of two algebraic expressions which involve one or more unknown quantities is known as an equation.

(ii) **Linear Equation :** An equation in which the maximum power of variable is one is called a linear equation.

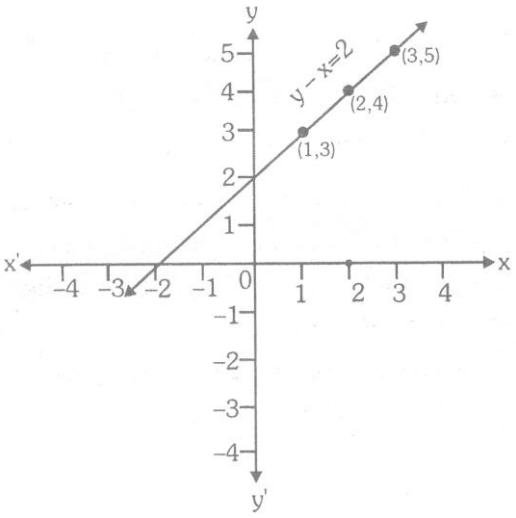
(iii) **Linear Equation in One Variable :** An equation of the form $ax + b = 0$ where x is a variable, a, b are real number and $a \neq 0$ is called a linear equation in one variable.

(iv) **Linear Equation in Two Variables :** An equation of the form $ax + by + c = 0$, where a, b, c are real number , $a \neq 0$, $b \neq 0$ and x, y are variables is called linear equation in two variables.
Any pair values of x & y which satisfies the equation $ax + by + c = 0$ is called a root or solution it.
Ex. (x = 1, y = 1) is a solution of $4x - y - 3 = 0$.
Remark : A linear equation in two variables have infinite number of solutions.

(v) **Graph of a Linear Equation in two Variables :** Assume $y - x = 2$ be a linear equation in two variables. The following table exhibits the abscissa and ordinates of points on the line represented by the equation $y - x = 2$

x	1	2	3
y	3	4	5

Plotting the points (1, 3), (2, 4) and (3, 5) one the graph paper and drawing the line joining them we obtain the graph of line represented by the given equation as shown in fig.



★ SIMULTANEOUS LINEAR EQUATIONS IN TWO RABBLES

A pair of linear equations in two variables is said to form a system of simultaneous linear equations.

General Form : $a_1x + b_1y\ c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, where a_1, a_2, b_1, b_2, c_1 and c_2 are era number ; $a_1^2 + b_1^2 \neq 0$ and $a_2^2 + b_2^2 \neq 0$ and x, y are variables.

Ex. Each of the following pairs of linear equations form a system of two simultaneous linear equations in two variables.

(i) $x - 2y = 3$, $2x + 5y = 5$ (ii) $3x+5y+7=0$, $5x + 2y + 9 = 0$

★ SOLUTION OF THE SYSTEM OF EQUATIONS

Consider the system of simultaneous linear equations : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

A pair of value of the variables x and y satisfying each one of the equations in a given system of two simultaneous linear equations in x and y is called a solution of the system .

Ex. x = 2, y = 3 is a solution of the system of simultaneous linear equations.

$2x + y = 7$, $3x + 2y = 12$

The given equations are $2x + y = 7$ ……(i)

$3x + 2y = 12$ ……(ii)

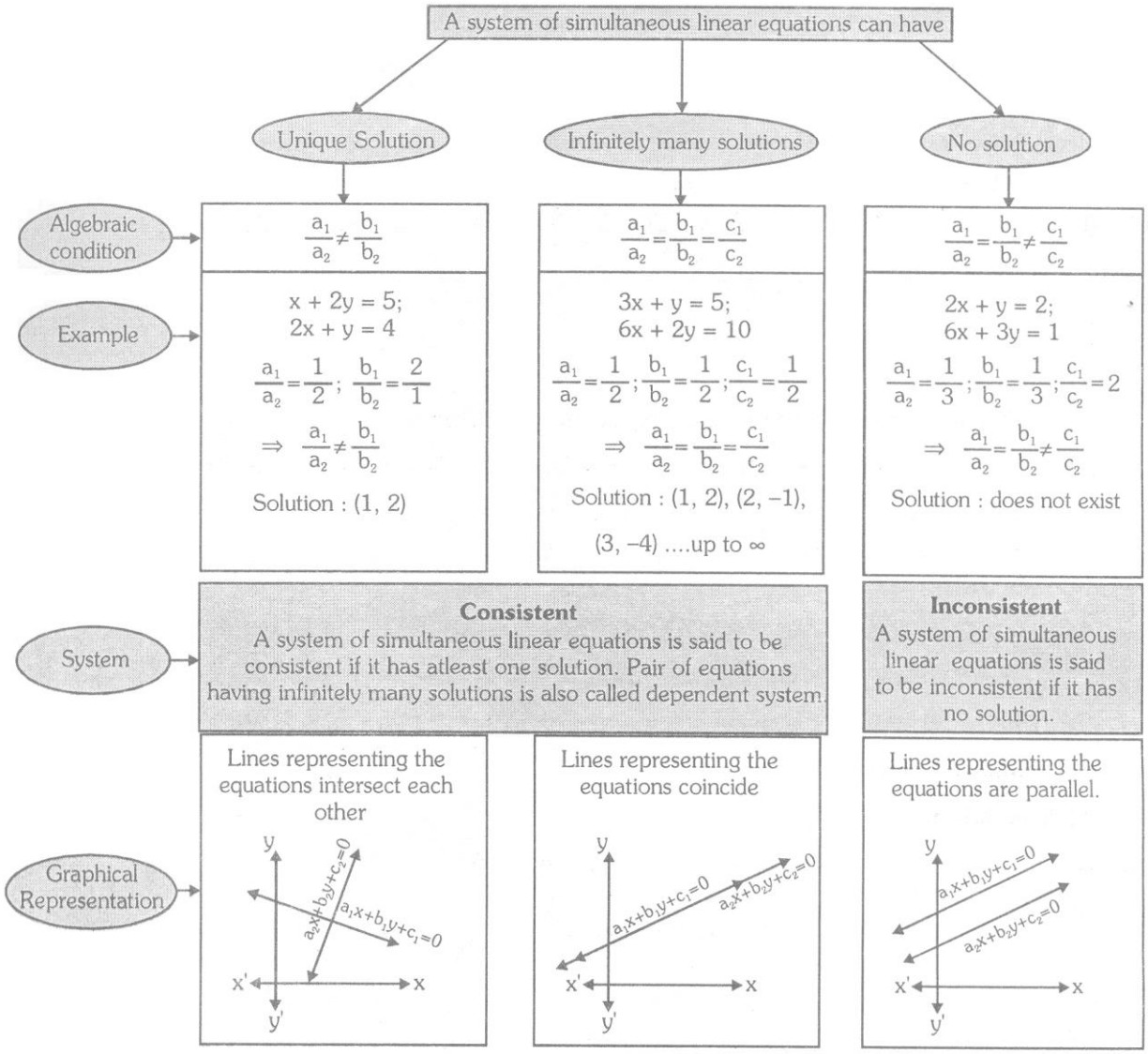
Put x = 2, y = 3 in LHS of equation (i), we get

LHS = $2 \times 2 + 3 = 7$ = RHS

Put x = 2, y = 3 in LHS of equation (ii), we get

LHS = $3 \times 2 + 2 \times 3 = 12$ = RHS

The value x = 2, y = 3 satisfy both equations (i) and (ii).



Hence x = 2, y = 3 is a solution of the given system. **Remark :** An equation involving two variables cannot give value of both the variables. For values of both the variables we required two equations. Similarly for three variables we require three equations and so on, i.e. to find n variables we need n equates.

★ HOMOGENEOUS SYSTEM OF EQUATIONS

A system of simultaneous equations is said to be homogenous, if all of the constant terms are zero.

General Form : $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$

Homogeneous equation of the form ax + by = 0 is a line passing through the origin.

Therefore, the system is always consistent.

(i) When $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ the system of equation has only one solution.

(ii) When $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ the system of equation has infinitely many solutions.

Ex.1 On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following points of linear equations are consistent of inconsistent.

(i) 3x + 2y = 5, 2x – 3y = 7 **(ii) 2x – 3y = 8, 4x – 6y = 9**

Sol. **(i)** We have, $3x + 2y = 5 \Rightarrow 3x + 2y - 5 = 0$ and $2x - 3y = 7 \Rightarrow 2x - 3y - 7 = 0$

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3} \text{ and } \frac{c_1}{c_2} = \frac{5}{7}$$

$$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, the given pair of linear equations is consistent.

(ii) We have, $2x - 3y = 8 \Rightarrow 2x - 8 = 0$ and $4x - 6y = 9 \Rightarrow 4x - 6y - 9 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given pair of linear equations is inconsistent .

Ex.2 **For what value of k, the system of equations x + 2y = 5, 3x + ky + 15 = 0 has**
(i) a unique solution
(ii) No solution ?

Sol. We have , $x + 2y = 5 \Rightarrow x + 2y - 5 = 0$ and $3x + kt + 15 = 0$.

(i) The required condition for unique solution is : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\therefore \quad \frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$$

Hence, for all real values of k except 6, the given system of equations will have a unique solution.

(ii) The required condition for no solution is : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\therefore \quad \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15} \Rightarrow \frac{1}{3} = \frac{2}{k} \text{ and } \frac{2}{k} \neq \frac{-5}{15}$$

$$\Rightarrow \quad k = 6 \text{ and } \frac{2}{k} \neq \frac{-1}{3} \Rightarrow k = 6 \text{ and } k \neq -6$$

Hence the given system of equations will have no solution when k = 6.

Ex.3 **Find the value of k for which the system of equations 4x + 5y = 0, kx + 10y = 0 has infinitely many solution .**

Sol. The given system is of the form $a_1x + b_1y = 0$, $a_2x + b_2y = 0$

$a_1 = 4$, $a_2 = k$, $b_1 = 5$, $b_2 = 10$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, the system has infinitely many solutions.

$$\frac{4}{k} = \frac{5}{10} \Rightarrow k = 8$$

Ex.4 **Find the value of a and b for which the given system of equations has an infinite number of solutions :**
2x + 3y = 7 ; (a + b + 1) x + (a + 2b + 2) y = 4 (a + b) + 1

Sol. We have $2x + 3y = 7 \Rightarrow 2x + 3y - 7 = 0$

and $(a + b + 1)\, x + (a + 2b + 2)\, y = 4\,(a + b) + 1$

$\Rightarrow$ (a + b + 1) x + (a + 2b + 2) y – {4 (a + b) + 1} = 0

The required condition for an infinite number of solutions is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \quad \frac{2}{a+b+1} = \frac{3}{a+2b+2} = \frac{-7}{-\{4(a+b)+1\}}$

$\Rightarrow \quad \frac{2}{a+b+1} = \frac{3}{a+2b+2}$ and $\frac{3}{a+2b+2} = \frac{7}{4(a+b)+1}$

$\Rightarrow$ 2a + 4b + 4 = 3a + 3b + 3 and 12a + 12b + 3 = 7a + 14b + 14

$\Rightarrow$ a – b – 1 = 0 and 5a – 2b = 11

$\Rightarrow$ a – b = 1 …(i)

and 5a – 2b = 11 …(ii)

Multiplying (i) by 2 we get 2a – 2b = 2 …(iii)

Subtracting (iii) from (ii) we get 3a $= \Rightarrow$ a = $\frac{9}{3}$ = 3

Put a = 3 in (i), we get 3 – b = $\Rightarrow$ b = 2

Hence, the given system of equations will have infinite number of solutions when a = 3 and b = 2.

★ GRAPHICAL METHOD OF SOLVING A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

To solve a system of two linear equations graphically,

(i) Draw graph of he first equation.

(ii) On the same pair of axes, draw graph of the second equation.

(iii)(a) If the two lines intersect at a point, read the coordinates of the point of intersection of obtain the solution and verify your answer.

(b) If the two lines are parallel, there is no point of intersection, write the system as inconsistent. Hence , no solution

(c) If the two lines have the same graph, then write the system as consistent with infinite number of solutions

Ex.5 Which of the following pairs of linear equations are consistent / inconsistent ? If consistent, obtain the solution graphically.

(i) x + 2y – 3 = 0, 4x + 3y = 2 (ii) 3x + y = 1, 2y = 2 – 6x (iii) 2x – y = 2, 2y – 4x = 2

Sol. (i) x + 2y – 3 = 0 $\Rightarrow$ y = $\frac{3-x}{2}$ | 4x + 3y $\Rightarrow$ y = $\frac{2-4x}{3}$

x	1	2	-3
y	1	0	3

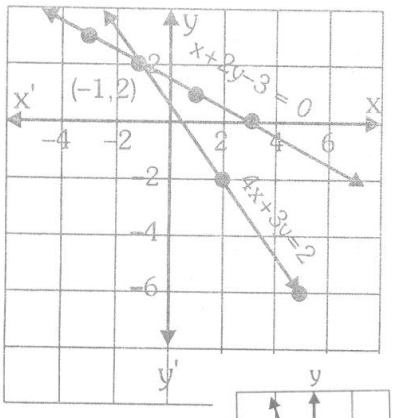
x	2	-1	5
y	-2	2	-6

Points are (1, 1), (3, 0), (-3, 3) | points are (2, -2), (-1, 2), (5, -6)

From the graph, we see that the two lines intersect at a point (-1, 2)

So the solution of the pair of linear equations is x = - 1, y = 2

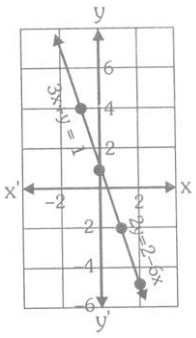
i.e., the given pair of equations is consistent.



(ii) 3x + y = 1 $\Rightarrow$ y = 1 – 3x | 2y = 2 – 6x $\Rightarrow$ y = $\frac{2-6x}{2}$

x	0	1	2
y	1	-2	-5

X	-1	1	-2
y	4	-2	7



Points are (0, 1), (1, -2), (2, -5) Points are (-1, 4), (1, -2), (-2, 7)

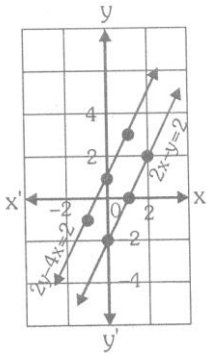
The two equations have the same graph. Thus system is consistent with infinite number of solutions, i.e., the system is dependent.

(iii) $2x - y = 2 \Rightarrow y = 2x - 2$ | $2y - 4x = 2 \Rightarrow y = \dfrac{4x+2}{2}$

x	0	1	2
y	-2	0	2

x	0	1	-1
y	1	3	-1

Points are (0, -2), (1, 0), (2, 2) | Points are (0, 1), (1, 3), (-1, -1)



The graph of the system consists of two parallel lines. Thus, the system is inconsistent. It has no solution.

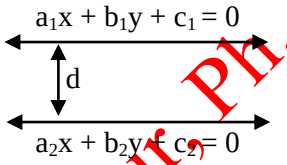
COMPETITION WINDOW

DISTANCE BETWEEN TWO PARALLEL LINES

Consider pair of parallel lines

$a_1x + b_1y + c_1 = 0$ …(i)

$a_2x + b_2y + c_2 = 0$ …(ii)



$\because$ The lines are parallel .

$\therefore \quad \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = k$ (say) $\Rightarrow a_1 = a_2 k$ & $b_1 = b_2 k$

Putting these values in (i), we get : $a_2kx + b_2ky + c_1 = 0$ or $a_2x + b_2y + \dfrac{c_1}{k} = 0$

or $a_2x + b_2y + c_3 = 0$ …(iii) $\left[c_3 = \dfrac{c_1}{k}\right]$

Clearly in equation (ii) and (iii), coefficients of x and y are same but the constant term is different in both the equations. The perpendicular distance (d) between the two lines can be calculated by using the following formula :

$$d = \frac{|c_2 - c_3|}{\sqrt{a_2^2 + b_2^2}}$$

e.g, The distance between the parallel liens $3x - 4y\ 9 = 0$ and $6x - 8y - 15 = 0$ can be calculated as follows :

$3x - 4y + 9 = 0$ …(i), $\quad 6x - 8y - 15 = 0$ or $3x - 4y - \dfrac{15}{2} = 0$ …(ii)

Required perpendicular distance, $d = \dfrac{\left|9 - \left(-\frac{15}{2}\right)\right|}{\sqrt{(3)^2 + (4)^2}} = \dfrac{\left|9 + \frac{15}{2}\right|}{\sqrt{25}} = \dfrac{33}{10}$

★ ALGEBRAIC METHOD OF SOLVING A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Some times, graphical number does not given an accurate answer. While reading the co-ordinate of a point on a graph paper we are likely to make an error. So we require some precise method to obtain accurate result. The algebraic methods are given below :

(i) Method of elimination by substitution.

(ii) Method of elimination by equating the coefficients.

(iii) Method of cross multiplication.

★ ALGEBRAIC SOLUTION BY SUBSTITUTION METHOD

To solve a pair linear equations in two variables x and y by substitution method, we follow the following steps :

Step – I : Write the given equations

$a_1x + b_1y + c_1 = 0$ ..(i)

and $a_2x + b_2y + c_2 = 0$ ...(ii)

Step –II : Choose one of the two equations and express y in terms of x (or x in terms of y), i.e. express, one variable in terms of the other.

Step –III : Substitute this value of y obtained in step-II, in the other equation to get a linear equation in x.

Step-IV : Solve the linear equation obtained in step-III and get the value of x.

Step-V : Substitute this value of x in the relation obtained in step-II and find the value of y.

Ex.6 Solve for x and y : 4x + 3y = 24, 3y – 2x = 6.

Sol. $4x + 3y = 24$ …(i)

$3y – 2x = 6$ …(ii)

From equation (i), we get

$$y = \frac{24 - 4x}{3} \quad \ldots(iii)$$

Substituting in equation (ii), we get

$$3\left(\frac{24 - 4x}{3}\right) - 2x = 6$$

$\Rightarrow 24 – 4x – 2x = 6$

$\Rightarrow - 6x = - 24 + 6$

$\Rightarrow 6x = 18$

$\Rightarrow x = 3$

Substituting x = 3 in (iii), we get

$$\Rightarrow y = \frac{24 - 12}{3}$$

$$\Rightarrow \frac{12}{3} = 4$$

Hence , x + 3, y = 4

Ex.7 Solve the following pair of linear equations by the substitution method.

$\sqrt{2}x + \sqrt{3}y = 0$ **and** $\sqrt{3}x - \sqrt{8}y = 0$

Sol. We have,

$$\sqrt{2}x + \sqrt{3}y = 0 \quad \ldots(i)$$

and $\sqrt{3}x - \sqrt{8}y = 0$ …(ii)

From (i), we get $y = \frac{-\sqrt{2}x}{\sqrt{3}}$ …(iii)

Substituting $y = \frac{-\sqrt{2}x}{\sqrt{3}}$ in (ii), we get $\sqrt{3}x - \sqrt{8}\left(\frac{-\sqrt{2}x}{\sqrt{3}}\right) = 0$

$\Rightarrow \quad \sqrt{3}x + \frac{4x}{\sqrt{3}} = 0 \Rightarrow 3x + 4y = 0 \Rightarrow 7x = 0 \Rightarrow x = 0$

Substituting x = 0 in (iii), we get $y = \frac{-\sqrt{2} \times 0}{\sqrt{3}} = 0$

Hence, the solution is x = 0 and y = 0.

★ ALGEBRAIC SOLUTION BY ELIMINATION METHOD

To solve a pair of linear equations x and y by elimination method, we follow the following steps :

Step-I : Write the given equation

$a_1x + b_1y + c_1 = 0$ …(i)

and $a_2x + b_2y + c_2 = 0$ …(ii)

Step-II : Multiply the given equations by suitable numbers so that the coefficient of one of the variables are numerically equal .

Stpe-III : If the numerically equal coefficients are opposite in sign , then add the new equations otherwise subtract

Step-IV : Solve the linear equations in one variable obtained in step–III and get the value of one variable .

Step-V : Substitute this value of the variable obtained in step-IV in any of the two equations and find the value of the other variable.

Ex.8 Solve the following pair of linear equations by elimination method : 3x + 4y = 10 and 2x – 2y = 2.

Sol. We have, $3x + 4y = 10$ …(i)

and $2x - 2y = 2$ …(ii)

Multiplying (ii) by 2, we get $4x - 4y = 4$ …(iii)

Adding (i) and (iii), we get $7x = 14 \Rightarrow x = 2$

Putting x = 2 in equation (ii), we get $2 \times 2 - 2y = 2 \Rightarrow y = 1$

Hence, the solution is x = 2 and y = 1.

Ex.9 Solve : ax + by = c, bx + ay = 1 + c

Sol. $ax + by = c$ …(i)

$bx + ay = 1 + c$ …(ii)

Adding (i) and (ii), we get

$(a + b)x + (a + b)y = 2c + 1$

$\Rightarrow \quad x + y = \frac{2c+1}{a+b}$ …(iii)

Subtracting (ii) and (i), we get

$(a-b)\,x-(a-b)\,y=-1$

$\Rightarrow \quad x-y=\dfrac{-1}{a-b}$ …(iv)

Adding (iii) and (iv), we get

$$2x=\frac{2x+1}{a+b0}-\frac{-1}{a-b}=\frac{2ac-2bc+a-b-a-b}{a^2-b^2}$$

$$\Rightarrow \quad 2x=\frac{2ac-2bc-2b}{a^2-b^2}$$

$$\Rightarrow \quad x=\frac{ac-bc-b}{a^2-b^2}$$

Subtracting (iv) from (iii) we get

$$2y=\frac{2c+1}{a+b}+\frac{-1}{a-b}=\frac{2ac-2bc+a-b+a+b}{a^2-b^2}$$

$$\Rightarrow \quad 2y=\frac{2ac-2bc+2a}{a^2-b^2}$$

$$\Rightarrow \quad y=\frac{ac-bc+a}{a^2-b^2}$$

Hence , $x=\dfrac{ac-bc-b}{a^2-b^2}, y=\dfrac{ac-bc+a}{a^2-b^2}$

★ ALGEBRAIC SOLUTIONS BY CROSS-MULTIPLICATION METHOD

Consider the system of linear equations

$a_1x + b_1y + c_1 = 0$ …(i)

$a_2x + b_2y + c_2 = 0$ …(ii)

To solve it by cross multiplication method , we follow the following steps :

Step-I : Write the coefficients as follows :

$$\frac{x}{b_1 \quad c_1}=\frac{y}{c_1 \quad a_1}=\frac{1}{a_1 \quad b_1} \quad \text{or} \quad \overbrace{a_1 \quad c_1}^{x}\ \overbrace{a_1 \quad b_1}^{y}\ \overbrace{}^{1}$$
$$b_2 \quad c_2 \qquad c_2 \quad a_2 \qquad a_2 \quad b_2 \qquad\qquad b_2 \quad c_2 \qquad b_2 \quad b_2$$

The arrows between the two numbers indicate that they are to be multiplied. The products with upward arrows are to be subtracted from the products with downward arrows.

To apply above formula, all the terms must be in left to the equal sign in he system of equations –
New, by above mentioned rule, equation (i) reduces to

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow \quad x=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \text{ and } y=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$$

Case-I : If $a_1b_2-a_2b_1\neq 0 \Rightarrow$ x and y have some finite value, with unique solution for the system of equations.

Case-II : If $a_1b_2 - a_2b_1 = 0 \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

Here two cases arise :

(a) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \lambda (\lambda \neq 0)$

Then $a_1 = a_2\lambda$, $b_1 = b_2\lambda$, $c_1 = c_2\lambda$

Put these values in equation $a_1x + b_1y + c_1 = 0$ …(i)

$\Rightarrow$ $a_2\lambda x + b_2\lambda + c_2\lambda = 0$

$\Rightarrow$ $\lambda(a_2x + b_2y + c_2) = 0$ but $\lambda \neq 0$

$\Rightarrow$ $a_2x + b_2y + c_2 = 0$ …(ii)

So (i) and (ii) are dependent, so there are infinite number of solutions.

(b) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow a_1b_2 - b_1a_2 = 0$

But $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow$ $x = \frac{\text{Finite value}}{0}$ = does not exist

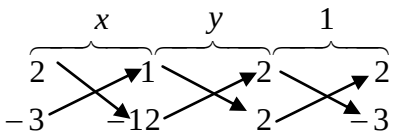
and $y = \frac{\text{Finite value}}{0}$ = does not exist

So system of equations is inconsistent.

Ex10 Solve by cross-multiplication method : x + 2y + 1 = 0 and 2x – 3y – 12 = 0

Sol. We have, x + 2y + 1 = 0 and 2x – 3y – 12 = 0

By cross-multiplication method , we have



$\therefore$ $\frac{x}{2\times(-12) - (-3)\times 1} = \frac{y}{1\times 2 - (-12)\times 1} = \frac{1}{1\times(-3) - 2\times 2}$

$\Rightarrow$ $\frac{x}{-24+3} = \frac{y}{2+12} = \frac{1}{-3-4} \Rightarrow \frac{x}{-21} = \frac{y}{14} = \frac{1}{-7}$

$\Rightarrow$ $x = \frac{-21}{-7} = 3$ and $y = \frac{14}{-7} = -2$

Hence the solution is x = 3 and y = – 2 .

Ex.11 Solve by cross-multiplicaiton method : (a – b) x + (a + b) y = 2 ($a^2 - b^2$), (a + b) x – (a – b) y = 4ab.

Sol. Writing the equations in the standard form, we get .

$(a - b)x + (a + b)y - (a^2 - b^2) = 0$

$(a + b)x - (a - b)y - 4ab = 0$

Applying the cross-multiplication method, we get

$$\frac{x}{(a+b) - 2(a^2 - b^2)} = \frac{y}{-2(a^2 - b^2)(a - b)} = \frac{1}{(a-b)(a+b)}$$

$-(a-b)$ $-4ab$ $-4ab$ $(a+b)$ $(a+b)-(a-b)$

Simplification of the expression under x :

$-4ab\,(a+b) - 2\,(a-b)\,(a^2-b^2)$

$= -2\,(a+b)\,[2ab + (a-b)^2]$

$= -2\,(a+b)\,(2ab + a^2 + b^2 - 2ab)$

$= -2\,(a+b)\,(a^2+b^2)$

Simplification of the expression under y :

$-2\,(a^2-b^2)\,(a+b) + 4ab\,(a-b)$

$= -2\,(a-b)\,[(a+b)\,(a+b) - 2ab]$

$= -2\,(a-b)\,(a^2+b^2+2ab-2ab)$

$= -2\,(a-b)\,(a^2+b^2)$

Simplification of the expression under 1 :

$-(a-b)^2 - (a+b)^2$

$= -(a^2+b^2-2ab) - (a^2+b^2+2ab)$

$= -2\,(a^2+b^2)$

Hence, $\dfrac{x}{-2(a+b)(a^2-b^2)} = \dfrac{y}{-2(a-b)(a^2-b^2)} = \dfrac{1}{-2(a^2+b^2)}$

$\Rightarrow \quad \dfrac{x}{a+b} = \dfrac{y}{a-b} = \dfrac{1}{1}$

$\Rightarrow$ x = (a + b) and y = (a – b)

★ EQUATIONS OF THE FORM ax + by = c AND bx + ay = d, WHERE a ≠ b.

To solve the equations of the form. :

$ax + by = c$ …(i)

and $bx + ay = d$ …(ii)

where $a \neq b$, we follow the following steps :

Step-I : Add (i) and (ii) and obtain $(a+b)x + (b+a)y = c + d$, i.e., $x + y = \dfrac{c+d}{a+b}$ …(iii)

Step-II : Subtract (ii) from (i) and obtain $(a-b)x - (a-b)y = c - d$, i.e., $x - y = \dfrac{c-d}{a-b}$ …(iv)

Step-III: Solve (iii) and (iv) to get x and y.

Ex.12 Solve for x and y : 47x + 31y = 63, 31x + 47y = 15.

Sol. We have,

47x + 31y = 63…(i) and 31x + 47y = 15…(ii)

Adding (i) and (ii), we get : 78x + 78y = 78 $\Rightarrow$ x + y = 1 …(iii)

Subtracting (ii) from (i), we get : 16x – 16y = 48 $\Rightarrow$ x – y = 3 …(iv)

Now , adding (iii) and (iv), we get : 2x = 4 $\Rightarrow$ x = 2

Putting x = 2 in (ii), we get : 2 + y = 1 $\Rightarrow$ y = - 1

Hence, the solution is x + 2 and y = - 1

★ EQUATIONS REDUCIBLE TO LINER EQUATIONS IN TWO VARIABLES

Equations which contain the variables, only in the denominators, are called reciprocal equations. These equations can be of the following types and can be solved by the under mentioned method :

Type-I : $\frac{a}{u}+\frac{b}{v}=c$ and $\frac{a'}{u}+\frac{b'}{v}=c' \forall a,b,c,a',b',c' \in R$

Put $\frac{1}{u}=x$ and $\frac{1}{y}=v$ and find the value of x and y by any method described earlier.

Then $u=\frac{1}{x}$ and $v=\frac{1}{y}$

Type-II : au +bv = cuv and a'u + b'v = c'uv $\forall$ a,b,c,a',b',c' $\in$ R

Divide both equations by uv and equations can be converted in the form explained in (i).

Type-III : $\frac{a}{lx+my}+\frac{b}{cx+dy}=k, \frac{a'}{lk+my}+\frac{b'}{cx+dy}=k' \forall a,b,k,a',b',k' \in R$

Put $\frac{1}{lx+my}=u$ and $\frac{1}{cx+dy}=v$

Then equations are au + vc = k and a'u + b'v = k

Find the values of u and v and put in Ix + my = $\frac{1}{u}$ and cx + dy = $\frac{1}{v}$

Again solve for x and y, by any method explained earlier.

Ex.13 Solve for x and y : $\frac{3a}{x}-\frac{2b}{y}+5=0$ **and** $\frac{a}{x}+\frac{3b}{y}-2=0 (x \neq 0, y \neq 0)$

Sol. We have, $\frac{3a}{x}-\frac{2b}{y}+5=0$ and $\frac{a}{x}+\frac{3b}{y}-2=0$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$. Then, the given equations can be written as

3au – 2bv = - 5…(i) and au +3bv = …(ii)

Multiplying (i) by 3 and (ii) by 2, we get

9au – 6bv = - 17 …(iii) and 2au + 6bv = 4…(iv)

Adding (iii) and (iv), we get 11au = - 11 $\Rightarrow u=\frac{-1}{a}$

Put $u=\frac{-1}{a}$ in equation (ii), we get $a\left(\frac{-1}{a}\right)+3bv=2 \Rightarrow 3bv=3 \Rightarrow v=\frac{1}{b}$

But $\frac{1}{x}=u$ and $\frac{1}{y}=v$

Therefore, $\frac{1}{x}=\frac{-1}{a} \Rightarrow x=-a$ and $\frac{1}{y}=\frac{1}{b} \Rightarrow y=b$ $\quad [\because u=\frac{-1}{a}, v=\frac{1}{b}]$

Hence the solution is x = - a and y = b.

Ex.14 Solve $\frac{57}{x+y}+\frac{6}{x-y}=5$ **and** $\frac{38}{x+y}+\frac{21}{x-y}=9.$

Sol. We have, $\frac{57}{x+y}+\frac{6}{x-y}=5 \Rightarrow \frac{57}{x+y}+\frac{6}{x-y}-5=0$

and $\frac{38}{x+y}+\frac{21}{x-y}=9 \Rightarrow \frac{38}{x+y}+\frac{21}{x-y}-9=0$

Let $\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$. Then. the given equations can be written as

57p + 6q – 5 = 0 and 38p + 21q – 9 = 0

By cross-multiplication method, we have

$$\begin{array}{ccccc} \overbrace{6} & & \overbrace{-5} & & \overbrace{57} \\ 21 & & -9 & & 38 \end{array} \quad \begin{array}{c} \overbrace{\quad} \\ 6 \\ 21 \end{array}$$

$$\therefore \quad \frac{p}{6\times(-9)-21\times(-5)} = \frac{q}{(-5)\times 38-(-9)\times 57} = \frac{1}{57\times 21-38\times 6}$$

$$\Rightarrow \quad \frac{p}{-54+105} + \frac{q}{-190+513} = \frac{1}{1197-228}$$

$$\Rightarrow \quad \frac{p}{51} = \frac{q}{323} = \frac{1}{969} \Rightarrow p = \frac{51}{969} = \frac{1}{18} \text{ and } q = \frac{323}{969} = \frac{1}{3}$$

But $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$. therefore

$$\frac{1}{x+y} = \frac{1}{19} \Rightarrow x+y = 19 \qquad \ldots(i)$$

and $$\frac{1}{x-y} = \frac{1}{3} \Rightarrow x-y = 3 \qquad \ldots(ii)$$

adding (i) and (ii) , we get $2x = 22 \Rightarrow x = 11$

Put x = 11 in (i),we get $11 + y = 19 \Rightarrow y = 8$

Hence, the solution is x = 11 and y = 8.

Ex.15 Solve for x and y : $\frac{7x-2y}{xy} = 5, \frac{8x+7y}{xy} = 15.$

Sol. $$\frac{7x-2y}{xy} = 5, \Rightarrow \frac{7}{y} - \frac{2}{x} = 5 \qquad \ldots(i)$$

$$\frac{8x+7y}{xy} = 15 \Rightarrow \frac{8}{y} + \frac{7}{x} = 15 \qquad \ldots(ii)$$

Putting $\frac{1}{y} = u$ and $\frac{1}{x} = v$, we get

$7u - 2v = 5$ …(iii)

$8u \;\; 7v = 15$ …(iv)

Multiplying (iii) by 7 and (iv) by 2 and adding we get

$49u - 14v = 35$

and $16u + 14v = 30$

$65u = 65 \Rightarrow u = 1 \Rightarrow \frac{1}{y} = 1$ or $y = 1$

Substituting u = 1 in (iii) we get : $7 - 2v = 5 \Rightarrow v = 1 \Rightarrow \frac{1}{x} = 1$ or x = 1

Hence, x = 1, y = 1.

★ APPLICATIONS OF LINEAR EQUATIONS IN TWO VARIABLES

In This section, we will study about some applications of simultaneous linear equations in solving variety of word problems related to our day-to-day life situations. The following examples are self-explanatory and will give some insight to the solution to such problems.

Type-1 :Based on Articles And Their Costs / Quantities

Ex.16 7 audio cassettes and 3 video cassettes cost Rs. 1110, which 5 audio cassettes and 4 video cassettes cost Rs. 1350. Find the cost of an audio cassette and a video cassette.

Sol. Let the cost of an audio cassette and a video cassette be Rs. x and Rs. y respectively .

The cost of 7 audio cassettes and 3 video cassettes = Rs. 1110

$\Rightarrow$ $7x + 3y = 1110$ …(i)

The cost of 5 audio cassettes and 4 video cassettes = Rs. 1350

$\Rightarrow$ $5x + 4y = 1350$ …(ii)

Multiplying (i) by 4 and (ii) by 3, we get

$28x + 12y = 4440$ …(iii)

$15x + 12y = 4050$ ..(iv)

Subtracting (iv) from (iii), we get $13x = 390 \Rightarrow x = 30$

Putting $x = 30$ in (i), we get $7 \times 30 + 3y = 1110 \Rightarrow 210 + 3y = 1110$

$\Rightarrow$ $3y = 900 \Rightarrow y = 300$

Hence, the cost of an audio cassette is Rs. 30 and that of a video cassette is Rs. 300.

Type-II: Based on numbers

Ex.17 The sum of the digits of a two-digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. Find the number .

Sol. Let the digit at ten's place by x and that at unit's place be y. Then,

$x + y = 12$ …(i)

And , the two digits number = $10x + y$

Now, according to the equation,

$(10y + x) = (10x + y) + 18 \Rightarrow 9y - 9x = 18 \Rightarrow y - x = 2$ …(ii)

Adding (i) and (ii), we get $2y = 14 \Rightarrow y = 7$

Put $y = 7$ in (i), we get $x + y = 12 \Rightarrow x = 5$

Hence, the enquired number is $(10 \times 5 + 7)$, i.e., 57.

Type-III: Based on Fractions

Ex.18 If we add 1 to ht numerator and subtract 1 from the denominator, a fraction reduces to 1, It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?

Sol. Let the required fraction be $\frac{x}{y}$. Then

$$\frac{x+1}{x-y} = 1 \Rightarrow x+1 = y-1 \Rightarrow x-y = -2 \quad \text{…(i)}$$

and $$\frac{x}{x+y} = \frac{1}{2} \Rightarrow 2x = y+1 \Rightarrow 2x-y = 1 \quad \text{…(ii)}$$

Subtracting (i) from (ii), we get $x = 3$

Put $x = 3$ in (i), we get $3 - y = -2 \Rightarrow y = 5$

Hence, the fraction is $\frac{3}{5}$

Type-IV : Based on Ages

Ex.19 Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of father and son.

Sol. Let the present ages of the father and the son be x years and y years respectively .

Two years ago, Father's age = $(x - 2)$ years and son's age = $(y - 2)$ years

$\therefore$ $(x - 2) = 5 (y - 2) \Rightarrow x - 5y = -8$ …(i)

Two years later, father's age = $(x + 2)$ years and son's age = $(y + 2)$ years

$\therefore$ $(x + 2) = 3 (y + 2) + 8 \Rightarrow x + 2 = 3y + 6 + 8 \Rightarrow x - 3y = 12$ …(ii)

Subtracting (i) from (ii), we get $2y = 20 \Rightarrow y = 10$

Putting $y = 10$ in (ii), we get $x - 3 \times 10 = 12 \Rightarrow x = 42$

Hence , the present ages of father and son are 42 years and 1 years respectively.

Type-III: Based on Geometrical Applications

Ex.20 The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Sol. Let the larger angle be x^0 and the smaller angle by y^0. Then,

$x + y = 180$ …(i)

and $x = y + 18 \Rightarrow x - y = 18$ …(ii)

Adding (i) and (ii) , we get $2x = 198 \Rightarrow x = 99$

Putting $x = 99$ in (i), we get $99 + y = 18 \Rightarrow y = 81$

Hence the required angles are 99^0 and 81^0.

Type-III: Based on Time, Distance and Speed.

Formulae to be used :

1. (a) Speed = $\frac{\text{Distance}}{\text{Time}}$

(b) Distance = Speed × Time

(c) Time = $\frac{\text{Distance}}{\text{Speed}}$

2. Let speed of a boat in still water = u km/h

and speed of the current = v km/h. Then ,

(a) Speed of a boat downstream = (u + v)km/h

(b) Speed of a boat upstream = (u – v)km/h.

Ex.21 A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car it takes him 4 hours. But, if he travels 130 km by train and rest by car, be takes 18 minutes, longer. Find the speed of the train and that of the car.

Sol. Let the speeds of the train and that of the car be x km/h and y km/h respectively .

$$\frac{250}{x} + \frac{120}{y} = 4 \qquad \left(\therefore \text{Time} = \frac{\text{Distance Speed}}{\text{Distance}}\right) \qquad \text{…(i)}$$

And if he covers 130 km by train and 240 km by car it takes 4 hours and 18 minutes.

Therefore ,

$$\frac{130}{x} + \frac{240}{y} = 4 + \frac{18}{60} \qquad \left(\therefore 18 \text{ minutes} = \frac{18}{60} \text{ hours}\right)$$

$$\frac{130}{x} + \frac{240}{y} = \frac{43}{10} \qquad \text{…(ii)}$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$ Then, the equations (i) and (ii), can be written as

$250u + 120v = 4$ …(iii)

and $130u + 140v = \frac{43}{10}$ …(iv)

Multiplying (iii) by 2, we get 500u + 240v = 8 …(v)

Subtracting (iv) from (v), we get 370u = $8-\frac{43}{10}\Rightarrow 370u=\frac{37}{10}\Rightarrow u=\frac{1}{100}$

Putting $u=\frac{1}{100}$ in (iii), we get $250\times\frac{1}{100}+120v=4\Rightarrow\frac{5}{2}+120v=4$

$\Rightarrow \quad 120v=4-\frac{5}{2}\Rightarrow v=\frac{3}{120\times 2}=\frac{1}{80}$

but $u=\frac{1}{x}$ and $v=\frac{1}{y}$.

Therefore, $\frac{1}{x}=\frac{1}{100}\Rightarrow x=100$ and $\frac{1}{y}=\frac{1}{80}\Rightarrow y=80$

Hence the speeds of the train and that of the car are 100 km/h and 80 km/h respectively .

Type-VII : Miscellaneous

Ex.22 8 man and 12 boys can finish is a piece of work in 10 days while 6 man and 8 boys can finish it in 14 days. Find the time taken by one men alone and that by one boy alone to finish the work .

Sol. Let one man alone can finish the work in x days and one alone can finish the work in y days. Then. the work done by one man in one day $=\frac{1}{x}$ and the work done by one boy in on e day $=\frac{1}{y}$

According to the question, $10\left(\frac{8}{x}+\frac{12}{y}\right)=1\Rightarrow\frac{1}{x}+\frac{3}{y}=\frac{1}{40}$ …(i)

Also, $14\left(\frac{6}{x}+\frac{8}{y}\right)=1\Rightarrow\frac{3}{x}+\frac{4}{y}=\frac{1}{28}$ …(ii)

Multiplying (i) by 4 and (ii) by 3, we get : $\frac{8}{x}+\frac{12}{y}=\frac{1}{10}$ …(iii) and $\frac{9}{x}+\frac{12}{y}=\frac{3}{28}$ …(iv)

Subtracting (iii) from (iv), we get $\frac{1}{x}=\frac{3}{28}-\frac{1}{10}\Rightarrow\frac{1}{x}=\frac{2}{280}\Rightarrow x=140$

Putting x = 140 (ii), we get $\frac{3}{140}+\frac{4}{y}=\frac{1}{28}\Rightarrow\frac{4}{y}=\frac{1}{28}-\frac{3}{140}$

$\Rightarrow \quad \frac{4}{y}=\frac{5-3}{140}\Rightarrow\frac{4}{y}=\frac{2}{140}\Rightarrow y=180$

Hence, one man alone can finish the work in 140 days and one by alone can finish the work in 280 days.

★ **SYNOPSIS**

(i) Two linear equations in the same two variables are called a pair of linear equations in two variables, or briefly, a linear pair. The most general form of a linear pair is :

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers , such that $a_1^2+b_1^2\neq a_2^2+b_2^2\neq 0$.

(ii) A pair of linear equations in two variables can be represented, and solved , by the

(a) Graphical method (b) Algebraic method

(iii) **Graphical Method :** The graph of the pair of linear equations in two variables is represented by a pair of lines.

(a) If the pair intersects at a point, then that point is the unique common solution of the two equations. In this case, the pair is consistent.

(b) If the pair coincide, then it has infinitely many solutions – each point on the line being a solution. In this case, the pair is consistent (dependent).

(c) If two lines are parallel, then the pair has no solution, and is called inconsistent.

(iv) **Algebraic Method :** We have discussed the following methods for finding the solutions (s) of a pair of linear equations

(a) Substitution method. (b) Elimination method. (c) Cross-multiplication method.

(v) If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise :

(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: **In this case the pair of linear equations is consistent.**

(b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: In this case the **pair of linear equations is inconsistent.**

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$: In this case the **pair of linear equations is dependent and consistent.**

(iv) There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a linear pair.

SOLVED NCERT EXERCISE

EXERCISE : 3 . 1

1. After tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three sears from now, I shall be three times as old as you will be". (Isn't this interesting ?) Represent this situation algebraically and graphically .

Sol. Let the present age of Aftab's daughter = x years.

and the present age of Aftab = y years (y > b)

According to the given conditions

Seven years ago, $(y - s) = 7 \times (x - 7)$

i.e., $y - 7 = 7x - 49$

i.e., $7x - y - 42 = 0$ …(i)

Three years later, $(y + 3) = 3 \times (x + 3)$

i.e., $y + 3 = 3x + 9$

i.e., $3x - y + 6 = 0$ …(ii)

Thus, the algebraic relations are $7x - y - 42 = 0$, $3x - y + 6 = 0$.

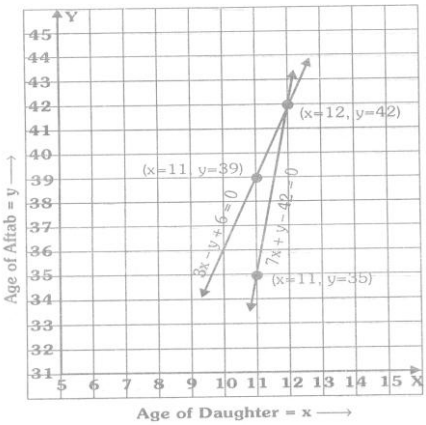
Now, we represent the problem graphically as below :

$7x - y - 42 = 0$ …(i)

Age of Aftab's daughter = x	11	12
Age of Aftab = y = 7x – 42	35	42

$3x - y + 6 = 0$ …(ii)

Age of Aftab's daughter = x	11	12
Age of Aftab = y = 3x + 6	39	42



From the graph, we find that x = 12

and $y = 42$

Thus, the present age of Aftab's daughter = 12 years

and the present age of Aftab = 42 years

2. The coach of a cricket team buys 3 bats and 6 balls for Rs. 3900. Later, she buys another bat and 3 more balls of the same kind for Rs. 1300. Represent this situation algebraically and geometrically .

Sol. [Try Yourself]

3. The cost of 2 kg of apples and 1 kg of grapes on a day found to be Rs. 160. After a month, the cost of 4 kg of apples and 2 g of grapes is Rs. 300. Represent the situation algebraically and geometrically .

Sol. [Try Yourself]

EXERCISE : 3 . 2

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys , find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

Sol. (i) Let the number of boys be x and the number of girls be y.

According to the given conditions

$x + y = 10$ and $y = x + 4$

We get the required pair of linear equations as

$x + y - 10 = 0,\ x - y + 4 = 0$

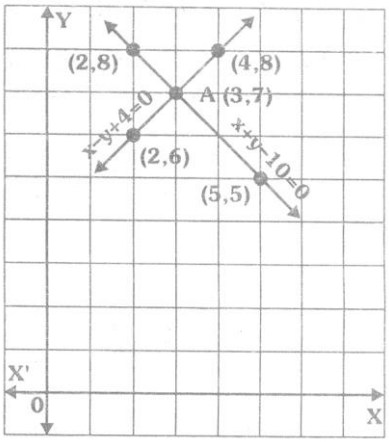
Graphical Solution

$x + y - 10 = 0$ …(i)

x	2	5
$y = 10 - x$	8	5

$x - y + 4 = 0$ …(ii)

x	2	5
$y = x + 4$	8	5



From the graph, we have : x = 3, y = 7 common solution of the two linear equations.

Hence, the number of boys = 3 and the number of girls = 7.

(ii) **[Try Yourself]**

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at point, are parallel or coincident .

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

Sol. (i) $5x - 4y + 8 = 0$ …(i)

$7x + 6y - 9 = 0$ …(ii)

$$\frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = -\frac{2}{3}$$

$$\Rightarrow \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ Lines represented by (i) and (ii)

Intersect at a point

[Rest Try Yourself]

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent, or inconsistent.

(i) 3x + 2y = 5 ; 2x – 3y = 7 **(ii) 2x – 3y = 8 ; 4x – 6y = 9**

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; 9x – 10y = 14 **(iv) 5x – 3y = 11 ; - 10x + 6y = - 22**

(v) $\frac{4}{3}x + 2y = 8$; 2x + 3y = 12

Sol. (i) $3x + 2y - 5 = 0$ …(i)

$2x - 3y - 7 = 0$ …(ii)

$\frac{a_1}{a_2} = \frac{3}{2}$; $\frac{b_1}{b_2} = -\frac{2}{3}$ $\Rightarrow \frac{a1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ The equations have a unique solution.

Hence, consistent.

[Rest Try Yourself]

4. Which of the following pairs of linear equations are consistent / inconsistent ? If consistent, obtain the solution graphically :

(i) x + y = 5, 2x + 2y = 10 **(ii) x – y = 8, 3x – 3y = 16**

(iii) 2x + y – 6 = 0, 4x – 2y – 4 = 0 **(iv) 2x – 2y – 2 = 0, 4x – 4y – 5 = 0**

Sol. (i) $x + y = 5$ …(i)

$2x + 2y = 10$ …(ii)

$\frac{a_1}{a_2} = \frac{1}{2}, = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$

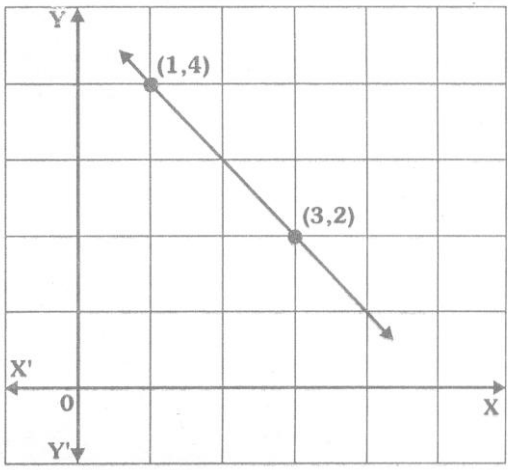
i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the pair of linear equations is consistent.

(i) and (ii) are same equations and hence the graph

Is coincident straight line.

x	1	3
y = 5 – x	4	2



[Rest Try Yourself]

5. Half the perimeter of a rectangular garden, whose length is 4 m more that its width, is 36 m. Find the dimensions of the garden

Sol. [Try Yourself]

6. Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(i) Intersecting **(ii) Parallel** **(iii) Coincident lines**

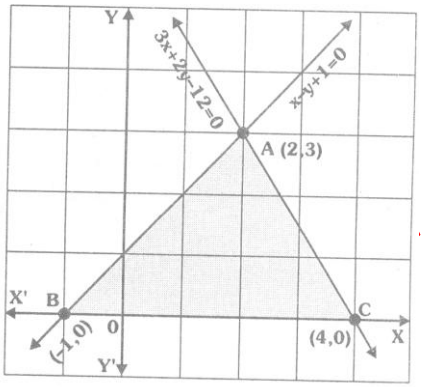
Sol. (i) $2x + 3y - 8 = 0$ (Given equation)

$3x + 2y + 4 = 0$ (New equation)

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the graph of the two equations will be two intersecting lines. **[Rest Try Yourself]**

7. **Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.**



Sol. x – y + 1 = 0 …(i)

x	1	3
y = x + 1	2	4

3x+2y – 12 = 0 …(ii)

x	0	4
$y = \frac{12-3x}{2}$	6	0

The vertices of the triangle are
A (2, 3), B (-1, 0) and C (4, 0)

EXERCISE : 3.3

1. **Solve the following pair of linear equations by the substitution method.**

(i) x + y = 14, x – y = 4

(ii) s – t = 3, $\frac{s}{3}+\frac{t}{2}=3$

(iii) 3x – y = 3, 9x – 3y = 9

(iv) 0.2x + 0.3y = 1.3, 0.4 + 0.5y = 2.3

(v) $\sqrt{2}x+\sqrt{3}y=0, \sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2, \frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

Sol. (i) x + y = 14 …(i)

x – y = 4 …(ii)

From (ii) y = x – 4 …(iii)

Substituting y from (iii) in (i), we get

x + x – 4 = 14 $\Rightarrow$ 2x = 18 $\Rightarrow$ x = 9

Substituting x = 9 in (iii), we get

y = 9 – 4 = 5,

i.e., y = 5

x = 9, y = 5

(ii) s – t = 3 …(i)

$\frac{s}{3}+\frac{t}{2}=6$ …(ii)

From (i) s = t + 3 …(iii)

Substituting s from (iii) in (ii), we get

$\frac{t+3}{3}+\frac{t}{2}=6 \quad \Rightarrow 2(t+3)+3t=36$

$\Rightarrow$ 5t + 6 = 36 $\Rightarrow$ t = 6

From (iii), s = 6 + 3 = 9

Hence, s = 9 , t = 6

[Rest Try Yourself]

2. **Solve 2x + 3y = 11 and 2x – 4y = - 24 and hence find the value of 'm' for which y = mx = 3**

Sol. **[Try Yourself]**

3. **Form the pair of linear equations for the following problems and find their solution by substitution method.**

(i) **The difference between two number is 26 and one number is three times the other. Find them.**

(ii) **The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.**

(iii) **The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. Later, she buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball.**

(iv) **The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 105 and for a journey of 15 km, the charge paid is Rs. 155. what are the fixed charges and the charge per kilometer ? How much does a person have to pay for travelling a distance of 25 km ?**

(v) **A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator is becomes $\frac{5}{6}$. Find the fraction.**

(vi) **Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their the present ages ?**

Hints (i) Let the two numbers be x and y (x > y). Then, x – y = 26 and x = 3y.

(ii) Let the supplementary angles by x and y (x > y) Then, x + y = 180 and x – y = 18.

(iii) Try Yourself

(iv) Let fixed charge be Rs x and charge per km be Rs y. Then, x + 10y = 105 and x + 15y = 155.

(v) Let $\frac{x}{y}$ be the fraction where x and y are positive integers. $\frac{x+2}{y+2}=\frac{9}{11}$ and $\frac{x+3}{y+3}=\frac{5}{6}$

(vi) Let x (in years) be the present age of Jacob's son and y (in years) be the present age of Jacob. Then, (x + 5) = 3 (x + 5) and (y – 5) = 7 (x – 5).

EXERCISE : 3 . 4

1. **Solve the following pair of equations by the elimination method and the substitution method.**

(i) **x + y = 5 and 2x – 3y = 4**

(ii) **3x + 4y = 10 and 2x – 2y = 2**

(iii) **3x – 5y – 4 = 0 and 9x = 2y + 7**

(iv) $\frac{x}{2}+\frac{2y}{3}=-1$ **and** $x-\frac{y}{3}=3$

Sol. (i) Solution By Elimination Method:

x + y = 5 …(i)

2x – 3y = 4 …(ii)

Multiplying (i) by 3 and (ii) by 1 and adding we get 3(x + y) + 1 (2x – 3y) = 3 × 5 + 1 × 4

$\Rightarrow$ 3x + 3y + 2x – 3y = 19

$\Rightarrow$ 5x = 19 $\Rightarrow$ x = $\frac{19}{5}$

From (i), substitution x = $\frac{19}{5}$, we get

$\frac{19}{5}$ + y = 5 $\Rightarrow$ y = 5 – $\frac{19}{5}$ $\Rightarrow$ y = $\frac{6}{5}$

Hence , x = $\frac{19}{5}$, y = $\frac{6}{5}$

[Rest Try Yourself]

(i) Solution By Substitution Method:

x + y = 5 …(i)

2x – 3y = 4 …(ii)

From (i), y = 5 – x …(iii)

Substituting y from (iii) in (ii),

2x – 3(5 – x) $\Rightarrow$ 2x – 15 + 3x = 4

$\Rightarrow$ 5x = 19 $\Rightarrow$ x = $\frac{19}{5}$

Then from (iii), y = 5 – $\frac{19}{5}$ $\Rightarrow$ y = $\frac{6}{5}$

Hence , x = $\frac{19}{5}$, y = $\frac{6}{5}$

2. **Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method.**

(i) **If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?**

(ii) **Five years ago Nuri was thrice as old as Sonu. Ten years late, Nuri will be twice as old as Sonu. How old are Nuri and Sonu ?**

(iii) **The sum of the digits of a tow-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.**

(iv) **Meena went to a bank to withdraw Rs. 2000. She asked the cashier to give her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received.**

(v) **A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs. 27 for a book kept for seven days, while Susy paid Rs. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.**

Hints (i) Let the fraction be $\frac{x}{y}$. Then $\frac{x+1}{y-1}=1$; $\frac{x}{y+1}=\frac{1}{2}$

(ii) Try yourself

(iii) Let x be the digit at unit place and y be the digit at tens place of the number. So, number = x + 10y. Then x + y = 9 and 9[x + 10y] = 2[y + 10x].

(iv) Let x and y be the number of Rs. 50 and Rs. 100 notes respectively.
Then, x + y = 25 and 50x + 100y = 2000

(v) Try yourself

EXERCISE : 3 . 5

1. **Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.**

(i) x – 3y – 3 = 0
3x – 9y – 2 = 0

(ii) 2x + y = 0
3x + 2y = 8

(iii) 3x – 5y = 20
6x – 10y = 40

(iv) x – 3y – 7 = 0
3x – 3y – 15 = 0

Sol. (i) x – 3y – 3 = 0, 3x – 9y – 2 = 0

$$\frac{a_1}{a_2}=\frac{1}{3},\frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3},\frac{c_1}{c_2}=\frac{3}{2}\Rightarrow\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Hence, no solution .

(ii) 2x + y = 5 …(i) and 3x + 2y = 8 …(ii)

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}\qquad\left(\frac{a_1}{a_2}=\frac{2}{3},\frac{b_1}{b_2}=\frac{1}{2}\right)$$

Here, we have a unique solution. By cross multiplication, we have

$$\frac{x}{\begin{vmatrix}1 & -5\\ 2 & -8\end{vmatrix}}=\frac{y}{\begin{vmatrix}-5 & 2\\ -8 & 3\end{vmatrix}}=\frac{1}{\begin{vmatrix}2 & 1\\ 3 & 2\end{vmatrix}}$$

$$\Rightarrow\quad\frac{x}{\{(1)(-8)-(2)(-5)\}}=\frac{y}{\{(-5)(3)-(-8)(2)\}}=\frac{1}{\{(2)(2)-(3)(1)\}}$$

$$\Rightarrow\quad\frac{x}{(-8+10)}=\frac{y}{(-15+16)}=\frac{1}{(4-3)}$$

$$\Rightarrow\quad\frac{x}{2}=\frac{y}{2}=\frac{1}{1}\qquad\Rightarrow\frac{x}{2}=\frac{1}{1}\text{ and }\frac{y}{1}=\frac{1}{1}$$

$\Rightarrow$ x = 2 and y = 1 **[Rest Try Yourself]**

2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions ?

$$2x + 3y = 7$$

$$(a - b)x + (a + b)y = 3a + b - 2$$

(ii) For which value of k will the following pair of linear equations have no solution ?

$$3x + y = 1$$

$$(2k - 1)x + (k - 1)y = 2k + 1.$$

Sol. (i) $2x + 3y - 7 = 0$ …(i)

$(a - b)x + (a + b)y - (3a + b - 2) = 0$ …(ii)

For infinite number of solutions, we have

$$\frac{a-b}{2} = \frac{a+b}{3} = \frac{3a+b-2}{7}$$

For first and second, we have

$$\frac{a-b}{2} = \frac{a+b}{3}$$

or $3a - 3b = 2a + 2b$

or $a = 5b$ …(i)

From second and third, we have

$$\frac{a+b}{3} = \frac{3a+b-2}{7}$$

or $7a + 7b = 9a + 3b - 6$

or $4b = 2a - 6$

From (i) and (ii), eliminating a,

$2b = 5b - 3 \Rightarrow b = 1$

Substituting b = 1 in (i), we get a = 5

(ii) **[Try Yourself]**

3. Solve following pair of linear equations by the substitution and cross-multiplication methods :

$8x + 5y = 9$, $3x + 2y = 4$

Sol. [Try Yourself]

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method .

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs. 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs. 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer , then Yash would have scored 50 marks. How many questions were there in the test ?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If travel towards each other, they meet in 1 hour. What are the speeds of the two cars ?

(v) The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and breadth is increased by 3 units . If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangles.

Sol. (i) Try Yourself

(ii) Try Yourself

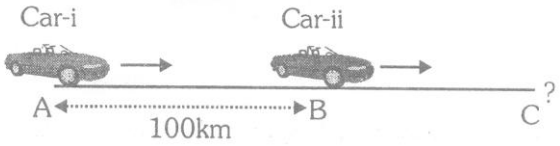
Hint (iii) number of right answers = x. Number of wrong answers = y

Then, $3x - y = 40$ and $4x - 2y = 50$

Hint (iv) Speed of car i = x km/hr

Speed of car ii = y km/hr

First case :



Two cars meet at C after 5 hrs.

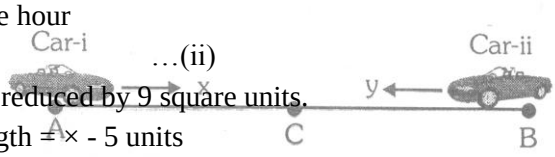
$AC - BC = AB$

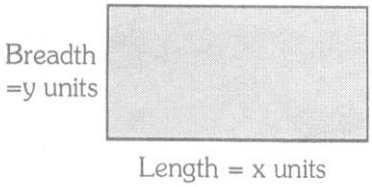
$\Rightarrow \quad 5x - 5y = 100 \quad \ldots(i)$

Second case:

Two cars meet at C after one hour

$x + y = 100 \quad \ldots(ii)$





Hint (v) In first case, area is reduced by 9 square units.

When length = x - 5 units

and breadth = y + 3 units

$\Rightarrow \quad xy - (x - 5) \times (y - 3) = 9 \quad \ldots(i)$

In second case area increases by 67 sq. units when length = x + 3 and breadth = y + 2.

$\Rightarrow \quad (x + 3) \times (y + 2) - xy = 67 \quad \ldots(ii)$

EXERCISE : 3 . 6

1. Solve the following pairs of equations by reducing them to a pair of linear equations :

(i) $\frac{1}{2x} + \frac{1}{3y} = 2, \ \frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2, \ \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

(iii) $\frac{4}{x} + 3y = 14, \ \frac{3}{x} - 4y = 23$

(iv) $\frac{5}{(x-1)} + \frac{1}{(y-2)} = 2, \ \frac{6}{(x-1)} - \frac{3}{(y-2)} = 1$

(v) $\frac{7x - 2y}{xy} = 5, \ \frac{8x + 7y}{xy} = 15$

(iv) $6x + 3y = 6xy, \ 2x + 4y = 5xy$

(vii) $\frac{10}{(x+y)} + \frac{2}{(x-y)} = 4, \ \frac{15}{(x+y)} - \frac{5}{(x-y)} = -2$

(viii) $\frac{1}{(3x+y)} + \frac{1}{(3x-y)} = \frac{3}{4}, \ \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

Sol. (i) $\frac{1}{2x} + \frac{1}{3y} = 2, \ \frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$

We get $\frac{1}{2}u+\frac{1}{3}v=2,\ \frac{1}{3}u+\frac{1}{2}v=\frac{13}{6}$

Multiplying by 6 on both sides, we get

$\Rightarrow$ $3u + 2v = 12$ …(i)

$2u + 3v = 13$ …(ii)

Multiplying (i) by 3 and (ii) by 2, then subtracting later from first, we get

$3(3u + 2v) – 2(2u + 3v) = 3 \times 12 – 2 \times 13$

$\Rightarrow$ $9u – 4u = 36 – 26 \Rightarrow u = 2$

Then substituting u =2 in (i), we get

$6 + 2v = 12 \Rightarrow v = 3$

Now, u = 2 and v = 3

$\Rightarrow$ $\frac{1}{x}=2$ and $\frac{1}{y}=3$ $\Rightarrow x=\frac{1}{2}$ and $y=\frac{1}{3}$

(ii) [Hint : Put $\frac{1}{\sqrt{x}}=u$ & $\frac{1}{\sqrt{y}}=v$],

(iii) Try Yourself

(iv) [Hint : Put $\frac{1}{x-1}=u$ and $\frac{1}{y-2}=v$ to get :

$5u + v = 2$ and $6u – 3v = 1$]

(v) [Hint : $\frac{7x-2y}{xy}=5,\ \frac{8x+7y}{xy}=15$

$\Rightarrow$ $\frac{7x}{xy}-\frac{2y}{xy}=5,\ \frac{8x}{xy}+\frac{7y}{xy}=15$

$\Rightarrow$ $\frac{7}{y}-\frac{2}{x}=5,\ \frac{8}{y}+\frac{7}{x}=15$

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$, we get

$7v – 2u = 5,\ 8v + 7u = 15$

[Rest Try Yourself]

2. Formulate the following problems as a pair of linear equations, and hence find their solutions :

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current .

(ii) **2 woman and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.**

(iii) **Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by has, she takes 10 minutes longer. Find the speed of the train and the bus separately .**

Sol. (i) [Hint Speed of Ritu in still water = x km/hr

Speed of current = y km/hr

Then speed downstream = (x + y) km/hr

Speed upstream = (x – y) km/hr

$\frac{20}{x+y}=2$ and $\frac{4}{x-y}=2$

$\Rightarrow$ x + y = 10 …(i)

x – y = 2 …(ii)

Hint (ii) Let 1 woman finish the work in x days and let 1 man finish the work in y days.

Work of 1 woman in 1 day $=\frac{1}{x}$

Work of 1 man in 1 day $=\frac{1}{y}$

Work of 2 woman and 5 men in one day $=\frac{2}{x}+\frac{5}{y}=\frac{5x+2y}{xy}$

The number of days required for complete work $=\frac{xy}{5x+2y}$

We are given that $\frac{xy}{5x+2y}=4$ …(i)

Similarly, in second case $\frac{xy}{6x+3y}=3$ …(ii)]

(iii) **[Try Yourself]**

EXERCISE – 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

Choose The Correct One

1. If A : Homogeneous system of linear equations is always consistent. R : x = 0, y = 0 ix always a solution of the homogeneous system of equations with unknowns x and y, then which of the following statement is true ?

(A) A is true and R is the correct explanation of A

(B) A is false and R is not a correct explanation of A

(C) A is true and R is false

(D) A is false and R true

2. If the pair of linear equations x – y = 1, x + ky = 5 has a unique solution x = 2, y = 1, then value of k is –

(A) -2 (B) 3 (C)-3 (D) 4

3. The pair of linear equations $2x + ky - 3 = 0$, $6x + \frac{2}{3}y + 7 = 0$ has a unique solution if –

(A) $k = \frac{2}{3}$ (B) $k \neq \frac{2}{3}$

(C) $k = \frac{2}{9}$ (D) $k \neq \frac{2}{9}$

4. The pair of linear equations 2kx + 5y = 7, 6x – 5y = 11 has a unique solution if –

(A) $k \neq -3$ (B) $k \neq 3$

(C) $k \neq 5$ (D) $k \neq -5$

5. The pair of equations 3x + 4y = k, 9x + 12y = 6 has infinitely many solutions if –

(A) k = 2 (B) k = 6

(C) k ≠ 6 (D) k = 3

6. The pair of linear equations 2x + 5y = k, kx + 15y = 18 has infinitely many solution if –

(A) k = 3 (B) k = 6 (C) k = 9 (D) k = 18

7. The pair of linear equations 3x + 5y = 3, 6x + ky = 8 do not have any solution if –

(A) k = 5 (B) k = 10 (C) k ≠ 10 (D) k ≠ 5

8. The pair of linear equations 3x + 7y = k, 12x + 28y = 4k + 1 do not have any solution if

(A) k = 7 (B) k = 14 (C) k = 21 (D) k = 28

9. The pair of linear equations $7x - 3y = 4$, $3x + \frac{k}{7}y = 4$ is consistent only when –

(A) k = 9 (B) k = -9 (C) k ≠ - 9 (D) k ≠ 7

10. The pair of linear equations kx + 4y = 5, 3x + 2y = 5 is consistent only when –

(A) k ≠ 6 (B) k = 6 (C) k ≠ 3 (D) k = 3

11. The pair of linear equations 7x + ky = k, 14x + 2y = k + 1 has infinitely many solution if –

(A) k = 1 (B) k ≠ 1 (C) k = 2 (D) k = 4

12. The pair of linear equations 13x + ky = k, 39x + 6y = k + 4 has infinitely many solutions if -

(A) k = 1 (B) k = 2 (C) k = 4 (D) k = 6

13. The pair of linear equations x + y = 3, 2x + 5y = 12 has a unique solution $x = x_1$, $y - y_1$ then value of x_1 is –

(A) 1 (B) 2 (C) -1 (D) -2

14. The pair of linear equations 3x – 5y + 1 = 0, 2x – y + 3 = 0 has a unique solution $x = x_1$, $y = y_1$ then y_1 =

(A) 1 (B) -1 (C) -2 (D) – 4

15. The pair of linear equations x + 2y = 5, 7x + 3y = 13 has a unique solution –

(A) x = 1, y = 2 (B) x = 2, y = 1

(C) x = 3, y = 1 (D) x = 1, y = 3

16. The pair of linear equations x + 2y = 5, 3x + 12y = 10 has –

(A) Unique solution

(B) No solution

(C) More than two solution

(D) Infinitely many solutions

17. If the sum of the ages of a father and his son in years is 65 and twice the difference of their ages in years is 50, then the age of father is –

(A) 45 years (B) 40 years (C) 50 years (D) 55 years

18. A fraction becomes $\frac{4}{5}$ when 1 is added to each of the numerator and denominator. However, if we subtract 5 from each then it becomes $\frac{1}{2}$. The fraction is –

(A) $\frac{5}{8}$ (B) $\frac{5}{6}$ (C) $\frac{7}{9}$ (D) $\frac{13}{16}$

19. Three chairs and two tables cost Rs. 1850 Five chairs and three tables cost Rs. 1850. Then the total cost of one chair and table is –

(A) Rs. 800 (B) Rs. 850 (C) Rs. 900 (D) Rs.950

20. Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son. The present age of the man is –

(A) 28 years (B) 30 years (C) 32 years (D) 34 years

OBJECTIVE				ANSWER KEY										EXERCISE - 1	
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**A**	**B**	**D**	**A**	**A**	**B**	**B**	**B**	**C**	**A**	**A**	**B**	**A**	**B**	**A**
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	**A**	**A**	**C**	**B**	**B**										

EXERCISE – 2 (FOR SCHOOL / BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions

1. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find for whether the following pair of liner equations are consistent or inconsistent .

(i) x – 3y = 4 ; 3x + 2y = 1 (ii) $\frac{4}{3}$x + 2y = 8 ; 2x + 3y = 12

(iii) $4x + 6y = 7$; $12x + 18y = 21$ (iv) $x - 2y = 3$; $3x - 6y = 1$

2. On comparing the ratio $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines representing is following pair of linear equations intersect at a point, are parallel or coincident :

(a) (i) $2x - y = 3$; $4x - y = 5$ (ii) $x + 2y = 8$; $5x - 10y = 10$ (iii) $3x + 4y = -2$; $12x + 16y = -8$

(b) (i) $6x + 3y = 18$; $2x + y = 6$ (ii) $x - 3y = 3$; $3x - 9y = 2$

(iii) $ax - by = c_1$; $bx + ay = c_2$, where $a \neq 0, b \neq 0$

3. For the linear equations given below, write anther linear equation in two variables, such that the geometrical representation of the pair so formed is -

(i) Intersecting (ii) Parallel lines (iii) Coincident lines

(a) $2x - 3y = +$ (b) $y = 2x + 3$

4. Find the value of k for which the given system of equations has a unique solution .

(a) $(k - 3)x + 3y = k$; $kx + ky = 12$ (b) $x - ky = 2$; $3x + 2y = -5$

5. Find the value of k for which the given system of equations has no solution .

(a) $kx + 2y - 1 = 0$; $5x - 3y + 2 = 0$

(b) (i) $x + 2y = 3$; $5x + ky + 7 = 0$ (ii) $kx + 3y = k - 3$; $12x + ky = k$

6. (a) Find the value (s) of k for which the system of equations $kx - y = 2$ and $6x - 2y = 3$ has

(i) A unique solution (ii) No solution

(b)Find the value of k for which system $kx + 2y = 5$ and $3x + y = 1$ has

(i) A unique solution (ii) No solution

7. Find the value of k for which the given system of equations has an infinite number of solutions.

(a) $5x + 2y = 2k$ and $2(k + 1)x + ky = (3k + 4)$

(b) (i) $x + (k + 1)y = 5$ and $(k + 1)x + 9y = 8k - 1$

(ii) $10x + 5y - (k - 5) = 0$ and $20x + 10y - k = 0$

(c) $kx + 3y = k - 3$ and $12x + ky = k$

8. Find the value of a and b for which the given system of linear equation has an infinite number of solutions :

(a) $2x + 3y = 7$ and $(a - b)x + (a + b)y = 3a + b - 2$

(b) $(a + b)x - 2by = 5a + 2b + 1$ and $3x - y = 14$

(c) $(2a - 1)x + 3y - 5 = 0$ and $3x + (b - 1)y - 2 = 0$

Short Answer Type Questions

Based on graphical solution of system of equations :

Solve graphically each of the following pairs of equations (1–9) :

1. $x + y = 4$, $2x - 3y = 3$

2. $x + y = 3$, $2x + 5y - 12 = 0$

3. $\frac{4}{9}x + \frac{1}{3}y = 1$, $5x + 2y = 13$

4. $2x + 3y = 4$, $x - y + 3 = 0$

5. $x + y = 7$, $5x + 2y = 20$

6. $x + 4y = 0$, $2x + 8y = 0$

7. x + 2y = 3, 2x + 4y = 15

8. 3x + 2y = 3, 6x + 4y = 15

9. 2x + 3y – 5 = 0, 6x + 9y – 15 = 0

10. Check whether the pair of equations x + 3y = 6, and 2x – 3y = 12 is consistent. If so, solve graphically .

11. Show graphically that the pair of equations 2x – 3y + 7 = 0, 6x – 9y + 21 = 0 has infinitely many solutions.

12. Show graphically that the pair of equations 8x + 5y = 9, 16x + 10y = 27 has no solution.

13. Find whether the pair of equations 5x – 8y + 1 = 0, $3x - \frac{24}{5}y + \frac{3}{5} = 0$ has no solution, unique solution or infinitely many solutions.

14. Show graphically that the pair of equations 2x – 3y = 4, 3x – 2y = 1 has a unique solution.

15. Show graphically that the pair of equations 3x + 4y = 6, 6x + 8y = 12 represents coincident lines.

16. Determine by drawing graphs whether the following pair of equations has a unique solution or no :
2x – 3y = 6, 4x – 6y = 9. If yes, find the solution also.

17. Determine graphically whether the pair of linear equations 3x – 5y = - 1 , 2x – y = - 3 has a unique solution or not. If yes, find the solution also .

18. Solve graphically the pair of equations x + 3y = 6, and 3x – 5y = 18. Hence, find the value of K if 7x + 3y = K.

19. Solve graphically the pair of equations 2x – y = 1, x + 2y = 8. Also find the points where the lines meet the axis of y.

20. Solve graphically the following pair of linear equations :
2x + 3y – 12 = 0, 2x – y – 4 = 0. Also find the coordinates of the points where the lines meet the y-axis.

21. Solve the following pair of equations graphically : x + y = 4, 3x – 2y = 3
Shade the region bounded by the lines representing the above equations and x-axis.

22. Solve the following pair of linear equations graphically : 2x + y = 8, 3x – 2y = 12.
From the graph, read the points where the lines meet the x-axis.

23. Solve graphically the following pair of equations : x – y = 1, 2x + y = 8. Shade the area bounded by these lines and the y-axis.

24. On the same axes, draw the graph of each of the following equations :
2y – x = 8, 5y – x = 14, y – 2x = 1. Hence, obtain the vertices of the triangle so formed.

25. Solve graphically the pair of linear equations : 4x – 3y + 4 = 0, 4x + 3y – 20 = 0. Find the area of the region bounded by theses lines and x-axis,

Based on substitution method :
Solve the following equations by the substitution method : (26-41)

26. 3x + 11y = 13, 8x + 13y = 2

27. x + 2y = 1.6, 2x + y = 1.4

28. 11x – 8y = 27, 3x+5y = - 7

29. 0.04x + 0.02y = 5, 0.5x – 0.4y = 30

30. 5x + 8y = - 1, 6y – x = 4y – 7

31. 12x – 16y = 20, 8x + 6y = 30

32. 8x – 5y + 40 = 0, 7x – 2y = 0

33. $\frac{1}{2}(9x + 10y) = 23$, $\frac{5x}{4} - 2y = 3$

34. $\sqrt{2}x + \sqrt{3}y = 0$, $\sqrt{3}x - \sqrt{8}y = 0$

35. $\frac{(3x - y)}{5} = 2y - 1$, $\frac{3x}{8} - \frac{y}{4} = \frac{1}{2}$

36. 3x + 15 = 4y, 3y + 17 = 2 + 3x

37. x + 6y = 2x – 16, 3x – 2y = 24

38. x = 3y – 19, y = 3x – 23

39. $5x + 2y = 14,\ x + 3y = 8$

40. $x + y = 27,\ \frac{3}{4}x + \frac{2}{3}y = 19$

41. $\frac{x+11}{7} + 2y = 10,\ \ 3x = 8 + \frac{y+7}{11}$

42. Solve $2x - y = 12$ and $x + 3y + 1 = 0$ and hence find the value of m for which $y = mx + 3$.

43. Solve $4x - 3y + 17 = 0$ and $5x + y + = 0$ and hence find the value of n for which $y = nx - 1$.

Based on substitution method :

Solve the following pairs of lines equations by elimination method : (44-52)

44. (a) $x + y = 5$ and $2x - 3y = 4$

(b) (i) $2x + 3y = 8$ and $4x + 6y = 7$

(ii) $11x + 15y + 23 = 0$ and $7x - 2y - 20 = 0$

(c) $78x + 91y = 39$ and $65x + 117y = 42$

45. (a) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$ (b) $\frac{x}{3} + \frac{y}{4} = 11$ and $\frac{5x}{6} - \frac{y}{3} + 7 = 0$

46. (a) $ax - by = a^2 + b^2$ and $x + y = 2a$ (b) $\frac{bx}{a} - \frac{ay}{b} + a + b = 0$ and $bx - ay + 2ab = 0$

47. $ax + by - 2a + 3b = 0$ and $bx - ay - 3a - 2b = 0$

48. (a) $2(ax - by) + (a + 4b) = 0$ and $2(bx + ay) + (b - 4a) = 0$

(b) $(bx + ay) = 0$ and $(a + b)x + (a - b)y = a^2 + b^2$

49. (a) $(a + c)x - (a - c)y = 2ab$ and $(a + b)x - (a - b)y = 2ab$

(b) $(a + 2b)x + (2a - b)y = 2$ and $(a - 2b)x + (2a + b)y = 3$

50. (a) $\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{5}x + \sqrt{2}y = 0$

(b) $\sqrt{7}x + \sqrt{11}y = 0$ and $\sqrt{3}x - \sqrt{5}y = 0$

51. $0.5x + 0.7y = 0.74$ and $0.3x + 0.5y = 0.5$

52. (a) $23x - 29y = 98$ and $29x - 23y = 100$

(b) (i) $217x + 131y = 913$ and $131x + 127y = 827$

(ii) $23x + 37y = 32$ and $37x + 23y = 88$

(c) (i) $65x - 33y = 97$ and $33x - 65y = 1$

(ii) $47x + 31y = 63$ and $31x + 47y = 15$

(iii) $99x + 101y = 499$ and $101x + 99y = 501$

Based on cross-multiplication method :

Solve each of the following pairs of equations by cross multiplication rule : (53-62)

53. $x - 2y = 10,\ 4x + y = 13$

54. $5x + 3y = 35,\ 2x + 4y = 28$

55. $4x - 3y + 1 = 0,\ 2x - 5y + 11 = 0$

56. $3x + 4y = 27,\ 5x - 3y = 16$

57. $2x + 3y = 46,\ 3x + 5y = 74$

58. $2x - y + 4 = 0,\ x + y - 1 = 0$

59. $\frac{x}{2} - \frac{y}{3} + 4 = 0,\ \frac{x}{2} - \frac{5y}{3} + 12 = 0$

60. $ax + by + a = 0,\ bx + ay + b = 0$

61. $\frac{x}{a} + \frac{y}{b} = 0,\ \frac{x}{a} - \frac{y}{b} = 4$

62. $x + y = a + b,\ ax - by = a^2 - b^2$

Based on equations reducible to linear equations :

Solve for x and y : (63-82)

63. $\frac{2}{x}+\frac{3}{y}=2;\ \frac{1}{x}-\frac{1}{2y}=\frac{1}{3}$

64. $\frac{4}{x}+\frac{7}{y}=29;\ \frac{3}{y}+\frac{1}{x}=11$

65. $\frac{1}{3x}-\frac{1}{7y}=\frac{2}{3};\ \frac{1}{2x}-\frac{1}{3y}=\frac{1}{6}$

66. $\frac{1}{5x}+\frac{9}{y}=4;\ \frac{3}{x}+\frac{27}{y}=24$

67. $\frac{11}{2x}-\frac{9}{2y}=-\frac{23}{2};\ \frac{3}{4x}+\frac{7}{15y}=\frac{23}{6}$

68. $x + y = 2xy\ ;\ x - y = xy$

69. $4x + 3y = 8xy\ ;\ 6x + 5y = 13xy$

70. $6x + 5y = 8xy\ ;\ 8x + 3y = 7xy$

71. $\frac{x-y}{xy}=9;\ \frac{x+y}{xy}=5$

72. $9 + 25xy = 53x\ ;\ 27 - 4xy = x$

73. $\frac{16}{x+3}+\frac{3}{y-2}=5;\ \frac{8}{x+3}-\frac{1}{y-2}=0$

74. $\frac{13}{x+y}-\frac{56}{x-y}=21;\ \frac{11}{x+y}-\frac{23}{x-y}=14\frac{2}{7}$

75. $\frac{24}{2x+y}-\frac{13}{3x+2y}=2;\ \frac{26}{3x+2y}+\frac{8}{2x+y}=3$

76. $\frac{29}{x-1}-\frac{81}{y+1}=-26;\ \frac{19}{y+1}-\frac{4}{x-1}=\frac{15}{2}$

77. $\frac{2}{x-1}+\frac{y-2}{4}=2;\ \frac{3}{2(x-1)}+\frac{2(y-2)}{5}=\frac{47}{20}$

78. $\frac{x-4}{x-3}=\frac{y+4}{y+7};\ \frac{x+5}{x+2}=\frac{y+1}{y-2}$

79. $\frac{3x-2}{3y+7}=\frac{5x-1}{5y+16};\ \frac{3x-15}{x-9}=\frac{6y-5}{2y+3}$

80. $\frac{x+y+3}{x-y-3}=-3;\ \frac{x-y-3}{x-y-3}=-2$

81. $\frac{x+y-1}{x-y+1}=7;\ \frac{y-x+1}{x-y+1}=35$

82. $\frac{x+2y=1}{2x-y+1}=2;\ \frac{3x-y+1}{x-y+3}=5$

- **Very Short Answer Type Questions**

1. (i) consistent (ii) consistent (iii) consistent (iv) inconsistent

2. (a) (i) intersect at point (ii) parallel (iii) coincident (b) (i) coincident (ii) parallel (iii) intersect at a point

3. (a) (i) $3x + 5y - 7 = 0$ (ii) $4x - 6y - 8 = 0$ (iii) $6x - 9y = 18$

(b) (i) 2x + 3y – 4 = 0 (ii) 4x – 2y + 8 = 0 (iii) 8x – 4y + 12 = 0 (many such examples may be given)

4. (a) $k \neq 6$ (b) $k \neq \frac{-2}{3}$ **5.** (a) $k = \frac{-10}{3}$ (b) (i) k = 10 (ii) k = - 6 **6.** (a) (i) $k \neq 3$ (ii) k = 3 (b)(i) $k \neq 6$(ii) k = 6

7. (a) k = 4 (b) (i) k = 2 (ii) k = 10 (c) k = 6 **8.** (a) a = 5, b = 1 (b) a = 5, b = 1 (c) a = $\frac{17}{4}$, b = $\frac{11}{5}$

- **Short Answer Type Questions**

1. x = 3, y = 1 **2.** x = 1, y = 2 **3.** x = 3, y = - 1 **4.** x = - 1, y = 2 **5.** x = 2, y = 5 **6.** Infinite number of solutions
7. No solution **8.** No solution **9.** Infinite number of solutions **10.** Yes ; x = 6, y = 0 **13.** Infinitely many solutions
16. No **17.** Yes ; x = - 2, y = - 1 **18.** x = 6, y = 0 ; K = 42 **19.** x = 2, y = 3 ; (0, - 1), (0, 4)
20. x = 3, y = 2, (0, 4), (0, - 4) **21.** x = 1, y = 3
22. x = 4, y = 0 ; The two lines meet at the x-axis at a common point (4, 0). **23.** x = 3, y = 2 **24.** (2, 5), (-4, 2), (1, 3)
25. 12 sq units. **26.** x = -3, y = 2 **27.** x = 0.4, y = 0.6 **28.** x = 1, y = - 2 **29.** x = 100, y = 50
30. x = 3, y = - 2 **31.** x = 3, y = 1 **32.** x = $\frac{80}{19}$, y = $\frac{80}{19}$ **33.** x = 4, y = 1 **34.** x = y = 0 **35.** x = 2, y = 1
36. x = 35, y = 30 **37.** x = 7, y = - 3/2 **38.** x = 11, y = 10 **39.** x = 2, y = 2 **40.** x = 12, y = 15
41. x = 3, y = 4 **42.** x = 5, y = - 2 , - 1 **43.** x = - 2, y = 3, - 2
44. (a) x = $\frac{19}{5}$, y = $\frac{6}{5}$ (b) (i) No solution (ii) x = 2, y = - 3 (c) x = $\frac{3}{13}$, y = $\frac{3}{13}$ **45.** (a) x = 2, y = -3 (b) x = 6, y = 36
46. (a) x = (a + b), y = (a – b) (b) x = - a, y = b **47.** x = 2, y = - 3 **48.** (a) x = $\frac{-1}{2}$, y = 2, (b) x = a, y = - b
49. (a) x = b, y = - b (b) x = $\frac{5b-2a}{10ab}$, y = $\frac{a+10b}{10ab}$ **50.** (a) x = 0, y = 0 (b) x = 0, y = 0 **51.** x = 0.5, y = 0.7
52. (a) x = 3, y = - 1 (b) (i) x = 3, y = 2 (ii) x = 3, y = - 1 (c) (i) x = 2, y = 1 (ii) x = 2, y = - 1 (iii) x = 3, y = 2
53. x = 4, y = - 3 **54.** x = 4, y = 5 **55.** x = 2, y = 3 **56.** x = 5, **57.** x = 8, y = 10 **58.** x = - 1, y = 2
59. x = - 4, y = 6 **60.** x = - 1, y = 0 **61.** x = 2a, y = - 2b **62.** x = a, y = b **63.** x = 2, y = 3 64. x = $\frac{1}{2}$, y = $\frac{1}{3}$
65. x = $\frac{1}{5}$, y = $\frac{1}{7}$ **66.** x = $\frac{1}{5}$, y = 3 67. x = $\frac{1}{2}$, y = $\frac{1}{5}$ **68.** x = 2, = $\frac{2}{3}$ **69.** x = $\frac{1}{2}$, y = 2 **70.** x = 1, y = 2
71. x = - $\frac{1}{2}$, y = $\frac{1}{7}$ **72.** x = 3, y = 2 **73.** x = 5, y = 3 **74.** x = - 3, y = 4 **75.** x = 3, y = 2 **76.** x = 3, y = 1
77. x = 3, y = 6 **78.** x = 7, y = 5 **79.** x = 3, y = 2 **80.** x = 3, y = 2 **81.** x = $\frac{2}{9}$, y = $\frac{7}{6}$ **82.** x = 13, y = 10

EXERCISE – 3 (FOR SCHOOL / BOARD EXAMS)

APPLICATIONS TO WORD PROBLEMS

Based On Articles And Their Costs :

Q.1 4 chairer and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs. 1750. Find the cost of a chair and a table separately .

Q.2 37 pens and 53 pencils together cost Rs. 320, while 53 pens and 37 pencils together cost Rs 40. Find the cost of a en and that of a pencil.

Q.3 4 tables and 3 chairs together cost Rs 2250 and 3 tables and 4 chairs cost Rs 1950. Find the cost of 2 chairs and 1 table.

Q.4 A and B each have certain number of oranges. A says to A says to B. “if you give me 10 of your oranges, I will have twice the number of oranges left with you. “B relies,” if you give me 10 of your oranges, If will have the same number of oranges as left with you.” Find the number of oranges with A and B separately.

Q.5 A and B each have a certain number of mangoes. A says to B, " if you give 30 of your mangoes, I will have twice as many as left with you." B replies, "if you give me 10, I will have thrice as many as left with you." How many mangoes does each have ?

Q.6 One says, " give me a hundred , friend ! I shall then become twice as rich as you, " The other replies," If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their respective capital ?

Q.7 Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Q.8 A man has only 20 paisa coins and 25 paisa coins in his purse. If he has 50 coins in all totaling Rs 11.25, how many cons of each kind does he have :

Q.9 A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Based on numbers :

Q.10 Sum of two numbers is 35 and their difference is 13. Find the numbers .

Q.11. The sum of two number is 8. If their sum is 4 times their difference. Find the number.

Q.12. The sum of two numbers is 1000 and the difference between their squares is 256000. Find the numbers .

Q.13 In a two digit number, the unit's digit is twice the ten's digit . If 27 is added to the number. the digits interchange their paces. Find the number.

Q.14. In a two digit number, the ten's digit is three times the unit's digit . When the number is decreased by 54, the digits are reversed. Find the number.

Q.15 The sum of the digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number .

Q.16 The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digit is 18. Find the number.

Q.17. The sum of a two digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of its digits in the first number. Find the first number .

Q.18 The sum of a two digit number and the number formed by interchanging its digits is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

Q.19 The sum of a two digit number and the number obtained by reversing the order of its digits is 121, and the two digits differ by 3. Find the number.

Q.20 A two digit number is 3 more than 4 times the sum of digits. If 18 is added to the number, the digits are reversed. Find the number.

Based On Fractions :

Q.21 A fraction becomes $\frac{4}{5}$, if 1 is added to both numerator and denominator. If however, 5 is subtracted from both numerator and denominator, the fraction becomes $\frac{1}{2}$. What is the fraction ?

Q.22 A fraction is such that if the numerator is multiplied by 3 and the denominator is reduced by 3, we get $\frac{18}{11}$. But , if the numerator is increased by 8 and the denominator is doubled, we get $\frac{2}{5}$. Find the fraction ?

Q.23 The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator . Determine the fraction.

Q.24 The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Q.25 The sum of the numerator and denominator of a fraction is 4 more than twice the numerator . If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction .

Q.26. The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator by 1, the numerator becomes half the denominator. Determine the fraction.

Based On ages :

Q.27. If twice the son's age in years is added to the father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son .

Q.28. A father is three times as old as his son. After twelve years, his age will be twice as that of his son then. Find their present ages.

Q.29. I am three times as old as my son. Five years later, I shall be two and a half times as old as my son. How old am I and how old is my son ?

Q.30 Ten years ago, father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be. Find their present ages.

Q.31 Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.

Q.32 The present age of a father is three years more than three times the age of the son. Three years hence, father's age will be 10 years more than twice the age the son. Determine their present ages.

Q.33 A and B are friends and their ages differ by 2 years. A's father D is twice as old as A and B is twice as old as his sister C. The age of D and C differ by 40 years. Find the ages of A and B.

Q.34 A is elder to B by 2 years. A's father F is twice as old as A and B is twice as old his sister S. If the ages of the father and sister differ by 40 years, find the age of A.

Q.35 Father's age is three times the sum of ages of his two children . After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Based On Time, Distance And Speed :

Q.36 Points A and B are 90 km. apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction, they meet in 9 hours and if they go in opposite directions, they meet in 9/7 hours. Find their speeds.

Q.37 Points A and B are 70 km. apart on a highway. A car starts from A and another from B simultaneously. If they ravel in the same direction, they meet in 7 hours but if they travel towards each other they meet in one hour. Find the speeds of the two cars.

Q.38 Points A and B are 80 km. apart from each other on a highway. A car starts from A and another from B at the same time. If they move in the same direction, they meet in 8 hours and if move in opposite directions, they meet in one hour and twenty minutes. Find the speeds of the two cars.

Q.39 Rahul travels 600 km to his home partly by train and partly by car. He takes 8 hours if he travels 120 km by train and rest by car. He takes 20 minutes longer if he travels 200 km by train and the rest by car. Find the speed of the train and the car.

Q.40 A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But , if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

Q.41 A boat covers 32 m upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of he boat in still water and that of the stream.

Q.42 A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current .

Q.43 The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours. It can go 40 km upstream and 55 km downstream. Determine the speed of stream and that of the boat in still water.

Q.44 A boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6^{1/2}$ hrs. Find the speed of the boat in still water and also speed of the stream.

Q.45 X takes 3 hours more than Y to walk 30 km, But, if X doubles his pace, he is ahead of Y by $1^{1/2}$ hours. Find their speed of walking.

Q.46 While covering a distance of 30 km. Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed, he would take 1 hour less than Amit. Find their speeds of walking .

Q.47 A man walks a certain distance with certain speed. If he walks $\frac{1}{2}$ km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. Find the distance covered by the man and his original rate of walking.

Q.48 A train covered a certain distances at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/h, it would have taken 6 hours more than the scheduled time. Find the length of the journey .

Based on geometrical applications :

Q.49 In a $\Delta ABC, \angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Q.50 Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (2x-1)^0, \angle B = (y+5)^0, \angle C = (2y+15)^0$ and $\angle D = (4x-7)^0$.

Q.51 In a $\Delta ABC, \angle A = x^0, \angle B = 3x^0$ and $\angle C = y^0$. If 3y – 5x = 30, prove that the triangle is right angled.

Q.52 The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units. Find the length and breadth of the rectangle .

Q.53 If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle .

Q.54 In a rectangle, if the length is increased by 3 meters and breadth is decreased by 4 metres, the area of the rectangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres. Find the dimensions of the rectangle .

Miscellaneous problems :

Q.55 A man starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was Rs.1500 after 4 years of service and Rs. 1800 after 10 years of service, what was his starting salary and what is the annual increment ?

Q.56 A railway half ticket costs half the full fare and the reservation charge is the same o half ticket as on full ticket. One reserved first class ticket from Mumbai to Ahmadabad costs Rs 216 and one full and one half reserved first class tickets cost Rs. 327. What is the basic first class full fare and what is the reservation charge ?

Q.57 Meena went to a bank to withdraw Rs. 2000. She asked the cashier to given her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received.

Q.58 Yash scored 40 marks in test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test ?

Q.59 The incomes of X and Y are in the ratio of 8 : 7 and their expenditures are in the ratio 19 : 16. If each saves Rs. 1250, find their incomes.

Q.60 The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them saves Rs. 200 per month, find their monthly incomes.

Q.61 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

Q.62 2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it ?

Q.63 2 women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman along to finish the embroidery, and that taken by 1 man alone.

Q.64 Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row, there would be 3 rows more. Find the number of students in the class.

Q.65 The students of a class are made to stand in rows. If 3 students are extra in raw, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

1. Cost of a chair = Rs. 15, Cost of a table = Rs. 500 **2.** Cost of a pen = Rs. 6.50, Cost of a pencil = Rs. 1.50
3. Rs. 150 **4.** A : 70 oranges, B : 50 oranges **5.** A : 34 mangoes, B : 62 mangoes

6. Rs. 40 , Rs. 170 **7.** Number of pens = 13, Number of pencils = 27 **8.** 25 coins of each kind.
9. Rs. 15, Rs. 3 **10.** 24, 11 **11.** 5, 3 **12.** 628, 372 **13.** 36 **14.** 93 **15.** 78 **16.** 53
17. 64 **18.** 48 **19.** 47 or 74 **20.** 35 **21.** $\frac{7}{9}$ **22.** $\frac{12}{25}$ **23.** $\frac{7}{18}$ **24.** $\frac{3}{7}$ **25.** $\frac{5}{9}$
26. $\frac{4}{7}$ **27.** Father's age = 40 years, Son's age = 15 years. **28.** Father's age = 36 years, Son's age = 12 years
29. My present age is 45 years and my son's present age is 15 fears.
30. Father's age = 34 years. Son's age = 12 years. **31.** Father's age = 40 years, Son's age = 10 years.
32. Father's age = 33 years, Son's age = 10 years.
33. A's age $= 27\frac{1}{3}$ years, B's age $= 29\frac{1}{3}$ years or A's age = 26 years, B's age = 24 years.
34. 26 years **35.** Father's age = 45 years **36.** 40 km/h & 30 km/hr
37. 40 km/h & 30 km/hr **38.** 35 km/h & 25 km/hr
39. Speed of train = 6 km/h & Speed of car = 80 km/hr **40.** Speed of train = 100 km/h & Speed of car = 80km/hr
41. Speed of boat = 10 km/h & Speed of steam = 2 km/hr **42.** Speed of sailor = 10 km/h & Speed of current =2km/hr
43. Speed of boat = 8 km/h & Speed of stream = 3 km/hr **44.** Speed of boat = 10 km/h & Speed of stream = 4 km/hr
45. X's speed $= \frac{10}{3}$ km/hr, Y's speed = 5 km/hr **46.** Ajeet's speed = 5 km/h & Amit's speed = 7.5 km/hr
47. Distance = 36 km, original speed = 4 km/hr **48.** 720 km
49. $\angle A = 20^0, \angle B = 40^0, \angle C = 120^0$ **50.** $\angle A = 65^0, \angle B = 55^0, \angle C = 115^0, \angle D = 15^0$
52. Length = 17 units breadth = 9 units **53.** 253 Sq. units
54. Length = 28 m, Bredth = 19 m
55. Starting salary = Rs. 1300, Annual increment = Rs. 50 **56.** Fare = Rs. 21, Reservation charge = Rs. 6
57. 10, 15 **58.** 20 **59.** X;s income = Rs. 6000, Y's income = Rs. 5250 **60.** Rs. 1800, Rs. 1400
61. Man : 140 days, Boy : 280 days. **62.** Man : 15 days, Boy : 60 days. **63.** Woman : 36 days, Man : 18 days
68. 60 **65.** 36

EXERCISE -4 (FOR SCHOOL / BOARD EXAMS)

PREVIOUS YEARS BOARD (SBSE) EMERSIONS

Short Answer Type – I

1. Find the value of k for which the following system of linear equations has infinite number of solutions
x + (k + 1) y = 5 ; (k + 1) x + 9y = 8k – 1 **[AI-2003]**

2. Find the value of k so that the system of linear equations will have infinite number of solutions :
x + (k + 2) y = 4 ; (2k – 1) x + 25y = 6k + 2 **[foreign-2003]**

3. Solve the following system of linear equations :
2(ax – by) + (a + 4b) = 0, 2(bx + ay) + (b – 4a) = 0.

OR

Two years ago, a father was five times as old as his son. Two years later , his age will be 8 more than three times the age of the son. Find the present ages of father and son. **[Delhi-2003]**

4. Solve the following system of linear equations : 6(ax + by) = 3a + 2b ; 6(bx – ay) = 3b – 2a .

OR

The sum of the digits of a two digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number **[AI-2004]**

5. Solve the following system of linear equations : 3(bx + ay) = a - +b, 3(ax – by) = - (6a + b).

OR

If 1 is added to each of numerator and denominator of a fraction, it becomes 2/3. However, if 1 is subtracted form each of numerator and denominator it becomes 3/5. Find the fraction. **[Foreign-2003]**

6. solve for x and y : $\frac{4}{x} + 3y = 14, \ \frac{3}{x} - 4y = 23$

OR

Solve for x and y : $\frac{b}{a}x+\frac{b}{b}y=a^2+b^2,\ x+y=2ab.$ **[Delhi-2004C]**

7. If (x – 4) is a factor of $x^3 + ax^2 + 2bx – 24$ and a – b = 8, find the values of a and b. **[Delhi-2004C]**

8. If (x + 3) is a factor of $x^3 + ax^2 – bx + 6$ and a + b = 7, find the values of a and b **[Delhi-2004C]**

9. If (x + 2) is a factor of $x^3 + ax^2 + 4bx + 12$ and a + b = - 4, find the values of a and b. **[Delhi-2004C]**

10. Solve for x and y : $\frac{2}{x}+\frac{3}{y}=13,\ \frac{5}{x}-\frac{4}{y}=-2, x, y\neq 0$

OR

Solve for x and y : ax + by – a + b = 0, bx – ay – a – b = 0 **[AI-2004C]**

11. If (x – 2) is a factor of x3+ax2+bx+18 and a – b = 7, find a and b. **[AI-2004C]**

12. Solve the following system of linear equations : ax + by = a – b, bx – ay = a + b. **[Delhi-2004C]**

13. Solve for x and y : $\frac{x}{a}+\frac{y}{b}=2,\ ax-by=a^2-b^2$

OR

A two digit number is four times the sum of its digits and twice the product of the digits. Find the number. **[AI-2005]**

14. Solve for x and y : $\frac{x}{a}-\frac{y}{b}=a-b,\ ax+by=a^3+b^3.$

OR

A number consisting of two digit, is equal to 7 times the sum of its digits. When 27 is subtracted from the number , the digit interchange places. Find the number . **[Foreign-2005]**

15. Solve for x and y : $\frac{2a}{x}+\frac{3b}{y}+1=0;\ \frac{3a}{x}-\frac{b}{y}-4=0$ **[Delhi -2005]**

16. Solve for x and y : $\frac{3a}{x}-\frac{2b}{y}+=0;\ \frac{a}{x}+\frac{3b}{y}-2=0$ **[AI-2005C]**

17. Solve for x and y ; 47x + 31y = 63, 31x + 47 = 15

OR

Solve for x and y : $\frac{ax}{b}-\frac{by}{a}=a+b; ax-by=2ab$ **[Delhi -2006]**

18. Solve the system of equations :

$\frac{bx}{a}-\frac{ay}{b}+a+b=0$ and $bx-ay+2ab=0$

OR

The sum of ht digits of a two digit number is 12. the number obtained by interchanging the two digits exceeds the given number by 18. Find the number **[AI-2006]**

19. Solve the system of equations for x ; $\frac{b^2x}{a}-\frac{a^2y}{b}=ab(a+b)$ and $b^2x-a^2y=2a^2b^2$

OR

A man sold a table and a chair together for Rs. 850 at a loss of 10% on the table and a gain of 10% on the chair. By selling them together for Rs. 950, he would have made a gain of 10% on the table and loss of 10% on the chair. Find the cost price of each . **[Foreign-2006]**

20. Solve the following equations for x and y : mx – ny = $m^2 + n^2$, x + y = 2m

OR

Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 2650 km by train and 240 km taxi he takes 6 minutes longer. Find the speed of the train and that of the taxi **[Delhi-2006C]**

21. Solve the following equations for x and y : $\frac{a^2}{x}-\frac{b^2}{y}=0\ ;\ \frac{a^2b}{x}+\frac{b^2a}{y}=a+bx, y\neq 0.$

OR

The sum of the numerator and the denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction. **[AI-2006C]**

22. Solve for x and y : $x+\frac{6}{y}=6,\ 3x-\frac{8}{y}=5$

OR

Solve for x and y : $\frac{x+1}{2}+\frac{y-1}{3}=8\ ;\ \frac{x-1}{3}+\frac{y+1}{2}=9$ **[Delhi -2007]**

23. Solve for x and y : 8x – 9y = 6xy ; 10x + 6y = 19xy

OR

Solve for x and y : $4x+\frac{y}{3}=\frac{8}{3};\ \frac{x}{2}+\frac{3y}{4}=-\frac{5}{2}$ **[AI-2007]**

24. Find the value of k so that the following system of equations has no solution :
3x – y = 5 ; 6x – 2y – k = 0 **[Delhi-2008]**

25. Find the value of k so that the following system of equations has infinite solutions :
3x – y – 5 = 0 ; 6x – 2y + k = 0 **[Delhi-2008]**

26. Find the value (s) of k for which the pair of linear equations kx + 3y = k – 2 and 12x + ky = k has no solution **[Delhi-2008]**

27. Find the number of solutions of the following pair of linear equations :
x + 2y – 8 = 0 ; 2x + 4y = 16 **[AI-2009]**

28. Write whether the following pair of linear equations is consistent or not :
x + y = 14 ; x – y = 4 **[Foreign-2009]**

29. Without drawing the graph find out whether the lines representing the following pair of liner equations intersect at a point, are parallel or coincident : 9x – 10y = 21 ; $\frac{3}{2}x-\frac{5}{3}y=\frac{7}{2}$ **[Foreign-2009]**

30. Without drawing the graph find out whether the lines representing the following pair of linear equations intersect at a point , are parallel or coincident : 48x – 7y = 24 ; $\frac{9}{5}x-\frac{7}{10}y=\frac{9}{10}$ **[Foreign-2009]**

31. Without drawing the graph, find out whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident : 5x + 3y – 6 = ; $\frac{9}{5}$x + 3y = 6 **[Foreign-2009]**

SHORT ANSWER TYPE –II

1. Solve the following system of linear equations graphically : 2x – 3y = 1, 3x – 4y = 1 Does the point (3, 2) lie on any of the lies ? Write its equation **[Delhi-2003]**

2. Solve for x and y : $\frac{4}{x}$ + 5y = 7, $\frac{3}{x}$ + 4y = 5

OR

Father's age is three times the sum of ages of his two children . After 5 years his age will be twice the sum of age of two children. Find the age of father . **[Delhi-2003]**

3. Solve the following system of liner equations graphically : 3x – 5y = 19, 3y – 7x + 1 = 0 Does the point (4, 9) lie on any of the lines ? Write its equations **[AI-2003]**

4. Solve the following system of linear equations graphically : 2x + y = 10, 4x – y = 8. Does the point (1, -4) lie on any of the lines ? Write its equation. **[Foreign-2003]**

5. The sum of numerator and denominator of a fraction is 8. is added to both the numerator and denominator the fraction becomes 3/4. Find the fraction. **[AI-2003]**

6. Solve the following system of equations : $\frac{a}{x}-\frac{b}{y}=0,\frac{ab^2}{x}+\frac{a^2b}{y}=a^2+b^2\ ;\ x,y\neq 0.$

OR

5 years hence the age of a father shall be three times the age of his son while 5 years earlier the age of the father was 7 times the age of his son. Find their present ages. **[Foreign-2003]**

7. Solve the following system of linear equations graphically : $4x - 5y - 20 = 0$, $3x + 5y - 15 = 0$. Determine the vertices of the triangle formed by the lines, representing the above equations, and the y-axis. **[Delhi-2004]**

8. Solve the following system of linear equations graphically : $5x - 6y + 30 = 0$, $5x + 4y - 20 = 0$. Also find the vertices of the triangle formed by the above two lines and x-axis. **[AI-2004]**

9. Solve the following system of linear equations graphically : $2x + y + 6 = 0$, $3x - 2y - 12 = 0$. Also find the vertices of the triangle formed by the lines representing the above equations and x-axis. **[Foreign-2004]**

10. Solve the following system of linear equations graphically : $2x + 3y = 4$, $3x - y = -5$. Shade the region bounded by the above lines and the x-axis. **[Delhi-2004C]**

11. Solve the following system of linear equations graphically : $3x + y = 1 = 0$, $2x - 3y + 8 = 0$ Shade the region bounded by the lines and the x-axis. **[AI-2004C]**

12. The monthly incomes of A and B are in the ratio of 9 : 7 and their monthly expenditures are in the ratio of 4 : 3 If each saves Rs. 1600 per month, find the monthly incomes of each. **[AI-2004C]**

13. Solve the following system of equations graphically : $x + 2y = 5$, $2x - 3y = -4$. Also find the points where the lines meet the x-axis. **[Delhi-2005]**

14. Solve the following system of equations graphically : $2x - y = 4$; $3y - x = 3$. Find the points where the lines meet the y-axis. **[AI-2005]**

15. Solve the following system of equations graphically : $3x - y = 3$, $x - 2y = -4$. Shade the are of the region bounded by the lines and x-axis. **[Delhi-2005C]**

16. Draw the graphs of the equations : $4x - y - 8 = 0$ and $2x - 3y + 6 = 0$. Also determine the vertices of the triangle formed by the lines and x-axis. **[Delhi-2006]**

17. Draw the graphs of the following equations : $3x - 4y + 6 = 0$; $3x + y - 9 = 0$. Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis. **[AI-2006]**

18. Draw the graphs of the equations : $4x - 3y - 6 = 0$; $x + 3y - 9 = 0$. Determine the co-ordinates of the vertices of the triangle formed by the lines and the y-axis. **[Foreign-2006]**

19. Solve the following system of linear equations graphically : $3x - 2y - 1 = 0$; $2x - 3y + 6 = 0$. Shade the region bounded by the lines and x-axis. **[Delhi-2006C]**

20. Solve the following system of equations graphically for x and y : $3x + 2y = 12$; $5x - 2y = 4$. Find the co-ordinates of the points where the lines meet the y-axis. **[AI-2006C]**

21. Solve the following system of equations graphically . $2x + 3y = 8$; $x + 4y = 9$. **[Delhi-2007]**

22. Solve the following system of linear equations graphically. $2x + 3y = 12$; $2y - 1 = x$ **[AI-2007]**

23. Represent the following system of linear equations graphically. From the graph, find the points where the lines intersect y-axis. $3x + y - 5 = 0$; $2x - y - 5 = 0$. **[Delhi-2008]**

24. Solve for x and y : $(a - b)\,x + (a + b)\,y = a^2 - 2ab - b^2$; $(a + b)\,(x + y) = a^2 + b^2$.

OR

Solve for x and y : $37x + 43y = 123$; $43x + 37y = 117$. **[AI-2008]**

25. Represent the following pair of equations graphically and write and co-ordinates of points where the lines intersect y-axis : $x + 3y = 16$; $2x - 3y = 12$. **[Foreign-2008]**

26. Solve the following pair of equations : $\frac{5}{x-1} + \frac{1}{y-2} = 2; \frac{6}{x-1} - \frac{3}{y-2} = 1$ **[Delhi-2009]**

27. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds. They meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars ? **[Delhi-2009]**

28. Solve the following pair of equations : $\frac{10}{x+y} + \frac{2}{x-y} = 4; \frac{15}{x+y} - \frac{2}{x-y} = -2$ **[Delhi-2009]**

29. Solve for x and y : $\frac{ax}{b} - \frac{by}{a} = a + b;\ ax - by = 2ab.$ **[AI -2009]**

SUBJECTIVE **ANSWER KEY** **EXERCISE-4 (X) CBSE**

- **Short Answer Type-I**

1. k = 2 **2.** k = 3 **3.** x = -1/2, y = 2 or 42yrs, 10yrs **4.** x = 1/2, y = 1/3 or 78 **5.** x = -2, y = 1/3 or 7/11
6. x = 1/5, y = -2 or x = ab, y = ab **7.** a = 1, b = - 7 **8.** a = 0, b = 7 **9.** a = -3, b = -1
10. x = 1/2, y = 1/3 or x = 1, y = - 1 **11.** a = -2, b = - 9 **12.** x = 1, y = - 1 **13.** x = a, y = b or 36
14. x = a^2, y = b^2 or 63 **15.** x = a, y = - b **16.** x = - a, y = b **17.** x = 2, y = -1 or x = b, y = - a
18. x = - a, y = b or 57 **19.** x = a^2, y = - b^2 or cast of table = Rs. 700, cost of chair = Rs 200
20. x = m + n, y = m – n or speed of train = 100 km/h. Speed of taxi = 80 km/h **21.** x = a^2, y = b^2 or $\frac{5}{7}$
22. x = $-\frac{14}{5}$, y = $\frac{1}{13}$ or x = 7, y = 13 **23.** x = 3/2, y = 2/3 or x = 1, y = - 4 **24.** k ≠ 10 **25.** k = - 10 **26.** k = ± 6
27. Infinite number of solutions **28.** Consistent **29.** Coincident lines **30.** Parallel **31.** Unique solution.

- **Short Answer Type-II**

1. (-1, -1) ; Yes ; 3x – 4y = 1 **2.** x = 1/3, y = - or 45 years **3.** (-2, -5) ; Yes ; 3y – 7x + 1 = 0 **4.** (3, 4) ; yes ; 4x – y = 8
5. 3/5 **6.** (a, b) or 40, 10 years **7.** (0, -4), (5, 0), (0, 3) **8.** (-6, 0), (0, 5), (4,0) **9.** (-3, 0), (0, -6), (4, 0)
12. A's = Rs. 14400, B's = Rs. 11200 **13.** (5, 0), (-2, 0) **14.** (0, -4), (0, 1) **16.** (-3, 0), (2, 0), (3, 4) **17.** (-2, 0), (2, 3), (3, 0)
18. (0, 3), (3, 2), (0, -2) **19.** x = 3, y = 4 **20.** x = 2, y = 3 **21.** x = 1, y = 2 **22.** x = 3, y = 2 **23.** (0, 5) and (0, -5)
24. x = a + b, y = $\frac{-2ab}{a+b}$ or x = 1, y = 2 **25.** (0, 2) and (0, -4) **26.** x = 4, y = 5 **27.** 60 km/h ; 40 km/h
28. x = 3 ; y = 2 **29.** x = b, y = - a

EXERCISE -5 (FOR OLYMPIADS)

Choose The Correct One

1. The number of solutions of the equation 2x + y = 40, where both x and y are positive integers and x ≤ y is :
(A) 7 (B) 13 (C) 14 (D) 18

2. A confused bank teller transposed the rupees and paise when he cashed a cherub for Mansi, giving her rupees instead of paisa and paise instead of rupees. After buying a toffee for 50 paise, Mansi noticed that she was left with exactly three times as much as the amount on the cheque. Which of the following is a valid statement about the cheque amount ?
(A) Over Rs. 4 but less than Rs. 5 (B) Over Rs. 13 but less than Rs. 14
(C) Over Rs. 7 but less than Rs. 8 (D) Over Rs. 18 but less than Rs. 19

3. John inherited $25000 and invested part of it in a money market account, part in municipal bounds, and part in a mutual fund. After one year, he received a total of $ 1620 in simple interest from the three investments. The money market paid 6% annually, the bonds paid 7% annually, and the mutual funds paid 8% annually. There was $ 6000 more invested in the bonds than the mutual funds. The amount John invested in each category are in the ratio :
(A) 15 : 8 : 2 (B) 11 : 13 : 1 (C) 2 : 2 : 1 (D) None of these

4. Which one of the following conditions must p,q and r satisfy so that the following system of linear simultaneous equations has at least one solution , such that p + q + r ≠ 0 ?
x + 2y – 3z = p ; 2x + 6y – 11z = q ; x - 2y + 7z = r
(A) 5p – 2q – r = 0 (B) 5p + 2q + r = 0 (C) 5p + 2q – r = 0 (D) 5p – 2q + r = 0

5. If x and y are integers, then the equation 5x + 19y = 64 has :
(A) No solution for x < 300 and y < 0 (B) No solution for x > 250 and y > - 100
(C) A solution for 250 < x < 300 (D) A solution for – 59 < y < - 56

6. The number of solutions of the equation 2x + y = 40, where both x and y are positive integers and x ≤ y is :

(A) 7 (B) 13 (C) 14 (D) 18

7. Study the question and statements given below : Decide whether any information provided in the statement (s) is redundant and / or can be dispensed with, to answer it.
If 7 is added to numerator and denominator each of fraction a/b. will the new fraction be less than the original one ? (Assume both a and b to be positive)
Statement-I : a = 73, b = 103
Statement-II : The average of a and b is less than b.
Statement-III : a – 5 is greater than b – 5.
(A) II and either I or III (B) Only I or III (C) Any two of them (D) Any one of them

8. A cyclist drove 1 km, with the wind in his back , in 3 min and drove the same way back, against the wind in 4 min. If we assume that the cyclist always puts constant force on the pedals, how much time would it take to drive 1 km without wind ?
(A) $2\frac{1}{3}$ min. (B) $3\frac{3}{7}$ min. (C) $2\frac{3}{7}$ min. (D) $3\frac{7}{12}$ min.

9. A person buys 18 local tickets for Rs. 110. Each first class ticket costs Rs. 10 and each second class ticket costs Rs. 3. What will another lot of 18 tickets in which the number of first class and second class tickets are interchanged cost ?
(A) Rs. 112 (B) Rs. 118 (C) Rs. 121 (D) Rs. 124

10. Rajesh walks to and fro to a shopping mall. He spends 30 min. shopping. If he walks at a speed of 10 km/h, he returns to home at 19:00h. It he walks at 15 km/h. he returns at 18:30 h. How fast must he walk in order to return home at 18:15 h ?
(A) 17 km/h (B) 17.5 km/h (C) 18 km/h (D) 20 km/h

11. A single reservoir supplies the petrol to the whole city, while the reservoir is fed by a single pipeline filling the reservoir with the stream of uniform volume. When the reservoir is full and if 40000 liters of petrol is used daily, the supply fails in 90 days. If 32000 liters of petrol is used daily, the supply fails in 60 days. How much petrol can be used daily without the supply ever failing ?
(A) 64000 litres (B) 56000 litres (C) 78000 litres (D) 60000 litres

12. Two horses start trotting towards each other, one from A to B and another from B to A . They cross each other after one hour and the first horse reaches B, 5/6 hours before the second hoarse reaches A. If the distance between A and B is 50 km. What is the speed of the slower hours ?
(A) 30 km/h (B) 15 km/h (C) 25 km/h (D) 20 km/h

13. A man row downstream at 12 km/h and upstream at 8 km/h. What is the speed of man in still water ?
(A) 12 km/h (B) 10 km/h (C) 8 km/h (D) 9 km/h

14. A motor boat takes 12 hours to go downstream and it takes 24 hours to return the same distance. What is the time taken by boat in still water ?
(A) 15 h (B) 16 h (C) 8 h (D) 20 h

15. Equation $xy^2 + xy^2 = 2xy$, $x, y \neq 0$ is
(A) Linear (B) Quadratic (C) Cubic (D) Not an equation

16. Sum of two integers is 88. If the greater is divided by the smaller, the quotient is 5 and the remainder is 10. the greater integer is :
(A) 13 (B) 75 (C) 65 (D) 23

17. The length of the sides of a triangle are $3x+2y$, $4x+\frac{4}{3}y$ and $3(x+1)+\frac{3}{2}(y-1)$. If the triangle is equilateral , then its side is
(A) 8 (B) 10 (C) 12 (D) 16

18. The largest angle of a triangle is twice the sum of the other two. The smaller angle is one fourth of the largest. The largest angle is :
(A) 90^0 (B) 60^0 (C) 120^0 (D) None of these

19. In town, $\frac{2}{3}$ of men are married to $\frac{3}{7}$ of the women . In the town total population is more than 1000. If all marriages happen within the town. The smallest possible number of total population is (assume there are only adults in the town) :
(A) 1012 (B) 1035 (C) 1058 (D) None of these

20. The solution of the equations : $\frac{x}{4}=\frac{y}{3}=\frac{z}{2}$, $7x+8y+5z=62$ is :

(A) (4, 3, 2) (B) (2, 3, 4) (C) (3, 4, 2) (D) (4, 2, 3)

21. If $\frac{1}{3}(x+y)2z=21$, $3x-\frac{1}{2}(y+z)=65$, $x+\frac{1}{2}(x+y-z)=38$, then its solution is :

(A) (24, 9, 5) (B) (2, 9, 5) (C) (4, 9, 5) (D) (5, 24, 9)

22. The solution of the equations : $\frac{xy}{y-x}=110$, $\frac{yz}{z-y}=132$, $\frac{zx}{z+x}=\frac{60}{11}$ is :

(A) (12, 11, 10) (B) (10, 11, 12) (C) (11, 10, 12) (D) (12, 10, 11)

23. Four men earn as much in a day as 7 women. 1 women earns as much as 2 boys. If 6 men, 10 women and 14 boys work together for 8 days to earn Rs. 2200, then what will be the earning of 8 men and 6 women working together is for 10 days ?

(A) Rs. 2000 (B) Rs. 1800 (C) Rs. 2400 (D) None of these

24. The point of intersection of the straight lines $2x - y + 3 = 0$, $3x - 7y + 10 = 0$ lies in :

(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant

25. A right-angled triangle is formed by the straight line : $4x + 3y = 12$ with both the axis. Then length of perpendicular from the origin to the hypotenuse is :

(A) 3.5 units (B) 2.4 units (C) 4.2 units (D) None of these

OBJECTIVE					ANSWER KEY								EXERCISE - 5		
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**B**	**D**	**A**	**A**	**C**	**B**	**C**	**B**	**D**	**D**	**B**	**D**	**B**	**B**	**A**
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**					
Ans.	**B**	**C**	**C**	**A**	**A**	**A**	**B**	**A**	**B**	**B**					

COMPETITION WINDOW

LINEAR IN EQUATIONS

Inequation : A statement involving variable (s) and the sign of inequality viz, $<, >, \leq$ or $\geq$ is called an inequation or an inequality.

An inequation may contain one or more variables. Also, is may be linear or quadratic or cubic etc.

E.g. (i) $3x - 2 < 0$ (ii) $2x + 3y < 1$ (iii) $x2 - 5x + 4 \leq 0$

Linear Inequation In One Variable : Let a be a non-zero real number and x be a variable. Then inequations of the form $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$ and $ax + b \geq 0$ are known as linear equations in one variable x.

E.g. $9x - 15 > 0$, $5x - 4 \geq 0$, $3x + 2 < 0$, $2x - 3 \leq 0$

Solving Linear Inequation In One Variable : In the process of solving an inequation, we use mathematical simplifications which are governed by the following rules :

Rule-I : Same number may be added (or subtracted from) both sides of an inequation without changing the sign of inequality

Rule-II: Both sides of an inequation can be multiplied (or divided) by the same positive real number without changing the sign of inequality. However the sign of inequality is reversed when both sides of an inequation are multiplied (or divided) by a negative number.

Rule-III : Any tern of an inequation may be taken to the other side with it's sign changed without affecting the sign of inequality

Ex. Solve : $5x - 3 < 3x + 1$, when (i) x is a real number (ii) x is an integer (iii) x is a natural number.

Sol. We have

$5x - 3 < 3x + 1$

$\Rightarrow$ $5x - 3x < 3x + 1$ [Transposing 3x on LHS and – 3 on RHS]

$\Rightarrow \quad 2x < 4 \Rightarrow \frac{2x}{2} < \frac{4}{2} \Rightarrow x < 2$

(i) If $x \in R$, then $x < 2 \Rightarrow x \in (-\infty, 2)$

(ii) If $x \in Z$, then $x < 2 \Rightarrow x = 1, 0, -1, -2, -3, -4, \ldots\ldots\ldots$

(iii) If $x \in N$, then $x < 2 \Rightarrow x = 1$

Ex. Solve ; $\frac{2x+4}{x-1} \geq 5$

Sol. We have,

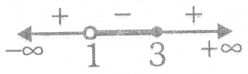
$$\frac{2x+4}{x-1} \geq 5$$

$\Rightarrow \quad \frac{2x+4}{x-1} - 5 \geq 0 \Rightarrow \frac{-3x+9}{x-1} \geq 0$

$\Rightarrow \quad \frac{3x-9}{x-1} \leq 0$ [Multiplying both sides by – 1]

$\Rightarrow \quad \frac{x-3}{x-1} \leq 0$ [Dividing both sides by 3]

$\Rightarrow \quad 1 < x \leq 3 \Rightarrow x \in (1, \ 3)$



EXERCISE-6 (FOR IIT-JEE/AIEEE)

Choose The Correct One

1. Solve : $3x - 7 > x + 1, x \in R$:

(A) $(4, \infty)$ (B) $[4, \infty)$

(C) $(-\infty, \ 4]$ (D) $(-\infty, \ 4]$

2. Solve : $\frac{x}{5} < \frac{3x-2}{4} - \frac{5x-3}{5}, x \in R$:

(A) $(2/9, \infty)$ (B) $[2/9, \infty)$

(C) $(-\infty, \ 2.9)$ (D) $(-\infty, \ 2.9)$

3. Solve : $\frac{2(x-1)}{5} \leq \frac{3(2+x)}{7}, x \in R$:

(A) $(44, \infty)$ (B) $[44, \infty)$

(C) $[-44, \infty)$ (D) $(-44, \ \infty)$

4. Solve : $\frac{5x}{2} + \frac{3x}{4} \geq \frac{39}{4}, x \in R$:

(A) $(-3, \infty)$ (B) $(3, \infty)$

(C) $(-\infty, \ 3)$ (D) $[3, \ \infty)$

5. Solve : $\frac{x-1}{3} + 4 < \frac{x-5}{5} - 2, x \in R$:

(A) $(50, \infty)$ (B) $(-\infty, \ -50)$

(C) $[50, \ \infty)$ (D) None of these

6. Solve : $\frac{2x+3}{4}-3<\frac{x-4}{3}-2, x\in R$:

(A) $(13/2, \infty)$ (B) $(-\infty, -13/2)$

(C) $(-13/2, \infty)$ (D) $[13/2, \infty)$

7. Solve : $\frac{5-2x}{3}<\frac{x}{6}-5, x\in R$:

(A) $(8, \infty)$ (B) $[8, \infty)$ (C) $(-\infty, -8)$ (D) $(-\infty, 8)$

8. Solve : $\frac{4+2x}{3}\geq\frac{x}{2}-3, x\in R$:

(A) $(26, \infty)$ (B) $(-\infty, 26]$ (C) $[-26, \infty)$ (D) $(-\infty, -26)$

9. Solve : $\frac{2x+3}{5}-2<\frac{3(x-2)}{5}, x\in R$:

(A) $(-1, \infty)$ (B) $[1, \infty)$ (C) $(-\infty, -1)$ (D) $(-\infty, 1)$

10. Solve : $x-2\leq\frac{5x+8}{3}, x\in R$

(A) $[-7, \infty)$ (B) $(7, \infty)$ (C) $(-\infty, 7)$ (D) $(-\infty, 7)$

11. Solve : $\frac{6x-5}{4x+1}<0, x\in R$:

(A) $(-1/4, 5/6)$ (B) $[-1/4, 5/6]$ (C) $(-\infty, -1/4)$ (D) $(5/6, \infty)$

12. Solve : $\frac{2x-3}{3x-7}>0, x\in R$

(A) $[3/2, 7/3]$ (B) $(3/2, 7/3)$

(C) $(-\infty, 3/2)\cup(7/3, \infty)$ (D) None of these

13. Solve : $\frac{3}{x-2}<1, x\in R$:

(A) $(2, 5)$ (B) $[-\infty, 2)\cup(5, \infty)$

(C) $(-\infty, -2)\cup(5, \infty)$ (D) None of these

14. Solve : $\frac{1}{x-1}\leq 2, x\in R$:

(A) $(-1, 3/2]$ (B) $(-\infty, -1)\cup(3/2, \infty)$

(C) $(-1, 3/2)$ (D) $(-\infty, 1)\cup[3/2, \infty)$

15. Solve : $\frac{4x+3}{2x-5}<6, x\in R$

(A) $(5/2, 33/8)$ (B) $(-\infty, -5)\cup(4, \infty)$

(C) $(-5/2, 33/8)$ (D) $(-\infty, 5/2)\cup(33/8, \infty)$

16. Solve : $\frac{5x-6}{x+6}<1, x\in R$:

(A) $(-6, -3)$ (B) $(6, \infty)$ (C) $(-6, 3)$ (D) None of these

17. Solve : $\frac{5x+8}{4-x}<2, x\in R$:

(A) $[0,, 4)$ (B) $[4, \infty)$ (C) $(-\infty, 4)$ (D) $(-\infty, 0)\cup(4, \infty)$

18. Solve : $\frac{x-1}{x+3} > 2, x \in R$:

(A) $(7, 3)$ (B) $[7, 3]$ (C) $(-7, -3)$ (D) $[-7, -3]$

19. Solve : $\frac{7x-5}{8x+3} > 4, x \in R$:

(A) $(17/25, 3/8)$ (B) $(17/25, 3/8]$

(C) $(-17/25, -3/8)$ (D) $[17/25, 3/8]$

20. $\frac{2x-3}{3x-7} > 0, x \in R$

(A) $(-5, 5)$ (B) $[-5, 5]$

(C) $(-\infty, -5) \cup (5, \infty)$ (D) None of these

OBJECTIVE					ANSWER KEY						EXERCISE -6				
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**A**	**C**	**C**	**D**	**B**	**B**	**A**	**C**	**A**	**A**	**A**	**C**	**B**	**D**	**D**
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	**C**	**D**	**C**	**C**	**C**										

TRIGONOMETRY

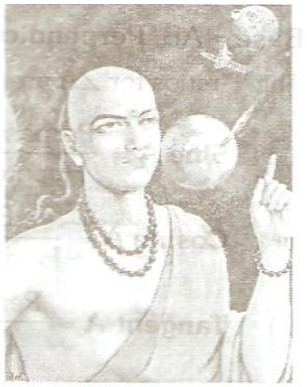
★ INTRODUCTION

Trigonometry is the branch of Mathematics which deals with the measurement of angles and sides of a triangle.

The word Trigonometry is derived from three Greek roots : 'trio' meaning 'thrice or Three', 'gonia' meaning an angle and 'metron' meaning measure. In fact, **Trigonometry is the study of relationship between the sides and the angles of a triangle.**
Trigonometry has its application in astronomy, geography, surveying, engineering and navigation etc. In the past, astronomers used it to find out the distance of stars and plants from the earth. Even now, the advanced technologies used in Engineering are based on trigonometric concepts.
In this chapter, we will define trigonometric ratios of angles in terms of ratios of sides of a right triangle. We will also define trigonometric ratios of angles of 0^0, 30^0, 45^0, 60^0, and 90^0. We shall also establish some identities involving these ratios.

★ HISTORICAL FACTS

Indian Mathematician has established keen interest in the study of Trigonometry since ages. They are known for their innovation in the use of size instead the use of choid. The most outstanding astronomer has been Aryabhatta.


ARYABHATTA (476 AD)

Aryabhatta was born in 476 A.D. in Kerala. He studied in the university of Nalanda. In mathematics, Aryabhatta's contribution are very valuable. He was the first mathematician to prepare tables of sines. His book 'Aryabhatta' deals with Geometry, Mensuration, Progressions, Square root, Cube root and Celestial sphere (spherical Trigonometry). This work, has won him recognition all over the world because of its logical and unambiguous presentation of astronomical observations.

Aryabhatta was the pioneer to find the correct value of the constant π with respect to a circle $\left(\frac{Circumference}{Diameter} = \pi\right)$ up to four decimals as 3.1416. he found the approximate value of π and indirectly suggested that π is an irrational number. His observations and conclusions are very useful and relevant today.


ANCIENT (85 – 165 AD)

Greek Mathematician Ptolemy, Father of Trigonometry proved the equation $\sin^2 A + \cos^2 A = 1$ using geometry involving a relationship between the chords of a circle. But ancient Indian used simple algebra to calculate sin A and cos A and proved this relation. Brahmagupta was the first to use algebra in trigonometry. Bhaskaracharya II (1114 A.D.) was very brilliant and most popular Mathematician. His work known as Siddhantasiromani is divided into four parts, one of which is Goladhyaya's spherical trigonometry.

★ BASE, PERPENDICULAR AND HYPOTENUSE OF A RIGHT TRIANGLE

In ΔABC, if $\angle B = 90^0$, then :
(i) For $\angle A$, we have :

Base = AB, **Perpendicular** = BC and **Hypotenuse** = AC.

(ii) For $\angle C$, we have :

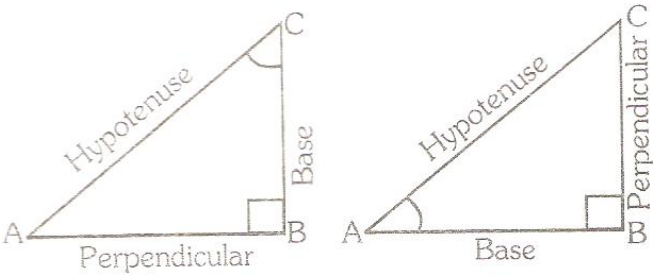
Base = BC, **Perpendicular** = AB and **Hypotenuse** = AC.

So, in a right angled triangle, for a given angle,

(i) The side opposite to the right angle is called **hypotenuse.**

(ii) The side opposite to the right angle is called **perpendicular.**

(iii) The third side (i.e., the side forming the given angle with the hypotenuse), is called **base.**



★ TRIGONOMETRICAL RARIOS (T-RATIOS) OF AN ANGLE

In ΔABC, let $\angle B = 90^0$ and let $\angle A$ be acute.

For $\angle A$, we have :

Base = AB, **Perpendicular** = BC and **Hypotenuse** = AC.

The T-ratios for $\angle A$ are defined as :

(i) **Sine A** = $\frac{Perpendicular(P)}{Hypotenuse(H)} = \frac{BC}{AC}$, written as **sin A.**

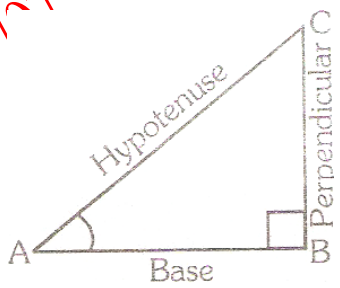
(ii) **Cosine A** = $\frac{Base(B)}{Hypotenuse(H)} = \frac{AB}{AC}$, written as **cos A.**

(iii) **Tangent A** = $\frac{Perpendicular(P)}{Base(B)} = \frac{BC}{AB}$, written as **tan A.**

(iv) **Cosecant A** = $\frac{Hypotenuse(H)}{Perpendicular(P)} = \frac{AC}{BC}$, written as **cosec A.**

(v) **Secant A** = $\frac{Hypotenuse(H)}{Base(B)} = \frac{AC}{AB}$, written as **sec A.**

(vi) **Cotangent A** = $\frac{Base(B)}{Perpendicular(P)} = \frac{AB}{BC}$, written as **cot A.**



Thus, there are six Trignometrical ratios based on the three sides of a right angled triangle.

Aid to Memory : The sine, cosine, and tangent ratios in a right triangle can be remembered by representing them as strings of letters, as in **SOH-CAH-TOA.**

Sine = **O**pposite ÷ **H**ypotenuse

Cosine = **A**djacent ÷ **H**ypotenuse

Tangent = **O**pposite ÷ **A**djacent

The memorization of this mnemonic can be aided by expanding it into a phrase, such as " **S**ome **O**fficers **H**ave **C**urly **A**uburn **H**air **T**ill **O**ld **A**ge".

★ RECIPROCAL RELATIONS

Clearly, we have :

$$\text{(i) } \operatorname{cosec}\theta = \frac{1}{\sin\theta} \qquad \text{(ii) } \sec\theta = \frac{1}{\cos\theta} \qquad \text{(iii) } \cot\theta = \frac{1}{\tan\theta}$$

Thus, we have :

$$\text{(i) } \sin\theta \operatorname{cosec}\theta = 1 \qquad \text{(ii) } \cos\theta \sec\theta = 1 \qquad \text{(iii) } \tan\theta \cot\theta = 1$$

★ QUOTAENT RELATIONS

Consider a right angled triangle in which for an acute angle θ, we have :

$\sin\theta = \frac{Perpendicular}{Hypotenuse} = \frac{P}{H}$: $\cos\theta = \frac{Base}{Hypotenuse} = \frac{B}{H}$

Now, $\frac{\sin\theta}{\cos\theta} = \frac{\frac{P}{H}}{\frac{B}{H}} = \frac{P}{H} \times \frac{H}{B} = \frac{P}{B}$ = **tan** θ (by def.)

and, $\frac{\cos\theta}{\sin\theta} = \frac{\frac{B}{H}}{\frac{P}{H}} = \frac{B}{H} \times \frac{H}{P} = \frac{B}{P}$ = **cos** θ (by def.)

Thus, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

★ POWER OF T-RATIOS

We denote :

(i) $(\sin\theta)^2$ by $\sin^2\theta$; (ii) $(\cos\theta)^2$ by $\cos^2\theta$; (iii) $(\sin\theta)^3$ by $\sin^3\theta$; (iv) $(\cos\theta)^3$ by $\sin^3\theta$; and so on.

REMARK :

(i) The symbol sin A is used as an abbreviation for 'the sine of the angle A'. Sin A is not the product of 'sin' and A. 'sin' separated from A has no meaning. Similarly, cos A is not the product of 'cos' and A. similar interpretations follow for other trigonometric ratios also.

(ii) We may write sin2 A, cos2 A, etc., in place of (sin A)2, (cos A)2, etc., respectively. But cosec A = (sin A)-1 ≠ sin-1 A (it is called sine inverse A). sin-1 A has a different meaning, which will be discussed in higher classes. Similar conventions hold for the other trigonometric ratios as well.

(iii) Since the hypotenuse is the longest side in a right triangle, the value of sin A or cos A is always less than 1 (or, in particular, equal to 1).

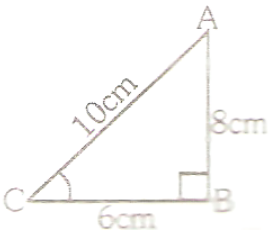
Ex.1 Using the information given in fig. write the values of all trigonometric ratios of angle C.

Sol. Using the definition of t-ratios,

$\sin C = \frac{AB}{AC} = \frac{8}{10} = \frac{4}{5}$; $\cos C = \frac{BC}{AC} = \frac{6}{10} = \frac{3}{5}$

$\tan C = \frac{AB}{BC} = \frac{6}{8} = \frac{4}{3}$; $\cot C = \frac{BC}{AB} = \frac{6}{8} = \frac{3}{4}$

$\tan C = \frac{AC}{BC} = \frac{10}{6} = \frac{5}{3}$ and $\cos C = \frac{AC}{AB} = \frac{10}{8} = \frac{5}{4}$



Ex.2 In a right ΔABC, if $\angle$A is acute and tan A = $\frac{3}{4}$. find the remaining trigonometric ratios of $\angle$A.

Sol. Consider a ΔABC in which $\angle$B = 90^0

For $\angle$A, we have :

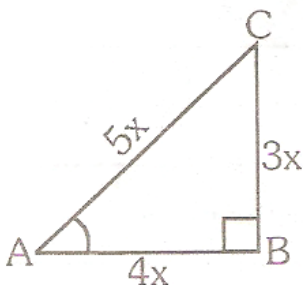
Base = AB, Perpendicular = BC and Hypotenuse = AC.

$\therefore$ $\tan A = \frac{Perpendicular}{Base} = \frac{3}{4}$

$\Rightarrow$ $\frac{BC}{AB} = \frac{3}{4}$

Let, BC = 3x units and AB = 4x units.

Then, AC = $\sqrt{AB^2 + BC^2}$



$= \sqrt{(4x)^2 + (3x)^2}$

$= \sqrt{25x^2}$ = 5x units.

Ex.3 In a Δ ABC, right angled at B, if tan A = $\frac{1}{\sqrt{3}}$, find the value of

(i) sinA cosC + cosA sinC

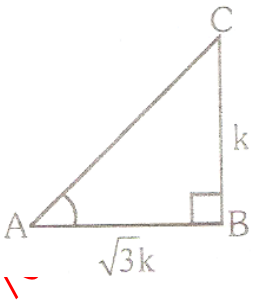
(ii) cosA cosC – sinA sinC. **[NCERT]**

Sol. We know that

$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

$\therefore$ BC : AB = 1 : $\sqrt{3}$

Let BC = k and AB = $\sqrt{3}$ k



Then, AC = $\sqrt{AB^2 + BC^2}$ …(Phythagoras theorem)

$= \sqrt{(\sqrt{3}k)^2 + (k)^2} = \sqrt{3k^2 + k^2}$

$= \sqrt{4k^2}$ = 2k

Now, $\sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3k}}{2k} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{AB}{AC} = \frac{\sqrt{3k}}{2k} = \frac{\sqrt{3}}{2}$

and $\cos C = \frac{BC}{AC} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C = \frac{1}{2}.\frac{1}{2} + \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{2}\frac{1}{2} - \frac{1}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

Ex.4 If sinA = $\frac{1}{2}$, verify that 2 sinA cosA = $\frac{2\tan A}{1+\tan^2 A}$

Sol. We know that

$\text{Sin}A = \frac{BC}{AC} = \frac{1}{2}$

Let BC = k and AC = 2k

$\therefore$ $AB = \sqrt{AC^2 - AB^2}$

$= \sqrt{(2k)^2 - k^2} = \sqrt{4k^2 - k^2} = \sqrt{3k^2} = \sqrt{3k}$

Now $\cos A = \frac{AB}{AC} = \frac{\sqrt{3k}}{2k} = \frac{\sqrt{3}}{2}$

and $\cos A = \frac{BC}{AB} = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$

Now $2 \sin A \cos A = 2.\frac{1}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ …(i)

and $\frac{2\tan A}{1+\tan^2 A} = \frac{2\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3}{4}}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$ …(ii)

Ex.5 In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Find the value of sinP, cosP and tanP. **[NCERT]**

Sol. We are given

$PR + QR = 25$ cm

$\therefore$ $PR = (25 - QR)$ cm

By Pythagoras theorem,

$PR^2 = QR^2 + PQ^2$

or $(25 - QR)^2 = QR^2 + 5^2$

or $625 + QR^2 - 50\,QR = QR^2 + 25$

or $50QR = 625 - 25 = 600$

$\therefore$ $QR = 12$ cm.

and $PR = (25 - 12)$ cm $= 13$ cm

Now $\sin P = \frac{QR}{PR} = \frac{12}{13}$

$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

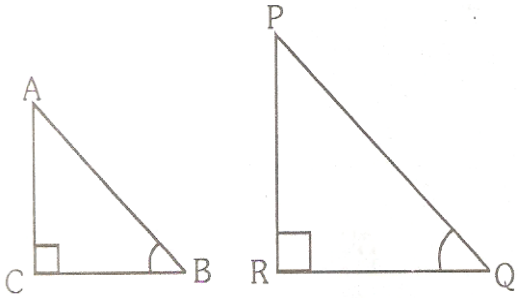
and $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

Ex.6 If $\angle A$ and $\angle Q$ are acute angles such that sin B = sinQ, then prove that $\angle B = \angle Q$. [NCERT]

Sol. Consider two right ΔABC and ΔPQR such that sin B = sinQ.

We have,

$\sin B = \frac{AC}{AB}$ and, $\sin Q = \frac{PR}{PQ}$



$\because$ sinB = sinQ

$\Rightarrow$ $\frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow$ $\frac{AC}{PR} = \frac{AB}{PQ}$ k, (say) …(i)

$\Rightarrow$ $AC = k\,PR$ and $AB = k\,PQ$ …(ii)

Using Pythagoras theorem in triangles ABC and PQR, we have

$AB^2 = AC^2 + BC^2$ and $PQ^2 = PR^2 + QR^2$

$\Rightarrow$ BC $= \sqrt{AB^2 - AC^2}$ and QR $= \sqrt{PQ^2 - PR^2}$

$\Rightarrow \quad \dfrac{BC}{QR} = \dfrac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \dfrac{BC}{QR} = \dfrac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}}$ [Using (ii)]

$\Rightarrow \quad \dfrac{BC}{QR} = \dfrac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k$

From (i) and (ii), we have,

★ TRIGONOMETRICAL RATIO OF STANDARD ANGLES

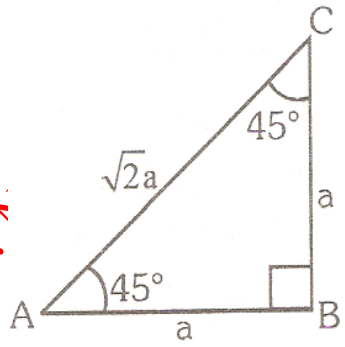
T-Ratios of 45^0

Consider a ΔABC in which $\angle B = 90^0$ and $\angle A = 45^0$

Then, clearly, $\angle C = 45^0$.

$\therefore$ AB = BC = a (say).

AC $= \sqrt{AB^2 + CB^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a.$



$\therefore$ **sin 45^0** $= \dfrac{BC}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}$:

cos 45^0 $= \dfrac{AB}{AC} = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}$:

tan 45^0 $= \dfrac{BC}{AC} = \dfrac{a}{a} = 1$

$\therefore$ **cosec 45^0** $= \dfrac{1}{\sin 45^0} = \sqrt{2}$; **sec 45^0** $= \dfrac{1}{\cos 45^0} = \sqrt{2}$; **cot 45^0** $= \dfrac{1}{\tan 45^0} = 1$

T-Ratios of 60^0 and 30^0

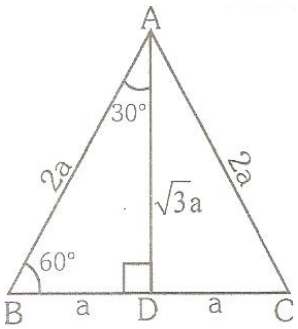
Draw an equilateral ΔABC with each side = 2a.

Then, $\angle A = \angle B = \angle C = 60^0$.

From A, draw AD $\perp$ BC.

Then, BD = DC = a, $\angle BAD = 30^0$ and $\angle ADB = 90^0$.

Also, AD $= \sqrt{AB^2 - BD^2} = \sqrt{4a^2 - a^2} = \sqrt{3a^2} = \sqrt{3}a$.



T-Ratios of 60^0

In ΔADB we have : $\angle ADB = 90^0$ and $\angle ABD = 60^0$.

Base = BC = a, Perp. = AD = $\sqrt{3a}$ and Hyp. AB = 2a.

$\therefore$ **sin 60^0** $= \dfrac{AD}{AB} = \dfrac{\sqrt{3}a}{2a} = \dfrac{\sqrt{3}}{2}$:

cos 60^0 $= \dfrac{BD}{AB} = \dfrac{a}{2a} = \dfrac{1}{2}$:

tan 60^0 $= \dfrac{AD}{BD} = \dfrac{\sqrt{3}a}{a} = \sqrt{3}$

$\therefore$ **cosec 60^0** = $\frac{1}{\sin 60^0} = \frac{2}{\sqrt{3}}$; **sec 60^0** = $\frac{1}{\cos 60^0} = 2$; **cot 60^0** = $\frac{1}{\tan 60^0} = \frac{1}{\sqrt{3}}$

T-Ratios of 30^0

In Δ ADB we have : $\angle$ ADB = 90^0 and $\angle$ ABD = 30^0.

$\therefore$ Base = AD = $\sqrt{3}a$, Perp. = BD = a and Hyp. AB = 2a.

$\therefore$ **sin 30^0** = $\frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$:

cos 30^0 = $\frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$:

tan 30^0 = $\frac{BD}{AD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$

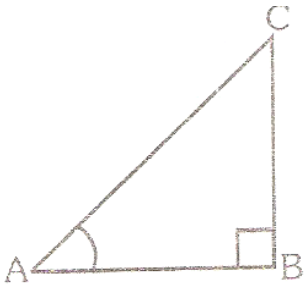
$\therefore$ **cosec 30^0** = $\frac{1}{\sin 30^0} = 2$; **sec 30^0** = $\frac{1}{\cos 30^0} = \frac{2}{\sqrt{3}}$; **cot 30^0** = $\frac{1}{\tan 30^0} = \sqrt{3}$

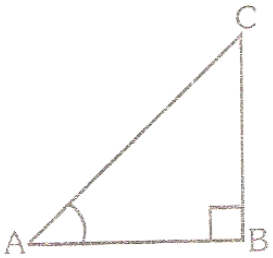
T-Ratios of 0^0 and 90^0

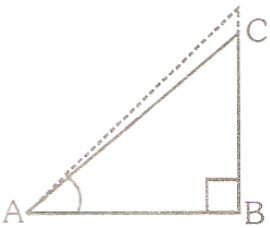
T-Ratios of 0^0

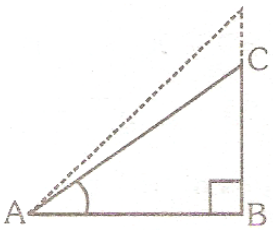
We shall see what happens to the trigonometric ratios of angle A, if it is made smaller and smaller in the right triangle ABC (see figure), till it becomes zero. As $\angle$A gets smaller and smaller, the length of the side BC decreases. The point C gets closer to point B, and finally when $\angle$A becomes very close to 0^0, AC becomes almost the same as AB.

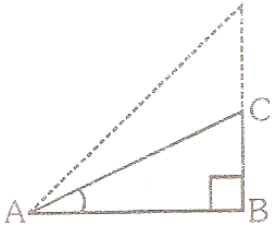
When $\angle$A is very close to 0^0, BC gets very close to 0 and so the value of sin A = $\frac{BC}{AC}$ is very close to O. Also when $\angle$A is very close to 0^0, AC is nearly same as AB and so the value of cos A = $\frac{AB}{AC}$ is very close to 1.

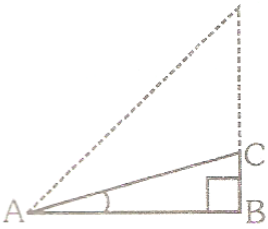


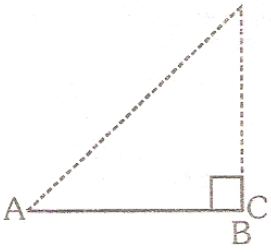












This helps us to see how we can define the values of sin A cos A when A = 0^0. We define :

sin 0^0 = 0 and **cos 0^0** = 1.

Using these, we have :

tan 0^0 = $\dfrac{\sin 0^0}{\cos 0^0} = 0$,

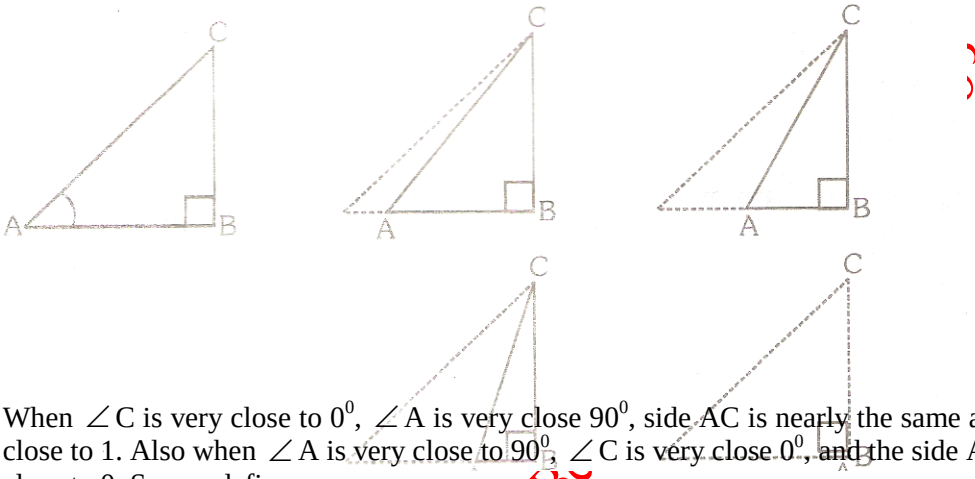
cot 0^0 = $\dfrac{1}{\tan 0^0} = \dfrac{1}{0}$ (not defined)

sec 0^0 = $\dfrac{1}{\cos 0^0} = 1$

and **cosec 0^0** = $\dfrac{1}{\sin 0^0} = \dfrac{1}{0}$ (not defined)

T-Ratios of 90^0

Now, we shall see what happens to the trigonometric ratios of $\angle A$ when it is made larger and larger in ΔABC till it becomes 90^0. As $\angle A$ gets larger and larger, $\angle C$ gets smaller and smaller. Therefore, as in the case above, the length of the side AB goes on decreasing. The point A gets closer to point B. Finally when $\angle A$ is very close 90^0, $\angle C$ becomes very close to 0^0 and the side AC almost coincides with side BC (see figure).



When $\angle C$ is very close to 0^0, $\angle A$ is very close 90^0, side AC is nearly the same as side BC, and so sin A is very close to 1. Also when $\angle A$ is very close to 90^0, $\angle C$ is very close 0^0, and the side AB is nearly zero, so cos A very close to 0. So, we define:

sin 90^0 = 1 and **cos 90^0** = 0.

Using these, we have :

tan 90^0 = $\dfrac{\sin 90^0}{\cos 90^0} = \dfrac{1}{0}$, (not defined)

cot 90^0 = $\dfrac{\cos 90^0}{\sin 90^0} = \dfrac{0}{1} = 0$

cosec 90^0 = $\dfrac{1}{\sin 90^0} = \dfrac{1}{1} = 1$

and **sec 90^0** = $\dfrac{1}{\cos 90^0} = \dfrac{1}{0}$ (not defined)

table for T-Ratios of Standard Angles

Angle θ / **Ratio**	**0^0**	**30^0**	**45^0**	**60^0**	**90^0**
Sin θ	**0**	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	**1**
cos θ	**1**	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	**0**
tan θ	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\sqrt{3}$	**Not defined**

cot θ	**Not defined**	$\sqrt{3}$	**1**	$\frac{1}{\sqrt{3}}$	**0**
sec θ	**1**	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	**2**	**Not defined**
cosec θ	**Not defined**	**2**	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	**1**

REMARK:

(i) As θ increases from 0^0 to 90^0, sin θ increases from 0 to 1.

(ii) As θ increases from 0^0 to 90^0, cos θ decreases from 1 to 0.

(iii) As θ increases from 0^0 to 90^0, tan θ increases from 0 to ∞.

(iv) The maximum value of $\frac{1}{\sec\theta}, 0^0 \le \theta \le 90^0$ is one.

(v) As $\cos\theta$ decreases from 1^0 to0, θ increases from 0 to 90^0.

(vi) $\sin\theta$ and cos θ can not be greater than one numerically.

(vii) $\sec\theta$ and cosec θ can not be less than one numerically.

(viii) $\tan\theta$ and cot θ can have any value.

COMPETITION WINDOW

T-RATIOS OF SOME ANGLES LESS THAN 90^0

Angle θ / Ratio	15^0	18^0	$22\frac{1^0}{2}$	36^0
Sin θ	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$
cos θ	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$
tan θ	$2-\sqrt{3}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$

Ex.7 In ΔABC, right angled at B, BC = 5 cm, $\angle BAC = 30^0$, find the length of the sides AB and AC.

Sol. We are given

$\angle BAC = 30^0$, i.e., $\angle A = 30^0$

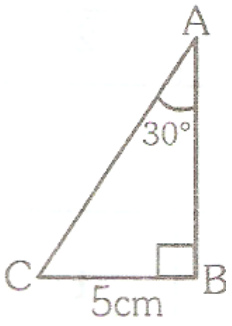
and BC = 5 cm

Now $\sin A = \frac{BC}{AC}$ or $\sin 30^0 = \frac{5}{AC}$

or $= \frac{5}{AC} = \frac{1}{2}$ …[$\because \sin 30^0 = \frac{1}{2}$]

or AC = 2 × 5 or 10 cm

To find AB, we have,



$$\frac{AB}{AC} = \cos A$$

or $$\frac{AB}{10} = \cos 30^0$$

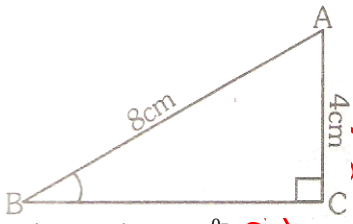
or $$\frac{AB}{AC} = \frac{\sqrt{3}}{2} \quad \ldots[\because \cos 30^0 = \frac{\sqrt{3}}{2}]$$

$\therefore$ $AB = \frac{\sqrt{3}}{2} \times 10$ or $5\sqrt{3}$ cm

Hence, $AB = 5\sqrt{3}$ cm and $AC = 10$ cm.

Ex.8 In ΔABC, right angled at C, if AC = 4 cm and AB = 8 cm. Find $\angle A$ and $\angle B$.

Sol. We are given, AC = 4 cm and AB = 8 cm



Now $\sin B = \frac{AC}{AB} = \frac{4}{8} = \frac{1}{2}$

But we know that $\sin 30^0 = \frac{1}{2}$

$\therefore$ $B = 30^0$

Now $\angle A = 90^0 - \angle B$ $\quad \ldots[\because \angle A + \angle B = 90^0]$

$= 90^0 - 30^0 = 60^0$

Hence, $\angle A = 60^0$ and $\angle B = 30^0$.

Ex.9 Evaluate : $\frac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 - \cot 45^0}$ **[NCERT]**

Sol. $\frac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 - \cot 45^0} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$

$= \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{(3\sqrt{3} - 4)}{(3\sqrt{3} - 4)}$

$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16} = \frac{43 - 24\sqrt{3}}{11}$

Ex.10 Find the value of θ in each of the following :

(i) $2 \sin 2\theta = \sqrt{3}$ (ii) $2 \cos 3\theta = 1$ (iii) $\sqrt{3} \tan 2\theta - 3 = 0$

Sol. (i) we have,

$2 \sin 2\theta = \sqrt{3}$

$\Rightarrow$ $\sin 2\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $\sin 2\theta \ \sin 60^0 \Rightarrow 2\theta \Rightarrow 60^0 \Rightarrow \theta = 30^0$

(ii) we have,

$2 \cos 3\theta = 1$

$\Rightarrow$ $\cos 3\theta = \frac{1}{2}$

$\Rightarrow$ $\cos 3\theta \ \cos 60^0 \Rightarrow 3\theta \Rightarrow 60^0 \Rightarrow \theta = 20^0$

(iii) we have,

$\sqrt{3} \tan 2\theta - 3 = 0$

$\Rightarrow \quad \sqrt{3}\ \tan 2\theta = 3$

$\Rightarrow \quad \tan 2\theta = \dfrac{3}{\sqrt{3}} = \sqrt{3}$

$\Rightarrow \quad \tan 2\theta\ \tan 60^0 \Rightarrow 2\theta \Rightarrow 60^0 \Rightarrow\ \theta = 30^0$

x^0	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	1' 2' 3' 4' 5 Mean Differences
43^0	0.6820								0.6921		8

COMPETION WINDOW

USING TRIGONOMETRIC TABLES

A Trigonometric Table consists of three parts :

(i) A column on the extreme left containing degrees from 0^0 to 89^0.

(ii) Ten column headed by 0', 6', 12', 18', 24', 30', 36', 42', 48', and 54',

(iii) Five column of mean differences, headed by 1', 2', 3', 4', and 5', The mean differences is added in case of sines, tangents and secants. The mean difference is subtracted in case of cosines, cotangents and cosecants.

The method of finding T-ratios of given angles using trigonometric tables, will be clear from the following example :

Find the value of sin 43^0 52'.

We have, $43^0 52' = 43^0 28' + 4'$

In the table natural sines, look at the numbers in the row against 43^0 and in the column headed 48' as shown below.

From Table of Natural Sines :

Now, sin $43^0 48' = 0.6921$

Mean difference for 4' = 0.0008 [To be added]

[See the number in the same row under 4']

$\therefore$ sin 43^0 52' = [0.6921 + 0 0.0008] = 0.6929

TO FIND THE ANGLE WHEN ITS T-RATIOS IS GIVEN

Find θ, when sin θ = 0.7114.

From the table, find the angle whose sine is just smaller than 0.7114.

We have sin θ = 0.7114
Sin 45^0 18' = <u>0.7108</u>
Diff. = 0.0006

Mean difference of 6 corresponds to 3'.

$\therefore$ Required angle = (45^0 18' + 3') = $45^0 21'$

Find θ, when cos θ = 0.5248

From the table, find the angle whose costing is just smaller than 0.5248

We have cos θ = 0.5248
cos 58^0 18' = <u>0.5255</u>
Diff. = 0.0007

And 7 corresponds to 3'.

$\therefore$ Required angle = $58^0\ 18' + 3' = 58^0\ 21'$

TRY OUT THE FOLLOWING

1. Using tables find the value of :
   (i) sin $83^0\ 12'$ (ii) cos $70^0\ 17'$ (iii) tan $24^0\ 14'$ (iv) cosec $(30.8)^0$ (v) sec $68^0\ 10'$ (vi) cot $39^0\ 15'$
2. Using tables find the value of θ if :
   (i) $\sin\theta = 0.42'$ (ii) $\cos\theta = 0.8092$ (iii) $\tan\theta = 2.91$ (iv) $\text{cosec}\,\theta = 2.8893$ (v) $\sec\theta = 1.2304$
   (vi) $\cot\theta = 0.1385$

ANSWERS

1. (i) 0.993. (ii) 0.3373 (iii) 0.4536 (iv) 1.9530 (v) 2.6892 (vi) 1.2283
2. (i) $24^0\ 50'$ (ii) $35^0\ 59'$ (iii) $71^0\ 2'$ (iv) $20^0\ 15'$ (v) $35^0\ 38'$ (vi) $82^0\ 7'$

★ T-RATIOS OF COMPLEMENTARY ANGLES

Complementary Angles

Two angles are said to be complementary, if their sum is 90^0.

Thus, θ^0 and $(90^0 - \theta)$ are complementary angles.

T-ratios of Complementary Angles

Consider ΔABC in which $\angle B = 90^0$ and $\angle A = \theta^0$.

$\therefore$ $\angle C = (90^0 - \theta)$.

Let AB = x. BC = y and AC = r.

When we consider the T-ratios of $(90^0 - \theta)$, then

Base = BC, **Perp.** = AB and **Hyp.** AC =

$\therefore$ **sin** $(90^0 - \theta) = \dfrac{AB}{AC} = \dfrac{x}{r} =$ **cos** θ.

cos $(90^0 - \theta) = \dfrac{BC}{AC} = \dfrac{y}{r} =$ **sin** θ.

tan $(90^0 - \theta) = \dfrac{AB}{BC} = \dfrac{x}{y} =$ **cot** θ.

$\therefore$ **cosec** $(90^0 - \theta) = \dfrac{1}{\sin(90^0 - \theta)} = \dfrac{1}{\sin\theta} =$ **sec** θ.

sec $(90^0 - \theta) = \dfrac{1}{\cos(90^0 - \theta)} = \dfrac{1}{\sin\theta} =$ **cosec** θ.

cot $(90^0 - \theta) = \dfrac{1}{\tan(90^0 - \theta)} = \dfrac{1}{\cot\theta} =$ **tan** θ.

(i) sin $(90^0 - \theta) =$ **cos** θ	**(ii) cos** $(90^0 - \theta) =$ **sin** θ	**(iii) tan** $(90^0 - \theta) =$ **cot** θ
(iv) cosec $(90^0 - \theta) =$ **sec** θ	**(v) sec** $(90^0 - \theta) =$ **cosec** θ	**(vi) cot** $(90^0 - \theta) =$ **tan** θ

Aid to memory:

Add co if that is not there

Remove co if that is there

Thus we have,

sine of $(90^0 - \theta)$ = cosine of $\theta \Rightarrow$ sine $(90^0 - \theta) = \cos\theta$

cosine of $(90^0 - \theta)$ = sine of $\theta \Rightarrow \cos(90^0 - \theta) = \sin\theta$
tangent of $(90^0 - \theta)$ = cotangent of $\theta \Rightarrow \tan(90^0 - \theta) = \cot\theta$
cotangent of $(90^0 - \theta)$ = tangent of $\theta \Rightarrow \cot(90^0 - \theta) = \tan\theta$
secant of $(90^0 - \theta)$ = cosecant of $\theta \Rightarrow \sec(90^0 - \theta) = \operatorname{cosec}\theta$
cosecant of $(90^0 - \theta)$ = secant of $\theta \Rightarrow \operatorname{cosec}(90^0 - \theta) = \sec\theta$

In other words :

sin (angle) = cos (complement) ; cos (angle) = sin (complement)
tan (angle) = cot (complement) ; cot (angle) = tan (complement)
sec (angle) = cosec (complement) ; cosec (angle) = sec (complement)

where complement = 90^0 – angle

Ex.11 Without using tables, evaluate :

(i) $\dfrac{\sin 53^0}{\cos 37^0}$ (ii) $\dfrac{\cos 49^0}{\sin 41^0}$ (iii) $\dfrac{\tan 66^0}{\cot 24^0}$

Sol. (i) $\dfrac{\sin 53^0}{\cos 37^0} = \dfrac{\sin(90^0 - 37^0)}{\cos 37^0} = \dfrac{\cos 37^0}{\cos 37^0} = 1$ $[\because \sin(90^0 - \theta) \cos\theta]$

(ii) $\dfrac{\cos 53^0}{\sin 37^0} = \dfrac{\cos(90^0 - 41^0)}{\sin 41^0} = \dfrac{\sin 41^0}{\sin 41^0} = 1$ $[\because \cos(90^0 - \theta) \sin\theta]$

(iii) $\dfrac{\tan 66^0}{\cot 24^0} = \dfrac{\tan(90^0 - 24^0)}{\cot 24^0} = \dfrac{\cot 24^0}{\cot 24^0} = 1$ $[\because \tan(90^0 - \theta) \cot\theta]$

REMARK : (i) The above example suggests that out of the two t-ratios, we convent one is term of the t-ratios of the complement.
(ii) For uniformity, we usually convert the angle greater than 45^0 in terms of its complement.

Ex.12 Without using tables, show that $(\cos 35^0 \cos 55^0 - \sin 35^0 \sin 55^0) = 0$.

Sol. LHS $= (\cos 35^0 \cos 55^0 - \sin 35^0 \sin 55^0)$
$= [(\cos 35^0 \cos 55^0 - \sin(90^0 - 55^0) \sin(90^0 - 35^0)]$
$= (\cos 35^0 \cos 55^0 - \cos 55^0 \cos 35^0) = 0 =$ RHS.
$[\because \sin(90^0 - \theta) \cos\theta$ and $\cos(90^0 - \theta) \sin\theta]$

Ex.13 Express $(\sin 58^0 + \operatorname{cosec} 85^0)$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

Sol. $(\sin 58^0 + \operatorname{cosec} 85^0) = \sin(90^0 - 5^0) \operatorname{cosec}(90^0 - 5^0) = (\cos 5^0 + \sec 5^0)$.

Ex.14 If $\tan 2A = \cot(A - 18^0)$, where 2A is an acute angle, find the value of A.

Sol. We are given,

$\tan 2A = \cot(A - 18^0)$
or $\cot(90^0 - 2A) = \cot(A - 18^0)$ …$[\because \cot(90^0 - 2A) = \tan 2A]$
$\therefore$ $90^0 - 2A = A - 18^0$
or $A + 2A = 90^0 - 18^0$
or $3A = 108^0$
$\therefore$ $A = 36^0$

Ex.15 Evaluate : $\dfrac{\sec 29^0}{\operatorname{cosec} 61^0} + 2\cot 8^0 \cot 17^0 \cot 45^0 \cot 73^0 \cot 82^0$.

Sol. $\dfrac{\sec 29^0}{\operatorname{cosec} 61^0} + 2\cot 8^0 \cot 17^0 \cot 45^0 \cot 73^0 \cot 82^0$.

$$= \frac{\sec 29^0}{\text{cosec}(90^0 - 29^0)} + 2\cot 8^0 \cot 17^0 (1)\cot(90^0 - 17^0)\cot(90^0 - 8^0)$$

$$= \frac{\sec 29^0}{\sec 29^0} + 2\cot 8^0 \cot 17^0 \tan 17^0 \tan 8^0 . \qquad [\because \cot(90^0 - \theta) = \tan\theta]$$

$$= 1 + 2\cot 8^0 \cot 17^0 . \frac{1}{\cot 17^0} . \frac{1}{\cot 8^0} \qquad \ldots\left(\because \tan\theta = \frac{1}{\cot\theta}\right)$$

Ex.16 For a triangle ABC, show that $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$, where A, B and C are interior angles of Δ ABC.

Sol. We know that $\angle A + \angle B + \angle C = 180^0$

Thus we have, $B + C = 180^0 - A$

or $\left(\frac{B+C}{2}\right) = 90^0 - \frac{A}{2}$ or $\sin\left(\frac{B+C}{2}\right) = \sin\left(90^0 - \frac{A}{2}\right)$ or $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$.

★ T-IDENTITIES

We know that an equation is called an identity when it is true for all value of the variables involved. Similarly, **an equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angle(s) the angle(s) involved.**

The three Fundamental Trigonometric Identities are –

(i) $\cos^2 A + \sin^2 A = 1 \; ; \; 0^0 \le A \le 90^0$

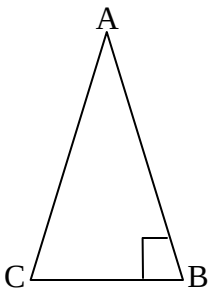
(ii) $1 + \tan^2 A + \text{cosec}^2 A = 1 \; ; \; 0^0 \le A < 90^0$

(iii) $1 + \cot^2 A + \text{cosec}^2 A = 1 \; ; \; 0^0 < A \le 90^0$

Geometrical Proof :

Consider a Δ ABC, right angled at B. Then we have :

$AB^2 + BC^2 = AC^2$ …(i) By Pythagoras theorem



(i) **$\cos^2 A + \sin^2 A = 1 \; ; \; 0^0 \le A \le 90^0$**

Dividing each term of (i) by AC^2, we get

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

i.e., $\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$

i.e., $(\cos A)^2 + (\sin A)^2 = 1$

i.e., **$\cos^2 A + \sin^2 A = 1$** …(ii)

This is true for all A such that **$0^0 \le A \le 90^0$**

So, this is a trigonometric identity.

(ii) **$1 + \tan^2 A = \sec^2 A \; ; \; 0^0 \le A < 90^0$**

Let us now divide (i) by AB^2. We get

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

or, $\left(\frac{AB}{AB}\right)^2+\left(\frac{BC}{AB}\right)^2=\left(\frac{AC}{AB}\right)^2$

i.e., $\mathbf{1+\tan^2 A=\sec^2 A}$ …(iii)

This equation is true for $A = 0^0$. Since tan A and sec A are not defined for $A = 90^0$, so (iii) is true for all A such that $\mathbf{0^0 \le A < 90^0}$

(iii) $\mathbf{1+\cot^2 A=\operatorname{cosec}^2 A : 0^0 < A < 90^0}$

Again, let us divide (i) by BC^2, we get

$\Rightarrow \quad \frac{AB^2}{BC^2}+\frac{BC^2}{BC^2}=\frac{AC^2}{BC^2}$

$\Rightarrow \quad \left(\frac{AB}{BC}\right)^2+\left(\frac{BC}{BC}\right)^2=\left(\frac{AC}{BC}\right)^2$

$\Rightarrow \quad \mathbf{1+\cot^2 A=\operatorname{cosec}^2 A}$ …(iii)

Since cosec A and cot A are not defined for $A = 0^0$, therefore (iv) is true for all A such that $\mathbf{: 0^0 < A < 90^0}$

Using the above trigonometric identities, we can express each trigonometric ratio in terms of the other trigonometric ratios, i.e., if any one of the ratios is known, we can also determine the value of other trigonometric ratios.

Fundamental Identities (Results)

$\sin^2\theta+\cos^2\theta=1$	$1+\tan^2\theta=\sec^2\theta$	$1+\cot^2\theta=\operatorname{cosec}^2\theta$
$\sin^2\theta=1-\cos^2\theta$	$\sec^2\theta-\tan^2\theta=1$	$\operatorname{cosec}^2\theta-\cot^2\theta=1$
$\cos^2\theta=1-\sin^2\theta$	$\tan^2\theta=\sec^2\theta-1$	$\cot^2\theta=\operatorname{cosec}^2\theta-1$

To prove Trignometrical Identities

The following methods are to be followed :

Method-I : Table the more complicated side of the identity (L.H.S. or R.H.S. as the case may be) and by using suitable trigonometric and algebraic formula prove it equal to the other side.

Method-II : When neither side of the identity is in a simple form, simplify the L.H.S. and R.H.S. separately by using suitable formulae (by expressing all the T-ratios occurring in the identity in terms of the sine and cosine and show that the results are equal).

Method-III : If the identity to the proved is true, transposing so as to get similar terms on the same side, or cross-multiplication, and using suitable formulae, we get an identity which is true.

Ex.17 Express sin A, sec A and tan A in terms of cot A. **[NCERT]**

Sol. We know that

sin A $= \frac{1}{\operatorname{cosec} A}=\frac{1}{\sqrt{\operatorname{cosec}^2 A}}=\frac{1}{\sqrt{1+\cot^2 A}}$

sec A $= \frac{1}{\cos A}=\frac{1}{\frac{\sin A}{\cos A}}$ …(Dividing num. and denom., by sin A)

$= \frac{\operatorname{cosec} A}{\cot A}=\frac{\sqrt{1+\cot^2 A}}{\cot A}$ and tan A $= \frac{1}{\cot A}$

Ex.18 Prove $\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}=\tan\theta+\cot\theta$

Sol. LHS $= \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \sqrt{(1+\tan^2\theta)+(1+\cot^2\theta)} = \sqrt{\tan^2\theta+\cot^2\theta+2\tan\theta\cot\theta}$

$= \sqrt{\tan\theta+\cot\theta} = \tan\theta+\cot\theta = \text{RHS}$

Hence, proved.

Ex.19 Prove $(\operatorname{cosec}A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

Sol. LHS $= (\operatorname{cosec}A-\sin A)(\sec A-\cos A)=\left(\dfrac{1}{\sin A}-\sin A\right)\left(\dfrac{1}{\cos A}-\cos A\right)$

$= \left(\dfrac{1-\sin^2 A}{\sin A}\dfrac{1-\cos^2 A}{\cos A}\right)=\dfrac{\cos^2 A}{\sin A}\dfrac{\sin^2 A}{\cos A}$ $[\because \sin^2 A+\cos^2 A=1]$

$= \sin A\cos A=\dfrac{\sin A\cos A}{\sin^2 A+\cos^2 A}$ $[\because \sin^2 A+\cos^2 A=1]$

$= \dfrac{\dfrac{\sin A\cos A}{\sin A\cos A}}{\dfrac{\sin^2 A}{\sin A\cos A}+\dfrac{\cos^2 A}{\sin A\cos A}}$ [Dividing the numerator and denominator by sin A cos A.]

$= \dfrac{1}{\tan A+\cot A} = \text{RHS}$

Hence, proved.

★ APPLICATIONS OF TRIGONOMETRY

Many times, we have to find the height and distances of many objects in real life. We use trigonometry to solve problems, such as finding the height of a tower, height of a flag mast, distance between two objects, where measuring directly is trouble, some and some times impossible. In those cases, we adopt indirect methods which involve solution of right triangles.

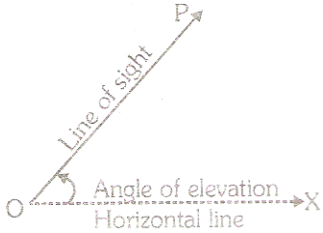
Thus Trigonometry is very useful in geography, astronomy and navigation. It helps us to prepare maps, determine the position of a landmass in relation to the longitudes and latitudes. Surveyors have made use of this knowledge since ages.

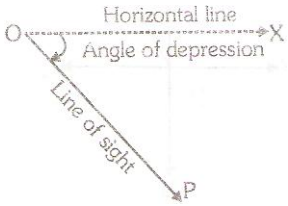
Angle of Elevation

The angle between the horizontal line drawn through the observer eye and line joining the eye to any object is called the angle of elevation of the object, if the object is at a higher level than the eye i.e., If a horizontal line OX is drawn through O, the eye of the observer, and P is an object in the vertical plane through OX, then if P is above OX, as in fig. $\angle$XOP is called the angle of elevation or the altitude of P as seen from O.

Angle of Depression

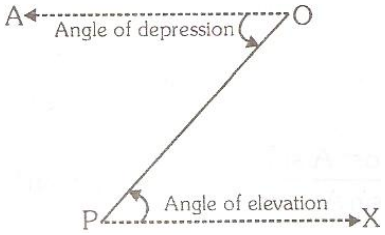
The angle between the horizontal line drawn through the observer eye and line joining the eye to any object is called the angle of depression of the object, if the object is at a higher level than the eye i.e., If a horizontal line OX is drawn through O, the eye of the observer, and P is an object in the vertical plane through OX, then if P is above OX, as in fig. $\angle$XOP is called the angle of depression of P as seen from O.





REMARK :

1. The angle of elevation as well as angle of depression are measured with reference to horizontal line.
2. All objects such as towers, mountains etc. shall be considered as linear for mathematical convenience, throughout this section.



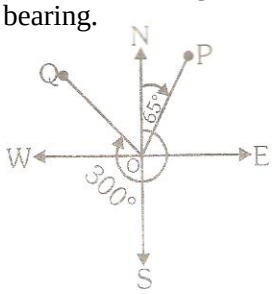
3. The height of the observer, is neglected, if it is not given in the problem.
4. Angle of depression of P as seen from O is equal to the angle of elevation of O, as seen from P.
   i.e., $\angle AOP = \angle OPX$.
5. To find one side a right angled triangle when another side and an acute angle are given, the hypotenuse also being regarded as a side.

   $\frac{Required}{Given side}$ = a certain T-ratio of the given angle.
6. The angle of elevation increases as the object moves towards the right of the line of sight.
7. The angle of depression increases as the object moves towards the right of the line of sight.

COMPETITION WINDOW

BEARING OF A POINT

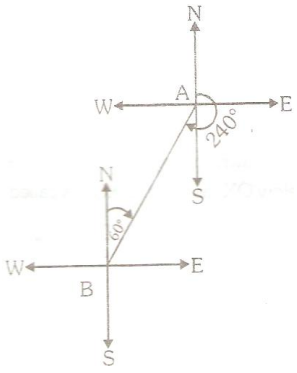
The true bearing to a point is the angle measured in degrees in a clockwise direction from the north line. We will refer to the true bearing simply as the bearing.



e.g. (i) the bearing of point P is 65^0 (ii) the bearing of point Q is 300^0

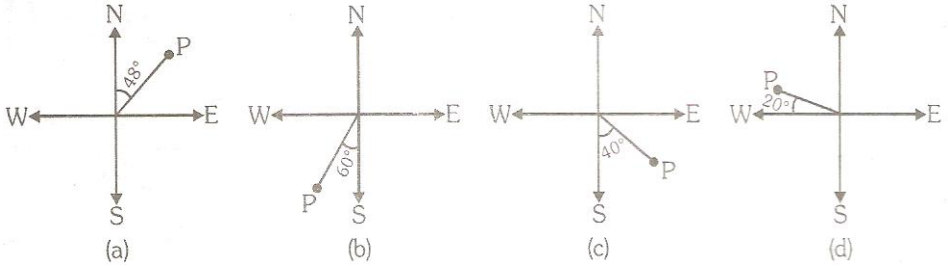
A bearing is used to represent the direction of one point relative to another point.

e.g., the bearing of A from B is 60^0. The bearing of B from A is 240^0.



TRY OUT THE FOLLOWING

State the bearing of the point P in each of the following diagrams :



ANSWERS

(a) 48^0 (b) 240^0 (c) 140^0 (d) 290^0

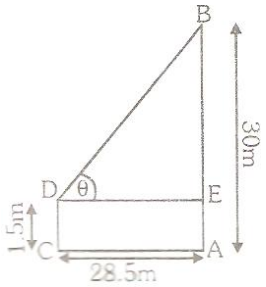
Ex.20 An observer 1.5 m tall, is 28 5 m away from a tower 30 m high. Determine the angle of elevation of the top of the tower from his eye. **(NCERT)**

Sol. Let AB be the height of the tower, CD the height of the observer with his eye at the point D, AB = 30 m, CD = 1.5 m.

Through D, draw DE ∥ CA than $\angle BDE = \theta$ where θ is the angle of elevation of the top of the tower from his eye.

AC = horizontal distance between the tower and the observer = 28.5 m

BE = AB – AE = (30 – 1.5) m = 28.5 m BDE is right triangle at E,



then $\frac{BE}{DE} = \tan\theta \Rightarrow \frac{28.5}{28.5} = \tan\theta \Rightarrow \tan\theta = 1$

$\Rightarrow \tan\theta = 1 = \tan 45^0 \Rightarrow \theta = 45^0$.

Required angle of elevation of the tower = $\theta = 45^0$.

Ex.21 A vertical post casts a shadow 21 m long when the altitude of the sun is 30^0. Find :

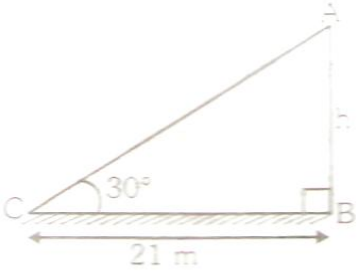
(a) the height of the post.

(b) the length of the shadow when the altitude of the sun is 60^0.

(c) the altitude of the sun when the length of the shadow is $7\sqrt{3}$ m.

Sol. Let AB be the vertical post and its shadow is 21 m when the altitude of the sun is 30^0.

(a) BC = 21 m, $\angle ACB = 30^0$, AB = h metres



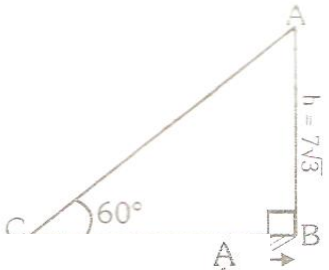
ABC is rt. $\Delta, \frac{AB}{BC} = \tan 30^0 \Rightarrow \frac{h}{21} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad h = \frac{21}{\sqrt{3}} = \frac{7 \times \sqrt{3} \times \sqrt{3}}{\sqrt{3}} = 7\sqrt{3}$

$\Rightarrow$ AB = h, Height of the pole = $7\sqrt{3}$

(b) In this case, we have,

$\angle ACB = 60^0$, BC = x m AB = $7\sqrt{3}$ m



ABC is rt. Δ, then :

$\frac{AB}{BC} = \tan 60^0 = \sqrt{3}$

$\Rightarrow \quad \frac{h}{x} = \sqrt{3} \Rightarrow \frac{7\sqrt{3}}{x} = \sqrt{3}$

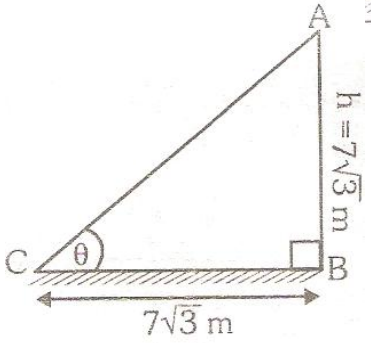
$\Rightarrow$ x = BC, Length of the shadow = 7 m.

(c) In this case :

AB = h = $7\sqrt{3}$

BC = The length of the shadow = $7\sqrt{3}$ m

when the altitude of the sun is θ



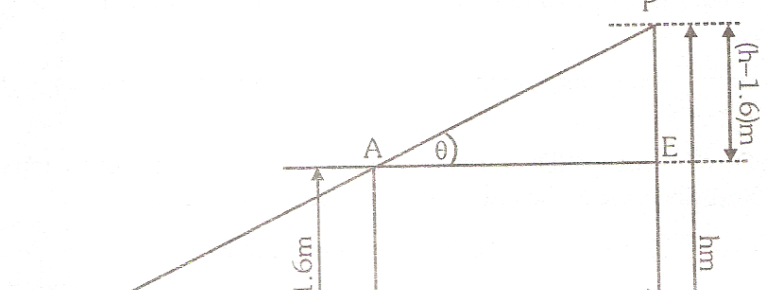
ABC is rt. Δ, then $\frac{AB}{BC} = \tan\theta = \frac{7\sqrt{3}}{7\sqrt{3}} = \tan\theta$

$\tan\theta = 1 = \tan 45^0 \Rightarrow \theta\ 45^0$

Altitude of the sun = $\theta = 45^0$

Ex.22 A 1.6 m tall girl stands at a distance of 3.2 m from the lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using (i) trigonometric ratios (ii) property of similar triangles. **(NCERT)**

Sol. Let PQ be the position of the lamp-post whose height is h metres. i.e., PQ = h metres. AB be the position of the tall girl such that AB = 16. m. Let BC be the shadow of AB such that BC = 4.8 m, $\angle ACB = \angle PAE = \theta$ (corr. $\angle$s)



(i) In right Δ ABC, $\frac{AB}{BC} = \tan\theta \Rightarrow \frac{1.6}{4.8} = \tan\theta \Rightarrow \tan\theta = \frac{1}{3}$

AB = EQ = 1.6 m. Also AE = BQ = 3.2 m

PE = PQ – EQ = (h – 1.6) m

In right Δ ABC, $\frac{PE}{AE} = \tan\theta \Rightarrow \frac{(h-1.6)}{3.2} = \tan\theta = \frac{1}{3} \Rightarrow \frac{h-1.6}{3.2} = \frac{1}{3}$

3h – 4.8 = 3.2 $\Rightarrow$ 3h = 4.8 + 3.2 = 8 $\Rightarrow$ 3h = 8

The height of the lamp-post = $\frac{8}{3}$ m = $2\frac{2}{3}$ m

(ii) In two Δs ACB and PCQ, we have :

$\angle$ ACB = $\angle$ ACB = θ (common)

$\angle$ ABC = $\angle$ PQC = $90^0 \Rightarrow \Delta$ ACB ~ Δ PAE (AA similar)

CQ = CB + BQ

= (4.8 + 3.2) m = 8 metres

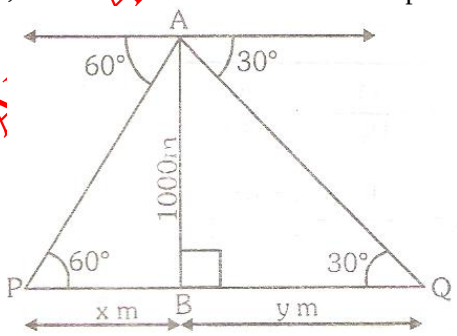
$\frac{AC}{PC} = \frac{CB}{CQ} = \frac{AB}{PQ} \Rightarrow \frac{BC}{CQ} = \frac{AB}{PQ}$

$\frac{BC}{CQ} = \frac{AB}{PQ} \Rightarrow \frac{4.8}{8} = \frac{1.6}{h} \Rightarrow \frac{3}{8} = \frac{1}{h} \Rightarrow 3h = 8$

Thus $3h = 8 \Rightarrow h = \frac{3}{8}m \Rightarrow h = 2\frac{2}{3}$ m

Required height of the lamp-post = PQ = h = $2\frac{2}{3}$ m

Ex.23 A captain of an airplane flying at an altitude of 1000 metres sights two ships as shown in the figure. If the angle of depressions is 60^0 and 30^0, find the distance between the ships.



Sol. Let A be the position of the captain of an airplane flying at the altitude of 1000 metres from the ground.

AB = the altitude of the airplane from the ground = 1000 m

P and Q be the position of two ships.

Let PB = x metres, and BQ = y metres.

Required : PQ = Distance between the ships = (x + y) metres.

ABP is rt. Δ at B

$\frac{AB}{PB} = \tan 60^0$

ABQ is rt. Δ at B

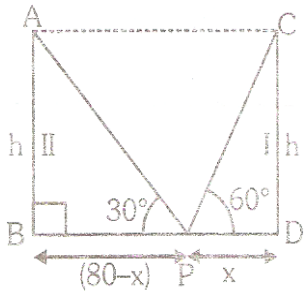
$\frac{AB}{BQ} = \tan 60^0$

$$\frac{1000}{x}=\sqrt{3}\Rightarrow x=\frac{1000}{\sqrt{3}} \qquad \frac{1000}{y}=\frac{1}{\sqrt{3}}\Rightarrow y=1000\sqrt{3}$$

$$x=\frac{1000(1.732)}{3}=577.3 \text{ m}$$

Required distance between the ships = (x + y) metres = (577.3 + 1732) m = 2309.3 m

Ex.24 Two poles of equal heights are standing opposite to each other on either side of a road, which is 80 metres wide. From a point between them on the road, the angles of elevation of their top are 30^0 and 60^0. Find the position of the point and also the height of the poles.



Sol. Let AB and CD be two poles of equal height standing opposite to each of them on either side of the road BD.

$\Rightarrow$ AB = CD = h metres.

Let P be the observation point on the road BD. The angles of elevation of their top are 30^0 and 60^0.

$\angle APB = 30^0$, $\angle CPD = 60^0$

The width of the road = BD = 80 m, let PD = x metres

Then BP = (80 - x) metres

Consider right Δ CDP, we have :

$$\frac{CD}{PD}\tan 60^0\Rightarrow\frac{h}{x}=\sqrt{3}\Rightarrow h=\sqrt{3}x \qquad \ldots(i)$$

In right Δ ABP, we have :

$$\frac{AB}{BP}\tan 30^0\Rightarrow\frac{h}{80-x}=\frac{1}{\sqrt{3}}\Rightarrow h=\frac{80-x}{\sqrt{3}} \qquad \ldots(ii)$$

From (i) and (ii), we get : $\left.\begin{matrix} h=\sqrt{3x} \\ h=\dfrac{80-x}{\sqrt{3}} \end{matrix}\right\} \Rightarrow (80-x)=3x\Rightarrow 4x=80\Rightarrow x=20$

Height of each pole = AB = CD = $\sqrt{3}$. x = 20. $\sqrt{3}$ = 20 (1.732) = 34.64 metres.

Position of point P is 20 m from the first and 60 m from the second pole.

i.e., position of the point P is 20 m from either of the poles.

★ **SYNOPSIS**

1. In a right triangle ABC, with right angle B,

$\sin A=\frac{Perpendicular}{Hypotenuse}$	$\cos A=\frac{Hypotenuse}{Perpendicular}$
$\cos A=\frac{Base}{Hypotenuse}$	$\sec A=\frac{Hypotenuse}{Base}$
$\tan A=\frac{Perpendicular}{Base}$	$\cot A=\frac{Base}{Perpendicular}$

2. $\cos=\frac{1}{\sin A}:\sec A=\frac{1}{\cos A}:\tan A=\frac{1}{\cot A}$

3. The value of sinA or cosA never exceeds 1, whereas the value of secA or cosecA is always greater than or equal to 1.

4. $\sin(90^0 - A) = \cos A$ $\quad$ $\cot(90^0 - A) = \tan A$

$\cos(90^0 - A) = \sin A$ $\quad$ $\sec(90^0 - A) = \operatorname{cosec} A$

$\tan(90^0 - A) = \cot A$ $\quad$ $\operatorname{cosec}(90^0 - A) = \sec A$

5. $\sin^2 A + \cos^2 A = 1 : 1 + \tan^2 A = \sec^2 : 1 + \cot^2 A = \operatorname{cosec}^2 A$

6. If one of the sides and any other part (either an acute angle or any side) of a right triangle is known, the remaining sides and angles of the triangle can be easily determined.

7. In a right triangle, the side opposite to 30^0 is half the side of the hypotenuse.

8. In a right triangle, the side opposite to 60^0 is $\frac{\sqrt{3}}{2}$ times the side of the hypotenuse.

9. (i) The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

(ii) The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level i.e., the case when we raise our head to look at the object.

(iii) The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level i.e., the case when we raise our head to look at the object.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. In ΔABC, $\angle B = 90^0$. If AB = 14 cm and AC = 50 cm then tan A equals :

(A) $\frac{24}{25}$ (B) $\frac{24}{7}$ (C) $\frac{7}{24}$ (D) $\frac{25}{24}$

2. If $\sin\theta = \frac{12}{13}$ then the value of the $\frac{2\cos\theta + 3\tan\theta}{\sin\theta + \tan\theta\sin\theta}$ is :

(A) $\frac{12}{5}$ (B) $\frac{5}{3}$ (C) $\frac{259}{102}$ (D) $\frac{259}{65}$

3. If $\sec\theta = \frac{\sqrt{p^2 + q^2}}{q}$ then the value of the $\frac{p\sin\theta + q\cos\theta}{p\sin\theta + q\cos\theta}$ is :

(A) $\frac{p}{q}$ (B) $\frac{p^2}{q^2}$ (C) $\frac{p^2 - q^2}{p^2 + q^2}$ (D) $\frac{p^2 + q^2}{p^2 - q^2}$

4. If angle A is acute and cos A = $\frac{8}{17}$ then cot A is :

(A) $\frac{8}{15}$ (B) $\frac{17}{8}$ (C) $\frac{15}{8}$ (D) $\frac{17}{15}$

5. sec θ is equal to –

(A) $\frac{1}{\sqrt{1 - \cos^2\theta}}$ (B) $\frac{\sqrt{1 + \cot^2\theta}}{\cot\theta}$ (C) $\frac{\cot\theta}{\sqrt{1 + \cot^2\theta}}$ (D) $\frac{\sqrt{\operatorname{cosec}^2\theta - 1}}{\operatorname{cosec}\theta}$

6. sin 30^0 + cos 60^0 equals :

(A) $\frac{1+\sqrt{3}}{2}$ (B) $\sqrt{3}$ (C) 1 (D) None of these

7. The value of $2\tan^2 60^0 - 4\cos^2 45^0 - 3\sec^2 30^0$ is :
(A) 0 (B) 1 (C) 12 (D) 8

8. The value of $\frac{3}{4}\tan^2 30^0 - 3\sin^2 60^0 + 3\,\text{cosec}^2 45^0$ is
(A) 1 (B) 8 (C) 0 (D) 12

9. $7\sin^2\theta + 3\cos^2\theta = 4$ then :
(A) $\tan\theta = \frac{1}{\sqrt{2}}$ (B) $\tan\theta = \frac{1}{2}$ (C) $\tan\theta = \frac{1}{3}$ (D) $\tan\theta = \frac{1}{\sqrt{3}}$

10. The solution of the trigonometric equation $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3, 0^0 < \theta < 90^0$:
(A) $\theta = 0^0$ (B) $\theta = 30^0$ (C) $\theta = 60^0$ (D) $\theta = 90^0$

11. If $\cot\theta + \cos\theta = p$ and $\cot\theta = q$, then the value of $p^2 - q^2$ is :
(A) $2\sqrt{pq}$ (B) $4\sqrt{pq}$ (C) 2 pq (D) 4 pq

12. The value of $\sin^2 15^0 + \sin 2\,30^0 + \sin^2 45^0 + \sin^2 60^0 + \sin^2 75^0$ is :
(A) 1 (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) 3

13. The value of $\frac{\sin 29^0}{\cos 61^0} - \frac{\sin 61^0}{\cos 29^0}$ is :
(A) Zero (B) 1 (C) $\frac{61}{29}$ (D) $\frac{29}{61}$

14. The values of x and y which make the following solutions true are: $\cos x^0 = \sin 52^0$ and $\cos y^0 = \sin(y^0 + 10)$
(A) $x = 52^0, y = 30^0$ (B) $x = 38^0, y = 40^0$ (C) $x = 48^0, y = 52^0$ (D) $x = 40^0, y = 50^0$

15. If $\alpha + \beta = 90^0$ and $\alpha = 2\beta$ then $\cos^2\alpha + \sin^2\beta$ equal :
(A) 1 (B) Zero (C) $\frac{1}{2}$ (D) 2

16. A flagstaff 6 metres high throws shadow $2\sqrt{3}$ metres long on the ground. The angle of elevation is :
(A) 30^0 (B) 45^0 (C) 90^0 (D) 60^0

17. An observer $\sqrt{3}$ m tall is 3 m away from the pole $2\sqrt{3}$ m high. The angle of the top of elevation of the top from the pole is :
(A) 45^0 (B) 30^0 (C) 60^0 (D) 15^0

18. An observer 1.5 m tall is 28.5 m away from. a chimney. The angle of elevation of the top of the chimney from her eyes is 45^0. The height of the chimney is :
(A) 30 m (B) 27 m (C) 28.5 m (D) None of these

19. The angle of elevation of the top of a tower from a distance 100 m from its foot is 30^0. The height of the tower is :
(A) $1000\sqrt{3}$ m (B) $\frac{200}{\sqrt{3}}$ m (C) $5\sqrt{3}$ m (D) $\frac{100}{\sqrt{3}}$ m

20. A kite is flying at a height of 60 m above the ground. The sting attached to the kite is temporarily the to a point on the ground. The inclination of the string with the ground is 60^0. The length of the staring is :
(A) $40\sqrt{3}$ m (B) 30 m (C) $20\sqrt{3}$ m (D) $60\sqrt{3}$ m

21. A tree is broken by the wind. Its top struck the ground at an angle 30^0 at a distance of 30 m from its foot. The whole height of the tree is :
(A) $10\sqrt{3}$ m (B) $20\sqrt{3}$ m (C) $40\sqrt{3}$ m (D) $30\sqrt{3}$ m

22. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30^0 and 45^0 respectively. If the bridge is at a height of 3 m from the banks then the width of the river is :
(A) $3(\sqrt{3}-1)$ m (B) $3(\sqrt{3}+1)$ m (C) $(\sqrt{3}+3)$ m (D) $(\sqrt{3}-3)$ m

23. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is :
(A) $\sqrt{5}$ m (B) $\sqrt{13}$ m (C) 6 m (D) 2.25 m

24. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angles of elevation from his eyes to the top of the building increases from 30 to 60^0 as he walks towards the building. The distance he walked towards the building is :

(A) $19\sqrt{3}$ m (B) $57\sqrt{3}$ m (C) $38\sqrt{3}$ m (D) $18\sqrt{3}$ m

25. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30^0 and 60^0. if one strip is exactly behind the other on the same side of the light-house then the distance between the two ships is :

(A) $25\sqrt{3}$ m (B) $75\sqrt{3}$ m (C) $50\sqrt{3}$ m (D) None of these

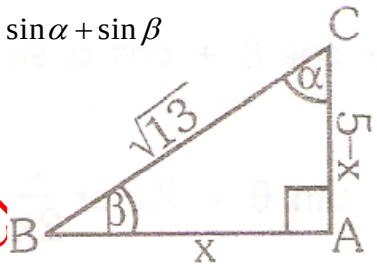
(OBJECTIVE)				ANSWER KEY					EXERCISE	
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	B	C	C	A	B	C	A	C	D	C
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	B	C	A	B	C	D	B	A	D	A
Que.	**2**	**122**	**23**	**24**	**25**					
Ans.	D	B	C	A	C					

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. In the adjoining fig, determine : $\sin\alpha + \sin\beta$



2. If 5 tan θ = 4, find the value of $\dfrac{5\sin\theta - 3\cos\theta}{\sin\theta + \cos\theta}$:

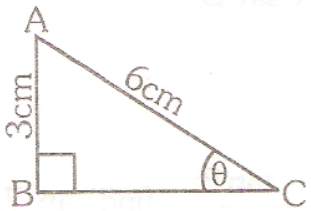
3. If A = 30^0. verify sin 2A = 2 sin A cos A :

4. Given that $\tan\theta = \dfrac{1}{\sqrt{5}}$, what is the value of $\dfrac{\text{cosec}^2 - \sec^2\theta}{\text{cosec}^2 + \sec^2\theta}$?

5. What is the maximum value of $\dfrac{1}{\sec\theta}$?

6. What is the value of θ if $\sin\theta = \cos\theta = \dfrac{\tan\theta}{\sqrt{2}}$

7. In the given fig ABC is right Δ at B such that AB = 3 cm and AC = 6 cm. Determine $\angle$ACB.



8. Evaluate : $\dfrac{2\tan 30^0}{1 - \tan^2 30^0}$

9. Evaluate : $\frac{\cos\theta}{\sin(90^0-\theta)}+\frac{\sin\theta}{\cos(90^0-\theta)}$

10. Evaluate : $\sin 25^0\cos 65^0+\cos 25^0\sin 65^0$

11. If tan A $=\frac{3}{4}$ and A + B = 90^0, then what is the value of cot B?

12. If tan A = cot B, prove that A + B = 90^0

13. Evaluate : $\frac{2\tan 80^0}{3\cot 10^0}$

14. The height of a tower is 10 m. Calculate the height of its shadow when sun's altitude is 45^0.

15. What is the angle of elevation of the sum when the length of the shadow of a pole is $\sqrt{3}$ times of the height of the pole ?

SHORT ANSWER TYPE QUESTIONS

1. If cos A = $\frac{3}{5}$. evaluate $\frac{5\sin A+3\sec A-3\tan A}{4\cot A+4\cos ecA+5\cos A}$

2. Given $\cos\theta=\frac{q}{\sqrt{p^2+q^2}}$, find $\frac{\cos ec\theta+\cot\theta}{\cos ec\theta-\cot\theta}$

3. If $\sin\alpha=\frac{4}{5}$ and $\cos\beta=\frac{12}{13}$ find $\sin\alpha\cos\beta+\cos\alpha\sin\beta$

4. If $\sec\theta=x+\frac{1}{4x}$, prove that $\sec\theta+\tan\theta=2x$ or $\frac{1}{2x}$.

5. If $\frac{1+\cos\theta}{1-\cos\theta}=\frac{16}{9}$, find $\frac{1+\cot\theta}{1-\cot\theta}$.

6. If $\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}=\frac{36}{49}$, find $\frac{\cos ec\theta-\sec\theta}{\cos ec\theta+\sec\theta}$.

7. If 2 sin θ + cos θ = 2, find sin θ

8. If $\frac{1+\cos x}{1-\cos x}=7+4\sqrt{3}$. find the value of $\frac{1+\sin x}{1-\sin x}$.

9. (i) If sin (A + B) = 1 and cos (A – B) = 1, find A and B.

(ii) If tan (2A + B) = $\sqrt{3}$ and cot (3A – B) = $\sqrt{3}$, find A and B.

10. Find x if : (i) cos (5x – 40^0) = sin 30^0. (ii) cosec (x + 30^0) = cot 45^0.

11. If A = 60^0. B = 30^0, verify each of the following :

(i) cos (A - B) cos A cos B + sin A sin B.

(ii) cos (A + B) $=\frac{\cot A\cot B-1}{\cot A+\cot B}$

12. (i) Assume that tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A\tan B}$, find tan 75^0 when A = 45^0, B = 30^0.

(ii) Assume cos (A - B) = cos A cos B + sin A sin B, find cos 15^0 when A = 45^0, B = 30^0.

13. Assume that sin (A + B) = cos A cos B + cos A sin B, if sin A = $\frac{1}{\sqrt{10}}$, sin B = $\frac{1}{\sqrt{5}}$. Then find angle (A + B)

14. Find the value of θ in each of the following if :

(i) 2 cos 3θ = 1 (ii) $2\sqrt{3}$ tanθ = 6 (iii) $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1}{2}$

15. Prove that : (i) sin 60^0 cos 30^0 + cos 60^0 sin 30^0 = sin 90^0 (ii) cos 90^0 = 4 $\cos^3$cos 30^0 – 3 cos 30^0

16. Evaluate the following :

(i) $\frac{3}{4}\cot^2 30^0 + 3\sin^2 60^0 - 2\,\mathrm{cosec}^2 60^0 - \frac{3}{4}\tan^2 30^0$

(ii) $\frac{1+\tan^2 30^0}{1-\tan^2 30^0} + \mathrm{cosec}^2 60^0 - \cos^2 45^0 + \sin^2 45^0 + \frac{1+\cot^2 60^0}{1-\cot^2 60^0}$

(iii) $\cos^2 30^0 \cos^2 45^0 + 4\sec^2 60^0 + \frac{1}{2}\cos^2 90^0 - 2\tan^2 60^0$

(iv) $4(\sin^4 30^0 + \cos^4 60^0) - 3(\cos^2 45^0 - \sin^2 90^0)$

17. Find the value of x in each of the following :

(i) $2x\tan^2 60^0 + 3x\sin^2 30^0 = \frac{27\cos^2 45^0}{4\sin^2 60^0}$.

(ii) $(x+1)(\sin^4 60^0 + \cos^4 30^0) - x(\tan^2 60^0 - \tan^2 45^0) + (x+2)\cos^2 45^0 = 1$.

(iii) $(x-4)\sin^2 60^0 + (x-5)\tan^2 30^0 - x\sin 45^0 \cos 45^0 = 0$.

(iv) $\tan x = \sin 45^0 \cos\ 45^0 + \sin 30^0$.

(v) $\sin 2x = \sin 60^0 \cos\ 30^0 - \cos 60^0 \sin 30^0$.

(vi) $\tan 3x = \sin 45^0 \cos\ 45^0 + \sin 30^0$.

18. If (sec x – 1) (sec x + 1) = 3 then find the value of x.

19. Prove : $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\,\mathrm{cosec}\theta - 2\sin\theta\cos\theta$

20. Prove : $(\tan\theta + \cot\theta + \sec)(\tan\theta + \cot\theta - \sec\theta) = \mathrm{cosec}^2\theta$.

21. Prove : $(\sec^2 A + \tan^2 A)(\mathrm{cosec}^2 A + \cot^2 A) = 1 + 2\sec^2 A\,\mathrm{cosec}^2 A$.

22. Prove : $\frac{1+\cot^2\theta}{1+\tan^2\theta} + \left(\frac{1+\cot\ \theta}{1+\tan\ \theta}\right)^2$

23. Prove : $\frac{\tan\ \theta}{(1+\tan^2\theta)^2} + \frac{\cot\ \theta}{(1+\cot^2\theta)} = \sin\theta\cos\theta$.

24. Prove : $(1-\cos\theta+\sin\theta)(1+\cos\theta+\sin\theta) = 2\sin\theta(1+\sin\theta)$

25. Prove : $\sin A(1+\tan A) + \cos A(1+\cot A) = \sec A + \mathrm{cosec}A$.

26. Prove : $\sec\theta(1+\sin\theta)(\sec\theta - \tan\theta) = 1$

27. Prove : $\sec\theta(1-\sin\theta)(\sec\theta + \tan\theta) = 1$

28. Prove : $\cos^2\theta(1+\tan^2\theta) + \sin^2\theta(1+\cot^2\theta) = 2$

29. Prove : $\frac{\cot\ \theta}{\cos^3\theta\,\mathrm{cosec}\theta + \sin\theta\cos\theta} = 1$

30. Prove : $(\sec\theta + \mathrm{cosec}\theta)(\sin\theta + \cos\theta) = \sec\theta\,\mathrm{cosec}\theta + 2$

31. Prove : $(\tan\theta + \sec\theta)^2 + (\tan\theta + \sec\theta)^2 = \frac{2(1+\sin^2\theta)}{\cos^2\theta}$

32. Prove : $\frac{(\sin\theta+\cos\theta)^2 - (\sin\theta-\cos\theta)^2}{(\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2} = 2\sin\theta\cos\theta$

33. Prove : $(1+\sin\theta+\cos\theta)^2 = 2(1+\sin\theta)(1+\cos\theta)$

34. Prove : $\sin^8\theta - cec^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

35. Prove : $\sec^6\theta - \tan^6\theta = 1 + 3\sec^2\theta\tan^2\theta$

36. Prove : $\sec^4\theta - \tan^4\theta = 2\sec^2 - 1$

37. Prove : $\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}-\frac{\cos^3\theta\sin^3\theta}{\cos\theta+\sin\theta}=2\sin\theta\cos\theta$

38. Prove : $\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}=\frac{2}{\sin^2 A-\cos^2 A}$

39. Prove : $\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=2\sec^2\theta$

40. Prove : $\frac{1}{\sec\theta-\tan\theta}+\frac{1}{\sec\theta+\tan\theta}=2\sec\theta$

41. Prove : $\frac{\cos\theta}{1+\sin\theta}+\frac{\cos\theta}{1-\sin\theta}=2\sec\theta$

42. Prove : $\frac{1}{\operatorname{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta-\cot\theta}$

43. Prove : $\frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}=2\operatorname{cosec}\theta$

44. Prove : $\frac{\sin\theta\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2}{\sin^2\theta-\cos^2\theta}=\frac{2\sec^2\theta}{\tan^2\theta-1}$

45. Prove : $\frac{\sin A-\sin B}{\cos A+\cos B}+\frac{\cos A-\cos B}{\sin\theta+\sin B}=0$

46. Prove : $\frac{1+\cot\theta}{1-\cot\theta}+\frac{1-\cot\theta}{1+\cot\theta}=\frac{2}{\sin^2\theta-\cos^2\theta}$

47. Prove : $\sqrt{\frac{\operatorname{cosec}\theta-1}{\operatorname{cosec}\theta+1}}+\sqrt{\frac{\operatorname{cosec}\theta+1}{\operatorname{cosec}\theta-1}}=2\sec\theta$

48. Prove : $\frac{\sec\theta+1-\tan\theta}{\sec\theta+1+\tan\theta}+\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=2\sec\theta$

49. Prove : $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}+=\operatorname{cosec}\theta-\cot\theta$

50. Prove : $\frac{1+\sin\theta}{1-\sin\theta}+=\frac{\cot^2\theta}{(\operatorname{cosec}\theta-1)^2}$

51. Prove : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}=\frac{\tan\theta}{\sec\theta+1}$

52. Prove : $\sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}}+=\sec\theta-\tan\theta$

53. Prove : $\frac{1-\sin\theta}{1+\sin\theta}=1+2\tan^2\theta-2\tan\theta\sec\theta$

54. Prove : $\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}=1+2\cot^2\theta+2\operatorname{cosec}\theta\cot\theta$

55. Prove : $\sqrt{\frac{\operatorname{cosec}\theta-\cot\theta}{\operatorname{cosec}\theta+\cot\theta}}=\frac{1}{\operatorname{cosec}\theta+\cot\theta}$

56. Prove : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}=\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2\operatorname{cosec}\theta$

57. Prove : $\sqrt{\frac{\text{cosec}\theta-\cot\theta}{\text{cosec}\theta+\cot\theta}}+\sqrt{\frac{\text{cosec}\theta+\cot\theta}{\text{cosec}\theta-\cot\theta}}=2\,\text{cosec}\theta$

58. Prove : $\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\sec A+\tan A=\frac{1+\sin A}{\cos A}\sqrt{\frac{1+\sin A}{1-\sin A}}$

59. Prove : $\frac{1+\cos A-\sin A}{1+\cos A+\sin A}=\sec A-\tan A=\frac{1-\sin A}{\cos A}\sqrt{\frac{1-\sin A}{1+\sin A}}$

60. Prove : $\frac{\cos A-1-\sin A}{\cos A-1+\sin A}=\text{cosec}A-\cot A=\frac{1+\cos A}{\sin A}\sqrt{\frac{1+\cos A}{1-\cos A}}$

61. What is the angle of elevation of a vertical flagstaff of height $100\sqrt{3}$ m from a point 100 m from its foot.

62. A ladder makes an angle of 60^0 with the floor and its lower end is 20 m from the wall. Find the length of the ladder.

63. The shadow of a building is 100 m long when the angle of elevation of the sun is 60^0. Find the height of the building.

64. A ladder 20 m long is placed against a vertical wall of height 10 metres. Find the distance between the foot of the ladder and the wall and also the inclination of the ladder to the horizontal.

65. What is the angle of elevation of the sun when the length of the shadow of the pole is $\frac{1}{\sqrt{3}}$ times the height of the pole ?

66. A flagstaff 6 metres high throws a shadow $2\sqrt{3}$ metres logn on the ground. Find the angle of elevation of the sun.

67. A tree $10(2+\sqrt{3})$ metres high is broken by the wind at a height $10\sqrt{3}$ metres from its root in shch a way that top struck the ground at certain angle and horizontal distance from the root of the tree to the point where the top meets the ground is 10 m. Find the angle of elevation made by the top of the tree with the ground.

68. A tree is broken at certain height and its upper part $9\sqrt{2}$ m long not completely separated meet the ground at an angle of 45^0. Find the height of the tree before it was broken and also find the distance from the root of the tree to the point where the top of the tree meets the ground.

LONG ANSWER TYPE QUESTIONS

1. The ladder resting against a vertical wall is inclined at an angle of 30^0 to the ground. The foot of the ladder is 7.5 m from the wall. Find the length of the ladder.

2. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30^0.

3. The length of a string between a kite and a point on the roof of a building 10 m high is 180 m. If the string makes an angle θ with the level ground such that $\tan\theta=\frac{4}{3}$, how high is the kite from the ground?

4. The angle of depression of a ship as seen from the top of 120 m high light house is 60^0. How far is the ship from the light house?

5. A boy 1.7 m tall, is 25 m away from a tower and observes the angle of elevation of the top of the tower to be 60^0. Find the height of the tower.

6. A man 1.8 m tall stands at a distance of 3.6 m from a lamp post and casts a shadow of 5.4 m on the ground. Find the height of the tower.

7. A straight highway leads to the foot of a tower of height 50 m. from the top of the tower, the angles of depression of two cars standing on the highway are 30^0 and 60^0. What is the distance between the two cars and how far is each car from the tower ?

8. Two points A and B are on opposite sides of a tower. The top of the tower makes an angle of 30^0 and 45^0 at A and B respectively. If the height of the tower is 40 metres, find the distance AB.

9. Two men on either side of a tower 60 metres high observe the angle of elevation of the top of the tower to be 45^0 and 60^0 respectively. Find the distance between the two men.

10. Two boats approach a light house in the middle of the sea from opposite directions. The angles of elevation of the top of the light house from two boats are α and β. If the distance between the two boats is x metres, prove that the height of the light house is

$$h = \frac{x}{\cot\alpha + \cot\beta}$$

(i) Find h if $\alpha = 60^0$, $\beta = 45^0$ and x = 250 m

(ii) Find h if $\alpha = 60^0$, $\beta = 30^0$ and x = 400 m

11. A boy standing on a horizontal plane finds a bird flying a distance of 100 m from him at an elevation of 30^0. A girl standing on the roof of 20 metres high building, finds the angle of elevation of the same bird to be 45^0. Both the boy and the girl are on opposite sides of the bird. Find the distance of the bird from the girl.

12. Two pillars of equal height stand on either side of a roadway which is 180 metres wide. The angle of elevation of the top of the pillars are 60^0 and 30^0 at a point on the roadway between the pillars. Find the height of the pillars and the position of the point.

13. Two lamp posts are 60 metres apart, and the height of the one is double that of the other. From the middle of the line joining their feet, an observer finds the angular elevation of their top to be complementary. Find the height of each lamp.

14. Two lamp posts are of equal height. A boy measured-the elevation of the top of each lamp-post from the mid-point of the line-segment joining the feet of lamp-post as 30^0. After walking 15 m towards one of them, he measured the elevation of its top at the point where he stands as 60^0. Determine the height of each lamp-post and the distance between them.

15. When the sun's altitude increases from 30^0 to 60^0, the length of the shadow of a tower decreases by 100 metres. Find the height of the tower.

16. The angle of elevation of the top of a tower from two points at distances a and b metres from the base and in the same straight lien with it are α and β respectively. Prove that the height of the tower is :

17. From the top of a church spire 96 m high, the angels of depression of two cars on a road, at the same level as the base of the spire and on the same side of it are θ and ϕ where $\tan\theta = \frac{1}{4}$ and $\tan\phi = \frac{1}{7}$ Calculate the distance between tow cars.

18. At a point on the level ground, the angle of the elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$. On waling 192 m towards the towards, the tangent of the angel is found to be $\frac{3}{4}$. Find the height of the tower.

19. AB is a straight road leading to C, the foot of a tower, A being at a distance of 120 m from C and B being 75 m nearer. It the angle of elevation of the tower at B be the double of the angle of elevation of the tower at A, find the height of the tower.

20. An aeroplane is observed at the same time by two anti-aircraft batteries distant 6000 m apart to be at elevation of 30^0 and 45^0 respectively. Assuming that the aeroplane is traveling directly towards the two batteries, find its height and its horizontal distance from the nearer battery.

21. AT.V tower stands vertically on a bank of canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60^0. From a point 20 m away from this point on the same bank, the angle of elevation of the top of the tower is 30^0. Find the height of the tower and the width of the canal.

22. A car is traveling on a straight road leading to a tower. From a point at a distance of 500 m from the tower as seen by the driver is 30^0. After driving towards the tower for 10 seconds, the angle of elevation of the top of the tower as seen by the driver is found to be 60^0. Find the speed of the car.

23. The height of a hill is 3300 metres. From a point P on the ground the angle of elevation of the top of the hill is 60^0. A balloon is moving with constant speed vertically upwards from P. After 5 minutes of its movement, a person sitting in it observes the angle of elevation of the top of the hill as 30^0. What is the speed of the balloon ?

24. A man in a boat rowing away from a light-house 100 m high, takes 2 minutes to change the angle of elevation of the top of the light-house from 60^0 to 45^0. Find speed of the boat.

25. From a point on the ground 40 m away from the foot of tower, the angle of elevation of the top of the tower is 30^0. The angle of elevation to the top of a water tank (on the top of the tower) is 45^0. Find
(i) The height of the tower (ii) The depth of the tank.

26. At a point on a level plane, a tower subtends an angle α and a man h metres high on its top an angle β. Prove that the height of the tower is : $\dfrac{h\tan\alpha}{\tan(\alpha+\beta)-\tan\alpha}$

27. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 12 metres. At a point on the plane, the angle of elevation of the bottom of the flagstaff is 45^0 and of the top of the flagstaff is 60^0. Determine the height of the tower:

28. The angles of elevation of the top and the bottom of a flagstaff fixed on a wall are 45^0 and 30^0 to a man standing on the other end of the road 20 metres wide. Find the height of the flagstaff and the height of the wall.

29. An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of the elevation of the two planes from the same point on the ground are 60^0 and 45^0 respectively. Find the vertical distance between the aeroplane at that instant.

30. An aeroplane when 6000 metres high passes vertically above another aeroplane at an instant when the angles of the elevation at the same observing point are 60^0 and 45^0 respectively. How many metres higher is the one than the other ?

31. Two aeroplane are observed to be in a vertical line. The angle of the upper plane is α and a that of the lower is β. If the height of the former be H metres, find the height of the latter plane if $\alpha = 60^0$, $\beta = 45^0$, H = 3500 m.

32. The angle of elevation of a Jet fighter from a point A on the ground is 60^0. After 10 seconds flight, the angle of the of elevation changes to 30^0. If the Jet is flying at a speed of 432 km/hour, find the height at which the jet is flying.

33. The angle of elevation of a Jet fighter from a point on the ground is 60^0. After 15 seconds flight, the angle of the of elevation changes to 30^0. If the Jet is flying at height of $1500\sqrt{3}$ m, find the speed of the Jet.

34. From the top of tower 60 metres high, the angle of depression of the top and bottom of a pole are observed to be 45^0 and 60^0 respectively. Find the height of the pole and distance of tower from the pole.

35. From the top of a building 60 metres high, the angle of depression of the top and bottom of a vertical lamp-post are observed to be 30^0 and 60^0 respectively. Find :
(i) The horizontal distance between the building and the lamp-post and
(ii) The difference between the height of the building and the lamp-post.

36. From the top of a cliff 200 metres high, the angles of depression of the top and bottom of a tower are observed to be 30^0 and 60^0. Find the height of the tower and calculate the distance between them.

37. A man on the deck of a ship is 12 m above water level. He observes that the angle of elevation, of the top of a cliff is 45^0 and the angle of depression of its base is 30^0. Calculate the distance of the cliff from the ship and the height of the cliff.

38. From a window (60 metres high above the ground) of a house in a street the angles of elevation and depression of the top and the foot of another house on opposite side of street are 60^0 and 45^0 respectively. Show that the height of the opposite house is 60 $(\sqrt{3}+1)$ metres.

39. A man on the deck of a ship, 16 m above water level, observes that the angle of elevation and depression respectively of the top and bottom of a cliff are 60^0 and 30^0. Calculate the distance of the cliff from the ship and the height of the cliff.

40. The angle of elevation of a cloud from a point 100 m above a lake is 30^0 and the angle of depression of its reflection in the lake is 60^0. Find the height of the cloud.

41. If the angle of elevation of a cloud from a point h metres above a lake be β, and the angle of depression of its reflection in the lake be α, prove that the height of the cloud is : $h\left(\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}\right)$

42. If the angle of elevation of a cloud from a point h metres above a lake be α, and the angle of depression of its reflection in the lake be β, prove that the distance (x) of the cloud from the point of observation is : $\frac{2h\sec\alpha}{\tan\beta-\tan\alpha}$. Find x if $\alpha = 30^0$, $\beta = 45^0$ and h = 250 m.

TRIGONOMETRY **ANSWER KEY** EXERCISE-2 (X)-CBSE

VERY SHORT ANSWER TYPE QUESTION :

1. $\frac{5}{\sqrt{3}}$ **2.** $\frac{5}{14}$ **4.** $\frac{2}{3}$ 5.1 **6.** 45^0 **7.** 30^0 **8.** $\sqrt{3}$ **9.** 2 **10.** 1 **11.** $\frac{4}{3}$ **13.** $\frac{2}{3}$ **14.** 10 m **15.** 30^0

SHORT ANSWER TYPE QUESTION :

1. $\frac{5}{11}$ **2.** $\frac{\sqrt{p^2+q^2}+q}{\sqrt{p^2+q^2}-q}$ **3.** $\frac{63}{65}$ **5.** $\frac{31}{17}$ **6.** $\frac{-71}{97}$ **7.** $1,\frac{3}{5}$ **8.** 3 **9. (i)** A = B 45^0 **(ii)** A = 18^0, B = 24^0

10. (i) 20^0 **(ii)** 60^0 **12. (i)** $2+\sqrt{3}$ **(ii)** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **13.** 45^0 **14. (i)** 20^0 **(ii)** 60^0 **(iii)** 30^0

16. (i) $\frac{10}{3}$ **(ii)** $\frac{16}{3}$ **(iii)** $\frac{83}{8}$ **(iv)** 2 **17. (i)** $\frac{2}{3}$, **(ii)** 3, **(iii)** 8, **(iv)** 45^0, **(v)** 15^0, **(vi)** 15^0 **18.** 60^0 **61.** 60^0 **62.** 40 m

63. 173.2 m **64.** 17.32 m, $\theta = 30^0$ **65.** 60^0 **66.** 60^0 **67.** 60^0 **68.** $9(\sqrt{2}+1)$ m , 9 m

LONG ANSWER TYPE QUESTION :

1. 8.66 m **2.** 10 m **3.** 154 m **4.** 69.28 m **5.** 45 m **6.** 3 m **7.** 86.5 m ; 57.67 m, 28.83 m **8.** 109.28 m **9.** 94.64 m
10. (i) 158.5 m **(ii)** 173.2 m **11.** 42.42 m **12.** 135 m from one end, h = 77.94 m
13. 21.21 m, 42.42 m **14.** Distance = 45m, height = 12.99 m **15.** 86.6 m **17.** 288 m **18.** 180 m **19.** 60 m
20. $3000(\sqrt{3}+1)$ m, $3000(\sqrt{3}-1)$ m, **21.** Height = 17.32 m, width = 10 m **22.** 120 km/hr **23.** 26.4 km/hr
24. 1.269 km/hr **25. (i)** 23.1 m **(ii)** 16.91 m **27.** 16.392 m **28.** 8.45 m, 11.55 m **29.** 1690.66 m, **30.** 2536 m
31. 2020.78 m **32.** 1039.2 m **33.** 720 km/hr **34.** h = 25.36 m, x = 34.64 m **35. (i)** 34.64 m, **(ii)** 20 m
36. Height = $133\frac{1}{3}$ m, Distance = 115.46 m **37.** Height = 32.784 m, Distance = 20.784 m
39. Height = 48 m, Distance = 27.71 m **40.** 200 m **42.** 1366 m

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD QUESTIONS

SHORT ANSWER TYPE 1

1. Without using tables, find the value of $14\sin 30^0 + 6\cos 60^0 = 5\tan 45^0$. **[ICSE-2004]**

2. Prove that : $\frac{\operatorname{cosec}A}{\operatorname{cosec}A-1}+\frac{\operatorname{cosec}A}{\operatorname{cosec}A+1}=2+2\tan^2 A$ **[CBSE-Al-2004C]**

3. Evaluate : $\frac{\sec\theta.\operatorname{cosec}\theta(90^0-\theta)-\tan\theta\cot(90^0-\theta)+\sin^2 55^0+\sin^2 35^0}{\tan 10^0\tan 20^0\tan 60^0\tan 70^0\tan 80^0}$ **[CBSE-Al-2004C]**

4. Without using mathematical tables, find the value of x if $\cos x = \cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$. **[ICSE-2005]**

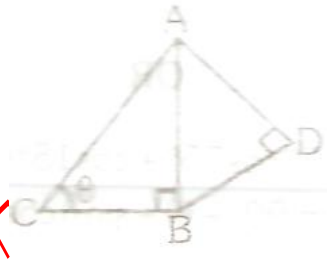
5. Without using trigonometric tables, evaluate : $\frac{2\tan 53^0}{\cot 37^0} - \frac{\cot 80^0}{\tan 10^0}$ **[ICSE-2006]**

6. Without using trigonometric tables, evaluate : $\frac{\sin 80^0}{\cos 10^0} + \sin 59^0 \sec 31^0$ **[ICSE-2007]**

7. Without using tables, evaluate : $\frac{\sin 25^0}{\sec 65^0} - \frac{\cos 25^0}{\operatorname{cosec} 65^0}$ **[ICSE-2008]**

8. Prove the $\frac{\sin A}{(1+\cos A)} = (\operatorname{cosec} A - \cot A)$ **[ICSE-2008]**

9. In the fig AD = 4 cm, BD = 3 cm and CB = 12 cm, find $\cot\theta$ **[CBSE-Delhi-2008]**



10. Without using the trigonometric tables, evaluate the following :

$\frac{11\sin 70^0}{7\cos 20^0} - \frac{4}{7}\frac{\cos 53^0 \operatorname{cosec} 37^0}{7\tan 15^0 \tan 35^0 \tan 55^0 \tan 75^0}$ **[CBSE-Delhi-2008]**

11. Without using the trigonometric tables, evaluate the following :

$\frac{\sin 18^0}{\cos 72^0} + \sqrt{3}\,[\tan 10^0 \tan 30^0 \tan 40^0 \tan 50^0 \tan 80^0]$ **[CBSE-Delhi-2008]**

12. If $\sin\theta = \cos\theta$, find the value of θ. **[CBSE-Al-2008]**

13. Without using the trigonometric tables, evaluate the following : $(\sin^2 25^0 + \sin^2 65^0) + \sqrt{3}(\tan 5^0 \tan 15^0 \tan 30^0 \tan 75^0 \tan 85^0)$. **[CBSE-Al-2008]**

14. Without using trigonometric tables, evaluate the following : $(\cos^2 25^0 + \cos^2 65^0) + \operatorname{cosec}\theta \sec(90-\theta) - \cot\theta \tan(90-\theta)$. **[CBSE-Al-2008]**

15. If $7\sin^2\theta + 3\cos^2\theta = 4$ show that $\tan\theta = \frac{1}{\sqrt{3}}$. **[CBSE-Al-2008]**

16. If $\tan A = \frac{5}{12}$, find the value of (sin A + cos A) sec A. **[CBSE-Foreign-2008]**

17. If sec 4A = cosec $(A - 20^0)$, where 4A is an acute angle, find the value of A. **[CBSE-Foreign-2008]**

OR

In a ΔABC, right angled at C, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of sin A cos B + cos A sin B.

18. If cos A = $\frac{7}{25}$, find the value of tan A + cot A. **[CBSE-Foreign-2008]**

19. If $\sin\theta = \frac{1}{3}$, find the value of $[2\cot 2\theta + 2)$ **[CBSE-Delhi-2009]**

20. Simplify : $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$ **[CBSE-Delhi-2009]**

21. If $\sec^2\theta(1+\sin\theta)(1-\sin\theta) = k$, then find the value of k. **[CBSE-Al-2009]**

22. If $\cot\theta = \frac{15}{8}$, then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

OR

Find the value of $\tan 60^0$ geometrically. **[CBSE-Al-2009]**

23. If sec A = $\frac{15}{7}$ and A + B = 90^0. find the value of cosec B. **[CBSE-Foreign-2009]**

24. Without using trigonometric tables, evaluate : $\frac{7\cos 70^0}{2\sin 20^0}+\frac{3}{2}\frac{\cos 55^0\, \text{cosec}\, 35^0}{2\tan 5^0\tan 25^0\tan 45^0\tan 85^0\tan 65^0}$ **[CBSE-Foreign-2009]**

SHORT ANSWER TYPE II

1. Evaluate : $\frac{\sec^2 54^0-\cot^2 36^0}{\text{cosec}^2 57^0-\tan^2 33^0}+2\sin^2 38^0\sec^2 52^0-\sin^2 45^0$ **[Al-2005]**

2. Prove that following : $(\tan A-\tan B)^2+(1+\tan A\tan B)^2=\sec^2 A\sec^2 B$. **[Foreign-2005]**

OR

Evaluate : $\sec^2 10^0-\cot^2 80^0+\frac{\sin 15^0\cos 75^0+\cos 15^0\sin 75^0}{\cos\theta\sin(90-\theta)+\sin\theta\cos(90-\theta)}$ **[Foreign-2005]**

3. Prove that: $\frac{\sec\theta+\tan\theta-1}{\tan\theta-\sec\theta+1}=\frac{\cos\theta}{1-\sin\theta}$.

OR

Without using trigonometric tables, evaluate the following :

$\frac{\cot(90-\theta).\sin\theta(90-\theta)}{\sin\theta}+\frac{\cot 40^0}{\tan 45^0}-(\cos^2 20^0+\cos^2 70^0)$. **[Delhi-2005C]**

4. Without using trigonometric tables, evaluate the following :

$\frac{\sec^2\theta-\cot^2(90-\theta)}{\text{cosec}^2 67^0-\tan^2 23^0}+(\sin^2 40^0+\sin^2 50^0)$. **[Al-2005C]**

5. Prove that: $(1+\tan A)^2+(1-\tan A)^2=2\sec^2 A$. **[ICSE-2005]**

6. Prove that: $\frac{\sin\theta\tan\theta}{1-\cos\theta}=1+\sec\theta$. **[ICSE-20056]**

7. Prove that: $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2\sec^2\theta}{\tan^2\theta-1}$. **[ICSE-20056]**

OR

Without using trigonometric tables : $\frac{\sec^2(90-\theta)-\cot^2\theta}{2(\sin^2 25^0+\sin^2 65^0)}+\frac{2\cos^2 60^0\tan^2 28^0\tan^2 62^0}{3(\sec^2 43^0-\cot^2 47^0)}$.

8. Prove that : $\frac{1}{\text{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\text{cosec}\theta+\cot\theta}$ **[Al-2006]**

OR

Without using trigonometric tables : $\frac{\text{cosec}^2(90-\theta)-\tan^2\theta}{4(\cos^2 48^0+\cos^2 42^0)}+\frac{2\tan^2 30^0\sec^2 52^0\sin^2 38^0}{\text{cosec}^2 70^0-\tan^2 20^0}$.

9. Without using trigonometric tables : $\frac{\sin^2\theta+\sin^2(90-\theta)}{3(\sec^2 61^0-\cot^2 29^0)}+\frac{3\cot^2 30^0\sin^2 54^0\sec^2 36^0}{2(\text{cosec}^2 65^0-\tan^2 25^0)}$. **[Foreign-2006]**

10. Without using trigonometric tables evaluate the following :

$\frac{\cos^2 20^0+\cos^2 70^0}{\sec^2 50^0-\cot^2 40^0}+2\text{cosec}^2 58^0-2\cot 58^0\tan 32^0-4\tan 13^0\tan 37^0\tan 45^0\tan 53^0\tan 77^0$

OR

Prove that : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2\text{cosec}\theta$ **[Delhi-2006C]**

11. Without using trigonometric tables evaluate the following :

(i) $\frac{\sec 39^0}{\text{cosec}\, 51^0}+\frac{2}{\sqrt{3}}\tan 17^0-\tan 38^0\tan 60^0\tan 52^0\tan 73^0-3(\sin^2 31^0+\sin^2 59^0)$ **[Al-2006C]**

(ii) $\frac{3\cos 55^0}{7\sin 35^0} - \frac{4(\cos 70^0.\operatorname{cosec} 20^0)}{7(\tan 5^0.\tan 25^0.\tan 45^0.\tan 65^0.\tan 85)}$ **[Delhi-2007]**

(iii) $\tan 7^0.\tan 23^0.\tan 60^0.\tan 67^0.\tan 83^0 + \frac{\cot 54^0}{\tan 36^0} + \sin 20^0.\sec 70^0 - 2$ **[Al-2007]**

12. Prove that : $\frac{\sin A - 1}{\sin A + 1} = \frac{1 - \cos A}{1 + \cos A}$ **[ICSE-2007]**

13. Prove that : $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

OR

Prove that : (1 + cot A – cosec A) (1 + tan A + sec A) = 2 **[CBSE (Delhi)-2008]**

14. Prove that : $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

OR

Prove that $\sin\theta(1 + \tan\theta) + \cos\theta\ (1 + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$ **[CBSE - Al-2008]**

15. Prove that : $(1 + \cot A + \tan A)(\sin A - \cos A) = \sin A \tan A - \cot A \cos tA$. **[CBSE-foreign-2008]**

OR

Without using trigonometric tables evaluate the following : $2\left[\frac{\cos 58^0}{\sin 32^0}\right] - \sqrt{3}\left[\frac{\cos 38^0 \operatorname{cosec} 52^0}{\tan 15^0 \tan 60^0 \tan 75^0}\right]$.

16. Find the value of $\sin 30^0$ geometrically.

OR

Without using trigonometric tables, evaluate : $\frac{\cos 58^0}{\sin 32^0} + \frac{\sin 22^0}{\cos 68^0} - \frac{\cos 38^0 \operatorname{cosec} 52^0}{\tan 18^0 \tan 35^0 \tan 60^0 \tan 72^0 \tan 55^0}$ **[CBSE-Delhi-2009]**

17. Evaluate : $\frac{2}{3}\operatorname{cosec}^2 58^0 - \frac{2}{3}\cot 58^0 \tan 32^0 - \frac{5}{3}\tan 13^0 \tan 37^0 \tan 45^0 \tan 53^0 \tan 77^0$. **[CBSE-Al-2009]**

18. Prove that : $\sec^2\theta - \frac{\sin^2\theta + 2\sin^4\theta}{2\cos^4\theta + \cos^2\theta} = 1$ **[CBSE-foreign-2009]**

LONG ANSWER TYPE

1. On a horizontal plane there is a vertical tower with a pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60^0 and 30^0 respectively. Find the height of the tower and flag pole mounted on it.

OR

From a building 60 metres high the angles of depression of the top and bottom of lamp-post are 30^0 and 60^0 respectively. Find the distance between lamp-post and building. Also find the difference of height between building and lamp-post. **[Delhi-2008]**

2. From the top of a cliff 92 cm high, the angle of depression of a buoy is 20^0. calculate to the nearest metre, the distance of the bucy from the foot of the cliff. **[ICSE-2005]**

3. The shadow of a vertical tower AB on level ground is increased by 10 m, when the altitude of the sun changes from 45^0 to 30^0. Find the height of the tower and give your answer correct to $\frac{1}{10}$ of a metre. **[ICSE-2006]**

4. The angle of depression of the top and the bottom of a building 50 metres high as observed from the top of a tower are 30^0 and 60^0 respectively. Find the height of the tower and also the horizontal distance between the building and the tower. **[Delhi-2006]**

OR

The angle of elevation of the top of a tower as observed from a point on the ground is $'\alpha'$ and on moving 'a' metres towards the tower, the angle of elevation is $'\beta'$ Prove that the height of the tower is $\frac{a\tan\alpha\tan\beta}{\tan\beta-\tan\alpha}$.

5. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly tower it. If it takes 12 minutes for the angle of depression to change from 30^0 to 45^0 how soon after this, will the car reach the tower? **[Al-2006C]**

6. A boy standing on a horizontal plane finds at a distance of 100 m from him at an elevation of 30^0. A girl standing on the roof of 20 metre high building, finds the angle of elevation of the same bird to be 45^0. Both the boy and the girl are on opposite sides of the bird. Find distance of bird from the girl. **[Delhi-2007]**

7. Statue 1.46 m tall, stands on the top of the pedestal . From a point on the ground, the angle of elevation of the top of the statue is 60^0 and from the same point, the angle of elevation of the top of the pedestal is 45^0. Find the height of the pedestal. (use $\sqrt{3}=1.73$) **[CBSE-Delhi-2008]**

8. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60^0. When he moves 40 m away from the bank, he finds the angle of elevation to be 30^0. Find the height of the tree and the width of the river. (use $\sqrt{3}$ = 1.732) **[CBSE-Delhi-2008]**

9. The angle of elevation of a jet fighter from a point A on the ground is 60^0. After a fight of 15 seconds, the angle of elevation changes to 30^0. If the jet is flying at a speed of 720 km/hour, find the constant height at which the jet is flying. (use $\sqrt{3}$ = 1.732) **[CBSE-Al-2008]**

10. The angle of elevation of an aeroplane from a point A on the ground is 60^0. After a flight of 30 seconds, the angel of elevation changed to 30^0. If the plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed in km/hour of the plane. **[CBSE-foreign-2008]**

11. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30^0, which is approaching the foot of the tower with a uniform speed. Six seconds later the angle of depression of the car is found to be 60^0. Find the time taken by the car to reach the foot of the tower from this point. **[CBSE-Delhi-2009]**

12. An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angels of elevation of the two planes from the same point on the ground are 30^0 and 60^0 respectively. Find the distance between the planes at that instant **[CBSE-Al-2009]**

13. A man is standing on the deck of a ship which is 25m above water level. He observes the angle of elevation of the top of a lighthouse as 60^0 and the angle of depression of the base of the light house as 45+0. Calculate the height of the lighthouse. **[CBSE-foreign-2009]**

SHORT ANSWER TYPE QUESTION-I

1 5 3. $\frac{2}{\sqrt{3}}$ **4.** 30^0 **5.** 1 **6.** 2 **7.** 1 **9.** $\frac{12}{5}$ **10.** 1 **11.** 2 **12.** 45^0 **13.** 2 **14.** 2 **16.** $\frac{17}{12}$ **17.** 22^0 or 1 **18.** $\frac{625}{168}$

19. 18 **20.** 1 **21.** 1 **22.** $\frac{625}{64}$ or $\sqrt{3}$ **23.** $\frac{15}{7}$ **24.** 5

SHORT ANSWER TYPE QUESTION-II

1. $\frac{5}{2}$ **2.** or 2 **3.** or 1 **4.** 2 **7.** or $\frac{2}{3}$ **8.** or $\left[\frac{-5}{12}\right]$ **9.** $-\frac{25}{6}$ **10.** 1 **11.** (i) 0 (ii) $\frac{-1}{7}$ (iii) $\sqrt{3}-1$ **15.** or 1

16. $\frac{1}{2}$ or $\frac{2\sqrt{3}-1}{\sqrt{3}}$ **17.** -1

LONG ANSWER TYPE QUESTION-II

1. 15.588 m, 5.196 m or 34.64 m and 20 m **2.** 253 m **3.** 13.66 m **4.** 75 m and 43.3 m **5.** 16 minutes 23 seconds
6. $30\sqrt{2}$ **7.** 2 m **8.** 34.64 m and 20 m **9.** 2598 m **10.** 864 km/hr **11.** 3 seconds **12.** 2083.33 m **13.** 68.25 m

EXERCISE – 4 (FOR OLYMPIADS)

CHOOSE THE CORRECT ONE

1. If $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2-b^2$ and $\frac{ax\sin\theta}{\cos^2\theta}-\frac{by\cos\theta}{\sin^2\theta}=0$ then $(ax)^{2/3}+(by)^{2/3}$ is equal to :

(A) $(a^2-b^2)^{2/3}$ (B) $(a^2+b^2)^{2/3}$ (C) $(a-b)^{2/3}$ (D) None of these

2. The sides of a right angled triangle form a geometric progression, find the cosines of the acute angles. (If a, b, c are in G.P. $\Rightarrow b^2=ac$) :

(A) $\frac{\sqrt{5}-1}{2}$ and $\sqrt{\frac{\sqrt{5}+1}{2}}$ (B) $\frac{\sqrt{5}+1}{2}$ and $\sqrt{\frac{\sqrt{5}+1}{2}}$

(C) $\frac{\sqrt{5}-1}{2}$ and $\sqrt{\frac{\sqrt{5}-1}{2}}$ (D) None of these

3. If $y=\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$, then $\frac{1-\cos\alpha+\sin\alpha}{1+\sin}$ is equal to :

(A) 1+ y (B) 1 – y (C) $\frac{1}{y}$ (D) None of these

4. $\cot 36^0 \cot 72^0$ is equal to :

(A) $\frac{1}{5}$ (B) $\frac{1}{\sqrt{5}}$ (C) 1 (D) None of these

5. The value of $\cos^2 15^0 - \cos^2 30^0 + \cos^2 45^0 - \cos^2 60^0 + \cos^2 75^0$ is :

(A) 2 (B) 0 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

6. If θ $x=\sin^2\theta\cos\theta$ and $y=\cos^2\theta\sin\theta$, then :

(A) $(x^2y)^{2/3}+(xy^2)^{2/3}=1$ (B) $\left[\frac{x^2}{y}\right]^{2/3}+\left[\frac{y^2}{x}\right]^{2/3}=1$

(C) $x^2 + y^2 = x^2y^2$ (D) None of these

7. If $x=\sec\theta-\tan\theta$ and $y=\operatorname{cosec}\theta+\cot\theta$, then $xy+1$ is equal to :

(A) x + y (B) x – y (C) 2x + y (D) y – x

8. If $5\sin\theta=3$, then $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$ is equal to :

(A) $\frac{1}{4}$ (B) 4 (C) 2 (D) None of these

9. The value of the expression $1-\frac{\sin^2 y}{1+\cos y}+\frac{1+\cos y}{\sin y}-\frac{\sin y}{1-\cos y}$ is equal to :

(A) cos y (B) 1 (C) 0 (D) sin y

10. If $\sec\theta = x + \frac{1}{4x}, x \in R, x \neq 0$, then the value of $\sec\theta + \tan\theta$ is :

(A) 2x (B) $\frac{1}{2x}$ (C) 2x or $\frac{1}{2x}$ (D) None of these

11. If $\tan\theta = \frac{p}{q}$, then the value of $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$ is :

(A) $\frac{p^2 - q^2}{p^2 + q^2}$ (B) $\frac{p^2 + q^2}{p^2 - q^2}$ (C) 0 (D) None of these

12. If m = $\tan\theta + \sin\theta$ and $n = \tan\theta - \sin\theta$, then $(m^2 - n^2)^2$ is equal to :

(A) mn (B) 4 mn (C) 16 mn (D) $4\sqrt{mn}$

13. If $x = \cos\theta + b\sin\theta$ and $y = a\sin\theta + \cos\theta$ then $a^2 + b^2$ is equal to :

(A) $x^2 - y^2$ (B) $x^2 + y^2$ (C) (x+ y) (D) None of these

14. If $\cos\theta + \frac{y}{b}\sin\theta + 1 = 0$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta - 1 = 0$ then $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is equal to :

(A) 2 (B) 0 (C) – 2 (D) 1

15. ABC is a triangle, right angled at A. If the length of hypotenuse is $2\sqrt{2}$ times the length of perpendicular from A on the hypotenuse, the other angles of the triangle are :

(A) $22.5^0, 67.5^0$ (B) $30^0, 60^0$ (C) $45^0, 45^0$ (D) None of these

16. If sin A + cos A = m and $\sin^3 A + \cos^3 A$ = n, then :

(A) $m^3 + 3m + 2n = 0$ (B) $m^3 - 3m + 2n =$

(C) $n^3 - 3n + 2m = 0$ (D) $m^3 - 3m + n = 0$

17. If $\sin^2\theta + 3\cos\theta - 2 = 0$, then $\cos^3\theta + \sec^3\theta$ is equal to :

(A) 18 (B) 9 (C) 4 (D) $\frac{1}{4}$

18. If $\sin\alpha + \cos\alpha = a$, then $\sin^6\alpha + \cos^6\alpha$ is equal to :

(A) $1 + \frac{3}{4}(a^2 - 1)^2$ (B) $1 - \frac{3}{4}(a^2 - 1)^2$ (C) $\frac{3 + 4(a^2 - 1)^2}{4}$ (D) $\frac{3 - 3(a^2 - 1)^2}{4}$

19. The quadratic equation whose roots are $\sin 18^0$ and $\cos 36^0$ is :

(A) $4x^2 + 2\sqrt{5}x + 1 = 0$ (B) $4x^2 - 2\sqrt{5}x - 1 = 0$

(C) $x^2 + 2\sqrt{5}x + 1 = 0$ (D) $4x^2 - 2\sqrt{5}x + 1 = 0$

20. If $\cos\theta + \sec\theta = 2$, then the value of $\cos^2\theta + \sec^2\theta$ is :

(A) 1 (B) 2 (C) 4 (D) None of these

21. If $\sin(A - B) = \cos(A + B) = \frac{1}{2}$, then the values of A and B lying between 0^0 and 90^0 are respectively:

(A) 30^0 and 60^0 (B) 60^0 and 30^0 (C) 45^0 15^0 (D) None of these

22. If $0 \leq x \leq \frac{\pi}{2}$ and $81^{\sin 2x} + 81^{\cos 2x} = 30$, then x is equal to :

(A) $\frac{\pi}{3}$ or $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ or 0 (C) $\frac{\pi}{2}$ or $\frac{\pi}{4}$ (D) None of these

23. If $m^2 + m'^2 + 2mn'\cos\theta = 1, n^2 + n'^2 + 2nn'\cos\theta = 1$, and $mn + m'n' + (mn' + m'n)\cos\theta = 0$, then $m^2 + n^2$ is equal to :

(A) $\sin^2\theta$ (B) $\text{cosec}^2\theta$ (C) $\cos^2\theta$ (D) None of these

24. If $\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos} = q$, then tan A is equal to :

(A) $\pm\frac{p}{q}\sqrt{\frac{q^2 - 1}{1 - p^2}}$ (B) $\pm\sqrt{\frac{q^2 - 1}{1 - p^2}}$ (C) $\pm\frac{p}{q}\sqrt{\frac{q^2 - 1}{1 - p^2}}$ (D) None of these

25. If $T_n = \sin^n\theta + \cos^n\theta$, then $\frac{T_3 - T_5}{T_1}$ is equal to :

(A) $\frac{T_5 - T_7}{T_3}$ (B) $\frac{T_3 - T_5}{T_7}$ (C) $\frac{T_9 - T_6}{T_4}$ (D) $\frac{T_6 - T_9}{T_4}$

26. The number of values of θ which lie between 0 and $\frac{\pi}{2}$ and satisfy the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$ is :

(A) 1 (B) 2 (C) 3 (D) None of these

27. The greatest angle of a cyclic quadrilateral is 3 times least. The circular measure of the least angle is :

(A) 60^0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) None of these

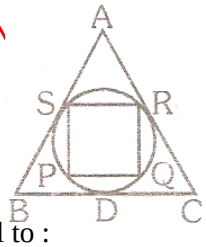
28. A circle is inscribed in an equilateral triangle of sides a, the area of any square inscribed in the circle is :

(A) $6a^2$

(B) $3a^2$

(C) $\frac{a^2}{6}$

(D) $\frac{a^2}{3}$



29. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12}x + 3\cos^8 x + \cos^6 x + 2\cos^4 x + \cos^2 x - 2$ is equal to :

(A) 0 (B) 1 (C) 2 (D) $\sin^2 x$

30. The angles of elevation of the top of a TV tower from three points A, B and C in a straight line (in the horizontal plane) through the foot of tower are α, 2α and 3α respectively. If AB = a, the height of tower is :

(A) a tan α (B) a sin α (C) a sin 2α (D) a sin 3α

31. The expression $\text{cosec}^2 A \cot^2 A - \sec^2 A \tan^2 A - (\cot^2 A - \tan^2 A)(\sec^2 A \text{cosec}^2 A - 1)$ is equal to :

(A) 0 (B) 1 (C) – 1 (D) None of these

32. $(1 + \tan\alpha\tan\beta)^2 + (\tan\alpha - \tan\beta)^2$ is equal to :

(A) $\cos^2\alpha\cos^2\beta$ (B) $\tan^2\alpha\tan^2\beta$ (C) $\tan^2\alpha + \tan^2\beta$ (D) $\sec^2\alpha\sec^2\beta$

33. From the top of a light house, 60 m high with its base at the sea level, the angle of depression of a boat is 15^0. The distance of the boat from the foot of the light house is :

(A) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ 60 m (B) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ m (C) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ 60 m (D) None of these

34. The angles of elevation of the top of a tower as observed from the bottom and top of a building of height 60 m are 60^0 and 45^0 respectively. The distance of the base of the tower from the base of the building is :

(A) $30(\sqrt{3}-1)$ m (B) $30(3+\sqrt{3})$ m (C) $30(3-\sqrt{3})$ m (D) $30(\sqrt{3}+1)$ m

35. $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$ is equal to :

(A) 0 (B) 1 (C) – 1 (D) None of these

36. If $0 < x < \frac{\pi}{2}$, then the largest angle of a triangle whose sides are 1, sin x, cos x is :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ – x (D) x

37. ABC is right angled at C, then tan A + tan B =

(A) $\frac{a^2}{bc}$ (B) $\frac{c^2}{ab}$ (C) $\frac{b^2}{ac}$ – x (D) a + b

38. A rectangle with an area of 9 square metre is inscribed in a triangle ABC having AB = 8 m, BC = 6 m and $\angle ABC = 90^0$. The dimensions of the rectangle (in metres) are :

(A) $2, \frac{9}{2}$ or $6, \frac{3}{2}$ (B) 1 , 9 or 3, 3 (C) 2, 4.5 (D) 4, 2.25

39. From the top of a light house, the angles of depression of two stations on opposite sides of it at distance 'a' apart are α and β. The height of the light house is :

(A) $\frac{a}{\cot\alpha\cot\beta}$ (B) $\frac{a}{\cot\alpha+\cot\beta}$ (C) $\frac{a\cot\alpha\cot\beta}{\cot\alpha+\cot\beta}$ (D) $\frac{a\tan\alpha\tan\beta}{\cot\alpha+\cot\beta}$

40. The value of the expression $\tan 1^0 \tan 2^0 \tan 3^0 \ldots\ldots \tan 89^0$ is equal to :

(A) 0 (B) Not defined (C) 1 (D) ∞

41. If $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$ then $\cos\theta_1+\cos\theta_2+\cos\theta_3$ is equal to :

(A) 3 (B) 2 (C) 1 (D) 0

42. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x$ is equal to :

(A) 0 (B) – 1 (C) 2 (D) 1

43. Which of the following is not possible ?

(A) $\sin\theta=\frac{5}{7}$ (B) $\cos\theta=\frac{1+t^2}{1-t^2}, t\neq 0$ (C) $\tan\theta=100$ (D) $\sec\theta=\frac{5}{2}$

44. $\cot\theta=2\sin\theta\cos\theta(0\le\theta\le 90^0)$ if θ equals :

(A) 45^0 and 90^0 (B) 45^0 and 60^0 (C) 45^0 only (D) 90^0 only

45. In a triangle ABC right angled at C, tan A and tan B satisfy the equation :

(A) $abx^2 - (a^2 + b^2)x - ab = 0$ (B) $abx^2 - c^2x + ab = 0$

(C) $c^2x^2 - abx + c^2 = 0$ (D) $ax^2 - bx + a = 0$

46. The area of the circle and the area of a regular polygon of n sides and of perimeter equal to that of the circle are in the ratio of :

(A) $\tan\left(\frac{\pi}{n}\right):\frac{\pi}{n}$ (B) $\cos\left(\frac{\pi}{n}\right).\frac{\pi}{n}$ (C) $\sin\left(\frac{\pi}{n}\right):\frac{\pi}{n}$ (D) $\cot\left(\frac{\pi}{n}\right):\frac{\pi}{n}$

47. If $\tan\theta+\sec\theta=\sqrt{3}, 0<\theta<\frac{\pi}{2}$, then θ is equal to :

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) None of these

48. A tower subtends an angle α at a point 'A' in the plane of it's base and the angle of depression of the foot of the tower at a height b just above A is B. Then the height of the tower is :

(A) $b\tan\alpha\cot\beta$ (B) $b\cot\alpha\tan\beta$ (C) $b\tan\alpha\tan\beta$ (D) $b\cot\alpha\cot\beta$

49. If $\sin x + \sin^2 x = 1$, then $\cos^2 x + \cos^4 x$ is equal to :

(A) 1 (B) – 1 (C) 2 (D) 0

50. The angle of elevation of a tower from a point A due south of it is x and from a point b due to east of A is y. if AB = ℓ, the height h of the tower is :

(A) $\frac{\ell}{\sqrt{\cot^2 y-\cot^2 x}}$ (B) $\frac{\ell}{\sqrt{\tan^2 y-\tan^2 x}}$ (C) $\ell\sqrt{\cot^2 y-\cot^2 x}$ (D) $\ell\sqrt{\tan^2 y-\tan^2 x}$

OBJECTIVE				**ANSWER KEY**							EXERCISE -4				
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	C	D	B	D	B	D	B	A	C	A	C	B	A	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	B	A	B	D	C	C	A	B	A	A	D	B	C	D	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	A	D	A	D	B	A	A	A	B	C	D	D	B	A	B
Que.	**46**	**47**	**48**	**49**	**50**										

Ans.	A	B	A	A	A	

COMPETITION WINDOW

LAW OF SINES

We use the sine rule for non-right angled triangles to find the lengths and angles. In trigonometry, the law of sines (also known as the sines law, sine formula, or sine rule) is an equation relating the lengths of the sides of an arbitrary triangle to the sines of it's angle. According to the law.

$$\boxed{\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}}$$

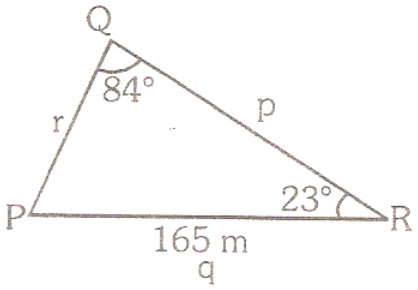
Where a, b and c are the lengths of the sides of a triangle, and A, B and C are the opposite angles. To use the sine rule, choose an appropriate pair, depending on what you know in the triangle

e.g., $\frac{a}{\sin A}=\frac{b}{\sin B}$ or $\frac{a}{\sin a}=\frac{c}{\sin C}$ or $\frac{b}{\sin B}=\frac{c}{\sin C}$

If you are finding an angle, you can invert the formulae.

e.g., $\frac{\sin A}{a}=\frac{\sin B}{b}$ or $\frac{\sin a}{a}=\frac{\sin C}{c}$ or $\frac{\sin B}{b}=\frac{\sin C}{c}$

E.g. Find the length of PQ in triangle PQR.



Use the sine rule

$$\frac{p}{\sin P}=\frac{q}{\sin Q}=\frac{r}{\sin R}$$

Use : $\frac{q}{\sin Q}=\frac{r}{\sin R}$

$$\Rightarrow \frac{165}{\sin 84^0}=\frac{r}{\sin 23^0}$$

$$\Rightarrow r=\frac{165\times \sin 23^0}{\sin 84^0}=64.8 \text{ metres}$$

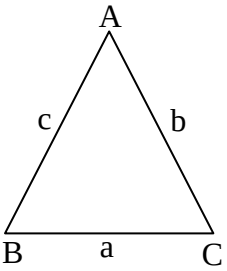
TRY OUT THE FOLLOWING

1. Find the length of BC in triangle ABC, if $\angle A=78^0, \angle C=18^0$ and AB = 26 cm. [Ans : 82.3 cm]

2. Find the length of AC in triangle ABC, if $\angle B=86^0, \angle A=74^0$ and AB = 35 cm. [Ans : 102.1 m]

RELATION TO THE CIRCUM RADIUS (R)

$$\boxed{\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R} \quad \ldots(i)$$



AREA OF A TRIANGLE

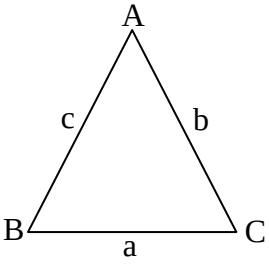
For any triangle ABC, the area is given by

$$\boxed{\Delta=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin C} \quad \ldots(ii)$$

From (i) $\sin C=\frac{c}{2R}$ …(iii)

From (ii) $\Delta=\frac{1}{2}$ ab sin C …(iv)

From (iii) and (iv) $R=\frac{abc}{4\Delta}$

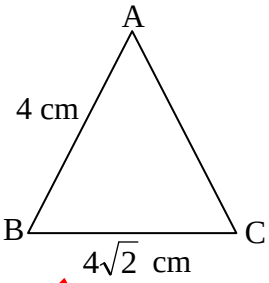


LAW OF COSINES

In any triangle ABC,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

To use the cosine rule, you need to know either two sides and the included angle or all three sides. e.g., Find the length of AC in Δ ABC



use : $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$$\cos 45^0 = \frac{(4)^2 + (4\sqrt{2})^2 - AC^2}{2 \times 4 \times 4\sqrt{2}}$$

$32 = 48 - AC^2$

AC = 4 cm

EXERCISE – 5 (FOR IIT-JEE/AIEEE)

CHOOSE THE CORRECT ONE

Based On Sine Rule (Q.No. 1-20)

1. In Δ ABC. AB = 30 cm and $\angle C = 45^0$. The length of the radius of circumcircle of Δ ABC is :

(A) $15\sqrt{2}$ cm (B) $5\sqrt{2}$ cm (C) $15\sqrt{3}$ cm (D) $5\sqrt{3}$ cm

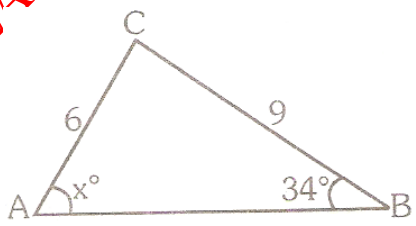
2. The radius of the circumcircle of Δ ABC is $\frac{2\sqrt{3}}{3}$ cm. If BC = 2 cm, the size of angle A is :

(A) 30^0 (B) 60^0 (C) 90^0 (D) 45^0

3. In Δ ABC, $\angle A : \angle B = 1 : 3 : 8$. If AB = 10 cm, the length of AC is : [Use : is $(180^0 - \theta) = \sin\theta$]

(A) $\frac{10\sqrt{6}}{3}$ cm (B) $\frac{10\sqrt{3}}{3}$ cm (C) $\frac{10\sqrt{3}}{6}$ cm (D) None of these

4. The measure of angle x in the triangle below is :



(A) 54^0 (B) 57.01^0 (C) 59^0 (D) None of these

5. In a circle of radius 7 cm, the are AB subtends an angle of 120^0 at the centre. The length of chord AB is :

(A) $7\sqrt{3}$ cm (B) $3\sqrt{2}$ cm (C) $5\sqrt{3}$ cm (D) $2\sqrt{3}$ cm

6. In a triangle ABC, a = 6, b = 12 and $B = 60^0$. The value of sin A is ;

(A) $\frac{\sqrt{3}}{4}$ cm (B) $\frac{1}{\sqrt{3}}$ cm (C) $\frac{1}{2}$ cm (D) None of these

7. In Δ ABC. a = 2, b = 3 and $\sin A = \frac{2}{3}$, then $\angle B$ is equal to :

(A) 30^0 (B) 60^0 (C) 90^0 (D) 120^0

8. In Δ ABC. a = 4, c = 12 and $\angle C = 60^0$, then the value of sin A is :

(A) $\frac{1}{2\sqrt{3}}$ cm (B) $\frac{-1}{2\sqrt{3}}$ cm (C) $\frac{\sqrt{2}}{3}$ cm (D) $\frac{\sqrt{3}}{2}$

9. In an isosceles triangle ABC, the base AB = 12 cm and the angle at the top is 30^0. D is a point on the side BC such that $\angle CAD : \angle DAB = 1 : 4$. The length of the radius of circumcircle of ΔABC is :

(A) $3\sqrt{2}$ cm (B) $5\sqrt{2}$ cm (C) $6\sqrt{2}$ cm (D) $10\sqrt{2}$ cm

10. The base of an isosceles triangle is 10 cm, and the angle at the base is 2a. the length of the angle bisector of one of the base angles is : [Use : $\sin(180^0 - \theta) = \sin\theta$]

(A) 10 sin 2a cos 2a (B) $\frac{10\sin 2a}{\sin 3a}$ (C) $\frac{10\sin 3a}{\sin 2a}$ (D) 1 0sin 4a

11. In the circumference with radius 50 cm is inscribed a quadrilateral. Two of its angles are 45^0 and 120^0. The length of diagonals is :

(A) $25\sqrt{2}$ cm; $25\sqrt{3}$ cm (B) $10\sqrt{2}$ cm; $10\sqrt{3}$ cm

(C) $50\sqrt{2}$ cm; $50\sqrt{3}$ cm (D) None of these

12. In ΔABC, $\angle A = 45^0$, $\angle B = 30^0$. M is a point on the side AB. The radius of the circumcircle of ΔAMC is R. The radius of the circumcircle of ΔMBC is :

(A) 2R cm (B) $R\sqrt{2}$ cm (C) $\frac{R}{\sqrt{2}}$ cm (D) None of these

13. The angle of a triangle are as 5 : 5 : 2, the ratio of the greatest side to the least side is :

(A) $2 + \sqrt{3} : 1$ (B) $2 + \sqrt{3} : 2 - \sqrt{3}$ (C) $\sqrt{3} - 1 : \sqrt{3} + 1$ (D) None of these

14. The perimeter of an acute angled triangle ABC is 6 times the arithmetic mean of the sines of its angles. If the side b is 2, the angle B is :

(A) 30^0 (B) 60^0 (C) 90^0 (D) None of these

15. If the angles of a triangle be in the ratio 1 : 4 : 5, then the ratio of it's greatest side to the smallest side is :

(A) 5 : 1 (B) $(\sqrt{5}+1) : 1$ (C) $1 : (\sqrt{5}-1)$ (D) None of these

16. In a ΔABC, if a sin A = b sin B, then the triangle is :

(A) Right angled (B) Equilateral (C) Right angled isosceles (D) Isosceles

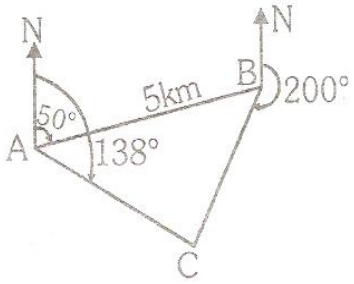
17. Points D, E are taken on the side BC of a triangle ABC such that BD = DE = EC. If $\angle BAD = x$, $\angle DAE = y$, $\angle EAC = z$, then the value of $\frac{\sin(x+y)\sin(y+z)}{\sin x \sin z}$ is equal to :

(A) 1 (B) 2 (C) 4b (D) None of these

18. In a triangle ABC, A = 45^0, B = 75^0, then $a + \sqrt{2}$ c is equal to :

(A) 2b (B) b (C) 4b (D) $\frac{b}{2}$

19. A hiker starts her journey at point A. She notices a farm house at point C and works out its bearing is at 138^0. She then walks for 5 kilometers and stops at point B. At point B the hiker looks again at the farm house and calculates its bearing now to be 200^0. The distance AC and BC respectively are :



(A) 3.28 km, 6.55 km (B) 2.66 km, 5.83 km

(C) 2.83 km, 5.66 km (D) None of these

20. The angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the largest side to the perimeter is (Use : sin $(180^0 - \theta) = \sin\theta$)

(A) $1 : (1+\sqrt{3})$ (B) $2 : 3$ (C) $\sqrt{3} : (2+\sqrt{3})$ (D) $1 : (2+\sqrt{3})$

Based On Cosine Rule (Q. No. 21-37)

21. In ΔABC, AB = 5 cm, AC = 6 cm, $\angle A = 60^0$. The length of the side BC is :
(A) $\sqrt{31}$ cm (B) $\sqrt{29}$ cm (C) 31 cm (D) 29 cm

22. Which of the following options contains the sides of a right angled triangle ?
(A) 13, 14, 15 (B) 12, 35, 37 (C) 13, 15, 24 (D) None of these

23. The size of $\angle C$ of ΔABC, if a = $2\sqrt{3}$ cm, b = 3 cm, c = $\sqrt{3}$ cm is :
(A) 90^0 (B) 60^0 (C) 30^0 (D) None of these

24. The size of $\angle C$ of ΔABC, if a = 11 cm, b = 60 cm, c = 61 cm is :
(A) 90^0 (B) 60^0 (C) 30^0 (D) None of these

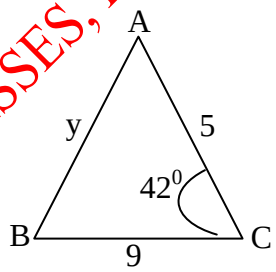
25. In ΔABC we have AC = 3 cm, BC = $\sqrt{5}$ cm, $\angle A = 45^0$. The length of the side AB is :
(A) $\sqrt{3}$ cm (B) $3\sqrt{3}$ cm (C) $\sqrt{2}$ cm or $2\sqrt{2}$ cm (D) $\sqrt{3}$ cm or $3\sqrt{3}$ cm

26. The length of a diagonal of a rectangle is 32 cm, and the angle between the diagonals is 135^0. The length of the sides of rectangle are :
(A) $4\sqrt{3-\sqrt{3}}$ cm and $4\sqrt{3+\sqrt{3}}$ cm (B) $16\sqrt{2-\sqrt{2}}$ cm and $16\sqrt{2+\sqrt{2}}$ cm
(C) 4 cm and 16 cm (D) None of these

27. The in centre of a right angled triangle is at distance $\sqrt{5}$ and $\sqrt{10}$ from the two ends of the hypotenuse. The length of the hypotenuse is :
(A) 5 cm (B) 10 cm (C) 15 cm (D) 7.5 cm

28. The in centre of ΔABC is at distance 7 and $3\sqrt{3}$ from the point A and B. If the angle at point C is 120^0, the length of the side AB is :
(A) $\sqrt{139}$ cm (B) $\sqrt{129}$ cm (C) $\sqrt{119}$ cm (D) None of these

29. Calculate the length y of the side in the triangle below :



(A) 5.25 (B) 4 (C) 6.25 (D) None of these

30. A ship sails from harbor and travels 25 km on a bearing of 300 before reaching a marker buoy. At this point the ship turns and follows a course on a bearing of 900 and travels for 32 km until it reaches an island. On the return journey, the ship is able to take the most direct route back to the harbor. The total distance travelled by the ship is :
(A) 105 km (B) 95 km (C) 112 km (D) 130 km

31. If the angles of a triangle ABC are in AP, then :
(A) $c^2 = a^2 + b^2 + ab$ (B) $a^2 + c^2 - ac = b^2$
(C) $c^2 = a^2 + b^2$ (D) None of these

32. If a = 4, b = 3 and $A = 60^0$, then c is a root of the equation :
(A) $x^2 - 3x - 7$ (B) $x^2 + 3x + 7 = 0$
(C) $x^2 - 3x + 7$ (D) $x^2 + 3x - 7 = 0$

33. If p1, p2, p3 are the altitudes of a triangle from the vertices A, B C and Δ, the area of the triangle, $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}$ = $\frac{ab(1+k)}{\Delta(a+b+c)}$, then k is equal to :

(A) cos C (B) cos A (C) cos B (D) None of these

34. In a ΔABC, 2ac $\sin\frac{A-B+C}{2}$ is equal to :

(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$
(C) $b^2 - c^2 - a^2$ (D) $c^2 - a^2 - b^2$

35. In a triangle the length of two larger sides are 10 and 9 respectively. If the angles are in A. P., then the third side can be : **[DCE-2001]**

(A) $5\pm\sqrt{6}$ (B) $5-\sqrt{6}$ (C) $3\sqrt{3}$ (D) 5

36. In a ΔABC if b = 20, c = 21 and sin A = $\frac{3}{5}$, then a = **[EAMCET-2003]**

(A) 12 (B) 13 (C) 14 (D) 15

37. In a ΔABC, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then cos C = **[Karnataka-CET-2003]**

(A) $\frac{5}{7}$ (B) $\frac{7}{5}$ (C) $\frac{16}{17}$ (D) $\frac{17}{36}$

Mixed Applications of Sine & Cosine Rule (Q.No. 38-41)

38. The sides of a triangle are $\sqrt{3}+1$ and $\sqrt{3}-1$ and the included angle is 60^0. The difference of the remaining angles is :

(A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

39. If two sides of a triangle and the included angle are given by a = $(1+\sqrt{3})$ cm, b = 2 cm, c = 60^0, the other two angles are :

(A) 90^0, 30^0 (B) 75^0, 45^0 (C) 60^0, 60^0 (D) None of these

40. In the previous Q., the third side is ;

(A) $\sqrt{6}$ cm (B) 6 cm (C) 9 cm (D) None of these

41. If $b^2 + c^2 = 3a^2$, then cot B + cot C – cot A =

(A) 1 (B) $\frac{ab}{4\Delta}$ (C) 0 (D) $\frac{ac}{4\Delta}$

Based On Area Of Triangle (Q.No. 42-46)

42. In a triangle ABC, B = 45^0, a = 2($\sqrt{3}+1$) and area of ΔABC = 6 + $2\sqrt{3}$ square units, then the side b is equal to

(A) $\frac{\sqrt{3}+1}{\sqrt{2}}$ (B) 4 (C) $\sqrt{2}(\sqrt{3}+1)$ (D) None of these

43. In any ΔABC, the expression $\frac{(a+b+c)(c+b-a)(c+a-b)(a+b-c)}{4b^2c^2}$ is equal to :

(A) $\cos^2 A$ (B) $\sin^2 A$ (C) 1 – cosA (D) 1 + cos A

44. In any ΔABC, the expression (a + b +c) (a + b – c) (b + c – a) (c + a – b) is equal to :

(A) 16Δ (B) $4\Delta^2$ (C) 4Δ (D) None of these

45. If x,y,z are perpendiculars drawn from the vertices of a triangle having sides a, b and c, then $\frac{bx}{c}+\frac{cy}{a}+\frac{az}{b}$ =

(A) $\frac{a^2+b^2+c^2}{2R}$ (B) $\frac{a^2+b^2+c^2}{R}$ (C) $\frac{a^2+b^2+c^2}{4R}$ (D) $\frac{2(a^2+b^2+c^2)}{R}$

46. In an equilateral triangle of each side $2\sqrt{3}$ cm, the radius of the circumcircle is :

(A) 2 cm (B) 1 cm (C) $\sqrt{3}$ cm (D) $2\sqrt{3}$ cm

47. A pole stands vertically inside a triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in Δ ABC, the foot of the pole is at the : **[IIT-2001]**

(A) Centroid (B) Circumcentre (C) In centre (D) Orthocenter

48. A man from the top of a 100 m high tower sees a car moving towards the tower at an angle of depression of 30^0. After some time, the angle of depression becomes 60^0. the distance (in metres) traveled by the car during this time is : **[II'T-Screening-2001]**

(A) 100 $\sqrt{3}$ (B) $\frac{200\sqrt{3}}{3}$ (C) $\frac{100\sqrt{3}}{3}$ (D) $200\sqrt{3}$

49. The value of k for which $(\cos x + \sin x)^2 + k$ in sin x cos x – 1 = 0 is an identity is : **[Kerala Engineering-2001]**

(A) – 1 (B) – 2 (C) 0 (D) 1

50. Which of the following pieces of does not uniquely determine an acute angled triangle ABC (R beign the radius of the circumcircle)?

(A) a, sinA, sinB (B) a, b, c (C) a, sinB, R (D) a, sinA, R.

51. The value of $\frac{1-\tan^2 15^0}{1+\tan^2 15^0} =$ **[II' Screening-2002]**

(A) a, sinA, sinB (B) a, b, c (C) a, sinB, R (D) a, sinA, R.

52. $\cos^2\frac{\pi}{12}+\cos^2\frac{\pi}{4}+\cos^2\frac{5\pi}{12}$ is equal to : **[Karnataka-CET-2002]**

(A) $\frac{2}{3+\sqrt{3}}$ (B) $\frac{2}{3}$ (C) $\frac{3+\sqrt{3}}{2}$ (D) $\frac{2}{3}$

53. If tanA + cotA = 4, then $\tan^4 A + \cot^4 A$ is equal to : **[Kerala Engineering-2002]**

(A) 110 (B) 191 (C) 80 (D) 194

54. If $\tan\theta+\sec\theta=e^x$, then $\cos\theta$ equals : **[AMU-2002]**

(A) $\frac{e^x+e^{-x}}{2}$ (B) $\frac{2}{e^x+e^{-x}}$ (C) $\frac{e^x-e^{-x}}{2}$ (D) $\frac{e^x-e^{-x}}{e^x+e^{-x}}$

55. In a Δ ABC, if a2 + b2 + c2 – ab – bc – ca = 0, then $\sin^2 A + \sin^2 B + \sin^2 C =$ **[Karnataka-CET-2003]**

(A) $\frac{4}{9}$ (B) $\frac{9}{4}$ (C) $3\sqrt{3}$ (D) 1

56. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the triangle ABC is : **[AIEEE-2003]**

(A) $\frac{64}{3}$ (B) $\frac{8}{3}$ (C) $\frac{32}{3}$ (D) $\frac{32}{3\sqrt{3}}$

57. The upper $\left(\frac{3}{4}\right)$ the portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through it's foot and at a distance 40 m from the foot. A possible height of the vertical pole is:

[Hint : Use the formula $\tan(\theta+\alpha)=\frac{\tan\theta+\tan\alpha}{1-\tan\alpha+\tan\theta}$] **[AIEEE-2003]**

(A) 60 m (B) 20 m (C) 40 m (D) 80 m

58. If θ and ϕ are acute angles, $\sin\theta=\frac{1}{2}.\cos\phi=\frac{1}{3}$, then the value of θ ϕ lies in : **[IIT-Screening-2004]**

(A) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{3},\frac{2\pi}{3}\right)$ (C) $\left(\frac{2\pi}{3},\frac{5\pi}{6}\right)$ (D) $\left(\frac{5\pi}{6},\pi\right)$

59. The sides of a triangle are in the ratio 1 : $\sqrt{3}$: 2, the angles of the triangle are in the ratio :

(A) 1 : 3 : 5 (B) 2 : 3 : 4 **[IIT-Screening-2004]**

(C) 3 : 2 : 1 (D) 1 : 2 : 3

60. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60^0 and when he retires 40 metres away from the tree the angle of elevation becomes 30^0. The breadth of the river is : **[AIEEE-2004]**

(A) 20 m (B) 30 m (C) 40 m (D) 60 m

61. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 300 and tan 150, then the value of $2 + q - p$ is

(A) 1 (B) 2 **[AIEEE-2006]**

(C) 3 (D) 0

62. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60^0 at foot of the tower, and the angle of elevation of the top of the tower from A or B is 30^0. The height of the tower is : **[AIEEE-2007]**

(A) $\frac{2a}{\sqrt{3}}$ (B) $2a\sqrt{3}$ (C) $\frac{a}{\sqrt{3}}$ (D) $a\sqrt{3}$

63. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angles of elevation of the point A from a certain point C on the ground is 60^0. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45^0. then the height of the pole is :

(A) $\frac{7\sqrt{3}}{2(\sqrt{3}-1)}$ m (B) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m **[AIEEE-2008]**

(C) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m (D) $\frac{7\sqrt{3}}{2(\sqrt{3}+1)}$ m

OBJECTIVE				ANSWER KEY							EXERCISE -5				
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	B	A	B	A	A	C	A	C	B	C	B	A	C	B
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	D	A	A	C	C	A	B	C	A	C	B	A	A	C	B
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**

Ans.	B	A	A	B	A	B	A	D	B	B	C	B	B	D	A
Que.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	A	B	B	B	D	C	D	D	B	B	D	C	B	D	A
Que.	**61**	**62**	**63**												
Ans.	C	C	B												

Sin x^0

Degree	0’ $0^0.0$	6’ $0^0.1$	12, $0^0.2$	18’ $0^0.3$	24’ $0^0.4$	30, $0^0.5$	36’ $0^0.6$	42’ $0^0.7$	48, $0^0.8$	54’ $0^0.9$	MEAN DEFFERENCES				
											1	2	3	4	5
.0	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
.1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
.2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
.3	0523	0541	0558	0576	0593	0610	0628	1645	0663	0680	3	6	9	12	15
.4	0698	0715	0732	0750	0767	0785	0802	1819	2837	0854	3	6	9	12	15
.5	0872	0889	0906	0924	0941	0958	0976	1993	1011	1028	3	6	9	12	14
.6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
.7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
.8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
.9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
.10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
.11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
.12	2079	2096	2113	2300	2147	2164	2181	2198	2215	2233	3	6	9	11	14
.13	2250	2267	2284	2130	2317	2334	2351	2368	2385	2402	3	6	8	11	14
.14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
.15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
.16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
.17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
.18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
.19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
.20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
.21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
.22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
.23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
.24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
.25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
.26	4384	4399	4415	4431	4436	4462	4478	4493	4509	4524	3	5	8	10	13

.27	4540	4555	4571	4586	4602	4617	4433	4648	4664	4679	3	5	8	10	13
.28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
.29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
.30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	5	8	10	13
.31	5150	5165	5080	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
.32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
.33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
.34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
.35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
.36	5878	5892	5906	5920	5934	5948	5962	5976	5850	6004	2	5	7	9	12
.37	6018	6032	6046	6060	6074	6088	6101	6115	5990	6143	2	5	7	9	12
.38	6157	6170	6184	6198	6211	6225	6239	6252	6129	6280	2	5	7	9	12
.39	6293	6307	6320	6334	6347	6361	6374	6388	6266	6414	2	4	7	9	11
.40	6428	6441	6455	6468	6481	6494	6508	6521	6401	6547	2	4	7	9	11
.41	6561	6574	6587	6600	6613	6626	6639	6652	6534	6678	2	4	7	9	11
.42	6691	6704	6717	6730	6743	6756	6769	6782	6665	6807	2	4	6	9	11
.43	6820	6833	6845	6858	6871	6884	6896	6909	6794	6934	2	4	6	8	11
.44	6947	6959	6972	6984	6997	7009	7022	7034	6921	7059	2	4	6	8	10
.45	7071	7083	7096	7108	7120	7133	7145	7157	7046	7181	2	4	6	8	10

Sin x^0

Degree	0’	6’	12,	18’	24’	30,	36’	42’	48,	54’	MEAN DEFFERENCES				
	$0^0.0$	$0^0.1$	$0^0.2$	$0^0.3$	$0^0.4$	$0^0.5$	$0^0.6$	$0^0.7$	$0^0.8$	$0^0.9$	1	2	3	4	5

.46	7193	7206	7218	7230	7242	7256	7266	7278	7290	7302	2	4	6	8	10
.47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
.48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
.49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
.50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
.51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
.52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
.53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
.54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
.55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
.56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
.57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
.58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
.59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
.60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
.61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
.62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
.63	8910	8318	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
.64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
.65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
.66	9135	9193	9150	9157	9194	9171	9178	9184	9191	9198	1	2	3	5	6
.67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
.68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
.69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
.70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
.71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
.72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
.73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
.74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
.75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
.76	9403	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
.77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
.78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
.79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
.80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
.81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
.82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
.83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
.84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
.85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
.86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
.87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
.88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
.89	9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0

Degree	0’	6’	12,	18’	24’	30,	36’	42’	48,	54’	MEAN EFFERENCES				
	$0^0.0$	$0^0.1$	$0^0.2$	$0^0.3$	$0^0.4$	$0^0.5$	$0^0.6$	$0^0.7$	$0^0.8$	$0^0.9$	1	2	3	4	5

.0	1.000	1.000	1.000	1.000	1.000	1.000	9999	9999	9999	9999	0	0	0	0	0
.1	.9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
.2	.9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
.3	.9986	9985	9984	9983	9982	9981	9980	9966	9978	9977	0	0	1	1	1
.4	.9976	9974	9973	9972	9971	9969	9968	9979	9965	9963	0	0	1	1	1
.5	.9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
.6	.9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
.7	.9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
.8	.9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
.9	.9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
.10	.9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
.11	.9816	9813	9810	9806	9805	9799	9796	9792	9789	9785	1	1	2	2	3
.12	.9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
.13	.9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
.14	.9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
.15	.9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
.16	.9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
.17	.9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
.18	.9511	9505	9500	9494	9489	9583	9478	9472	9466	9461	1	2	3	4	5
.19	.9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
.20	.9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
.21	.9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
.22	.9272	9265	5259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
.23	.9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
.24	.9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	3	5	6
.25	.9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
.26	.8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
.27	.8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
.28	.8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
.29	.8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
.30	.8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
.31	.8575	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
.32	.8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
.33	.8387	8377	8368	8358	8343	8339	8329	8320	8310	8300	2	3	5	6	8
.34	.8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
.35	.8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
.36	.8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
.37	.7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
.38	.7880	7859	7869	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
.39	.7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
.40	.7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
.41	.7547	7536	7424	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
.42	.7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
.43	.7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
.44	.7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10
.45	.7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10

Degree	0’ $0^0.0$	6’ $0^0.1$	12, $0^0.2$	18’ $0^0.3$	24’ $0^0.4$	30, $0^0.5$	36’ $0^0.6$	42’ $0^0.7$	48, $0^0.8$	54’ $0^0.9$	MEAN EFFERENCES				
											1	2	3	4	5
.46	.6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11

.47	.6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
.48	.6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
.49	.6428	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
.50	.6428	6414	6401	3688	6374	6361	6347	6334	6320	6307	2	4	7	9	11
.51	.9293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
.52	.6157	6124	6129	6115	6101	6088	6074	6060	6064	6032	2	5	7	9	12
.53	.6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
.54	.5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
.55	.5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
.56	.5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
.57	.5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
.58	.5299	5284	5270	5255	5240	5225	5210	5135	5180	5165	2	5	7	10	12
.59	.5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	12
.60	.5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
.61	.4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
.62	.4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
.63	.4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
.64	.4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
.65	.4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
.66	.4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
.67	.3907	3891	3875	3859	3843	3887	3811	3795	3778	3762	3	5	8	11	14
.68	.3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
.69	.3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
.70	.3420	3404	3387	3371	3335	3338	3322	3305	3289	3272	3	5	8	11	14
.71	.3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
.72	.3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
.73	.2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
.74	.2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
.75	.2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
.76	.2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
.77	.2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
.78	.2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
.79	.1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
.80	.1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
.81	.1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
.82	.1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
.83	.1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
.84	.1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
.85	.0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	14
.86	.0698	0680	0663	0645	0623	0610	0593	0576	0558	0541	3	6	9	12	15
.87	.0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
.88	.0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
.89	.0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15

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Degree	0’ $0^0.0$	6’ $0^0.1$	12, $0^0.2$	18’ $0^0.3$	24’ $0^0.4$	30, $0^0.5$	36’ $0^0.6$	42’ $0^0.7$	48, $0^0.8$	54’ $0^0.9$	MEAN DEFFERENCES				
											1	2	3	4	5
.0	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
.1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
.2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
.3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
.4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
.5	0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
.6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
.7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
.8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
.9	1584	1602	1620	1638	1635	1673	1691	1709	1727	1745	3	6	9	12	15
.10	1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
.11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
.12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
.13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
.14	2493	2512	2530	2549	2568	2586	2605	2623	6242	2661	3	6	9	12	16
.15	2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
.16	2867	2886	1905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
.17	3057	3076	3096	3115	3134	3253	3172	3191	3211	3230	3	6	10	13	16
.18	3249	3269	3288	3307	3327	3541	3365	3385	3404	3424	3	6	10	13	16
.19	3443	3463	3482	3502	3522	3739	3561	3581	3600	3620	3	7	10	13	16
.20	3640	3659	3679	3699	3719	4142	3759	3779	3799	3819	3	7	10	13	17
.21	3839	3859	3869	3899	3919	4348	3959	3979	4000	4020	3	7	10	13	17
.22	4040	4061	4081	4101	4122	4557	4163	4183	4204	4224	3	7	10	14	17
.23	4245	4265	4286	4307	4327	4770	4369	4390	4411	4431	3	7	10	14	17
.24	4452	4473	4494	4515	4536	4986	4578	4599	4621	4642	4	7	11	14	18
.25	4663	4684	4706	4727	4748	5206	4791	4813	4834	4856	4	7	11	14	18
.26	4877	4809	4921	4942	4962	5430	5008	5029	5051	5073	4	7	11	15	18
.27	5095	5117	5139	5161	5184	5658	5228	5250	5272	5295	4	7	11	15	18
.28	5317	5340	5362	5384	5407	5890	5452	5475	5498	5520	4	8	11	15	19
.29	5543	5566	5589	5612	5635	6128	5681	5704	5727	5750	4	8	12	15	19
.30	5774	5797	5820	5844	5867	6371	5914	5938	5961	5985	4	8	12	16	20
.31	6009	6032	6056	6080	6104	6619	6152	6176	6200	6224	4	8	12	16	20
.32	6249	6273	6297	6322	6346	6873	6395	6420	6445	6469	4	8	12	16	20
.33	6494	6519	6544	6569	6594	7133	6644	6669	6694	6720	4	8	13	17	21
.34	6745	6771	6796	6822	6874	7400	6899	6924	6950	6976	4	9	13	17	21
.35	7002	7028	7054	7080	7107	7673	7159	7186	7212	7239	4	9	13	18	22
.36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
.37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
.38	7513	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
.39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
.40	8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
.41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
.42	9004	9036	9067	9099	9131	9163	9195	9228	6260	9293	5	11	16	21	27
.43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
.44	9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29
.45	1.000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30

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Degree	0’ $0^0.0$	6’ $0^0.1$	12, $0^0.2$	18’ $0^0.3$	24’ $0^0.4$	30, $0^0.5$	36’ $0^0.6$	42’ $0^0.7$	48, $0^0.8$	54’ $0^0.9$	MEAN EFFERENCES				
											1	2	3	4	5
.46	1.0355	0392	0428	0464	0501	0838	0575	0612	0649	0686	6	12	18	25	31
.47	1.0724	0761	0299	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
.48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
.49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
.50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
.51	1.2349	2393	2437	2484	2527	2572	2617	2662	2708	2753	8	15	23	30	38
.52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
.53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
.54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
.55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
.56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
.57	1.5399	5458	4938	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
.58	1.6003	6066	5517	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
.59	1.6643	6709	6128	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
.60	1.7321	7391	6775	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
.61	1.8040	8115	7461	8265	8341	8418	8405	8572	8650	8728	13	26	38	51	64
.62	1.8807	8887	8190	9047	9128	9210	9262	9375	9458	9542	14	27	41	55	68
.63	1.9626	9711	8967	9883	9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
.64	2.0503	0594	9797	0778	0872	0965	1060	1155	1155	1348	16	31	47	63	78
.65	2.1445	1543	0686	1742	1842	1943	2045	2148	2148	2355	17	34	51	68	85
.66	2.2460	2566	1642	2781	2889	2998	3109	3220	3220	3445	18	37	55	73	92
.67	2.3559	3673	2673	3906	4023	4142	4262	4383	4383	4627	20	40	60	79	99
.68	2.4751	4876	3789	5129	5257	5386	5517	5649	5649	5916	22	43	65	87	108
.69	2.6051	6187	5002	6464	6605	6746	6889	7034	7034	7326	24	47	71	95	119
.70	2.7475	7625	6325	7929	8083	8239	8397	8556	8556	8878	26	52	78	104	133
.71	2.9042	9208	7776	9544	9714	9887	3.0061	3.0237	3.0237	3.0595	29	58	87	116	145
.72	3.0777	0961	9375	1334	1524	1716	1910	2106	2106	2506	32	64	96	129	161
.73	.02709	2914	1146	3332	3544	3759	3977	4197	4197	4646	36	72	108	144	180
.74	3.4874	5105	3122	5576	5816	6059	6305	6554	6554	7062	41	81	122	163	204
.75	3.7321	7583	5339	8118	8391	8667	8947	9232	9232	9812	46	93	139	186	232
.76	4.0108	0408	7848	1022	1335	1653	1976	2303	2303	2972	53	107	160	213	267
.77	4.3315	3662	0713	4374	4737	5107	5483	5864	5864	6646					
.78	4.7046	7453	4015	8288	8716	9152	9594	5.0045	5.0045	5.0970					
.79	5.1446	1923	7867	2924	3435	3955	4486	5026	5026	6140					
.80	5.6713	7297	2422	8502	9124	9758	6.0405	6.1066	6.1066	6.2432					
.81	6.3138	3859	7894	5350	6122	6912	7720	8548	8548	7.0264					
.82	7.1154	2066	4596	3962	4947	5958	6996	8062	8062	8.0285					
.83	8.1443	2636	3002	5126	6427	7769	9152	9.0579	9.0579	9.3572					
.84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.78	11.20					
.85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.30	13.95					
.86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.34	18.46					
.87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	24.90	27.27					
.88	28.64	30.14	31.82	33.69	35.80	38.19	40.62	44.07	44.07	52.08					
.89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	191.0	573.0					

Important Notes

SURFACE AREAS AND VOLUMES

★ INTRODUCTION

In this chapter, shall discuss problems on conversion of one of the solids like cuboid, cube, right circular cylinder, right circular cone and sphere in another.

In our day-to-day life we come across various solids which are combinations of two or more such solids, For example, a conical circus tent with cylindrical base is a combination of a right circular cylinder and a right circular cone, also an ice-cream cone is a combination of a cone and a hemi-sphere. We shall discuss problems on finding surface areas and volumes of such solids. We also come across solids which are a part of a cone. For example, a bucket , a glass tumbler, a friction clutch etc. these solids are known as frustums of a cone. In the end of the chapter, we shall discuss problems on surface area and volume of frustum of a cone.

★ UNITS OF MEASUREMENT OF AREA AND VOLUME

The inter-relationships between various units of measurement of length, area and volume are listed below for ready reference:

LENGTH

1 Centimetre (cm)	=	10 milimetre (mm)
1 Decimetre (dm)	=	10 centimetre
1 Metre (m)	=	10 dm = 100 cm = 1000mm
1 Decametre (dam)	=	10 m = 1000 cm
1 Hectometre (hm)	=	10 dam = 100 m
1 Kilometre (km)	=	1000 m = dam = 10 hm
1 Myriametre	=	10 kilocetre

AREA

$1\ cm^2 = 1\ cm \times 1\ cm = 10\ mm \times 10\ mm = 100\ mm^2$
$1\ dm^2 = 1\ dm \times 1\ dm = 10\ cm \times 10\ cm = 100\ cm^2$
$1\ m^2 = 1\ m \times 1\ m = 10\ dm \times 10\ dm = 100\ dm^2$
$1\ dam^2 = 1\ dam \times 1\ dam = 10\ m \times 10\ m = 100\ m^2$
$1\ hm^2$ 1 hectare $= 1\ hm \times 1\ hm = 100\ m \times 100\ m = 10000\ m^2 = 100\ dm^2$
$1\ km^2 = 1\ km \times 1km = 10\ hm \times 10\ hm = 100\ hm^2$ or 100 hectare

VOLUME

$1\ cm^3 = 1\ ml = 1\ cm \times 1\ cm \times 1cm = 10\ mm \times 10\ mm \times 10\ mm = 1000\ mm^3$
$1\ litre = 1000\ ml = 1000\ cm^3$
$1\ m^3 = 1\ m \times 1\ m \times 1\ m = 100\ cm \times 100\ cm \times 100\ cm = 10^6\ cm^3 = 1000\ litre = 1\ kilolitre$
$1dm^3 = 1000\ cm^3$
$1\ m^3 = 1000\ dm^3$
$1\ km^3 = 10^9\ m^3$

★ CUBOID

A rectangular solid bounded by six rectangular plane faces is called a cuboid. A match box, a tea-packet, a brick, a book, etc., are all examples of a cuboid.

A cuboid has 6 rectangular faces, 12 edges and 8 vertices.

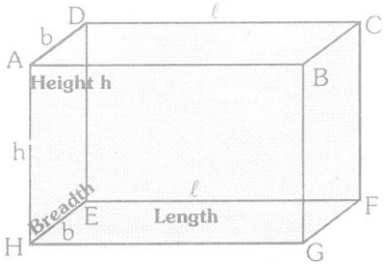
The following are some definitions of terms related to a cuboid.

(i) The space enclosed by a cuboid is called its **volume.**

(ii) The line joining opposite comers of a cuboid is called its **diagonal.**

A cuboid has four diagonals.

A diagonal of a cuboid is the length of the longest rod that can be placed in the cuboid.



(iii) The sum of areas of all the six faces of a cuboid is known as its **total surface area.**

(iv) The four faces which meet the base of a cuboid are called the **lateral faces** of the cuboid.

(v) The sum of areas of the four walls of a cuboid is called its lateral **surface area.**

Formulae

For a cuboid of length = units, breadth = b units and height = h units, we have:

Sum of lengths of all edges = 4 $(\ell + b + h)$ **units.**

Diagonal of cuboid = $\sqrt{\ell^2 + b^2 + h^2}$ **units.**

Total Surface Area of cuboid = 2 $(\ell b + bh + \ell h)$ **sq. units.**

Lateral Surface Area of cuboid = [2 $(\ell + b) \times h$ **] sq. units.**

Area of four walls of a room = [2 $(\ell + b) \times h$ **] sq. units.**

Volume of cuboid = $(\ell \times b \times h)$ **cubic units.**

REMARK: For the calculation of surface area, volume etc. of a cuboid, the length, breadth and height must be expressed in the same units.

★ CUBE

A cuboid whose length, breadth and height are all squal

is called a cube .

Ice-cubes, Sugar, Dice, etc. are all examples of a cube.

Each edge of a cube is called its side.

Formulae

For a cube of edge = a units, we have;

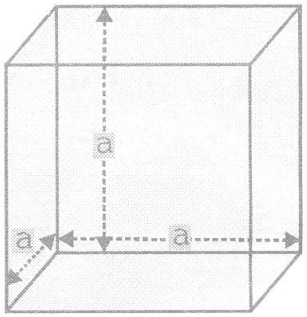
Sum of length of all edges = 12 a units.

Diagonal of cube = $(a\sqrt{3})$ **units.**

Total Surface Area of cube = (6a^2) sq. units.

Lateral Surface Area of cube = (4a^2) sq. units.

Volume of cube = a^3 cubic units.



Cube

★ CROSS SECTION

A cut which is made through a solid perpendicular to its length is called its cross section. If the cut has the same shape and size at every point of its length, then it is called **uniform cross-section.**

Volume of a solid with uniform cross section = (Area of its cross section) x (length).

Lateral Surface Area of a solid with uniform cross section

= (Perimeter of cross section) x (length).

Ex.1 The length, breadth and height of a rectangular solid are ratio 6 : 5 : 4. If the total surface area is 5328 cm^2, find the length, breadth and height of the solid.

Sol. Let length = (6x) cm, breadth = (5x) cm and height = (4x) cm.

Then, total surface area = [2(6x x 5x + 5x x 4x + 4x x 6x)] cm^2 = [2(30x^2 + 20x^2)] cm^2
= (148x^2)cm^2.

$\therefore$ 148x2 = 5328 ⇨ x^2 = 36 ⇨ x = 6.

Hence, length = 36 cm, breadth = 30 cm, height = 24 cm.

Ex.2 **An open rectangular cistern is made of iron 2.5 cm thick. When measured from outside, it is 1 m 25 cm long. 1 m 5 cm broad and 90 cm deep.**

Find: **(i) the capacity of the cistern in litres:**
(ii) the volume of iron used;
(iii) the total surface area of the cistern.

Sol. External dimensions of the cistern are :
Length = 125 cm, Breadth = 105 cm and Depth = 90 cm.
Internal dimensions of the cistern are :
Length = 120 cm, Breadth = 100 cm and Depth = 87.5 cm.

(i) Capacity = Internal volume = (120 x 100 x 87.5) $cm^3 = \left(\frac{120\times100\times87.5}{1000}\right)$ litres = 1050 litres.

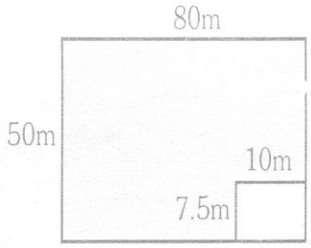
(ii) Volume of iron = (External volume) – (Internal volume) = [(125 x 105 x 90) – (120 x 100 x 87.5)] cm^3
= (1181250 – 1050000) cm^3 = 131250 cm^3.

(iii) External area = (Area of 4 faces) + (Area of the base) = ([2(125 + 105) x 90] + (125 x 105)} cm^2.

= (41400 + 13125) cm^2 = 54525 cm^2.
Internal area = ([2(120 + 100) x 87.5] + (120 x 100)} cm^2 = (38500 + 12000) cm^2 = 50500 cm^2.
Area at the top = Area between outer and inner rectangles = [(125 x 105) – (120 x 100)] cm^2
= (13125 – 12000) cm^2 = 1125 cm^2.
Total surface area = (54525 + 50500 + 1125) cm^2 = 106150 cm^2.

Ex.3. **A field is 80 m long and 50 m broad. In one corner of the field, a pit which is 10 m long, 7.5 m broad and 8 m deep has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.**

Sol. Area of the field = (80 x 50) m^2 = 4000 m^2



Area of the pit = (10 x 7.5) m^2 = 15 m^2

Area over which the earth is spread out = (4000 – 75) m2 = 3925 m2

Volume of earth dug out = (10 x 7.5 8) m^3 = 600 m^3.

$\therefore$ Rise in level = $\left(\frac{volume}{Area}\right) = \left(\frac{600}{3925}\right)m = \left(\frac{600\times100}{3925}\right)cm = 15.3cm$

Ex.4. **A room is half as long again as it is broad. The cost of carpeting the room at Rs 18 per m^2 is Rs 972 and the cost of white-washing the four walls at Rs 6 per m^2 is Rs 1080. Find the dimensions of the room.**

Sol. Let breadth = (x) in. Then, length = $(\frac{3}{2}\times)$ m.

Let height of the room = y m.

Area of the floor = $\left(\frac{colt\ of\ carpeting}{Rate}\right) = \left(\frac{972}{18}\right) = 54\ m^2$

$\therefore$ $x\times\frac{3}{2}x = 54 \Rightarrow x^2 = \left(54\times\frac{2}{3}\right) = 36 \Rightarrow x = 6$.

So, breadth = 6 m and length = $\left(\frac{3}{2}\times6\right)$ m = 9 m.

Now, area of four walls $= \left(\frac{\text{cost of white-washing}}{\text{Rate}}\right) = \left(\frac{1080}{6}\right) m^2 = 180\, m^2$.

$\therefore \quad 2(9+6)\times y = 180 \Rightarrow 30y = 180 \Rightarrow y = \left(\frac{180}{30}\right) = 6$.

Hence, length = 9 m, breadth = 6 m, height = m.

Ex.5. The water in a rectangular reservoir having a base 80 x m 60 m, is 6.5 m deep. In what time can the water be emptied by a pipe of which the cross section is a square of side 20 cm, if water runs through the pipe at the rate of 15 km/ hr ?

Sol. Volume of water in the reservoir = (80 x 60 x 6.5) m^3 = 31200 m^3.

Area of cross section of the pipe $= \left(\frac{20}{100}\times\frac{20}{100}\right) m^3 = \frac{1}{25}\, m^2$.

Volume of water emptied in 1 hr $= \left(\frac{1}{25}\times 15000\right) m^3 = 600\, m^3$.

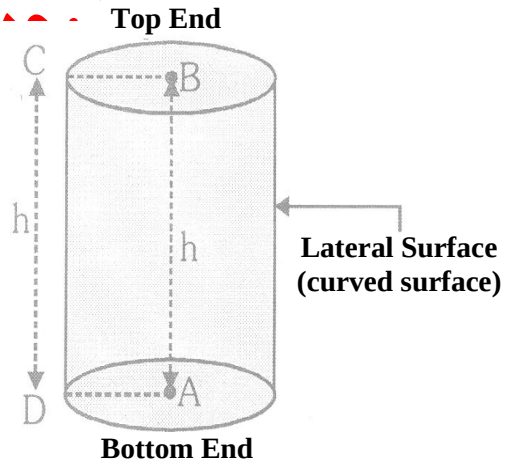
Time taken to empty the reservoir $= \left(\frac{31200}{600}\right)$ **hrs = 52 hrs.**

RIGHT CIRCULAR CYLINDER

Solids like circular pillars, circular pencils, measuring jars, road rollers and gas cylinders, etc., are said to be in cylindrical shape.

In mathematical terms, **a right circular cylinder is a solid generated by the revolution of a rectangle about its sides.**

Let the rectangle ABCD revolve about its side AB, so as to describe a right circular cylinder as shown in the figure.

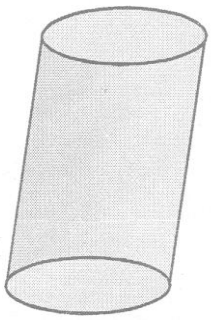


You must have observed that the cross-section of a right circular cylinder are circles congruent and parallel to each other.

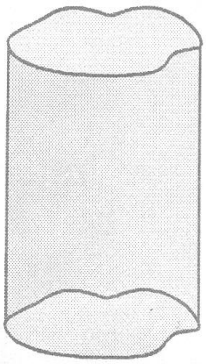
Cylinders Not Right Circular

There are two cases when the cylinder is not a right circular cylinder.

Case-I : In the following figure, we see a cylinder, which is certainly, but is not at right angles to the base. So we cannot say it is a right circular cylinder,



Case-II : In the following figure, we see a cylinder, with a non-circular base as the base is not circular. So we cannot say it is a right circular cylinder,



REMARK : Unless stated otherwise, here in this chapter the word cylinder would mean a right circular cylinder.

The following are definitions of some terms related to a right circular cylinder :

(i) The radius of any circular end is called the **radius** of the right circular cylinder. Thus, in the above figure, AD as well as BC is a radius of the cylinder .

(ii) The line joining the centres of circular ends of the cylinder, is called the axis of the right circular cylinder. In the above figure, the line AB is the axis of the cylinder. Clearly, the axis is perpendicular to the circular ends.

REMARK : If the line joining the centres of circular ends of a cylinder is not perpendicular to the circular ends, then the cylinder is not a right circular cylinder.

(iii) The length of the axis of the cylinder is called the **height or length** of the cylinder.

(iv) The curved surface joining the two bases of a right circular cylinder is called its **lateral surface.**

Formulae

For a right circular cylinder of radius = r units & height = h units, we have :

Area of each circular end = πr^2 sq. units.

Curved (Lateral) Surface Area = ($2\pi rh$) sq. units.

Total Surface Area = Curved Surface Area
+ Area of two circular ends.
= ($2\pi rh + 2\pi r^2$) sq. units.
= [$2\pi r (h + r)$] sq. units.

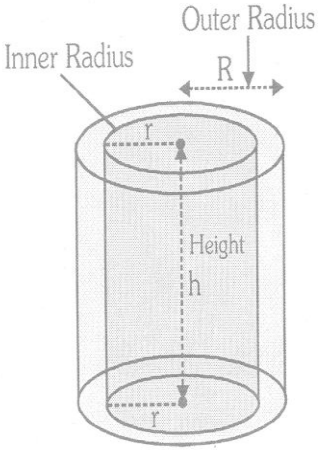
Volume of cylinder = $\pi r^2 h$ cubic units.

The above formulae are applicable to solid cylinders only.

Hollow Right Circular Cylinders

Solids like iron pipes, rubber tubes, etc., are in the shape of hollow cylinder.

A solid bounded by two coaxial cylinders of the same height and different radii is called a hollow cylinder



Formulae

For a hollow cylinder of height h and with external and internal radii R and r respectively, we have :

Thickness of cylinder = (R – r) units.
Area of a cross-section = ($\pi R^2 - \pi r^2$) sq. units.
= $\pi(R^2 - r^2)$ sq. units.

Curved (Lateral) Surface Area = (External Curved Surface Area) + (Internal Curved Surface Area)
= (2πRh + 2πrh) sq. units = 2πh (R + r) sq. units.
Total Surface Area = (Curved Surface Area) + 2 (Area of Base Ring)
= [(2πRh + 2πrh) + (πR² – πr²)] sq. units
= 2π(Rh + rh +vR² – r²) sq. units
Volume of Material = π(R² – r²) h cubic units
Volume of Hollow region = πr²h cubic units

Ex. 6 **2.2 cu dm of brass is to drawn into a cylindrical wire of diameter 0.50 cm. Find the length of the wire.**

Sol. Volume of brass = 2.2 cu dm = (2.2 x 10 x 10 x 10) cm^3 = 2200 cm^3. Let the required length of wire be x cm.

Then, its volume = $(\pi r^2 x)cm^3 = \left(\frac{22}{7}\times 0.25\times 0.25\times x\right) cm3$

$\therefore \quad \frac{22}{7}\times 0.25\times 0.25\times x = 2200$

$\Rightarrow \quad x = \left(2200\times\frac{7}{22}\times\frac{1}{0.25\times 0.25}\right) = 11200\,cm = 112\,m.$

Hence, the length of wire is 112 m.

Ex. 7 **A well 14 m diameter is dug 8 m deep. The earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. Find the height of the embankment.**

Sol. Volume of earth dug out from the well = π r² h = $\left(\frac{22}{7}\times 7\times 7\times 8\right) m^3 = 1232\,m^3$.

Area of the embankment $= \pi(R^2 - r^2) = \frac{22}{7}\times\{(28)^2\}m^2 = \left(\frac{22}{7}\times 35\times 21\right) m^2 = 2310\,m^2.$

Height of the embankment $= \frac{Volume\ of\ earth\ dug\ out}{Area\ of\ embankment} = \left(\frac{1232}{2310}\times 100\right) cm. = 53.3\,cm.$

Ex. 8 **The difference between the outside and inside surface of a cylinder metallic pipe 14 cm long is 44 cm^2. If the pipe is made of 99 cu cm of metal, find outer and inner radii of the pipe.**

Sol. Let, external radius = R cm and internal radius = r cm.

Then, outside surface = π Rh = $\left(2\times\frac{22}{7}\times R\times 14\right) cm^2 = (88R)\ cm^2.$

Inside surface = 2 π rh = $\left(2\times\frac{22}{7}\times r\times 14\right) cm^2 = (88r)\ cm^2.$

$\therefore \quad (88R - 88r) = 44 \Rightarrow (R - r) = \frac{44}{88} = \frac{1}{2} \Rightarrow (R - r) = \frac{1}{2}$

Internal volume = π R²h = $\left(\frac{22}{7}\times R^2\times 14\right) cm^3 = (44R^2)\ cm^3$

$\therefore \quad (44R^2 - 44r^2) = 99 \Rightarrow (R^2 - r^2) = \frac{99}{44} \Rightarrow (R^2 - r^2) = \frac{9}{4}$

On dividing (ii) by (i), we get: (R + r) = $\left(\frac{9}{4}\times\frac{2}{1}\right) \Rightarrow (R + r) = \frac{9}{2}$

Solving (i) and (ii), we get, R = 2.5 and r = 2.

Hence, outer radius = 2.5 cm and inner radius = 2 cm.

Ex. 9 **A solid iron rectangular block of dimensions 4.4 m, 2.6 m and 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.**

Sol. Volume of iron = (440 x 260 x 100) cm^3.
Internal radius of the pipe = 30 cm.
External radius of the pipe = (30 + 5) cm = 35 cm.

Let the length of the pipe be h cm.
Volume of iron in the pipe = (External volume) – (Internal volume)

$$= [\pi \times (35)^2 \times h - \pi \times (30)^2 \times h] cm^3 = (\pi h)\{(35)^2 - (30)^2\} cm^3$$

$$= (65 \times 5)\pi h\, cm^3 = (325\pi h)\, cm^3 .$$

$$\therefore \quad 325\pi h = 440 \times 260 \times 100 \qquad \Rightarrow h = \left(\frac{440 \times 260 \times 100}{325} \times \frac{7}{22}\right) cm$$

$$\Rightarrow h = \left(\frac{11200}{100}\right) m = 112 m.$$

Hence, the length of the pipe is 112 m.

Ex. 10 A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow in kilometers per hour.

Sol. Volume of water that flows per hour = (192.50 x 60) liters = (192.5 x 60 x 1000) cm^3.
Inner radius of the pipe = 3.5 cm.
Let the length of column of water that flows in 1 hour be h cm.

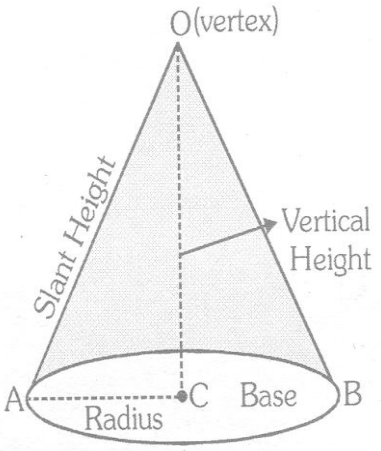
Then, $\frac{22}{7} \times 3.5 \times 3.5 \times h = 192.5 \times 60 \times 1000$

$$\Rightarrow h = \left(\frac{192.5 \times 60 \times 1000 \times 7}{3.5 \times 3.5 \times 22}\right) cm = 300000 cm = 3 \text{ km}$$

Hence, the rate of flow = 3 km per hour.

★ **RIGHT CIRCULAR CONE**

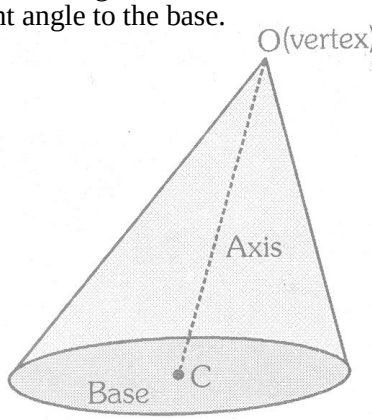
Solids like an ice-cream cone, a conical tent, a conical vessel, a clown's cap etc. are said to be in conical shape.
In mathematical terms, **a right circular cone is a solid generated by revolving a right-angled triangle about one of the sides containing the right angle.**
Let a triangle AOC revolve about it's OC, so as to describe a right circular cone, as shown in the figure.

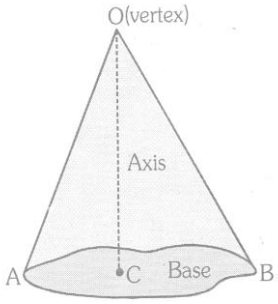


Cones Not Right circular

There are two cases when we cannot call a right circular cone.

Case-1 : The figure shown blown below is not a right circular cone because the line joining its vertex to the centre of its base is not at right angle to the base.





Case-II: The figure shown below is not a right circular cone because the base is not circular.

REMARK : Unless stated otherwise, by 'cone' in this chapter, we shall mean 'a right circular cone

The following are definitions of some terms related to right circular cone :

(i) The fixed point O is called the **vertex** of the cone.

(ii) The fixed line OC is called the **axis** of the cone.

(iii) A right circular cone has a plane end, which is in circular shape. This is called the **base** of the cone. The vertex of a right circular cone is farthest from its base.

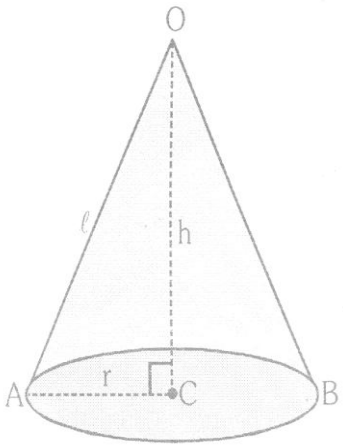
(iv) The length of the line segment joining the vertex to the centre of the base is called the **height** of the cone. Length OC is the height of the cone.

(v) The length of the line segment joining the vertex to any point on the circular edge of the base, is called the **slant height** of the cone.

(vi) The radius AC of the base circle the **radius** of the cone.

Relation Between Slant Height, Radius and Vertical Height.

Let us take a right circular cone with vertex at O, vertical height h, slant height ℓ and radius r. A is any point on the rim of the base of the cone and C is the centre of the base. Here, OC = h, AC = r and OA = ℓ. The cone is right circular and therefore, OC is at right angle to the base of the cone. So, we have $OC \perp CA$, i.e., ΔOCA is right angled at C.



Then by Pythagoras theorem, we have :

$$\ell^2 = r^2 + h^2$$

Formulae

For a right circular cone of Radius = r, Height = h & Slant Height = ℓ, we have :

Area of the curved (lateral) surface = $(\pi r \ell)$ **sq. units. =** $\left(\pi r\sqrt{h^2+r^2}\right)$ **sq. units**

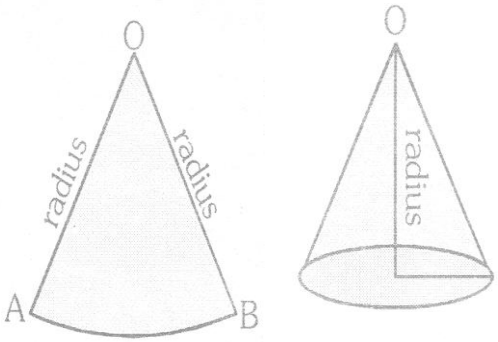
Total Surface Area of cone = (Curved surface Area + Area of Base)

$= (\pi r \ell + \pi r^2)$ **sq. units =** π **r** $(\ell + r)$ **sq. units.**

Volume of cone = $\left(\frac{1}{3}\pi r^2 h\right)$ **cubic units.**

Hollow Right Cricular Cone

Suppose a sector of a circle is folded to make the radii coincide, then we get a hollow right circular cone. In such a cone;



(i) Centre of the circle is vertex of the cone.

(ii) Radius of the circle is slant height of the cone.

(iii) Length of arc AB is the circumference of the base of the cone.

(iv) Area of the sector is the curved surface area of the cone.

Ex.11 The total surface area of a right circular cone of slant height 13 cm us 90π cm^2.

Calculate : (i) its radius in cm, (ii) its volume in cm^3, in terms of π.

Sol. Given : slant height, $\ell = 13$cm.
Let, radius = r cm and height = h cm.

(i) Total surface area = π **r** $(\pi + r) = [\pi r(13 + r)]$ **cm^2.**

$\therefore \quad \pi r(13+r) = 90\pi \Rightarrow r^2 + 13r - 90 = 0 \Rightarrow (r+18)(r-5) = 0$

$\Rightarrow$ r = 5 [Neglecting r = - 18, as radius cannot be negative]

$\therefore$ Radius of the cone = 5 cm.

(ii) $h = \sqrt{\ell^2 - r^2} = \sqrt{(13)^2 - (5)^2}$

= Volume of the cone $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times 5 \times 5 \times 12\right) cm^3$

= 100π cm^3.

Ex.12 A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find :

(i) its radius, (ii) its slant height.

Sol. Height of cylindrical bucket, H = 32 cm.

Radius of cylindrical bucket, R = 18 cm.

Volume of sand = $\pi R^2 H = \left(\frac{22}{7} \times 18 \times 18 \times 32\right)$ cm^3.

(i) Height of conical heap, h = 24 cm.

Let the radius of the conical heap be r cm.

Then, volume of conical heap $= \frac{1}{3}\pi r^2 h \left(\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24\right)$ cm^3.

Now, Volume of conical heap = Volume of sand

$\Rightarrow \quad \left(\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24\right) = \left(\frac{22}{7} \times 18 \times 18 \times 32\right)$

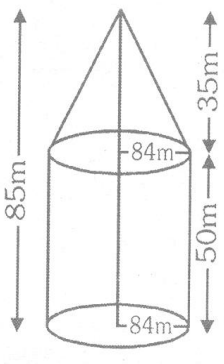
$\Rightarrow \quad r^2 = \left(\frac{18 \times 18 \times 32}{24}\right) = (18 \times 18 \times 4)$

$\Rightarrow \quad r = \sqrt{(18 \times 18 \times 4} = (18 \times 2)$ cm. = 36 cm.

$\therefore$ Radius of the heap = 36 cm.

(ii) Slant height, $\ell = \sqrt{h^2 + r^2} = \sqrt{(24)^3 + (36)^2} = \sqrt{1872} = 12\sqrt{13}$ cm.

Ex.13 An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for folds and stitching.



Given your answer to the nearest m^2.

Sol. Radius of the tent, $r = \left(\frac{168}{2}\right)$ m = 84 m.

Height of the tent = 85 m.
Height of the cylindrical part, H = 50 m.
Height of the conical part, h = (85 – 50) m = 35 m.

Slant height of the conical part, $\ell = \sqrt{h^2 + r^2} = \sqrt{(35)^2 + (84)^2} = \sqrt{8281}$ m = 91m.

Quantity of canvas required = Curved surface area of the tent
= Curved surface area of the cylindrical part
+ Curved surface area of the conical part
$= 2\pi rH + \pi r\ell = \pi r(2H + \ell)$

$= \left[\frac{22}{7} \times 84(2 \times 50 + 91)\right] m^2 = (22 \times 12 \times 191)\ m^2 = 50424 m^2$.

Area of canvas required for folds and stitching = (20% of 50424) $m^2 = \left(\frac{20}{100} \times 50424\right) m^2 = 10084.80\ m^2$.

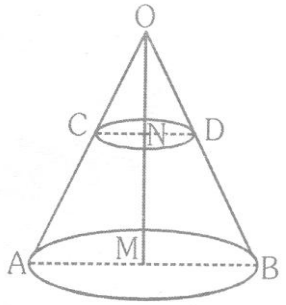
$\therefore$ Total quantity of canvas required to make the tent
= (50424 + 10084.80) m^2 = 60508.80m^2 = 60509 m^2. (to the nearest m^2)

Ex.14 The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height, above the base is the section cut?

Sol. Let OAB be the given cone of height, H 30 cm and base radius R cm. Let this cone be cut by the plane CND to obtain the cone OCD with height h cm and base radius r cm.

Then, $\Delta OND \sim \Delta OMB$.

So, $\frac{ND}{MB} = \frac{ON}{OM} \Rightarrow \frac{r}{R} = \frac{h}{30}$ …(i)



Volume of cone OCD $= \frac{1}{27} \times \frac{1}{3}\pi R^2 \times 30$

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{1}{27} \times \frac{1}{3}\pi R^2 \times 30$

$\Rightarrow \quad \left(\frac{r}{R}\right)^2 = \frac{10}{9h} \Rightarrow \left(\frac{h}{30}\right)^2 = \frac{10}{9h}$ [From (i)]

$\Rightarrow \quad 9h^3 = 9000 \Rightarrow h^3 = 1000 \Rightarrow h = 10.$

$\therefore$ Height of the cone OCD = 10 cm.

Hence, the section is cut at the height of (30 – 10) cm, i.e., 20 cm from the base.

Ex.15 From a solid cylinder of height 30 cm and radius 7 cm, a conical cavity of height 24 cm and of base radius 7 cm is drilled out. Find the volume and the total surface of the remaining solid.

Sol. Radius, r = 7 cm.
Height of the cylinder, H = 30 cm.
Height of the cone, h = 24 cm.

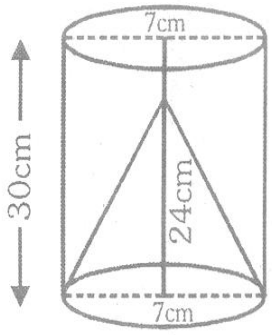
Slant height of the cone, $\ell = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (7)^2} = \sqrt{625} = 25\text{cm}$

(i) Volume of the remaining solid

= (Volume of the cylinder) – (Volume of the cone)

$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \pi r^2\left(H - \frac{h}{3}\right)$

$= \left[\frac{22}{7} \times 7 \times 7 \times \left(30 - \frac{24}{3}\right)\right] cm^3 = \left[\frac{22}{7} \times 7 \times 7 \times 22\right]$ cm^3



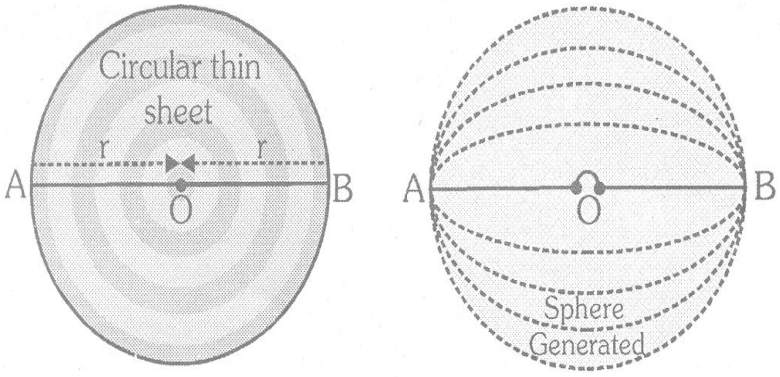
$= (22\times 7\times 22)\, cm^3 = 3388\, cm^3$.

(ii) Total surface area of the remaining solid
= Curved surface area of cylinder + Curved surface area of cone
+ Area of (upper) circular base of cylinder

$$= 2\pi rH + \pi r\ell + \pi r^2 = \pi r(2H + \ell + r) = \left[\frac{22}{7}\times 7\times(60+25+7\right] cm^2 = (22\times 92)\, cm^2 = 2024 cm^2.$$

★ **SPHERE**

Objects like football, volleyball, throw-ball etc. are said to have the shape of a sphere.
In mathematical terms, **a sphere is a solid generated by revolving a circle about any of its diameters.**
Let a thin circular disc of card of card board with centre O and radius r revolve about its diameter AOB to describe a sphere as shown in figure.



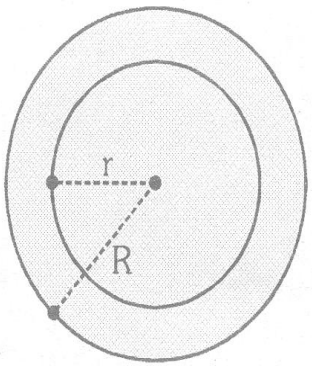
Here, O is called the **centre of the sphere** and r is **radius of the sphere.** Also, the line segment AB is a **diameter of the sphere.**

Formulae
For a solid sphere of radius = r, we have :

Surface area of the sphere = ($4\pi r^2$) sq. units.

Volume of the sphere = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.

SPHERICAL SHELL
The solid enclosed between two concentric spheres to called a spherical shell.

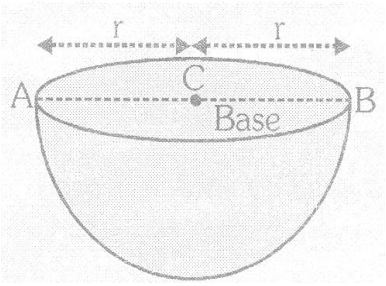


Formula
For a spherical shell with external radius = R and internal radius = r, we have :

Thickness of shell = (R – r) units.
Outer surface area = $4\pi R^2$ sq. units.
Inner surface area = $4\pi r^2$ sq. units.
Volume of material = 4/3 $\pi (R^3 - r^3)$sq. units.

HEMISPHERE

When a plane through the centre of a sphere cuts it into two equal parts, then each part is called a hemisphere.
Form a solid sphere, the obtained hemisphere is also a solid and it has a base as shown in fig.

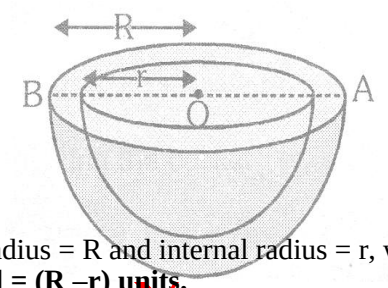


Formula
For a hemisphere of radius r, we have :

Curved surface area = 2 π r² sq. units.
Total Surface area = (2 π r² + π r²) = 3 π r² sq. units.
Volume $= \frac{2}{3}\pi r^3$ **cubic units.**

HEMISPHERICAL SHELL

The solid enclosed between two concentric hemispheres is called a hemispherical shell.



Formulae
For a hemispherical shell of external radius = R and internal radius = r, we have :

Thickness of the shell = (R –r) units.
Outer curved surface area = (2πR²) sq. units.
Inner curved surface area = (2πr²) sq. units.
Total surface area = 2πR² + 2πr² + π(R³ – r³) = π(3R² + r²) sq. units.

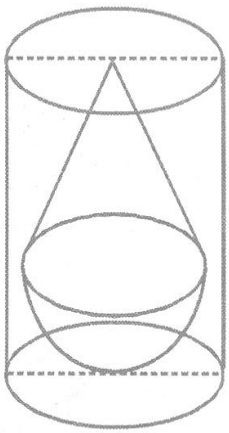
Ex.16 A solid consisting of a right circular cone, standing on a hemisphere, is placed upright, in a right circular cylinder, full of water, and touches the bottom. Find the volume of water left in the cylinder., having given that the radius of the cylinder is 3 cm and its height is 6 cm: the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to the nearest cm³ (Take π = 22/7)

Sol. Radius of the cylinder = 3 cm and its height = 6 cm.

Volume of water in the cylinder, when full $= \left[\pi \times (3)^2 \times 6\right] cm^3 = (54\pi)\ cm^3$.

Volume of solid consisting of cone hemisphere = (Volume of hemi-sphere) + (Volume of cone)

$$= \left[\frac{2}{3}\pi \times (2)^3 + \frac{1}{3}\pi \times (2)^2 \times 4\right] cm^3 = \left(\frac{32\pi}{3}\right) cm^3.$$



Volume of water displaced from cylinder

= Volume of solid consisting of cone and hemisphere

$$= \left(\frac{32\pi}{3}\right) cm^3$$

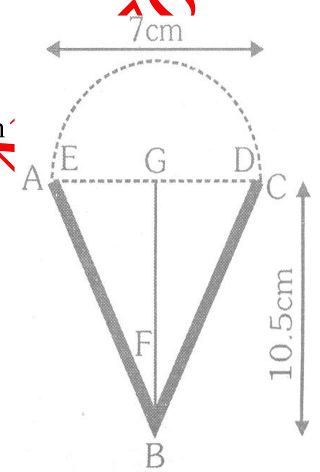
Volume of water left in the cylinder after placing the solid into it

$$\left(54\pi - \frac{32}{3}\right) cm^3 = \left(\frac{130\pi}{3}\right) cm^3 = \left(\frac{130}{3} \times \frac{22}{7}\right) cm^3 = 136.19 cm^3$$

Hence, the volume of water left in the cylinder to the nearest cm^3 is 136 cm^3.

Ex.17 The given figure shows the cross-section of an ice-cream cone consisting of a cone surmounted by a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 10.5 cm. The outer shell ABCDEF is shaded and is not filled with ice-cream. AE = DC = 0.5 cm, AB ∥ EF and BC ∥ FD. Caclculate:
(i) the volume of the ice-cream in the cone (the unshaded poration including the hemisphere) in cm^3;
(ii) the volume of the outer shell (the shaded portion) in cm^3. Give your answer to the nearest cm^3.

Sol. Radius of hemisphere, R= AG = 3.5 cm.



External radius of conical shell, R = AG = 3.5 cm.
Internal radius of conical shell, r = EG = (AG – AE) = (3.5 – 0.5) cm = 3 cm
Now, ΔΔ BG ~ Δ EFG.

$$\therefore \quad \frac{FG}{BG} = \frac{EG}{AG} \Rightarrow \frac{FG}{1.05} = \frac{3}{3.5} \Rightarrow FG = 9 \text{ cm.}$$

So, internal height of conical shell, h = FG = 9 cm.

(i) Volume of ice-cream

= Volume of hemisphere + Internal volume of conical shell

$$= \frac{2}{3}\pi R^3 + \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2R^3 + r^2 h)$$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times \{2 \times (3.5)^3 + (3)^2 \times 9\}\right] cm^3 = \left[\frac{1}{3} \times \frac{22}{7} \times \left(\frac{343}{4} + 81\right)\right] \text{cm}^3$$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times \frac{667}{4}\right) cm^3 = \left(\frac{7337}{42}\right) cm^3 = 174.69 cm^3 = cm^3$$. (to the nearest cm^3)

(i) Volume of the shell = External volume – Internal volume

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(R^2 H - r^2 h)$$

$$= \frac{1}{3}\pi\left[(3.5)^2 \times 10.5 - (3)^2 \times 9\right] cm^3 = \frac{1}{3}\pi\left[\left(\frac{7}{2}\right)^2 \times \left(\frac{21}{2}\right) - (9 \times 9)\right] \text{cm}^3$$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times \left(\frac{1029}{8} - 81\right)\right] cm^3 = \left(\frac{1}{3} \times \frac{22}{7} \times \frac{381}{8}\right) cm^3 \left(\frac{1397}{28}\right) cm^3 = 49.89 cm^3$$

= 50 cm^3 (to the nearest cm^3)

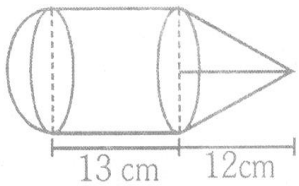
Ex.18 A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical part are the same as that of the cylindrical part. Calculate the surface area of the height of the conical part is 12 cm.

Sol. The toy is in the shape shown below :

Radius of the hemispherical part = 5 cm,

∴ Curved surface area of the Hemispherical part

$2\pi r^2 = [2\pi \times (5)^2\, cm^2 = (50\pi)\, cm^2$.



Cylindrical part has radius = 5 cm and height = 13 cm.

∴ Curved surface area of the cylindrical part =

$\pi rh = (2\pi \times 5 \times 13)\, cm^2 = (130\pi)\, cm^2$.

Conical part has radius = 5 cm and height = 12 cm.

∴ Its slant height $\sqrt{5^2 + (12)^2} = \sqrt{25+144} = \sqrt{169} = 13 cm.$

∴ Curved surface area of the conical part = $\pi r\ell$

$= (\pi \times 5 \times 13)\, cm^2 = (65\pi)\, cm^2$

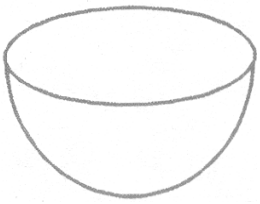
Hence, the surface area of the toy = (50π + 130 π + 65 π) cm^2 = (245 π) cm^2.

$= \left(245 \times \frac{22}{7}\right) cm^2 = 770 cm^2$.

Also, volume of the toy = $\left(\frac{2}{3}\pi r^3 + \pi r^2 h + \frac{1}{3}\pi r^2 H\right) cm^3 = \left(\frac{250\pi}{3} + 325\pi + 100\pi\right) cm^3 = \left(\frac{1525}{3}\right) cm^3$

Ex.19 The outer and inner diameters of a hemispherical bowl are 17 cm and 15 cm respectively. Find cost of polishing it all over at 25 paise per cm^2. (Take π = 22/7).

Sol. Outer radius $= \frac{17}{2}$ cm, Inner radius $= \frac{15}{2}$ cm.



Area of outer surface = 2 πR^2 = $\left[2\pi \times \left(\frac{17}{2}\right)^2\right] cm^2 = \left(\frac{289\pi}{2}\right) cm^2$.

Area of inner surface = 2 π r^2 = $\left[2\pi \times \left(\frac{15}{2}\right)^2\right] cm^2 = \left(\frac{225\pi}{2}\right) cm^2$.

Area of the ring at the top = π (R^2 – r^2) = π [(8.5)^2 - (7.5)^2] cm^2 = (16 π) cm^2.

∴ Total area to be polished $= \left(\frac{289\pi}{2} + \frac{225\pi}{2} + 16\pi\right) cm^2$.

$= (273\pi)\, cm^2 = \left(273 \times \frac{22}{7}\right) cm^2 = 858 cm^2$.

∴ Cost of polishing the bowl = Rs $\left(\frac{858 \times 25}{100}\right)$ = Rs. 214. 50.

Ex.20 A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. What fraction of water overflows?

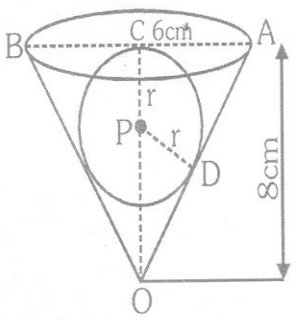
Sol. Radius of the conical vessel, R = AC = 6 cm.

Height of the conical vessel, H = OC = 8 cm.

Let the radius of the sphere be r.

Then, PC = PD = 6 cm.

[∵ lengths of two tangents from an external point to a circle are equal]



OA = $\sqrt{OC^2 + AC^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm.

OD = (OA – AD) = (10 – 6)

OP = (OC – PC) =(8 – R).
In right angled ΔODP, we have :

$OP^2 = OD^2 + PD^2$

$\Rightarrow \quad (8-R)^2 = 4^2 + r^2 \Rightarrow 64 - 16r + r^2 = 16 + r^2$

$\Rightarrow \quad 16r = 48 \Rightarrow r = \frac{48}{16} = 3.$

Volume of water overflown = volume of sphere $= \frac{4}{3}\pi r^3 = \left[\frac{4}{3}\pi \times (3)^3\right] cm^3 = (36\pi)cm^3.$

Volume of water in the cone before immersing the sphere

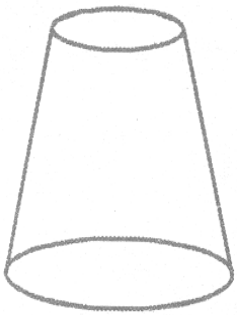
= Volume of cone $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times (6)^2 \times 8\right) cm^3 = (96\pi)cm^3.$

$\therefore$ Fraction of water overflown $= \frac{Volume\ of\ water\ overflown}{Original\ volume\ of\ water} = \frac{(36\pi)}{96\pi} = \frac{3}{8}$

★ FRUSTUM

FRUSTUM OF A RIGHT CIRCULAR CONE

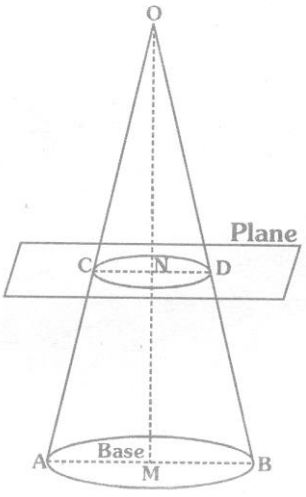
In our day-to-day life we come across a number of solids of the shape as shown in the figure. For example, a bucket or a glass tumbler. We observe that this type of solid is a part of a right circular cone and is obtained when the cone is cut by, a plane parallel to the base of the cone.



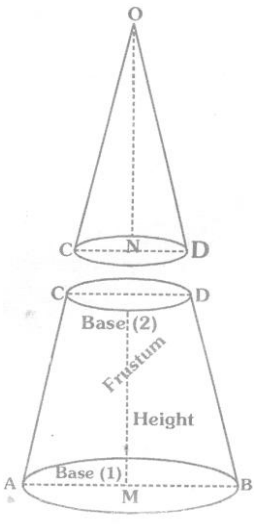
If right circular cone is cut off by a plane parallel to its base, the portion of the cone between the plane and the base of the cone is called a frustum of the cone.

We can see this process from the figures given below:

The lower portion in figure is the frustum of the cone. It has two parallel flat circular bases, mark as Base (1) and Base (2). A curved surface joins the two bases.



(i)



(ii)

The line segment MN joining the centres of the two bases is called the height of the frustum. Diameter CD of Base (2) is parallel to diameter AB of base (1). Each of the line segments AC and BD is called the slant height of the frustum. We observe from the figures (i) and (ii) that,

1. **Height of the frustum = (the height of the cone OAB) – (the height of the cone OCD)**
2. **Slant height of the frustum = (the height of the cone OAB) – (the height of the cone)**

⏩ Volume of a Frustum of a Right Circular Cone

Let h be the height ; r_1 and r_2 be the radii of the two bases ($r_1 > r_2$) of frustrum of a right circular cone.
The frustum is made from the complete cone OAB by cutting off the conical part OCD. Let h_1 be the height of the cone OAB and h_2 be the height of the cone OCD.

Here, $h_2 = h_1 - h$.

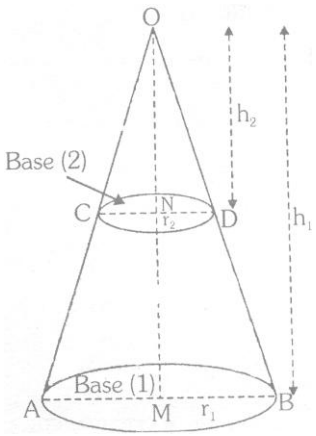
Since right angled triangles OND and OMB are similar, therefore, we have:

$$\frac{h_2}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{h_1 - h}{h_1} = \frac{r_2}{r_1} \qquad \Rightarrow \quad 1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1} \qquad \Rightarrow \quad h_1 = \frac{hr_1}{r_1 - r_2}$$

and $\quad h_2 = h_1 - h = \frac{hr_1}{r_1 - r_2} - h \Rightarrow \quad h_2 = \frac{hr_2}{r_1 - r_2}$



Volume V of the frustum of cone = Volume of the cone OAB – volume of the cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3}\pi r_1^2 \times \frac{hr_1}{(r_1 - r_2)} - \frac{1}{3}\pi r_2^2 \times \frac{hr_2}{(r_1 - r_2)}$$

$$= \frac{1}{3}\pi h\left\{\frac{r_1^3 - r_1^3}{r_1 - r_2}\right\} = \frac{1}{3}\pi h\{r_1^2 + r_1 r_2 + r_2^2\}$$

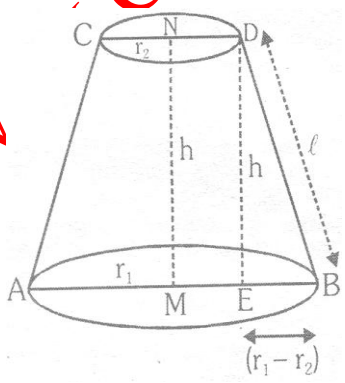
$$\therefore \quad V = \frac{1}{3}\pi h\{r_1^2 + r_1 r_2 + r_2^2\}$$

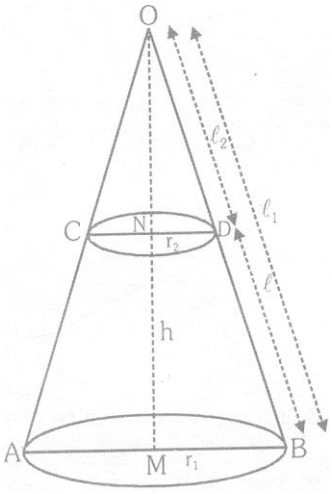
Note : Volume $V = \frac{1}{3}\pi h\{r_1^2 + r_1 r_2 + r_2^2\}$

$$= \frac{h}{3}(\pi r_1^2 + \pi r_2^2 + \pi r_1 r_2) = \frac{h}{3}\{\pi r_1^2 + \pi r_2^2 + \sqrt{(\pi r_1^2)(\pi r_2^2)}\}$$

$$= \frac{h}{3}\{(area\,of\,base) + (area\,of\,base2) + \sqrt{(area\,of\,base)(area\,of\,base2)}\}$$

⏩ Curbed Surface Area of a Frustum of a Right Circular Cone





Let h be the height, ℓ be the slant height and r_1,r_2 be the radii of the bases where $r_1 > r_2$.

In figure (i), we observe EB = $r_1 - r_2$

Aad $\ell^2 = h^2 + (r_1 - r_2)^2$

$\therefore$ $\ell = \sqrt{h^2 + (r_1 - r_2)^2}$

In figure (ii, we have OAB as the complete cone from which cone OCD is cut off to make the frustum ABDC.
Let ℓ be the slant height of the cone OAB and ℓ_2 be the slant height of the cone OCD.
Since, ΔOMB are similar,

$$\frac{\ell_2}{\ell_1} = \frac{r_2}{r_1} \quad \Rightarrow \quad \frac{\ell_1 - \ell}{\ell_1} = \frac{r_2}{r_1} \quad \Rightarrow \quad \ell_1 = \frac{\ell r_1}{r_1 - r_2}$$

Now, $\ell_2 = \ell_1 - \ell = \frac{\ell r_1}{r_1 - r_2} - \ell \quad \Rightarrow \quad \ell_2 = \frac{\ell r_2}{r_1 - r_2}$

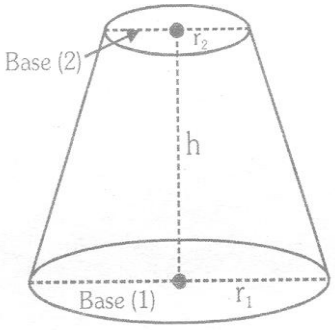
Curved surface area of frustum ABCD
= (Curved surface area of cone OAB) – (Curved surface area of cone OCD)

$$= \pi r_1 \ell_1 - \pi r_2 \ \ell_2 = \pi r_1 \times \frac{\ell r_1}{(r_1 - r_2)} - \pi r_2 \times \frac{\ell r_2}{(r \ - r_2)} = \pi \ell \left\{ \frac{r_1^2 - r_2^2}{r_1 - r_2} \right\}$$

Therefore, curved surface area of frustum = $\pi \ell (r_1 + r_2)$.

Total surface Area of a Frustum of a solid Right Circular Cone

Let h be the height, ℓ be the slant height and r_1, r_2 the radii of the bases where $r_1 > r_2$ as shown in figure.



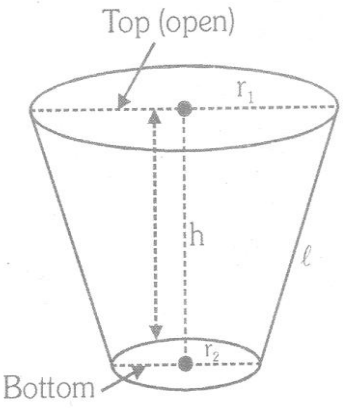
Total surface area of this frustum

= Curved surface area + Area of Base 1 + Area of Base 2

= $\pi \ell (r_1 + r_2) + \pi r_2^2$

Area of the Metal Sheet Used To Make a Bucket

A bucket is in the shape of a frustum of a right circular hollow cone.
Let h be the depth, ℓ be the slant height, r_1 be the radius of the top and r_2 be the radius of the bottom as shown in figure



The area of the metal sheet used for making the bucket

= Outer (or inner) curved surface area + Area of bottom

= $\pi \ell (r_1 + r_2) + \pi r_2^2$

Ex.21 **A bucket of height 16 cm and made up of metal sheet is in the form of frustum of a right circular cone with radii of its lower and upper ends as 3 cm and 15 cm respectively. Calculate:**

(i) the height of the cone of which the bucket is a part.

(ii) the volume of water which can be filled in the bucket.

(iii) the slant height of the bucket.

(iv) the area of the metal sheet required to make the bucket.

Sol. Let ABCD be the bucket which is frustum of a cone with vertex 0 (as shown in figure). Let ON = x cm

$\Delta OAB - \Delta OMC$

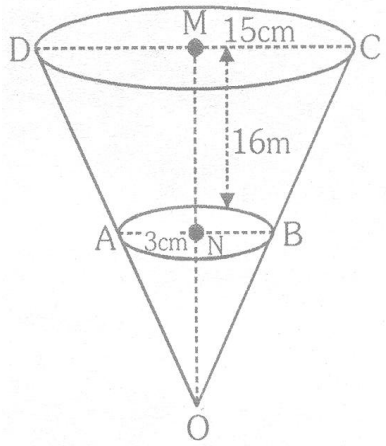
$\therefore \quad \frac{x}{16+x} = \frac{3}{15} \qquad \left\{\because \frac{ON}{OM} = \frac{NB}{MC}\right\}$

$\Rightarrow \quad \frac{x}{16+x} = \frac{1}{5} \quad \Rightarrow \quad 5x = 16 + x1$

$\Rightarrow \quad 4x = 16 \quad \Rightarrow \quad x = 4$

$\therefore$ ON = 4 cm and OM = 4 + 16 = 20 cm

$\therefore$ the height of the cone = 20 cm



volume of the bucket $= \frac{1}{3}\pi(15)^2 \times 20 - \frac{1}{3}\pi(3)^2 \times 4 cm^3$

{i.e., Volume of the large cone – Volume of the small cone}

$= \frac{1}{3}\pi[225 \times 20 - 36]cm^3$

$= \pi[75 \times 20 - 12]cm^3$

$= 1488\pi\, cm^3$

Slant height of cone of radius 15 cm

$= \sqrt{(15)^2 + (20)^2}\, cm = \sqrt{625}\, cm = 25cm$

Slant height of cone of radius 3 cm

$= \sqrt{(4)^2 + (3)^2}\, cm = 5cm$

$\therefore$ Slant height of bucket $= (25-5)cm = 20cm, i.e., \ell = 20cm$

$\therefore$ The area of the metal sheet $= \pi\ell(R+r) + \pi r^2$

$= \pi \times 20 \times (15+3) + \pi(3)^2\, cm^2 = 360\pi + 9\pi\, cm^2$

$= 369\pi\, cm^2$

Note. The area of the metal sheet used = C.S. of larger cone – C.S. of smaller cone + Area of the base of the bucket

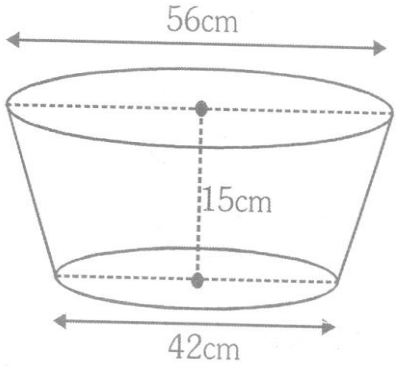
$= [\pi \times 25 \times 15 - \pi \times 5 \times 3 + \pi \times (3)^2]cm^2 = [375\pi - 15\pi + 9\pi]cm^2$

$= 369\pi\ cm^2$

Ex.22 **A bucket is in the form of a frustum of a cone, depth is 15 cm and the diameters of the top and the bottom are 56 cm and 42 cm respectively. Find how many liters of water can the bucket hold ?**
(Take $\pi = 22/7$)

Sol. R = 28 cm
r = 21 cm

h = 15 cm



Capacity of the bucket $= \frac{1}{3}\pi h\{R^2 + r^2 + R\,r\}$

$= \frac{1}{3} \times 22 \times 15 \times \{(28)^2 + (21)^2 + (28)(21)\} cm^3$

$= \frac{22}{7} \times 5 \times \{784 + 441 + 588\} cm^3$

$= \frac{22}{7} \times 5 \times 1813 cm^3 \qquad = 22 \times 5 \times 259 \text{cm}^3$

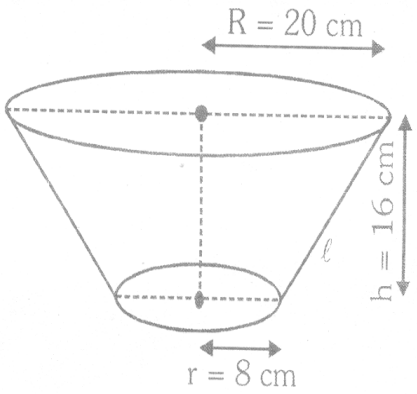
$= 28490 \text{ cm}^3 \qquad = \frac{28490}{1000}$ liters

$= 28.49$ liters

Ex.23 A container made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container at the rate of Rs. 15 per liter and the cost of the metal sheet used, if it costs Rs. 5 per 100 cm^2. (Take π 3.14)

Sol. R = 20 cm, r = 8 cm, h = 16 cm

$\ell = \sqrt{h^2 + (R-r)^2} = \sqrt{256 + 144}\, cm = 20 cm$



Volume of contamer $= \frac{1}{3}\pi h\{R^2 + r^2 + Rr\}$

$= \frac{1}{3} \times (3.14) \times 16\{400 + 64 + 160\}\, cm^3$

$= 3.14 \times \frac{16}{3}\{624\}\, cm^3$

$= 3.14 \times 16 \times 208 cm^3$

$= 10449.92 cm^3$

Therefore, the quantity of milk in the container $= \frac{10449.92}{1000}$ liters = 10.45 liters

Colt of milk at the rate of Rs. 15 per liter = Rs. $\{10.45 \times 15\}$ = Rs. 156.75

Surface area of the metal sheet used to make the container

$= \pi\ell(R + r) + \pi r^2 = \pi\{\ell(R + r) + r^2\}$

$= (3.14) \times \{20 \times 28 + 64\} cm^2$

$= (3.14) \times 624 cm^2 = 1959.36 cm^2$

Therefore, the cost of the metal sheet at rate of Rs. 5 per 100 cm^2

$= Rs. \frac{1959.36 \times 5}{100} = Rs.\ 97.97$ approx.

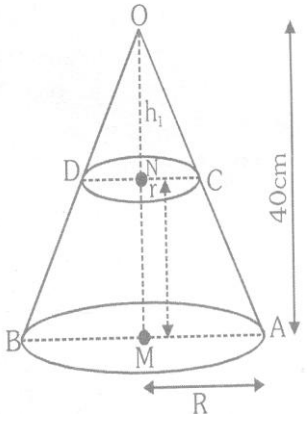
Ex.24 The height of a cone is 40 cm. A small cone is cut off at the top by a plane parallel to the bases. It the volume of the small cone be $\frac{1}{64}$ of the volume of the given cone, at what height above the base is the section made

Sol. Let R be the radius of the given cone, r the radius of the small cone, h be the height of the frustum and h_1 be the height of the small cone.

In figure 13.49, ΔONC and ΔOMA are similar (ΔONC - ΔOMA)

$\therefore \quad \frac{ON}{OM} = \frac{NC}{MA} \quad \Rightarrow \quad \frac{h_1}{40} = \frac{r}{R}$

$\Rightarrow \quad h_1 = \left(\frac{r}{R}\right)40 \quad \ldots(i)$



We are given that $\frac{Volume\ of\ small\ cone}{Volume\ of\ given\ cone} = \frac{1}{64}$

$\Rightarrow \quad \frac{\frac{1}{3}\pi r^2 \times h_1}{\frac{1}{3}\pi R^2 \times 40} = \frac{1}{64}$

$\Rightarrow \quad \frac{r_2}{R_2} \times \frac{1}{40} \times \left\{\left(\frac{r}{R}\right)40\right\} = \frac{1}{64} \quad$ (By 1)

$\Rightarrow \quad \left(\frac{r}{R}\right)^3 = \frac{1}{64} = \left(\frac{1}{4}\right)^3 \Rightarrow \frac{r}{R} = \frac{1}{4} \quad \ldots(2)$

From (i) and (ii) $h_1 = \frac{1}{4} \times 40 = 10cm$

Therefore, $h = 40 - h_1 = (40 - 10)$ cm

$\Rightarrow \quad h = 30$ cm

Ex.25 The radius of the base of a right circular cone is r. It is cut by a plane parallel to the base a height h from the base. The slant height of the frustum is $\sqrt{h^2 + \frac{4}{9}r^2}$. Show that volume of the frustum is $\frac{13}{27}\pi r^2 h$.

Sol. In figure 13.50, $\ell = \sqrt{h^2 + \frac{4}{9}r^2}$ is the slant height of frustum of the given cone having base radius r. O is the centre of the base and O' is the centre of the top of the frustum.

OO' = h (given)

AOB and COD are diameters of the lower and upper faces of the frustum. Draw DE $\perp$ OB. Let O' P = x

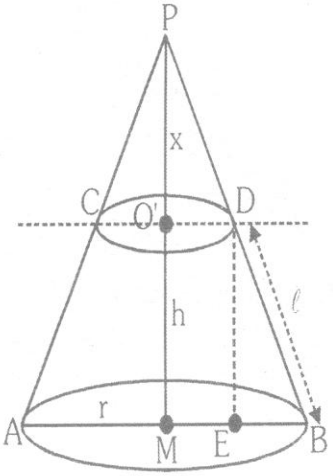
In right angled ΔDEB,

$DB^2 = BE^2 + DE^2$

$\Rightarrow \quad \ell^2 = BE + h^2 \quad (\because DE = OO' = h)$

$\Rightarrow h^2 + \frac{4}{9}r^2 = h^2 + BE^2 \quad \Rightarrow \quad BE^2 = \frac{4}{9}r^2$

$\Rightarrow \quad BE = \frac{2}{3}r \quad \Rightarrow \quad OE = r - \frac{2}{3}r = \frac{1}{3}r$



$\Rightarrow$ $O'D = \frac{1}{3}r$ is the radius of the top face of the frustum.

Now, $\Delta PO'D \sim \Delta POB$

$\Rightarrow$ $\frac{PO'}{O'D} = \frac{PO}{PB}$ $\Rightarrow$ $\frac{x}{\frac{1}{3}r} = \frac{h+x}{r}$

$\Rightarrow$ $3x = h + x$ $\Rightarrow$ $x = \frac{1}{2}h$.

Volume of the frustum = Volume of the cone PAB – Volume of the cone PCD

$$= \frac{1}{3}\pi \times (r^2) \times OP - \frac{1}{3}\pi \times \left(\frac{1}{3}r\right)^2 \times O'P$$

$$= \frac{1}{3}\pi r^2 \times (h+x) - \frac{1}{27}\pi r^2 \times x$$

$$= \frac{1}{3}\pi r^2 \times \left(h + \frac{1}{2}h\right) - \frac{1}{27}\pi r^2 \times \frac{1}{2}\ h$$

$$= \left(\frac{1}{2}\pi r^2 - \frac{1}{54}\pi r^2\right)h = \frac{26}{54}\pi r^2 h = \frac{13}{27}\pi r^2 h$$

Hence, the required volume is $\frac{13}{27}\pi r^2 h$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

1. If the volume of a cube is 1728 cm^3, the length of its edge is equal to
(a) 12 cm (b) 14 cm (c) 16 cm (d) 24 cm

2. Two cubes each of 10 cm edge are joined end to. The surface area of the resulting cuboid is
(a) 1200 cm^2 (b) 1000 cm^2 (c) 800 cm^2 (d) 1400 cm^2

3. A rectangular sheet of paper 44 cm $\times$ 18 cm is rolled along its length and a cylinder is formed. The volume the cylinder so formed is equal to $(Take\ \pi = \frac{22}{7})$
(a) 2772 cm^3 (b) 2505 cm^3 (c) 2460 cm^3 (d) 2672 cm^3

4. If the radius and height of a cylinder are in ratio 5 : 7 and its volume is 550 cm^3, then its radius is equal to $(Take \pi = \frac{22}{7})$

(a) 6 cm (b) 7 cm (c) 5 cm (d) 10 cm

5. If the curved surface area of a solid right circular cylinder of height h and radius r is one-third of its total surface area, then

(a) $h = \frac{1}{3}r$ (b) $h = \frac{1}{3}r$ (c) $h = r$ (d) $h = 2r$

6. A hollow cylindrical pipe is 21 cm long. If its outer and inner diameters are 10 cm and 6 cm respectively, then the volume of the metal used in making the pipe is $(Take \pi = \frac{22}{7})$

(a) 1048 cm^3 (b) 1056 cm^3 (c) 1060 cm^3 (d) 1064 cm^3

7. If the radius and slant height of a cone are in the ratio 4 : 7 and is curved surface area is 792 cm^2, then its radius is $(Take \pi = \frac{22}{7})$

(a) 10 cm (b) 8 cm (c) 12 cm (d) 9 cm

8. If the radius of the base and the height of a right circular cone are respectively 21 cm and 28 cm, then the curved surface area of the cone is $(Take \pi = \frac{22}{7})$

(a) 3696 cm^2 (b) 2310 cm^2 (c) 2550 cm^2 (d) 2410 cm^2

9. A conical tent with base-radius 7 m and height 24 m is made from 5 m wide canvas. The length of the canvas used is $(Take \pi = \frac{22}{7})$

(a) 100 m (b) 105 m (c) 110 m (d) 115 m

10. The total surface area of a solid hemisphere of radius 3.5 m is covered with canvas at the rate of Rs. 20 per m^2. The total cost to cover the hemisphere is $(Take \pi = \frac{22}{7})$

(a) Rs. 2210 (b) Rs. 2310 (c) Rs. 2320 (d) Rs. 2420

11. If the volume of a vessel in the form of a right circular cylinder is 448π cm^3 and its height is 7 cm, then the curved surface area of the cylinder is

(a) 224π cm^2 (b) 212π cm^2 (c) 112π cm^2 (d) none of these

12. If the curved surface area of a right circular cone is 12320 cm^2 and its base-radius is 56 cm, then its height is $(Take \pi = \frac{22}{7})$

(a) 42 cm (b) 36 cm (c) 48 cm (d) 50 cm

13. If a solid metallic sphere of radius 8 cm is melted and recasted into spherical solid balls of radius 1 cm, then n =

(a) 500 (b) 510 (c) 512 (d) 516

14. If the diameter of a metallic sphere is 6 cm, it melted and a wire of diameter 0.2 cm is drawn, then the length of the wire made shall be

(a) 24 m (b) 28 m (c) 32 m (d) 36 m

15. If n coins each of diameter 1.5 cm and thickness 0.2 cm are melted and a right circular cylinder of height 10 cm and diameter 4.5 cm is made, then n =

(a) 336 (b) 450 (c) 512 (d) 545

16. A tent is in the form of a cylinder of diameter 8 m and height 2 m, surmounted by a cone of equal base and height 3 m. The canvas used for making the tent is equal to

(a) 36π m^2 (b) 28π m^2 (c) 24π m^2 (d) 32π m^2

17. A toy is in the form of a cone mounted on a hemisphere with same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. The surface area of the toy is
(a) 36π cm^2 (b) 33π cm^2 (c) 35π cm^2 (d) 24π cm^2

18. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm. The volume of the frustum is
(a) 3328π cm^3 (b) 3228π cm^3 (c) 3240π cm^3 (d) 3340π cm^3

19. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm has lateral surface area equal to
(a) 540π cm^2 (b) 580π cm^2 (c) 560π cm^2 (d) 680π cm^2

20. A solid metal cone with base-radius 12 cm and height 24 cm, is melted to form solid spherical balls, each of diameters 6 cm. The number of such balls made is
(a) 32 (b) 36 (c) 48 (d) none of these

OBJECTIVE			ANSWER				EXERCISE-4			
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	A	B	A	A	B	B	C	B	C	B
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	C	A	C	D	B	A	B	A	C	A

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CONVERSION OF SOUPS

1. Two cubes of the side 10 cm are joined end to end. Find the surface area of the resulting rectangular shaped solid

2. Three cubes each of side 4 cm are joined end to end. Find the surface area of the resulting rectangular cuboid.

3. The six cube marked I, II, III, IV, V, VI each of side 3 cm are placed as shown in fig. It takes the shape of a cuboid. Find the surface area of the cuboid.

I	II	II
I	V	V

4. A rectangular solid metallic cuboid 18 cm $\times$ 15 cm $\times$ 4.5 cm is melted and recast into solid cubes each of side 3 cm. How many solid cubes can be made ?

5. A rectangular solid metallic cuboid 32 cm $\times$ 27 cm $\times$ 15 cm is melted and recast into solid cubes each of side 6 cm. How many solid cubes can be made from the metal.

6. Two rectangular solid metallic cuboid 12 cm $\times$ 10 cm $\times$ 5 cm and 12 cm $\times$ 5 cm $\times$ 4 cm are melted together and recast into solid cubes each of side 2 cm. How many solid cubes can be made from the metal.

7. Three rectangular solid metallic cuboid 20 cm × 10 cm × 5 cm, 15 cm ×10 cm × 4 cm and 15 cm ×12 cm × 5 cm are melted together and recast into solid cubes each of side 2 cm. How many solid cubes can be made from the metal.

8. The side of a metallic cube 35 cm. The cube is melted and recast into 1000 equal solid dice. Determine the side of the dice.

9. Two solid metallic cube sides 40 cm and 30 cm are melted together into 160 equal solid cubical dice. Determine the side of the dice.

10. Three solid metallic cubes 60 cm, 50 cm and 30 cm are melted together and recast into 875 equal solid cubical dice. Determine the side of the dice.

11. The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform circular cross-section. If the length of the wire is 36 m, find the radius of its cross-section.

12. The diameter of a metallic sphere is 18 cm. The sphere is melted and drawn into a wire having diameter of the cross-section as 0-4 cm. Find the length of the wire.

13. How many balls, each of radius 0.5 cm, can be made from a solid sphere of metal of radius 10 cm by melting the sphere ?

14. A spherical ball of lead 5 cm in diameter is melted and recast into three spherical balls. The diameters of two of these balls are 2 cm and 2 $(1405)^{1/3}$ cm. Find the diameter of the third ball.

15. How many bullets, can be made out of a solid cube of lead whose edge measures 44 cm cm and diameter of each bullet being 4 cm.

16. How many spherical lead shots each 4.2 cm in dimater can be obtained from a rectangular solid (cuboid)of lead with dimensions 66 cm, 42 cm, 21 cm.(Take $\pi = 22/7$)

17. H**ow** many spherical balls each of 5 cm in diameter can be cast from a rectangular block of metal 11× dm ×10 dm ×5 dm ? (1 dm = 10 cm)

18. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 32 m of uniform thickness (diameter). Find the thickness of the wire.

19. 56 circular plates, each of radius 5 cm and thickness 0.25 cm, are placed one above another to form a solid right circular cylinder. Find the curved surface and the volume of the cylinder so formed.

20. The diameter of a metallic sphere is 4.2 cm. It is melted and recast into a right circular cone of height 8.4 cm, Find the radius of the base of the cone.

21. A right circular metallic cone of height 20 cm and radius of base 5 cm is melted and recast into a sphere. Find the radius of the sphere.

22. A right circular cone of height 81 cm and radius of base 16 cm is melted and recast into a right circular cylinder of height 48 cm. Find the radius of the base of the cylinder.

23. A spherical shell of lead, whose external diameter is 24 cm, is melted and recast into a right circular cylinder, whose height is 12 cm and diameter 16 cm. Determine the internal diameter of the shell.

24. The internal and external radii of a metallic spherical shell are 4 cm and 8 cm, respectively. It is melted and recast into a solid right circular cylinder of height $9\frac{1}{3}$ cm. Find the diameter of the base of the cylinder.

25. A right circular cone is of height 3.6 cm and radius of its base 1.6 cm. It is melted and recast into a right circular cone with radius of its base 1.2 cm. Find the height of the cone so formed.

26. A solid metallic right circular cylinder 1.8 m high with diameter of its base 2 m is melted and recast into a right circular cone with base of diameter 3 m. Find the height of the cone.

27. Find the number of coins, 1.5 cm in diameter and 0.2 cm thickness, to be melted to form a right circular cylinder o f height 10 cm and diameter 4.5 cm.

28. A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylinder vessel with internal radius 10 cm. Find the height to which the water rises.

29. A well, whose diameter is 4 m, has been dug 16 m deep and the earth dug out is used to form an embankment 8 m wide around it. Find the height of the embankment.

30. A well, whose diameter is 3.5 m, has been dug 16 m deep and the earth dug out is used to form a platform 27.5 m by 7 m just near the site of the well. Find the height of the platform. (Take $\pi = 22/7$)

31. The base radius and height of a right circular solid cone are 12 cm and 24 cm respectively. It is melted and recast into spheres of diameter 6 cm each. Find the number of spheres so formed.

32. The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. It is melted and recast into a solid cone of base diameter 14 cm. Find the number of spheres so formed.

33. A solid metallic sphere of diameter 28 cm is melted and recasted into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find the number of cones so formed.

34. A conical flask is full of water. The flask has base-radius 3 cm and height 15 cm. The water is poured into a cylindrical glass tube of uniform inner radius 1.5 cm, placed vertically and closed at the lower end. Find the height of water in the glass tube.

35. Find the depth of a cylindrical tank of radius 10.5 m, if its capacity is equal to that of a rectangular tank of size 15 m $\times$ 11 m $\times$ m $\times$ 10.5 m. (Take $\pi = 22/7$)

36. A rectangular tank 28 m ling and 22 m wide is required to receive entire water from a full cylindrical tank of internal diameter 28 m and depth 4 m. Find the least height of the tank that will serve the purpose(Take $\pi = 22/7$)

37. A conical flask is full of water. The flask has base-radius a and height 2 a. The water is poured into a cylindrical flask of base-radius $\frac{2a}{3}$ Find the height of water in the cylindrical flask.

38. A sphere of diameter 2 a is dropped into a cylindrical vessel partly filled with water. The diameter of the base of the vessel is $\frac{8a}{3}$ If the sphere is completely submerged, by how much will the level of water rise ?

39. The rain water from a roof 44 m $\times$ 20 m drains into a cylindrical vessel having diameter 2 m and height 2.8 m. If the vessel is justfull, find the rainfall in cm.

40. An agricultural field is in the form of a rectangle of length 20 m and width 14 m. A 10 m deep well of diameter 7 m is dug in a corner of the field and the earth taken out of the well is spread evenly over the remaining part of the field. Find the rise in its level. (Take $\pi = 22/7$)

41. 600 persons took dip in a rectangular tank which is 60 m long and 40 m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m^3 ?

42. The largest sphere is curved cut of a cube whose edge is of length ℓ units. Find the volume of the sphere

43. The largest right circular cone is curved out of a cube whose edge is o length p units. Find the volume of the cone.

44. Two solid right circular cones have same height. The radii of their bases are 4 cm and 3 cm. They are melted and recast into a right circular cylinder of same height. Find the radius of the base of the cylinder. $\left(Take \frac{1}{\sqrt{3}} = .577\right)$

45. Water is being pumped out through a circular pipe whose diameter is p cm. If the flow of water is 14 p cm per second, how many litres of water are being pumped cut in one hour? $(Take\ \pi = 22/7)$

46. Water flow out through a circular pipe, whose internal diameter is $1\frac{1}{3}$ cm, at the rate of 0.63 m per second into a cylinder tank, the radius of whose base is 0.2 m. By how much will the level of water rise in one hour.

47. Water in a canal 4 m wide and 1.5 m deep is flowing with velocity 12 km per hour. How much area will it irrigate in 30 minutes, if 9 cm of standing water is required for irrigation?

48. Water flow at the rate of 15 m per minute through a cylindrical pipe having its diameter 1.2 cm. How much time will it tank to fill a conical vessel whose diameter of base is 40 cm and depth 81 cm ?

49. A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to half-empty the tank, if the tank is 3 metres in diameter $(Take\ \pi = 22/7)$

50. A conical tank is full of water. Its base-radius is 1.75 m and height 2.25 m. It is connected with a pipe which empties it at the rate of 7 litres per second. How much time will it take to empty the tank completely ?
$(Take\ \pi = 22/7)$

51. Two solid metallic right circular cones have same height h. The radii of their bases are r_1 and r_2. The two cones ae melted together and recast into a right circular cylinder of height h. Show that radius of the base of the cylinder is $\sqrt{\frac{1}{3}(r_1^2 + r_2^2)}$

52. The radii of the bases of two right circular solid metallic cones of same height h are r_1 and r_2. The cones are

melted together and recast into a solid sphere of radius R. Show that $h = 4\left(\frac{R^3}{r_1^2 + r_2^2}\right)$

53. The radii of the solid metallic spheres are r_1 and r_2. The sphere are melted together and recast in a solid cone of height $(r_1 + r_2)$. Show that the radius of the cone is $2 \times \sqrt{r_1^2 + r_2^2 + r_1 r_2}$

54. The radii of a solid metallic sphere is r. A solid metallic cone of height h has base radius r. The two are melted together and recast into a solid right circular cone with base radius r. Prove that the height of the resulting cone is 4 r + h.

55. A solid metallic right circular cylinder and a solid metallic right circular cone are giver. The cylinder and cone both have same height h and same base radii r. The two solids are melted together and recast into a solid cylinder of radius $\frac{1}{2}$ r. Prove that the height of the cylinder is $\frac{16}{3}$ h.

SURFACE AREAS & VOLUMES OF COMBINATIONS OF SOLIDS

1. A solid is in the form of a cone mounted on a right circular cylinder both having same radii of their bases. Base of the cone is placed on the top base of the cylinder. If the radius of the base and height of the cone be 4 cm and 7 cm respectively and the height of the cylindrical part of the solid is 3.5 cm, find the volume of the solid.

2. A solid is in the form of a right circular mounted on a solid hemisphere of radius 14 cm. The radius of the base of the cylindrical part is 14 cm and the vertical height of the complete solid is 28 cm. Find :

(i) The volume of the solid

(ii) The surface area of the solid

(iii) Cost of painting the solid at the rate of Rs. 0.80 cm^2.

3. A solid is in the form of a cone of vertical height 9 cm mounted on the top base of a right circular cylinder of height 40 cm. The radius of the base of the cone and that of the cylinder are both equal to 7 cm. Find the weight of the solid if 1 cm^3 of the solid weight 4 gm.

4. A solid is in the form of a right circular cone mounted on a solid hemisphere with same radius is made from a piece of metal. The radius of the hemisphere is $\frac{1}{3}$ of the vertical height of the conical part. If the radius of the base of the cone is r, prove that the volume of the piece of metal is $\frac{5}{3}\pi r^3$

5. A solid wooden toy is in the shape of a right circular cone mounted on a solid hemisphere with same radius. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy. $(Take\ \pi = 22/7)$

6. A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and diameter of the base is 4 cm. If a right circular cylinder circumscribes the solid, find how much more space it will cover. $(Take\ \pi = 3.14)$

7. A cylindrical tub of radius 5 cm and height 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed completely into the tub. If the radius of the hemisphere is 3.5 cm and the height of the conical part is 5 cm, find the volume of water left in the tub. $(Take\ \pi = 22/7)$

8. A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The ice-cream is to be distributed to 10 children in equal cones with hemisphere tops. If the height the height of the conical portion is 4 times the radius of its base, find the radius of the ice-cream cone.

9. A right circular cylinder having diameter 18 cm and height 20 cm is full of ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

10. A circus tent has cylindrical shape surmounted by a conical reef. The radius of the cylindrical base is 40 m. The heights of the cylindrical and conical portions are 6.3 m and 4.2 m, respectively. Find the volume of the tent. $(Take\ \pi = 22/7)$

11. A circus tent is cylindrical up to a height of 3 m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53 m, find the total cast of the canvas used to make the tent when the cost per square metre of the canvas is Rs. 10. $(Take\ \pi = 22/7)$

12. A tent of height 11 m is in the form of a right circular cylinder with diameter of base 30 m and height 3 m, surmounted by a right circular cone of the same base. Find the cost of the canvas of the tent at the rate of Rs. 25 per m^2. $(Take\ \pi = 22/7)$

13. A tent is in the form of a cylinder of diameter 15 m and height 2.4 m, surmounted by a cone of equal base and height 4 m. Find the capacity of the tent and the cost of the canvas at Rs. 50 per square metre. $(Take\ \pi = 22/7)$

14. A iron pillar has some part in the form of a right circular and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if cone cubic cm of iron weight 7.8 gracs.

15. The interior of a building is in the form of a right circular of diameter 4.2 m and height 4m, surmounted by a cone. The vertical height of the cone is 2.1 m. Find the outer surface area and volume of the building. $(Take\ \pi = 22/7)$

16. The in interior of a building is in the form of a right circular of diameter 4.3 m and height 3.8 m, surmounted by a cone whose vertical angel is right angle. Find the area of the surface and the volume of the building. $(Take\ \pi = 22/7)$

17. A vessel is in the form of a hemispherical bowl, surmounted by a hollow cylinder. The diameter of the hemisphere is 12 cm and the total height of the vessel is 16 cm. Find the capacity of the vessel. $(Take\ \pi = 22/7)$ Also find the internal surface area of the vessel by taking $\pi = 3.14$.

18. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid. $(Take\ \pi = 22/7)$

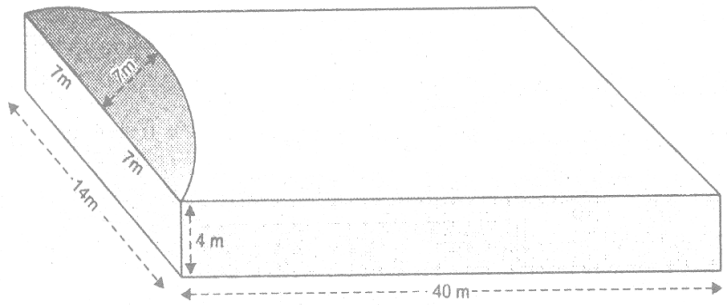
19. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm and the diameter of the hemisphere ends is 36 cm, find the cost of polishing the surface of the solid at the rate of 10 paise per cm^2. $(Take\ \pi = 22/7)$

20. A solid toy is in the form of a right circular cylinder with a hemispherical shape at cone end and a cone at the other end. Their common diameter is 4.2 cm and the heights of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the solid. $(Take\ \pi = 22/7)$

21. A petrol tank is a cylinder of base diameter 28 cm and length 24 cm fitted with conical ends each of axis-length 9 cm. Determine the capacity of the tank.

22. Form a solid right circular cylinder with height h and radius of the base r, a right circular cone of the same height And same base is removed. Find the volume of the remaining solid.

23. A right circular cone with sides 12 cm and 16 cm is revolved around its hypotenuse. Find the volume of the double cone so formed.



24. A godown building as shown in figure is made in the form of a cuboidal base with dimensions 40 m × 14 m ×4 m, surmounted by a half cylindrical curved roof having same length as that of the base. The diameter of the cylinder is 14 m. Find the volume of the building and its total outer surface area.

25. Find the mass of a 3.5 m long lead pipe, if the external diameter of the pipe is 2.4 cm, thickness of the metal is 2 mm and mass of 1 cm^3 of lead is 11.4 g. $(Take\ \pi = 22/7)$

26. A cylindrical vessel of diameter 16 cm and height h cm is fixed symmetrically inside a similar vessel of diameter 20 cm and height h cm. The total space between the two vessels is filled with cork dust. How many cubic centimeters of cork dust is used.

27. The interior of a building is in the form of cylinder of radius 4 m and height 3.5 m, surmounted by a cone of vertical angle 90^0. Find the surface area of the interior of the building (excluding the flooring area of the building). Also find the cost of painting the interior of the building at the rate of Rs. 5 per m^2. Use $\pi = 22/7 \; and \sqrt{2} = 1.414$

28. A solid is in the form of a cone of vertical height h mounted on a right circular cylinder of height 2 h and both having same radii of their bases. Base of the cone is placed on the top base of the cylinder. If V cube units be the volume of the solid, prove that the radius of the cylinder is $\sqrt{\frac{3V}{7\pi h}}$.

29. A solid is in the form of a cone of vertical height h mounted on the top base of a right circular cylinder of height $\frac{1}{3}h$. The circumference of the base of the cone and that of the cylinder are both equal to C. If V be the volume of the solid, prove that $C = 4\sqrt{\frac{3\pi V}{7h}}$.

30. A conical vessel of radius 12 cm and depth 16 cm is completely filled with water. A sphere is lowered into the water and its size is such that than when it touches inner curved surface of the vessel, it is just immersed upto the topmost point of the sphere. How much water over flows out of the vessel out of the total volume V cubic nuits.

FRUSTUM OF A RIGHT CIRCULAR CONE

1. A bucket of height 3 cm and made up of metal sheet is in the form of frustum of a right circular cone with radii of its lower and upper ends as 6 cm and 10 cm respectively. Calculate:

(i) the height of the cone of which the bucket is a part.
(ii) the volume of water which can be filled in the bucket.
(iii) the slant height of the bucket.
(iv) the area of the metal sheet required to make the bucket.

2. The radii of the circular ends of a frustum of a right circular cone are 5 cm and 8 cm and its lateral height (slant height) is 5 cm. Find the volume of the frustum. $(Take\ \pi = 22/7)$

3. The radii of the circular ends of a bucket frustum of a right circular cone are 14 cm an d2 cm and is thickness is 9 cm. Find the lateral surface of the frustum. $(Take\ \pi = 22/7)$

4. If the radii of the circular ends of a bucket 24 cm high are 5 cm and 15 cm respectively, find the inner surface area of the bucket (i.e., the area of the metal sheet required to make the bucket) $(Take\ \pi = 3.14)$

5. A bucket is in the form of a cone, its depth is 30 cm and the diameters of the top and the bottom are 42 cm and 14 cm respectively. Find how many litres of water can the bucket hold ? $(Take\ \pi = 22/7)$

6. A container made up of a metal sheet is in the form of a frustum of a cone of height 12 cm with radii of its lower and upper ends as 3 cm and 12 cm respectively. Find the cost of metal sheet used, if it costs Rs. 4 per 100 cm^2. $(Take\ \pi = 22/7)$

7. A vessel is in the form of a frustum of a cone of height 21 cm with radii of its lower and upper ends as 8 cm and 18 cm respectively. Find the cost of milk which can completely fill the vessel at the rate of Rs. 10 per litre.

8. The perimeters of the ends of a frustum are 48 cm and 36 cm. If the height of the frustum be 11 cm, fid the volume of the frustum. $(Take\ \pi = 22/7)$

9. The slant height of the frustum of a cone is 4 cm. If the perimeters of its circular bases be 18 cm and 6 cm, find the curved surface area of the frustum and also find the colt of painting its total surface at the rate of Rs. 12.50 per 100 cm^2.

10. The height of a cone is 30 cm. A frustum is cut off from this cone by a plane parallel to the base of the cone. If the volume of the frustum is $\frac{19}{27}$ of the volume of the cone, find the height of the frustum.

11. The height of a cone is 10 cm. The cone is divided into two parts by drawing a plane through the midpoint of the axis of the cone, parallel to the base. Compare the volume of the two parts.

12. A hollow cone is cut by a plane parallel to the base and upper part is removed. If the curved surface of the remainder is $\frac{15}{16}$ of the curved surface of the whole cone, find the ratio of the line-segments into which the cone's altitude is divided by the plane.

13. A right circular cone is cut by a plane parallel to the base of the cone and the upper portion is removed. If the curved surface of the frustum is $\frac{8}{9}$ of the curved surface of the whole given cone, prove that the height of the frustum is $\frac{2}{3}$ of the height of the whole cone.

14. The altitude of a right circular cone is trisected by two parallel planes, drawn parallel to the base of the cone. The cone is cut into three parts. The topmost part is a right circular cone, the middle one and last one at the bottom are two frustums. If V_1 be the volume of the small cone, V_2 be the volume of the middle portion frustum and V_3 be the volume of the frustum made at the bottom, prove the $V_1 : V_2 : V_3 = 1 : 7 : 19$.

15. A right circular cone is divided by a plane parallel to its base into a small cone of volume V_1 at the top and a frustum of volume V_2 as second part at the bottom. If $V_1 : V_2 = 1 : 3$, find the ratio of the height of the altitude of small cone and that of the frustum.

CONVERSION OF SOLIDS

1. 1000 cm^2 **2.** 224 cm^2 **3.** 198 cm^2 **4.** 45 **5.** 60 **6.** 105 **7.** 20 **8.** 3.5 cm **9.** 2.5 cm
10. 2 cm. **11.** 0.1 cm **12.** 243 m **13.** 8000 **14.** 1 cm **15.** 2541 **16.** 1500 **17.** 8400 **18.** 0.05 cm
19. 1100 cm^3 **20.** 2.1 cm **21.** 5 cm **22.** 12. cm **23.** 8 $(18)^{1/3}$cm **24.** 16 cm **25.** 6.4 cm
26. 2.4 cm **27.** 450 **28.** 2 cm **29.** 75 cm **30.** 80 cm **31.** 32 **32.** 4 cm **33.** 672 **34.** 20 cm
35. 5 m **36.** 4 m **37.** $\frac{3}{2}$a **38.** $\frac{3a}{4}$ **39.** 1 cm **40.** 1.6 cm approax. **41.** 1cm **42.** $\frac{\pi \ell^3}{6}$
43. $\frac{\pi p^3}{12}$ **44.** 2.885 cm **45.** 39.6 p^3 litres **46.** 2.52 m **47.** 400000 m^2 **48.** 20 min.
49. 16.5 min. **50.** $17\frac{3}{16}$ m.

SURFACE AREAS AND VOLUMES OF COMBINATIONS OF SOLIDS

1. $293\frac{1}{3}cm^3$ **2.** (i) $14373\frac{1}{3}cm^3$ (ii) 3080 cm^2 (iii) Rs. 2464 **3.** 26.488 kg **5.** 266.11 cm3 **6.** 25.12 cm3
7. 616 cm^3 **8.** 3 cm **9.** 30 **10.** 48720 m^3 **11.** Rs. 97350 **12.** Rs. 27082.5 **13.** 660 m3 ; Rs. 15675
14. 395. 4 kg approx. **15.** 72. 4 m^2 ; 65. 142 m^3 **16.** 71.83 m^2, **17.** 1584 cm^3 ; 602.88 cm^2
18. 641 66 cm^3 ; 418 cm^2 **19.** Rs. 1221.94 **20.** 218.064 cm^3 **21.** 18480 cm^3 **22.** $\frac{2}{3}\pi r^2 h$
23. 798 .816 cm^3 **24.** 4320 m^3 ; 1096 m^2 **25.** 50518 kg **26.** $36\pi h cm3$ **27.** Rs. 159.104 m^2 ; Rs. 795.52
28. $\frac{3}{8}V$

FRUSTRUM OF A RIGHT CIRCULAR CONE

1. 7.5 cm, 196 π cm^3, 5 cm 116 π cm^2. **2.** 540.57 cm^3 **3.** 753.6 cm^2 **4.** 1711.30 cm^2 **5.** 20.02 liters
6. Rs. 47.52 **7.** Rs. 117.04 **8.** 1554 cm3 **9.** Rs. 9.58 **10.** 10 cm **11.** 1 : 7 **12.** 1 : 3 **15.** 1 : $(4^{1/3} - 1)$

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PEREVIOUS YEARS BOARD (CBSE) QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

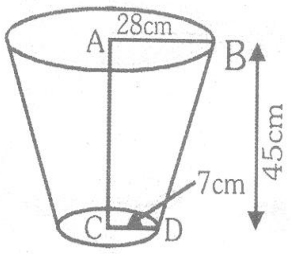
1. The surface area of a sphere is 616 cm^2. Find its radius. **[Foreign – 2008]**
2. A cylinder and a cone area of same base radius and of same height. Find the ratio of the volume of cylinder to that of the cone. **[Delhi – 2009]**
3. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm. write the height of the frustum. **[Al – 2010]**

SHORT ANSWER TYPE QUESTIONS

1. A solid metallic sphere of diameter 21 cm is melted and recasted into a number of smaller cones, each of diameter 7 cm and height 3 cm. Find number of cones so formed. **[Delhi – 2004]**
2. A solid metallic sphere of diameter 28 cm is melted and recasted into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find number of cones so formed. **[Delhi – 2004]**
3. A hemispherical bowl of internal diameter 30 cm contains some liquid. This liquid is to be filled into cylindrical shaped bottles each of diameter 5 cm and height 6 cm. Find the number of bottles necessary to empty the bowl. **[Al – 2004]**
4. Solid spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 18 cm and the water rises by 40 cm, find the number of solid spheres dropped in the water. **[Foreign – 2004]**
5. A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy. [use π = 22 / 7] **[Delhi – 2007]**
6. A toy is in the form of a cone mounted on a hemisphere with same radius. The diameter of the base of the conical portion is 7 cm and total height of the toy is 14.5 cm. Find the volume of the toy. [use π = 22 / 7] **[Al – 2007]**

LONG ANSWER TYPE QUESTOINS

1. If the radii of the circular ends of a bucket, 45 cm high are 28 cm and 7 cm (as shown in given fig.), find the capacity of the bucket. **[Al – 2004]**



2. A hollow cone is cut by a plane parallel to the base and the upper is removed. If the curved surface of the remainder is 8/7th of the curved surface of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane.

OR

If the radii of the ends of a bucket, 45 cm high, are 28 cm and 7 cm, find its capacity and surface area. **[Delhi – 2004C]**

3. A well, of diameter 3m, is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4m, to form an embankment. Find the height of the embankment. [use π = 22 / 7] **[Al – 2004C]**
4. If the radii of the ends of a bucket, 45 cm high are 28 cm and 7 cm, determine the capacity and total surface area of the bucket. **[Al – 2005]**
5. The rain water form a roof 22 m × 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the vessel is just full, find the rainfall in cm. **[Delhi – 2006]**

6. Water flows at the rate of 10 m per minute through a pipe having its diameter as 5 mm? How much time will it take to fill a conical vessel whose diameter of base is 40 cm and depth 24 cm? **[Foreign – 2006]**

7. A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

OR

A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter of the sphere. **[Delhi – 2007]**

8. A hemispherical bowl of internal diameter 36 cm is full of some liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. Find the number of bottles needed to empty the bowl.

OR

Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm/second into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will be level of water rise in the tank in half an hour. **[Al – 2007]**

9. A gulab jamun, when ready for eating, contains sugar syrup of about 30% of its volume. Find approximately how much syrup would be found in 45 such gulab jamuns, each shaped like a cylindrical with two hemispherical ends, if the complete length of each of them is 5 cm and it's diameter is 2.8 cm.

OR

A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. This ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream. **[Delhi – 2008]**

10. A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with diameters of it's lower and upper ends as 16 cm and 40 cm respectively. Find the volume of the bucket. Also find the cost of the bucket if the cost of metal sheet used is Rs. 20 per 100 cm^2 [use π = 3.14]

OR

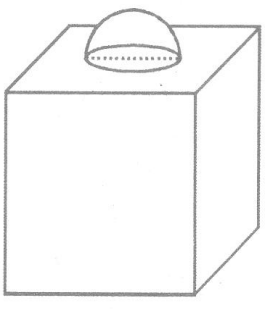
A farmer connects a pipe of internal diameter 20 cm from a canal in to a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 6 km/hr, in how much time will be tank be fillrd? **[Delhi – 2008]**

11. A tent consists of a frustum of a cone, surmounted by a cone. If the diameter of the upper and lower circular ends of the frustum be 14 m and 26 m respectively, the height of the frustum be 8 m and the slant height of the surmounted conical portion be 12 m, find the area of canvas required to make the tent. (Assume that the radii of the upper circular end of the frustum and the base of surmounted conical portion are equal). **[Al – 2008]**

12. If the radii of the circular ends of a conical bucket, which is 16 cm high, are 20 cm and 8 cm, find the capacity and total surface area of the bucket. [use π = 22 / 7] **[Foreign – 2008]**

13. Form a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid cored to two places of decimals. Also, find the total surface of the remaining solid. [Take π = 3.1416] **[Delhi – 2009]**

14. In figure, a decorative block which is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere, fixed on the top, has a diameter of 4.2 cm. Find the total surface area of the block. [Take π = 22 / 7] **[Al – 2009]**



15. A spherical copper shell, of external diameter 18 cm is melted and recast into a solid cone of base radius 14 cm and height $4\frac{3}{7}$ cm. Find the inner diameter of the shell.

OR

A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of metal sheet used in making it. [use π = 22 / 7]

16. The rain-water collected on the roof of a building, of dimensions 22 m × 20 m, is drained into a cylindrical vessel having base diameter 2m and height 3.5 m. If the vessel is full up to the brim, find the height of rain-water on the roof. [use π = 22 / 7] **[AI – 2010]**

SURFACE AREAS AND VOLUMES | ANSWER KEY | EXERCISE-2 (X)-CBSE

- **VERY SHORT ANSWER TYPE QUESTIONS**

1. 7 cm **2.** 3 : 1 **3.** 3 cm

- **SHORT ANSWER TYPE QUESTIONS**

1. 126 **2.** 672 **3.** 60 **5.** 858 cm^2 **6.** 231 cm^3

- **ANSWER TYPE QUESTIONS**

1. 48510 cm^2 **2.** 1 : 2 or 48510 cm^3, 5616.38 cm^2 **3.** 1.125 m 4. 48510 cm^3, 8079.5 cm^2 **5.** 2.5 cm
6. 51.2 min 7. 18 cm or 6 cm **8.** 72 or 90 cm **9.** 338.184 cm^3 or 10 cones **10.** Rs. 391.87 or 50 minutes

11. (284π) m^2 **12.** $\frac{73216}{7}cm^3, \frac{13728}{7}cm^2$ **13.** 150.79 cm^3, 259.55 cm^2 **14.** 163.86 cm^2
15. 16 cm or cm ; 2160.32 cm^2 **16.** 2.5 cm

EXERCISE-4 FOR OLYMPIADS

1. One cubic metre piece of copper is melted and recast in to a square cross-section bar, 36 m long. An exact cube is cut off from this. If cubic metre of copper cost Rs. 108, then the cost of this cube is :
(A) 50 paisa (B) 75 paisa (C) One paisa (D) 1.50 paisa

2. If the surface areas of two spheres are in the ratio 4 : 9, then the ratio of their volume is :
(A) 8 : 25 (B) 8 : 26 (C) 8 : 27 (D) 8 : 28

3. In a shower 10 cm of rain fall. The volume of water that falls on 1.5 hectares of ground is :
(A) 1500 m^3 (B) 1400 m^3 (C) 1200 m^3 (D) 1000 m^3

4. The radius of base and the volume of a right circular cone are doubled. The ratio of the length of the larger cone to that of the smaller cone is :
(A) 1 : 4 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1

5. A cone and a hemisphere have equal base diameter and equal volume. The ratio of their heights is :
(A) 3 : 1 (B) 2 : 1 (C) 1 : 2 (D) 1 : 3

6. If the lateral surface of a right circular cone is 2 times its base, then the semi-vertical angle of the cone must be :
(A) 15^0 (B) 30^0 (C) 45^0 (D) 60^0

7. The slant height of a conical tent made of canvas is $\frac{14}{3}$ m. The radius of tent is 2.5 m. The width of the canvas is 1.25 tube. If the height of the tube is 15 cm, then the diameter of the tube (in Rs.) is :
(A) 726 (B) 950 (C) 960 (D) 968

8. A hemispherical basin 150 cm in diameter holds water one hundred and twenty times as much a cylindrical m. If the rate of canvas per metre is Rs. 33, then the total cost of the canvas required for the tube (in cm) is :
(A) 23 (B) 24 (C) 25 (D) 26

9. A river 3 m deep and 60 m wide is flowing at the rate of 2.4 km/h. The amount of water running into the sea per minute is :
(A) 6000 m^3 (B) 6400 m^3 (C) 6800 m^3 (D) 7200 m^3

10. If a solid right circular cylinder is made of iron is heated to increase its radius and height by 1 % each, then the volume of the solid is increased by :
(A) 1.0 % (B) 3.03 % (C) 2.02 % (D) 1.2 %

11. If the right circular cone is separated into three solids of volumes V_1, V_2, and V_3 by two planes which are parallel to the base and trisects the altitude, then $V_1 : V_2 : V_3$ is :
(A) 1 : 2 : 3 (B) 1 : 4 : 6 (C) 1 : 6 : 9 (D) 1 : 7 : 19

12. Water flows at the rate of 10 m per minute from a cylindrical pipe 5 mm in diameter. A conical vessel whose diameter is 40 cm and depth 24 cm is filled. The time taken to fill the conical vessel is :
(A) 50 min (B) 50 min 12 sec. (C) 51 min 12 sec. (D) 51 min 15 sec.

13. A cylinder circumscribes a sphere. The ratio of their volume is :
(A) 1 : 2 (B) 3 : 2 (C) 4 : 3 (D) 5 : 6

14. If form a circular sheet of paper of radius 15 cm, a sector of 144^0 is removed and the remaining is used to make a conical surface, then the angle at the vertex will be :

(A) $\sin^{-1}\left(\frac{3}{10}\right)$ (B) $\sin^{-1}\left(\frac{6}{5}\right)$ (C) $2\sin^{-1}\left(\frac{3}{5}\right)$ (D) $2\sin^{-1}\left(\frac{4}{5}\right)$

15. A right circular cone of radius 4 cm and slant height 5 cm is curved out from a cylindrical piece of wood of same radius and height 5 cm. The surface area of the remaining wood is:
(A) 84 π (B) 70 π (C) 76 π (D) 50 π

16. If h, s, V be the height, curved surface area and volume of a cone respectively, then $(3\pi Vh^3 + 9V^2 - s^2h^2)$ is
(A) 0 (B) π (C) $\frac{V}{sh}$ (D) $\frac{36}{V}$

17. If cone is cut into two parts by a horizontal plane passing through the mid point of its axis, the ratio of the volume of the upper part and the frustum is :
(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 7

18. A cone, a hemisphere and a cylinder stand on equal bases of radius R and have equal heights H. Their whole surfaces are in the ratio:
(A) $(\sqrt{3}+1) : 3 : 4$ (B) $(\sqrt{2}+1) : 7 : 8$ (C) $(\sqrt{2}+1) : 3 : 4$ (D) None of these

19. If a sphere is placed inside a right circular cylinder so as to touch the top, base and the lateral surface of the cylinder. If the radius of the sphere is R, the volume of the cylinder is :
(A) $2\pi R^3$ (B) $8\pi R^3$ (C) $\frac{4}{3}\pi R^3$ (D) None of these

20. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of one end and the other end as its base. The volumes of the cylinder, hemisphere and the cone are respectively in the ratio of :
(A) $3 : \sqrt{3} : 2$ (B) 3 : 2 : 8 (C) 1 : 2 : 3 (D) 2 : 3 : 1

21. A hollow sphere of outer diameter 24 cm its cut into two equal hemisphere. The total surface area of one of the hemisphere is $1436\frac{2}{7} cm^2$. Each one of the hemisphere is filled with water. What is the volume of water that can be filled in each of the hemisphere?

(A) $3358\frac{2}{3}cm^3$ (B) $3528\frac{2}{3}cm^3$ (C) $2359\frac{2}{3}cm^3$ (D) $9335\frac{2}{3}cm^3$

22. A big cube of side 8 cm is formed by rearranging together 64 small but identical cubes each of side 2 cm. Further, if the corner cubes in the topmost layer of the big cube are removed, what is the change in total surface area of the big cube ?
(A) 16 cm^2, decreases (B) 48 cm^2, decreases
(C) 32 cm^2, decreases (D) Remains the same as previously

23. A large solid sphere of diameter 15 m is melted and recast into several small spheres of diameter 3 m. What is the percentage increase in the surface area of the smaller sphere over that of the large sphere?
(A) 200 % (B) 400 % (C) 500 % (D) Can't be determined

24. A cone is made of a sector with a radius of 14 cm and an angle of 60^0. What is total surface area of the cone ?
(A) 119.78 m^2 (B) 191.87 m^2 (C) 196.5 m^2 (D) None of these

25. If a cube of maximum possible volume is cut off from a solid sphere of diameter d, then the volume of the remaining (waste) material of the sphere would be equal to :

(A) $\frac{d^3}{3}\left(\pi-\frac{d}{2}\right)$ (B) $\frac{d^3}{3}\left(\frac{\pi}{2}-\frac{1}{\sqrt{3}}\right)$ (C) $\frac{d^3}{4}\left(\sqrt{2}-\pi\right)$ (D) None of these

26. A piece of paper is in the form of a right angle triangle in which the ratio of base and perpendicular is 3 : 4 and hypotenuse is 20 cm. What is the volume of the biggest cone that can be formed by taking right angle vertex of the paper as the vertex of the cone?
(A) 45.8 m^3 (B) 56.1 m^3 (C) 61.5 m^3 (D) 48 m^3

27. In a particular country the value of diamond is directly proportional to the surface area (exposed) of the diamond. For thieves steel a cubical diamond piece and then divide equally in four parts. What is the maximum percentage increase in the value of diamond after cutting it ?
(A) 50 % (B) 66.66 % (C) 100 % (D) None of these

28. In a bullet the gun powder is to be filled up inside the metallic enclosure. The metallic enclosure is made up of a cylindrical base and conical top with the base of radius 5 cm. The ratio of height of cylinder and cone is 3 : 2. A cylindrical hole is drilled through the metal solid with height two-third the height of metal solid. What should be the radius of the hole, so that the volume of the hole (in which gun powder is to be filled up) is one third the volume of metal solid after drilling ?

(A) $\sqrt{\frac{88}{5}}$ cm **(B)** $\sqrt{\frac{55}{8}}$ cm **(C)** $\frac{55}{8}$ cm **(D)** 33π cm

29. A cubical cake is cut into several smaller cubes by dividing each edge in 7 equal parts. The cake is cut from the top along the two diagonals forming four prisms. Some of them get cut and rest remained in the cubical shape. A complete cubical (smaller) cake was given to adults and the cut off part of a smaller cake is given to a child get the cake?
(A) 343 (B) 448 (C) 367 (D) 456

30. In a factory there are two identical solid blocks of iron. When the first block is melted and recast into spheres of equal radii Y, then 14 cc of iron was left. The volumes of the solid blocks and all the spheres are in integers. What is the volume (in cm^3) of each of the large sphere of radius '2r'?
(A) 176 (B) 12π (C) 192 (D) Data insufficient

31. Initially the diameter of a balloon is 28 cm. It can explode when the diameter becomes 5/2 times of the initial diameter. Air is blown at 156 cc/s. It is knows that the shape of balloon always remains spherical. In how many seconds the balloon will explode?
(A) 1078s (B) 1368s (C) 1087s (D) None of these

32. The radius of a cone is $\sqrt{2}$ times the height of the cone. A cube of maximum possible volume is cut from the same cone. What is the ratio of the volume of the cone to the volume of the cube?

(A) $2\sqrt{3}$ ft (B) $(2+\sqrt{3})$ ft (C) $(3+\sqrt{2})$ ft (D) $(2+2\sqrt{3})$ ft

34. A blacksmith has a rectangular sheet of iron. He has to cut out 7 circular discs from this sheet. What is the minimum possible width of the iron sheet if the radius of each disc is 1 ft?

(A) $\frac{1}{11}$ (B) $\frac{2}{17}$ (C) $\frac{3}{22}$ (D) None of these

35. Barun needs an open box of capacity 864 m^3. Actually where he lives, the rates of paints are soaring high so he wants to minimize the surface area of the box keeping the capacity of the box same as required. What is the base area and height of such a box?

(A) 36 m2, 24 m (B) 216 m2, 4 m (C) 144 m2, 6 m (D) None of these

36. There are two cylindrical containers of equal capacity and equal dimensions. If the radius of one of the containers a increased by 12 ft and the height of another container is increased by 12 ft, then the capacity of both the containers is equally increased by K cubic ft. If the actual heights of the container be 4 ft, then find the increased volume of each of the container :

(A) 1680 π cu ft (B) 2304 π cu ft (C) 1480 π cu ft (D) Can't be determined

OBJECTIVE				ANSWER KEY						EXERCISE-4					
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	C	A	B	B	B	D	C	D	B	D	C	B	C	C
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	A	D	C	A	B	A	D	B	A	B	B	C	B	B	A
Que.	**31**	**32**	**33**	**34**	**35**	**36**									
Ans.	A	B	B	A	C	B									

STATISTICS

★ INTRODUCTION

In class IX, we have studied about the presentation of given data in the form of ungrouped us well as grouped frequency distributions. We have also studied how to represent the statistical data in the form of various graphs such as bar graphs, histograms and frequency polygons. In addition, we have studied the measure of central tendencies such as mean, median and mode of ungrouped data.

In this chapter, we shall discuss about mean, median and mode of grouped data. We shall also discuss the concept of cumulative frequency, cumulative frequency distribution and cumulative frequency curve (ogive).

★ MEAN OF UNGROUPED DATA

We know that the mean of observations is the sum of the values of all the observations divided by the total number of observations i.e., if $x_1, x_2, x_3, \ldots\ldots, x_n$ are n observations, then

mean, $\bar{x} = \dfrac{x_1 + x_2 + x_3 + \ldots.. + x_n}{n}$ or $\bar{x} = \dfrac{\sum_{1=1}^{n} x_1}{n}$, where $\sum_{1=1}^{n} x_1$ denotes the sum $x_1 + x_2 + x_3 + \ldots\ldots + x_n$.

- Direct method
- Short-cut method or Assumed-mean method
- Step-deviation method.

★ MEAN OF GROUPED DATA

• Direct method

If $x_1, x_2, x_3, \ldots\ldots, x_n$ are n observations with respective frequencies $f_1, f_2, f_3, \ldots\ldots\ldots, f_n$ then mean, $(\bar{x})$ defined by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \ldots.. + f_nx_n}{f_1 + f_2 + f_3 + \ldots. + f_n} \text{ or } \bar{x} = \frac{\sum_{1=1}^{n} f_1x_1}{\sum_{1=1}^{n} f_1}, \text{ where } \sum_{1=1}^{n} f_1 = f_1 + f_2 + f_3 + \ldots.. + f_n .$$

To find mean of grouped Data

The following steps should be followed in finding the arithmetic mean of grouped data by direct method.

STEP-1: Find the class mark (x_i) of each class using, $x_i = \dfrac{lower\lim it + Upper\lim it}{2}$

STEP-2: Calculate f_ix_i for each i

STEP-3: Use the formula : mean, $\bar{x} = \dfrac{\sum_{1=1}^{n} f_1x_1}{\sum_{1=1}^{n} f_1}$,

• SHORTCUT METHOD OR ASSUMED MEAN METHOD

In this case, to calculate the mean, we follow the following steps :

STEP-1: Find the class mark (x_i) of each class using

$$x_i = \frac{lower\lim it + Upper\lim it}{2}$$

STEP-2: Choose a suitable value of x_i in the middle as the assumed mean and denote it by 'a'.

STEP-3: Find $d_i = x_i - a$ for each i

STEP-4: Find $f_i \times d_i$ for each i

STEP-5: Find n = $\sum f_1$

STEP-6: Calculate the mean, $(\bar{x})$ by using the formula $\bar{x} = a + \dfrac{\sum f_i d_i}{N}$.

• STEP-DEVIATION METHOD

Sometimes, the values of x and f are so large that the calculation of mean by assumed mean method becomes quite inconvenient. In this case, we follow the following steps:

STEP-1: Find the class mark (x_i) of each class using, $x_i = \frac{lower\,limit + Upper\,limit}{2}$

STEP-2: Choose a suitable value of x_i in the middle as the assumed mean and denote it by 'a'.

STEP-3: Find h = (upper limit – lower limit) for each class.

STEP-4: Find $u_i = \frac{x_i - a}{h}$ for each class.

STEP-5: Find $f_i u_i$ for each i.

STEP-6: Calculate, the mean by using the formula $\bar{x} = a + \left\{\frac{\sum f_i \times u_i}{N}\right\} \times h$, where $N = \sum f_i$

Ex.1 Find the mean of the following data:

Class Interval	0-8	8-16	16-24	24-32	32-40
Frequency	6	7	10	8	9

Sol. We may prepare the table as given below :

Class Interval	**Frequency (f_i)**	**Class mark (x_i)**	**$f_i x_i$**
0-8	6	4	24
8-16	7	12	84
16-24	10	20	200
24-32	8	28	224
32-40	9	36	324
	$\sum f_i = 40$		$\sum f_i x_i = 856$

$\therefore$ Mean, $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{856}{40} = 21.4$

Ex.2 The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs. 18. Find the missing frequency f.

Daily pocket allowance (in Rs.)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of children	7	6	9	13	f	5	4

Sol. We may prepare the table as given below :

Daily pocket Allowance	**Number of Children (f_i)**	**Class mark (x_i)**	**$f_i x_i$**
11-13	7	12	84
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	20f
21-23	5	22	110
23-25	4	24	96
	$\sum f_i = 44 + f$		$\sum f_i x_i = 752 + 20f$

$\therefore$ Mean, $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{752 + 20f}{44 + f}$

Given, mean = 18

$\therefore \quad 18 = \frac{752+20f}{44+f} \Rightarrow 792+18f = 752+20f \Rightarrow f = 20$

Ex.3 Find the missing frequencies f1 and f2 in the table given below, it is being given that the mean of the given frequency distribution is 50.

Class	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

Sol. We may prepare the table as given below :

Class	**Number of (f_i)**	**Class mark (x_i)**	**f_ix_i**
0-20	17	10	170
20-40	f_1	30	$30f_1$
40-60	32	50	1600
60-80	f_2	70	$70f_2$
80-100	19	90	1710
	$\sum f_i = 68+f_1+f_2$		$\sum f_ix_i = 3480+30f_1+70f_2$

$\therefore \quad$ Mean, $\bar{x} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum f_i} = \frac{3480+30f_1+70f_2}{68+f_1+f_2}$

Given, mean = 50

$\therefore \quad 50 = \frac{3480+30f_1+70f_2}{68+f_1+f_2} \Rightarrow 3400+50f_1+50f_2 = 3480+30f_1+70f_2$

$\Rightarrow \quad 20f_1 - 20f_2 = 80 \Rightarrow f_1 - f_2 = 4 \quad \ldots(i)$

And $\sum f_i = 68+f_1+f_2$

$\therefore \quad 120 = 68 + f_1 + f_2 \qquad [\because \sum f_1 = 120]$

$\Rightarrow \quad f_1 + f_2 = 52 \qquad \ldots(ii)$

Adding (1) and (2), we get $2f_1 = 56 \Rightarrow f_1 = 28 \qquad \therefore \quad f_2 = 24$

Hence, following missing frequencies f1 and f2 are 28 and 24 respectively.

Ex.4 The following table gives the marks scored by 100 students in a class test :

Mark	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	12	28	27	20	17	6

Sol. We may prepare the table with assumed mean, a = 35 as given below :

Mrks	**No.of students (f_i)**	**Class mark (x_i)**	**$d_i = x_i - a = xi - 35$**	**f_id_i**
0-10	12	5	– 30	– 360
10-20	18	15	– 20	– 360
20-30	27	25	– 10	– 270
30-40	20	30 = a	0	0
40-50	17	45	10	170
50-60	6	55	20	120
	N = 100			$\sum f_id_i = -700$

$\therefore$ Mean, $\bar{x} = a + \frac{\sum f_i d_i}{N} = 35 + \frac{(-700)}{100} = 35 - 7 = 28$

Ex.5 Thirty women were examined in a hospital by a doctor and the number of heart beats per minute, were recorded and summarized as follows. Find the mean heart beats per minute for these women, by using assumed.

No. of heart beats per minute	65-68	68-71	71-74	74-77	77-80	80-83	83-86
Frequency	2	4	3	8	7	4	2

Sol. We may prepare the table with assumed mean, a = 35 as given below :

No. of heart beats per minute	**No.of women (f_i)**	**Class mark (x_i)**	**$d_i = x_i - a$ = xi – 75.5**	**$f_i d_i$**
65-68	2	66.5	– 9	– 18
68-71	4	69.5	– 6	– 24
71-74	3	72.5	**– 3**	– 9
74-77	8	75.5 = a	0	21
77-80	7	78.5	3	24
80-83	4	81.5	6	18
83-86	2	84.5	9	
	N = 30			$\sum f_i d_i = 12$

$\therefore$ Mean, $\bar{x} = a + \frac{\sum f_i d_i}{N} = 75.5 + \frac{12}{30} = 75.5 + \frac{2}{5} = 75.9$

Ex.6 Find the mean of the following distribution by step-deviation method :

Class	50-70	70-90	90-110	110-130	130-150	150-170
Frequency	18	12	13	27	8	22

Sol. We may prepare the table with assumed mean a = 120 as given below :

Class	**Frequency (f_i)**	**Class mark (x_i)**	$u_i = \frac{x_i - a}{h} = \frac{x_i - 120}{20}$	**$f_i u_i$**
50-70	18	60	– 3	– 54
70-90	12	80	– 2	– 24
90-110	13	100	– 1	– 13
110-130	27	120 = a	0	0
130-150	8	140	1	8
150-170	22	160	2	44
	N = 100			$\sum f_i u_i = -39$

$$\therefore \quad \text{Mean, } \bar{x} = a + \frac{\sum f_i u_i}{N} \times h = 120 + \frac{(-39) \times 20}{100} = 20 - \frac{39}{5} = \frac{561}{5} = 112.2$$

Ex.7 Find the mean marks from the following data :

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80	Below 90	Below 100
No. of Students	5	9	17	29	45	60	70	78	83	z85

Sol. We may prepare the table as given below :

Marks	**No. of students**	**Class Interval**	**f_i**	**Class mark (x_i)**	**$f_i x_i$**
Below 10	5	0-10	5	5	25
Below 20	9	10-20	4	15	60
Below 30	17	20-30	9	25	225
Below 40	29	30-40	12	35	420
Below 50	45	40-50	16	45	720
Below 60	60	50-60	15	55	825
Below 70	70	60-70	10	65	650
Below 80	78	70-80	8	75	600
Below 90	83	80-90	5	85	425
Below 100	85	90-100	2	95	190
			N = 85		$\sum f_i x_i = 4140$

$$\therefore \quad \text{Mean, } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{4140}{85} = 48.41$$

Ex.8 Find the mean marks of students from the adjoining frequency distribution table.

Marks	**No. of Students**
Above 0	80
Above 10	77
Above 20	72
Above 30	65
Above 40	55
Above 50	43
Above 60	23
Above 70	16
Above 80	10
Above 90	8
Above 100	0

Sol. We may prepare the table as given below :

Marks	No. of students	Class Interval	f_i	Class mark (x_i)	$f_i x_i$
Above 0	80	0-10	3	5	15
Above 10	77	10-20	5	15	75
Above 20	72	20-30	7	25	175
Above 30	65	30-40	10	35	350
Above 40	55	40-50	12	45	540
Above 50	43	50-60	20	55	1100
Above 60	23	60-70	7	65	455
Above 70	16	70-80	6	75	450
Above 80	10	80-90	2	85	170
Above 90	8	90-100	8	95	760
Above 100	0	100-110	0	105	0
			N = 80		$\sum f_i x_i = 4090$

$\therefore$ Mean, $\bar{x} = \dfrac{\sum f_i x_i}{N} = \dfrac{4090}{80} = 51.125 = 51.1$ (approax)

Ex.9 Find the arithmetic mean of the following frequency distribution.

Class	25-29	30-24	35-39	40-44	45-49	50-54	55-59
Frequency	14	22	16	6	5	3	4

Sol. The given series is in inclusive form. We may prepare the table in exclusive form with assumed mean a = 42 as given below :

Class	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - a = x_i - 75.5$	$f_i u_i$
24.5-29.5	14	27	– 15	– 210
29.5-34.5	22	32	– 10	– 220
34.5-39.5	16	37	– 5	– 80
39.5-44.5	6	42 = a	0	0
44.5-49.5	5	47	5	25
49.5-54.5	3	52	10	30
54.5-59.5	4	57	15	60
	N = 70			$\sum f_i d_i = -395$

$\therefore$ Mean, $\bar{x} = a\dfrac{\sum f_i d_i}{N} = 42 + \dfrac{(-395)}{70} = \dfrac{2940 - 395}{70} = \dfrac{2545}{70} 36.36$ (approx)

★ **MEDIAN OF A GROUPED DATA**

MEDIAN : It is a measure of central tendency which gives the value of the middle most observation in the data. In a grouped data, it is not possible to find the middle observation by looking at the cumulative frequencies as the

middle observation will be some value in a class interval. It is, therefore, necessary to find the value inside a class that divides the whole distribution into two halves.

MEDIAN CLASS : The class whose cumulative frequency is greater than $\frac{N}{2}$ is called the median class.

To calculate the median of a grouped data, we follow the following steps :

STEP-1: Prepare the cumulative frequency table corresponding to the given frequency distribution and obtain $N = \sum f_i$.

STEP-2: Find $\frac{N}{2}$

STEP-3: Look at the cumulative frequency just greater than $\frac{N}{2}$ and find the corresponding class (Median class).

STEP-4: Use the formula Median, $M = \ell + \left\{\frac{\frac{N}{2} - C}{f}\right\} \times h$

Where ℓ = Lower limit of median class.
F = Frequency of the median class.
C = Cumulative frequency of the class preceding the median class.
h = Size of the median class.
$N = \sum f_i$

Ex.10. Find the median of the following frequency distribution :

Marks	0-10	10-20	20-30	30-40	40-50	Total
No. of Students	8	20	36	24	12	100

Sol. At first we prepare a cumulative frequency distribution table as given below :

Marks	**Number of students (f_i)**	**Cumulative frequency**
0-10	8	8
10-20	20	28
20-30	36	64
30-40	24	88
40-50	12	100
	N = 100	

Here, N = 100

$\therefore \quad \frac{N}{2} = 50$

The cumulative frequency just greater than 50 is 64 and the corresponding class is 20-30.
So, the median class is 20-30.

$\therefore \quad \ell = 20$, N = 100, C = 28, f = 36 and h = 10

Therefore, median $= \ell + \left\{\frac{\frac{N}{2} - C}{f}\right\} \times h$

$$= 20 + \left(\frac{50-28}{36}\right) \times 10 = 20 + \frac{22 \times 10}{36} = 20 + \frac{55}{9} = \frac{180+55}{9} = \frac{235}{9} = 36.1$$

Ex.11 A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60

No. of policy Holders	2	6	24	45	78	89	92	98	100

Sol. From the given table we can find the frequency and cumulative frequency as given below :

Age (in years)	Number of students (f_i)	Cumulative frequency
15-20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100
	N = 100	

Here, N = 100

$\therefore \quad \frac{N}{2} = 50$

The cumulative frequency just greater than 50 is 78 and the corresponding class is 35-40.
So, the median class is 65-40.

$\therefore \quad \ell = 20$, N = 100, C = 45, f = 33 and h = 5

Therefore, median $= \ell + \left\{\frac{\frac{N}{2} - C}{f}\right\} \times h$

$= 35 + \left(\frac{50-45}{33}\right) \times 5 = 35 + \frac{5 \times 5}{33} = \frac{1155+25}{33} = \frac{1180}{33} = 35.76$

Hence, the median age is 35.76 years.

Ex.12 The length of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table. Find the median length of the leaves.

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No. of leaves	3	5	9	12	5	4	2

Sol. The given series is in inclusive form. We may prepare the table in exclusive form and prepare the cumulative frequency table as given below :

Length (in mm)	Number of leaves (f_i)	Cumulative frequency
117.5-126.5	3	3
126.5-135.5	5	8
135.5-144.5	9	17
144.5-153.5	12	29
153.5-162.5	5	34
162.5-171.5	4	38
171.5-180.5	2	40
	N = 40	

Here, N = 40

$\therefore \quad \frac{N}{2} = 20$

The cumulative frequency just greater than 20 is 29 and the corresponding class is 144.5-153.5
So, the median class is 144.5-153.5

$\therefore \quad \ell = 144.5$, N = 40, C = 17, f = 12 and h = 9

Therefore, median $= \ell + \left\{\frac{\frac{N}{2} - C}{f}\right\} \times h$

$$= 144.5 + \frac{(20-17)}{12} \times 9 = 144.5 + \frac{3 \times 9}{12} = 144.5 + 2.25 = 146.75$$

Hence, median length of leaves is 146.75 mm.

Ex.13 Calculate the missing frequency 'a' from the following distribution, it is being given that the median of the distribution is 24.

Age (in mm)	0-10	10-20	20-30	30-40	40-50
No. of persons	5	25	a	18	7

Sol. At first we prepare a cumulative frequency distribution table as given below :

Age (in years)	0-10	10-20	20-30	30-40	40-50	Total
No. of persons (f_i)	5	25	a	18	7	55+a
Cumulative frequency	5	30	30+a	48+a	55+a	

Since the median is 24, therefore, the median class will be 20-30.
Hence, $\ell = 20$, N = 55+a, C = 30, f = a and h = 10

Therefore, median $= \ell + \left\{\frac{\frac{N}{2} - C}{f}\right\} \times h$

$$\Rightarrow \quad 24 = 20 + \left(\frac{\frac{55+a}{2} - 30}{a}\right) \times 10$$

$$\Rightarrow \quad 24 = 20 + \frac{(a-5)}{2a} \times 10$$

$$\Rightarrow \quad 4 = \frac{(a-5)}{2a} \times 5$$

$\Rightarrow \quad 4a = 5a - 25 \Rightarrow a = 25$

Hence, the value of missing frequency a is 25.

Ex.14 The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class Interval	Frequency (f_i)
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4
	N = 100

Sol. At first we prepare a cumulative frequency distribution table as given below :

Class Interval	**frequency (f_i)**	**Cumulative frequency**
0-100	2	2
100-200	5	7
200-300	x	7+x
300-400	12	19+x
400-500	17	36+x
500-600	20	56+x
600-700	y	56+x+y
700-800	9	65+x+y
800-900	7	72+x+y
900-1000	4	76+x+y
	N = 100	

We have N = 100

$\therefore 76+x+y=100 \Rightarrow x+y=24$ …(i)

Since the median is 525, so, the median class is 500 – 600

$\therefore \ell = 500$, N = 100, C = 36 + x, f = 20 and h = 100

Therefore, median $= \ell + \left\{\dfrac{\frac{N}{2}-C}{f}\right\} \times h$

$\Rightarrow \quad 525 = 500 + \left(\dfrac{50-36-x}{20}\right) \times 100 \Rightarrow 25 = (14-x) \times 5$

$\Rightarrow \quad 5 = 14 - x \Rightarrow x = 9$

Also, putting x = 9 in (1), we get 9 + y = 24 $\Rightarrow$ y = 15

Hence, the values of x and y are 9 and 15 respectively.

★ MODE OF A GROUPED DATA

MODE : Mode is that value among the observations which occurs most often i.e., the value of the observation having the maximum frequency.

In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequency.

MODAL CLASS : The class of a frequency distribution having maximum frequency is called modal class of a frequency distribution .

The mode is a value inside the modal class and is calculated by using the formula.

$$\text{Mode} = \ell + \left\{\frac{f_1 f_0}{2f_1 - f_0 - f_2}\right\} \times h$$

Where ℓ = Lower limit of the modal class.
h = Size of class interval.
f_1 = Frequency of modal class.
f_0 = Frequency of the class preceding the modal class
f_2 = Frequency of the class succeeding the modal class

Ex15 The following data gives the information on the observed lifetimes (in hours) of 225 electrical components :

Lifetimes (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Sol. Here the class 60-80 has maximum frequency, so it is the modal class.

$\therefore \quad \ell = 60, h = 20, f_1 = 61, f_0 = 52$ and $f_2 = 38$

Therefore, mode $= \ell + \left\{\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right\} \times h$

$$= 60 + \left(\frac{61-52}{2\times 61 - 52 - 38}\right) \times 20 = 60 + \frac{9}{20} \times 20 = 60 + 5.625 = 65.625$$

Hence, the modal lifetimes of the components is 65.625 hours.

Ex.16 Given below is the frequency distribution of the heights of players in a school.

Heights (in cm)	160-162	136-165	166-168	169-171	172-174
No. of students	15	118	142	127	18

Find the average height of maximum number of students.

Sol. The given series is in inclusive form. We prepare the table in exculsive form, as given below :

Heights (in cm)	159.5-162.5	162.5-165.5	165.5-168.5	168.5-171.5	171.5-174.5
No. of students	15	118	142	127	18

We have to find the mode of the data.

Here, the class 165.5-168.5 has maximum frequency, so it is the modal class.

Ex.17 The mode of the following series is 36. Find the missing frequency f in it.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-71
Frequency	8	10	f	16	12	6	7

Sol. Since the mode is 36, so the modal class will be 30-40

$\therefore \quad \ell = 30, h = 10, f_1 = 16, f_0 = f$ and $f_2 = 12$

Therefore, mode = $\ell + \left\{\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right\} \times h$

$\Rightarrow \quad 66 = 30 + \left(\frac{61 - f}{2 \times 16 - f - 12}\right) \times 10 \Rightarrow 6 = \frac{(16 - f)}{(20 - f)} \times 10$

$\Rightarrow$ 120 – 6f = 160 – 10f $\Rightarrow$ 4f = 40 $\Rightarrow$ f = 10

Hence, the value of the missing frequency f is 10.

★ GRAPHICAL. REPRESENTATION OF CUMULATIVE FREQUENCY DISTRIBUTION

- **CUMULATIVE FREQUENCY POLYGON CURVE (OGIVE)**

Cumulative frequency is of two types and corresponding to these, the ogive is also of two types.

- **LESS THAN SERIES**
- **MORE THAN SERIES**

- **LESS THAN SERIES** To construct a cumulative frequency polygon and an ogive, we follow these steps :

STEP-1 : Mark the upper class limit along x-axis and the corresponding cumulative frequencies along y-axis.

STEP-2 : Plot these points successively by line segments. We get a polygon, called cumulative frequency polygon.

STEP-3 : Plot these points successively by smooth curves, we get a curve called cumulative frequency or an ogive.

★ APPLICATION OF AN OGIVE

Ogive can be used to find the median of a frequency distribution. To find the median, we follow these steps.

METHOD –I

STEP-1 : Draw anyone of the two types of frequencies curves on the graph paper.

STEP-2 : Compute $\frac{N}{2}$ $(N = \sum f_i)$ and mark the corresponding points on the y-axis.

STEP-3 : Draw a line parallel to x-axis from the point marked in step 2, cutting the cumulative frequency curve at a point P.

METHOD –II

STEP-1 : Draw less than type and more than type cumulative frequency curves on the graph paper.

STEP-2 : Mark the point of intersecting (P) of the two curves draw2n in step 1.

STEP-3 : Draw perpendicular PM from P on the x-axis. The x-coordinate of point M gives the median .

Ex.18 The following distribution gives the daily income of 50 workers of a factory.

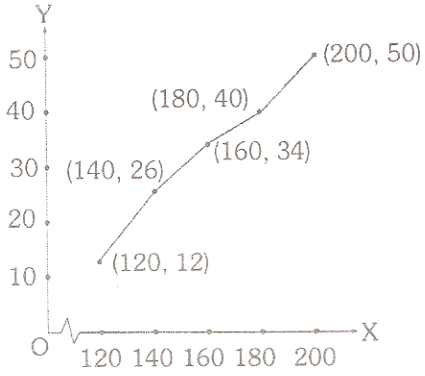
Daily income (in Rs.)	100-120	120-140	140-160	160-180	180-200
No. of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Sol. From the given table, we prepare a less than type cumulative frequency distribution table, as given below :

Join these points by a

Income less than (in Rs)	120	140	160	180	200
Cumulative frequency	12	26	34	40	50



freehand curve to get an ogive of 'less than' type.

Ex.19 The following table gives production yield per hectare of wheat of 100 farms of a village.

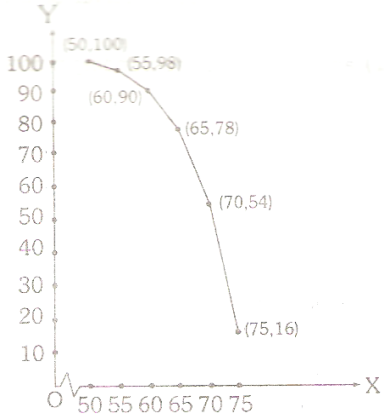
Production yieid (in kg/ha)	50-55	55-60	60-65	65-70	70-75	75-80
No. of farms	2	8	12	24	38	16

Change the distribution to more than type distribution and draw its ogive.

Sol. From the given table, we may prepare more than type cumulative frequency distribution table, as given below :

Production more than (in kg/ha)	50	55	60	65	70	75
Cumulative frequency	100	98	90	78	54	16

Now, plot the points (50, 100), (55,98), (60,90), (65,78), (70,54) and (75,16).
Join these points by a freehand curve to get an ogive of 'more than' type.



Ex.20 The annual profits earned by 30 shops of a shopping complex in a locality gives rise to the following distribution

Profit (in lakhs Rs.)	No. of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Draw both ogives for the data above. Hence, obtain the median profit.

Sol. We have a more than type cumulative frequency distribution table. We may also prepare a less than type cumulative frequency distribution table from the given data, as given below :

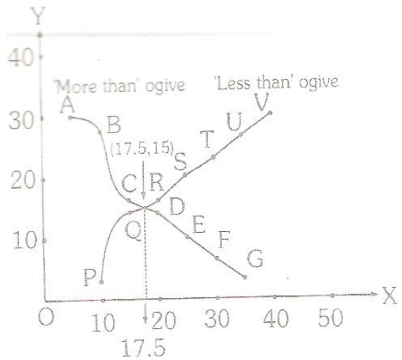
'More than' type

Profit more than (Rs. in lakhs)	No. of shops
5	30
10	28
15	16
20	14
25	10
30	7
35	3

'Less than' type

Profit less than (Rs. in lakhs)	No. of shops
10	2
15	14
20	11
25	20
30	23
35	27
40	30

Now, plot the points A(5,30), B(10,28), C(15,16), D(20,14), E(25,10), F(30,7) and G(35,3) for the more than type cumulative frequency and the points P(10,2), Q(15,14), R(20,16), S(25,20), T(30,23), U(35,27) and V(40,30) for the less then type cumulative frequency table.



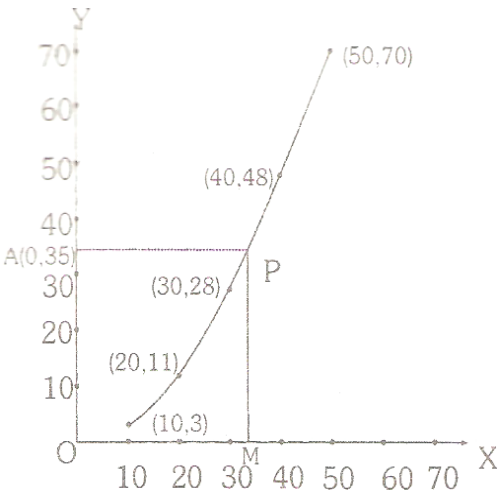
Join these points by a freehand to get ogives for 'more than' type and 'less than' type.
The tow ogives intersect each other at point (17.5, 15).
Hence, the median profit is Rs. 17.5 lakhs.

Ex.21 The following data gives the information on marks of 70 students in a periodical test :

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50
No. of students	3	11	28	48	70

Draw a cumulative frequency curve for the above data and find the median.

Sol. We have a less than cumulative frequency table. We mark the upper class limits along the x-axis and the corresponding cumulative frequency (no. of students) along the y-axis. Now, plot the points (10,3), (20,11), (30,28), (40,48) and (50,70). Join these points by a freehand curve to get an ogive of 'less than' type.



Here, N = 70

$\therefore \frac{N}{2} = 35$

Take a point A(0,35) on the y-axis and draw AP ∥ x-axis, meeting the curve at P.

Draw PM ⊥ x-axis, intersecting the x-axis, at M.

Then, OM = 33.

Hence, the median marks is 33.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. Which of the following is a measure of central tendency ?

(A) Frequency (B) Cumulative frequency

(C) Mean (D) Class limit

2. Class mark of a class is obtained by using –

(A) Class mark (B) $\frac{1}{2}$ [upper limit – lower limit]

(C) $\frac{1}{2}$ [upper limit + lower limit] (D) $\frac{1}{2}$ [upper limit + lower limit] – 1

3. The value of $\sum_{i=1}^{n} x_i$ is –

(A) $\frac{\bar{x}}{2}$ (B) 2 $\bar{x}$ (C) n $\bar{x}$ (D) $\frac{\bar{x}}{n}$

4. The mean of the following data $1^2, 2^2, 3^2, \ldots\ldots n^2$ is –

(A) $\frac{(n+1)(2n+1)}{6}$ (B) $\frac{n(n-1)(2n+1)}{6}$ (C) $\frac{n(n+1)(2n-1)}{6}$ (D) $\frac{n(n-1)(2n-1)}{6}$

5. The mean of following distribution is –

x_i	10	12	15	25
f_i	2	3	7	8

(A) 18.50 (B) 18.50 (C) 18.15 (D) 18.25

6. The mean of following data is 18.75 then the value of p is –

x_i	10	15	p	25	30
f_i	5	10	7	8	2

(A) 21 (B) 20.6 (C) 20 (D) 22

7. To find mean, we use the formula.

(A) $\sum_{i=1}^{n} f_i x_i$ (B) $N\sum_{i=1}^{n} f_i x_i$ (C) $\frac{1}{N}\sum_{i=1}^{n} f_i x_i$ (D) $\sum_{i=1}^{n}\left(\frac{f_i x_i}{N}\right)$

8. Which of the following can not be determined graphically –

(A) Mean (B) Median (C) Mode (D) Standard deviation

9. If the median of the following data is 40 then the value of p is –

Class	0- 10	10-30	30-60	60-80	80-90
Frequency	5	15	30	p	2

(A) 7 (B) 8 (C) 9 (D) 7.6

10. Which of the following is true?

(A) Mode = 2median – Mean (B) Mode = 3median + 2Mean

(C) Mode = 3median – 2Mean (D) None of these

11. Mode is –

(A) Most frequent value (B) Least frequent value

(C) Middle most value (D) None of these

12. Which of the following is true –

(A) Mode = 2median + Mean (B) Median = Mode + $\frac{3}{2}$ [Mean – Median]

(C) Mean = Mode + $\frac{3}{2}$ [Median – Mode] (D) Median = Mode + $\frac{3}{2}$ [Mean +Median]

13. In the formula for mode of a grouped data, mode = $\ell + \left\{\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right\} \times h$, where symbols have their usual meaning f_0 represents :

(A) Frequency of modal class
(B) Frequency of median class
(C) Frequency of the class preceding the modal class
(D) Frequency of class succeeding the modal class

14. Median of a given frequency distribution is found with the help of a –
(A) Bar graph (B) Ogive (C) Histogram (D) None of these

15. The measure of central tendency which is given by the x-coordinate of the point of intersection of the 'more than' ogive and 'less than' ogive is –
(A) Mean (B) Median (C) Mode (D) None of these

OBJECTIVE				ANSWER KEY				EXERCISE		
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	C	C	C	A	B	C	C	A	B	C
Que.	**11**	**12**	**13**	**14**	**15**					
Ans.	A	C	C	B	B					

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

(A) MEAN OF A GROUPED DATA

1. Find the mean of the following data :

(a)

Class Interval	0-6	6-12	12-18	18-24	24-30
Frequency	6	8	10	9	7

(b)

Number of Plant	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of house	1	2	1	5	6	2	3

2. Find the mean of the following distribution :

(a)

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	3	5	9	5	3

(b) (i)

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	12	16	6	7	9

(ii)

Class Interval	100-120	120-140	140-160	160-180	180-200
Frequency	12	14	8	6	10

(iii)

Class Interval	0-100	100-200	200-300	300-400	400-500
Frequency	6	9	15	12	8

3. (a) The arithmetic mean of the following frequency distribution is 25.25. Determine the value of p :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	7	8	p	15	4

(b) The arithmetic mean of the following frequency distribution is 47. Determine the value of p :

Class Interval	0-20	20-40	40-60	60-80	80-100
Frequency	8	15	20	p	5

4. Find the value of f, the missing frequency, if the mean of the following distribution is 67.

Class Interval	25-35	35-45	45-55	55-65	65-75	75-85	85-98
Frequency	10	6	4	f	4	12	26

5. (a) Find the missing frequencies f_1 and f_2 if the frequency distribution is 62.8 and the sum of all frequency is 50

Class	0-20	20-40	40-60	60-80	80-100	100-120	Total
Frequency	5	f_1	10	f_2	7	8	50

(b) Find the missing frequencies f_1 and f_2 in the following data if the mean is $166\frac{9}{26}$ and the sum of the observation is 52.

Class	140-150	150-160	160-170	170-180	180-190	190-200
Frequency	7	f_1	20	f_2	7	8

(c) The mean of following frequency table is 53. But the frequency f_1 and f_2 in the classes 20-40 and 60-80 are missing

Age (in years)	0-20	20-40	40-60	60-80	80-100	Total
No. of people	15	f_1	21	f_2	17	100

6. (a) Find the mean of the following data, by using the assumed mean method.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	7	8	12	13	10

(b)

Marks	0-100	100-200	200-300	300-400	400-500	500-600
No. of students	2	8	12	20	5	3

7. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

No. of days	0-6	6-10	10-14	14-20	20-28	28-38	38-40
No. of students	11	10	7	4	4	3	1

8. (a)

Class	4-8	8-12	12-16	16-20	20-24	24-28	28-32	32-36
No. of students	2	12	15	25	18	12	13	3

Find the arithmetic mean of the following frequency distribution by using step deviation method :

(b) The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45-55	55-65	65-75	75-85	85-95
No. of cities	3	10	11	8	3

(c) The distribution show the number of wickets taken by bowlers in one day cricket matches. Find the mean number

No. of wickets	20-60	60-100	100-150	150-250	250-350	350-450
No. of bowlers	7	5	16	12	2	3

9. (a) The following table gives the distribution of expenditures of different families on education. Find the mean expenditure on education of a family.

Expenditure (in Rs.)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
No. of families	24	10	33	28	30	22	16	7

(b) (i) To find the concentriation of SO_2 in the air (in per million), the data was collected for 30 localities in a certain city and is presented below :

Concentration of SO_2 (in ppm.)	0.00-0.04	0.04-0.08	0.08-0.12	0.12-0.16	0.16-0.020	0.20-0.24
Frequency	4	9	9	2	4	2

Find the mean concentration of SO_2 in the air.

(ii) The following table shows that the daily expenditure on food of 25 house holds in a localities. Find the mean daily expenditure on food by a suitable method.

Daily expenditure (in Rs.)	100-150	150-200	200-250	250-300	300-350
No. of house holds	4	5	12	2	2

10. (a) Find the mean marks from the following data :

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of students	4	10	18	28	40	70

(b) Compute the mean for the following data :

Marks	Less than 10	Less than 30	Less than 50	Less than 70	Less than 90	Less than 110	Less than 130	Less than 150
No. of students	0	10	25	43	65	87	96	100

11. (a) Find the average marks of student from the following data :

Marks	No. of Students	Marks	No. of Students
Above 0	80	Above 60	23
Above 10	77	Above 70	16
Above 20	72	Above 80	10
Above 30	65	Above 90	8
Above 40	55	Above 100	0
Above 50	43		

(b) Find the mean wage of the following data :

Wages (in Rs.)	No. of Workers
0 and above	120
20 and above	108
40 and above	90
60 and above	75
80 and above	50
100 and above	24
120 and above	9
140 and above	0

12. (a) In a retail market, fruit vendors selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

No. of mangoes	50-52	53-55	56-58	59-61	62-64
No. of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a pocket box.

(b) The following data shows that the age distribution of patients of malaria in a village during a particular month. Find the average age of the patients.

Age (in years)	5-14	15-24	25-34	35-44	45-54	55-64
No. of cases	6	11	21	23	14	5

(B) MEDIAN OF A GROUPED DATA

1. Find the median for the following frequency distribution :

(a)

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	9	14	2	19	10

(b) (i)

Class Interval	25-35	35-45	45-55	55-65	65-75
Frequency	20	25	5	7	4

(ii)

Class Interval	0-8	8-16	16-24	24-32	32-40	40-48
Frequency	8	10	16	24	15	7

(c) 100 surnames were randomly picket up from a local telephone directly and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows :

No. of letters	1-4	4-7	7-10	10-13	13-16	16-19
No. of Surnames	6	30	40	16	4	4

Find the median number of letters in the surnames. Find the mean number of letters in the surnames.

2. (a) Find the median from the following data :

Class groups	110-120	120-130	130-140	140-150	150-160	160-170	170-180	180-190	190-200
Frequency	6	25	48	72	116	60	38	22	3

(b) (i) The following distribution gives the weights of 30 students of a class. Find the median weight of the student

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
No. of students	2	3	8	6	6	3	2

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

(ii) Find the median of the following frequency distribution :

Marks	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency	2	5	9	12	17	20	15	9	7	4

(c) The following table gives the distribution of the life time of 400 neon lamps :

Life Time (in hours)	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
No. of lamps	14	56	60	86	74	62	48

Find the median life time of a lamp.

3. (a) A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 years.

Age in years	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
No. of policy holders	2	6	24	45	78	89	92	98	100

(b) A survey regarding the heights (in cm) of 51 girls of class X of a school was conducted and the data obtained follows :

Heights (in cm)	Less than 140	Less than 145	Less than 150	Less than 155	Less than 160	Less than 165
No. of girls	4	11	29	40	46	51

Find the median height.

4. (a) The following table gives the marks obtained by 50 students in a class test :

Marks	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
No. of Students	2	3	6	7	14	12	4	2

Find the median.

(b) The following table gives the population of males in different age groups :

Age group (in years)	5-14	15-24	25-34	35-44	45-54	55-64	65-74
No. of males	447	307	279	220	157	91	39

Find their median age.

5. (a) The following table gives the distribution of IQ of 100 students. Find the median IQ.

IQ	75-84	85-94	95-104	105-114	115-124	125-134	135-144
Frequency	8	11	26	31	18	4	2

(b) The length of 70 leaves of a plant are measured correct to the nearest millimeter and the data obtained is represented in the following table :

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No. of leave	10	8	13	22	7	6	4

Find the median length of the leaves.

6. Calculate the missing frequency f from the following distribution, it being given that the median of the distribution is 24.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	25	f	18	7

7.

Variable	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	f_1	65	f_2	25	18

(a) If the median of the following frequency distribution is 28.5, find the missing frequencies.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	f_1	20	15	f_2	5	60

(b) If the median of the following frequency distribution is 32.5, find the values of f_1 and f_2.

Class interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	7	10	x	13	y	10	14	9

(c) (i) An incomplete distribution is given below :
If median value is 46 and the total number of items is 230.

(α) Find the missing frequencies f_1 and f_2.

(β) Find the arithmetic mean (AM) of the completed distribution.

(ii) The median of the following data is 20.75 Find the missing frequencies x and y, if the total frequency is 100

(C) MEDIAN OF A GROUPED DATA

1. (a) Calculate the mode for the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

(b) A student noted the number of cars passing through spot on a road for 100 periods each of 3 minutes and summarized it in the table given below. Find the mode of the data .

No. of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

2. (a) The given distribution shows the number of runs scored by some top batsmen of the world in one day international cricket matches :

Runs Scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000	10000-11000
No. of batsman	4	18	9	7	6	3	1	1

Find the mode of the data.

(b) (i) The following tables gives the ages of the patients admitted in a hospital during a year.

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
No. of patients	6	11	21	23	14	5

Find the mode and the mean of the data

(ii) The following data gives the distribution of total monthly house hold expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure

Expenditure (in Rs.)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
No. of families	24	40	33	28	30	22	16	7

(c) (i) The following distribution gives the state-wise teacher student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret two measures.

No. of students per teacher	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
No. of state/U.T.	3	8	9	10	3	0	0	2

(ii) The following table shows the marks obtained by 100 students of Class X in school during a particular academic session. Find the mode of this distribution

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80
No. of students	7	21	34	46	66	77	92	100

3. (a) Compute the mode of the following data :

Class Interval	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
Frequency	3	8	13	18	28	20	13	8	6	4

(b) Compute the mode of the following data :

Score	80-90	90-100	100-110	110-120	120-130	130-140	140-150
No. of pupil	18	27	48	39	12	6	16

4. Calculate the mode of the following data :

Wages (In Rs.)	51-56	57-62	63-68	69-74	75-80	81-86	87-92
No. of workers	12	24	40	30	18	8	20

5. The mode of the following data is 85.7 Find the missing frequency in it.

Size	45-55	55-65	650-75	75-85	85-95	95-105	105-115
Frequency	7	12	17	f	32	6	10

(C) GRAPHICA REPRESENTATION OF CUMULATIVE FREQUENCY DISTRIBUTION

1. The following distribution gives the mark obtained by 102 students of class X.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	9	10	25	50	5	3

Convert the above distribution to a less than type cumulative frequency distribution and draw its ogive.

2. The following table gives the distribution of IQ of 60 pupils of class X in a school.

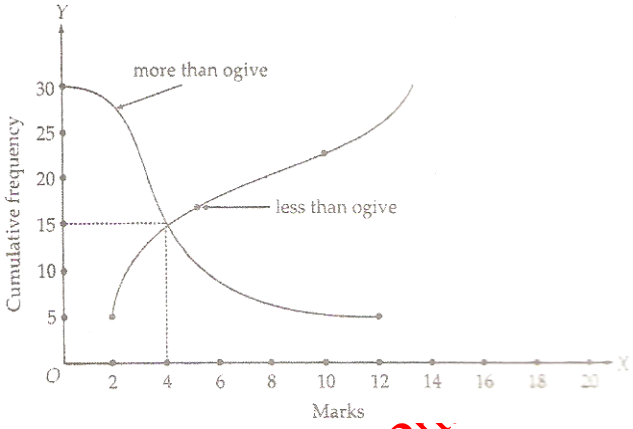
IQ	60-70	70-80	80-90	90-100	1005-110	110-120	120-130
No. of pupils	2	3	5	16	14	13	7

Convert the above distribution to a more than type cumulative frequency distribution and draw its ogive.

3. (a) The following table gives the height of trees :

Height	Less than 140	Less than 145	Less than 150	Less than 155	Less than 160	Less than 165
No. of trees	4	11	29	40	46	50

(b) What is the value of the median of the data using the graph in the given figure, of less than ogive and more than ogive?

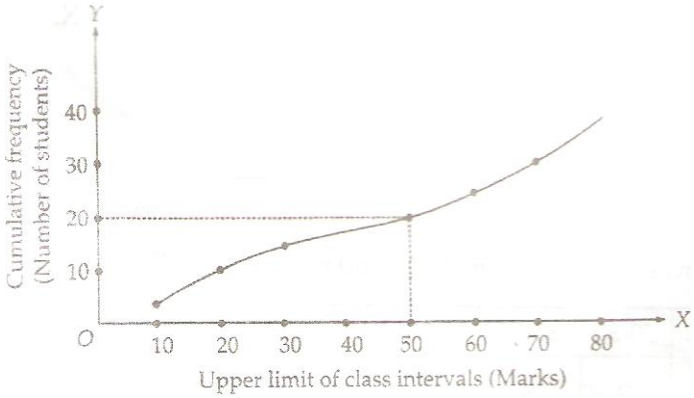


Draw both ogives for the data above. Hence, obtain the median of the data.

4. (a) Following is the age distribution of a group of students. Draw a cumulative frequency curve for the data and find the median.

Age in years	**No. of students**
Less than 5	36
Less than 6	78
Less than 7	136
Less than 8	190
Less than 9	258
Less than 10	342
Less than 11	438
Less than 12	520
Less than 13	586
Less than 14	634
Less than 15	684
Less than 16	700

(b) A student draws a cumulative frequency curve for the marks obtained by 40 students of a class as shown below. Find the median marks obtained by the students of the class.



5. The table given below shows the frequency distribution of the scores obtained by 200 candidates in a MCA entrance examination.

Score	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of students	30	15	45	20	25	40	10	15

Draw cumulative curve of more than type and hence find median.

(A) MEAN OF A GROUPED DATA :

1. (a) 15.45, (b) 8.1 **2.** (a) 25, (b) (i) 22 (ii) 145.20 (iii) 264 **3.** (a) 6, (b) 12 **4.** 23.71
5. (a) 8, 12, (b) $f_1 = 7$, $f_2 = 10$, (c) $f_1 = 18$, $f_2 = 29$, **6.** (a) 27.2 (b) 304 **7.** 12.48 days
8. (a) 19.92 (b) 69.43%, (c) On an average the number of wickets taken by bowers in one day cricket is 152.89.

9. (a) 2823.53 (b) (i) 0.099 ppm (ii) Rs.211 **10.** (a) $40\frac{5}{7}$ marks (b) 74.80

11. (a) 51.1 (b) (i) 69.34 **12.** (a) 57.19 (b) 34.87 years

(B) MEDIAN OF A GROUPED DATA :

1. (a) 35 (b) (i) 39.2 (ii) 26 (c) Median = 8.05, Mean = 8.32
2. (a) 153.8 (b) (i) 56.67 kg (ii) 532.5 (c) 3406.98 hours
3. (a) 35.76 years (b) 149.03 cm **4.** (a) 33 (b) 25.07 years
5. (a) 106.1 (b) 146.14 m **6.** 25
7. (a) 8, 7 (b) 3, 6 (c) (i) (α) 34 & 46 (β) 45.87 (ii) x = 17, y = 20

(C) MODE OF A GROUPED DATA :

1. (a) 46.67 (b) 44.7 cars
2. (a) 4608.7 runs
(b) (i) mode = 36.8, mean = 35.37 years, (ii) Rs. 1847.83, Rs. 2662.5
(c) (i) mode = 30.6, mean = 29.2 Most states U. T., have a student teacher ratio of 30.6 and on an average, this ratio is 29.2
(ii) 44.7
3. (a) 23.28 (b) 107 4. 66.2 5. 30 (approx.)

(D) GRAPHICAL REPRESENTATION OF CUMULATIVE FREQUENCY DISTRIBUTION :

3. (a) Median = 148.9 (b) Median = 4
4. (a) Median = 10 (b) Median marks = 50
5. Median = 375

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. Which measure of central tendency is given by the x-coordinate of the point of intersection of the "more than ogive" and "less than ogive"? **Delhi-2008**
2. Find the median class of the following data : **Al-2008**

Marks Obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	10	12	22	30	18

3. Write the median class of the following distribution : **Delhi-2009**

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

4. What is the lower limit of the modal class of the following frequency distribution? **Foreing-2009**

Age (in years)	0-10	10-20	20-30	30-40	40-50	50-60
Number of patients	16	13	6	11	27	18

SHORT ANSWER TYPE QUESTIONS

1. The mean of the following frequency distribution is 57.6 and the sum of observations is 50. Find the missing frequencies f_1 and f_2 : **Al-2004**

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	7	f_1	12	f_2	8	5

2. The following table gives the distribution of expenditure of different families on education. Find the mean expenditure on education of a family : **Delhi-2004C**

Expenditure (in Rs.)	Number of families
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

3. Find the mean of the following distribution : **Delhi-2005**

Class	4-8	8-12	12-16	16-20	20-24	24-28	28-32	32-36
Number of students	2	12	15	25	18	12	13	3

4. If the mean of the following data is 18.75 find the value of p : **Al-2005**

x_i	10	15	p	25	30
f_i	5	10	7	8	2

5. The Arithmetic Mean of the following frequency distribution is 50. Find the value of p : **Delhi-2006**

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	p	32	24	19

6. If the mean of the following is 50, find the value of f_1 : **Delhi-2006**

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	28	32	f_1	19

7. The mean of the following frequency distribution is 62.8. Find the missing frequency x. **Delhi-2007**

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	x	12	7	8

LONG ANSWER TYPE QUESTIONS

1. A survery regarding the heights (in cm) of 50 girls of class x of a school was conducted and the following data was obtained : **Delhi-2008**

Height in cm	120-130	130-140	140-150	150-160	160-170	Total
Number of girls	2	8	12	20	8	50

Find the mean, median and mode of the above data.

2. Find the mean, mode and median of the following data. **Al-2008**

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	10	18	30	20	12	5

3. Find the mean, median and mode of the following data. **Foreign-2008**

Class	Frequency
0-50	2
50-100	3
100-150	5
150-200	6
200-250	5
250-300	3
300-350	1

4. The following table gives the daily income of 50 workers of a factory : **Delhi-2009**

Daily income (in Rs.)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Find the mean, mode and median of the above data.

5. During the medical check-up of 35 students of a class their weights were recorded as follows : **Al-2009**

Weight (in kg)	Number of students
38-40	3
40-42	2
42-44	4
44-46	5
46-48	14
48-50	4
50-52	3

Draw a less than type and a more than type ogive from the given data. Hence obtain the median weight from the graph.

6. Find the mode, median and mean for the following data : **Foreign-2009**

Marks obtained	Number of students
25-35	7
35-45	31
45-50	33
50-55	17
55-65	11
65-75	1

VERY SHORT ANSWER TYPE QUESTION

1. Median **2.** 30-40 **3.** 17.5 and 45 **4.** 30-40 5. 40

SHORT ANSWER TYPE QUESTION

1. $f_1 = 8$, $f_2 = 10$ **2.** Rs. 2662.5 **3.** 19.92 **4.** p = 20 **5.** p = 28 **6.** $f_1 = 24$ **7.** 10

LONG ANSWER TYPE QUESTION

1. mean = 150.25 ; Median = 151.5 ; Mode = 154. **2.** mean = 35.76 ; Median = 35.66 ; Mode = 35.44
3. mean = 59.9 ; Median = 61.6 ; Mode = 65. **4.** mean = 145.20 ; Median = 138.57 ; Mode = 125
5. 42.2 kg **6.** mean = 49.7 ; Median = 48.5 ; Mode = 46.1

SCIENCE

(CHEMISTRY, PHYSICS AND BIOLOGY)

CHEMICAL REACTIONS AND EQUATIONS

1. Chemical Reactions: The transformation of chemical substance into a new chemical substance by making and breaking of bonds between different atoms is known as chemical reaction.

1.1. Signs of a Chemical Reaction: These factors denote that a chemical reaction has taken place- change of state of substance, change of color of substance, evolution of heat, absorption of heat, evolution of gas and evolution of light.

2. Chemical Equation: The representation of chemical reaction by means of symbols of substances in the form of formulae is called chemical equation. Ex- H2 + O2 → H2O (Hydrogen and oxygen are reactants whereas water is product)

2.1. Forward Arrow (→) when reaction proceeds in forward direction

2.2. Double Arrow (⇄) when reaction is reversible

2.3. Equilibrium Arrow (⇌) when a reversible reaction is at equilibrium.

3. Balanced Chemical Equation: A balanced chemical equation has number atoms of each element equal on both left and right sides of the reaction. According to law of conservation of mass, mass can neither be created nor destroyed in a chemical reaction. To obey this law, the total mass of elements present in reactants must be equal to the total mass of elements present in products.

4. Types of Chemical Reactions:

Name and Definition	Example
In a combination reaction, two or more reactants combine to give single product.	$CaO(s)+H2O(l)\longrightarrow Ca(OH)2(aq)$
In a decomposition reaction, a single reactant breaks down into two or more simpler products.	$AB \longrightarrow A + B$ aqueous $NaCl(s) \longrightarrow Na^{+}(aq)\ Cl^{-}(aq)$
When decomposition reaction is carried out by heating, it is called thermal decomposition reaction.	$CaCO_3(s)\longrightarrow CaO(s)+CO_2(g)$
When decomposition reaction is carried out in the presence of sunlight, the process is called photochemical decomposition.	$2AgBr(s) \longrightarrow 2Ag(s) + Br_2(g)$
Electrolysis: When decomposition reaction is carried out with the help of electric current, the process is called electrolysis.	$2H_2O(l)\longrightarrow 2H_2(g)+O_2(g)$
In a displacement reaction, a more reactive element displaces a less reactive element from a less compound.	$Fe(s)+CuSO_4(aq)\longrightarrow BaSO_4(aq)+Cu(s)$
Those reactions in which the different atoms or groups of atoms are displaced by other atoms or groups of atom, i.e., two compounds exchange their ions and one of the products formed is insoluble, are said to be double displacement reactions.	$Na_2SO_4(aq)+BaCl_2(aq)\longrightarrow BaSO_4(s)+ 2NaCl(aq)$
The reactions in which acid or acidic oxide reacts with the base or basic oxides to form salt and water are called neutralization reactions.	$2NaOH+H_2SO_4 \longrightarrow Na_2SO_4 + H_2O(l)$
Precipitation the insoluble compound called precipitate forms in this reaction.	$AgNO_3(aq)+ KCl(aq)\longrightarrow AgCl(s)+KNO_3(aq)$
Exothermic reactions which produce energy are called exothermic reaction. Most of the	$H_2(g)+Cl_2(g)\longrightarrow 2HCl(g)\ (\Delta H > 0)$

decomposition reactions are exothermic.	
Endothermic reactions which absorb energy are called endothermic reaction. Most of the combination reactions are endothermic.	$C(s)+H_2O(l) \longrightarrow CO(g)+H_2(g)$ $(\Delta H > 0)$
Oxidation is gain of oxygen or removal of hydrogen or metallic element from a compound is known as oxidation.	$MnO_2(s)+ 4HCl(aq) \longrightarrow MnCl_2(aq)+Cl_2(g)+ 2H_2O(l)$
Reduction is addition of hydrogen or removal of oxygen from a compound is called reduction.	
Redox is chemical reactions where oxidation and reduction both take place simultaneously are also known as Redox reaction.	$2\ NaOH(aq)+HCl(aq) \longrightarrow NaCl(s)+H_2O(l)$

5. Rusting: When iron reacts with oxygen and moisture forms a red substance called rust.
Reaction: $4Fe + 3O_2 + 2H_2O \longrightarrow 2Fe_2O_3.H_2O$ (rust)
6. Rancidity: Oils and fats when get oxidized on exposure to air show a change in taste and smell. Due to this reason packets of chips are flushed with gas such as nitrogen that acts as antioxidant and prevents spoilage of the fried chips.

Sample Questions for Practice:

Q1. What happens chemically when quick lime is added to water?
Q2. How will you test for the gas which is liberated when HCL reacts with an active metal?
Q3. What is an oxidation reaction? Is it exothermic or endothermic? Give one example of oxidation Reaction.
Q4. Give an example of photochemical reaction.
Q5. Give an example of a decomposition reaction. Describe any activity to illustrate such a reaction by heating.
Q6. Why is respiration considered as exothermic process?
Q7. Balance the following chemical equation.
$Fe(s) + H_2O(g) = Fe_3O_4 + H_2(g)$
$MnO_2 + HCL = MnCl_2 + Cl_2 + H_2O$
$HNO_3 + Ca(OH)_2 = Ca(NO_3)_2 + H_2O$
Q8. On what basis is a chemical equation balanced?
Q9. State any two observations in an activity suggesting the occurrence of a chemical reaction.
Q10. Name a reducing agent which may be used to obtain manganese from manganese dioxide.
Q11. What change in colour is observed when silver chloride is left exposed to sunlight? Also mention the type of chemical reaction.
Q12. Define a combination reaction. Give one example of an exothermic combination reaction.
Q13. What is observed when a solution of potassium iodide is added to lead nitrate solution? What type of reaction is this? Write a balanced chemical equation for this reaction.
Q14. Distinguish between an exothermic and an endothermic reaction.
Q15. Distinguish between a displacement and a double displacement reaction.
Q16. Identify the type of reaction in the following:
$Fe + CuSO_4(aq) = FeSO_4(aq) + Cu(s)$
$2H_2 + O_2 = 2H_2O$

Important Questions for Practice:

1. What is a redox reaction?
2. What is corrosion? Explain its advantage and disadvantage.
3. What is rancidity? How can we reduce the problem of rancidity?
4. How is corrosion different from rusting?
5. What is meant by endothermic and exothermic reactions? Give suitable example for each.
6. Define different types of chemical reaction and give examples for each.
7. Why is photosynthesis considered as an endothermic reaction?
8. In electrolysis of water, why is the volume of gas collected over one electrode double that of the other electrode?
9. What happens when water is added to solid calcium oxide taken in a container? Write a chemical formula for the same.
10. Give one use of quick lime.
11. Give three types of decomposition reaction.
12. Name the compound used for testing CO^2 gas.

ACIDS, BASES AND SALTS

1. Acids: Those substances which turn blue litmus solution red are called acids. Acids are sour in taste. They give H^+ ions in aqueous solution. Some acid are organic while others inorganically created. A concentrated acid is one which contains the minimum amount of water in it. A dilute acid is obtained by mixing the concentrated acid with water. Example: HCl, H_2SO_4, HNO_3, CH_3COOH

1.1. Types:

(a) Weak Acid: weak acids → release a smaller number of H^+ ions → acetic acid

(b) Strong Acid: strong acids → release more H^+ ions → HCl

1.2. Reactions:

(a) Neutralization: Acid + Base → Salt + Water ⇒ $HCl(aq)+NaOH(aq)\longrightarrow H2O(l)+$ Salt

(b) With Water: Acids produce H^+ ions when dissolved in water. H^+ ions cannot exist alone. They combine with water molecule (H_2O) to form H_3O^+ (Hydronium ions). It conducts electricity. Decrease in H_30^+ ions concentration per unit volume results in formation of dilute acids. It is a highly exothermic reaction. Acids when dissolved in water release large amount of heat. If water is added to concentrated acid, then the heat generated may cause the mixture to splash out and cause burns. Hence to avoid burns acid must be added drop wise into water with constant stirring.

(c) With Metals: Acid + Metal → Salt + Hydrogen Gas ⇒ $2HCl + Zn \rightarrow ZnCl_2 + H_2$

(d) With Metal Oxides: Acid + Metal Oxide → Salt + Water ⇒ $2HCl + CuO \rightarrow CuCl_2 + H_2O$

(e) With Metal Carbonate: Acid + Metal Carbonate → Salt + CO_2 + H_2O ⇒ $2HCl + Na_2CO_3 \rightarrow 2NaCl + CO_2 + H_2O$

2. Bases: They are bitter in taste and soapy to touch. Sea water and detergents are some examples of substances that are basic. Many bases are oxide or hydroxide compounds of metals. Strong bases can also burn ones skin. Ex- Caustic soda, NaOH; Caustic potash, KOH; Milk of magnesia, $Mg(OH)_2$; Liquor ammonia, NH_3; Washing powder, Tooth paste.

2.1. Types:

(a) Weak Base: weak base → gives less OH^- ions → NH_4OH

(b) Strong Base: strong base → give more OH^- ions → NaOH

2.2. Reactions:

(a) With Water: Bases produce OH^- ions when dissolved in water. Bases soluble in water are called alkalis. It conducts electricity. Decrease in OH^- ions single concentration per unit volume results in formation of dilute bases. It is an exothermic reaction. To make basic solution, base must be added drop wise into water with constant stirring, so that the heat generated spreads over in water.

(b) With Metals: Base + Metal → Salt + H_2 Gas ⇒ $2NaOH + Zn \rightarrow Na_2ZnO_2 + H_2$

(c) With Non-Metal Oxides: Base + Non-metallic oxide → Salt + H_2O

3. Salts: Salts are obtained by treating an acid with a base. Salts consist of both positive ions or 'cations', and negative ions or 'anions'.

The cations are called basic radicals and are mostly obtained from metallic ions (ammonium ion being one exception), while the anions are called acidic radicals and are obtained from acids. Salts can be classified into the following types:

3.1. Normal Salts: A salt that is formed by the complete replacement of the replaceable hydrogen ions of an acid by a metal ion or ammonium ion is called a normal salt. Examples: NaCl, Na_2SO_4, Na_3PO_4, NH_4Cl, K_2CO_3

3.2. Neutral Salts: A neutral salt arises due to the neutralization reaction. Here, salts of strong acid and strong base combine to form such salts that show a neutral pH of 7.

Acid	Uses
Organic acids	
Citric acid ($C_6H_8O_7$)	1. As a preservative for food 2. As a flavoring agent
Ascorbic acid ($C_6H_8O_6$) (also called vitamin C)	1. In the treatment of bone marrow and scurvy diseases
Acetic acid (CH_3COOH)	1. Added to pickles to make them sour
Tartaric acid ($C_4H_6O_6$)	1. A component of baking powder (baking powder is a mixture of sodium hydrogen carbonate and tartaric acid)
Inorganic acids	
Hydrochloric acid (HCl)	1. 1Its presence in the gastric juice helps digestion of food which we eat. 2. As a bathroom cleaner 3. Manufacturing of PVC
Nitric acid (HNO_3)	1. Nitric acid present in rainwater forms nitrates in the soil which are then used by plants to obtain nitrogen. 2. In manufacturing fertilizers like ammonium nitrate. 3. Making explosives like TNT and dynamite
Sulphuric acid (H_2SO_4)	1. In storage batteries 2. In manufacturing fertilizers, paints, fibers, HCl and alum
Phosphoric acid (H_3PO_4)	1. In fertilizer and detergent industries
Boric acid (H_3BO_3)	1. In the manufacture of glass, glazes and enamels, leather, paper, adhesives and explosives 2. In detergents 3. As grain preservative
Bases	Uses
Sodium hydroxide (NaOH)	1. In the manufacture of soaps, textile, paper, medicines 2. In the refining of petroleum
Ammonium hydroxide $[Al(OH)_3]$	1. As a reagent in the laboratory 2. In making fertilizers, rayon, plastics and dyes
Calcium hydroxide $[Ca(OH)_2]$	1. In making cement and mortar 2. In making bleaching powder 3. In whitewashing 4. In removing acidity of soils
Salts	Uses
Sodium chloride (NaCl)	1. An essential requirement of our food 2. In the preservation of food 3. In curing fish and meat 4. In making a freezing mixture which is used by ice-cream vendors 5. In the manufacture of soaps

Sodium carbonate (Na_2CO_3)	1. As washing soda for cleaning clothes 2. Used in the manufacture of glass, paper, textiles, caustic soda, etc. 3. In the refining of petroleum 4. In fire extinguishers
Sodium bicarbonate ($NaHCO_3$)	1. Used as baking soda 2. In fire extinguishers 3. As an antacid in medicine
Potassium nitrate (KNO_3)	1. To make gun powder, fireworks and glass 2. As a fertilizer in agriculture
Copper sulphate ($CuSO_4$)	1. Called 'blue vitriol', used as a fungicide to kill certain germs 2. In electroplating 3. In dyeing
Potash alum [$KAl(SO_4)_2 .12H_2O$]	1. Used to purify water; makes suspended particles in water settle down 2. As an antiseptic and in Dying

4. Baking Soda: The chemical name of baking soda is sodium hydrogen carbonate ($NaHCO_3$). Chlorine produced on passing electricity through brine solution undergoes reaction with ammonia produces baking soda.

$NaCl + H_2O + CO_2 + NH_3 \rightarrow NH_4Cl + NaHCO_3$

4.1 On Heating: $NaHCO_3 \rightarrow Na_2CO_3 + H_2O + CO_2$

4.2 Uses: As a component of baking powder. In fire extinguishers, In medicines as a mild antiseptic for skin diseases and to neutralize the acidity of stomach, As a reagent in laboratory.

5. Plaster of Paris: Chemical name is Calcium sulphate sodium carbonate hydrated. Formula: $CaSO_4 . \frac{1}{2}H_2O$

5.1 On Reaction with Water: $CaSO_4. \frac{1}{2}H_2O + \frac{3}{2}H_2O \longrightarrow CaSO_4.2H_2O$

5.2 Uses: It is in manufacturing chalks, toys, making walls of homes, medical use etc.

6. Bleaching Powder: Chlorine produced on passing electricity through brine solution undergoes reaction with dry slaked to produce bleaching powder. Formula: $CaOCl_2$

6.1 Reaction: $Ca(OH)_2 + Cl_2 \rightarrow CaOCl_2 + H_2O$

6.2 Uses: For bleaching cotton and linen in the textile industry, to bleach wood pulp in paper manufacturing industry. To bleach washed clothes in laundry, to disinfect drinking water and make it germfree.

7. Washing Soda: Chemical name is sodium carbonate hydrated. The heating of baking soda produces sodium carbonate. This sodium carbonate undergoes recrystalization to give off washing soda. Formula: $Na_2CO_3.10H_2O$

7.1 On recrystalization: $Na_2CO_3 + 10H_2O \rightarrow Na_2CO_3.10H_2O$

7.2 Uses: In glass, soap and paper manufacturing industries. It is also used in the manufacture of sodium compounds like borax. It is also used as a cleaning agent and in removing permanent hardness of water.

8. pH Indicator: The acidity or basicity (alkalinity) of a solution is usually expressed in terms of a function of the H+ ion concentration.

This function is called the pH of a solution.

8.1. Types of indicators:

(a) Olfactory indicators: Those substances whose odor changes in acidic or basic media e.g., clove, vanilla, onion.

(b) Natural indicators: Turmeric, litmus (obtained from lichen).

(c) Synthetic indicators: Methyl orange, phenolphthalein

8.2. pH scale Value: The scale lies between 0 to 14

(a) The pH of a neutral solution is 7.

(b) The pH of an acidic solution is less than 7.

(c) The pH of an alkaline solution is more than 7.

(d) The pH of an aqueous solution is the negative logarithm of its H+ ion concentration. That is, pH = – log $[H^+]$.

(e) pOH = –log $[OH^-]$.

8.3. Rules for pH scale (at 298 K)

(a) Acidic solutions have pH less than 7.

(b) The lower the pH, the more acidic is the solution.

(c) Neutral solutions or pure water has pH equal to 7.

(d) Basic solutions have pH greater than 7.

(e) The higher the pH, the more basic is the solution.

Sample Questions for Practice:

1. How will you test for a gas which is liberated when HCL reacts with an active metal? **(CBSE 2008)**

2. What is baking powder? How does it make the cake soft and spongy? **(CBSE 2008)**

3. When fresh milk is changed into curd will its pH value increase or decrease? Why?

4. Give Arrhenius definition of an acid and a base. **(CBSE 2009)**

5. What happens chemically when quick lime is added to water? **(CBSE 2008)**

6. Name the gas evolved when dilute HCL reacts with Sodium hydrogen carbonate. How is it recognized? **(CBSE 2008)**

7. How does the flow of acid rain water into a river make the survival of aquatic life in the river difficult? **(CBSE 2008)**

8. How is the pH of a solution of an acid influenced when it is diluted? **(CBSE 2008 F)**

9. How does the pH of the solution change when a solution of base is diluted? **(CBSE 2008 F)**

10. Arrange these in increasing order of their pH values- NaOH, blood, lemon juice. **(CBSE 2008 F)**

11. Two solutions of A and B have pH values of 5 and 8. Which solution will be basic in nature? **(CBSE 2008 C)**

12. Why does tooth decay start when pH of mouth is lower than 5.5? **(CBSE 2009)**

13. What would be the colour of litmus in a solution of sodium carbonate? **(CBSE 2009)**

14. Name the products obtained when sodium hydrogen carbonate is heated. Write the chemical equation for the same. **(AI CBSE 2009)**

15. Write the chemical formula of washing soda and baking soda. Which one of these two is an ingredient of antacids? How does it provide relief in stomachache? **(CBSE 2008 F)**

16. What do you mean by "water of crystallization" of a substance? Describe an activity to show that blue copper sulphate crystals contain water of crystallization. **(CBSE 2009 F)**

17. How can washing soda be obtained from baking soda? Name an industrial use of washing soda other than washing clothes. **(AI CBSE 2008)**

18. Why does 1 M HCL solutions have a higher concentration of H^+ ions than 1M CH_3COOH solution? **(AII CBSE 2009)**

Important Questions for Practice:

1. Why is Plaster of Paris stored in a moisture proof container?

2. What do you mean by neutralization reaction? Give two examples.

3. Mention two uses of baking soda and washing soda.

4. Why does a milkman add a small amount of baking soda to fresh milk to shift the pH of fresh milk from 6 to slightly alkaline?

5. Why do acids not show acidic behavior in the absence of water?
6. Rain water conducts electricity but distilled water does not. Why?
7. Why don't we keep sour substances in brass and copper vessels?
8. What is the common name of CaOCl2?
9. Name the compound used for softening hard water.
10. What happens when baking soda is heated?
11. Give the properties and uses of bleaching powder.
12. Give a few uses of acids, bases and salts respectively.

METALS AND NON-METALS

1. Chemical Properties of Metals:

1.1. Metals react with air or oxygen to form metal oxide. Ex: Copper reacts with oxygen to form copper oxide.

$$\text{Metal} + O_2 \rightarrow \text{Metal oxide}$$

1.2. Oxides of metals can react with both acids and bases to produce salt and water. Such oxides are known as amphoteric Oxides.

$$Al_2O_3 + 6HCl \rightarrow 2AlCl_3 + H_2O$$

1.3. Metals also react with water to form metal oxide. Metal oxide in turn can react with water to form metal hydroxide.

$$2Na + 2H_2O \rightarrow 2NaOH + 1H_2$$

1.4. Metals also react with dilute acids to form salt and hydrogen. Metal + Acid → Metal Salt + Hydrogen

$$Mg + 2HCl \rightarrow MgCl_2 + H_2$$

2. Chemical Properties of Non-metals:

2.1. Non-metals react with oxygen to form non-metal oxide. Non-metal + Oxygen → Non-metal oxide

$$C + O_2 \rightarrow CO_2$$

2.2. Non-metals do not react with water and acids to evolve hydrogen gas.

2.3. Non-metals can react with salt solution; more reactive element will displace the less reactive non-metal.

$$2NaBr\ (aq) + Cl_2(aq) \rightarrow 2NaCl\ (aq) + Br_2\ (aq)$$

2.4. Non-metals can also react with hydrogen to form hydrides.

$$H_2(g) + S(l) \rightarrow H_2S(g)$$

3. Reactivity Series:

3.1. The series in which metals are arranged in the decreasing order of reactivity.

3.2. K (most reactive) > Na > Ca > Mg > Al > C > Zn > Fe > Sn > Pb > H > Cu > Ag > Au > Pt (least reactive)

4. Ionic Compounds: Compounds formed due to the transfer of electrons from a metal to a non-metal. They are generally solid, with a high melting and boiling point. They are soluble in water but insoluble in inorganic solvents like ether. They conduct electricity in molten and solution state.

5. Occurrence of Metals: Elements or compounds which occur naturally in earth crust are known as minerals. Minerals from which pure metals can be extracted are known as mineral ores.

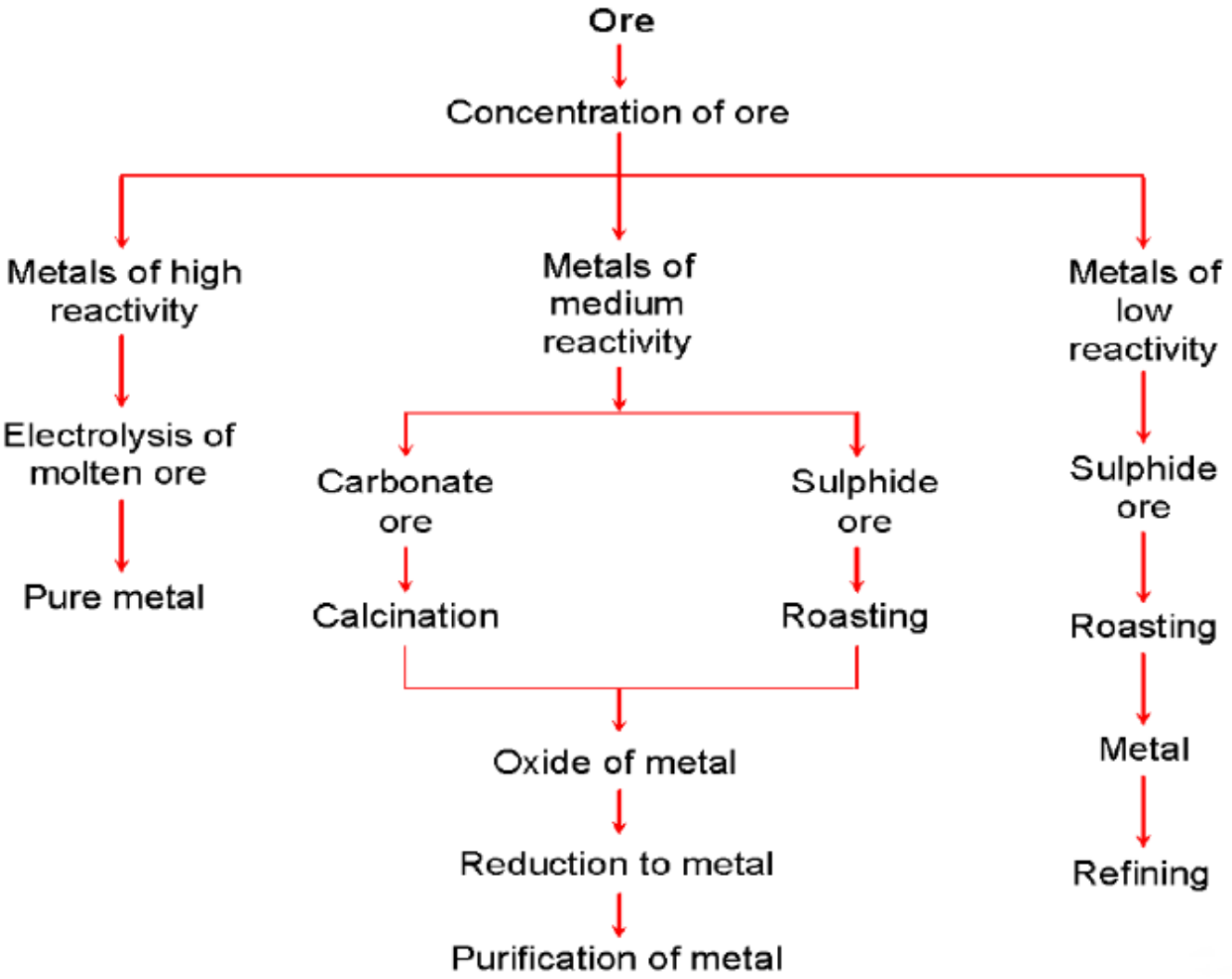
6. Extraction: Extraction of pure metals from its ores start with enrichment of the ore, extraction of metals, refining of metal.

6.1. The metals at the top of the activity series (K, Na, Ca, Mg and Al) are so reactive that they are never found in nature as free elements.

6.2. The metals in the middle of the activity series (Zn, Fe, Pb, etc.) are moderately reactive. They are found in the earth's crust mainly as oxides, sulphides or carbonates.

6.3. Below copper lies gold and silver that are found in Free State. These metals have low reactivity.

7. Gangue: Ores contain different impurities in it such as sand, soil etc. Gangue + Flux = Slag
8. Refining of Metals: Refining of impure metal is done using electrolytic refining. Impure copper is used as anode and strip of pure copper is used as cathode. Acidified copper sulphate is used as electrolyte. When electric current is passed through this, impure metal from the anode gets deposited in the electrolyte solution, whereas pure metal from the electrolyte is deposited at cathode.
9. Extracting Metals which are Low in Activity Series: Metals which are low in activity series are unreactive. The oxides of such metals can be reduced to metals by heating alone. For example, Cinnabar (HgS)
10. Extracting Metals in the Middle of the Activity Series: These metals are moderately reactive. They exists as sulphides or carbonates in nature. Before reduction, metal sulphides and carbonates must be converted into metal oxides. Sulphide ores are converted into oxides by heating strongly in presence of excess air, this is known as roasting. Carbonate ores are converted into oxides by heating in limited air. This is known as Calcination.



$$2HgS(s) + 3O_2(g) \xrightarrow{\text{Heat}} 2HgO(s) + 2SO_2(g)$$

$$2HgO(s) \xrightarrow{\text{Heat}} 2Hg(l) + O_2(g)$$

Roasting:

$$2ZnS(s) + 3O_2(g) \xrightarrow{\text{Heat}} 2ZnO(s) + 2SO_2(g)$$

Calcination:

Heat

$ZnCO_3$ (s)——→ZnO (*s*)+CO_2 (*g*)

Reduction-metal oxides can be reduced to metals using reducing agent such as such as carbon.

11. Extracting Metals towards the Top of the Activity Series: The metals are highly reactive. They cannot be obtained by heating.

Ex- Sodium, magnesium and calcium are obtained by the electrolysis of their molten chlorides.

At cathode $Na^+ + e^- \rightarrow Na$

At anode $2Cl^- \rightarrow Cl_2 + 2e^-$

12. Corrosion: Metals when exposed to moist air for a long period of time, they become corroded. Ex: Silver reacts with moist air and becomes black in colour due to silver sulphide coating.

Iron + oxygen → Iron (III) oxide = Fe + O_2 → Fe_2O_3

12.1. Prevention of Corrosion:

(a) Rusting of iron can be prevented by oiling, galvanizing, painting, greasing etc.

(b) To protect steel and iron from rusting, a thin layer of zinc is coated on them, this is known as galvanization.

(c) Sacrificial Protection: 'Rusting' can be prevented by connecting iron to a more reactive metal

(d) Electroplating: Coating the surface with metals like tin, chromium, nickel etc.

(e) Alloying: Iron or steel along with other metals can also be protected by 'alloying' or mixing with other metals (e.g., chromium) to make non-rusting alloys.

Sample Questions for Practice:

1. Name a reducing agent that may be used to obtain manganese from manganese dioxide. (CBSE 2009)

2. From amongst the metals sodium, calcium, aluminium, copper and mangnesium, name the metal (I) which reacts with metal only on boiling and

(II) another which does not react even with steam. (CBSE 2008)

3. (a) Show the formation of NaCl from sodium and chlorine atoms by the transfer of electrons.

(b) Why sodium chloride has a high melting point?

(c) Name the anode and cathode used in electrolytic refining of impure copper metal. (CBSE 2008)

Why are ionic compounds usually hard? How is it that ionic compounds in the solid state do not conduct electricity but they do so when in molten state? (CBSE 2008)

On adding dilute HCL acid to copper oxide powder the solution formed is blue-green. Predict the new compound formed which imparts a blue-green colour to the solution. (CBSE 2008)

4. (a) Show on a diagram the transfer of electron between the atoms in the formation of MgO.

(b) Name the solvent in which ionic compounds are generally soluble.

(c) Why are aqueous solutions of ionic compounds able to conduct electricity? (CBSE 2008)

5. What are amphoteric oxides? Choose the amphoteric oxides from- Na_2O, ZnO, Al_2O_3, CO_2, H_2O

6. Why is it that non-metals do not displace hydrogen from dilute acids? (AI CBSE 2008)

7. Show the electronic transfers in the formation of $MgCl_2$ from its elements. (CBSE 2008 F)

8. Which of the following metals will melt at body temperature: gallium, magnesium, caesium, aluminium? (CBSE 2008 C)

9. Name the two metals which react violently with cold water. Write any three observations you would make when such a metal is dropped into water. How would you identify the gas evolved, if any? (AI CBSE 2008)

10. Give reasons for the following: (i) Gold and silver are used for jewellery making.

(ii) carbonate and sulphide ores are usually converted into oxides prior to reduction during the process of extraction. (CBSE 2008 C)

11. Give reasons for the following: (i) Aluminium oxide is considered as an amphoteric oxide.

(ii) Ionic compounds conduct electricity in molten state. (CBSE 2008 C)

12. Give reasons for the following:

(i) Metals can be given different shapes according to our needs.

(ii) Hydrogen is not evolved when a metal reacts with nitric acid.

Important Questions for Practice:

1. A metal 'X' loses two electrons and a non-metal 'Y' gains one electron. Show the electron dot structure of compound formed between them. Is ionic or covalent? Does it have high melting point or low? Will it conduct electricity in solid state or in aqueous solution and why? Will it be soluble in water?

2. A student was given Mn, Zn, Fe and Cu metals. Identify which of them

(a) will not displace H_2 from dil. HCl.

(b) will react only with steam to give H2(g).

(c) Will give H_2 with 5% HNO_3.

Write the chemical reactions involved.

3. Compound X and aluminium are used to join railway tracks.

(a) Identify the compound X.

(b) Name the reaction.

(c) Write down its reaction.

4. Samples of five metals 'A', 'B', 'C', 'D' and 'E' were taken and added to the following solution one by one. The results obtained have been tabulated as follows.

Metal- $FeSO_4$,$CuSO_4$,$ZnSO_4$,$AgNO_3$,$Al_2(SO_4)_3$,$MgSO_4$

A. No reaction, Displacement, No reaction, Displacement, No reaction, No reaction

B. Displacement, Displacement, No reaction, Displacement, No reaction, No reaction

C. No reaction, No reaction, No reaction, Displacement, No reaction, No reaction

D. No reaction, No reaction, No reaction, No reaction, No reaction, No reaction

E. Displacement, Displacement, Displacement, Displacement, No reaction, No reaction

Use the above table to answer the following questions about the given metals.

(a) Which of them is most reactive and why?

(b) What would you observe if 'B' is added to $CuSO_4$?

(c) Arrange 'A', 'B', 'C', 'D' and 'E' in the increasing order of reactivity.

(d) Container of which metal can store zinc sulphate and silver nitrate solution?

(e) Which of the above solution(s) can be stored in a container made of any of these metals and why?

5. A metal A, which is used in Thermite process, when heated with oxygen gives an oxide B, which is a amphoteric in nature? Identify A and B. Write down the reactions of oxide B with HC1 and NaOH.

6. A non-metal A is an important constituent of our food and forms two oxides B and C. Oxide B is toxic whereas C causes global warming.

(a) Identify A, B and C.

(b) To which group of periodic table does A belong?

7. An element A reacts with water to form a compound B which is used in white washing. The compound B on heating forms an oxide which on treatment with water gives back B. Identify A, B and C and give the reactions involved.

8. A non-metal A which is the largest constituent of air, when heated with H2 in 1 : 3 ratio in the presence of catalyst (Fe) gives a gas B. On heating with O2, it gives an oxide C. If this oxide is passed into water in the presence of air, it gives and acid D which act as a strong oxidizing agent.

(a) Identify A, B, C and D.

(b) To which group of periodic table does this non-metal belong?

9. An element A burns with golden flame in air. It reacts with another element B, atomic number 17 to

give a product C. An aqueous solution of product C on electrolysis gives a compound D and Liberates hydrogen. Identify A, B, C and D. Also write down the equations for the reactions involved.

CARBON & ITS COMPOUND

INTRODUCTION:

The compound obtained from 'Carbon' is widely used as clothes, medicines, books fertilizer, fuel etc. al living structures are carbon based.

The amount of carbon present in the earth's crust and in the atmosphere is quite merge. The earths crust has only 0.02% carbon in the form of mineral (like carbonates, hydrogen-carbonates, coal and petroleum) and the importance has 0.03% of carbon dioxide. In spite of this small amount of carbon available in nature, the importance of carbon seems to be immense.

Carbon forms a large number of compounds with hydrogen which are known as hydrocarbons. In addition to hydrogen, Carbon compound may also contain some other element such as oxygen, halogen, nitrogen, phosphorus, sulphur etc.

The number of compounds of carbon is more than three million which is much larger than the compounds formed by all other element put together.

BONDING IN CARBON COMPOUNDS:

Carbon forms **covalent bonds** in its compounds with other atoms. In each compound the valency of carbon is four. That is, carbon has **tetravalent c**haracter. But what is covalent bond and what is meaning of tetravalent?

Why does a carbon atom form only covalent bond?

- The atomic number of carbon is 6 and first shell contains just two electrons and second shell (Outermost shell) contains four electrons.
- Carbon atom can attain the noble gas configuration by sharing its valence electrons with other atoms of carbon or with atoms of other elements and form covalent bond.

COVALENT BOND:

A chemical bond formed between two atoms of the same element or two atoms of different elements by sharing of electron is called a **covalent bond.**

Necessary conditions of the formation of covalent bond:

- The combining atoms should have **nonmetallic** character.
- The combining atoms should contain **4 to 7 electrons** in their respective valence shell.
- In **hydrogen** there is only **1 valence electron,** but it also forms **covalent bond.**
- The combining atoms need **1, 2,3, or 4 electrons** to complete their **octet (hydrogen completes its duplet)**
- The combining atoms should contribute equal number of electrons to form pair of electrons to be **shared.**
- After sharing the pair of electrons each combining atoms should attain stable electronic configuration like its nearest noble gas.

CLASSIFICATION OF COVALENT BOND:

On the basis of the number of electrons shared by two combining atoms, the covalent bond are of three types.

- **Single Covalent Bond :** A single covalent bond is formed by the sharing of one pair of electrons between the two atoms. It is represented by one short line (—) between the two atoms.
  Example: H—H, Cl—Cl, H—Cl, CH_3— CH_3.
- **Double Covalent Bond:** A double covalent bond is formed by the sharing of two pairs of electron between the two combining atoms. It is represented by putting (=) two short lines between the two bonded atoms.
  Examples: O = O (O_2), CO_2 (O = C = O), $H_2C = CH_2$
- **Triple covalent bond:** A triple bond is formed by the sharing of three pair of electrons between the two combining atoms. It is represented by putting three short line () between two bonded atom.
  Example: N_2(N≡N), CH≡CH.

FORMATION OF SINGLE COVALENT COMPOUNDS:

- **Formation of hydrogen molecule (H_2) :**

A molecule of hydrogen is composed to two H-atoms. The electronic configuration of H-atom is.

Shell - K
Electron-1
} incomplete duplet (unstable)

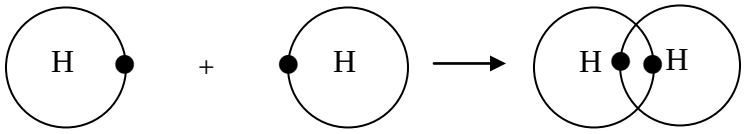
Electronic configuration of He atom

Shell - K
Electrons-2
} complete duplet (stable)

$H^{\cdot} + {}^{\cdot}H \longrightarrow H:H \longrightarrow H\text{-}H \longrightarrow H_2$

K(1) K(1) Shared electron Covalent Hydrogen
Atom Pair Bond molecule

H – H Bond in terms of energy shells (orbits)



H Atom H Atom H_2 molecule

- **Formation of chlorine molecule** (Cl_2): The atomic number of chlorine is 17, thus there 17 electrons in an atom of chlorine.

Electronic configuration of Cl atom –

Shells	K	L	M'
Electrons	2	8	7

} Incomplete octet

Electronic configuration of Ar atom –

Shells	K	L	M
Electrons	2	8	8

} Complete octet

Chlorine atom needs one electron more to complete its octet –

$:\ddot{Cl}\bullet + \bullet\ddot{Cl}: \longrightarrow :\ddot{Cl}\!:\!\ddot{Cl}: \longrightarrow Cl - Cl \longrightarrow Cl_2$

Atom Atom Shared electrons Covalent Bond Chlorine molecule

Cl – Cl bond in terms of energy shell orbits

Cl Atom Cl Atom Cl_2 Molecule

Formation of hydrochloric acid (HCl) :

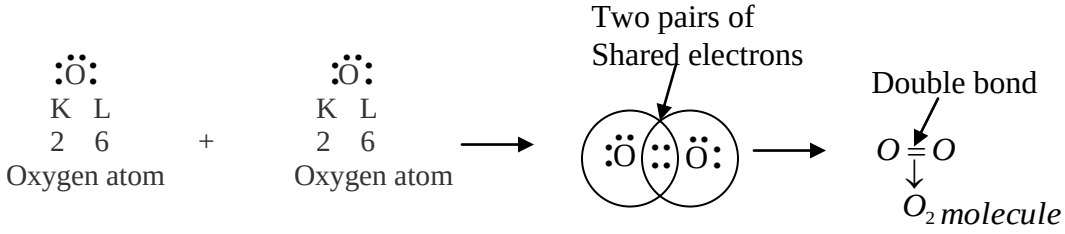
- H atom has one valence electron. It needs 1 electron more to complete its duplet and chlorine has 7 valence electrons. It need 1 electron more to complete its octet and acquire stable electronic configuration (2,8,8) like noble gas argon.

$H + :\ddot{Cl}: \longrightarrow H:\ddot{Cl}: \longrightarrow H - Cl \longrightarrow HCl$

Shared pair of electrons Covalent bond Hydrogen chloride molecule

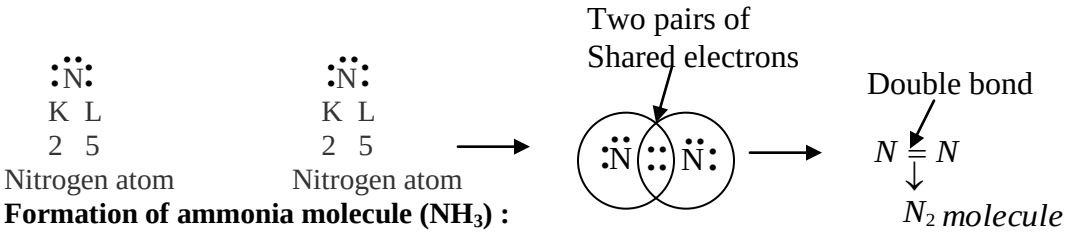
- **Formation of oxygen(O_2) :**

The atomic number of O atom is 8. There are 6 electron in the valence shell of oxygen atom it needs 2 more electrons to attain the nearest stable inert gas Neon (2,8) configuration :



- **Formation of nitrogen molecule (N_2) :**

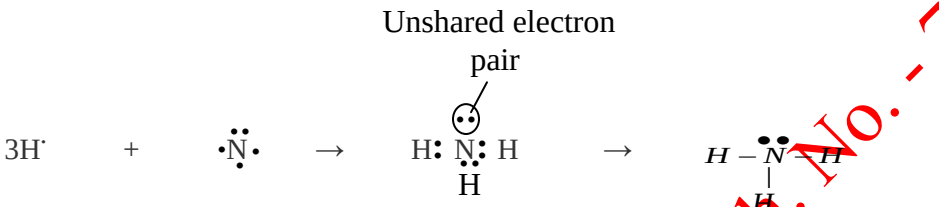
The atomic number of nitrogen is 7 and its electronic configuration is K(2). L(5). It needs 3 electrons more to complete its octet like noble gas neon (2,8).



Nitrogen atom Nitrogen atom

- **Formation of ammonia molecule (NH_3) :**

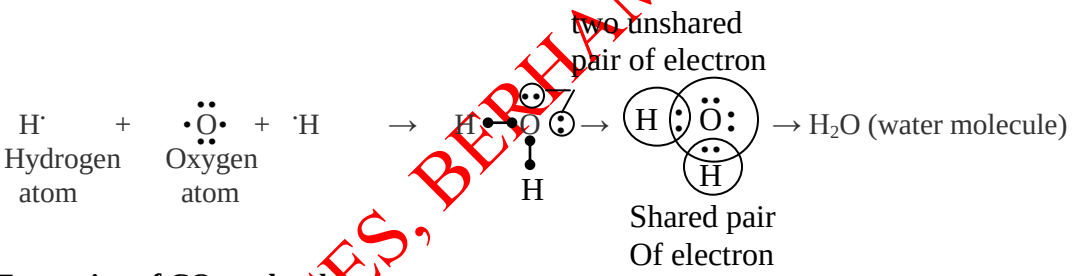
The atomic number of N is 7. It's electronic configuration is 2,5 there are 5 electrons in its valence shell. It needs 3 electrons more to complete its octet like noble gas neon (2,8).



There hydrogen Atom | Nitrogen atom | Shared electron pair in ammonia | Ammonia molecule

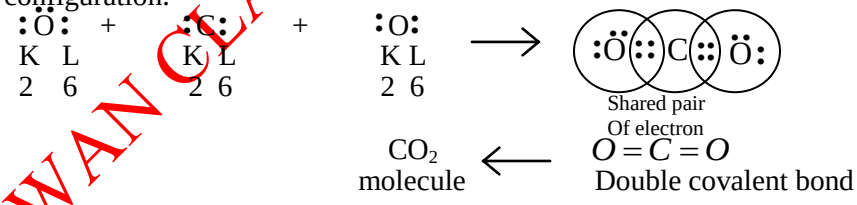
- **Formation of H_2O molecule :**

The electronic configuration of hydrogen is K (1) and that of oxygen is K(2) L(6) thus each hydrogen require one and oxygen required two electrons to achieve the stable electronic configuration.



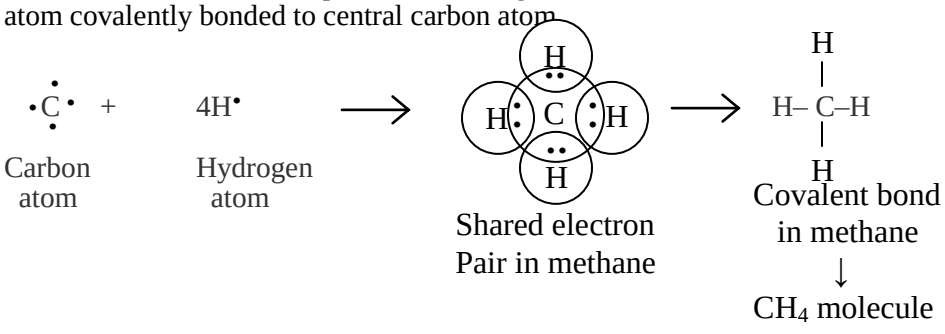
Formation of CO_2 molecule :

The atomic number of C is 6 and the electronic configuration of C is K(2). L(4) and that of oxygen is K(2), L(6) thus each carbon require 4 and oxygen require two electrons to achieve the stable electronic configuration.



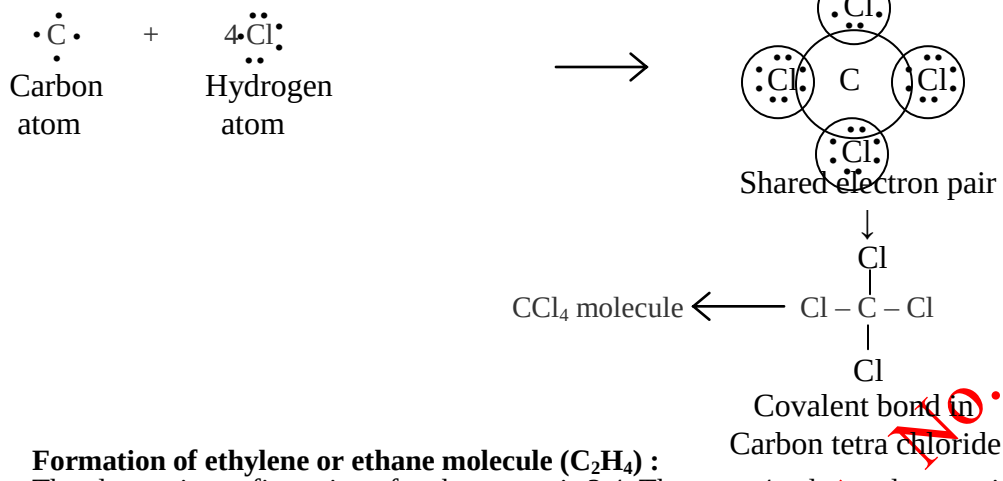
- **Formation of CH_4 molecule :**

Methane is a covalent compound containing 4 covalent bond. It contains one carbon atom and four hydrogen atom covalently bonded to central carbon atom.

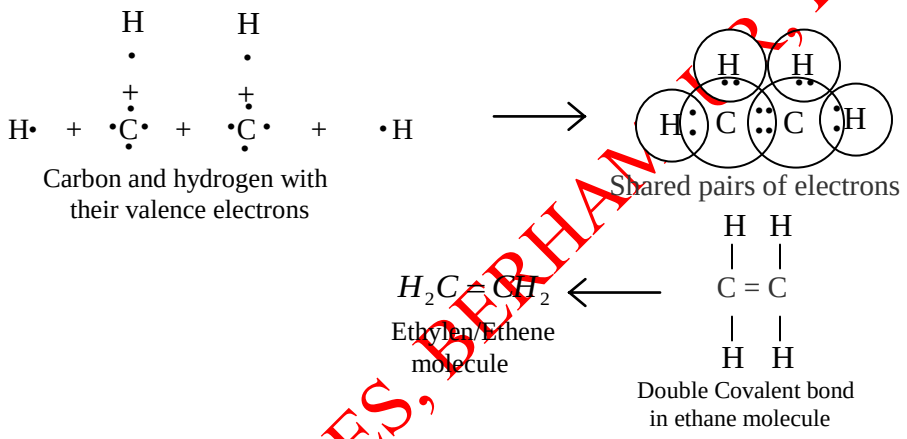


Carbon atom | Hydrogen atom | Shared electron Pair in methane

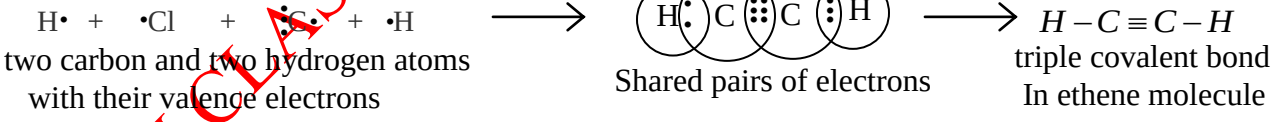
- **Formation of carbon tetrachloride molecule (CCl_4) :**
  The electronic configuration of carbon and chlorine atoms are (2,4) and (2,8,7) respectively. Carbon atom needs four electrons and chlorine atom needs one electron to attain the stable electronic configuration.

- 

- **Formation of ethylene or ethane molecule (C_2H_4) :**
  The electronic configuration of carbon atom is 2,4. There are 4 valence electrons in one C atom. Each H atom contains 1 valence electron. Thus, there are 12 valence electrons present in ethane molecule.



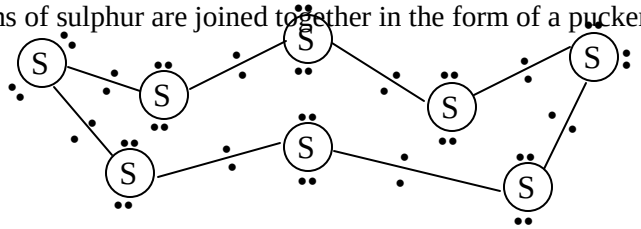
Formation of Acetylene or ethyne molecule (C_2H_2)



Q 1. What would be the electron dot structure of carbon dioxide which has formula CO_2? **[NCERT]**

Q 2. What would be the electron dot structure of a molecule of sulphur which is made up of eight atoms of sulphur ?

Ans. The eight atoms of sulphur are joined together in the form of a puckered ring. **[NCERT]**



Q. Explain the nature of the covalent bond using the bond formation in CH_3Cl. **[NCERT]**

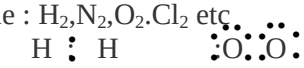
Q. Draw the electron dot structure for **[NCERT]**

(a) Ethanoic acid (b) H_2S (c) Propanone (d) F_2

NON POLAR AND POLAR COVALENT COMPOUNDS :

Non polar covalent bond :

A covalent bond formed between **two atoms** of the same element or same **Electronegativity** is called a **non-polar covalent bond.** Example : H_2,N_2,O_2.Cl_2 etc.



the shared pair electron lies exactly midway between the two atoms

Polar covalent bond :

The covalent bond between the atoms of **two elements** having **different electro negativities** is called a **polar covalent bond.** Molecule in which the atom are bonded by a polar covalent bond are called **polar molecules.**

Note : In a polar covalent bond, the shared pair of electrons lies more toward the atom which is more electronegative.

Example : HCl, H_2O & NH_3

Shared pair of electrons lie nearer to Cl because Cl is more electronegative

Note : δ means partial

CHARACTERISTICS OF COVALENT BOND AND COVALENT COMPOUNDS :

Characteristics of covalent bond :

- Covalent bond are formed by mutual **sharing** of electrons
  Note : Shared pair of electrons is also called **bonding pair of electrons.**

- Covalent bond is directional in nature because shared pair of electrons remain localized in a definite space between the two atoms.

Characteristics of covalent compounds :

Physical Store : The covalent compounds are generally **gases or liquids.** but compounds with high molecular masses are **solids.**

Example : **Solid :** Urea, Glucose, Naphthalene.

Liquids : Water, ethanol, benzene.

Gases : Methane, chlorine, hydrogen, oxygen

- **Melting and boiling points :** Covalent compounds have **low melting** and **low boiling points** because **intermolecular forces** (cohesive forces) in covalent compounds **are weaker** than those in ionic compounds.

Note : Some exception like diamond and graphite which are covalent solids have very high M.P. & B.P.

- **Solubility :**

Covalent compounds generally dissolve readily in organic solvents but they are **less soluble in water.**

For example : Naphthalene which is an organic compound dissolves readily in organic solvents like ether but is insoluble in water. However some covalent compounds like urea, glucose, sugar etc. are soluble in water. Some polar covalent compounds like ammonia and hydrochloric acid are soluble in water.

- **Conductivity :**

Covalent compounds **do not conduct electricity** because they contain neither the ions nor free electrons necessary for conduction. So they do not conduct electricity

For example : Covalent compounds like glucose, alcohol. Carbon tetrachloride do not conduct electricity.

Differences between ionic and covalent compounds :

S.N.	Electrovalent (Ionic) Compounds	Covalent Compounds
1	Formed by transfer of electrons, (only single bond network exist)	Formed by sharing of electrons, (single, double & triple are formed)
2	Usually crystalline solid	Usually gasses or liquid only a few of them are solid
3	Generally have high melting and boiling points	Generally have low M.P. and B.P.
4	Soluble in water but insoluble in organic solvents	Soluble in organic solvent but insoluble or soluble in water
5	Conduct electricity in solution or molten state	Usually non conductor of electricity
6	Highly polar and ionize in water eg. $NaCl \rightarrow Na^+ + Cl^-$	Usually Non-polar and do not ionize in water but few compounds are polar in nature and ionize in water eg. $HCl \rightarrow H^+ + Cl^-$

ORGANIC COMPOUNDS :

The chemical compounds which are present in living organisms (plant and animal) are called **organic compounds.** The belief that formation of organic compounds was possible only in plants and animals led the scientists of early days to propose that **Vital Force** was necessary for the formation of such compounds. But the experimental work of **Friedrich Wohler** (German chemist) denied the idea of vital force when he prepared urea in his laboratory. (urea is an organic compound and waste product of urine).

Q. Name the organic compound which was prepared by Wohler in his laboratory. **[NCERT]**

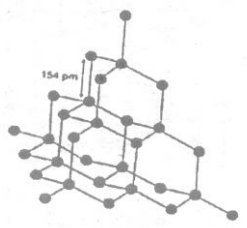
Allotropy/allotrops of carbon :

The phenomenon of existence of allotropic forms of an element is called allotropy. Allotropes are the different forms of the same element having different physical properties but almost similar chemical properties. There are three allotropes of carbon these are diamond, graphite and fullerene.

DIAMOND : Diamond is a **crystalline allotrope** of carbon. Its atomic symbol & empirical formula is 'C'.

Structure : In diamond, each carbon atom is covalently bonded to four other carbon atoms in a tetrahedral arrangement. This tetrahedral arrangement of carbon atoms gives a rigid, three dimensional structure to diamond It is due to this rigid structure that diamond.

- is very hard crystalline structure.
- Has high melting point.
- Is non conductor of heat and electricity.



Structure of Diamond

Properties : Pure diamond is a transparent and colorless solid.

- Polished diamond sparkles brightly because it reflects most of the light (refractive index of diamond is 245)
- Diamond are not attacked by acids. alkalis and solvents like water, ether, benzene or carbon tetrachloride but diamond is attacked by fluorine at 750^0C.

$$C \text{ (Diamond)} + 2F_2 \xrightarrow{750^0 C} CF_4$$

Carbon Fluorine Carbon Tetra fluoride

- The density of diamond is 3.51 g per cm^3 at 20^0C.

Uses :

- A saw fitted with diamond is used for sawing marbles.
- A chip diamond is used for glass cutting.
- Black diamonds are used in making drill.
- Diamonds are used for making dice for drawing very thin wires of harder metals.
- Diamonds are also used for making high precision tools for use in surgery such as, for the removal of cataract.
- Diamond are used for making precision thermometers and protective windows for space crafts.

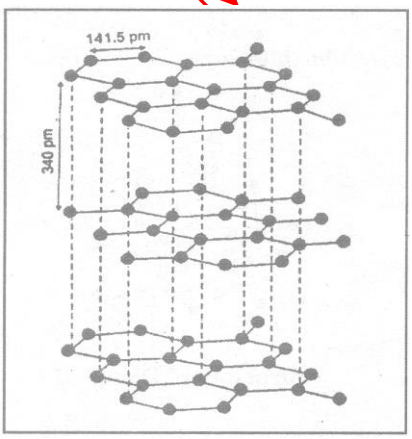
GRAPHITE:

Graphite is also known as **black lead** it marks paper work black. The name graphite has been taken from the Greek word "**graphein**" (which means to write) in reference to its uses as 'lead' in lead pencils.

Structure :

Graphite is an opaque and dark grey solid. In a crystal of graphite the carbon atoms are arranged in hexagonal patterns in parallel planes. In a layer of graphite each carbon atom is strongly bonded to three carbon atoms by covalent bonds. Thus, one valence electron of each carbon atoms is free in every layer of graphite crystal. Thus free electron makes graphite a good conductor of electricity.

Each layer is bonded to the adjacent layers by weak forces. As a result, each layer can easily slide over the other.



Properties :

- Graphite is grayish-black, opaque material having metallic (shiny) **luster.**
- It is soft and has a **soapy** (slippery) touch.
- Graphite is lighter than diamond. The **density of graphite** is 2.26 g per cm^3 at 20^0C.
- Graphite is a **good conductor** of heat and electricity.
- Graphite has a very **high melting point.**
- Graphite is **insoluble** in all common solvent.

Uses:

- For making electrodes in dry cells and electric arc furnaces.
- Graphite is a **good dry lubricant** for those parts of machines where grease and oil cannot be used.
- For making crucibles for melting metals.
- For manufacturing lead pencils.
- Graphite is used as neutron moderator in nuclear reactors.

- For the manufacture of gramophone records and in electrotyping.
- For the manufacture of artificial diamond.

Fullerene :

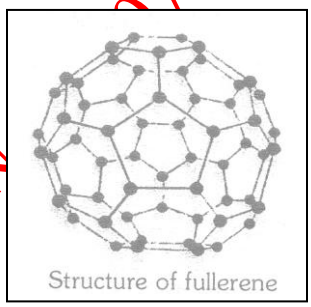
- Fullerene was discovered in 1985 by Robert F. Curl Jr, Harold Kroto and Richard E.Smally.
- This molecule containing sixty atoms of carbon has been named Buckminster fullerene. Fullerenes have been named after American architect and engineer **R. Buckminster-fuller** whose geodesic domes follow similar building principles.

Type of fullerene :

C_{60},C_{70},C_{74} and C_{78} are the members of the fullerene family. But C_{60} is the most stable and most studied from of fullerenes.

Structure of fullerene :

- Buckminster fullerene molecule (C_{60}) is nearly spherical.
- It consists of 12 pentagonal faces and 20 hexagonal faces giving it 60 corners. Thus, Buckminster fullerene has a hollow, cage-like structure.
- In figure, ball like molecules containing C atoms.


Structure of fullerene

Preparation :

- By electrically heating a graphite rod in atmosphere of helium.
- By vaporizing graphite by using laser.

Properties :

- Fullerene is soluble in benzene and forms deep violet colour solution.
- Crystalline fullerene has semiconductor properties.
- Compounds of fullerene with alkali metals are called fullerides and they are superconductors.

Uses :

- As a superconductor.
- As a semiconductor.
- As a lubricants and catalyst.
- As fibres to reinforce plastics.

VERSATILE NATURE OF CARBON :

About three million (or thirty lakh) compounds of carbon are known. The existence of such a large number of organic compounds is due to the following characteristic features of carbon.

(1) CATENATION : Tendency to form Carbon-Carbon bond:

"The property of forming bonds with atoms of the same element is called **catenation".**

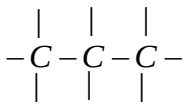
Carbon has the maximum tendency for catenation in the periodic table. This is because of strong carbon bonds as compared to other atoms.

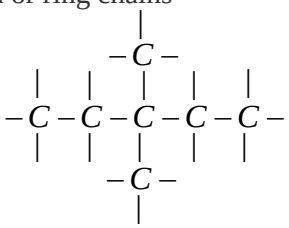
- When two or more carbon atoms combine with one another, they form different types of chain such as

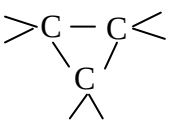
(i) Straight chains

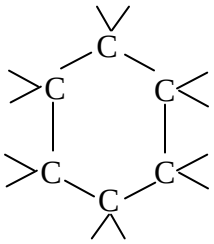
(ii) Branched chains

(iii) Closed chain or ring chains


Straight chain

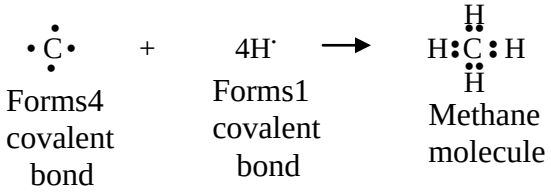

Branched chain


(Minimum 3C required in closed chain structure)


Closed chain

(2) Tetravalency of Carbon :

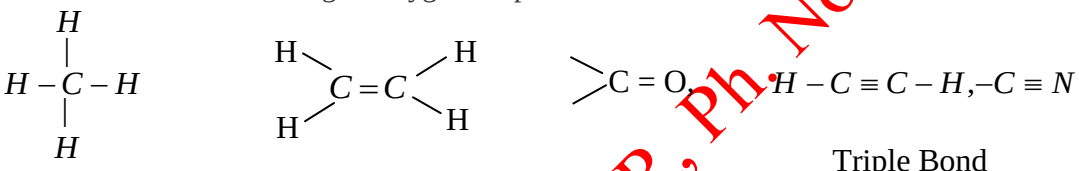
- The atomic number of carbon is **6.**
- The electronic configuration of carbon atom is **$1s^2$,$2s^2$,$2p^2$.**
- It has four electrons in the outermost shell, therefore its valency is four. Thus carbon forms four covalent bonds in its compounds.



Forms4 covalent bond | Forms1 covalent bond | Methane molecule

(3) Tendency to form multiple bonds :

Due to small size, carbon can easily form double or triple bonds**(called multiple bonds)** with itself and with the atoms of other elements as nitrogen, oxygen, sulphur etc.

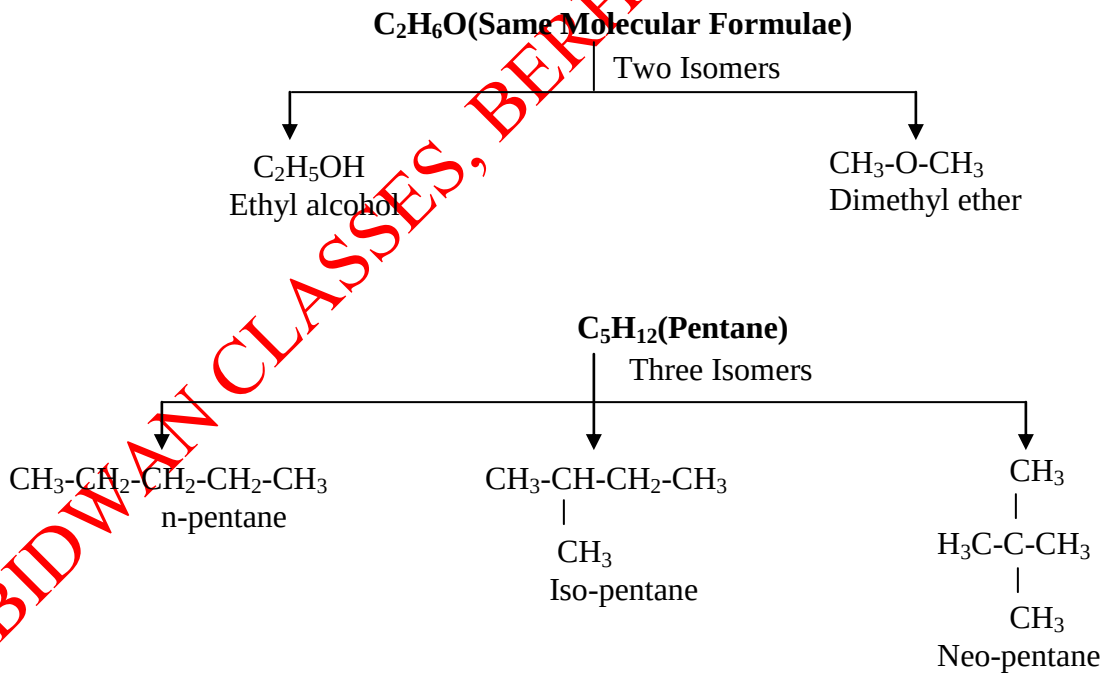


Single bond | Double bond | Triple Bond

(4) Isomerism :

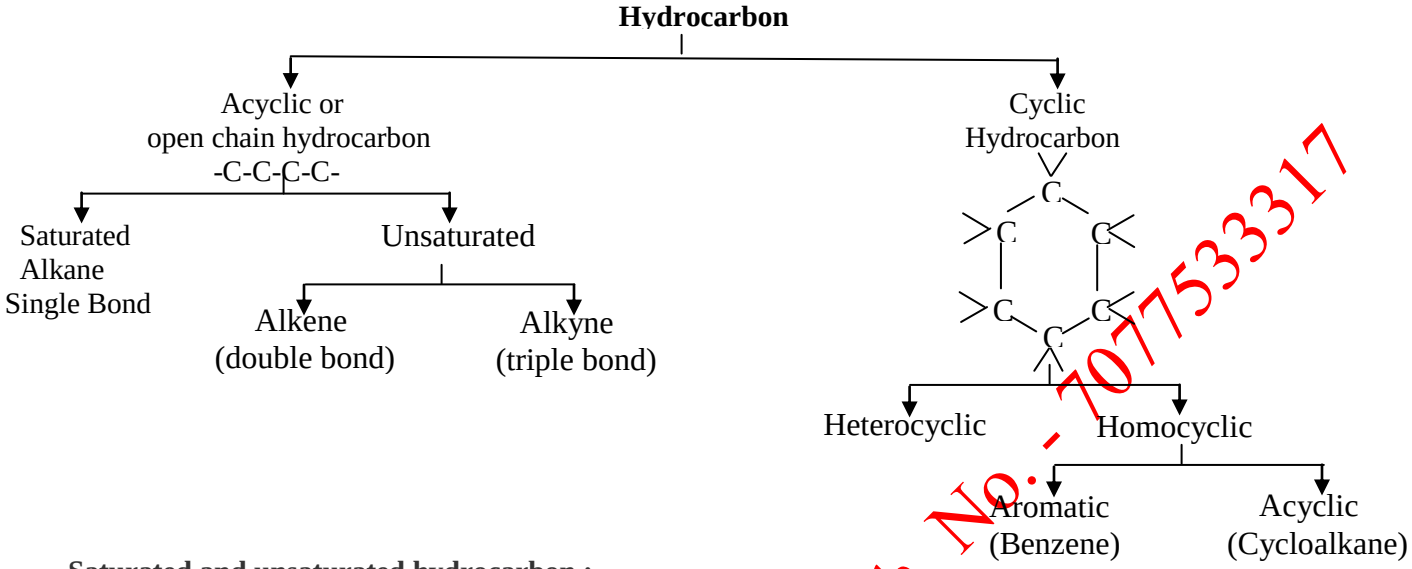
Compounds having same molecular formula but different structural formulae are known as Isomers and the phenomenon of existence of isomers is termed as **isomerism.**

Example :



HYDROCARBON :

Compounds formed from combination of carbon and hydrogen are known as hydrocarbon. Hydrocarbon the basis of chain are mainly classified into two parts.

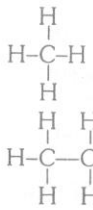
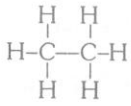
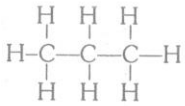
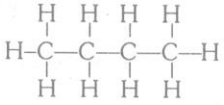
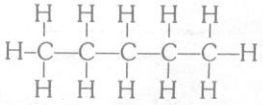
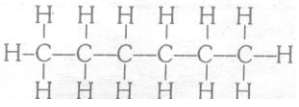


Saturated and unsaturated hydrocarbon :

(1) Saturated Hydrocarbon :

- The hydrocarbons which contain only single carbon-carbon covalent bonds are called **saturated hydrocarbons.**
- They are also called **alkanes.**
- General formula for alkanes is C_nH_{2n+2} where 'n' is the number of carbon atoms.

General formula of saturated hydrocarbon (C_nH_{2n+2})

No. of 'C' atoms	Name	Formula	Structure
1	Methane	CH_4	
2	Ethane	C_2H_6	
3	Propane	C_3H_8	
4	Butane	C_4H_{10}	
5	Pentane	C_5H_{12}	
6	Hexane	C_6H_{14}	

(2) Unsaturated hydrocarbons :

The hydrocarbon in which two carbon atoms are bonded to each other by a double (=) ior a triple (≡) bond is called an unsaturated hydrocarbon.

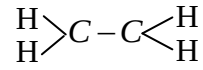
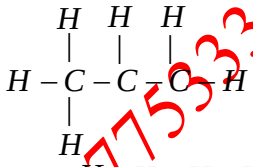
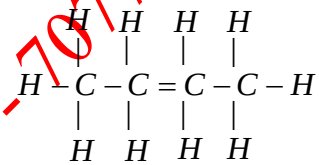
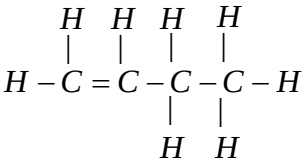
- Unsaturated hydrocarbons are of two types viz. alkenes and alkynes.

$$(-\overset{|}{C}=\overset{|}{C}-)$$

(1) Alkenes :

- The hydrocarbon in which the two carbon atoms are bonded by a double bond are called **alkenes.**
- Their general formula is C_nH_{2n} where "n" is the number of carbon atoms.

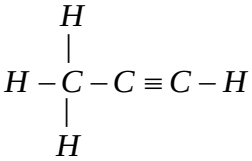
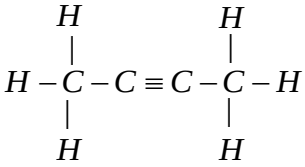
General formula of alkenes : C_nH_{2n}

No. of C atoms	Name	Formula	Structure
2.	Ethene or Ethylene	C_2H_4 $CH_2=CH_2$	
3.	Propene or Propylene	C_3H_6 CH_3-$CH=CH_2$	
4.	Butene or Butylene	C_4H_8 CH_3-$CH=CH$-CH_3 or $CH_2=CH$-CH_2-CH_3	 or 

(II) Alkyne $(-C \equiv C-)$

- The hydrocarbon in which two carbon atoms are bonded by a triple bond are called **alkyne.**
- Their general formula is C_nH_{2n-2} where 'n' is the number of carbon atoms.

General formula of alkynes : C_nH_{2n-2}

No. of 'C' atoms	Name	Formula	Structure
2	Ethyne or Acetylene	C_2H_2 or $HC \equiv CH$	
3	Propyne or Methyl acetylene	C_3H_4 or $H_3C-C \equiv C-H$	
4	Butylene or Dimethyl acetylene	C_4H_6 or $H_3C-C \equiv C-CH_3$	

Q. Give a test that can be used to differentiate chemically between butter and cooking oil. **[NCERT]**

CHAINS, BRANCHES AND RINGS :

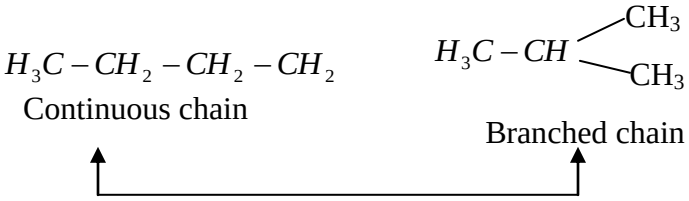
The hydrocarbon may also have branched, closed chains or ring or cyclic structures.

Branched structure :

The alkanes containing three or less carbon atoms do not form branches.

CH_4	CH_3-CH_3	CH_3-CH_2-CH_3
Methane	Ethane	Propane

- The Alkane containing four carbon atoms (C_4H_{10}) has two types of arrangement of carbon atoms.

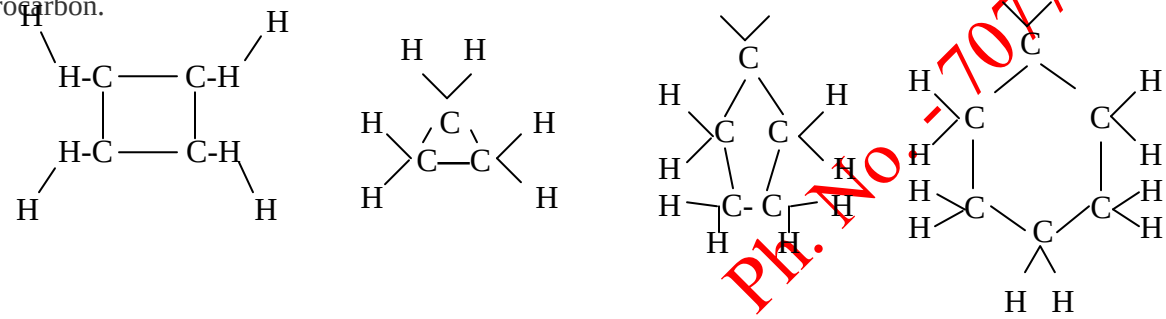


- **Closed chains or cyclic hydrocarbon :** Isomers

These hydrocarbons contains closed chain or rings of atoms in their molecules. These are of two types :

(A) Salicylic hydrocarbon :

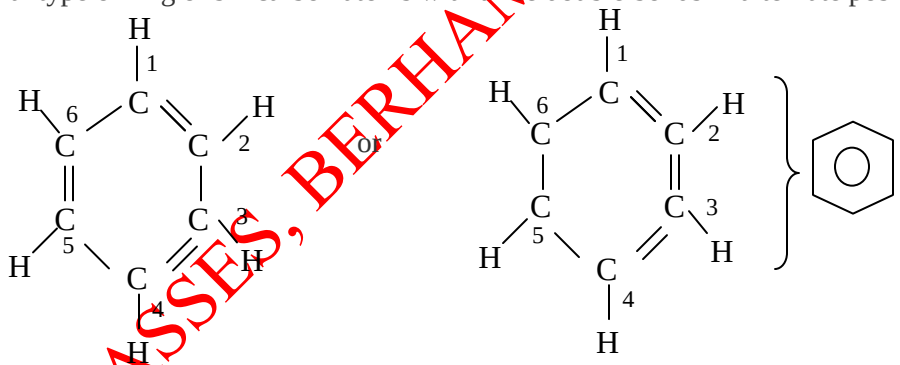
- These hydrocarbon contain a ring chain of three or more carbon atoms.
- These cyclic compounds are named by prefixing cyclo before the name of corresponding straight chain hydrocarbon.



Cyclobutane (C_4H_8) Cyclopropane (C_3H_6) Cyclopentane (C_5H_{10}) Cyclohexane (C_6H_{12})

(B) Aromatic hydrocarbon :

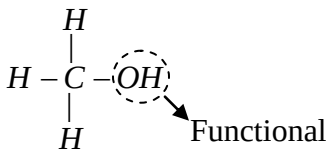
- These have at least one benzene ring in their molecules.
- It is a special type of ring of six carbon atoms with three double bonds in alternate positions.



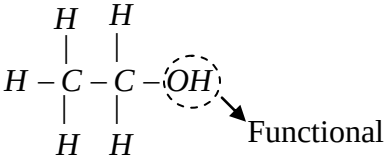
Will you be may friend ? (functional group) :

- Carbon forms many compounds with hydrogen. But carbon also forms bonds with other atoms such as halogen, oxygen, nitrogen and sulphur. Therefore, carbon is said to be very friendly element.
  These compounds are obtained by replacing one or more hydrogen atoms by other atoms such that the valency of carbon remains satisfied. The atom replacing the hydrogen atom is called **heteroatom** or **Functional** group.
- Different organic compounds having same functional group have almost same properties these are called **families.**

Example :



Methanol



Ethanol

- Properties of CH_3-OH and CH_3-CH_2OH are similar and it is due to the presence of –OH (hydroxyl) group .
- This group is known as **alcoholic group.**
- Family of compounds having –OH group is called **alcohols.**

SOME FUNCTIONAL GROUPS IN CARBON COMPOUNDS

Hetero atom	Functional Group	Formula of Functional Group
Halogen atom	Halo	-X
(F, Cl, Br, I)	(Fluoro, Chloro, Bromo, Iodo)	(-F,-Cl,-Br,-I)
Oxygen	1. Alcohol	-OH
	2. Aldehydes	$-\overset{\overset{H}{|}}{\underset{\underset{O}{||}}{C}}\ or\ -CHO$
	3. Ketones	$\diagdown\!\!\diagup C=O\ or\ -\overset{|}{C}O$
	4.Carboxylic acid	$-\overset{\overset{O}{||}}{C}-OH\ or\ -COOH$
Nitrogen	1. Nitro	$-NO_2$
	2. Amines	$-NH_2$

HOMOLOGOUS SERIES :

"A series of organic compounds having similar structures and similar chemical properties in which the successive members differ in their molecular formula by $-CH_2$ group".

The different members of the series are called **homologous.**

Characteristics of Homologous Series :

- All the member of a homologous series can be described by a common general formula.
  Example : All Alkane can be described by the general formula $\mathbf{C_nH_{2n+2}}$**.**
- Each member of a homologous series differ from its higher and lower neighboring members by a common difference of $-CH_2$ group.
- Molecular masses of the two adjacent homologues differ by 14 mass units, because molecular mass of $-CH_2$ group is 12 + 2 = 14.
- All the members of a homologous series show similar chemical properties.
- All the members of the series can be prepared by similar methods known as the general method of preparation.

Table : Some members of Alkane, Alkene and alkyne homologous series.

Alkane		**Alkene**		**Alkyne**	
C_nH_{2n+2}		C_nH_{2n}		C_nH_{2n+2}	
Homologous series		Homologous series		Homologous series	
Name	**Formula**	**Name**	**Formula**	**Name**	**Formula**
Methane	CH_4	-	-	-	-

Ethane	C_2H_6	Ethene	C_2H_4	Ethyne	C_2H_2
Propane	C_3H_8	Propene	C_3H_6	Propyne	C_3H_4
Butane	C_4H_{10}	Butene	C_4H_8	Butyne	C_4H_6
Pentane	C_5H_{12}	Pentene	C_5H_{10}	Pentyne	C_5H_8
Hexane	C_6H_{14}	Hexene	C_6H_{12}	Hexyne	C_6H_{10}

Activity : Calculate the difference in the formulae and molecular masses for (a) CH_3OH and C_2H_5OH (b) C_2H_5OH and C_3H_7OH and (c) C_3H_7OH and C_4H_9OH

Q. It there any similarity in these three ? **[NCERT]**

Q. Arrange these alcohols in the order of increasing carbon atoms to get a family. Can we call this family a homologous series ? **[NCERT]**

Q. What is homologous series ? Explain with an example. **[NCERT]**

Solution :

Formula	**Molecular Mass (Calculated)**	**Difference In**	
		Formula	**Molecular mass**
(a) CH_3OH	12 + 3 + 16 + 1 = 32	$-CH_2$	14
C_2H_5OH	24 + 5 + 16 + 1 = 46		
(b) C_2H_5OH	24 + 5 + 16 + 1 = 46	$-CH_2$	14
C_3H_7OH	36 + 7 + 16 + 1 = 60		
(c) C_3H_7OH	36 + 7 + 16 + 1 = 60	$-CH_2$	14
C_4H_9OH	49 + 9 + 16 + 1 = 74		

Conclusion :

(i) Yes, all these compounds are the members of a homologous series for alcohols.

(ii) CH_3OH, C_2H_5OH, C_3H_7OH and C_4H_9OH –increasing carbon atoms. These four compounds from a homologous series.

HOMOLOGOUS SERIES CONTAINING FUNCTIONAL GROUPS.

- **Aldehydes** :
  HCHO, CH_3CHO, CH_3CH_2CHO, $CH_3CH_2CH_2CHO$
- **Carboxylic acids :** HCOOH, CH_3COOH, CH_3CH_2COOH, $CH_3CH_2CH_2COOH$
- **Amines :** CH_3NH_2, $CH_3CH_2NH_2$,$CH_3CH_2CH_2NH_2$.
- **Ketones :** CH_3COCH_3,$CH_3COCH_2CH_3$,$CH_3COCH_2CH_2CH_3$
- **Holoalkanes :** CH_3X,CH_3CH_2X,$CH_3CH_2CH_2X$,$CH_3CH_2CH_2$-CH_2X

How do physical properties change in a homologous series of hydrocarbons.

The physical properties of the various members of a homologous series change regularly with an increases in the molecular mass.

(i) **Melting and boiling points :** Melting point and boiling of hydrocarbon in a homologous series increases with an increase in molecular mass.

(ii) **Physical State :**

- Hydrocarbons containing lesser number of carbon atoms are **gases.**
- Hydrocarbons containing large number of carbon are **solids.**
- Hydrocarbon containing intermediate number of carbon atoms are **liquid.**

Example : Hydrocarbon containing 1-4 carbon atoms are gases, these containing 5-13 carbon atoms are liquid and those containing more than 14 carbon atoms are solids,

Nomenclature of carbon compounds :

Carbon compounds can be called by their common names, but, then remembering millions of compounds by their individual names may be very difficult. Due to this reason, **the International Union of Pure and Applied chemistry (IUPAC)** has devised a very systematic method of naming these compounds.

Naming a carbon compound can be done by the following methods.

- The number of carbon atoms in the molecule of a hydrocarbon is indicated by the following stems.

No. of carbon atom :	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Stem	Meth	Eth	Prop	But	Pent	Hex	Hept	Oct	Non	Dec.

Example; Saturated hydrocarbon.

Alkane → Meth +ane = Methane

Unsaturated hydrocarbon

Alkene → Eth + ene = Ethene

Alkyne → Eth + yne = Ethyne

- In case of functional group is present, it is indicated in the name of compound with either a prefix or a suffix.
- Identify the longest continuous chain of carbon atoms. This gives the name of parent hydrocarbon.
- In the case of any substituent appropriate prefix is added before the name of parent hydrocarbon.
- In the case of a functional group. the ending 'e' in the name of the parent hydrocarbon is replaced by the appropriate suffix.

Functional Group :

"Functional group may be defined as an atom or a group of atoms which is responsible for most of the characteristic chemical properties of an organic compound".

The prefixes and suffixes of some substituent/functional group

Substituent/ Functional group	**Prefix**	**Suffix**	**Example**	
			Structure	**Name**

1. Halogen : Chlorine Bromine Iodine	Chloro Bromo Iodo	- - -	$CH_3CH_2-CH_2-Cl$ $CH_3-CH_2-CH_2-Br$ $CH_3-CH_2-CH_2-I$	Chloropropane Bromopropane Iodo propane
2. Alcohol	-	ol	$CH_3-CH_2-CH_2-OH$	propanol
3. Aldehydes	-	al	$CH_3-CH_2-\overset{H}{\overset{\vert}{C}}=O$	propanal
4. Ketone	-	one	$H_3C-\overset{O}{\overset{\Vert}{C}}-CH_3$	Propanone
5. Carboxylic acid	-	oic acid	$H_3C-CH_2-\overset{O}{\overset{\Vert}{C}}-OH$	Propanoic acid
6. Single bond (Alkane**)**	-	ane	$CH_3-CH_2-CH_3$	Propane
7. Double bond (Alkene**)**	-	ene	$CH_3-CH=CH_2$	Propene
8. Triple bond (Alkyne**)**	-	yne	$CH_3-CH\equiv CH$	Propyne

Q. How many structure isomers can you draw for pentane?
Q. What will be the formula and electron dot structure of cyclopentane?
Q. Draw the structure for the following compounds :
(i) Ethanoic acid (ii) Bromopentane (iii) Butanone (iv) Hexanal
Q. Draw the possible structural isomers for bromopentane.
Q. How would you name the following compounds ?

(i) CH_3-CH_2-Br (ii) $\overset{H}{\overset{\vert}{H-C}}=O$

(ii) $H-\overset{H}{\overset{\vert}{C}}-\overset{H}{\overset{\vert}{C}}-\overset{H}{\overset{\vert}{C}}-\overset{H}{\overset{\vert}{C}}-C\equiv C-H$

CHEMICAL PROPERTIES OF CARBON COMPOUND:

All carbon compounds show more common characteristic properties. As most of the fuels we use are either carbon or its compounds. Some such properties are described here :

COMBUSTION :

Combustion is a chemical process in which heat and light (in the form of flame) are given out
The process of combustion. is a rapid oxidation reaction of any substance in which heat and light are produced.

Combustion of some common substance :

- **Combustion of Carbon :** Carbon (or charcoal) burn in air or oxygen to give CO_2 productin heat and light.

$$C(s) + O_2(g) \xrightarrow{Combusion} CO_2(g) + Heat + light$$

Carbon Oxygen Carbon dioxide

Q. Why carbon and its compounds are used as fuels for most application ?

- **Combustion of Hydro Carbon :** Hydrocarbons burn to produce carbon dioxide (CO_2), water (H_2O) and heat and light.

$$CH_4(g) + O_2(g) \xrightarrow{Combusion} CO_2(g) + H_2O(g) + Heat + Light$$

Methane

Note: Natural gas and biogas contain methane. So, burning of natural gas and biogas are also combustion reactions.

Burning of LPG (Butane) produces CO2,H2O heat and light.

$$C_4H_{10}(g) + \frac{13}{2}O_2(g) \xrightarrow{Combustion} 4CO_2(g) + 5H_2O(g) + Heat + Light$$

Butane/LPG

- **COMBUSTION OF CELLULOSE:**

Combustion of cellulose (like wood, cotton cloth and paper) gives CO_2, H_2O heat and light. Cellulose is a carbohydrate and can be described by the formula $(C_6H_{10}O_5)_n$.

$$(C_6H_{10}O_5)_n(s) + 6nO_2(g) \xrightarrow{burn} 6nCO_2(g) + 5nH_2O(g) + Heat + light$$

Cellulose

- Combustion of alcohol :

$$C_2H_5OH(l) + 3O_2(g) \xrightarrow{burn} 2CO_2(g) + 3H_2O(g) + Heat + light$$

Ethanol oxygen (in air)

Activity : To observe the combustion of given organic compounds.
Materials : Benzene, naphthalene, Camphor, alcohol (ethanol). Spirit, acetone.
Procedure:
1. Take each compound on iron spatula and burn them in Bunsen burner.
2. Record the type of flame produced.
3. Put a metal plate above the flame and observe whether or not there in black carbon deposition.

Observation :

Compound used	Flame Produced	Deposit
Benzene	Smoky flame	Carbon deposited
Naphthalene	Smoky flame	Carbon deposited
Camphor	Smoky flame	Carbon deposited
Alcohol	Non-Luminous flame	No Carbon deposited
Spirit	Non-Luminous flame	No Carbon deposited
Acetone	Non-Luminous flame	No Carbon deposited

Conclusion :

Benzene, naphthalene, camphor burn with smoky flame and carbon particles get deposited they undergo incomplete combustion due to excess of carbon content.

- Alcohol, spirit and acetone burn with non-Luminous flame and no carbon gets deposited. They under go complete combustion, therefore produce more heat.

Activity : To study the different types of flames/presence of smoke.

Material required : Bunsen burner.

Procedure:
1. Light the Bunsen burner.
2. Close the air hole and observe the colour of the flame.
3. Put a metal plate over it and observe the nature of deposit.
4. Open the air regulator to allow flow of air.
5. Observe the colour of flame.
6. Put a metal plate and observe the nature of deposit.

Observation:

Air Regulator	Colour of flame	Nature of deposit	Nature of flame	Temperature
Closed	Yellow sooty flame	Black carbon deposited	Reducing flame	low
Open	Bluish flame	No black carbon deposited	Oxidizing flame	High

Conclusion : Keep the air regulator open to get oxidizing, non-sooty flame which has high temperature and does not lead to black deposits.

COMBUSTION AND THE NATURE TO FLAME :

(i) Saturated hydrocarbon such as, methane, ethane, propane, butane and natural gas and LPG burn with a blue flame in the presence of sufficient/excess of air/oxygen.

(ii) In the presence of limited amount/of air/oxygen, saturated hydrocarbon, such as, methane, butane, etc give smoky flame.

(iii) Unsaturated hydrocarbon such as ethane, ethyne etc. burn with a luminous/yellow smoky flame.

(iv) The gas/kerosene stove used at home has inlets for air so that a sufficiently oxygen rich mixture is burnt to give a clean blue flame. If you carefully observe the bottoms of vessels getting blackened, it is clear indication that the air holes are blocked and the fuel is getting wasted.

(v) Fuels, such as coal and petroleum, have some amount of nitrogen and sulphur in them. Combustion of coal and petroleum results in formation of oxides of sulphur and nitrogen (such as sulphur dioxide, nitric oxide. Nitrogen peroxide) which are major pollutants in the environment.

FORMATION OF COAL AND PETROLEUM :

Coal and petroleum have been formed from biomass which has been subjected to various biological can geological processes.

Coal is a naturally occurring black mineral and is a mixture of free carbon and compounds of carbon containing hydrogen, oxygen, nitrogen and sulphur. It is not only a good fuel but is also a source of many organic compounds.

It is found in coal mines deep under the surface of earth.

Coal is believed to be formed from fossils which got buried inside the earth during earthquakes and volcanoes which occurred about 300 million years ago. Due to huge pressure and temperature inside the earth and in the absence fo air the fossils fuels (vegetable matter or wood, etc.) were converted into coal. The slow chemical processes of the conversion of wood into coal is called **carbonization.** Since coal is formed by slow carbonization. of plants and fossils, it produces many important carbonization products like peat, lignite, bituminous and anthracite etc. and is itself known as **fossil fuel.** Coal is also a **non-renewable source** of energy.

Petroleum is complex mixture containing various hydrocarbons (compounds of carbon and hydrogen) in addition to small amounts of other organic compounds containing oxygen, nitrogen, and sulphur. It is a dark colored. viscous and foul smelling crude oil. The name petroleum is derived from latin words: **"petra"** meaning rock and **"Oleum"** meaning oil. Since petroleum is found trapped between various rocks, it is also known as **rock oil.**

OXYDATION:

Carbon and its compounds can be easily oxidized on combustion (or burning). During combustion/burning, the compounds gets oxidized completely to different products, depending upon the nature of the oxidizing agents.

- Carbon gives carbon monoxide or carbon dioxide depending upon the oxygen available.

$$2C(s) + O_2(g) \rightarrow 2CO(g)$$

Carbon Oxygen(limited) Carbon monoxide

- $C(s) + O_2(g) \rightarrow CO_2(g)$

(excess) Carbon dioxide

- Hydrocarbon when oxidized give different product as follows :

$$CH_4(g) + 2O_2(g) \xrightarrow{Complete\ oxidation} CO_2(g) + 2H_2O(g)$$

Methane Oxygen(excess)

- $2CH_4(g) + 3O_2(g) \xrightarrow{Incomplete\ oxidation} 2CO(g) + 4H_2O(g)$

Methane Oxygen (Limited)

- Alcohols also give different products on oxidation depending upon the reaction conditions.

Example:

Alcohols on oxidation with certain oxidizing agents such as chromic anhydride in acetic acid, yield corresponding aldehydes, where as on oxidation with alkaline potassium permanganate (or acidified potassium dichromate) corresponding carboxylic acid is formed, as given below:

$$CH_3CH_2OH(l) + [O] \xrightarrow[CH_3COOH]{CrO_3in} CH_3CHO + H_2O$$

Ethanol Nascent oxygen Ethanal (an aldehyde)

$$CH_3CH_2OH(l) + 2[O] \xrightarrow[KMnO_4, Heat]{Alkaline} CH_3COOH + H_2O$$

Ethanoic acid

Activity: To study the reaction of ethanol with alkaline potassium permanganate:

Material required : Ethanol, alkaline $KMnO_4$, test tube.

Procedure:

- Take about 3 ml of ethanol in a test tube.
- Add 5% solution of alkaline $KMnO_4$ drop by drop into this solution.
- Observe the colour of alkaline $KMnO_4$ after adding initially as well as finally.

Observation : The colour of $KMnO_4$ gets discharged in the beginning. When excess of $KMnO_4$ is added, the colour of $KMnO_4$ does not disappear because whole of ethanol gets oxidized to Ethanoic acid.

$$CH_3CH_2OH \xrightarrow[or\ K_2Cr_2O_7/H_2SO_4, Heat)]{Alkaline, KMnO_4, Heat} CH_3COOH + H_2O$$

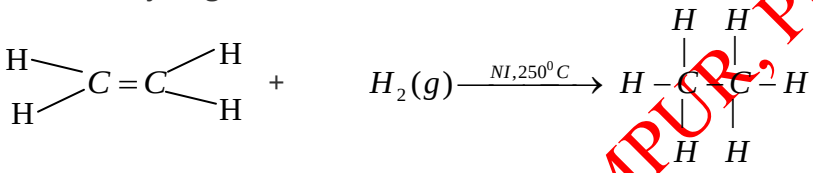
ADDITION REACTION :

All unsaturated hydrocarbons (unsaturated carbon compounds) react with a molecule like H2. X2. H2O etc. to form another saturated compounds are called **addition reactions.**

Unsaturated hydrocarbons add hydrogen, in the presence of catalysts, such as nickel or palladium to give saturated hydrocarbons.

Note: Catalysts are substance that cause a reaction to occur or proceed at a different rate without the reaction it say being affected.

- **Addition of hydrogen to ethene :**

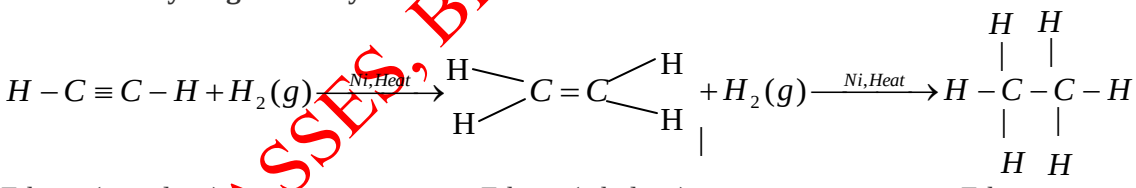


Ethene (Double bond containing unsaturated carbon compound)

Ethane (Saturated hydrocarbon)

Q. Which of the following hydrocarbons undergo addition reactions : C_2H_6, C_3H_8, C_3H_6, C_2H_2 and CH_4. **[NCERT]**

- **Addition of hydrogen to ethyne :**



Ethyne (acetylene) Ethene (ethylene) Ethane

- Addition of hydrogen to a unsaturated carbon compound is called **hydrogenation reaction.**
Certain vegetable oils such as ground nut oil, cotton seed oil and mustard oil, contain double bonds (C = C) are liquids at room temperature. Because of the unsaturation, the vegetable oils undergo hydrogenation. like alkenes, to from saturated products called vanaspati ghee. Which is semi-solid at room temperature.
Vegetable oils (Unsaturated oil)+Hydrogen $\xrightarrow{Ni, Heat}$ Vanaspati Ghee (Saturated ghee)

Q. What is hydrogenation? What is its industrial application? **[NCERT]**

Q. If a molecule Y contain two –C = C- double bonds, then how many moles of H_2 are required for completed hydrogenation of one mole of Y? **[NCERT]**

Q. Write the industrial application of hydrogenation. **[NCERT]**

SUBTITUTION REACTION:

The reactions in which one or more hydrogen atoms of a hydrocarbon are replaced by some other atoms or groups are called **substitution reaction.**

Example :
Methane reacts with chlorine (or bromine) in the presence of sunlight and undergo substitution reaction. It is called **photochemical reaction** because it takes place in presence of sunlight.

$$CH_4(g) + Cl_2(g) \xrightarrow{Sunlight} CH_3Cl(g) + HCl(g)$$
Chloromethane

$$CH_3Cl(g) + Cl_2(g) \xrightarrow{Sunlight} CH_2Cl_2(g) + HCl(g)$$
Dichloromethane

$$CH_2Cl_2(g) + Cl_2(g) \xrightarrow{Sunlight} CHCl_3(g) + HCl(g)$$
Trichloromethane(chloroform)

$$CHCl_3(g) + Cl_2(g) \xrightarrow{Sunlight} CCl_4(l) + HCl(g)$$
Tetra chloromethane
(carbon tetra chloride)

Q. Why is the conversion of ethanol to Ethanoic acid an oxidation reaction? **[NCERT]**
Q. A mixture of oxygen and ethyne is burnt for welding. Can you tell why a mixture of ethyne and air is not used ? **[NCERT]**

SOME IMPORTANT CARBON COMPOUNDS :

ETHANOL (ETHYL ALCOHOL. C_2H_5OH) :
Ethanol is the second member of the homologous series of alcohols.

Preparation : By the fermentation of carbohydrates (sugar or starch).

Ethanol is prepared on commercial scale by fermentation of sugar Fermentation is allowed to take place at 298-303 K in the absence of air. This is ethanol (ethyl alcohol) gets oxidized to ethanoic acid (acetic acid) in the presence of air.

$$C_{12}H_{22}O_{11} + H_2O \xrightarrow{Invertase} C_6H_{12}O_6 + C_6H_{12}O_6$$

Sucrose — Glucose — Fructose

$$C_6H_{12}O_6 \xrightarrow{Zymase} 2C_2H_5OH + 2CO_2(g)$$

Glucose/Fructose — Ethanol (ethyl alcohol) — Carbon dioxide

PHYSICAL PROPERTIES :

- Physical state /colour and odour : Pure ethanol is a colorless liquid having a pleasant smell and a burning taste.
- **Boiling and Freezing points :** It is a volatile liquid with a boiling point of 78.1^0C, and freezing point is -118^0C.
- **Density :** Ethanol is lighter than water as its density is 0.79 g ml^{-1} at 293 K.
- **Solubility :** Ethanol is miscible with water in all proportion, due to the formation of hydrogen bonds with water molecules.
- **Conductivity :**Ethanol is a covalent compound and does not ionize easily in water. Hence it is a neutral compound.
- **Action on Litmus :** Ethanol is a neutral compound So, it has no effect on the colour of litmus.

CHEMICAL PROPERTIES OF ETHANOL :

- **Combustion (or burning) :** Ethanol is highly inflammable liquid and readily burn in air with blue flame to form water vapor, carbon dioxide and evolving heat. Thus, combustion of ethanol is an exothermic reaction.

$$C_6H_5OH(l) + 3O_2(g) \xrightarrow{Combustion} 2CO_2(g) + 3H_2O(g) + Heat$$

Q.	Carbon and its compounds are used as fuels. Give its main reason**.**	**[NCERT]**

- **Reaction with sodium metal :** Ethanol reacts with sodium metal to produce sodium ethoxide and hydrogen gas is evolved.

$$2C_2H_5OH(l) + 2Na(s) \rightarrow 2C_2H_5ONa + H_2(g)$$

Ethanol — Sodium metal — Sodium ethoxide — Hydrogen

Activity : To study the reaction of ethanol with sodium metal.
Materials : Ethanol, dry piece of sodium metal test tube.

Procedure :

- Take ethanol in a test tube.
- Add a dry piece of sodium metal.
- Bring a burning matchstick near the gas evolved to test it and record observation.

Observation : The gas burns in air with a pop sound which is the characteristics of hydrogen gas.
Conclusion : Alcohol react with sodium metal to liberate hydrogen gas.

- **Reaction with Ethanoic acid (Etherification reaction) :**

  The reaction in which an alcohol reacts with acetic acid in the presence of conc. H2SO4 to form an ester is called etherification.

$$CH_3COOH(l) + C_2H_5OH(l) \underset{}{\overset{\text{Conc. } H_2SO_4, \text{ Heat}}{\rightleftharpoons}} CH_3COOC_2H_5(l) + H_2O(l)$$

ethanoic acid (acetic acid) | ethanol | ethyl ethanoate (ethyl acetate) Sweet smelling compound

Note : Ester are sweet-smelling compounds and are used for making perfumes.

- **Reaction with conc. sulphuric acid (Dehydration) :**

  Ethanol when heated with excess of concentrated sulphuric acid at 443 K, gets dehydrated to give ethene.

$$C_2H_5OH(l) + H_2SO_4(Conc.) \xrightarrow{443K} H_2C=CH_2(g) + H_2O(l)$$

ethanol | excess | ethene

Note : The concentrated sulphuric acid can be regarded as a dehydrating agent which remove water from ethanol.

Use of ethanol :

- Ethanol is present in alcoholic beverages such as beer, wine, whisky.
- As a solvent for paints, varnishes-dyes, cosmetics, perfumes, soaps and synthetic rubber etc.
- Ethanol is used in cough syrups, digestive syrups and tonics.
- A mixture of 80% rectified spirit and 20% petrol is called **power alcohol.** It is used as fuel in cars and airplanes.
- A mixture of ethanol and water has lower freezing point than water this mixture is known as **antifreezing** and is used in radiators of vehicles in cold countries and at hill stations.
- As an antiseptic to sterilize wounds and syringes in hospitals.
- For the manufacture of terylene and polythene.
- As a preservative for biological specimens.
- Ethyl alcohol is used as hypnotic (induces-sleep).

Harmful effects of Alcohols:

- Consumption of small quantities of dilute ethanol causes drunkenness. Even though this practice is condemned, it is a socially widespread practice. However, intake of even a small quantity of pure ethanol (called **absolute alcohol**) can be lethal. Also long-term consumption of alcohol leads to many health problems.
- When large quantities of ethanol are consumed, it tends to slow metabolic processes and to depress the central nervous system. This results in lack of coordination, mental confusion, drowsiness, lowering of normal inhibitions and finally stupor(unconscious state of wild)
- Drinking of alcohol over a long period of time damages liver.

Denatured Alcohol :

Ethanol to which certain poisonous and nauseating substances like methyl alcohol, pyridine etc. have been added is termed **denatured alcohol.**

Note: To prevent the misuse of ethanol (Alcohol), industrial alcohol is colored blue to that it can be recognized easily.

Harmful effects of denatured alcohol:

- Methanol is highly poisonous compounds for human beings. Methanol when taken, even in small amount, can cause death.
- Methanol gets oxidized to methanol in the liver, which causes coagulation of protoplasm.
- Methanol also effects the optic nerve and cause blindness.

ETHANOIC ACID (ACETIC ACID) CH_3COOH:

- Ethanoic acid is commonly called acetic acid and belongs to the homologous series of carboxylic acid and is represented as CH_3COOH.
- 5-8% solution of acetic acid in water is called **vinegar** and is used for preservation foods like sausage, pickles. **Physical properties :**
- At ordinary temperature, Ethanoic acid is a colorless liquid with a strong pungent smell and sour taste.
- Its boiling point is 391 K and its density at 273 K is 1.08 (heavier than water).
- It is miscible with water due to the formation of hydrogen bonds with water molecules.
- On cooling at 289.6K, it turns in ice-like crystals, hence named as glacial acetic acid.
- It dissolves sulphur, iodine and many other organic compounds.
- It immerse when dissolved in benzene.
- $2CH_3COOH \rightleftharpoons (CH_3COOH)_2$

Ethanoic acid Dimer

Activity : To determine pH of acetic acid and hydrochloric acid.
Material : Acetic acid (1M).HCl (1M), blue litmus paper, universal indicator.
Procedure : Take two strips of blue litmus paper.

- Put a few drops of HCl on one of them and few drops of acetic acid on the other.
- Observe the change in colour.
- Take 1 ml of acetic acid in a test tube and add a few drops of universal indicator.
- Take 1ml of HCl in a test tube and add few drops of universal indicator.

Observation : Both acetic acid and HCl turn blue litmus red showing that they are acidic in nature. pH of acetic acid and HCl are not equal.

Conclusion : HCl is strong acid than CH_3COOH, therefore, pH of HCl will be lower than that of acetic acid.

CHEMICAL PROPERTIES :

- **Reaction with alcohols (Etherification reaction):**

Ethanoic acid reacts with ethanol in the presence of cons. H_2SO_4 to form ethyl ethanoate which is an ester.

$$CH_3COOH(l) + C_2H_5OH(l) \xrightarrow{Conc, H_2SO_4, heat} CH_3COOC_2H_5 + H_2O$$

Ethanoic acid Ethanol Ethyl ethanoate (ester)

The reaction of carboxylic acid with an alcohol to form an ester is called **"estrification".**

Note : Ester can be hydrolyzed in the presence of an acid or a base to give back the parent carboxylic acid and the alcohol.

Example :

(i) Ethyl ethanoate on acid hydrolysis gives Ethanoic acid and ethanol.

$$CH_3COOC_2H_5(l) \quad + \quad H_2O(l) \quad \rightarrow \quad CH_3COOH(aq.) \; + \; C_2H_5OH$$

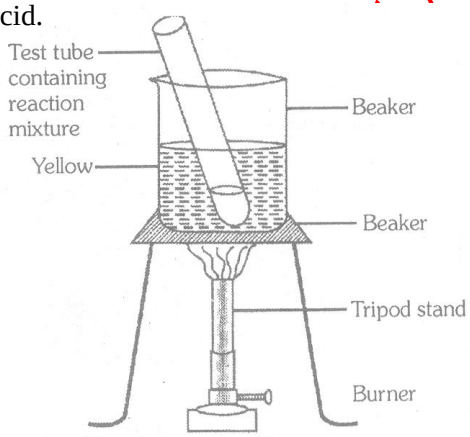
(ii) Hydrolysis of ester in the presence of base (alkali) is called **"Saponification reactions".**

$$CH_3COOC_2H_5(l) \quad + \quad NaOH(aq) \rightarrow CH_3COONa \quad + \; C_2H_5OH$$

Ethyl ethanoate Sodium Hydroxide Sodium ethanoate Ethanol

Note: Alkaline hydrolysis of higher esters is used in the manufacture of soaps.

Activity : To study the etherification process using ethanol and acetic acid.



Materials : Beaker, water, test tube, ethanol, acetic acid. Conc. H_2SO_4 etc.

Procedure : Take 2ml of ethanol in a test tube.

- Add 2ml of Ethanoic acid (acetic acid) in to it.
- Add few drops of conc. H_2SO_4.
- Warm it in a beaker containing water.

Observation : Pleasant fruity smelling compound (called ester) is formed.
Conclusion : Acetic acid reacts with alcohol in presence of conc. H2SO4 which act as a dehydrating agent to form ester.

Reaction with sodium carbonate and sodium hydrogen carbonate :

Ethanoic acid decomposes sodium hydrogen carbonate and sodium carbonate with a rapid evolution of carbon dioxide gas.

$$NaHCO_3(aq) + CH_3COOH(aq) \rightarrow CH_3COONa(aq) + H_2O(l) + CO_2(g)$$

Sodium Hydrogen carbonate Ethanoic acid Sodium ethanoate

$$Na_2CO_3(aq) + \; 2CH_3COOH(aq) \rightarrow 2CH_3COONa(aq) + H_2O + CO_2(g)$$

Sodium carbonate Ethanoic acid Sodium ethanoate

Note : Reactions of Ethanoic acid with NaOH, NaHCO3, Na2CO3 and active metals show that the hydrogen present in the carboxy(-COOH) group is acidic in nature.
Activity : To study the reaction of carboxylic acid with sodium carbonate and sodium hydrogen carbonate.
Material : Ethanoic acid, Sodium carbonate, Sodium hydrogen carbonate.

Procedure :

- Take 1g of Na2CO3 and 2ml of Ethanoic acid into it.
- Pass the gas formed through lime water and note down the observation.
- Repeat the same procedure with sodium hydrogen carbonate and record observation.

Observation: Brisk effervescence due to carbon dioxide formed which turns lime water milky.
Conclusion: Acetic acid react with Na_2CO_3 and $NaHCO_3$ to liberate CO_2 gas.

USES OF ETHANOIC ACID :

- Ethanoic acid is used in the manufacture of various dyes, perfumes and rayon.
- It is used for making vinegar.
- It is used for making white lead [$2PbCO_3$. $Pb(OH)_2$]which is used in white paints.
- Its 5% solution is bactericidal (destroys bacteria).
- It is used in preparation of cellulose acetate which is used for making photographic film.

- It is used for coagulation of the latex.
- It is used for preparation of 2,4-dichloro phenoxy Ethanoic acid which is used as **herbicide.**
- Aluminium acetate and chromium acetate are used as mordents in dyeing and water proofing of fabrics.

Q. How would you distinguish experimentally between an alcohol and a carboxylic acid? **[NCERT]**

Q. What are oxidizing agents? **[NCERT]**

Q. How can ethanol and ethanoic acid be differentiated on the basis of their physical and chemical properties? **[NCERT]**

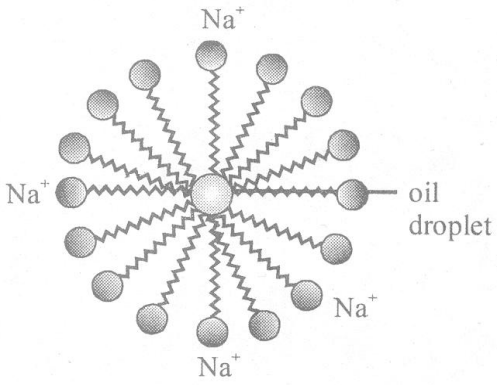
SOAP AND DETERGENTS :

Soap and detergents are substances which are used for cleaning. There are two types of detergents:

1. Soap 2. Synthetic detergents

Soap : A soap is the sodium or potassium salt of a long-chain fatty acids (carboxylic acid or glycerol).

Activity :



Formation of micelles

Take about 10mL of water each in two test fubes.

Add a drop of oil (cooking oil) to both the test tubes and label them as A and B.

To test tube B, add a few drops of soap solution. Now shake both the test tubes vigorously for the same period of time.

Can you see the oil and water layers separately in both the test tubes immediately after you stop shaking them.

Leave the test tubes undisturbed for some time add observe. Does the oil layer separate out ? In which test tube does this happen first ?

This activity demonstrate effect of soap in cleansing as we know that most of the dirt is oily in nature and oil does it dissolve in water.

But know the question arise what are soap? What is the detergent which one is more effect. How the work.

Soap is sodium or potassium salt a long change fatty acid (Carboxylic acid or Glycerol)

A soap has large non ionic hydrocarbon group and an ionic group. COONa.

Ex. of soap are :

(1) Sodium stearate ($C_{17}H_{35}COONa$)

(2) Sodium plamitate($C_{15}H_{31}COONa$)

Soap are basic in nature so soap solution turn red litmus to blue.

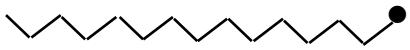
Preparation of Soap :

The soap is prepared by heating animal fats or vegetable oils (olive oils, castor oil or palm oil) with sodium hydroxide or potassium hydroxide.
The process of formation of soap by the hydrolysis of fat or oil with alkali is called saponification.
Oil or Fat + Sodium hydroxide → Soap + glycerol

Structure :

- A soap molecule contains two parts that interact differently with water. One part is a long hydrocarbon (non-polar) chain, and other belongs to the-COONa group (Hydrophilic).

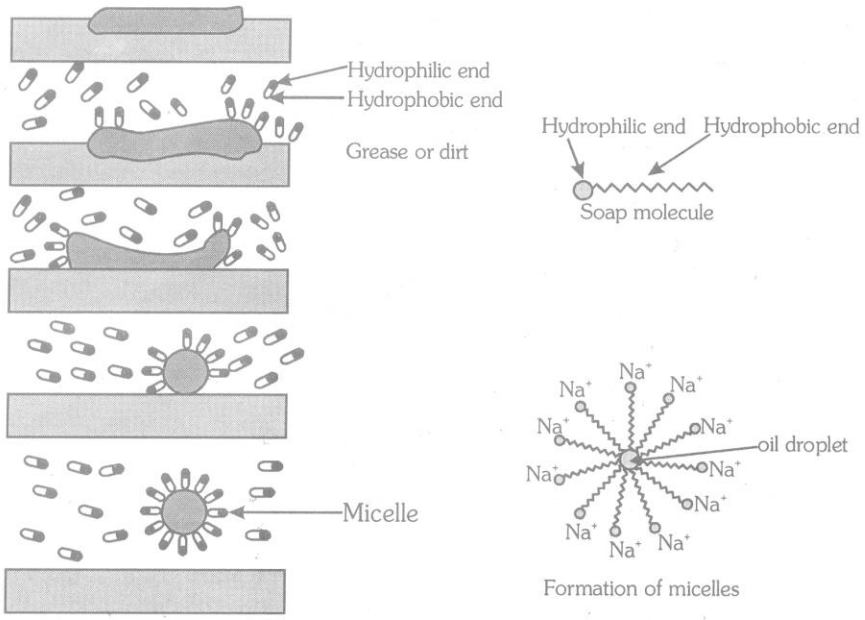
A soap molecule may be represented as :



$$R-\underset{\underset{O}{\|}}{C}-O^{-}\ Na^{+}$$

Cleansing action of soap :

The molecules of soap are sodium or potassium salts of long chain carboxylic acids, The ionic end of soap dissolves in water while the carbon chain dissolves in oil. The soap molecules. thus form structures called micelles where one end of the molecules is towards the oil droplet while the ionic end faces outside. This form and emulsion in water. The soap micelle thus helps in dissolving the dirt in water and we can wash out clothes clean.



Effect of soap in cleaning

Q.	Why does micelle formation take place when soap is added to water? Will a micelle be formed in other solvents such as ethanol also?	**[NCERT]**
Q.	Explain the mechanism of the cleaning action of soaps.	**[NCERT]**

Activity : Take two clean test tubes and label them as 'A' and 'B'. Now put 10ml hard water in each of the two test tubes. Add five drops of soap solution in test tube 'A' and five drops of detergent solution in test tube 'B' Shake the two test tubes for the same period and observe if both observe if both the test tubes have the same amount of foam. Find out in which of the two test tubes a curdy white mass is formed.

In which test due do you get more form ?

We get more foams in test tube…… .

A white curdy precipitate is formed in test tube…….

Result (Conclusion): Soaps are not effective in acidic medium.

When soaps is used for washing clothes with hard water, a large amount of soap is wasted in reacting with the calcium and magnesium ions of hard water to form an insoluble precipitate called scum, before it can be used for the real purpose of washing soap. A large amount of soap is needed for washing clothes. When the water is hard.

Activity :

Take two test tubes with a about 10 mL of hard water in each.

Add five drops of soap solution to one and five drops of detergent solution to the other.

Shake both test tubes for the same period

Do both test tubes have the same amount of foam?

In which test tube is a curdy solid formed?

Observation :

Test tube in which ……..is present contain more amount of foam.

Curdy solids is form in the test tube containing……

Conclusion : Detergents have better cleansing action than soap. Detergents are generally ammonium or sulphonate salts of long chain carboxylic acids. The charged ends of these compounds do not form insoluble precipitates with the calcium and magnesium ions in hard water. Thus, they remain effective in hard water. Detergents are usually used to make shampoos and products for cleaning clothes.

Q. What change will you observe if you test soap with litmus paper (red and blue)? **[NCERT]**
Q. Would you be able to check if water is hard by using a detergents?
Q. People use a variety to methods to wash clothes. Usually after adding the soap, they 'beat 'beat' the clothes on a stone, or beat it with a paddle, scrub with a brush or the mixture is agitated in a washing machine. Why is agitation necessary to get clean clothes ?

DIFFERENCES BETWEEN SOAPS AND SYNTHETIC DETERGENTS:

Soaps	**Synthetic detergents**
1. Soaps are sodium salts of long chain fatty acid (carboxylic acids)	**1.** Synthetic detergents are the sodium salts or long-chain benzene sulphonic acids or the sodium salt of a long. chain alkyl hydrogen sulphate
2. The ionic part of soap is $-COO^{-}Na^{+}$	**2.** The ionic part in a synthetic detergents is $-SO_3H-Na^{+}$
3. They are prepared from animal fats or plant based oils.	**3.** The are prepared from hydrocarbons extracted from coal or petroleum.
4. Their efficiency decreases in hard water	**4.** Their efficiency is unaffected in hard water.
5. Soaps take more time to dissolve in water.	**5.** Synthetic detergents dissolve faster than soaps in water
6. They are biodegradable	**6.** Some synthetic detergents are not biodegradable.
7. Examples : Sodium state, sodium palmitate	**7. Example :** Sodium lauryl sulphate, sodium dodecyl benzene sulphonate.

SOAP :

$$C_{17}H_{35}COO-CH_2$$
$$|$$
$$C_{17}H_{35}COO-CH \quad + \quad 3NaOH \quad \rightarrow \quad 3C_{17}H_{35}COONa$$
$$|$$
$$C_{17}H_{35}COO-CH_2$$

Glycerol stearate (fatty acid) — Sodium hydroxide — Sodium stearate(soap)

Structure:

- The hydrocarbon chain is non-polar and water-hating (hydrophobic), while the other part is polar or water loving (hydrophilic).
- Hydrophilic part makes the soap soluble in water and hydrophobic part makes the soap insoluble.

Hydrophobic "Tail" (Non-polar part) — Hydrophilic "head" (Polar part)

- When soap is added to water, the soap molecules assume a configuration which increases the interaction of the water loving heads with the water molecules, and decreases the interaction between the water hating tails with the water molecules.
- The hydrophobic part of the soap molecules traps the dirt and the hydrophilic part makes the entire molecules soluble in water. Thus, the dirt gets washed away with the soap.
- The water-hating, non polar tails clump together in a radial fashion with the water-loving. polar heads remaining at the periphery of the clump, these clumps or droplets of soap molecules are called micelles.

Disadvantage of soap :

- **Soaps are not effective in hard water :** Hard water contains calcium ions (Ca^{2+}) and magnesium ions (Mg^{2+}) These ions react with the carboxylate ions (RCCO-) of the soap forming an insoluble precipitate called scum, For example, soap like sodium stearate (C17H35COONa) reacts with calcium and magnesium ions as per the following chemical equation.

$$2C_{17}H_{35}COONa + Ca^{2+}(aq) \quad \rightarrow \quad (C_{17}H_{35}COO)_2Ca \downarrow + \quad 2Na^{+}(aq)$$

Sodium stearate (In hard water) — Calcium stearate — Sodium ion

$$2C_{17}H_{35}COONa + Mg^{2+}(aq) \quad \rightarrow \quad (C_{17}H_{35}COO)_2Mg \downarrow + \quad 2Na^{+}(aq)$$

Sodium stearate (In hard water) — Magnesium stearate(scum) — Sodium ion

The scum gets attached to the clothes, utensils and even skin and thus, interferes with the cleansing ability of the additional soap and makes the cleansing of clothes difficult. Moreover, large amount of soap is wasted in reacting with calcium and magnesium ions present in hard water.

Q.	Explain the formation of scum when hard water is treated with soap.	**[NCERT]**

- **Soaps are not effective in acidic medium :** In presence of hydrogen ions (H ions), i.e. in acidic medium, the carboxylate ions of soap ($RCOO^-$ion)interact with hydrogen ions (H^+) to form dissociated (free) fatty acid as represented below:

$$C_{17}H_{35}COO^-(aq) + H^+ \rightleftharpoons C_{17}H_{35}COOH$$

carboxylate ion Carboxylic acid (Unionized)

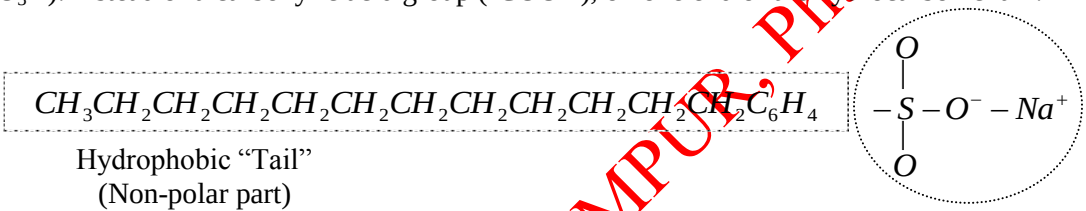
As the fatty are weak acids, so they do not get ionized and hence, micelle formation is hindered, thus, adversely affecting the cleansing property of soaps.

You will observe that the amount of foam in the two test tubes in different. The foam is formed to a greater extent in test tube 'B' (containing detergent solution), while formation of a curdy white mass will be observed in test tube 'A'. This activity clearly indicates that detergents can be used for cleansing purpose, even with hard water.

SYNTHETIC DETERGENTS :

- Synthetic detergents are called soap less soap because they are not prepared from **fatty acid** and **alkali.**

- Synthetic detergents are sodium salts of sulphonic acids, i.e. detergents contain a sulphonic acid group (-SO_3H). instead of a carboxylic acid group (-COOH), on one end of the hydrocarbon chain.



Hydrophobic "Tail"
(Non-polar part)

Hydrophilic "head"
(Polar part)

Properties of synthetic detergents :

- Synthetic detergents do not react with the ions present in hard water. Hence synthetic detergents have no problem in forming lather with hard water, i.e. their efficiency is not affected by hard water.

- Synthetic detergents can be used even in acidic solution and sea water, whereas soap cannot be used in the acidic solution (due to precipitation of free acids)

- Synthetic detergents do not form insoluble salts of calcium or magnesium with hard water. Hence, lesser amounts of synthetic detergents are required for washing.

Washing powder :

- Washing powders used for washing clothes contain only about 15 to 30 percent detergents by mass. The remaining part is made of the following.

(1) Sodium sulphate and sodium silicate which keep the power dry,

(ii) Sodium tripolyphosphate or sodium carbonate which maintains alkalinity for removing dirt.

(iii) Carboxymethylcelulose (CM-Cellulose) which keep the dirt particle suspended in water.

(iv) Sodium perborate (a mild bleaching agent) which impart whiteness to the materials (clothes, etc.) being washed.

PREVIOUS YEARS' BOARD QUESTIONS :

1. An organic compound 'A' has molecular formula $C_2H_4O_2$ and is acidic in nature. On heating with alcohol and conc. sulphuric acid, vapours with pleasant or fruity smell are given out, What is this chemical compound 'A' and what is the chemical equation involved in the reaction ? **(CBSE ALL India 1999)**

Ans. The compound 'A' with molecular formula $C_2H_4O_2$ is Ethanoic acid (acetic acid). Upon heating with ethanol (ethyl alcohol) and concentrated sulphuric acid, ethyl ethanoate (ethyl acetate) is formed as the product. It is an ester with pleasant or fruity smell. The reaction is known as etherification reaction.

$$CH_3COOH + C_2H_5OH \xrightarrow{H_2SO_4(conc.)} CH_3COOC_2H_5 + H_2O$$

Ethanoic acid (A) Ethanol Ethyl ethanoate (Ester)

2. An organic compound 'A' is a constituent of antifreeze and has the molecular formula C_2H_6O. Upon reaction with alkaline $KMnO_4$, the compound 'A' is oxidized to another compound 'B' with formula C_2H_6O, Identify the compounds 'A' and 'B'. Write the chemical equation for the reaction which leads to the formation of 'B'. **(CBSE ALL India 2000 Comptt.)**

Ans. The compound 'A' is ethanol and with alkaline KMnO4, it is oxidized to ethanoic acid 'B'. The chemical equation for the reaction is :

$$C_2H_5OH \xrightarrow[KMnO_4]{Alkaline} CH_3COOH$$

Ethanol (A) Ethanoic acid (B)

3. Name the functional groups present in the following compounds :

(i) $CH_3-CH_2-CH_2-OH$ (ii) $CH_2-CH_2-CH_2-COOH$

(iii) CH_3-CH_2-CHO (iv) $CH_3-CO-CH_2-CH_3$

Ans. (i) $-OH\,(ol)$ (ii) $-COON\,(oic\ acid)$ (iii) $-CHO\,(al)$ (iv) $-CO-(one)$

4. Write the formulae of the compounds and name the functional groups present in each of them

(i) Ethanoic acid (ii) Propanone (iii) Nitro methane **(C.B.S.E. Delhi 2005)**

Ans. (i) Ethanoic acid : CH_3COOH (oic acid)

(ii) Propanone : CH_3COCH_3 (one)

(iii) Nitro methane : CH_3NO_2 (nitro)

5. Name the enzyme which converts :

(i) milk into curd (yogurt)

(ii) cane sugar into glucose and fructose

(iii) glucose into ethanol.

Ans. (i) Lactase converts milk into curd

(ii) Invertase converts cane sugar (sucrose) into glucose and fructose

(iii) Zymase converts glucose into ethanol. **(C.B.S.E. Foreign 2005)**

6. (i) Name the gas evolved during fermentation process. **(C.B.S.E. Delhi 2006)**

(ii) List the two products formed when enzyme invertase acts on sugar present in molasses.

Ans. (i) CO_2 gas is evolved accompanied by brisk effervescence.

(ii) Glucose and fructose are the products when enzyme invertase acts on sucrose ($C_{l2}H_{22}O_{11}$) present in molasses.

7. Complete the following equations and write the names of the products formed. **(C.B.S.E. Delhi 2007)**

(i) $CH_3COOH + NaOH \xrightarrow{Heat}$ (ii) $C_2H_5OH + O_2 \xrightarrow[KMnO_4]{Alkaline}$

(iii) $CH_3COOH + C_2H_5OH \xrightarrow{conc.H_2SO_4}$

Ans. (i) $CH_3COOH + NaOH \xrightarrow{Heat} CH_3COOH + H_2O$

Ethanoic acid Sod. ethanoate

(ii) $C_2H_5OH + O_2 \xrightarrow[KMnO_4]{Alkaline} CH_3COOH + H_2O$

Ethanol Ethanoic acid

(iii) $CH_3COOH + C_2H_5OH \xrightarrow{conc.H_2SO_4} CH_3COOC_2H_5 + H_2O$

Ethanoic acid Ethanol Ethyl ethanoate

8. Name the organic compound present in vinegar. Write a chemical equation which represents the commercial method for the preparation of this compound from methanol. **(C.B.S.E. All India 2007)**

Ans. The organic compound present in vinegar is Ethanoic acid also called acetic acid. For its commercial preparation .

9. (a) Why does carbon form compounds by covalent bonding?

(b) An organic acid 'X' is a liquid which often freezes during winter time in cold countries. It has the molecular formula $C_2H_4O_2$. On warming with ethanol in the presence of a few drops of sulphuric acid, a compound 'Y' with sweet smell is formed.

(i) Identify X and 'Y'. (ii) Write chemical equation for the reaction involved. **(C.B.S.E. Delhi 2008)**

Ans. (a) Carbon forms a large number of organic compounds due to the self linking property known as catenation.

(b) The available information suggests that the organic acid X with molecular formula C2H4O2 is Ethanoic acid (CH_3COOH) . It reacts with ethanol in the presence of a few drops of sulphuric acid on warming to give ethyl ethanoate ester with a plesant smell.

$$CH_3COOH + C_2H_5OH \xrightarrow[Warm]{H_2SO_4(Conc)} CH_3COOC_2H_5 + H_2O$$

($C_2H_4O_2$) Warm Ethyl ethanoate (Y)

Ethanoic acid (X)

10. Why do covalent compounds have low melting and boiling points ? **(C.B.S.E. All India 2008)**

Ans. In covalent compounds, the atoms are linked by covalent bonds formed by electron sharing. Since no ions are present in these, the attractive forces are quite weak. As a result, the covalent compounds have low melting and boiling points.

11. (i) How ae carboxylic acids different from mineral acids from ionization point of view ?

(ii) Describe an activity to find how Ethanoic acid reacts with sodium carbonate. Name the gas evolved. How can it be tested ? **(C.B.S.E All India 2008)**

Ans. (i) Carboxylic acids (organic acids) are less ionized in solution as compared to mineral acids (HCl,HNO3,H2SO4 etc.) Due to this reason, these are weaker acids than the mineral acids.

(ii) Take a small volume of Ethanoic acid in a tube. Add a few drops of sodium carbonate (Na2CO3) solution prepared in water to the tube. A colorless gas with brisk effervescence will evolve. When the gas is passed through lime water, it will become milky.

$$Na_2CO_3(aq) + CH_3COOH(aq) \rightarrow CH_3COONa(aq) + H_2O(l) + CO_2(g)$$

$$Ca(OH)_2(aq) + CO_2(g)) \rightarrow CaCO_3(s) + H_2O(l)$$

Lime water (Milky)

12. (a) What is a functional group in a carbon compound ? Identify the functional group present in CH_3COOH and C_2H_5OH.

(b) State the principle on which the cleansing action of soap is based. **(C.B.S.E. Foreign 2008)**

Ans. (a) Functional group may be defined as an atom or group of atoms upon which the properties of a particular organic compound are based. Different families differ in the functional groups.

Functional group in CH_3COOH : $(-COOH)$

Functional group in C_2H_5OH : $(-OH)$

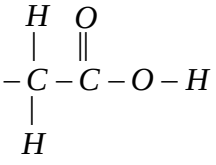
(b) The cleansing action of soap is based on its tendency to act as a bridge between water and oil drops containing dirt particles. As a result. oil and water get mixed. They form a stable emulsion also called micelle. This helps in removing oil drops containing dirt particles from clothes. The clothes become clean.

13. (a) Draw the structure of the following compounds

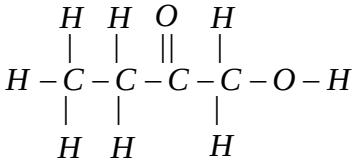
(i) Ethanoic acid (ii) Butanone.

(b) Why is conversion of ethanol to Ethanoic acid considered an oxidation reaction? **(C.B.S.C. Foreign 2008)**

Ans. (a)



Ethanoic acid



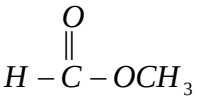
Butanone

(b) When ethanol (C2H5OH) changes with Ethanoic acid (CH3COOH)

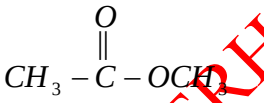
- There is a decrease in the number of hydrogen atoms by two.
- There is an increase in the number of oxygen atoms by one. Therefore, the conversion represents an oxidation reaction.

14. (a) What are esters ? How are they formed ? (b) Write two uses of ester ? **(CBSE Foreign 2008)**

Ans. (a) Esters are the group of organic compounds which contain the function group $(-COOR)$ called ester group. The value of R may change as $-CH_3$, $-C_2H_5$, $-C_3H_7$ etc. A few example of esters are :



Methyl methanoate



Methyl ethanoate

Esters are formed as a result of chemical reaction called Etherification.

Uses of esters

(i) Esters have pleasant smell. These are used as flavoring agents and also in perfumes.

(ii) Esters of glycerol known as triglycerides are used in the manufacture of soaps. This reaction is called Saponification reaction.

15. Distinguish between Etherification and saponification reactions of organic compounds.

(C.B.S.E. All India 2008)

Ans. (a) In the Etherification reaction an acid reacts with alcohol in the pressure of conc. H2SO4 to form an ester with a pleasant or fruity smell. For example,

$$CH_3COOH + C_2H_5OH \xrightarrow{H_2SO_4(Con.)} CH_3COOC_2H_5 + H_2O$$

Ethanoic acid Ethanol Ethyl ethanoate (ester)

Saponification is quite different from Etherification because in this case an ester reacts with an alkali (NaOH or KOH) to form salt of acid and alcohol. For example,

$$C_3COOC_2H_5 + NaOH \rightarrow CH_3COONa + C_2H_5OH$$

Ethyl ethanoate Sod. ethanoate Ethanol

16. (a) In organic compounds, which part largely determines the physical and chemical properties.

(CBSE All India, 2008)

(b) Write chemical equation to represent the reaction of ethanol with acidified solution of potassium dichromate.

Ans. (a) In organic compounds, it is the functional group which largely determines the physical and chemical properties of compounds. Actually, and organic compound is made up of two parts. These are alkyl group and the functional group. Whereas the alkyl group remains the same (size may change) but the functional groups change. These are responsible for the characteristics of the compounds. For example, the properties of alkanols ($-OH$ is the functional group) are different from those of alkanoic acid ($-COONa$ is the functional group). For more details, consult text part.

(b) Acidified solution of potassium dichromate ($K_2O_2O_7$) forms chromic acid (H_2CrO_4). It releases oxygen to bring about the oxidation of ethanol first to ethanal and then to Ethanoic acid.

$$CH_3-CH_2-OH \xrightarrow[-H_2O]{(O)} CH_3-\underset{\displaystyle O}{\overset{\displaystyle H}{\overset{|}{C}}}=O \xrightarrow{(O)} CH_2-\underset{\displaystyle O}{\overset{\displaystyle OH}{\overset{|}{C}}}=O$$

Ethanol Ethanal Ethanoic acid

17. Give reason for the following:

(a) Air holes of the gas burners have to be adjusted when heated vessels get blackened by the flame.

(b) Use of synthetic detergents causes pollution problems. **(C.B.S.E. Delhi, 2009)**

Ans. (a) In case the vessel where cooking is done get blackened from outside, this means that combustion is incomplete. As a result, the carbon particles in the form of soot get deposited and the vessel becomes black from outside. In order to check this, oxygen or air supply must be increased. This can be done only by adjusting the air holes of the gas burner.

(b) The pollution problems caused by the synthetic detergents is due to their non-biodegradable nature. These are actually long chain organic compounds which do not break or decompose in water. Naturally, this will result in pollution problems. Some of the detergents are even of toxic nature and will make water unfit for drinking.

ROUND UP

- Carbon is versatile element that forms the basis of all living things.
- Carbon can from a vast variety of compounds because of its Tetravalency and the property of catenation.
- Covalent bonds are formed between two similar or different atoms by sharing electron in their valence shell, such that both of them can achieve the structure of nearest noble gas.
- Carbon forms covalent bonds with itself as well as atoms of hydrogen, oxygen, nitrogen, sulphur and halogens.
- Carbon can form compounds having a straight chain between carbon atoms with a single bond, or double bond or triple bond. It can also form compounds with branched chains and closed chains.
- Homologous series of carbon compounds is a group of carbon compounds having the same functional group with the same general formula.

- The functional groups such as alcohols, aldehydes, Ketones, carboxylic acids and halogens impart characteristic properties to the carbon compounds.
- Carbon and its compounds are the major sources of fuels.
- Ethanol and Ethanoic acid are most important compounds of carbon in our daily life.
- The soaps and detergents have cleansing action, because of the presence of hydrophobic and hydrophilic groups in their molecules, which help in emulsifying oil, and hence, in the removal of dirt.

EXERCISE #1 CARBON &ITS COMPOUNDS

1. Ethane, with the molecular formula C_2H_6 has (NCERT)
(a) 6 covalent bonds (b) 7 covalent bonds (c) 8 covalent bonds (d) 9 covalent bonds

2. Butanone is a four-carbon compound with the functional group (NCERT)
(a) Carboxylic acid (b) aldehyde (c) ketone (d) alcohol

3. While cooking. If the bottom of the vessel is getting blackened on the outside, it means that (NCERT)
(a) the food is not cooked completely (b) the fuel is not burning completely
(c) the fuel is wet (d) the fuel is burning completely

4. A covalent bond is formed by
(A) complete transfer of electrons (B) one sided sharing of electron
(C) mutual sharing of electron (D) all of the three above.

5. Which of the following compounds does not contain a multiple bond?
(A) Ethane (B) Ethene (C) Ethyne (D) Benzene

6. Which of the following is not a saturated hydrocarbon?
(A) Cyclohexane (B) Benzene (C) Butane (D) Isobutane

7. Benzene with molecular formula, C_6H_6, has
(A) 6 single bonds and 6 double bonds (B) 12 single bonds and 3 double bonds
(C) 18 single bonds only (D) 12 double bonds only

8. The functional group in methanol and methanol and methanal respectively are :
(A) $-OH,-CHO$ (B) $-CHO,-OH$ (C) $-OH,-COOH$ (D) $-CHO,-COOH$

9. Which of the following is not an allotropic form of carbon?
(A) Coal (B) Fullerene (C) Diamond (D)Graphite

10. Graphite is a soft lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite
(A) has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interpolate bonds.
(B) is a non-crystalline substance
(C) is an allotropic form of carbon

(D) has only single bonds between carbon atoms

11. Which of the following represent the correct order of unsaturation?
(A) Alkanes, alkenes, alkynes (B) Alkanes, alkynes, alkenes
(C) Alkenes, alkynes, alkenes (D) Alkynes, alkanes, alkenes

12. The general formula of alcohol is
(A) $C_nH_{2n+2}OH$ (B)$C_nH_{2n+1}OH$ (C) C_nH_{2n-1} OH) (D)$C_nH_{2n+4}OH$

13. Wine contains
(A) CH_3OH (B) C_6H_5OH (C)C_2H_5OH (D) CH_3COOH

14. The acid present in vinegar is
(A) CH_3COOH (B) HCOOH
(C) CH_3CH_2COOH (D) $CH_3CH_2CH_2COOH$

15. The reaction $2C_2H_5OH + 2Na \rightarrow 2C_2H_5ONa + H_2$ suggests that ethanol is
(A) Acidic in nature (B) Basic in nature (C) Amphoteric (D) Neutral

16. Which of the following substance is added to denature ethanol?
(A) Methanol (B) Benzene (C) Copper nitrate (D) Poison

17. Which of the following substances cannot be used to distinguish ethanol from Ethanoic acid ?
(A) Na metal (B) $NaHCO_3$
(C) hot alkaline $KMnO_4$ (D) hot acidified $K_2Cr_2O_7$ solution

18. An example of soap is
(A) CH_3COONa (B) CH3ONa (C) $C_{17}H_{35}COONa$ (D) $C_{17}H_{35}COOC_2H_5$

19. Detergents are sodium or potassium salts of long chain
(A) aldehydes (B) Ketones (C) carboxylic acids (D) sulphonic acids

20. Which of the following salts when dissolved in water produce hard water?
(A) Calcium sulphate (B) Magnesium bicarbonate
(C) Calcium chloride (D) Any of the above

21. Which of the following represents Lewis structure of N_2 molecule?

(A) $\overset{\times}{\underset{\times}{}}N \equiv N\overset{\times}{\underset{\times}{}}$ (B) $\overset{\times\times}{\underset{\times}{}}N \equiv N\overset{\times\times}{\underset{\times}{}}$ (C) $\overset{\times\times}{\underset{\times}{}}N - N\overset{\times\times}{\underset{\times}{}}$ (D) $\overset{\times\times}{\underset{\times}{}}N = N\overset{\times\times}{\underset{\times}{}}$

22. Which of the following has the shortest carbon-carbon bond length?
(A) C_2H_2 (B) C_2H_4
(C) C_2H_6 (D) All have the same bond length

23. Which of the following has the weakest carbon-carbon bond strength?
(A) C_2H_2 (B) C_2H_4
(C) C_2H_6 (D) All have the same bond length

24. The hydrocarbon with the general formula C_nH_{2n+n} is an-

(A) Alkane (B) Alkene
(C) Alkyne (D) unsaturated compounds

25. Which of the following is an alkyne?
(A) C_6H_6 (B) C_6H_{12} (C) C_6H_{10} (D) C_6H_{14}

EXERCISE #2 CARBON &ITS COMPOUNDS

1. Which of the following will not decolorize bromine water?
(A) C_4H_8 (B) C_3H_4 (C) C_3H_8 (D) C_4H_6

2. Compounds made up of carbon and hydrogen only are called
(A) Alkanes (B) Alkenes (C) Alkynes (D) Hydrocarbons

3. Open-chain saturated hydrocarbons are called
(A) Paraffin (B) Alkenes (C) Alkynes (D) Alkyl groups

4. The characteristic reaction of alkanes is
(A) Addition (B) Substitution (C) Polymerization (D) Isomerization

5. The major constituent of biogas is
(A) Propane (B) Acetylene (C) Methane (D) Benzene

6. n-butane and isobutane are
(A) Alkenes (B) Alkynes (C) Isomers (D) None of these

7. Methane is a major constituent of
(A) Coal gas (B) Water gas (C) Petroleum (D) Biogas

8. The major constituent of of natural gas is
(A) Butane (B) Methane (C) Propane (D) Ethane

9. Ethanol on oxidation gives
(A) Ethane (B) Formalin (C) Ethanoic acid (D) Methane

10. The functional group present in carboxylic acids is
(A) –OH (B) –CHO (C) –COOH (D) –CO

11. A dilute solution of Ethanoic acid in water is called
(A) Tincture of iodine (B) Fehling's solution (C) Vinegar (D) Tollen's reagent

12. Which of the following will undergo addition reactions?
(A) C_2H_4 (B) C_2H_6 (C) CH_4 (D) C_3H_8

13. Which of the following formula represent alkenes?

(A) C_nH_{2n} (B) C_nH_{2n+2} (C) C_nH_{2n-2} (D) C_nH_{2n+1}

14. The general formula of cyclic alkanes is

(A) C_nH_{2n+2} (B) C_nH_{2n-2} (C) C_nH_{2n-1} (D) C_nH_{2n}

15. A carboxylic group is present in

(A) Ethylene (B) Methanoic acid (C) Formaldehyde (D) Ethanol

16. The functional group in an alcohol is

(A) $-\overset{\overset{O}{\|}}{C}-O-$ (B) $-\overset{\overset{O}{\|}}{C}-OH$ (C) $-OH$ (D) $-\overset{\overset{H}{|}}{C}=O$

17. Which of the following will react with sodium metal?

(A) Ethanol (B) Ethanal (C) Ethene (D) Ethane

18. Which of the following will give a pleasant smell of ester when heated with ethanol and a small quantity of sulphuric acid?

(A) CH_3COOH (B) CH_2CH_2OH (C) CH_3OH (D) CH_3CHO

19. The functional group in aldehydes is

(A) $-CHO$ (B) $-\overset{\overset{O}{|}}{C}=O$ (C) $-COOH$ (D) $-COOR$

20. Ethanol on complete oxidation gives

(A) CO_2 and water (B) Acetaldehyde (C) Acetic acid (D) Acetone

21. Which class of organic compounds give effervescence with $NaHCO_3$ solution?

(A) Esters (B) Alcohols (C) Carboxylic acids (D) Aldehydes

22. Carboxylic acids are obtained from alcohols by -

(A) Oxidation (B) Reduction (C) Hydrolysis (D) Pyrolysis

23. Soaps are prepared by alkaline hydrolysis of-

(A) Carboxylic acids (B) Lower esters (C) Higher esters (D) None of these

EXERCISE #3 CARBON &ITS COMPOUNDS

FILL IN THE BLANKS TYPE QUESTION :

1. The ability of carbon atom to link with other carbon atom is known as ……

2. The hydrocarbons containing only single bonds are known as ……

3. Aliphatic hydrocarbons have been classified as alkanes, …… and alkynes.

4. …….. hydrocarbons undergo addition reaction.

5. Isomers have some molecular formula but different …….. formula.

6. Hydrocarbons are insoluble in ……..

7. The next homologue of ethene is ……..

8. During the formation of hydrogen molecule from its atoms, energy is ……

9. Bond between hydrogen and chlorine in HCl is ……

10. $CH_2 = CH_2 + H_2 \rightarrow CH_3 - CH_3$

11. The general formula of the homologous series of alcohols is ……..

12. The functional group present in ethanol is ……..

13. When ethanol is warmed with alkaline potassium permanganate, the product formed is ……..

14. When a piece of sodium metal is added to ethanol, the gas formed with effervescence is ……..

15. Ethene and ethylene are examples of …… hydrocarbons.

16. The …….. group in a soap molecule is hydrophilic.

TRUE & FALSE :

1. Methane belongs to the homologous series of alkanes.

2. The compound having the molecular formula C_4H_{10} does not show isomerism.

3. The methane molecule has a pyramidal shape.

4. Methane undergoes substitution reaction.

5. The consumption of ethanol increases the activity of the body.

6. Ethanoic acid is used in the manufacture of textiles.

7. Alkenes as well as alkynes decolorize bromine water.

8. Vanaspati ghee is obtained by the hydrogenation of vegetable oil.

9. Alkanes undergo substitution reactions.

10. Alkenes and alkynes are unsaturated compounds.

11. Ethanol is oxidized by alkaline $KMnO_4$ to oxalic acid.

12. Detergents give scum with hard water.

13. The polar end in soap is called hydrophilic end.

14. Methanol is safe to be used for drinking purpose.

15. The reaction of ethanol with conc. H_2SO_4 gives ethane.

16. Carboxylic acids react with alcohols to form esters.

VERY SHORT ANSWER TYPE QUESTIONS :

1. Name an allotrope of carbon which has 60 carbon atoms.

2. What is combustion?

3. What is the nature of substances that produce a flame?

4. The molecular formula of a compound is C_2H_6O. Name its homologous series.

5. What type of reactions are given by alkanes?

6. What is the composition of natural gas used for cooking?

7. Which of the following open chain compounds can have a double bond?

8. Name an allotrope of carbon which contains both single and double bonds between carbon atoms.

9. Write the name and molecular formula of alcohol derived from butane.

10. What is rectified spirit?

11. Give the name and structural formula of one homologue of HCOOH.

12. Why does hard water not produce foam with soap easily?

13. Name the hydrophobic and hydrophilic ends of soap.

14. What is a detergent?

15. An organic compound burns with a sooty flame. Is it saturated or unsaturated compound?

SHORT ANSWER TYPE QUESTION :

1. Name any four substance obtained from organic compounds which are used in our daily life.

2. Hydrogen and chlorine react to form hydrochloric acid. Will they form and ionic or covalent bond?

3. How many electrons a carbon atom shared with other carbon atom in the formation of acetylene molecule?

4. Give examples of covalent compounds which contain :

(a) one unshared pair of electron (b) two unshared pairs of electrons.

5. What is meant by the term functional group?

6. What is meant by homologous series? Give its three important characteristics.

7. Name the functional group present in : (i) CH_3CH_2OH, and (ii) HCOOH.

8. The molecular formula of a hydrocarbon is C_5H_{10}. Name its homologous series.

9. Write structural formulae of the following compounds.

(i) Ethyl alcohol (ii) Acetaldehyde (iii) Propionic acid (iv) Butanone

10. What is the IUPAC name of fourth member of alcohol series? Draw its two structural isomers.

11. Write the general formulae of the following homologous series.

(i) Aldehydes (ii) Ketones (iii) Alkyne

12. How do alcohols differ structurally from alkanes?

13. Give the common and IUPAC names of the following alcohols:

(i) CH_3OH (ii) CH_3CH_2OH (iii) $CH_3CHOHCH_3$

14. Identify the functional group in CH_3CH_2OH . Give IUPAC name of the compound.

15. Ethanol can be oxidized to Ethanoic acid. Write the equation involved in the reaction.

16. Name the oxidizing agent which can oxidize:

(i) Ethanol to ethanal (ii) Ethanol to Ethanoic acid.

17. How would you distinguish experimentally between an alcohol and a carboxylic acid?

18. A neutral organic compound 'A' having molecular formula C_2H_6O,undergoes oxidation with acidified potassium permanganate to give an acidic compound 'B' . The organic compound 'A' reacts with compound 'B' on warming and in the presence of concentrated sulphuric acid to form a sweet-smelling substance 'C'. Identify A, B and C.

19. An organic compound 'A' having molecular formula $C_2H_6O_2$ turns blue litmus red and gives brisk effervescence with sodium hydrogen carbonate. Give the name and formula of 'A'.

20. What happens when (Give equations of the chemical reaction)

(i) Sodium ethanoate is heated with soda lime.

(ii) Ethanoic acid is warmed with methanol in presence of concentrated sulphuric acid.

(iii) A pinch of sodium hydrogen carbonate is added to ethanoic acid.

(iv) Ethanol is oxidized with acidified potassium dichromate.

LONG ANSWER TYPE QUESTION :

1. What is covalent bond ? Draw electron dot structure for the following :

(i) Methane (ii) Carbon dioxide (iii) Ammonia (iv) Water

2. Explain the following reaction with one example each :

(i) Substitution reactions (ii) Addition reactions

(iii) Combustion reactions (iv) Oxidation reactions

3. (i) What are alcohols? What is functional group ?
(ii) Write names and formulae of first four members of alcohols family.
(iii) How does the second member of alcohol family react with
(i) Sodium metal (ii) Ethanoic acid

4. What are soaps and synthetic detergents ? How do they differ ? Discuss their cleaning actions.

5. Which properties of carbon make it a versatile element . Discuss its bonding in saturated and unsaturated hydrocarbon.

6. What are harmful effect of drinking alcohol ?

7. Write short not on :

(i) Catenation (ii) Glacial acetic acid (iii) Power alcohol (iv) Saponification

8. Give chemical test to detect the presence of (i) ethanol. (ii) Ethanoic acid and (iii) and ester.

9. Discuss briefly two physical and three chemical properties of ethanol.

10. Discuss briefly the physical and chemical properties of Ethanoic acid.

EXERCISE # 1

- ***Objective type questions***

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	B	C	A	B	B	A	A	A
Qus.	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	C	A	A	A	A	C	D	D
Qus.	21	22	23	24	25					
Ans	A	A	C	A	C					

EXERCISE # 2

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	C	D	A	B	C	C	D	B	C	C
Qus.	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	A	D	B	C	A	A	A	A
Qus.	21	22	23							
Ans	C	A	C							

EXERCISE # 3

- ***Fill in the blanks***

1. Catenation **2.** Alkanes/saturated hydrocarbon **3.** Alkenes
4. Unsaturated **5.** Structural **6.** Water **7.** Propene
8. Released **9.** Covalent **10.** Ni catalyst **11.** $C_nH_{2n+1}OH$.
12. Hydroxy **13.** Ethanoic acid **14.** Hydrogen **15.** Unsaturated
16. Polar/$COONa^+$

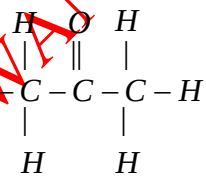
- ***True or False***

1. T **2.** F **3.** F **4.** T
5. F **6.** T **7.** T **8.** T
9. T **10.** T **11.** F **12.** F
13. T **14.** F **15.** F **16.** T

NCERT SOLVED QUESTIONS

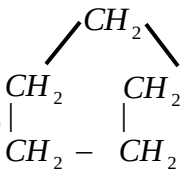
1. Write the chemical formula of simplest ketone.

Ans.



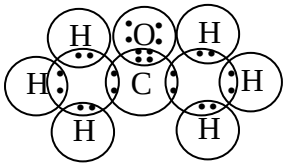
2. What will be formula and electron dot structure of cylopentane?

Ans. Formula for cylopentane is C5H10. Its electron dot structure can be represented as C_5H_{10}



3. Propanone.

Ans. Propanone $CH_3 - \overset{\overset{O}{\|}}{C} - CH_3$



4. What will happen if a piece of diamond is dropped in fire ? Write chemical equation for the reaction.

Ans. Diamond will burn in fire because it is an allotrope of carbon. On combustion, it will produce carbon dioxide.

$$\underset{\text{Diamond}}{C} + \underset{\text{Oxygen}}{O_2} \longrightarrow \underset{\text{Carbon dioxide}}{CO_2}$$

5. Write two main difference between the nature of diamond and graphite.

Ans. Two main difference between diamond and graphite are

(i) Diamond is very hard whereas graphite is a soft substance.

(ii) Diamond is a poor conductor of electricity whereas graphite is a good conductor of electricity.

6. Write the formulae for the given compounds and name the functional groups present in each of them.

(i) Ethanoic acid (ii) Propanone

Ans. (i) Compound : Ethanoic acid

Formula : $CH_3 - COOH$

Functional group : $-\overset{\overset{O}{\|}}{C} - OH$

(Carboxylic acid group)

(ii) Compound : Propanone

Formula : $CH_3 - \overset{\overset{O}{\|}}{C} - CH_3$

Functional group : $-\overset{\overset{O}{\|}}{C}-$

(Ketone group)

7. Mention any two test which can be used of defect carboxylic acid group in an organic compound.

Ans. **(i) Litmus test :-** Carboxylic acids turn blue litmus paper red.

(ii) Sodium bicarbonate test : Carboxylic acids give brisk effervescence due to the formation of carbon dioxide, when treated with sodium bicarbonate solution.

$$CH_3COOH + NaHCO_3 \rightarrow H_2O + CO_2 + CH_3COONa$$

8. What is hydrogenation ? What is its industrial application ?

Ans. Unsaturated hydrocarbons add hydrogen in the presence of palladium or nickel. This reaction is called hydrogenation.

$$C = C \quad + H_2 \xrightarrow{Pd \text{ or } Ni} R - \overset{\overset{H}{|}}{\underset{\underset{R}{|}}{C}} - \overset{\overset{H}{|}}{\underset{\underset{R}{|}}{C}} - R$$

This reaction is industrially used in the conservation of vegetable oils into vegetable ghee.

9. Define the following giving one example of each :
(i) Etherification
(ii) Addition reaction

Ans. (i) The reaction of carboxylic acid with an alcohol in presence of sulphuric acid yields esters and the reaction is known as Etherification.

$$CH_3-\overset{\overset{O}{\|}}{C}-OH + CH_3-CH_2-OH \xrightarrow[\text{heat}]{H_2SO_4}$$

(ii) Reactions in which two or more atoms are added across a double or triple bond, are called addition reactions.

$$CH_3=CH_2 + H_2 \xrightarrow{Pd.Pt\ or\ Ni} CH_3-CH_3$$

10. An organic compound 'X' is a constituent of wine. This compound on heating with acidified potassium dichromate forms another compound 'Y'. Name the compound 'X' and 'Y' and write the chemical equation of the reaction involved.

Ans. Compound 'Y' is ethanol.
Compound 'Y' is Ethanoic acid.

$$CH_3CH_2OH \xrightarrow[heat]{K_2Cr_2O_7} CH_3-\overset{\overset{O}{\|}}{C}-OH$$

Ethanol Ethanoic acid

11. (a) Identify the compounds A and B in the following reaction :

(i) $C_2H_5OH + CH_3COOH \xrightarrow{H_2SO_4}$ (ii) $CH_3COOC_2H_5 + NaOH \rightarrow B + C_2H_5OH$

(b) Name the type of reaction taking place in the above cases.

Ans. (a) (i) $CH_3-\overset{\overset{O}{\|}}{C}-O-CH_2-CH_3$ Ethyl ethanoate (ii) $CH_3-\overset{\overset{O}{\|}}{C}-ONa$ Sodium ethanoate

(b) (i) Etherification

12. Differentiate between a soap and a detergent on the basis fo their chemical constituents. For cleansing action why is a detergent preferred to a soap ?

Ans. Soaps are sodium salts of long chain fatty acids whereas detergents are sodium salts of alkyl benzene sulphonic acids. Basically, the structure soap is ~~~COONa whereas that of detergents is ~~~SO_3Na. If water is hard i.e., it contains calcium or magnesium chlorides then detergent is preferred to a soap.

13. Why does micelle formation take place when soap is added to water ? Will a micelle be formed in other solvents like ethanol also?

Ans. A molecule of soap has two dissimilar ends. At one end is the hydrocarbon chain which is water repellent. At the other end carboxylate anion is present which is water soluble end. 'When soap is dissolved in water, many molecules come together and form a group called micelle, these micelles are formed because their hydrocarbon chains come together and the polar ends are projected outward. Micelle formation in ethanol will not occur because the hydrocarbon chain end of the soap will also dissolve in ethanol.

14. Explain the formation of scum when hard water is treated with soap.

Ans. Hard water contains calcium and magnesium ions. When soap comes is contact with hard water these ions form calcium and magnesium salts of fatty acids which are insoluble in water. These calcium or magnesium salts precipitate out in the form of scum.

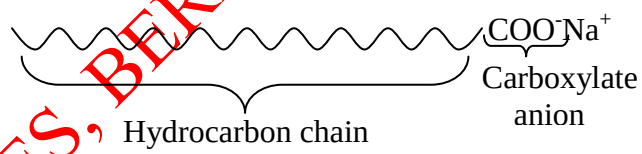
15. How can ethanol and Ethanoic acid be differentiated on the basis of their physical and chemical properties?

Ans.

	Property	**Ethanol**	**Ethanoic acid**
(i)	Melting point	156 K	290 K
(ii)	Action on litmus	No action	Turns blue litmus into red
(iii)	Reaction with $NaHCO_3$ solution	No reaction	Effervescence due to the formation of CO_2

16. Explain the mechanism of the cleansing action of soaps.

Ans. A molecule of soap has two dissimilar ends. At one end is the hydrocarbon, chain which is water repellent and the other and is carboxylate anion which is polar end.



When soap is dissolved in water, many molecules come together and from a group called micelle. These micelles are formed because their hydrocarbon chains come together and the polar ends are projected outward.

When a cloth with a spot of oil soaked into a soap solution. Soap dissolves tiny oil droplets by the hydrophobic end in the middle of the micelle. Due to the outer polar ends, these micelles dissolve in water and are washed away- In this way cloth gets cleaned.

17. What is soap ? How would you make soap in the laboratory ? State one advantage and one disadvantage of soap over synthetic detergents.

Ans. Soaps are sodium or potassium salts of long chain fatty acids Soap can be prepared in laboratory by heating the naturally occurring. Oil or fat with sodium hydroxide. On heating, sodium salt of the fatty acid soap is formed along with glycerol. This process is known as saponification.

$$Oil\,or\,fat + NaOH \rightarrow Soap + Glycerol$$

The soap is precipitated by adding common salt to the above solution. Soap being lighter, floats on water and is filtered. It is cooled and then cut into pieces.

Soaps have the advantage over detergents that they are biodegradable and do not cause water pollution.
The disadvantage of soaps over detergents is that, they cannot be used in hard water.

18. What happens when a small piece of sodium metal is dropped into ethanol.

Ans. Hydrogen gas is evolved when a piece of sodium metal is dropped into ethanol.

$$C_2H_5OH + Na \rightarrow C_2H_5ONa + \frac{1}{2}H_2$$

19. Give reasons for the following observations :

Ans. (a) The element forms a very number of compounds.

(b) Air holes of a gas burner have to be adjusted when heated vessels get blackened by the flame.

(c) Use of synthetic detergents causes pollution of water.

Ans. (a]) The two main properties of carbon responsible for a large number of compounds are catenation and Tetravalency of carbon.

(b) When the heated vessel is blackened by the flame, it shows that complete combustion of the gas is not occurring due to the less supply of oxygen or air. So air holes adjusted to increase the supply of air.

(c) Use of synthetic detergents causes pollution of water because most of them are non-biodegradable.

Important notes

PERIODIC CLASSIFICATION OF ELEMENTS

INTRODUCTION :

Before the beginning of the **eighteenth century** when there are only 30 elements were known, it was easier to study and remember their properties. In later years when number of elements discovered were increased then it become difficult to study them. So scientist fell the need of simple method to facilitate the study of the properties of various elements and their compounds. After numerous attempts they got success & elements were arranged in such manner that similar elements were grouped together and different elements were separated. This arrangement of elements is known as classification of elements which led to the formation of periodic table.

Periodic table may be defined as the arrangement of all the known elements according to their properties in such way that the elements of similar properties are grouped together in a tabular form.

Earlier attempts of classification of elements (development of periodic table) :

Earlier attempts to classify the elements resulted in grouping as metals and non-metals. Later on they were classified on the basis of their atomic masses.

DOBEREINER TRIADS RULE :

In 1817, **Johann Wolfgang Döbereiner.** A German chemist, arranged the elements is group of three elements and in a manner that the atomic mass of middle element was roughly the average of the atomic masses of the other two elements of the triad.

Example : Element : Lithium, Sodium and Potassium.

Atomic mass 7 23 39

Average of the atomic masses of Lithium and Potassium is $\frac{7+39}{2} = 23$ only three triads could be arranged in this manner at that time. They were :

Triads	Elements	Atomic masses	Average of the atomic masses of first and third element
1	Li Na K	7 23 39	$\frac{7+39}{2} = 23$
2.	Ca Sr Ba	40 87.5 137	$\frac{40+137}{2} = 88.5$
3.	Cl Br I	35.5 80 127	$\frac{35.5+127}{2} = 81.2$

SHORT COMING OF DOBEREINER'S TRIADS RULE :

This classification was not found satisfactory as it could be applied to the limited number of elements. Now a days some more triads have been made they are

(i)	Potassium K	Rubidium Rb	Cesium Cs

(ii)	Phosphorous P	Arsenic As	Antimony Sb
(iii)	Sulphur S	Selenium Se	Tellurium Te
(iv)	Hydrogen H	Fluorine F	Chlorine Cl
(v)	Scandium Sc	Itrium Y	Lanthanum La

For a **Döbereiner's triad** all the three elements should belong to the same group and the difference in atomic number should be 8 or 18.

New lands law of octave : In 1866. J.A.R. Newlands correlated the chemical properties of the elements with the increasing order of atomic masses. i.e. to arrange the element having lowest atomic mass (H) firstely and ended to at secondly the element having highest atomic mass. **(Thorium** which was 56th known element at that time).

Definition : When the elements are arranged in order of their increasing atomic masses, every eighth element has the properties similar to those of the first elements like the eighth note of an octav in music. Thus according to this law, the physical & chemical properties are repeated after an interval of eight elements and this is similar to eight notes of an octave on a musical scale shown below :

Sa (do)	re (re)	ga (mi)	ma (fa)	pa (so)	da (la)	ni (ti)
H	Li	Be	B	C	N	O
F	Na	Mg	Al	Si	P	S
Cl	K	Ca	Cr	Ti	Mn	Fe
Co and Ni	Cu	Zn	Y	In	As	Se
Br	Rb	Sr	Ce and La	Zr	-	-

The properties of Lithium are similar to that of 8th element i.e. Na, Be is similar to Mg and so on.

Limitations :

1. Law of octaves was applicable only up to calcium. It worked well with lighter elements only.
2. At that time only 56 elements were existed in nature, but later several elements were discovered which can not be kept in periodic table as per this law. Their properties were not in accordance with the law of octaves.

Law of octaves :

3. (i) In order to fit element in to his table New lands adjusted two elements in the same column. For example cobalt and nickel were placed in the same position and in the same column as fluorine and bromine.
   (ii) Iron which resembles cobalt and nickel in properties were placed far away from these elements.
4. After the discovery of inert gases & included in the periodic table it becomes the eighth elements from alkali so this law has to be dropped out.

Ques. Did Döbereiner's triads also exist in the columns of Newland's Octaves ? Compare and find out.
Ques. What were the limitation of Döbereiner's Classification ?
Ques. What were the limitations of Newlands' Law of Octaves ?

Mendeleev's Periodic table :

In the year 1861, **D Mitri Ivanovich Mendeleev** arranged all the known elements (63 elements) in the form of a table in which elements were arranged in the increasing order of their atomic mass and also on the similarities of chemical properties.

❒ The arrangement of element was based on the physical and chemical properties of the elements and also the formulae of the compounds they formed with oxygen and hydrogen. He selected hydrogen and oxygen as they are very reactive and formed compound with most elements.

The table which classifies the elements in such a way that elements having similar properties are placed in same vertical column or group in known as periodic table. The term periodic means repetition of elements having similar properties after a certain regular interval. The periodic table consists of vertical columns which are called as groups and horizontal rows called as periods. **Mendeleev's** periodic table had six periods and eight groups as shown in the table, he arranged all the elements horizontally fin the order of their increasing atomic masses and vertically according to their similarities in properties. Each group was further sub divided into two sub groups A & B.

Group	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**
Oxide Hydride	R_2O RH	RO RH_2	R_2O_3 RH_2	RO_2 RH_3	R_2O_5 RH_3	RO_3 RH_2	R_2O_7 RH	RO_4
↓ Period	A B	A B	A B	A B	A B	A B	A B	Transition Series
1.	H 1.008							
2.	Li 6.939	Be 9.012	B 10.81	C 12.011	N 14.007	O 15.999	F 18.998	
3.	Na 22.99	Mg 24.31	Al 29.98	Si 28.09	P 30974	S 32.06	Cl 35.453	
4. First series second series	K 39.102 Cu 63.54	Ca 40.08 Cu 65.37	Sc 44.96 Ga 69.72	Ti 47.90 Ge 72.59	V 50.94 As 74.92	Cr 50.20 Se 78.96	Mn 54.94 Br 79.909	Fe Co Ni 55.85 58.93 58.71
5. First series second series	Rb 85.47 Ag 107.87	Sr 87.62 Cd 112.40	Y 88.91 In 114.69	Zr 91.22 Sn 118.69	Nb 92.91 Sb 121.75	Mo 95.94 Te 127.60	Tc 99 I 126.90	Rh 102.91 Ru Pd 101.07 106.4
6. First series second series	Cs 132.9. As 196.97	Ba 137.34 Hg 200.59	La 138.91 Hg 200.59	Hf 178.49 Pb 207.19	Ta 183.85 Bi 208.98	W 183.85		Ir Os 192.2 190.2 Pt 195.09

Achievements of the Mendeleev's periodic table :

1. **Systematic study of the elements :** All the elements in general were arranged systematically in increasing order of their atomic masses. This arrangement helped to study the properties of various elements. If the nature of the element present in a group is known, it become easier to predict or guess the expected properties of other elements.
2. **Prediction of new elements :** Mendeleev predicted the properties of some unknown elements and left gaps for these elements to be filled as and when discovered For eg. Scandium, Gallium and Germanium were not known at that time but Mendeleev already named these elements as **eka-boron, eks-aluminium** and **eka-silicon.** When these elements were later on discovered, they were found to have more or less similar properties as predicted by Mendeleev.
3. **Position of Noble gasses :** When noble gases were discovered they were placed in a new group without disturbing the existing order.
4. **Correction of atomic masses :** Atomic masses of several elements were corrected on the basis of periodic table. eg. Atomic mass of Beryllium was corrected from 135 to 9. Mendeleev predicted that atomic mass of gold is incorrect. Later on it was found to be so. Similarly atomic masses of Indium, Uranium and Platinum were also corrected.

Drawbacks of Mendeleev's periodic table :

Position of Hydrogen is uncertain becomes it resemble with IA group alkali metals elements and VII A (halogens) group elements.

(i) Isotopes : Isotopes of an element have similar chemical properties but different atomic masses.

(ii) Position of isotopes : Since basis of periodic table was increasing atomic mass. So isotope should be placed separately but no separate place ws given to isotopes.

(iii) Anomalous pairs of certain elements : Certain elements were not arranged according to their increasing atomic mass eg.

(a) Argon (Atomic mass 39.9) was placed before potassium (atomic mass 39.0)
(b) Cobalt (58.95) before Nickel (58.70)
(c) Tellurium (127.6) before Nickel (126.9)
(d) Thorium (232) before **Protactimum** (231)

(iv) Similar elements were placed in different groups. eg.

(a) Silver and thallium
(b) Barium and lead
(c) Copper and mercury
(d) Platinum and gold.

(v) Dissimilar elements were placed in same group eg. silver and gold were placed in a same group while there is little similarity in physical and chemical properties.

(vi) Cause of periodicity : Mendeleev did not explain the cause of periodicity in the physical and chemical properties of the elements.

(vii) Metals have not been separated from non-metals.

(viii) Position for elements of group (VIII) : There is no proper position for the elements of group (VIII) consisting of elements in three triads. These elements are placed out side the main structure of the periodic table.

The modern periodic table :

In 1913 **Henry Moseley** showed that properties of the elements are determined by atomic numbers instead of the atomic mass. It formed the basis of modern periodic law. The law is –

"The physical and chemical properties of the elements are periodic function of their atomic numbers". Since atomic mass is a nuclear property where as atomic number implies for the no of electrons in neutral atom or no. of protons in nucleus. Nucleus is deep seated in the atoms and does not take part in chemical reactions. Therefore the physical and chemical properties depends upon the no. of electrons and their electronic configuration which in turn depends upon atomic number (Z) . So when elements are arranged in the increasing order of atomic numbers. After an regular interval elements have similar no of valence electrons therefore chemical properties are repeated i.e. periodicity in the chemical properties of the elements of the elements occurs.

Ques. Use Mendeleev's periodic table to predict the formulae for the oxides of the following elements : K, C, Al, Si, Ba

Ques. Besides gallium, which other elements have since been discovered that were left by Mendeleev in his periodic table ? (any two)

Ques. What were the criteria used by Mendeleev in creating his periodic table ?

Ques. Why do you think the noble gases are placed in a separate group ?

Modern periodic table or long form of the periodic table :

- It is also called as **Bohr, Bury** & **Rang.** Werner periodic table

(1) It is based on the **Bohr-Bury** electronic configuration concept and atomic number.

(2) This model is proposed by Rang and Werner.

This table is based on modern periodic law, the elements are arranged in the increasing order of atomic numbers in such a way that elements having the same number of valence electrons are placed in the same

vertical column. It consists of 18 vertical columns and seven horizontal rows. Vertical columns of periodic table are known as groups while horizontal rows are known as periods.

The co-relation between the groups in long form of periodic table and in modern form of periodic table are given below :-

IA 1	IIA 2	IIB 3	IVB 4	VB 5	VIB 6	VIIB 7	VIII 8,9,10	IB 11	IIB 12
IIIA 13	IVA 14	VA 15	VIA 16	VIIA 17	O 18				

Elements belonging to same group having same number of electrons in the outer most shell so their properties are similar.

Description of periods :-

Description of periods ;

Period	n	Sub shell	No. of elements	Element	Name of Period
1.	1	1s	2	$_1H$,$_2He$	Shortest
2.	2	2s, 2p	8	$_3Li$ - $_{10}Ne$	Short
3.	3	3s,3p	8	$_{11}Na$ - $_{18}Ar$	Short
4.	4	4s,3d,4p	18	$_{19}K$ - $_{36}Kr$	Long
5.	5	5s,4d,5p	18	$_{37}Rb$ - $_{58}Xe$	Long
6.	6	6s,4f,5d,6p	32	$_{55}Cs$ - $_{86}Rn$	Longest
7.	7	7s,5f,6d,	26	$_{87}Fr$ - $_{112}Uub$	Incomplete

Description of Groups :

1st /IA/Alkali metals

H = $1s^1$

Li = $1s^2$, $2s^1$

Na = $1s^2$, $2s^2$ $2p^6$, $3s^1$

K = $1s^2$, $2s^2$ $2p^6$, $3s^2$ $3p^6$, $4s^1$

General electronic configuration = ns1 (n= Number of shell)

Number of valence shell e^- = 1

2nd /IIA/Alkali earth metals :

Be = $1s^2$, $2s^2$

Mg = $1s^2$, $2s^2$, $2p^6$, $3s^2$

Ca = $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^6$, $4s^2$

General electronic configuration = **ns^2**

Number of valence shell e^- = 2

13th /IIIA/Boron Family ;

B = $1s^2$, $2s^2$, $2p^1$

Al = $1s^2$, $2s^2$. $2p^6$, $3s^2$, $3p^1$

Ga = $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^6$, $3d^{10}$, $4s^2$, $4p^1$

General electronic configuration = **ns^2 np^1**

14th /IVA/Carbon Family :

C = $1s^2$, $2s^2$, $2s^2$

Si = $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^2$

Ge = $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^6$, $4s^2$, $3d^{10}$, $4p^2$

General electronic configuration = $ns^2\ np^2$

Number of valence e^-s= 4

15th /VA/Nitrogen family/Pnicogen : (Used in fertilizer as urea)

N = $1s^2, 2s^2, 2p^3$

P = $1s^2. 2s^2. 2p^6, 3s^2, 3p^3$

As = $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^{10}, 4p^3$

General electronic configuration = $ns^2\ np^3$

Number of valence shell e^- = 5

16th /VIA/ Oxygen family/ Chalcogen : (Ore forming)

O = $1s^2, 2s^2, 2p^4$

S = $1s^2, 2s^2, 2p^6, 3s^2, 3p^4$

Se = $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10}, 4s^2, 4p^4$

General electronic configuration **:** $ns^2\ np^4$

Number of valence e^-s=6

17th /VIIA/ Fluorine family/ Halogens : (Salt forming)

F = $1s^2, 2s^2, 2p^5$

Cl = $1s^2, 2s^2, 2p^6, 3s^2, 3p^5$

Br = $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^{10}, 4p^5$

General electronic configuration = $ns^2\ np^5$

Number of valence e^-s=7

18th /Zero group/ Inert gases / Noble gases :

Ne = $1s^2, 2s^2, 2p^6$

Ar = 1s+, $2s^2, 2p^6, 3s^2, 3p^6$

Kr = $1s^2, 2s^2, 3p^6, 3s^2, 3p^6, 3d^{10}, 4s^2, 4p^6$

General electronic configuration **=** $ns^2\ np^6$ (except. He)

Number of valence shell e^-= 8

Elements of group 16 are known as Chalcogen Elements of group 17 are known as halogens.

Classification on the basis of sub shell in which last electron (e^-) enters

Periodicity in properties :

The electronic configurations of atoms display a periodic variation with increase in atomic number. Since the properties of elements depends upon the electronic configurations. So the elements exhibits periodic variation of physical & chemical properties. Some properties of elements are :-

(A) **Valency :-** It is defined as the combining capacity of the element. Valency is determined by the number of electrons present in outer most shell. These electron are known as valence electrons.

Variation of valency across a period :- The number of valency electron increases from 1 to 8 on moving across a period. The valency of an element with respect to hydrogen and halogen increases from 1 to 4 and then decreases from 4 to zero with respect to oxygen valency increases from 1 to 7.

Variation of valency along a group :- On moving down a group. The no. of valence electrons remains same so the valence of all the elements of a group is same.

Group (1) elements have valency – 1

Group (2) elements have valence – 2

Atomic size :- Atomic size means atomic radius of an atom which is defined as the distance between the centre of the nucleus of an atom and valence shell containing electrons in an isolated atom since it is very difficult to measure the atomic radius because –

(i) The isolation of single atom is very difficult.

(ii) There is no well defined boundary for the atom.

So the more accurate definition of atomic radius is –

(Half the inter nuclear distance between the two atoms in a homo atomic molecule is known as atomic radius)

This inter nuclear distance is also known as bond length. It depends upon the type of bond by which two atoms combine. Based on chemical bonds, atomic radius is divided in to four categories.

(a) Covalent radius (Single bonded covalent radius) For homo atoms

It is half of the inter nuclear distance between two singly bonded homo atoms.

(b) Covalent radius for hetero atoms.

(i) In case of hetero atomic molecule (A-B), if the Electronegativity difference is less. Then covalent radius of oxygen, nitrogen and carbon is taken from the compound H_2O_2, N_2H_4 and C_2H_6 respectively.

This radii is subtracted from the bond length of A-B molecules.

eg. C-I (Electronegativity is almost some 205)

Inter nuclear distance C-I is 2.13 Å, covalent radius of carbon in compound C_2H_6 is 0.77 Å covalent radius of I^- will be.

$d_{C-I}=r_C+r_I$ (covalent radius of iodine)

i.e.$2.13 = 0.77 + r_1$

$r_1 = 2.13 - 0.77 = 1.36$ Å

(ii) When Electronegativity difference is more. Then bond length is determined by the schole maker and Stevenson law-

$d_{A-B} = R_A+R_B - 0.09(X_A-X_B)$

where d_{A-B} = Bond length of d_{A-B} molecule

X_A= Electronegativity of A

X_B = Electronegativity of B

Example - Bond length of F_2 = 1.44 Å

i.e. d_{F-F} = 1.44 Å $\quad r_F = \frac{1.44}{2} = 0.72Å$

d_{H-H} = 0.74 Å $\quad r_H = \frac{0.74}{2} = 37Å$

Electronegativity of Fluorine is 4.0 and Electronegativity of Hydrogen is 2.1

$$d_{H-F} = r_F + r_H + 0.09(X_F - X_{H)}$$

= 0.72 +0.37 – 0. 09(4-2.1)

= 1.09 – (0.09×1.9)

= 1.09 – 0.171 = 0.919 Å

(B) Ionic Radius -

(i) Cationic radius (ii) Anionic radius

(i) Cationic Radius –

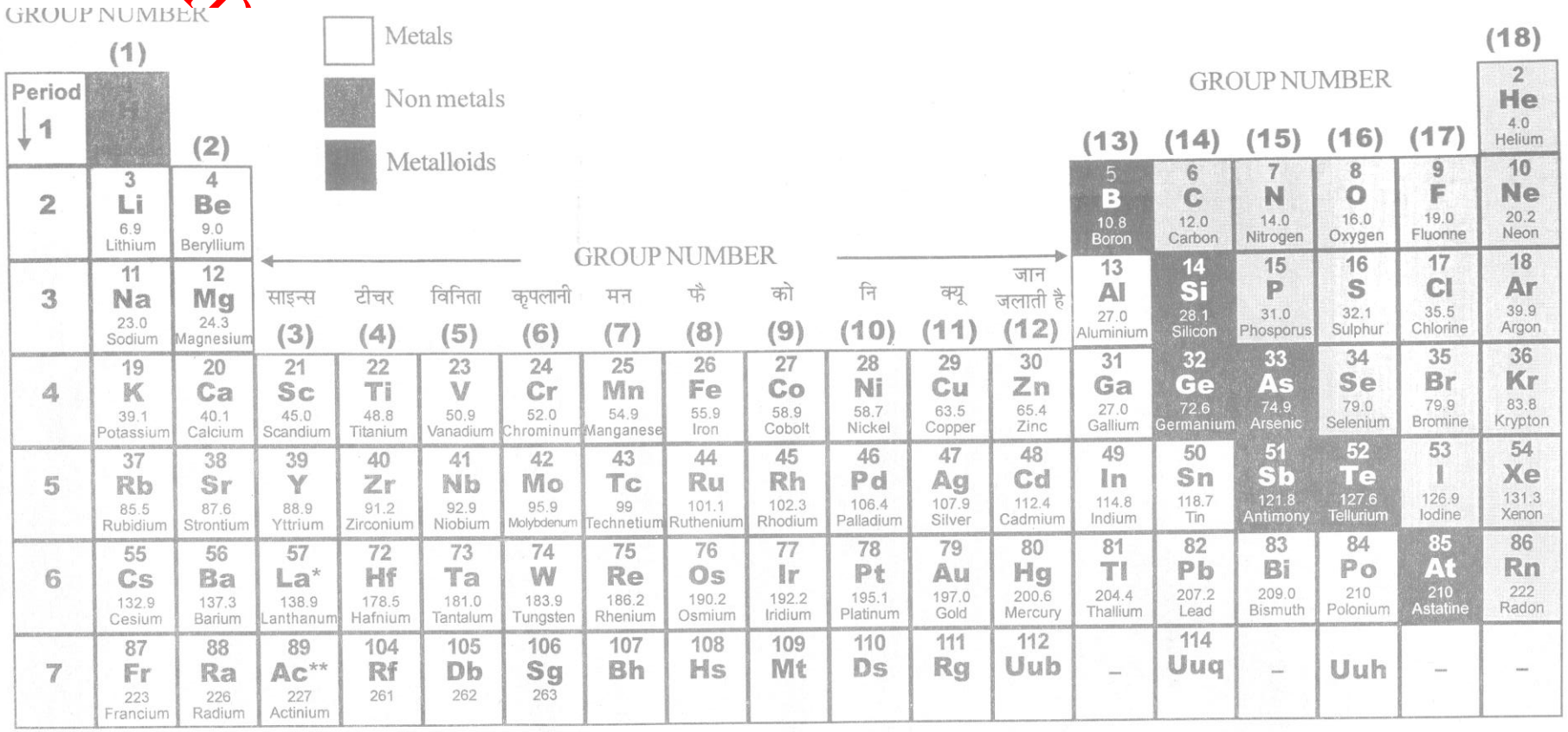
$$Sizeof\ cation\alpha \frac{1}{magnitudeof\ the\ charge\ or\ z_{eff}}$$

e.g. $Fe > Fe^{+2} > Fe^{+3}$

(ii) Anionic radius -

Anionic radius is always greater than atomic radius because in an anion electrons are more than the protons so effective nuclear charge reduces and inter electronic repulsion increases so size anion also increases.

MODERN PERIODIC TABLE



Important Question :

1. 1 period contains ……. And II period contains …… elements.
2. Group 17 elements are called ……
3. Group 18 elements are …….. Valent.
4. Which one has the bigger size
   Na (11) or Cl (17) : Cl (17) or F (9) ?
5. Name two elements whose valences are equal to their group numbers.
6. How many elements are there in the 4^{th} period.
7. Give two examples of elements of Groups 1,2, 16 and 17.
8. Group 2 elements are known as ……

(C) Metallic Radius -

Half of the inter nuclear distance between two adjacent metallic atoms.

$$Metallic radius \alpha \frac{1}{Metallic bond strength}$$

(D) Vander Waal's radius -

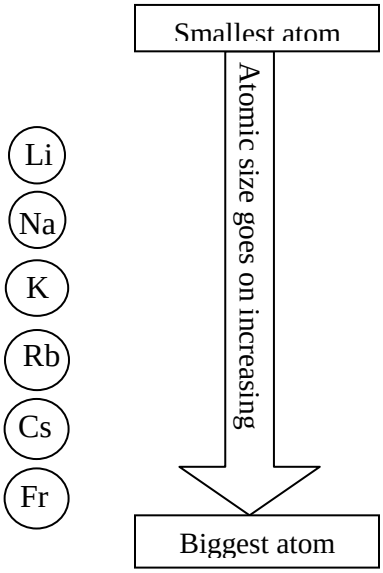
Those atoms which are not bonded with each other experiences a weak attractive to come nearer. Half of the distance between the nuclei of adjacently placed atoms in solid state of a noble gas is vanderwaal's radius.

Vander wall radius = 2 × Covalent Radius

Variation of Atomic size in a group :

On moving down a group of periodic table, the size of the atom increases.

Group-I Elements	Atomic Radii (pm)
Lithium (Li)	152
Sodium (Na)	186
Potassium (R)	231
Rubidium (Rb)	244
Cesium (Cs)	262
Francium (Fr)	-

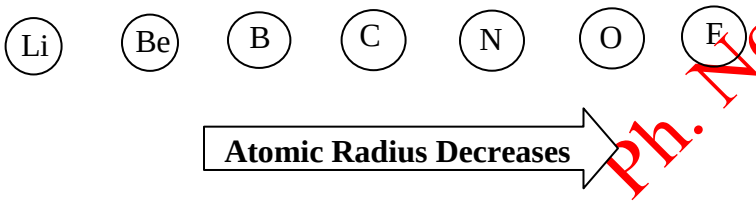


When we move from top to bottom in a group a new shell of electron is added in each period. This addition increases the size.

Variation of atomic size in period :

In general atomic radii decreases across a period from left to right eg. In IInd period. L atom is largest and Fluorine is the smallest atom because nuclear charge increases with increase in atomic number. Electrons are also increasing but these are added to the same shell.

Element	Li	Be	B	C	N	O	F
Atomic Number	3	4	5	6	7	8	9
Nuclear charge	+3	+4	+5	+6	+7	+8	+9
Electronic configuration	2 , 1	2 , 1	2 , 3	2 , 4	2 , 5	2 , 6	2 , 7
Radius (pm)	152	111	88	77	74	66	64



Atomic Size :- Decreases along the period. Increases down the group.

Metallic and Non-metallic character :

Metallic character is the tendency of atoms of the elements to lose electrons and form positive ions. It can be expressed as:

$$M \rightarrow M^{+} + e^{-}$$

Therefore metals are also called as electropositive elements.

The metallic character increases from top to bottom in a group the metallic character of the element goes on increasing eg. Li is least metallic element while cesium is most metallic element.

Elements	Metallic character
Lithium (Li)	Least Metallic
Sodium (Na)	
Potassium (K)	
Rubidium (Rb)	
Cesium (Cs)	Most Metallic

Metallic or electropositive Character increases

If we use the term electropositive in place of metallic character, we can say that electropositive character goes on increasing as we move from top to bottom in the periodic table. If we consider the electronegative character, it goes on decreasing as we move down in a group of the periodic table.

Ionization Enthalpy :

The minimum amount of energy required to remove the most loosely bounded electron from an isolated gaseous neutral atom to form gaseous electropositive ion called Ionization enthalpy. Its unit is kilo joules per mole (kJ/mol)

$$M_{(g)} + Energy \rightarrow M^{+}{}_{(g)} + e^{-}$$

It is a measure of tendency to lose electrons by atoms. The tendency to lose electron from top to bottom in a group and it decreases on moving left to right in a period.

Electron gain enthalpy :

It is defined as the amount of energy released when as isolated atom in the ground state accepts an electron to form gaseous negative ion i.e. and anion. It is a measure of tendency of an atom to accept an extra electron to form an anion. Its unit is kilo joule mole (kJ/mole). Electron gain enthalpy of elements goes on increasing as we move from left to right in a period. In group it decreases from to bottom.

PREVIOUS YEARS' BOARD QUESTIONS :

1 Which physical and chemical properties of the elements were used by Mendeleev in creating his periodic table? List two observation which posed a challenge of Mendeleev's periodic law. **(C.B.S.E 2009)**

Ans. The creation of Mendeleev's periodic table was based upon certain physical and chemical properties.
Physical properties : The atomic masses of the elements was taken into account and the elements were arranged in order of increasing atomic masses. The influences of their physical properties such as melting points, boiling points, density etc.
Chemical properties : The distribution of the elements into different groups was linked with formation of hydrides by combining with hydrogen and formation of oxides by combining with oxygen. This is linked with the valency of the elements.
The two main observations which posed challenge to Mendeleev's periodic table are as follows.
(i) Position of isotopes : Since the isotopes of an element differ in their atomic masses, they must be assigned separate slots or positions in the periodic table.
(ii) Anomalous positions of some elements : In the Mendeleev's periodic table, certain elements with higher atomic masses precede or placed before the elements with lower atomic masses. For example, the element Ar (Atomic mass = 39.9) is placed before the element K (Atomic mass = 39.1)

2. Using the part of the periodic table given below, answer the questions that follow.

Group Period	I	II	III	IV	V	VI	VII	Zero
1	H							He
2	Li	Be	B	C	N	O	F	Ne
3	Na	Mg	Al	Si	P	S	Cl	Ar
4	K	Ca						

(i) Na has physical properties similar to which elements and why?
(ii) Write the electronic configuration of N and P
(iii) State one property common to fluorine and chlorine. **(C.B.S.E. All India 2008)**

Ans. (i) Na has physical properties similar to Li and K. All the three elements have one electron each in the valence
of their atoms. These are known as alkali metals.
(ii) Electronic configuration of N(z = 7) = 2 ,5
Electronic configuration of P (z = 15) = 2,8,5
(iii) Both the elements have seven electrons in the valence shells as their atoms
Fluorine (z = 19) = 2, 7
Chlorine (z = 17) = 2,8,7

3. Table given below shows a part of the periodic table

H							He
Li	Be	B	C	N	O	F	Ne
Na	Mg	Al	Si	P	S	Cl	Ar

Using this table explain why
(a) Li and Na are considered as active metals.
(b) Atomic size of Mg is less than that of Na.
(c) Fluorine is more reactive than chlorine. **(C.B.S.E. Foreign 2008)**

Ans. (a) Both Li and Na are active elements since their atoms have only one, electron in their valence shells. They readily lose this electron to have the configuration of the nearest noble gas element.
(b) Mg is placed after Na in the same period (third.) As the atomic size decreases along a period, the size of Mg is less than that of Na.
(c) Both F and Cl belong to group 17 (halogen family). Since fluorine is more electronegative than chlorine, it is therefore more reactive also.

4. (a) Why do all the elements of the same group have similar properties ?

(b) How will the tendency to gain electrons change as we go from left to right across a period ? Why ? **(C.B.S.E. All India, 2009)**

Ans. (a) The properties of the elements are linked with the valence shell electronic configuration of their atoms The
elements with the sam configuration are expected to have similar properties. In a group, the element are separated by definite gaps of atomic numbers and have same number of electrons in the valence shells of their atoms. For example, the alkali metals in group I have one electron each. They have similar properties. For further details, consult text part.

(b) In moving from left towards the right across a period, the tendency of the elements to gain electrons increases

Explanation. In general, the atoms of all the elements have a desire or urge to have stable electronic configuration of the nearest noble gas elements or to have eight electrons in their outermost or valence shell. Now, across a period the valence electrons are added one by one from left to the right. This is supported by the electronic configuration of the elements present in period 3 or third period.

Element	**Na**	**Mg**	**Al**	**St**	**P**	**S**	**Cl**	**Ar**
No. of valence electrons	1	2	3	4	5	6	7	8
No. of electrons needed in Valence shells	7	6	5	4	5	2	1	0

This clearly shows that the element chlorine needs one electron while oxygen requires two to have to stable electronic configuration. Thus, tendency to gain electrons increases from left to right across a period.

- Element are classified on the basis of similarities in their physical and chemical properties.
- **Döbereiner** grouped elements into **triads.**
- **Newland** grouped elements on the basis of law of **octaves.**
- **Mendeleev** grouped elements in the increasing order of their atomic masses and similarity in chemical properties.
- **Mendeleev** was able to predict the existence of some elements on the basis of gaps in the periodic table.
- Moseley discovered that fundamental property of an element is its atomic number. rather than atomic mass.
  He revised **Mendeleev Periodic Table** on the basis of **atomic numbers** of elements and removed some of its anomalies.
- Elements in the long form of Modern Periodic Table are arranged in **18 vertical columns** called groups and **7 horizontal** rows called periods.
- The elements arranged in the **long form of periodic table** show (i) periodicity of properties (ii) atomic size (iii) valency (iv) metallic and non-metallic character.

EXERCISE

OBJECTIVE TYPE QUESTIONS :-

1. According to IUPAC recommendations, the number of groups in the long form of the periodic table is :-

(A) 7 (B) 8 (C) 16 (D) 18

2. The law of modern periodic table was proposed by :-

(A) D. I. Mendeleev (B) Döbereiner (C) H.G.I Moseley (D) Newlands

3. the least metallic element of group 1 is :-

(A) Lithium (B) Sodium (C) Potassium (D) Caesium

4. Which of the following properties generally increases on moving from top to bottom ?

(A) Ionization energy (B) Non-metallic character

(C) Atomic size (D) Valency

5. Which of the following statement is not correct about the trends when going from left to right across the periodic table ?

(A) The elements become less metallic in nature (B) The number of valence electrons increases

(C) The atoms lose their electrons more easily (D) The oxides becomes more acidic

6. The law of octaves was proposed by :-

(A) New lands (B) Luther Meyer (C) Döbereiner (D) Mendeleev

7. The number of periods in the long form of the periodic table is :-

(A) 6 (B) 7 (C) 10 (D) 18

8. Which of the following has the maximum non-metallic character ?

(A) F (B) Cl (C) Br (D) I

9. Which of the following sets of elements do not belong to the same group ?

(A) F, Cl, Br (B) Na, K, Rb (C) P, S, Cl (D) C, Si, Ge

10. Which of the following has lowest number of electrons in the valency shell ?

(A) O (B) C (C) N (D) B

11. Which of the following pairs of elements does not belong to same group ?

(A) Cl, Br (B) N, p (C) Mg, Ca (D) Al, Si

12. Which of the following has largest atomic size ?

(A) Be (B) C (C) O (D) F

13. Which of the following belongs to group 18 ?

(A) Sr (B) I (C) Ar (D) Rb

14. What is the basis of long form of the periodic table ?

(A) Atomic mass (B) Atomic number

(C) Atomic size (D) Metallic and Non-metallic character

15. Which one is more metallic element ?

(A) Na (B) Mg (C) Al (D) Si

16. Element X forms a chloride with the formula XCl_2. Which is a solid with a high melting point, X would most likely be in the same group of the periodic table as :-

(A) Na (B) Mg (C) Al (D) Si

FILL IN THE BLANKS :-

1. The horizontal rows in the periodic table are known as …….

2. The vertical columns of in the periodic table are known as ……

3. A very short period contains …….. elements.

4. The element having electronic configuration (2, 8 ,3) belongs to …….group.

5. In a group atomic radii …… from top to bottom and in a period atomic radii …… from left to right.

6. Size of Na^+ is …… than sodium atom.

7. Size of Cl^- is …… than Cl atom.

8. An element 'B' belongs to the second period and group 13,formula of its oxide is ……

9. Elements in the same group have similar ……

10. Elements in the same group have similar …… in their outer most shell.

11. The alkaline earth metal with the smallest atomic number is ……

12. A,B & C are the elements of the Döbereiner's triads. If the atomic mass of A is 7 and that of B is 23, then the atomic mass of 'C' will be …….

13. When Mendeleev made the periodic table the number of elements discovered till then were ……

14. Among alkali metal …….. has the smallest atomic radius.

15. Among halogens…….. has the smallest atomic radius.

16. The amount of energy released when a neutral gaseous atom gains one electron is called……

17. The energy required to remove an electron from an isolated gaseous atom is called……

18. Non-metallic character …….. from left to right in a period.

19. Metallic character …….. down a group.

20. Ionization energy …… down a group and …….. along a period.

21. Atomic size …….. form left to right in a period.

GIVE THE ANSWER FOLLOWING QUESTIONS :-

1. Why did Mendeleev leave some gaps in the periodic table of element ? Give you answer with examples.

2. Explain why the properties of the 8^{th} element are repeated in case of elements arranged in 2nd and 3rd period of the long from of the periodic table.

3. Where in period 3 of the modern periodic table do we fine :

(a) non-metals (b) elements forming negative ions

(c) elements with high melting points (d) elements forming positive ions

(e) metals

(f) elements with low boiling points ?

Mention their atomic number only.

4. If you look at the long of periodic table, you will find that the elements Li, B, C, N, O, F and Ne are present in the second period. Write down their electronic configurations.

5. Examine elements of the third period and classify them as metals and non-metals.

6. Nitrogen (atomic number 7) and phosphorus (atomic number 15) belong to Group 15 of periodic table. Write the electronic configuration of these elements. Which of these will be more electronegative ? Why ?

7. How could the modern Periodic Table remove various anomalies of Mendeleev's Periodic Table ?

8. Name two element you would expect to show chemical reactions similar to magnesium. What is the basis for your choice ?

9. Name

(a) Three elements that have a single electron in their outermost shells.

(b) Two elements that have two electrons in their outermost shells.

(c) Three elements with filled outermost shells.

10. (a) Lithium, sodium, potassium are all metal that react with water to liberate hydrogen gas is thre any similarity in the atoms of these atoms.

11. In the Modern Periodic Table , which are the metals among the first ten elements ?

12. By considering their position in the periodic table, which one of the following elements would you expect to have maximum metallic characteristic.

Ga Ge As Se Be

13. Which of the following statements is not a correct statement about the trends when going from left to right across the periods of periodic table.

(a) The elements become less metallic in nature.

(b) The number of valence electrons increase

(c) The atoms lose their electrons more easily

(d) The oxides become more acidic

14. Element X forms a chloride with the formula XCl_2, which is a solid with a high melting point X would most likely be in the same group of the periodic table

15. (a) What property do all elements in the same column of the periodic table as boron have in common ?

(b) What property do all elements in the same column of the periodic table as fluorine have in common ?

16. An atom has electronic configuration 2.8.7

(a) What is the atomic number of this element ?

(b) To which of the following elements would it be chemically similar ? (Atomic number are given in parenthesis)

N(7) F(9) P(15) Ar(18)

17. The position of three elements A, B and C in the periodic table are shown below :

Group 16	**Group 17**
-	-
-	A
-	-
B	C

(a) Nitrogen whether A is a metal or non-metal

(b) State whether C is more reactive or less reactive than A.

(c) Will C be larger or smaller in size than B ?

(d) Which type of ion, Cation or anion, will be formed by element A

18. How does the electronic configuration of an atom relate to its positions in the Modern Periodic Table ?

19. In the Modern Periodic Table, calcium (atomic number 20) is surrounded by elements with atomic numbers 12, 19 21 and 38. Which of these have physical and chemical properties resembling calcium ?

20. Compare and contrast the arrangement of element in Mendeleev periodic table and the Modern Periodic Table.

PERIODIC CLASSIFICATION OF ELEMENTS	ANSWER KEY	EXERCISE

- ***Objective type questions***

1. D 2. A 3. A 4. D 5. C 6. A 7. B 8. A 9. C 10. D

11. B 12. A 13. C 14. B 15. A 16. B

- ***Fill in the blanks***

1. Periods 2.Groups 3. 2 4. 13 5. Increases, decreases

6. Smaller 7. Larger 8. B_2O_3 9. Properties 10. electrons

11. 4 12. 15 13. 63 elements 14. Li

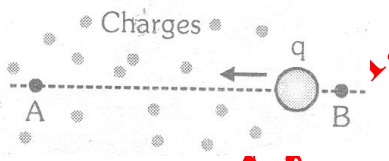
15. F 16. Electron affinity 17. Ionization energy 18. Increases

19. Increases 20. Decreases, Increases 21. Decreases

ELECTRICITY

The source of all electricity is charge. As charge is the basis of all electrical phenomena, we need to know the amount of charge on a body. It is measured in coulombs. The coulomb is the SI unit of charge and its symbol is C. Matter is generally made of protons, electrons and neutrons. Each proton carries a charge of 1.6×10^{-19} coulomb, and each negative charge. Neutrons do not carry any net charge. Normally, a body has equal number of protons and electrons, and is therefore, electrically neutral. In certain situations, the balance of charge in a body is disturbed. For example, when a glass rod is rubbed with a silk cloth, some electrons get transferred from the glass rod to the silk. The silk cloth which gains electrons, becomes negatively charged. And the glass rod, which is left with more protons than electrons, becomes positively charged. Charged particles or object can exert forces on each other. While like (similar) charges repel each other, unlike charge attract. Another important thing about charged particles is that they can flow, i.e., they can move in a particular direction, this flow of charged particles is called an electric current. Charged particles such as electrons are present in all substances. But they do not flow on their own. For flow of charges, there has to be a potential difference.

POTENTIAL DIFFERENCE AND THE FLOW OF CHARGE



The potential difference between two points A and B is the work done per unit charge in taking a charge from B to a. We express this mathematically as

$$V = V_A - V_B = \frac{W}{q}$$

Here, V is the potential difference between the poets A and B, and V_A and V_B are the potentials at these points.

The potential at infinity is chosen as zero.

If B be the reference point, the potential at B is V_B = O. From Equation, the potential at A is V_A = W/q. So, the potential at a point is the work done per unit charge in taking a charge to that point from a chosen reference point. Equation may also be written as

$$W = qV.$$

The work done on the charge q is stored as the electric potential energy (U) of the group of charges. So,

$$U = qV$$

UNIT OF POTENTIAL DIFFERENCE

The unit of potential difference (and potential) is the volt, whose symbol is V. One volt is the potential difference between two points in a current carrying conductor when 1 joule of work is done to move a charge of 1 coulomb from one point to the other.

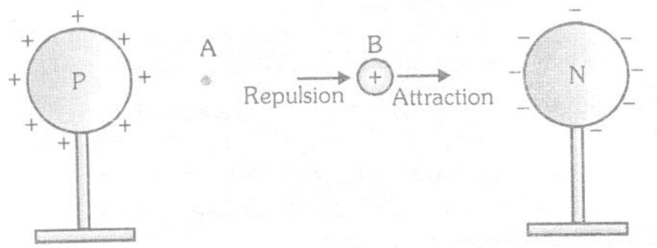
$$\frac{1 \text{ Joule}}{1 \text{ Coulomb}} = 1 \text{ volt or } 1V = 1 JC^{-1}$$

The potential difference between two points is sometimes also called the voltage.

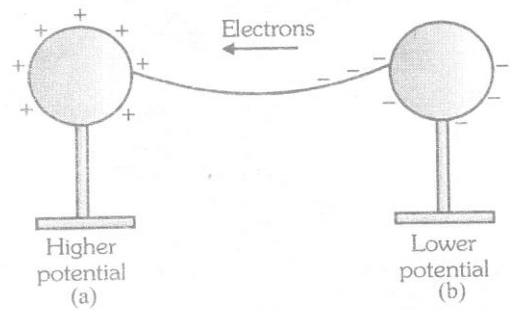
FLOW OF CHARGE

Consider two identical metallic spheres P and N, carrying equal amounts of positive and negative charges respectively. A positive charge is to be taken from B to A. It is attracted by the negatively charged sphere N and repelled by the positively charged sparer P. So, to move the charge towards A , one has to apply a force on it towards the left. Thus, he work done is positive. Hence, the potential difference $V_A - V_B$ is positive. This means $V_A > V_B$.

As one moves towards P, the work done increases; so, the potential increases; And on moving towards N, the potential decreases. So, the potential of P is higher than of N. In general, the potential of a positively charged body is taken as higher than of a negatively charged body.



What happens when a free-to-move charge is placed between the spheres ? A positive charge will move towards the negatively charged sphere. And a negative charge will move towered the positively charged sphere. That is, a free positive charge moves towards lower potential. And a free negative charge moves towards higher potential
If the two spheres are connected by a metal wire, electrons from the negatively charged sphere (at a lower potential) will flow to the positively charged sphere (at a higher potential). Eventually, the flow of electrons causes the charges on the spheres to become balanced. When that happens, the spheres no longer carry a net charge, and therefore, have equal potential. So, the flow of electrons stops. So we can say that **a potential difference causes charges to flow.**

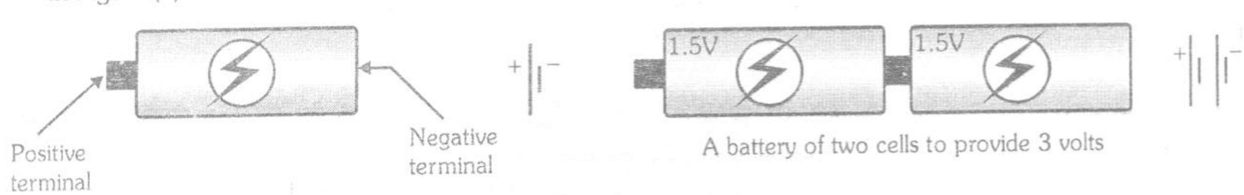


A CELL PROVIDES A CONSTANT POTENTIAL DIFFERENCE

The potential difference provided by things like charged spheres reduces to zero quickly once charges start to flow. So, we have to use cells to provide constant potential difference for a long time. Cells have chemicals inside. Reactions in the cell cause positive and negative charge to gather separately. This creates a potential difference between the terminals of the cell. The terminal at a higher potential is called the positive terminal and the one at a lower potential is called the negative terminal.
The cells that we commonly use are called dry cells (Figure). In a common dry cell, the small metallic cap at one end is the positive terminal, while the flat metallic plate at the other end is the negative terminal. It provides a potential difference of 1.5 V. A cell is represented by the symbol shown in fig (b) . The larger line represents the positive terminal, while the shorter line represents the negative terminal.

A COMBINATION OF CELLS IS CALLED A BATTERY

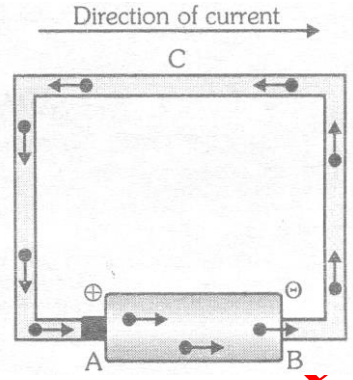
Quite often, multiple cells are combined to get a potential difference that is higher than that of a single cell. For example, we connect two 1.5V cells to get a potential difference of 3V (Figure (c)) This is shown using symbols in Figure (d) . in Figure (d).



(a) (b) (c) (d)

ELECTRIC CURRENT

Consider a metallic wire ACB connoted across a cell of potential difference V. Since the end A is connected to the positive terminal, it is at a higher potential than the end B. In metals, some electrons are loosely bound to the atoms, and can move within it. These are called free electros. In the metallic wire, these electrons (negative charges) move from the low-potential side B to the high-potential side A. After reaching A, they enter the cell. The chemical reactions in the cell drive these electrons to the negative terminal. From there, they re-enter the wire at the end B. Thus, there is a continuous flow of electrons in the wire from B to C to A. We say that there is an electric current in the wire. In a metal, the flow of negative charges constitutes the current .



Current in a wire connected to a to a cell

An electric current can also be a flow of positive charges. So, a flow of charge is called an electric current. By convention, the direction of current is taken as the direction of flow of positive charges. Thus, the direction of current is opposite to the direction of flow of negative charges. So, **when a wire is connected to a cell, the current in the wire is from the positive-terminal end to the negative-terminal end.**

MEASUREMENT OF CURRENT

The charge passing per unit time through a given place (area) is the magnitude of the electric current at that place. Thus,

$$i = \frac{Q}{t}$$

Here Q is the charge that passes through a place in time t.

Unit of current From Equation, we find that current is charge divided by time. The SI unit of charge is the coulomb and that of time is the second. The SI unit of current, therefore, is coulomb/second. This unit is called the ampere, whose symbol is A. Thus, if one coulomb of charge passes through a place in one second, the current there is 1 ampere.

CONDUCTOR AND INSULATORS

Materials that conduct electricity easily are called good conductors or simply, conductors, And, materials that do not conduct electricity easily are called insulators.

All metals conduct electricity because they have some loosely bound free electrons, which flow when a potential difference is applied. However, some metals conduct electricity better than others, Silver is the best conductor.

But because of the high cost of silver, electric wires are made of copper, or in some cases aluminum.

Most nonmetallic solids do not conduct electricity. Although diamond and graphite are both forms of carbon (a nonmetal,) graphite is a conductor while diamond is an insulator. Insulators do conduct electricity because their electrons are tightly bound to the atoms. Rubber, plastics, wood glass and porcelain are some examples of insulators. Insulators have many uses. For example, they are used as protective covers on electric wires and electrician's tools. Certain liquids also conduct electricity. While distilled water is an insulator, addition of certain slats, acids or bases allows it to conduct electricity. Under normal circumstances, gases do not conduct electricity.

ELECTRIC CIRCUITS AND MEASURING INSTRUMENTS

A closed path in which a current can flow is called an electric circuit. An electric circuit may have one or more electric elements such as bulbs (or lamps), cells, switches (or plug keys), metal wires, etc. Each element of a circuit has a specific function to play. For example, wires can be used to connect one element to the next. And a plug key or a switch can be used to either complete or break the closed path, thereby starting or stopping the current in the circuit.

Some common circuit elements and their symbols are shown in Figure.

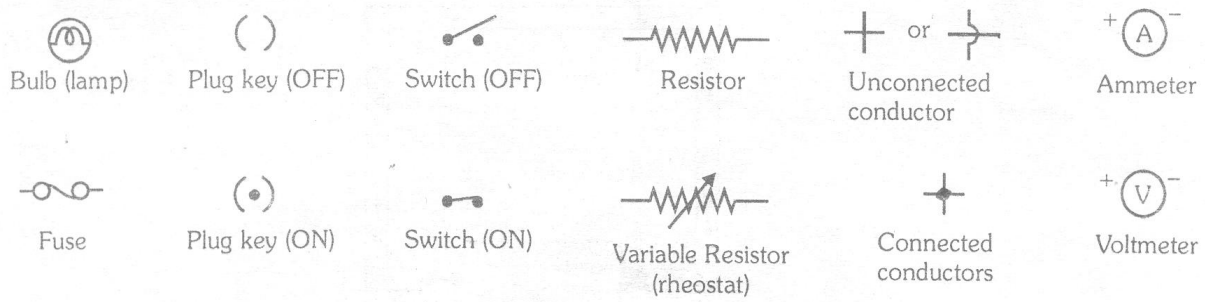


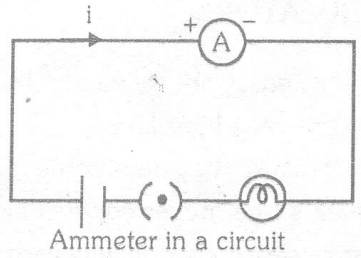
Fig. Some symbols used in circuit diagrams

COMMON MEASURING INSTRUMENTS

The electric current in a circuit is measured by an instrument called the ammeter, and the potential difference between two points in it is measured by a voltmeter (in voltage stabilizers). In these meters, a needle moving over a graduated scale gives the value of the measured quantity. Each meter has two terminals. The terminal marked '+' is connected by a wire to the higher-potential side of a circuit, while the terminal marked '-'is connected to the lower-potential side.

USING AN AMMETER TO MEASURE CURRENT

To measure the current through an element of a circuit, an ammeter is connected in such a way that the current flowing through it also flows through the element. Such a connection is called a series connection. In Figure, the current I flowing through the lamp also flows through the ammeter. The reading of the ammeter gives the current through the lamp. Note that if the ammeter is removed, there will be a gap, and the current through the circuit will stop .

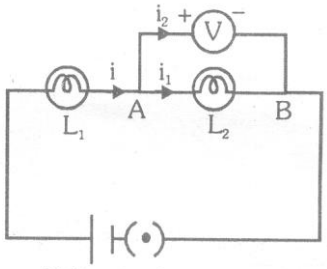


Ammeter in a circuit

Two or more electric elements are said to be connected in series if the current flowing through one also flows through the rest.
An ammeter is always connected in series in a circuit.

USING A VOLTMETER TO MEASURE POTENTIAL DIFFERENCE

Figure shows a circuit that two lamps connected to a cell. We want to measure the potential difference across the lamp L_2, i.e., between the points A and B. As A is on the side of the positive terminal of the cell, its potential is higher than to B. So, the '+' terminal of the voltmeter is connected to A, and the '-'terminal, to B. The reading of the voltmeter gives the potential difference across L_2. The current flowing through the voltmeter is different from those flowing through the other elements of the circuit. Also, even if the voltmeter is removed, the current continues to flow in the circuit. Note that the potential difference across L_2 and the voltmeter is the same. Such a connection is called a parallel connection.



Voltmeter in a circuit

Two or more electric elements are said to be connected in parallel if the same potential difference exists across them.

OHM'S LAW

The electric current through a metallic element or wire is directly proportional to the potential difference applied between its ends, provided the temperature remains constant.

If a potential difference V is applied to an element and a current i pass through it,

$$i \propto V$$

$$\text{or } i = \left(\frac{1}{R}\right)V$$

Thus **Ohm's Law** **V = iR**

here R is a constant for the given element (metallic wire) at a given temperature and is called its resistance. It is the property of a conductor to resist the flow of charges through it.

RESISTANCE

From equation, $i = \frac{V}{R}$

So, for a given potential difference $i \propto \frac{1}{R}$

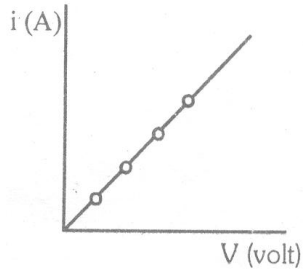
Thus, for a given potential difference, the current is inversely proportional to the resistance. The higher is the resistance, the lower is the current. If the resistance is doubled, the current is halved. Good conductors have low resistance, while insulators have very high resistance.

UNIT OF RESISTANCE

Potential difference is measured in volts, and current is measured in amperes. From Equation, R = V/i. So, the unit of resistance is volt/ampere. This unit is called the ohm, and its symbol is Ω. We can define one ohm as follows.

If a potential difference of 1 volt is applied across an element, and a current of 1 ampere passes through it, the resistance of element is called 1 ohm,

$$1 ohm = \frac{1 volt}{1 ampere}$$



ON WHAT DOES RESISTANCE DEPEND?

The resistance of the conductor depends (i) on its length, (ii) on its area of cross-section, and (iii) on the nature of its material. Resistance of a uniform metallic conductor is directly proportional to its length (ℓ) and inversely proportional to the area of cross-section (A).

$$R \propto \ell \quad \text{and} \quad R \propto \frac{1}{A}$$

Combining equations, we get $$R \propto \frac{\ell}{A}$$

or $$R = \rho \frac{\ell}{A}$$

where, ρ (rho) is a constant of proportionality and is called electrical of the material of the conductor. The SI unit of resistively is Ω m.

SERIES AND PARALLEL CONNECTIONS OF RESISTORS

A conducting material (e.g., a wire) of a particular resistance meant for use in a circuit is called a resistor. A resistor is sometimes simply referred to as a resistance. It is represented by the symbol ⊣⊢. Two or more resistors can be connected in series, in parallel or in a manner that is a combination of these two.

SERIES CONNECTION OF RESISTORS

Two or more resistors are said to be connoted in series if current flowing through one also flows through the rest. The total potential difference across the combination of resistors connected in series is equal to the sum of the potential differences across the individual resistors.

$$V = V_1 + V_2 + V_3$$

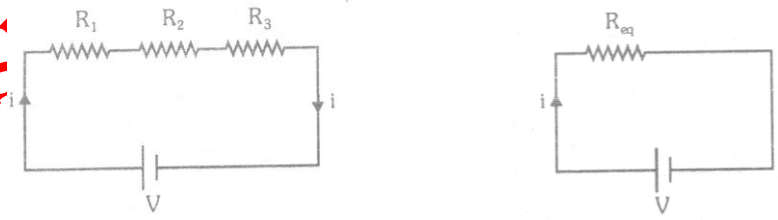
EQUIVALENT RESISTANCE IN SERIES CONCESSION

Figure (a) shows three resistors of resistances R_1, R_2 and R_3 connected in series. The cell connected across the combination maintains a potential difference V across the combination. The current through the cell is i. The same current i flows through each resistor.

Let us replace the combination of resistors by a single resistor R_{eq} such that current does not change, i.e., it remains i. This resistance is called the **equivalent resistance** of the combination, and its value is given by

Ohm's law as $R_{eq} = V / i$

Thus $V = iR_{eq}$.



(a) (b)

The potential differences V_1, V_2 and V_3 across the resistors R_1, R_2 and R_3 respectively are given by

Ohm's law as : $V_1 = iR_1$, $V_2 = iR_2$, $V_3 = iR_3$

Since the resistors are in series, $V = V_1 + V_2 + V_3$

Substituting the values of the potential differences in the above equation,

$iR_{eq} = iR_1 + iR_2 + iR_3$

or $iR_{eq} = i(R_1 + R_3)$

or $\boxed{R_{eq} = R_1 + R_2 + R_3}$

Similarly, for n resistors connected in series,

Equivalent resistance of resistors in series : $\boxed{\mathbf{Req = R_1 + R_2 + R_3 + \ldots\ldots\ldots + R_n}}$

PARALLEL CONNECTION OF RESISTORS

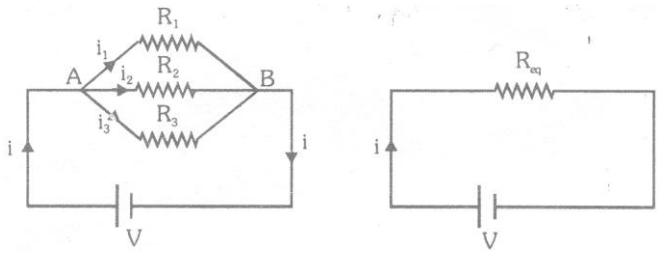
The total current flowing into the combination is equal to the sum of the currents passing through the individual resistors.

$$i = i_1 + i_2 + i_3$$

If resistors are connected in such a way that the same potential difference gets applied to each of them, they are said to be connected in parallel.

EQUIVALENT RESISTANCE IN PARALLEL CONNECTION

Figure (a) shows three resistors of resistances R_1, R_2 and R_3 connected in parallel across the points A and B . The cell connected across these two points maintains a potential difference V across each resistor. The current through the cell is i. It gets divided at A in to three parts i_1, i_2 and i_3. which flow through R_1, R_2 and R_3 respectively.



(a) (b)

Let us replaced the combination of resistors by an equivalent resistor R_{eq} such that the current i in the circuit does not change (Fig). The equivalent resistance is given by Ohm's law as $R_{eq} = V/I$.
Thus,

$$i = \frac{V}{R_{eq}} \text{ i}$$

The currents i_1, i_2 and i_3 through the resistor R_1, R_2 and R_3 respectively are given by Ohm's law as

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad i_3 = \frac{V}{R_3}$$

Since the resistors are in parallel ,

$$i = i_1 + i_2 + i_3$$

Substituting the values of the currents in above equation,

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Similarly, if there are n resistors connected in parallel , their equivalent resistance R_{eq} is given by

Equivalent Resistance of resistors in parallel : $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \text{.....} + \frac{1}{R_n}$$

For two resistances R_1 and R_2 connected in parallel.

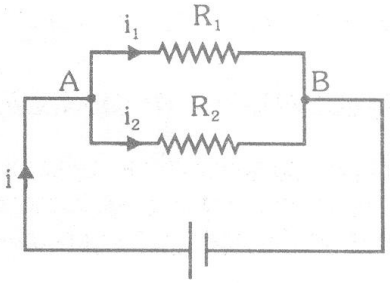
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \qquad \text{or} \qquad R = \frac{R_1 R_2}{R_1 + R_2}$$

The equivalent resistance in parallel connection is less than each of the resistances.
When a resistance is joined parallel to a comparatively smaller resistance, the equivalent resistance is very close to the value of the smaller resistance.
NOTE : If a resistor connected in series with others is removed or fails, the current through each resistor becomes zero. On the other hand, if a resistor connected in parallel with others fails or is removed, the current continues to flow through the other resistors.

DISTRIBUTION OF CURRENT IN TWO RESISTORS IN PARALLEL

Consider the circuit in fig. The resistors R_1 and R_2 are connected in parallel. The current i gets distributed in the two resistors.

$$i = i_1 + i_2 \qquad \text{...(i)}$$



Applying Ohm's law to the resistor R_1

$$V_A - V_B = R_1 i_1 \qquad \ldots(ii)$$

And applying Ohm's law to the resistor R2

$$V_A - V_B = R_2 i_2 \qquad \ldots(iii)$$

From (ii) and (iii), $R_1 i_1 = R_2 i_2$ or

$$i_2 = \frac{R_1}{R_2} i_1$$

Substituting for i_2 in (i), we have

$$i = i_1 + \frac{R_1}{R_2} i_1 = i_1 \left(1 + \frac{R_1}{R_2}\right) = i_1 \frac{R_1 + R_2}{R_2}$$

or $$i_1 = \frac{R_2}{R_1 + R_2} i$$

Similarly, $$i_2 = \frac{R_1}{R_1 + R_2} i$$

Thus, $$\frac{i_1}{i_2} = \frac{R_2}{R_1}$$

The current through each branch in parallel combination of resistors is inversely proportional to its resistance

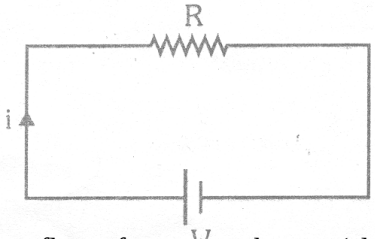
DEVICES IN SERIES AND PARALLEL

You must have seen tiny bulbs strung together for decorating buildings during festivals like Diwali, and occasions like marriages, etc. These bulbs are connected in series , and the mains voltage is applied to the combination. The potential difference (V) of the mains gets divided across the bulbs ($V = V_1 + V_2 + V_3 + \ldots$). So, a small potential difference exists across each bulb, close to that required to make the bulb work. However, the same current flows all the bulbs. So, if one bulb goes bad. The current through it strops. and this stops the current through the rest of the bulbs as well. To make the chain of lights work, we have to find and replace the defective bulb. This problem does not occur with the lights in our house. That is because **in houses, lights, fans, etc., are connected in parallel.** In parallel connection, the same mains voltage gets applied to each device, but the current through each is different. If one of them goes bad, the current in the other branches of the parallel connection does not stop. Another advantage of parallel connection is that, unlike series connection, each device can draw a different current, as per its requirement.

HEATING EFFECT OF ELECTRIC CURRENT

When an electric current passes through a bulb , the filament gets so hot that it glows and emits light. When a current passes through the filament of an electric iron, the iron becomes very not. This increase in temperature is due to what is called 'the heat produced due to current'. Suppose a resistor R is connected to a cell. The cell maintains a potential difference V across the resistor, driving a current i through it.

So, $V = iR$ …(i)



The current through the resistor is actually a flow of negative charges (electrons). Inside the cell, the negative charges flow from the positive to the negative terminal. The cell does work = QV to take a charge through the potential difference V between its terminals. This increases the energy of the charge by QV. This increased energy gets converted to heat in the resistor. So, the energy appearing as heat is given by

$U = QV$ …(ii)

The charge that passes through the wire in time is

Q = it …(iii)

Using (i), (ii) and (iii), we find that the heat produced in the wire in time t is

U = QV = (it)(iR) = i^2 Rt.

From Equation the heat produced is proportional to the square of the current, if R and t remain constant. So, if the current passing for a given time through a given resistance is doubled, the heat produced becomes four times. Similarly, for a given i and t, the heat produced is proportional to R. If the same current i passes through two resistances in a given time, more heat will be produced in the larger resistance. The heat produced can also be written as.

$$U = i^2 Rt = \left(\frac{V}{R}\right)^2$$

or

$$U = \frac{V^2}{R} t$$

For a given V and t, the heat produced is inversely proportional to R. So, if the same potential difference is applied across two resistances, more heat will be produced in the smaller resistance.

We have seen above that the increased energy of a charge gets converted to heat in the resistor. The increases in energy come from the work done by the cell. This uses up the chemical energy of the cell. So, the energy appearing as heat in the resistor ultimately comes at the expense of the chemical energy of the cell.

Not always is the work done by a cell converted to hart immediately after a motor is connected to a cell, the speed of the shaft of the motor increases. A part of the work done by the cell goes into producing the increase in kinetic energy. And a part is used to overcome friction, etc. When the motor achieves a constant speed, its kinetic energy does not change. So the work done by the cell is only used to overcome friction. etc. This appears as heat. That is why the cover over a motor becomes warm on use.

ELECTRIC POWER

Power is the rate of doing work , or the rate at which energy is produced or consumed . The electrical energy produced or consumed per unit time is called electric power. In an electric circuit, the power is

$$P = \frac{U}{t} = \frac{i^2 Rt}{t} = i^2 R$$

Using iR = Vi

P = Vi

$$P = \frac{V^2}{R}$$

The energy consumed and power are related as

U = Pt.

UNIT OF POWER

The SI unit of energy is the joule, and that of times is the second. The SI unit of power is therefore joule/ second. This unit is called the watt, whose symbol is W.

APPLICATIONS OF THE HEATING EFFECT OF CURRENT

The heating effect of electric current has many uses. Electric bulbs, room heaters, electric irons, immersion heaters, toasters, electric fuses and a number of other appliances work in this principle. In all of these, a wire of suitable resistance, commonly called the heating element, is connected to the power supply. The current passing through the element produces heat in it, which is used for same specific purpose.

ELECTRIC BULB

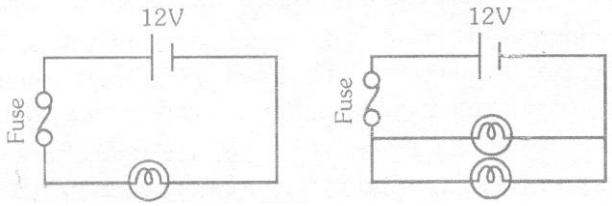
An electric bulb has a simple structure. It consists of a sealed glass bulb that has a tungsten filament connected to two electrical contacts. The bulb is filled with an uncreative gas like argon or nitrogen. To produce white light, the filament has to be heated to about 3000^0 C by passing a currant though it. Obviously, the material of the filament should such that it does not melt at this temperature. Tungsten is used for the filament because its melting point is about 3400^0 C. The sealed glass bulb serves two purposes. First, it protects the filament from oxidation and the effects of humidity. Secondly, the small enclosed volume makes it easier to maintain the required temperature, as without it the loss of heat would be more.

FUSE

A fuse is a safety device that does not allow excessive current to flow through an electric circuit. It consists of a metallic wire of low melting, fixed between the two terminals of a fuse plug. The fuse plug fits into a fuse socket connoted in the circuit. Fuses are available in various shapes. The fuse plug is used in household wiring. it is made f porcelain.

A fuse is connected in series with an appliance (such as a TV) or a group of appliances (such as the lights and fans in room). So, the current through the fuse is the same as the current through the appliance or the group of appliance. If this current exceeds a safe value, the heat produced in the fuse wire causes it to melt immediately. This brakes the circuit, preventing any damage. Figure shows examples of how a fuse is connected in circuits.

Good-quality fuse wires are made of tin, as it has low melting point. Some fuse wires are made of an alloy of tin and copper. The thickness of the fuse wire depends on the circuit in which it is to be used. If a section of the circuit is meant to carry a maximum of 5A current, the fuse wire should also be able to carry currents up to 5A. Similarly, for wiring meant for 15A, the fuse wire should be thicker, and should be able to carry current up to 15 A.



(a) (b)

DISADVANTAGES OF THE HEATING EFFECT OF CURRENT

A current always produces some neat, whether we use the heat or not. If the neat produced cannot be utilized, it represents a wastage of energy. A considerable amount of energy is thus wasted in the transmission of electricity from the generating station to out homes. Sometimes, the heat reduced in a device is so much that it can damage the device, unless power proper cooling arrangements are made. To dissipate the heat produced in TV sets, monitors, etc., their cabinets have grills for air to pass. Certain components of computer get so not that they have fans to cool them.

RATING OF ELECTRIC APPLIANCES

Take an electric bulb and see what is written on it. Apart from the name and the symbol of the company, we will find value of power and potential difference. For example, it could be 60W, 220V. It means that 220V should be applied across this bulb, and when 220V is applied, the power consumed will be 60W. We will find similar markings on all electric appliances. For an electric appliance, the values of power and voltage taken together form what is called the rating of the appliance?

From the rating of an appliance, you can easily calculate its resistance by using the equation $P = \frac{V^2}{R}$. Note that higher the power rating, smaller the resistance. So, a 1000W heater has less resistance than a 100W bulb.

We can also calculate the current drawn by an appliance by using relation $i = \frac{P}{V}$.

KILOWATT HOUR

Power is the rate of energy consume or produced. If 1 joule of energy is used per second, the energy is used at the rate of 1 watt. In other words, if energy is used at the rate of 1 watt, the total energy used in 1 second is 1 joule . How much energy is used in 1 hour if it is used at the rate of 1000 watt ?

It is (1000 watt) × (3600 second) = 3, 600,00 joule.

This amount of energy is called 1 kilowatt hour, written in short as kWh. Thus, **1kWh = 3, 600,00j=3.6×10^6J**. The electrical energy used in hosed, factories , etc., is measured in kilowatt hours. The cost of electricity is fixed per kilowatt hour. **One kilowatt hour of electrical energy is called one unit.**

NCERT EXERCISE

1. A piece of wire of resistance R is cut into equal parts. These parts are then connected in parallel. If the equivalent resistance of this combination is R', then the ratio R/R' is :

(A) 1/ 25 (B) 1/5 (C) 5 (D) 25

Ans. Resistance of each one of the five parts $= \frac{R}{5}$

Resistance of five parts connected in parallel is given by

$$\frac{1}{R'}=\frac{1}{R/5}+\frac{1}{R/5}+\frac{1}{R/5}+\frac{1}{R/5}+\frac{1}{R/5} \quad \text{or} \quad \frac{1}{R'}=\frac{5}{R}+\frac{5}{R}+\frac{5}{R}+\frac{5}{R}+\frac{5}{R}=\frac{25}{R}$$

or $\frac{R}{R'}=25$

Thus, (D) is the correct answer.

2. Which of the following terms does not represent electrical power in a circuit :

(A) I^2R (B) IR^2 (C) VI (D) V^2/R

Ans. Electrical power , $P=VI=(IR)R=t^2R=V\left(\frac{V}{R}\right)=\frac{V^2}{R}$

Obviously , IR^2 does not represent electrical power in a circuit.

Thus, (B) is the current an answer .

3. An electric bulb is rated 220 V and 100 W. When it is operated on 110 V, the power consumed will be :

(A) 100W (B) 75 W (C) 50W (D) 25W

Ans. Resistance of the electric bulbs, $R=\frac{V^2}{P}$ ($P = V^2/R$)

or $R=\frac{(220)^2}{100}=484\Omega$

power consumed by the bulb when it is operated at 110 V is given by

$$P'=\frac{V'^2}{R}=\frac{(110)^2}{484}=\frac{110\times110}{484}=25W \qquad (V' = 100\ V)$$

Thus, (D) is the correct answer .

4. Two conducting wires of the same material and of equal lengths and equal diameters are first connected in series and then in parallel in electric circuit . The ratio of the heat produced in series and parallel combinations would be :

(A) 1 : 2 (B) 2 : 1 (C) 1 : 4 (D) 4 : 1

Ans. Since both the wires are made of the material and have equal lengths and equal diameters, these have the same resistance. Let it be R.

When connected in series, their equivalent resistance is given by

$R_s = R + R = 2R$

When connected in parallel, their equivalent resistance is given by

$$\frac{1}{R_p}=\frac{1}{R}+\frac{1}{R}=\frac{2}{R} \text{ or } R_p=\frac{R}{2}$$

Further, electrical power is given by $P=\frac{V^2}{R}$

Power (or heat produced) in series, $P^s=\frac{V^2}{R_s}$

Power (or heat produced) in parallel, $P_p=\frac{V^2}{R_p}$

Thus, $\frac{p_s}{P_p}=\frac{V^2/R_s}{V^2/R_p}=\frac{R_p}{R_s}=\frac{R/2}{2R}=\frac{1}{4}$

or $P_s : Pp : : 1 : 4$

thus, (C) is the correct answer

5. How is voltmeter connected in the circuit to measure potential difference between two points ?

Ans. A voltmeter is always connected in parallel across the points between which the P.D. is to be determined.

6. A copper wire has a diameter of 0.5 mm and a resistively of 1.6×10^{-6} ohm cm. How much of this wire would be enquired to make a 10 ohm coil ? How much does the resistance change if the diameter is doubled ?

Ans. We are given that, Diameter of the wire, $D = 0.5\ mm = 0.5 \times 10^{-3}\ m$

resistivity of copper, $\rho = 1.6 \times 10^{-3}$ ohm cm $= 1.6 \times 10^{-8}$ ohm m

required resistance, $R = 10$ ohm

As $R = \frac{\rho\ell}{A}, \ell = \frac{RA}{\rho} = \frac{R(\pi D^2/4)}{\rho} = \frac{\pi RD^2}{4\rho} [A = \pi r^2 = \pi(D/2)^2 = \pi D^2/4]$

or $\ell = \frac{3.14\times10\times(0.5\times10^{-3})^2}{4\times1.6\times10^{-8}} m = 112.7\ m$

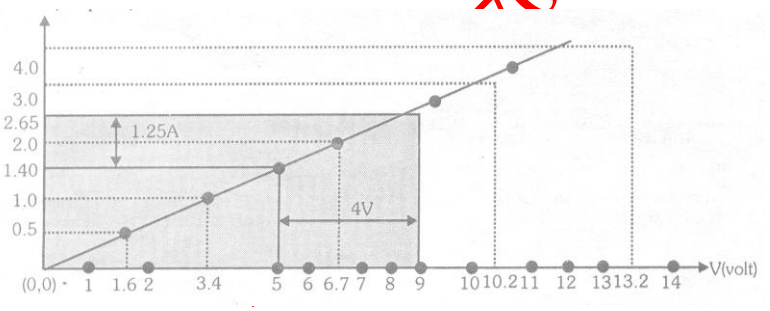
Since, $R = \frac{\rho\ell}{\pi D^2/4} = \frac{r\rho\ell}{\pi D^2}. R \propto 1/D^2$. When D is doubled, R becomes $\frac{1}{4}$ times.

7. The value of current, I, flowing in a given resistor for the corresponding value of potential difference, V, across the resistor are given below :

I (ampere) :	0.5	1.0	2.0	3.0	4.0
V (volt)	1.6	3.4	6.7	10.2	13.2

Plot a graph between V and I and calculate the resistance of resistor.

Ans. The V-I graph is as shown in fig.



For V = 4V (i.e., 9V – 5V), I = 1.25 A (i.e., 2.65 A – 1.40 A). Therefore, $R = \frac{V}{I} = \frac{4V}{1.25A} = 3.2\ \Omega$

The value of R obtained from the graph depends upon the accuracy with which the graph is plotted.

8. When a 12 V battery is connected across an unknown resistor, there is a current of 2.5 mA in the circuit. Find the value of the resistance of the resistor.

Ans. Here, V = 12V, I = 2.5 mA = 2.5×10^{-3} A

Resistance of the resistor $R = \frac{V}{I} = \frac{12V}{2.5\times10^{-3}A} = 4800\ \Omega = 4.8\ k\Omega$

9. A battery of 9 V is connected in series with resistors of 0.2Ω, 0.3Ω, 0.4Ω, 0.5Ω and 12Ω. How much current would flow through the 12Ω resistor ?

Ans. Since all the resistors are in series, equivalent resistance.

$R_s = 0.2\Omega + 0.3\Omega + 0.4\Omega + 0.5\Omega + 12\Omega = 13.4\Omega$

Current through the circuit, $I = \frac{V}{R_s} = \frac{9V}{13.4\Omega} = 0.67A$

In series, same current (I) flows through all the resistors. Thus, current flowing through 12Ω resistor = 0.67 A

10. How many 176Ω resistors (in parallel) are required to carry 5 A in 220 V line ?

Ans. Here, I = 5A, V = 220 V.

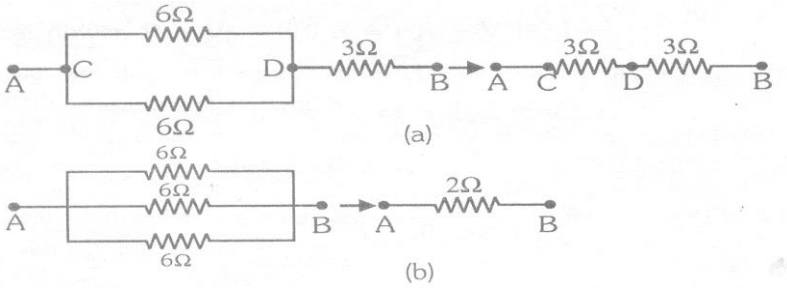
Resistance required in the circuit , $R = \frac{V}{I} = \frac{220V}{5A} = 44\Omega$, resistance of each resistor, r = 176 Ω If n resistors, each of resistance r, are connected in parallel to get the required resistance R, then $R = \frac{r}{n}$ or $44 = \frac{176}{n}$ or $n = \frac{176}{44} = 4$

11. Show how you would three resistors , each of resistance 6Ω, so that the combination has a resistance of (i) 9Ω (ii) 2Ω.

Ans. (i) In order to get a resistance of 9Ω from three resistors , each of resistance 6Ω, we connect two resistors in Parallel combination (or resistance 3Ω) in series with the third resistor as shown in fig.

(ii) In order to get a resistance of 2Ω from three resistors, each of resistance 6Ω, we connect all the three resistors in parallel as shown in fig (b) as equivalent resistance in parallel combination, i.e., R_p is given

by $R_p = \frac{6\Omega}{3} = 2\Omega$



12. Several electric bulbs designed to be used on a 220 V electric supply line, are rated 10 W. How many lamps can be connected in parallel with each other across the two wires of 220 V line if the maximum allowable current is 5 A ?

Ans. Resistance of each bulb, $r = \frac{V^2}{P} = \frac{(220)^2}{10} = 4840\Omega$

Total resistance in the circuit, $R = \frac{220V}{5A} = 44\Omega$

Let n be he number of bulb (each of resistance r) to be connected in parallel to obtain a resistance R.

Clearly , $R = \frac{r}{n}$ or $n = \frac{r}{R} = \frac{4848\Omega}{44\Omega} = 110$

13. A hot plate of an electric oven connected to a 220 V line has two resistance coils A and B, each of 24 Ω resistance, which may be used separately, in series, or in parallel. What are the currents in the three cases?

Ans. Here, potential difference, V = 220 V

Resistance of each coil, r = 24 Ω

(i) When each of the coils A or B is connected separately, current through each foil, i.e.,

$$I = \frac{V}{r} = \frac{220V}{24\Omega} = 9.2A$$

(ii) When coils A and B are connected in series, equivalent resistance in the circuit,

R_s = r + r + r = 48 Ω

Current through are series combination , i.e, $I_s = \frac{V}{R_s} = \frac{220V}{48\Omega} = 4.6A$

(iii) When the coils A and B are connected in parallel, equivalent resistance in the circuit,

$$R_p = \frac{r}{2} = \frac{24\Omega}{2} = 12\Omega$$

Current through the parallel combination, i.e, $I_p = \frac{V}{R_p} = \frac{220V}{12\Omega} = 18.3A$

14. Compare the power used in the 2Ω resistor in each of the following circuits:

(i) a 6V battery in series 1Ω and 2Ω resistors, and (ii) a 4 V battery in parallel with 12Ω and 2Ω resistors.

Ans. (i) Since 6V battery is in series with 1Ω and 2Ω resistors, current in the circuit.

$$I = \frac{6V}{1\Omega + 2\Omega} = \frac{6V}{3\Omega} = 2A$$

Power used in 2Ω resistor, $P_1 = i^2 R = (2A)^2 \times 2\ \Omega = 8W$

(ii) Since 4 V battery is in parallel with 12Ω and 2Ω resistors, pd across 2Ω resistor, V = 4V. Power used in 2Ω resistor, $P_2 = \frac{V^2}{R} = \frac{(4V)^2}{(2\Omega)} = 8W$

Clearly, $\frac{P_1}{P_2} = \frac{8W}{8W} = 1$

15. Two lamps, one rated 100 W at 220 V, and the other 60 W at 220 V, are connected in parallel to the electric mains supply. What current is drawn from the line if the supply voltage is 220 V ?

Ans. Resistance of first lamp , $r_1 = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\Omega$

Residence of the second lamp, $r_2 = \frac{V^2}{P} = \frac{(220)^2}{60} = 806.7\Omega$

Since the two lamps are connected in parallel , the equivalent resistance is given by

$$\frac{1}{R_p} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_2 + r_1}{r_1 r_2}$$

or $$R_p = \frac{r_2 r_1}{r_1 + r_2} = \frac{484 \times 806.7}{484 + 806.7} = \frac{3904428}{1290.7} = 302.6\Omega$$

Current drawn from the line, i.e., $I = \frac{V}{R_p} = \frac{220V}{302.6\Omega} = 0.73A$

16. Which uses more energy, a 250 W TV set in 1 h, or 1200 W toaster in 10 minutes ?

Ans. Energy used by 250 W TV set in 1 h = 250 W × 1h = 250 Wh
Energy used by 1200 W toaster in 10 min. (i.e., 1/6 h) = 1200 W ×(1/6) h = 200 Wh
Thus, a 250 W TV set uses more power in 1h than a 1200 W toaster in 10 minutes.

17. An electric heater of resistance 8Ω draws 15 A from the service mains for 2 hour. Calculate the rate at which heat is developed in the heater

Ans. Here, I = 15A, R = 8Ω , t = 2h
Rate at which heat is developed , i.e, electric power , $P = I^2 R = (15)^2 \times 8 = 1800$ W J/s.

18. Explain the following :

(a) Why is tungsten used almost exclusively for filament of incandescent lamps ?
(b) Why are the conductors of electric heating devices, such as toasters and electric irons, made of an alloy rather than a pure metal ?
(c) Why is the series arrangement not used for domestic circuits ?
(d) How does the resistance of a wire vary with its cross-sectional area ?
(e) Why are copper and aluminum wires usually employed for electricity transmission.

Ans. (a) Tungsten has a high milting point (3380^0C) and becomes incandescent (i.e., emits light at a high temperature) at 2400 K.
(b) The resistivity of an alloy is generally higher than that of pure metals of which it is made of .
(c) In series arrangement, if any one of the appliances fails or is swathed off, all the other appliances stop working because the same current is passing through all the appliances.
(d) The resistance of wire (R) varies inversely as its cross-sectional area (A) as $R \propto 1/A$.
(e) Copper and Aluminium wires possess low resistivity and as such are generally used for electricity transmission.

EXERCISE – 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOE THE CORRECT ONE

1. The space around a charge in which its effect can be felt is called its
(A) potential (B) field (C) field intensity (D) potential difference

2. The force acting on a unit positive test charge at a point inside an electric field is called
(A) potential (B) field
(C) field intensity (D) potential difference.

3. Work done in moving a unit positive test charge from infinity to a point inside an electric field , is called
(A) electric potential (B) field
(C) field intensity (D) potential difference.

4. Work done is moving a unit positive test charge from point to other insider an electric field , is called
(A) potential (B) field
(C) field intensity (D) potential difference.

5. Electricity constituted by electric charge at rest on the surface of a conductor, is called
(A) positive electricity (B) negative electricity (C) current electricity (D) static electricity.

6. Electricity constituted by moving electric charges, is called
(A) positive electricity (B) negative electricity (C) current electricity (D) static electricity.

7. Time rate of flow of electric charge measures electric
(A) circuit (B) current (C) potential difference (D) cell.

8. The condition for an electric charge to flow one point to other is that the two points must have electric
(A) circuit (B) current (C) potential difference (D) cell.

9. The device that maintains electric potential difference between two points is called electric
(A) circuit (B) current (C) potential difference (D) cell.

10. The path connecting two points at different potentials, to make the electric charges flow, is called electric
(A) circuit (B) current (C) potential difference (D) cell.

11. The device which measures electric potential difference between two points is called
(A) ammeter (B) voltmeter (C) manometer (D) water meter

12. The device which measures electric current through a conductor is called
(A) ammeter (B) voltmeter (C) manometer (D) water meter

13. Electric current is produced by flow of
(A) electrons (B) protons (C) negative ions (D) positive ions

14. Direction of flow of conventional current is taken from
(A) negative to positive (B) positive to negative
(C) any of the above two (D) none of the above two

15. The law which gives a relation between electric potential difference and electric current is called
(A) Faraday's law (B) Oersted's law (C) Ohm's law (D) Newton's law

16. With increase in temperature, resistance of a conductor
(A) decreases
(B) increases
(C) may decreases or increase according to the situation
(D) no particular observation

17. In series combination, total resistance
(A) decreases (B) increases
(C) may decrease or increases according to the situation (D) no particular observation

18. In parallel combination, total resistance
(A) decreases (B) increases
(C) may decrease or increase according to the situation (D) no particular observation.

19. In series combination, resistance increases due to increase in
(A) temperature (B) humidity
(C) length (D) area of cross-section

20. In parallel combination, persistence decreases due to increase in
(A) temperature (B) humidity (C) length (D) area of cross-section

21. Central part of an atom is called
(A) molecule (B) proton (C) ion (D) nucleus.

22. In an atom, particle having no charge, is called
(A) neutron (B) proton (C) electron (D) ion

23. In an atom, particle having a positive charge is called
(A) neutron (B) proton (C) electron (D) ion

24. In an atom, particle having a negative charge, is called
(A) neutron (B) proton (C) electron (D) ion

25. Substances whose atoms have more free electrons, are called
(A) electrolytes (B) conductors (C) insulators (D) semiconductors

26. Substances whose atoms have no free electrons, are called
(A) electrolytes (B) conductors (C) insulators (D) semiconductors

27. Substances whose atoms have only few free electrons, are called
(A) electrolytes (B) conductors (C) insulators (D) semiconductors

28. A neutral body has
(A) both types of positive and negative charges (B) only positive charge
(C) only negative chare (D) no charge at all

29. A body gets positively charged by losing
(A) neutron (B) protons (C) electrons (D) α – particles

30. A body gets negatively charged by gaining
(A) neutron (B) protons (C) electrons (D) α – particles

31. Time rate of work done or electrical energy developed or consumed by a generator or appliance, is called electrical
(A) current (B) power (C) potential (D) energy

32. The unit of electrical power is
(A) watt (W) (B) ampere (A) (C) joule (J) (D) ohm (Ω)

33. In series combination of electrical appliances, total electrical power
(A) increases
(B) decreases
(C) may increase or decrease according to the situation
(D) no definite observation.

34. In parallel combination of electrical appliances, total electrical power
(A) increases
(B) decreases
(C) may increase or decrease according to the situation
(D) no definite observation.

35. Power voltage rating of an electric bulb is 100 W 200 V. Current drawn by it will be
(A) 1.0 A (B) 0.8 A (C) 0.5 A (D) 0.4 A

36. The total work done by an electrical appliance during its operation, is called electrical
(A) current (B) power (C) potential (D) energy

37. The unit of electrical energy is
(A) watt (W) (B) ampere (A) (C) joule (J) (D) ohm (Ω)

38. Number of joules in 1 kWh is
(A) 3.6×10^7 (B) 3.6×10^6 (C) 3.6×10^5 (D) 3.6×10^4

39. When electric current flow through a conductor the conductor becomes
(A) cold (B) hot (C) liquid (D) vapor

40. When electric current flows through a concoctor
(A) free electrons move
(B) atoms move
(C) atoms attract free electrons
(D) atoms repel free electrons

41. Heating of a current carrying conductor is due to
(A) loss of kinetic energy of moving atoms
(B) loss of kinetic energy of moving electrons
(C) attraction between electrons and atoms
(D) repulsion between electrons and atoms

42. The correct relation between heat produced (H) and electric current (I) flowing
(A) $H \propto I$ (B) $H \propto 1/I$ (C) $H \propto I^2$ (D) $H \propto 1/I^2$.

43. In Q. 42, the relation between H and I is called
(A) Newton's law
(B) Faraday's law

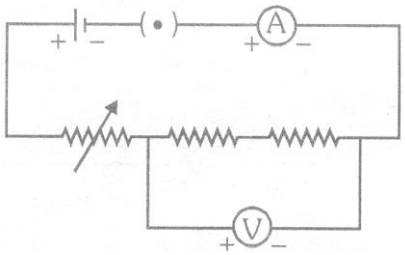
(C) Ohm's law
(D) Joule's law

44. In electric heating appliances the material of the heating element is
(A) brass (B) nichrome (C) silver (D) copper.

45. In domestic electric circuits, the cheapest appliance used is
(A) fuse (B) bulb (C) fan (D) television.

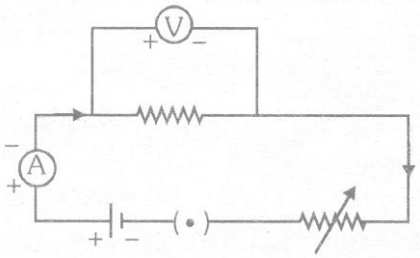
46. An ammeter has 20 divisions between mark 0 and mark 2 on its scale. The least count of the ammeter is
(A) 0.02 A (B) 0.01 A (C) 0.2 A (D) 0.1 A

47. To determine the equivalent resistance of two resistors when connected in series , a student arranged the circuit components as shown in the diagram. But he did not succeed to achieve the objective .



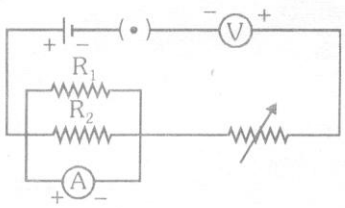
Which of the following mistake has been committed by him in setting up the circuit ?
(A) Position of ammeter is incorrect
(B) Position of voltmeter is incorrect
(C) Terminals of voltmeter are wrongly connected
(D) Terminals of ammeter are wrongly connected

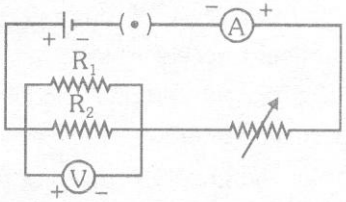
48. Which two circuit components are connected in parallel in the following circuit diagram ?

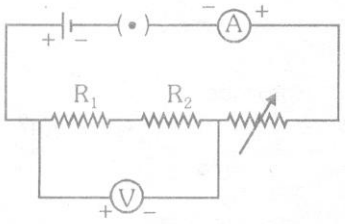


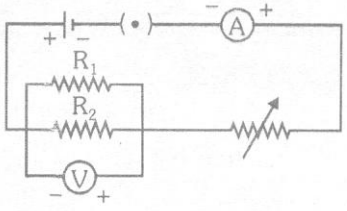
(A) rheostat and voltmeter (B) voltmeter and resistor
(C) ammeter and resistor (D) voltmeter and ammeter

49. The correct set up for determining the equivalent resistance of two resistors R_1 and R_2 when connected in parallel is :

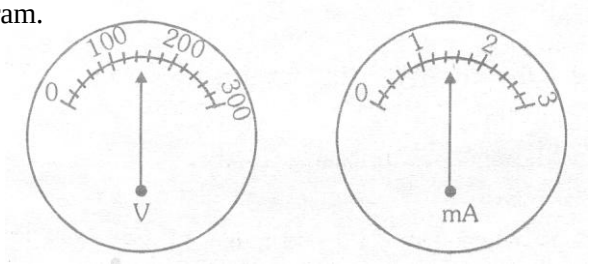
(A)


(B)


(C)


(D)


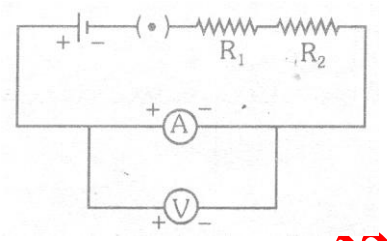
50. The current flowing though a resistor connected in electrical circuit and the potential difference developed across its ends are shown in the following diagram.



The value of resistance of the resistor in ohm is

(A) 10 (B) 15 (C) 20 (D) 25

51. To determine the equivalent resistance of a series combination of two resistors R_1 and R_2, a student arranges the following set up



Which one of the following statement will be true of this circuit ?

(A) incorrect reading for current I as well as potential difference V

(B) correct reading for current I but incorrect reading for potential difference V

(C) correct reading difference V but correct reading for current

(D) correct reading for both current as well as potential difference.

52. In a voltmeter there are 20 divisions between 0 mark and 0.5 mark. The least count of the voltmeter is

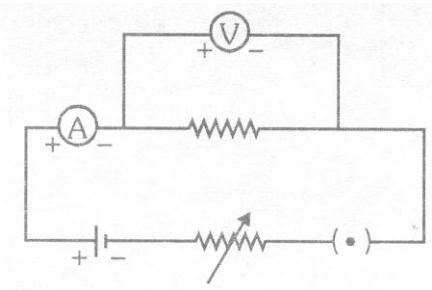
(A) 0.020 V

(B) 0.025 V

(C) 0.050 V

(D) 0.250 V

53. The following circuit diagram shown the experimental set up for the study of dependence of current on potential difference. Which two circuit components are connected in series ?



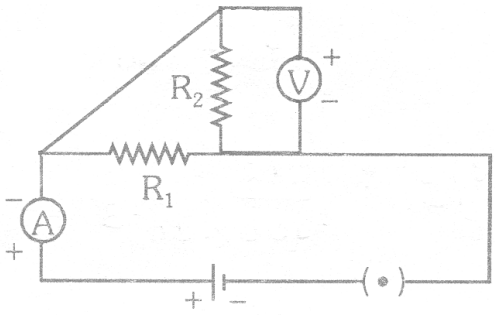
(A) Battery and voltmeter

(B) Resistor and voltmeter

(C) Ammeter and rheostat

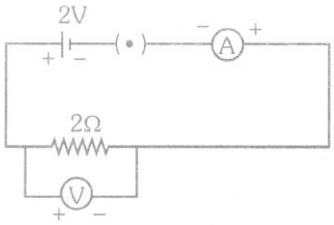
(D) Ammeter and voltmeter

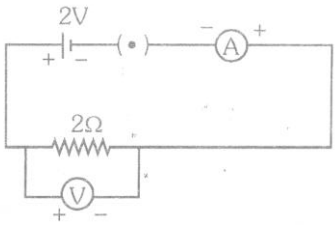
54. Which of the circuit components in the following circuit diagram are connected in parallel ?



(A) R_1 and R_2 only
(B) R_2 and V only
(C) R_1 and V only
(D) R_1 , R_2 and V only

55. For the circuit diagram shown in figures I and II voltmeter reading would be





(A) 2 V in circuit (I) and 0 V in circuit (II)

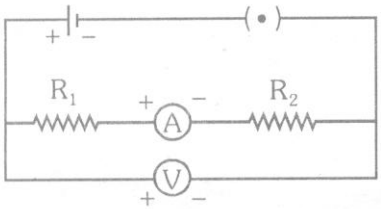
(B) 0 V in both circuits

(C) 2 V in both circuits

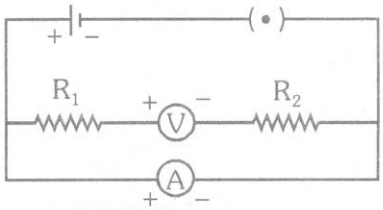
(D) 0 V in circuit (I) and 2 V in circuit (II).

56. In an experiment to determine equivalent resistance of two resistors R_1 and R_2 in series, which one of the following circuit diagrams shows the correct way of connecting the voltmeter in the circuit ?

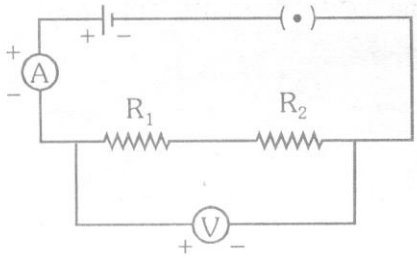
(A)



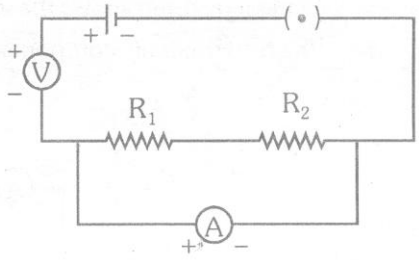
(B)



(C)



(D)



57. Current following through a conductor and the potential difference across its two ends are as per reading of the ammeter and voltmeter shown below. The resistance of the conductor and could be

(A) 0.02 Ω

(B) 20.0 Ω

(C) 0.024 Ω

(D) 24.0 Ω

58. The following instruments are available in a laboratory

Milli ammeter A_1 of least count 10 mA and range 0-300 mA

Milli ammeter A_2 of least count 20 ma and range 0-200 mA

voltmeter V_1 of least count 0.2 V and range 0-5 V

voltmeter V_2 of least count 0.3 V and range 0-3 V

Out of the following pairs of instruments which pair would be the best choice for carrying out the experiment to determine the equivalent resistance of two resistors connected in series ?

(A) milli ammeter A_1 and voltmeter V_2

(B) milli ammeter A_2 and voltmeter V_1

(C) milli ammeter A_1 and voltmeter V_1

(D) milli ammeter A_2 and voltmeter V_2

EXERCISE – 2 (FOR SCHOOL / BOARD EXAMS)

VERY SHORT ANSWER QUESTIONS

1. Write the unit of electric potential .

2. Define the potential a point.

3. Define the potential difference between two points.

4. A dry cell usually has a small cap at one and end and a flat surface at the other end. Which of the two is at a higher potential ?

5. Name the instruments used to measure electric current and potential difference respectively. Which of these is connected in series and which is connected in parallel in a circuit ?

6. What is the shape of graph between V and i, where V is the potential difference between the ends of a wire and i is the current in it ?

7. Consider the units volt, and ampere. One of them is the same as the product of other two. Which one is this ?

8. Name three electrical appliances in which the heating effect of electric current is used.

9. Two bulbs have ratings 100W, 220V and 60W, 220V. Which one has a grater resistance ?

10. You have two resistors of resistances 3 Ω and 60 Ω . What resistances can you get by combining the two ?

11. Dray a diagram to show two resistors R_1 and R_2 connected in series.

12. Two resistors of 5Ω and 10Ω are connected in series in a circuit . How does the current passing through them compare ?

13. A wire of resistance 10Ω is bent to form a closed circle. What is the resistance across a diameter of the circle ?

SHORT ANSWER QUESTIONS

14. What is the difference between a conductor and an insulator ? Give one example of each .

15. The current in a wire is one ampere. Explain this statement in terms of the charge flowing through the wire.

16. When do you say that the resistance of a wire is 1 Ω ?
17. Draw a circuit diagram for a circuit in which two resistors A and B are joined in series with a battery, and a voltmeter is connected to measure the potential difference across the resistor A.
18. When are resistors said to be connected in series ?
19. When are resistors said to be connoted in parallel ?
20. Why is tungsten suitable for making the filament of a bulb ?
21. Why is tungsten not used as a fuse wire ?
22. Alloys are preferred over metals for making the heating element of heaters. Why ?
23. Silver is a better conductor of electricity than copper. Why then do we use copper wire for conducting electricity ?
24. State Ohm's law. How can it be verified ?
25. When the terminals of a cell are connected to the ends of an iron rod, electric current flows through the rod. When the terminals are connected to the ends of a wooden rod, no current flows. Explain why, when the wooden rod also has a large number of electrons.
26. Define electric current and state its unit. How can Ohm's law be used to define ohm ?
27. Deduce the expression for the equivalent resistance of the parallel combination of two resistances R_1 and R_2.
28. Deduce the expression of the equivalent resistance of the two resistances R1 ad R2 connected in series.
29. Derive and expression for the heat produced in time t in a wire of resistance R_1 which is carrying a current it .

FILL IN THE BLANKS

1. The diameter of atom is of the order of ……m.
2. The diameter of nucleus of the order of ……m.
3. The negative charge on an electron is ……C.
4. The sign of charge on a proton is ……
5. The value of charge on a neutron is ……
6. Conductors have ……free electrons.
7. Insulators have……free electrons.
8. Semiconductors have……free electrons.
9. The sign of charge on a body which has gained electrons is……..
10. The sign of charge on a body which has lost electrons is……
11. The charge that produces electric field is called a……..charge.
12. The charge that measures the intensity of electric at a point is called a ……charge.
13. Electric potential at a point in an electric field is measured as the ……..done in bringing, a unit positive test charge from infinity to that point.
14. Static electricity is constituted by electric charges…….on the surface of a conductor.
15. Current electricity is constituted when the charges……..in a conductor.
16. The expression for eclectic current is……..
17. The S.I. unit of electric current is ……
18. The ratio V/I is called ……..
19. S.I. unit of resistance is…….
20. A voltmeter measures ……between two points.
21. An ammeter measures ……through a conductor.
22. Resistance increases in ……combination.
23. Resistance decreases in …….combination.
24. In a series combination, resistance increases due to increase in……
25. In a parallel combination, resistance decreases due to increase in……..
26. Watt is the S.I. unit of electrical …….

27. Joule is the S.I. unit of electrical …….
28. In series combination, power ……....
29. In parallel combination, power ……
30. Decrease of power of combination, is due to increase of ……..
31. Increase of power of combination in due to decrease of ……..
32. Filament of an electric bulb of low power has …….resistance.
33. The power is and electric bulb which takes 0.25 A current at 20 V is ……W.
34. An electric current heats a conductor due to loss of kinetic energy of ……
35. For same battery, heating of a wire will……if is resistance is increased.

MARK THE STATEMENT TRUE (T) OR FALSE (F)

1. An electron has a negative charge of 1.6×10^{-19} C.
2. A neutron has a positive charge of 1.6×10^{-19} C.
3. Conductors have more free electrons .
4. Insulators have few free electrons.
5. Proton has a positive charge.
6. Neutral atom has no charge.
7. A field charge has its own electric field.
8. The field intensity is measured by the field charge.
9. A point insider an electric field has an electric potential .
10. No work is done moving a test charge between two points at different potential.
11. Electric potential is a scalar quantity.
12. Electric field intensity is also a scalar quantity
13. Electric current is due to flow of electrons.
14. In electric current, electros flow from positive (higher) to negative (lower) potential/
15. A voltmeter measures electric potential difference between two poets.
16. A voltmeter has low resistance.
17. An ammeter measures electric current flowing through a resistance.
18. An ammeter has a high resistance.
19. In series combination, total resistance is more than the highest resistance.
20. In parallel combination, total resistance lies in between the lowest and the highest resistance.
21. Electrical power is time rate of production or consumption of electrical energy .
22. Kilo-watt-hour is the unit of electrical power.
23. To increase total power, we connect the appliances in parallel .
24. In house light, connecting the appliances in series will be easier and economical.
25. In parallel a 100W bulb glows more that a 25W bulb.
26. In series, a 25 W bulb glows less than a 100W bulb.
27. A high power bulb takes more current.
28. A high power bulb has more resistance.
29. More bulbs connected in parallel produce more light.
30. More bulbs connected in series produce more light .

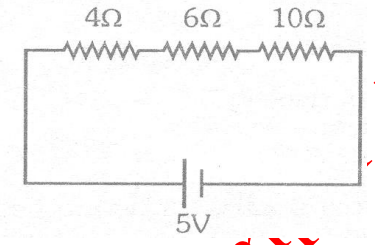
NUMERICALS

1. How much work will be done in bringing a charge of 5.0 milli coulombs from infinity to a point P at which the potential is 12V ?
2. A particle with a charge of 1.5 coulombs is taken from a point A at a potential of 50V to another point B at a potential of 120 V. Calculate the work done.

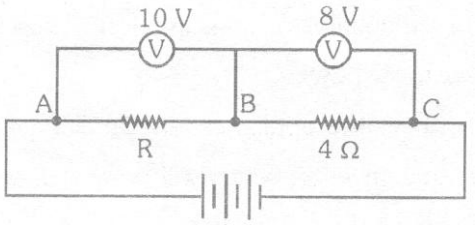
3. How many electrons are required to get 1C of negative charge ?
4. Calculate the current a wire if 900C of charge passes through it in 10 minutes .
5. How much current will flow through a resistor of resistance 12Ω if a battery of 18 V is connected across it ?
6. Calculate the resistance of a copper wire of length 1m and area of cross section $2mm^2$. Resistively of copper is $1.7 \times 10^{-8}\Omega$ m.
7. A copper wire has a resistance of $0.5\ \Omega$. Another copper wire of the same as the first one is double in length of the first. Find the resistance of the second wire.
8. In an experiment to verify Ohm's law, the current through a resistor and the potential difference across it are measured. From the values given below, plot a graph of i versus V. Show that the data confirms Ohm's law, and find the resistance of the resistor.

Current (A)	**0.1, 0.2, 0.3, 0.4**
Potential difference (V)	**1.2, 2.4, 3.6, 4.8**

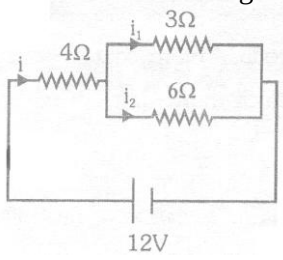
9. When a potential difference of 20 V is applied across a resistor, it draws a current of 3A. If 30 V is applied across the same resistor, what will be the current .
10. How will the resistance of a wire change if its diameter (d) is doubled , its length remaining the same ?
11. Calculate the potential difference across each resistor in the circuit shown in figure.



12. Three identical bulbs are connected in parallel with a battery. The current drawn from the battery is 6A. If one of the bulb gets fused, what will be the total current drawn from the battery ?
13. A uniform wire of resistance R is cut into three equal pieces. and these pieces are joined in parallel. What is the resistance of the combination ?
14. Consider the circuit shown in figure. The voltmeter on the left reads 10 V and that on the right reads 8V. Find **(a)** The current through the resistance R, **(b)** the value of R, and **(c)** the potential difference across the battery .



15. Three resistors of resistances 10Ω, 20Ω and 30 are connected in parallel with a 6V cell. Find (a) The current through each resistor, (b) the current supplied by the cell, and (c) the equivalent resistance of the circuit.
16. Consider the circuit shown in fig. Calculate the current through the 3Ω resistor .



17. When two resistors are joined in series, the equivalent resistance is 90Ω. When the same resistors are joined in parallel, the equivalent resistance is 20Ω. Calculate the resistance of the two resistors.

18. (a) How will you join three resistance of resistance 4Ω, 6Ω and 12Ω to get an equivalent resistance of 8Ω.
(b) What would be the highest and the lowest equivalent resistances possible by joining these resistors.

19. How many bulbs of resistance 6 ohms should be joined in parallel to draw a current of 2 amperes from a battery of 3 volts ?

20. A current 4A passes through a resistance of 100Ω for 15 minutes, Calculate the heat produced in calories.

21. A 12V battery is connected to a bulb. The battery sends a current of 2.5a through it. Calculate
(a) the power delivered to the bulb , and
(b) the energy transferred to the bulb in 5 minutes.

22. A current is passed through a resistor for some time. It produces 400 cal of heat in this period. If the current is doubled, how much heat will be produced for the same duration.

23. Calculate the wattage of an electric heater which draws 5A current connected to a 220V power supply.

24. A bulb draws 24W when connected to a 12V supply. Find the power if it is connected to a 6V supply. (Neglect resistance change due to unequal heating in the two cases.)

25. Two identical resistance R are connected in series with a battery of potential difference V for time t. The resistor are later connoted in patroller and the same battery is connected across the combination for time t. Compare the heat produced in the two cases .

26. A bulb is rated 40W, 220V. Find the current drawn by it when connoted to a 220V supply.

27. A bulb is rated 60W, 240V. Calculate its resistance when it is on. If the voltage drops to 192V, what will be the power consumed and the current drawn ?

28. A room has two the lights, a fan and a TV. Each tube light draws 40W. the fan drawn 80W, and the TV draws 60W. On the average, the tube lights are kept on for five hours. The fan for twelve hours and the TV for eight hours every day. The rate for electrical energy is Rs. 3.10 per kWh. Calculate the cost of electricity used in this room in a 30 day month.

29. When a particle of charge $10\,\mu$C is brought from infinity to a point P, 2.0 mJ of work is done by the external forces. What is the potential at P ?

30. Calculate the work done in taking a charge of 0.02C from A to B if the potential at A is 20V, and that at B is 30V.

31. How much charge flows through a wire in 10 minutes if the current through it is 2.5A ?

32. A 2V cell is connected to a 1Ω resistor. How many electrons same out of the negative terminal of the cell in 2 minutes ?

33. The amounts of charge passing through a cell in 4 seconds is 12C. What is the current supplied by the cell ?

34. A 6V battery is connected across a 5Ω resistor. Calculate the current passing through the resistor.

35. When a 24V battery is connected to a resistor, the current in it is 0.4A. What is the resistance of the resistor ? What would be the current through it when it is connected to a battery of 6V ?

36. In an experiment, the current flowing through a resistor and the potential difference across it are measured. The values are given below. Show that these values confirm Ohm's law, and the resistance of the resistor,

i (ampere)	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**
V (volt)	**4.0**	**6.0**	**8.0**	**10.0**	**12**

37. The resistivity of copper is $1.7 \times 10^{-8}\ \Omega m$.
(a) What length of copper wire of diameter 0.1 mm will have a resistance of $34\ \Omega$.
(b) Another copper wire of same length but of half the diameter as the first is taken. What is the ratio of its resistance to that of the first wire ?

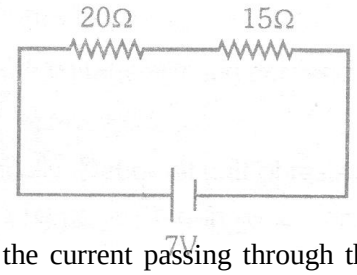
38. Three resistors, each of resistance 12Ω, are connected in parallel. What is the equivalent resistance ?

39. A uniform wire of resistance R is cut into two equal pieces. and these pieces are joined in parallel. What is the resistance of the combination ?

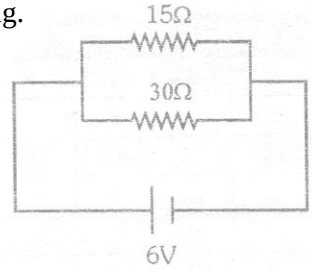
40. You have three resistors of 9 ohms each. By combining them what can be (a) the highest resistance, and (b) the lowest resistance ? (c) How can you combine them to get a resistance of $12\,\Omega$?

41. How will you join the resistors of resistance $3\,\Omega$, $6\,\Omega$ and $8\,\Omega$ to get an equivalent resistance of $10\,\Omega$?

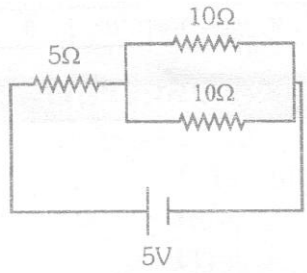
42. Find the current through the circuit shown in fig. Also find the potential difference across the $20\,\Omega$ resistor.



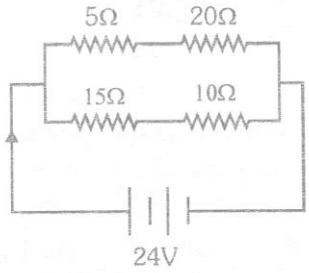
43. Find (a) the equivalent resistance, (b) the current passing through the cell, and (c) the current passing through the $30\,\Omega$ resistor in the circuit shown in fig.



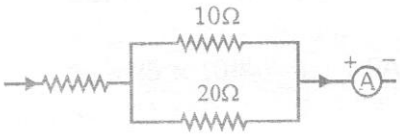
44. Find the current supplied by the cell in the circuit shown in fig.



45. In the circuit shown below, calculate the total resistance of the circuit and the current flowing through it.



46. Fig shows a part of an electric circuit. The reading of ammeter is 3.0A. Find the currents through the $10\,\Omega$ and $20\,\Omega$ resistors.



47. A 12V battery connoted to a bulb drives a current of 2.0A through if. Find the energy supplied by the battery in 10 minutes.

48. A current of 1.5A flows through a wire of $8\,\Omega$. Find the amount of heat produced in 10 seconds.

49. A current of 2A produces 200J of heat in a wire in a given period of time. If the current is increases to 4A how much heat will be produced in the same time.

50. A bulb is rated 5.0V, 100mA. Calculate its rated power and resistance .

51. Calculate the resistance of a bulb rated 40W, 230 V when in ON condition.

52. Calculate the current passing through a bulb rated 60W, 240V when it is connected to a 240V power supply.

53. Two resistors of resistances $10\,\Omega$ and $20\,\Omega$ are joined in series. A potential difference of 12V is applied across the combination. Find the power consumed by each resistor .

54. Two resistors of resistances $10\,\Omega$ and $20\,\Omega$ are joined in parallel. A potential difference of 12V is applied across the combination. Find the power consumed by each resistor.

55. Calculate the energy consumed in kilowatt hours by a 60W fan in 2 hours .

56. A heater draws 1100W at 220W.

(a) Find the resistance of the heater when in ON condition.

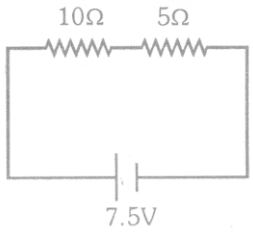
(b) Calculate the kilowatt hours consumed in a week if the heater is used daily for four hours at the rated voltage.

57. A bulb used in a car is rated 12V, 48W. Find the energy consumed in one minute when the bulb is connoted to (a) a 12V battery and (b) a 6V battery .

58. 6×10^7 electrons cross through an area per minute. What is the electric current ?

59. A 4V battery is connected to a lamp of resistance $4\,\Omega$. Calculate the current through the lamp.

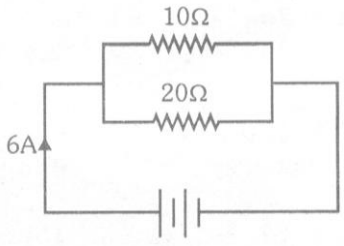
50. Calculate (a) the equivalent resistance, (b) the electric current, and (c) the potential difference across each resistor in circuit shown in Figure.



61. Two resistances of $3\,\Omega$ and $6\,\Omega$ are connected in parallel. Calculate their equivalent resistance.

62. A $1\,\Omega$ Resistor is connected in parallel to a $10\,\Omega$ resistor. Calculate the equivalent resistance.

63. Two resistors of resistances $10\,\Omega$ and $20\,\Omega$ are connected in parallel. A battery supplies 6A of current to the combination, as shown in Fig. Calculate the current in each resistor.



64. A 3V battery is connected across a $5\,\Omega$ resistance. Calculate the heat produced in 5 seconds.

65. An electric kettle is rated 500W, 220 V. It is used to heat water for 30 seconds. Assuming the voltage to be 220V, calculate the heat produced.

ANSWERS — ELECTRICITY

EXERCISE – 1															
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**B**	**C**	**A**	**D**	**D**	**C**	**B**	**C**	**D**	**A**	**B**	**A**	**A**	**B**	**C**
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	**B**	**B**	**A**	**C**	**D**	**D**	**A**	**B**	**C**	**B**	**C**	**D**	**A**	**C**	**C**
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	**B**	**A**	**B**	**A**	**C**	**D**	**C**	**B**	**B**	**A**	**B**	**C**	**A**	**B**	**As**
Que.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**		
Ans.	**C**	**A**	**B**	**B**	**A**	**B**	**B**	**C**	**D**	**D**	**C**	**C**	**C**		

EXERCISE - 2

FILL IN THE BLANKS

1. 10^{-10} **2.** 10^{-15} **3.** 1.6×10^{-19} **4.** positive
5. zero **6.** more **7.** No. **8.** few
9. negative **10.** positive **11.** field **12.** fest
13. work **14.** at rest **15.** flow **16.** I = V/R or q/t
17. ampere **18.** resistance **19.** ohm **20.** potential difference
21. current **22.** series **23.** parallel **24.** length
25. area of cross-section **26.** power **27.** energy **28.** decreases
29. increases **30.** resistance **31.** resistance **32.** more
33. 50

TRUE AND FALSE

1. True	**2.** False	**3.** True	**4.** False	**5.** True	**6.** False	**7.** True	**8.** False
9. True	**10.** False	**11.** True	**12.** False	**13.** True	**14.** False	**15.** True	**16.** False
17. True	**18.** False	**19.** True	**20.** False	**21.** True	**22.** False	**23.** True	**24.** False
25. True	**26.** False	**27.** True	**28.** False	**29.** True	**30.** False	**31.** False	

NUMERICAL

1. 0.06 J **2.** 105 J **3.** 6.25×10^{18} **4.** 1.5A **5.** 1.5A
6. $8.5 \times 10^{-3}\,\Omega$ **7.** $2.4\,\Omega$ **8.** $12\,\Omega$ **9.** 4.5 A **11.** 1V, 1.5V, 2.5V
12. 4A **13.** $\frac{R}{9}$ **14.** 2A, $5\,\Omega$, 18V **15.** 5.5 Ω **16.** 1.33A
17. $30\,\Omega$ **19.** 4 **20.** 3.4×10^{5} cal **21.** 30W, 9000J **22.** 1600 cal
23. 1100W **24.** 6W **26.** $\frac{2}{11}A$ **27.** 38.4W, 0.2A **28.** Rs. 171.12
29. 200 V **30.** 0.2 J **31.** 1500C **32.** 1.5×10^{21} electrons
33. 3A **34.** 1.2A **35.** $60\,\Omega$, 0.1A **36.** $4\,\Omega$
37. (a) 15.71 m, (b) 4 : 1 (4 times) **38.** 4W **39.** $\frac{R}{4}$

40. (a) $27\,\Omega$ (b) $3\,\Omega$ (c) one resistor connected in series to a combination of two resistors in parallel
41. $8\,\Omega$ resistor connected in series to a parallel combination of $6\,\Omega$ and $3\,\Omega$
42. 0.2A, 4V **43.** (a) $10\,\Omega$ (b) 0.6A (c) 0.2A **44.** 0.5A **45.** 12.5 W, 1.92 A
46. 2A, 1A **47.** 14,400J **48.** 180J or 43 cal **49.** 800 J
50. 0.5 W $50\,\Omega$ **51.** $1322.5\,\Omega$ **52.** 0.25A **53.** 1.6 W, 3.2W **54.** 14.4W, 7.2 W
55. 012 kWh **56.** (a) $44\,\Omega$ (b) 30.8 kWh **57. (a)** 2880 J (b) 720 J
58. 1.6 mA. **59.** 1A **60.** (a) $15\,\Omega$. (b) 0.5A (c) 5V, 2.5V
61. $2\,\Omega$ **62.** $0.99\,\Omega$ **63.** 4A, 2A **64.** 9J **65.** 15,000 J

EXERCISE -3 (FOR SCHOOL / BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

VERY SHORT ANSWER QUESTIONS (CARRYING 1 MARK EACH)

1. Define resistivity of a material. **[2004]**

2. A cylinder of a material is 10 cm long and has a cross-section of 2 cm^2. If its resistance along the length be 20Ω, what will be its resistivity in number and units. **[2004]**

3. Why is tungsten metal selected for making filaments of incandescent lamps ? **[2005]**

4. A resistance of 10 ohm is bent in the form of a closed circle. What is the effective resistance between the two points at the ends of any diameter of this circle ? **[2005]**

5. A wire of resistance 5Ω is bent in the form of a closed circle. What is the resistance between two points at the ends of any diameter of the circle ? **[2005]**

6. A wire of resistance 20Ω is bent in the form of a closed circle. What is the effective resistance between the two points at the ends of any diameter of the circle ? **[2005]**

7. Why is much less heat generated in long electric cables than in filaments of electric bulbs ? **[2005]**

8. State which has a higher resistance : a 50 W or a 25W lamp bulb and how many times ? **[2005]**

9. What is the power of torch bulb rated at 2.5 V and 500 mA ? **[2005]**

10. There are two electric bulbs, (i) marked 60 W, 220 V and (ii) marked 100 W ; 220 V. Which one of them has higher resistance ? **[2006]**

11. Out of the two, a toaster of 1 kW and an electric heater of 2 kW, which has a greater resistance ? **[2006]**

12. What is the SI unit of electrical potential ? **[2007]**

13. What is meant by the statement "potential difference between two points A and B in an electric circuit is 1 volt ? **[2007]**

14. Why is series arrangement not used for connecting domestic electrical appliances in a circuit **[2008]**

15. Out of 60 W and 40 W lamps, which one has a higher resistance when in use ? **[2008]**

SHORT ANSWER QUESTIONS (CARRYING 2 MARKS EACH)

16. An electric bulb draws a current of 0.2 A when the voltage is 220 volts. Calculate the amount of charge flowing through it in one hour. **[2004]**

17. An electric iron draws a current of 0.5 A when the voltage is 200 volts. Calculate the amount of electric charge flowing through it in one hour. **[2004]**

18. An electric appliance draws a current of 0.4 A when the voltage is 200 volts. Calculate the amount of charge flowing through it in one hour. **[2004]**

19. Calculate the amount of charge that would flow in 1 hour through the electric bulb drawing a current of 0.2 A. **[2004]**

20. Calculate the amount of charge that would flow in 2 hours through the element of an electric bulb drawing a current of 0.25 A. **[2004]**

21. Calculate the amount of charge that would flow in 1 hour through the element of an electric iron drawing a current of 0.4 A. **[2004]**

22. Derive the relation $R = R_1 + R_2 + R_3$ when resistors are joined in series **[2005]**

23. (i) Draw a circuit diagram to show how two resistors are connected in series. **[2006]**
(ii) In a circuit if the two resistors of 5Ω and 10Ω are connected in series, how does the current passing through the two resistors compare ?

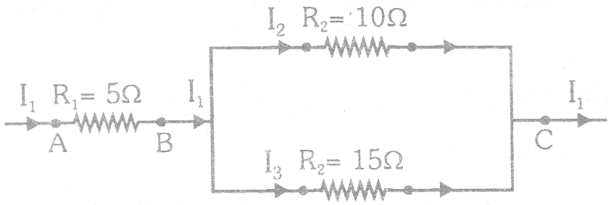
24. A bulb is rated at 5.0 V, 100 mA. Calculate its (i) power and (ii) resistance. **[2006]**

25. An electric iron has a rating of 750 W, 220V. Calculate **[2007]**
(ii) current passing through it, and (ii) its resistance, when in use.

26. An electric lamp in marked 100W, 220V. It is used for 5 hours daily. Calculate **[2007]**
(i) its resistance while glowing (ii) energy consumed in kWh per day.

SHORT ANSWER QUESTIONS (CARRYING 3 MARKS EACH)

27. Three resistors are as shown in Fig. Through a resistor of 5 ohm, a current of 1 ampere is flowing .



(a) What is the potential difference across AB and across AC ?
(b) What is the current through the other two resistors ?
(c) What is the total resistance ?

28. An electric bulb is rated at 200 V-100 W. What is its resistance ? Five such bulbs burn for 4 hours. What is electrical energy consumed ? Calculate the cost if the rate is 50 Paise unit. **[2003]**

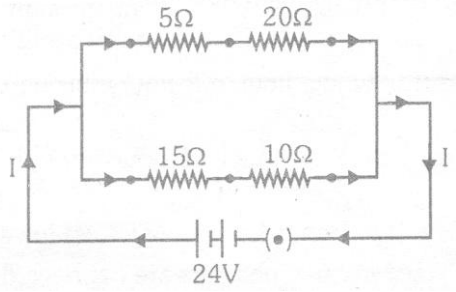
29. State the formula co-relating the electric current flowing in a conductor and the voltage applied across it. Also show this relationship by drawing a graph .
What would be the resistance of a conductor if the current flowing through it is 0.35 ampere when the potential difference across it is 1.4 volt. **[2004]**

30. (i) State the formula showing how the current I in a conductor varies when the potential difference V applied across it is increased stepwise **[2004]**
(ii) Show this relationship also on a schematic graph.
(iii) Calculate the resistance of a conductor if the current flowing through it is 0.2 ampere when the applied potential difference is 0.8 volt.

31. When a potential difference of 1.2 volt is applied across a conductor , the current flowing through it is 0.25 ampere. Calculate the resistance of the conductor. **[2004]**

32. A torch bulb is rated 5.0 V and 500 mA. Calculate its (i) power (ii) resistance and (iii) energy consumed when it is lighted for 4 hours. **[2005]**

33. If a 12 V battery is connected to the arrangement of resistance given in Fig. (with 5Ω replaced by 10Ω , 15Ω replace by 5Ω and 10Ω replaced by 25Ω). Calculate (i) the total effective resistance of the arrangement and (ii) the total current flowing in the circuit . **[2005]**

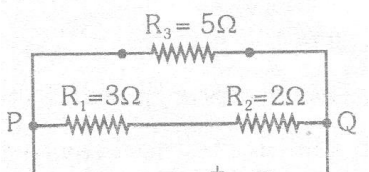


34. Two electric lamps of 100 W and 25 W respectively are joined to a supply of 200V. Calculate the total current flowing through the circuit . **[2005]**

35. Two identical resistors, each of resistance 2 Ω , are connoted in turn (i) in series, and (ii) in parallel to a battery of 12V. Calculate the ratio of power consumed in the two cases . **[2005]**

36. Two identical resistors, each of resistance 10Ω are connected (i) series, and (ii) in parallel, in turn to a battery of 10 V. Calculate the ratio of power consumed in the combination of resistors in the two cases. **[2005]**

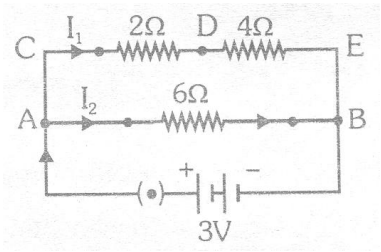
37. In the given circuit, calculate (i) total resistance of the circuit , and (ii) current shown by the ammeter. **[2005]**



38. (i) Draw a schematic diagram of a circuit consisting of a battery of five 2 V cells, a 5Ω resistor, a 10Ω resistor and a 15Ω resistor, and a plug key, all connected in series . **[2006]**
(ii) Calculate the current passing through the above circuit when is closed .

39. In a household, 5 tube lights of 40 W each are used for 5 hours and an electric press of 500 W for 4 hour each day. Calculate the total energy consumed by the tube lights and press in a month of 30 days. **[2006]**

40. In the circuit shown in Fig. calculate : (a) total resistance in arm CE, (b) total current drawn from the battery , and (c) current in each arm, i.e. , AB and CE of the circuit. **[2006]**



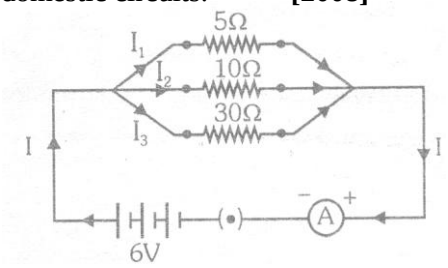
41. (a) What is meant by 'Electric Resistance' of a conductor ? **[2007]**
(b) A wire of length L and resistance R is stretched so that its length is doubled and area of cross, section is halved. How will its : (i) resistance change ? (ii) resistivity change ?

42. (a) State Ohm's law. **[2007]**
(b) Draw a schematic diagram of the circuit for studying Ohm's law .

43. Two lamps. One rated 60 W at 220 V and the other 40 W at 220 V, are connected in parallel to the electric supply at 220 V. **[2008]**
(a) Draw a circuit diagram to show the connections.
(b) Calculate the current drawn from the electric supply.
(c) Calculate the total energy consumed by the two lamps together when they operate for one hour.

44. (a) Distinguish between the terms ' overloading' and 'short-circuiting' as used in domestic circuits. **[2008]**
(b) Why are the coils of electric toasters made of an alloy than a pure metal ?

45. For the circuit shown in Fig. calculate **[2008]**
(a) the value of current through each resistance
(b) the total current in the circuit
(c) the total effective resistance of the circuit.



LONG ANSWER QUESTIONS (CARRYING 5 MARKS EACH)

46. (a) Express Ohm's law by a mathematical formula. **[2004]**
(b) Draw a circuit diagram to verify Ohm's law.
(c) resent the relationship between the voltage applied across a conductor and the current flowing through it graphically .

47. State Ohm's law. Express it mathematically. Define SI unit of resistance. Derive and expression for the equivalent resistance of three resistors R_1, and R_2 connected in series (or in parallel). **[2004]**

48. (a) Express Ohm's law both a mathematical formula and by a graph line . **[2004]**
(b) State SI units of (i) resistance and (ii) resistivity .
(c) What will be the equivalent resistance of two resistors R_1 and R_2 (i) connected in series and (ii) connected in parallel

49. (a) What is meant by saying that potential difference between two points is 1 volt ? Name a device that helps to measure the potential difference a conductor. **[2008]**
(b) Why does the connection cord of an electric heater not glow hot while the heating element does ?

(c) Electrical resistivites of some substances at 20^0C are given below :

Silver	$1.60 \times 10^{-8}\ \Omega$ m
Copper	$1.62 \times 10^{-8}\ \Omega$ m
Tungsten	$5.20 \times 10^{-8}\ \Omega$ m
Iron	$10.0 \times 10^{-8}\ \Omega$ m
Mercury	$94.0 \times 10^{-8}\ \Omega$ m
Nichrome	$100 \times 10^{-6}\ \Omega$ m

Answer the following questions in relation to them :

(i) Among silver and copper, which one is better conductor ? Why ?

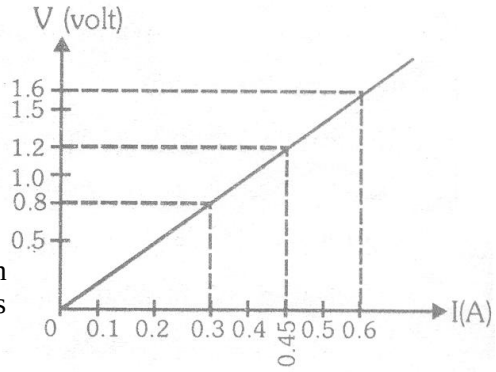
(ii) Which material would you advise to be used in electrical heating devices ? Why ?

50. (a) Name an instrument that measures electric current in a circuit. Define the unit of electric current **[2008]**

(b) What do the following symbols mean in circuit diagram ?

(i) 

(ii) 



(c) An electric circuit consisting of a 0.5 m long nichrome wire XY, an ammeter, a voltmeter, four cells of 1.5 V each and a plug key was set up.

(i) Draw a diagram of this electric circuit to study the relation between the potential difference maintained between the points 'X' and 'Y' and the electric current flowing through XY.

(ii) Graph shown Fig. was plotted between V and I values.

What would be the value of V/I ratios when the potential difference is 0.8 V, 1.2 V and 1.6 V respectively ?

What conclusion do you draw from these values ?

EXERCISE – 4 (FOR OLYMPIADS)

1. If a charged body attracts another body, the charge on the other body :-

(A) must be negative (B) must be positive

(C) must be zero (D) may be negative or positive or zero

2. 1 MeV is equal to :-

(A) 1.6×10^{-19} J (B) 1.6×10^{-14} J (C) 1.6×10^{-13} J (D) 1.6×10^{-13} J

3. A man has five resistors each of value $\frac{1}{5}\Omega$. What is the maximum resistance he can obtain by connecting them ?

(A) 1 Ω (B) 5Ω (C) $\frac{1}{2}\Omega$ (D) $\frac{1}{5}\Omega$

4. Materials which allow larger currents to flow through them are called :-

(A) Insulators (B) Conductors (C) Semiconductors (D) Alloys

5. If I is the current through a wire and e is the charge of electrons, then the number of electrons in t seconds will be given by :-

(A) $\frac{Ie}{t}$ (B) Ite (C) e/It (D) It/e

6. Conventionally, the direction of the current is taken as –

(A) the direction of flow to negative charge (B) the direction of flow of atoms

(C) the direction of flow of molecules (D) the direction of flow of positive charge

7. The unit of specific resistance is :-

(A) ohm (B) mho (C) ohm-metre (D) ohm per metre

8. If the length of a wire is doubled and its cross-section is also doubled, then the resistance will –

(A) increase eight times (B) decrease four times

(C) become four times (D) remain unchanged

9. A suitable unit for expressing the strength of electric field is –

(A) V / C (B) C / m (C) N / C (D) C/ N

10. 1 volt equals :-

(A) 1 joule (B) 1 joule per coulomb

(C) 1 coulomb per metre (D) 1 Newton per coulomb

11. 1 Vm^{-1} equals –

(A) 1 NC^{-1} (B) 1 NC^{-2} (C) 1 Jm^{-1} (D) I Jm^{-2}

12. The reciprocal of resistance is conductance. If the unit of resistance is ohm, the unit of conductance will be –

(A) ohm (B) volt (C) mho (D) ohm $metre^{-1}$

13. Good conductors have many loosely bound –

(A) atoms (B) molecules (C) protons (D) electrons

14. One ampere equals :-

(A) $10^{6}\ \mu A$ (B) $10^{-6}\ \mu A$ (C) $10^{-3}\ \mu A$ (D) 10 A

15. How many electrons constitute a current of one microampere ?

(A) 6.25×10^{6} (B) 6.25×10^{12} (C) 6.25×10^{9} (D) 6.25×10^{15}

16. If a wire of resistance $1\,\Omega$ is stretcher to double its length, then the resistance will become :-

(A) $\frac{1}{2}\,\Omega$ (B) $2\,\Omega$ (C) $\frac{1}{4}\,\Omega$ (D) $4\,\Omega$

17. The SI unit of specific resistance is :-

(A) ohm m (B) ohm m^{-1} (C) ohm m^{2} (D) $(ohm)^{-1}$

18. The effective resistance of a circuit containing resistances in parallel is –

(A) equal to the sum of the individual resistances

(B) smaller than any of the individual resistance

(C) greater than any of the individual resistances

(D) sometimes grater and sometimes smaller than the individual resistances

19. Electric intensity is

(A) a scalar quantity (B) a vector quantity

(C) neither scalar nor vector (D) sometimes scalar and sometimes vector

20. Electric potential is

(A) A scalar quantity (B) a vector quantity

(C) neither scalar nor vector (D) sometimes scalar and sometimes vector

21. In Coulomb's law, the constant of proportionality k has the units –

(A) N (B) Nm^{2} (C) NC^{2} / m^{2} (D) Nm^{2} / C^{2}

22. The variable resistance is called :-

(A) resistor (B) rheostat (C) open switch (D) none of these

23. How much work is done in moving a charge of two coulombs from a point at 118 volts to a point at 128 volts ?

(A) 10 J (B) 20 J (C) $\frac{1}{10}$ J (D) None of these

24. The law that governs the force between electric charges is called –

(A) Ampere's law (B) Coulomb's law (C) Faraday's law (D) Ohm's law

25. A charge q is placed at the center of the line joining two equal charges Q. The system of the three charges will be in equilibrium, if q is equal to -

(A) $-\frac{Q}{2}$ (B) $-\frac{Q}{4}$ (C) $+\frac{Q}{4}$ (D) $+\frac{Q}{2}$

26. Two small spheres each carrying a charge q are placed r mere apart. If one of the spheres is taken around the other one in a circular path of radius r, the work done will be equal to $\approx$.

(A) Force between them × r (B) Force between them × 2π r

(C) Force between them $/ 2\pi r$ (D) Zero

27. The force between two electrons separated by distance r varies as :-

(A) r^2 (B) r (C) r^{-1} (D) r^{-2}

28. When the distance between the charged particles is halved, the force between them becomes –

(A) One-fourth (B) Half (C) Double (D) Four times

29. There are two charge +1 μC and +5 μC. The ratio of the forces action on them will be –

(A) 1 : 5 (B) 1 : 1 (C) 5 : 1 (D) 1 : 25

30. Which one of the following is the unit of electric field intensity ?

(A) V × metre (B) V / joule (C) V × joule (D) V / metre

31. A charge q_1 exerts some force on second charge q_2. if third charge q_3 is brought near, the force of q_1 exerted on, q_2 :-

(A) Decreases

(B) Increases

(C) Remains unchanged

(D) Increases if q_3 is of the same sign as q_1 and decreases if q_3 is of opposite sign

32. If the charge is moved against the coulomb force of an electric field –

(A) Work is done by the electric field (B) Energy is used from some outside force

(C) The strength of the field is decreased (D) The energy of the system is decreased

33. The ratio of the forces between two small spheres with constant charge (a) in air (b) in a medium of dielectric constant K is -

(A) 1 : K (B) K : 1 (C) $1 : K^2$ (D) $K^2 : 1$

34. Two charges are placed at a distance. If a glass slab is placed between them, force between them will be

(A) Zero (B) Increased (C) Decreased (D) Remains same

35. Electric intensity and electric potential are related with each other by the relation –

(A) $V = \frac{dE}{dx}$ (B) $E = \frac{dV}{dx}$ (C) $E = \frac{dV}{dx}$ (D) E = potential × distance

36. If a unit positive chare is taken from one point to another over an equipotential surface then –

(A) Work is done on the charge (B) Work is done by the charge

(C) Work done is constant (D) No work is done

37. Electric lines of force about negative point charge are –

(A) Circular, anticlockwise (B) Circular, clockwise

(C) Radial, inward (D) Radial, outward

38. Electric intensity at a place due to a, charge conductor is a –

(A) Scalar quantity (B) Vector quantity

(C) Neither scalar vector (D) None of these

39. If a glass rod is rubbed with silk, it acquires a positive charge because –

(A) Protons are added to it (B) Electrons are added to it

(C) Protons are removed from it (D) Electrons are removed from it

40. The magnitude of electric field intensity E is such that, an electron placed in it would experience an electrical force equal to its weight is given by -

(A) mge (B) $\frac{mg}{e}$ (C) $\frac{e}{mg}$ (D) $\frac{e^2}{m^2}.q$

41. Two resistors of resistance R_1 and R_2 having $R_1 > R_2$ are connected in parallel. For equivalent resistance R, the correct statement is -

(A) $R > R_1 + R_2$ (B) $R_1 > R < R_2$

(C) $R_2 < R < (R_1 + R_2)$ (D) $R < R_1$

42. If a 0.1% increases in length due to stretching, the percentage increases in its resistance will be –

(A) 0.2% (B) 2% (C) 1% (D) 0.1%

43. Two unequal resistances are connected in parallel. Which one of the statement is correct –

(A) The current flowing is same in both

(B) More current will flow from higher resistance piece

(C) The potential drop is same in both

(D) The conductivity of lower resistance is less

44. When the length and area of cross-section both are doubled, then its resistance-

(A) Will become half (B) Will be doubled

(C) Will remain the same (D) Will become four times

45. A galvanometer can be converted into an ammeter by connecting –

(A) Low resistance in series (B) High resistance in parallel

(C) Low resistance in parallel (D) High resistance in series

46. There are 8 equal resistances R. Two are connected in parallel, such four groups are connected in series, the total resistance

(A) R / 2 (B) 2R (C) 4R (D) 8R

47. In a conductor 4 coulombs of charge flows for 2 seconds. The value of electric current will be –

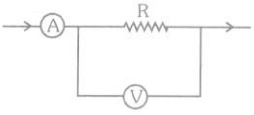
(A) 4 V (B) 4 A (C) 2 A (D) 2 V

48. In a conductor, the flow of current is :-

(A) Flow of molecules (B) Flow of free electrons

(C) Flow of positive charge (D) Flow of ions

49. Three resistances of magnitude 2, 3 and 5 ohm are connected in parallel to a battery of 10 volts and of negligible resistances. The potential difference across 3Ω resistance will be –



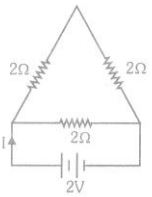
(A) 2 V (B) 3 V (C) 5 V

(D) 10 V

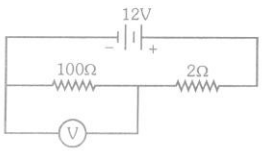
50. Which of the following have highest conductivity

(A) Cu (B) Insulator (C) Semiconductor

(D) All are equal



51. In the circuit shown below, the ammeter A reads 5A and the voltmeter V reads 20 V (Fig.). The correct value of resistance R is :-



(A) exactly 4Ω

(B) slightly greater than 4Ω

(C) slightly less than 4Ω

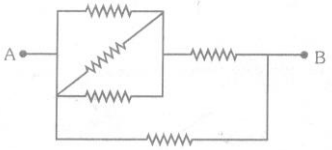
(D) zero

52. What is the current in the circuit shown (Fig.)-

(A) 1.5 A (B) 0.5 A (C) 2.5 A
(D) none these

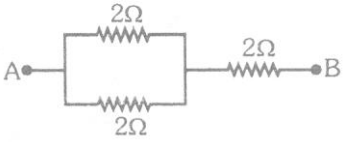
53. In the circuit shown in Fig., the reading of the voltmeter V will be :-
(A) 4 V (B) 2 V (C) 6 V
(D) 3 V

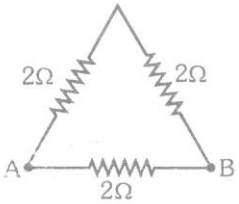
54. Five identical resistance coils are connected in the network as shown in fig. and ht resistance measured between A and B is $1\,\Omega$. Then the individual coils must have a resistances of :-

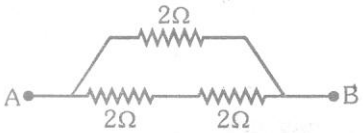


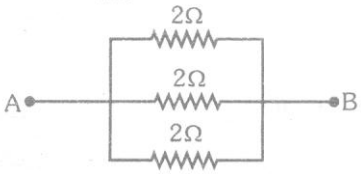
(A) $1\,\Omega$ (B) $\frac{1}{4}\,\Omega$ (C) $\frac{7}{4}\,\Omega$ (D) $\frac{4}{7}\,\Omega$

55. Which of the following networks yields maximum effective resistance between A and B?

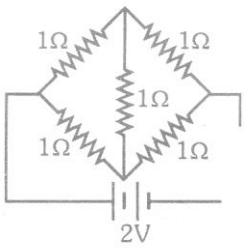
(A) 

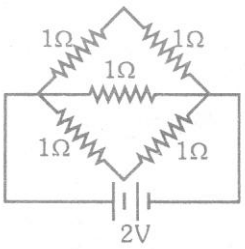
(B) 

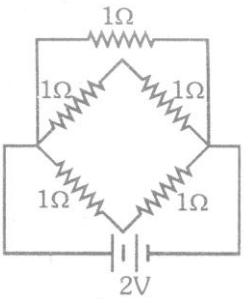
(C) 

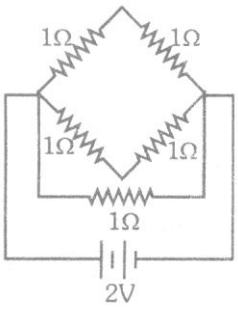
(D) 

56. What of the following network yields minimum current ?

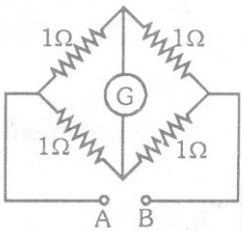
(A) 

(B) 

(C) 

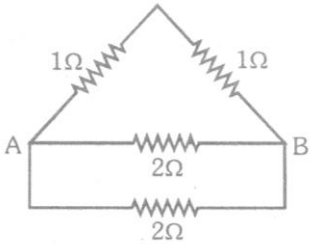
(D) 

57. What is the total resistance between A and B in the given network (Fig.)? (Given the resistance of the galvanometer as $40\,\Omega$)



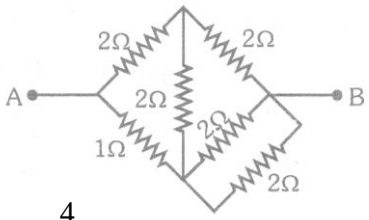
(A) $1\,\Omega$ (B) $2\,\Omega$ (C) $\frac{1}{2}\,\Omega$ (D) None of these

58. What is the resistance between A and B in the given network (Fig.)?



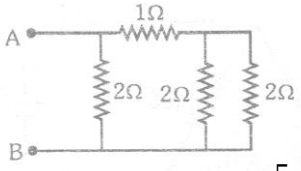
(A) $2\,\Omega$ (B) $2\,\Omega$ (C) $\frac{3}{2}\,\Omega$ (D) $\frac{2}{3}\,\Omega$

59. What is the resistance between A and B in the given network [Fig.] ?



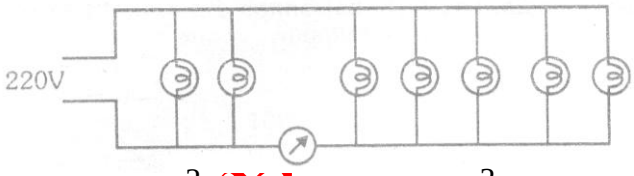
(A) $\frac{3}{4}\,\Omega$ (B) $\frac{4}{3}\,\Omega$ (C)$2\,\Omega$ (D) $\frac{1}{2}\,\Omega$

60. The equivalent resistance between A and B (Fig.) will be :-



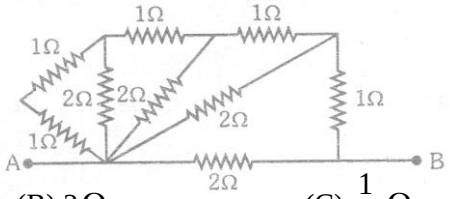
(A) $7\,\Omega$ (B) $2\,\Omega$ (C) $\frac{5}{3}\,\Omega$ (D) $1\,\Omega$

61. Seven identical lamps of resistance $220\,\Omega$ each are connected to a 220 V line as shown in Fig. Then the reading in the ammeter will be :-



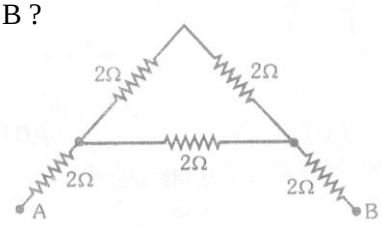
(A) $\frac{1}{10}A$ (B) $\frac{2}{5}A$ (C) $\frac{3}{10}A$ (D) $\frac{1}{2}A\,\Omega$

62. What is the resistance between A and B in the following circuit ?



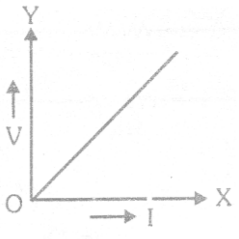
(A) $1\,\Omega$ (B) $2\,\Omega$ (C) $\frac{1}{2}\,\Omega$ (D) $\frac{3}{2}\,\Omega$

63. What is the resistance between A and B ?



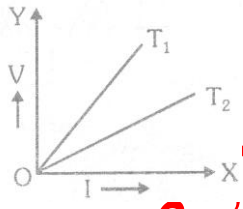
(A) $\frac{3}{4}\,\Omega$ (B) $\frac{4}{3}\,\Omega$ (C) $\frac{16}{3}\,\Omega$ (D) infinity

64. The slope of voltage (V) versus current (I) is called :-



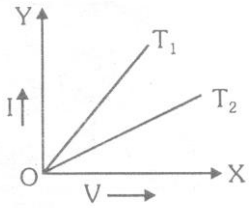
(A) resistance (B) conductance (C) resistivity
(D) conductivity

65. For a metallic conductor, current versus voltage graph is drawn at two different temperatures T_1 and T_2. From the graph it follows:-



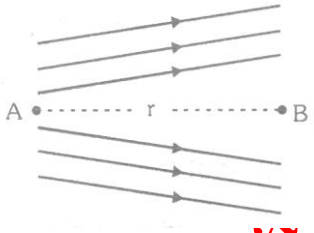
(A) $T_1 = T_2$
(B) $T_1 > T_2$
(C) $T_1 < T_2$
(D) None of these

66. For a metallic conductor, voltage versus current graph is drawn at two different temperatures T_1 and T_2. From the graph follows :-



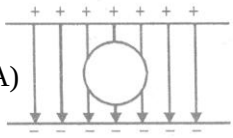
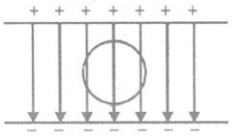
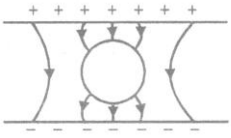
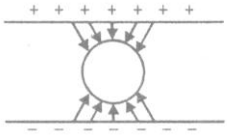
(A) $T_1 = T_2$
(B) $T_1 > T_2$
(C) $T_1 < T_2$
(D) None of these

67. Figure shows the electric lines of force emerging from a charged body. If the electric field at A and B are E_A and E_B respectively and if the displacement between A and B is r, then –

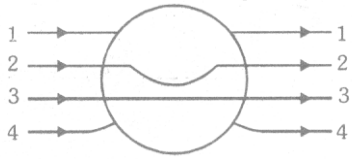


(A) $E_A > E_B$ (B) $E_A < E_B$ (C) $EA = \frac{E_B}{r}$ (D) $E_A = \frac{E_B}{r^2}$

68. An uncharged sphere of metal is placed in between two charged plates as shown. The lines of force look like

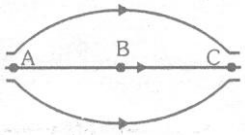
(A)  (B)  (C)  (D) 

69. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in figure as –



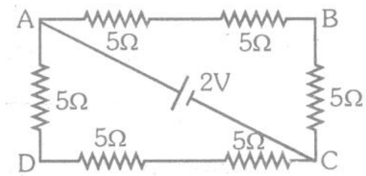
(A) 1 (B) 2 (C) 3 (D) 4

70. The figure shows some of the electric field lines corresponding to an electric field. The figure suggests



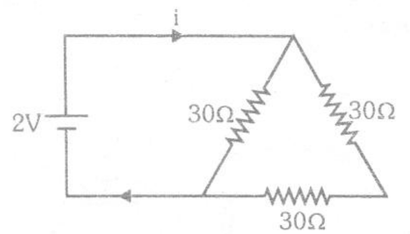
(A) $E_A > E_B > E_C$ (B) $E_A = E_B = E_C$
(C) $E_A = E_C > E_B$ (D) $E_A = E_C < E_B$

71. The potential difference between points A and B of adjoining figure is



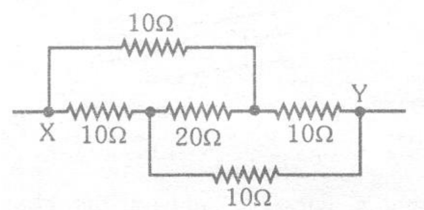
(A) $\frac{2}{3}$ V (B) $\frac{8}{9}$ V (C) $\frac{4}{3}$ V (D) 2 V

72. The current in the adjoining circuit will be –



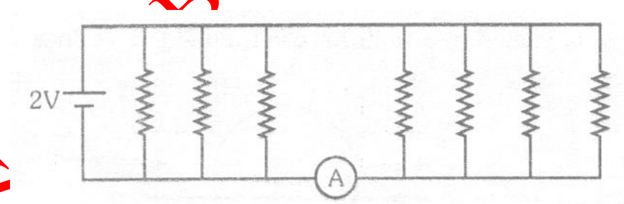
(A) $\frac{1}{45}A$ (B) $\frac{1}{15}A$ (C) $\frac{1}{10}A$ (D) $\frac{1}{5}A$

73. In the circuit shown five resistances are connected. The equivalent resistance between the two points X and Y will be -



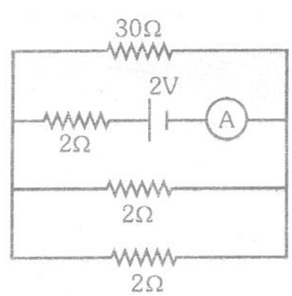
(A) 10 Ω (B) 20 Ω (C) 22 Ω (D) 50 Ω

74. Seven resistances each of $20\,\Omega$ are connected with 2 volt battery as shown in figure. The reading of ammeter will be :-



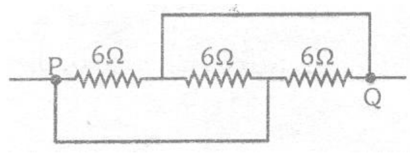
(A) 1 / 10A (B) 3 / 10A (C) 4 / 10A (D) 7 / 10A

75. The reading of the ammeter as per figure shown is :-



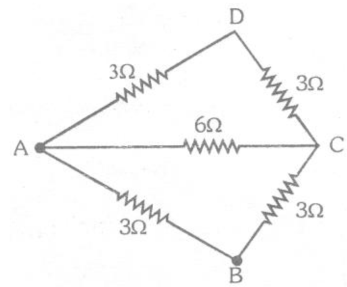
(A) $\frac{1}{8}$ A (B) $\frac{3}{4}$ A (C) $\frac{1}{2}$ A (D) 2A

76. The resultant resistance between P and Q as per the figure shown is :-



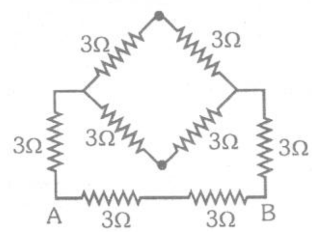
(A) 2Ω (B) 3Ω (C) 6Ω (D) 18Ω

77. The effective resistance between the points A and B in the figure is :-



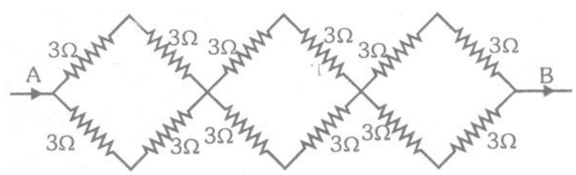
(A) 5Ω (B) 2Ω (C) 3Ω (D) 4Ω

78. Equivalent resistance between A and B will be –



(A) 2Ω (B) 18Ω (C) 6Ω (D) 3.6Ω

79. In the network of resistors shown in the adjoining figure, the equivalent resistance between A and B is :-



(A) 54Ω (B) 18Ω (C) 36Ω (D) 9Ω

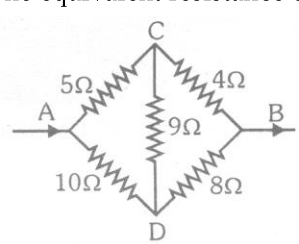
80. Five resistors are connected as shown in the diagram . The equivalent resistance between A and B is –

(A) 6Ω

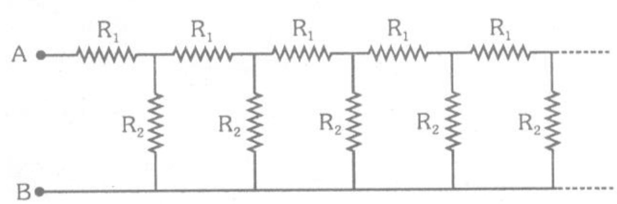
(B) 9Ω

(C) 12Ω

(D) 15Ω



81. An infinite sequence of resistance is shown in the figure. The resultant resistance between A and B will be when R_1 = 1Ω and R_2 = 2Ω :-

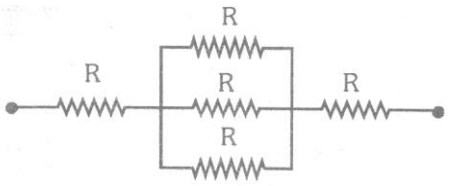


(A) Infinity (B) 1Ω (C) 2Ω (D) 1.5Ω

82. The resistance wires are made of the material having :-

(A) Low specific resistance and low temperature coefficient of resistance .
(B) High specific resistance and low temperature coefficient of resistance .
(C) Low specific resistance and high temperature coefficient of resistance .
(D) High specific resistance and high temperature coefficient of resistance .

83. The resistance between points A and B Fig. is :-



(A) $\frac{7}{3}$ R (B) 3 (C) 5 R (D) $\frac{4}{3}$ R

84. Two wires of same material and same mass have their lengths in the ration 1 : 2. Their electrical resistances are in the ratio

(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 1 : 4

85. A 24 V potential difference is applied across a parallel combination or four 6-ohm resistors. The current in each resistor is

(A) 1A (B) 4 A (C) 16 A (D) 36 A

86. Three resistors in parallel have an effective resistance of 1 ohm. When they are connected in series their resistance is 9 ohm. The resistance of each resistor is :

(A) 4, 4, 1 ohm (B) 6, 2, 1 ohm (C) 3, 3, 3, ohm (D) 2, 3, 4 ohm

87. A letter 'A' is constructed of a uniform wire of resistance 1 ohm per cm. The sides of the letter are 20 cm and the cross piece in the middle is 10 cm long. The resistance between the ends of the legs will be

(A) 32.4 ohm (B) 28.7 ohm (C) 26.7 ohm (D) 24.7 ohm

88. A 12 V car battery has an internal resistance of 0.2 ohm. What P.D. must be applied across its terminals by a battery has in order that the charging current be 5 A ?

(A) 13 V (B) 12 V (C) 11.4 V (D) 10.6 V

89. Four 20 ohm resistors are connected together to form a square. The resistance between opposite corners will be :-

(A) 20 ohm (B) 22 ohm (C) 24 ohm (D) 24.8 ohm

90. The cost of electricity is about 30 Paise per unit for household use. This unit is the same as

(A) ohm (B) ampere (C) volt (D) kilowatt-hour

91. A fuse wire should have

(A) low resistance and low melting point (B) low resistance and high melting point.
(C) high resistance and low melting point. (D) high resistance and high melting point.

92. Two lamps X and Y are connected in series. The lamp X glows lights than Y. Then

(A) the resistance of X is greater than the resistance of Y.
(B) the resistance of X is lesser than the resistance of Y.
(C) the resistance of X is equal to the resistance of Y.
(D) there is no relation between the resistances of X and Y.

93. A one-ohm and half-ohm resistor are connected in parallel across a 3 volt battery. Total energy given out per second is

(A) 27 J (B) 9 J (C) 4.5 J (D) 3 J

94. If it takes 8 minutes to boil a quantity of water electrically, how long will it take to boil the same quantity of water using the same heating coil but with the current doubled?

(A) 32 minutes (B) 16 minutes (C) 4 minutes (D) 2 minutes

95. Two electric lamps each of 100 watts 220 V are connected in series to a supply of 220 volts. The power consumed would be

(A) 100 watts (B) 200 watts (C) 25 watts (D) 50 watts

96. A galvanometer can be converted into' a voltmeter by connecting

(A) A high resistance in series with the galvanometer

(B) A high resistance in parallel with the galvanometer

(C) A low resistance in series with the galvanometer

(D) A low resistance in parallel with the galvanometer

97. A rheostat can be used in an electrical circuit as a

(A) standard resistance (B) potential divider (C) heat controller (D) on-off switch

98. Alternating current rather than direct current is normally used for the transmission of power over long distances because

(A) it can be rectified (B) it is easier to generate

(C) loss of energy can be minimized. (D) no question of polarity arises with electrical equipments.

99. A conductor has a positive charge of 3.2×10^{-7} coulombs.

(A) The conductor has 3.2×10^{7} electrons in excess.

(B) The conductor has 2×10^{26} electrons in excess.

(C) The conductor is deficient with 3.2×10^{7} electrons.

(D) The conductor is deficient with 2×10^{26} electrons

110. Two bulbs of rating 220 V, 50 W and 220 V, 110 V, 110 W are put in series across 220 V mains. Then :

(A) both bulbs will glow equally (B) neither bulb will glow

(C) 100 W bulb will glow brighter (D) 50 W bulb will glow brighter

ANSWERKEY **Exercise -4**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	C	A	B	D	D	C	D	C	B	A	C	D	A	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	D	A	B	B	A	D	B	B	B	B	D	D	D	B	D
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	C	B	B	D	C	D	C	B	D	C	D	A	C	C	C
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	B	C	B	D	A	B	A	A	C	A	A	A	D	B	D
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	D	A	C	A	B	C	A	C	D	C	D	C	A	C	A
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	A	B	D	D	A	C	B	A	C	B	C	C	A	A	D
Que.	91	92	93	94	95	96	97	98	99	100					
Ans.	A	B	A	D	D	A	B	C	D	D					

Important Notes

MAGNETIC EFFECT OF ELECTRIC CURRENT

MAGNETISM

The property due to which a substance attracts pieces of iron, nickel and cobalt towards itself, is called magnetism. A naturally occurring iron ore (black iron oxide $-Fe_2O_3$) having properties of attracting iron pieces, was found in Magnesia in the upper part of Greece. The name magnetism has been taken from the name of that place.
The substance having property of magnetism, is called magnetic substance and body up of a magnetic substance, is called a magnet.
Magnets are found is various shapes and sizes. A bar magnet is a long rectangular bar uniform cross-section, which can attract pieces of iron, steel, cobalt and. Magnet can be natural or artificial.

Poles of a magnet : When a magnet is dipped in iron filings, then maximum filings stick to its ends and almost no filings stick to its centre. It means that in magnets, centers of attraction are located near the ends only. These centers of attraction near the ends of a magnet, are called poles. Since a magnet has two poles hence it is also called magnetic dipole.
When this magnet is freely suspended, its two ends point in north south direction. The pole near the end pointing towards North (north-seeking end) is called North pole. The pole near the end pointing towards South (south-seeking end) is called South pole.
The magnetic poles exert forces on each other. Like poles repel each other, i.e, a north pole will repel another north pole or a south pole will repel another south pole. Unlike poles attract each other, i.e, a south pole will attract a north pole and vice versa .

Magnetic field of earth . Earth behaves as a huge magnet (or a giant solenoid). The source of this huge magnetism is given as the molten charged metallic giving rise to a current flowing inside the core of the earth. This core has a radius of about 3500 km (Earth's radius is 6400 km). Its strength is of the order of one gauss. Shape of the earth's magnetic field resembles with that of a bar magnet of length one fifth of earth's diameter buried at its centre.

Now it is believed that earth's magnetism is due to the magnetic effect of current which is flowing in the molten core at the centre of earth. Hence, earth is a huge electromagnet.

MAGNETIC FIELD AND MAGNETIC FIELD LINES (MAGNETIC LINES OF FORCE)

MAGNETIC FIELD

It is the space around a magnetic pole or a magnet in which its effect is experienced by another magnetic pole or magnet. Magnetic field is a quantity which has both direction and magnitude.

MAGNETIC LINES OF FORCE

A magnetic line of force is a line, straight or curved, in the magnetic field tangent to which at any point gives the direction of the magnetic field at that point.

OR

A line such that the tangent a any point on it gives the direction of the magnetic field at that point is called a magnetic field or magnetic line of force.

A free unit north pole (test pole) will move along the magnetic line of force in direction of the field if it is free to do so. Direction of the magnetic line of force at any point is the direction of the force acting on unit (north) pole (unit magnetic pole) when placed at that point. Since a free unit north pole (test pole) will move away from a north (N) pole, magnetic lines of force have outward direction [Fig. (a)]. Since the free unit north pole will move towards a south (S) pole, magnetic lines of

force have inward direction [Fig. (b)]. A small magnetic compass when moved along the lines of force always sets itself parallel to the line of force.

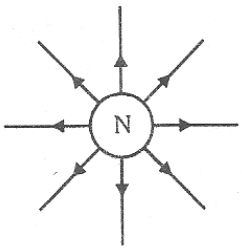


Fig.(a) due to north pole

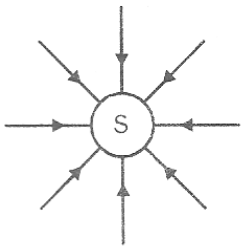


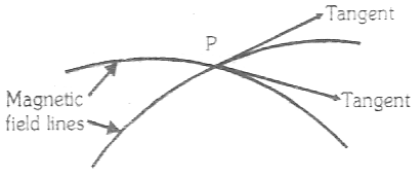
Fig.(b) due to south pole

Magnetic lines of force have the following properties :

1. All field lines are closed curves. They come out of magnet from the side of the north pole and go into it on the side of the south pole. i.e. They start from a north (positive) pole and end at a south (negative) pole. They continue inside the magnet too. Inside the magnet the direction of field lines is from its south pole to its north pole.

2. They are always normal to the surface of the magnet at every point .

3. Two lines of force do not intersect each other. If they intersect at a point, it would mean compass needle placed at the point of intersection would point towards two directions at that point which is not possible .



4. The field lines are close together near the poles and spread out away from them. The field is stronger where the field lines are more closely spaced. So the field is stronger near the poles than at other point.

5. The number of magnetic lines of force passing normally per unit area about a point, gives the intensity of the magnetic field at that point.

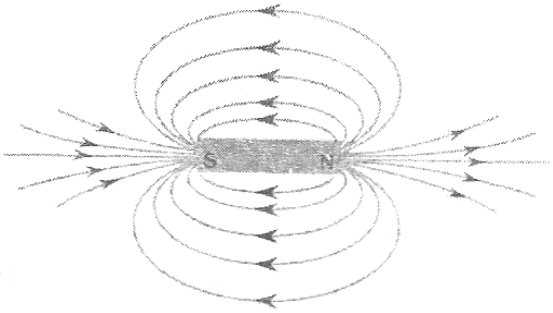
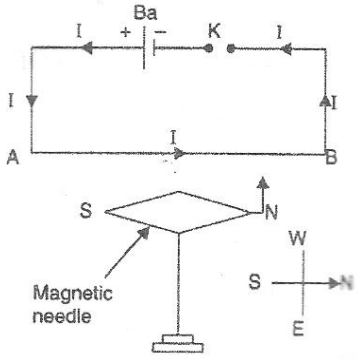


Fig. Magnetic field lines around a bar magnet

MAGNETIC FIELD DUE TO ELECTRIC CURRENT (OERSTED'S EXPERIMENT)



Hans Christian Oersted (1820) was the first to discover the magnetic effect of current, He found that if a compass needle is placed near a current carrying wire, needle gets deflected. He said that as compass needle is a tiny magnet and can be deflected only by some other magnetic field, hence a current carrying wire produces some magnetic field around itself. magnetic effect of current is also known as electromagnetism. Magnetic effect of current is very useful in electric motors, generators, telephone etc.

Experimental arrangement used by Oersted is shown in Fig. A straight wire AB is connected to a battery Ba and key K. The wire is held horizontally north-south over a magnetic needle. In this arrangement, when key is closed, current flows in the wire in the direction as shown in Fig. The north pole of the needle gets deflected towards west. When key is taken out and current in the wire becomes zero and the needle returns back to its initial position (S – N) . This shows that a magnetic field is associated with an electric current.

When direction of current in the wire is reversed, direction of deflection of needle also gets reversed. If direction of current is kept same and the wire is put under the needle then, direction of deflection of needle again gets reversed.

Amount of deflection depends on the distance of the needle from the current carrying wire.

But we know that a magnetic needle is deflected by a magnetic field only. Hence we can conclude that current flowing in a wire gives rise to some magnetic field around it.

Position of the wire (conductor) carrying the current, direction of current and direction of deflection of the needle can be related by SNOW rule given below.

If current flows in the conductor from South towards North, with conductor kept over the needle, then North pole of the needle will be deflected towards West.

MAGNETIC FIELD DUE TO CURRENT CARRYING STRAIGHT

CONDUCTOR (WIRE) AND CURRENT CARRYING CIRCULAR COIL

Case 1. When Current carrying conductor is Straight, Magnetic Field is Circular.

It means that when the current flows in a straight wire, the magnetic field produced has circular lines of force surrounding the wire, having their centres at the wire as shown in Fig. This can be shown by sprinkling iron filings on the cardboard C When current flows through the conductor, iron filings get magnetized and now if the cardboard C is tapped gently iron filings arrange themselves in circles around the wire. Hence we can say that magnetic lines of force around a straight current carrying conductor are circular. The plane of circular lines is perpendicular to the length of the wire. Their direction is marked by arrows.

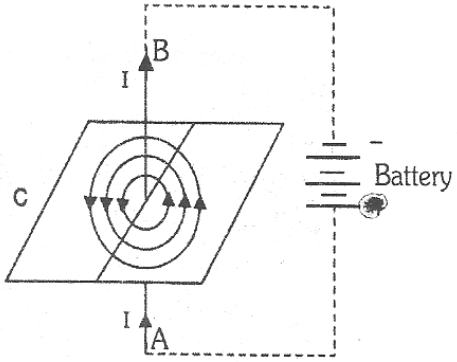


Fig.Current straight, magnetic field circular

When current I flows through a straight wire, the magnetic field strength (B) at a small distance r from it is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

From the above expression we see that magnitude of magnetic produced by a straight carrying wire at a given point is :

(i) directly proportional to the quantity of current flowing through the wire.

(ii) inversely proportional to the distance of point from the wire. Thus, if current is more, magnetic field will be stronger and vice versa.

DIRECTION OF MAGNETIC FIELD

The direction of the magnetic lines of force is related with the direction of the current by the **right hand thumb rule.**

This rule states that :

Curl the four fingers of the right hand on the palm, keeping the thumb stretched out at right angles. The thumb is straight and the fingers are circular, then

(i) **If thumb represents the direction of the current in the straight wire then curling of fingers represents the direction of the circular magnetic lines of force. (Fig. (a))**

(ii) **If curled fingers represents the direction of the current in circular wire then thumb represents the direction of the straight magnetic lines of force. (Fig. (b))**

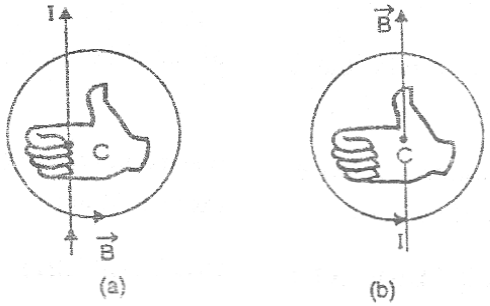


Fig. Right hand thumb rule for direction of magnetic field.

Maxwell's right hand thumb rule is also known as Maxwell cork screw rule . Cork screw is a device consisting of a handle and a spiral metal rod as shown in fig. (c).

When the cork screw is moved in the direction of current then the direction in which its handle is turned gives us the direction of the magnetic field. In short we can say that **when current flows vertically upward then direction of magnetic field produced is anticlockwise and when current flows vertically downward then direction of magnetic field is clockwise.**

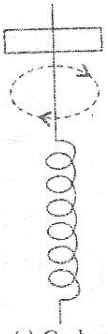


Fig. (c) Cork screw

Case 2. When Current is Circular, Magnetic Field is Straight.

When the current flows in a circular wire (coil), the magnetic field produced has straight lines of force near the centre of the coil, as shown in Fig. The parallel lines are lines are in a plane perpendicular to the plane of the coil. Their direction is marked by the arrows.

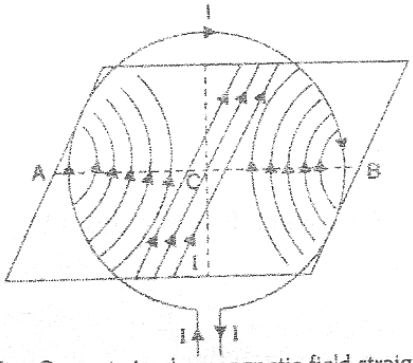
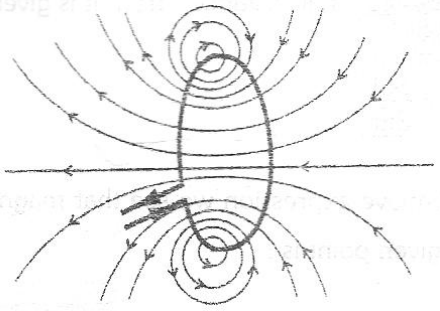


Fig. Current circular, magnetic field straight



Straight lines at the middle of the coil are due to the fact to the that each small segment of the circular coil is surrounded by the magnetic lines of force. At the centre of the coil, all lines of force add to each other resulting in the increase in strength.

It is found that magnitude of the magnetic field (B) at the centre of a circular coil carrying currents is directly proportional to amount of current flowing through the wire (I), inversely proportional to the radius of the coil (r) and directly proportional to the number of turns in the coil.

Mathematically,

$$B = \frac{\mu_0 nI}{2r}$$ Here, n is number of turns of the coil.

MAGNETIC FIELD DUE TO CURRENT IN A SOLENOID

A coil of many turns of wire wrapped in the shape of a cylinder is called a solenoid i.e., a solenoid is a long cylindrical coil wound over a hollow cylinder (non conducting). It is shown in Fig.(a).

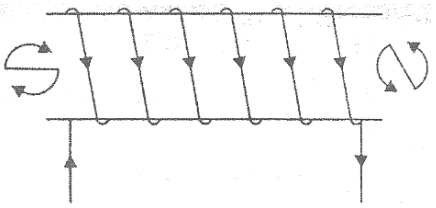


fig. (a) Solenoid carrying a current and polarity of its face

A solenoid differs from a circular coil in that the length of the solenoid is much greater than its diameter.

A solenoid behaves as if a large number of coils have been put one behind the other on the same axis over a length i.e., magnetic field produced by a current carrying solenoid is similar to the magnetic field produce by a bar magnet .

Current in solenoid produces magnetic field in each turn, which becomes **additive**.

Polarity of magnetic field exists only at the ends of the solenoid, as shown in Fig.(a).

One end of the coil acts as a north pole while the other end acts like a south pole.

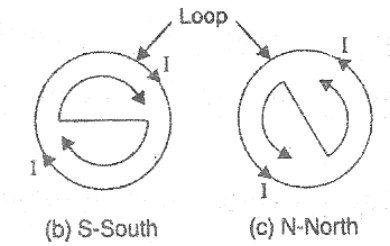


Fig.(b & c) Direction of current in loop.

Inside the solenoid the magnetic field is uniform (same at all points). It is represented by parallel and straight field lines. Magnetic field outside the solenoid is non-uniform.

Magnetic lines of force inside the solenoid are from south pole to north pole, while outside the solenoid these lines are from north pole to south pole.

RULE FOR POLARITY AT THE ENDS : If at any end, the current in the coil (or loop) is clockwise, the face of the coil towards the observer behaves as a south pole. [Fig. (b)]

If at any end, the current in the coil is anticlockwise, the face of the coil towards the observer behaves as a north pole [Fig. (c)]

Fig. (d) shows the magnetic field around a current carrying solenoid.

(i) Number of turns per unit length (n) of the solenoid and $B \propto n$

(ii) Strength of the current (I) in the solenoid and $B \propto I$

(iii) Nature (relative permeability, μ_r) of the core material of the solenoid and $B \propto \mu_r$

or $B \propto \mu_r \; nI$ or $\boxed{B = \mu_0 \mu_r nI}$

where μ_0 represents permeability of free space.

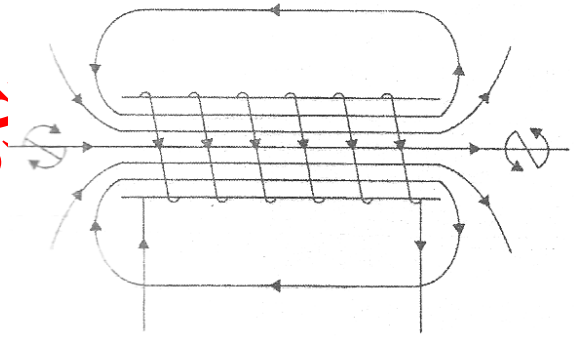


Fig. (d) Magnetic field around a solenoid : the-field is straight inside it.

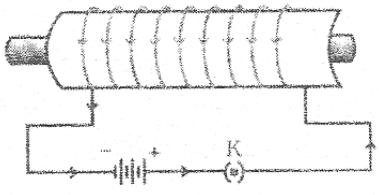
Note : Permeability is a magnetic property a material related to magnetism.

Que. How are the fields of a current-carrying solenoid and coil different ?

Sol. The field inside a current carrying solenoid is quite uniform for the most part. It only decreases near the ends. The field of a current carrying coil is not uniform. It changes with distance from the centre.

ELECTROMAGNET

Strong magnetic field which can be produced due to the flow of current can be used to magnetize a piece of magnetic material like steel rod, soft iron etc, when placed inside the coil. Magnet so produced is called electromagnet. Thus we can say that an electromagnet consists of many turns of insulated copper wire wound over a soft iron core or horse-shoe-magnet .



Strength of the electromagnet depends on :

(i) Number of turns in the coil : If number of turns in the coil are more, electromagnet will be strong.

(ii) Current flowing in the coil : Strength of electromagnet is directly proportional to the current flowing through the coil, and

(iii) Length of air gap between the pole : Strength of the electromagnet is inversely proportional to the distance between its poles.

Electromagnets are used to extract bullets from the body, to lift big machines and in electronic devices. Large values of μ_r n and I can produce very strong magnetic field inside the solenoid.

Electromagnets are better than permanent magnets as their strength can be controlled by changing the number of turns in the coil or by changing the current flowing through the coil.

Permanent magnets are usually made of some alloys of steel of because permanent magnets made from alloys are very strong than those made from ordinary steel.

Difference between an Electromagnet and a Permanent magnet

	Electromagnet	Permanent magnet
1	It behaves as a magnet as long as electric current passes through the solenoid surrounding it. It is demagnetized when electric current stops passing through the solenoid. Thus, electromagnet is a temporary magnet.	It cannot be demagnetized easily
2	Electromagnet gives a strong magnetic field and the strength of the magnetic field produced by these electromagnet can be increased or decreased by increasing or decreasing electric current through the solenoid.	Magnetic field of a permanent magnet is weak. The strength of magnetic field of the permanent magnet cannot be changed.
3	The polarity (i.e. North and South poles) of an electromagnet can be reversed by reversing the direction of electric current through the solenoid.	The polarity of a permanent magnet cannot be reversed.

DO YOU KNOW

- Hard steel, alnico (an alloy of aluminum, nickel, cobalt, iron and Nipermag (an alloy of aluminum, iron and titanium) are used to make permanent magnets.
- Soft iron is used to make electromagnets.
- A steel rod can be made a permanent magnet if it is placed inside a solenoid carrying direct current.
- Permanent magnets are used in loudspeakers, galvanometers, voltmeters, ammeters and speedometers.
- Permanent magnet can be demagnetized by heating it.

FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN A MAGNETIC FIELD AND FLEMING'S LEFT HAND RULE

A current carrying conductor produces a magnetic field around it. When it is placed in a magnetic field, the two magnetic fields interact with each other and a net force acts on the conductor.

If the conductor of length ℓ carrying current I is lying inside a magnetic field of intensity B and is making an angle θ with it, then force acting on the conductor is given by

$$F = I\ell B\sin\theta$$

If the conductor is lying perpendicular to the magnetic field, then $\theta = 90^0$ [sin θ = 1] and the force becomes F = I ℓ B.

This force acts in a direction which is perpendicular to the plane containing the conductor and the magnetic field (Fig.) is maximum.

If the conductor is lying parallel to the magnetic field, then θ = 0^0 (sin θ = 0) and the torque becomes zero and is minimum.

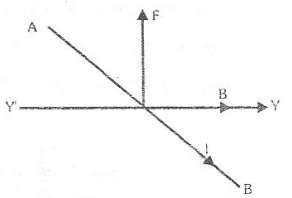


Fig. Force on a conductor

FLEMING'S LEFT HAND RULE

Direction of the force action on a current carrying conductor when placed in a magnetic field is given by Fleming's left hand rule, which states that :

If the forefinger, second finger and thumb of the left hand are stretched at right angles to each other, with the forefinger in the direction of the field and the second finger in the direction of the current then the thumb indicates the direction of the force.

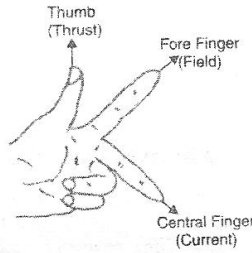


Fig.Fleming's left hand rule

FACTORS ON WHICH THE FORCE ACTING ON THE CURRENT CARRYING CONDUCTOR DEPENDS

The force acting on a current carrying conductor is placed in the magnetic field depends upon :

(i) **The strength of the magnetic field :** If the conductor is placed in a strong magnetic field , it experiences a large force. That is, F $\propto$ B (strength of magnetic field)

(ii) **The strength of the electric current :** If large current flows through the conductor placed in the magnetic field , it experiences a large force. That F $\propto$ I.

(iii) **The length of the conductor :** A long conductor experiences a greater force than the short conductor, when placed in the magnetic field. That is , F $\propto$ ℓ .

That is $F \propto BI\ell$

or $F = kBI\ell$

If k = 1 , $F = BI\ell$

then $B = \dfrac{F}{I\ell}$

If I = 1 amp. and ℓ = 1m then B = F

Thus, magnetic field strength (B) is defined as the force acting per unit current per unit length of a conductor placed perpendicular to the direction of the magnetic field.

SI unit of magnetic field strength is Tesla.

$$1 Tesla = \frac{1 Newton}{1 Ampere \times 1 metre} = 1 N\ A^{-1} m^{-1}$$

ELECTROMAGNETIC INDUCTION

The phenomenon in which an electric current is induced in a circuit because of a changing magnetic field is called electromagnetic induction.

When a straight metallic wire is moved up and down in a magnetic field between the two poles of a horse shoe magnet then magnetic flux (number of magnetic lines of force) linked with the wire changes and an electric current is produced in the wire. A galvanometer connected across the two ends of the wire shows some deflection indicating that some current is produced. This current lasts only as long as the change in the flux continues.

This phenomenon is called electromagnetic induction. In short we can say that electromagnetic induction means production of electricity from magnetism. The electric current produced, is called induced current. The e.m.f. which produces this current , is called induced e.m.f.

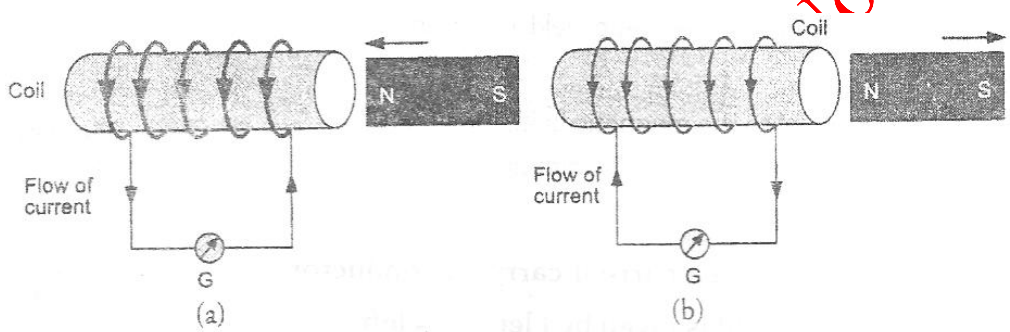


Fig. – Electromagnetic induction

FACTORS OF WHICH INDUCED CURRENT DEPENDS

The value of induced current depends upon :

(i) The number of turns in the coil : If coil has large number of turns, then large induced current is produced in the closed coil.

(ii) The strength of the magnet : A strong magnet moved towards or away from the closed coil produced a large induced current.

(iii) The speed with which the magnet moves towards the coil : If the magnet moves very quickly, then large induced current is produced in the closed coil.

FARADAY'S KISCOVERY OF ELECTROMAGNETIC INDUCTION

Michael Faraday in 1831, discovered phenomenon of electromagnetic induction. His experimental set up is shown in fig.

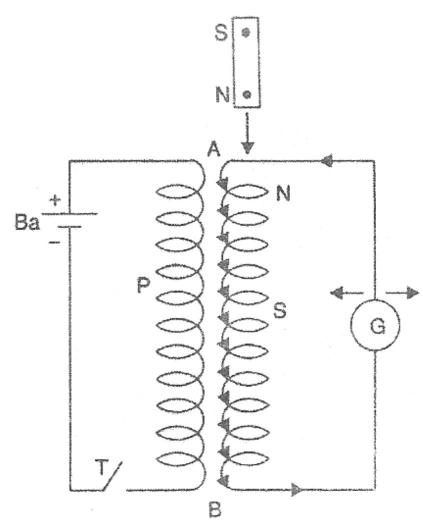


Fig.Faraday's Experiment

When north pole of a magnet was held above end A of secondary coil S, no deflection was seen in galvanometer G. As the pole was moved towards the coil, galvanometer showed some deflection to the right. When magnet was left inside coil, galvanometer deflection become zero. When he took the magnet out of the coil, deflection in the galvanometer to the left side was observed. Hence current flow

only when magnet was in motion and not when it was at rest near the coil or inside the coil.

Faraday performed a similar experiment by putting a primary coil P near coil AB. He completed primary circuit with a battery Ba and tapping key T.
So long as key was open, no deflection was seen in the galvanometer G. As the key was pressed and primary circuit was closed, galvanometer showed some deflection to the right. Deflection became zero when key was kept pressed. When key was left to open, deflection in the galvanometer to the left side was observed.
The closing and opening of circuit changed the current in primary coil, which produced changing magnetic flux in the primary. This changing magnetic flux become linked with secondary and produces induced current (just as done by moving magnet), in the secondary.

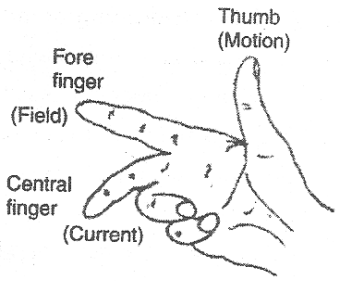


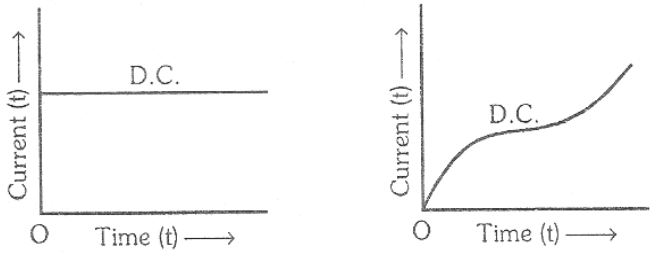
Fig.Fleming's Right Hand Rule

Direction of the induced current is given by **Fleming Right Hand rule which states that** Stretch the thumb, the fore finger and central finger of right hand in such a way that they are mutually perpendicular to each other. Then, if fore finger represents direction of Field and thumb represents direction of Motion of the conductor, then Central finger will represent the direction of induced Current (fig.).

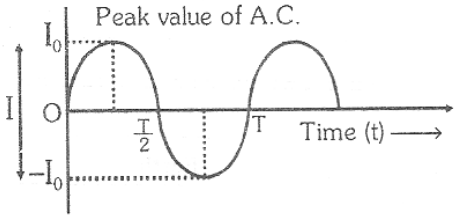
DIRECT AND ALTERNATING CURRENT

DIRECT CURRENT (D.C.) : A current which has a constant magnitude and same direction, is called a direct current

Current due to a cell or a battery is a direct current .



ALTERNATING CURRENT (A.C.) : A current which changes in magnitude and direction at regular intervals of time is called an alternating current .
The frequency of household supply of a.c. in India is 50 Hz. This means, a.c. completes 50 cycles in one second. Thus, a.c. changes direction after every $\frac{1}{100}$ second. In other words, a.c. used in India changes direction 100 times in one second.
Current changes direction after each rotation of the coil.



Frequency : Frequency of A.C. is the number of cycles per second completed by the current. One cycle is completed when the A.C. rises from zero to maximum positive then back to zero and then the maximum negative and zero again .

ADVANTAGES AND DISADVANTAGES OF A.C. OVER D.C.

A.C. has following merits (advantages) over D.C. :

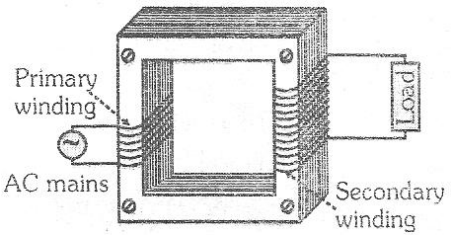
1. A.C. can be transmitted over long distances without much loss of energy.
2. A.C. can be produced easily and cheaply than D.C.
3. A.C. voltage can be transformed to any desired value with the help of a transformer.
4. Transmission of A.C. at "high-voltage" and "low-current' reduces line losses.
5. A.C. motors or other A.C. appliances are easier to operate.
6. A.C. can easily by converted in D.C. when required.

A.C. has following demerits (disadvantages) over D.C. :

1. A.C. attracts a person who touches its line whereas D.C. gives a repelling shock.
2. A.C. gives a huge and sudden shock which becomes fatal.
3. A.C. is conducted over the surface of a conductor (skin effect). It increases effective resistance of the conductor.
4. Commercial generators do not produce pure A.C.
5. In certain applications like electroplating, battery charging etc. only D.C. is required.
6. A.C. is more dangers than D.C.

DO YOU KNOW

Transformer is a device which changes small a.c. potential difference to a large a.c. potential difference. Transformer consists of two coils of wire wound around a core of soft iron. One coil is known as primary coil and the other is known as secondary coil as shown in figure.



Transformer is used to transmit a.c. to distance places through conductors (called transmission lines) without much loss of electric energy or power.

DOMESTIC ELECTRIC CIRCUITS

SUPPLY FROM POWER STATIONS : Electricity is generated at power station. In our homes, we receive the supply of electric power either supplied through overhead poles or underground cables using two thick aluminum wires.

MAIN BOARD : It is provided outside the building under a covered poach (verandah or poach). It contains the meter (energy-meter) and the main switch. (Fig.)

From the street electric pole, a thick rubber insulated cord reaches the main board. It contains two thick copper or aluminum wires, one covered with red and the other covered with black (or brown) plastic covering.

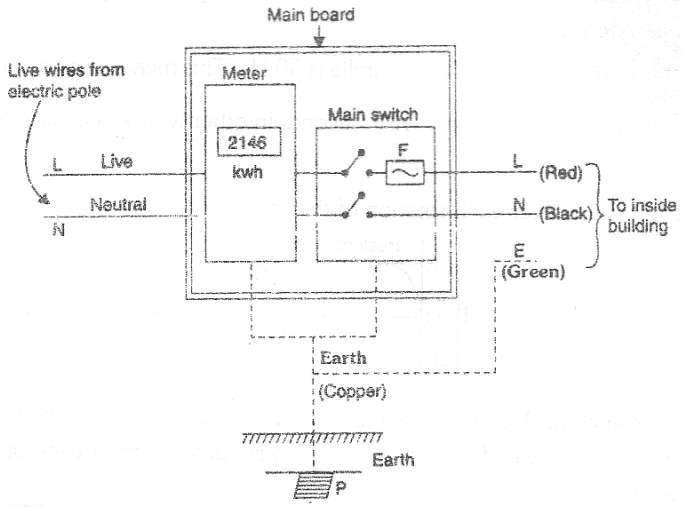


Fig . Main board outside the building

They form the lifeline wire (L) and neutral line wire (N) respectively.

Live line wire has a potential or 220 V whereas the neutral wire has zero potential (with respect to the earth). They enter the main board and are connected to the meter.

Wiring ahead is provided by the house owner himself. These wires are also red black plastic covered. From the meter the wires enter the main switch. In the main switch, a fuse F is provided in the path of live wire.

From outside the main switch, the wires become free to be used inside the building as required.

A third wire is a thick bare wire of copper. Called earth wire E. It is connected to an earth connection which consists of a thick copper plate P buried deep inside the moist earth.

INSIDE THE BUILDING : It is a well known fact that inside the house, connections to all the devices are made in parallel, each having independent switch and fuse (if necessary). Thus, whenever some fault occurs in circuit of one particular device in one room, devices in other rooms do not suffer.

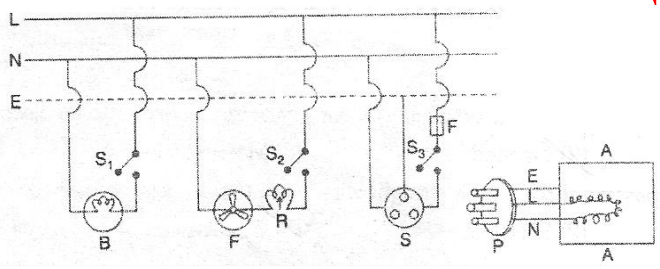


Fig. Circuit inside the building .

As shown in Fig. , connection to low power devices like bulb B and fan F are made with lines N and L only, putting switch in line L. For devices of more power and with whom the body remains in contact (like electric press or refrigerator), we use connections through a 3 pin plug socket (shoe) system .

A three pin plug P and three pin socket S are shown in Fig. . The three points of the socket are connected to the three lines as shown in the diagram. A fuse F is also introduced to avoid damage to the appliance.

The three pin plug uses a three wire cord which has three plastic wires inside a single rubber insulating cover. The wires are colored : red, black (or brown) and green respectively to serve as an extension of live, neutral and earth wires for the appliance. The three wire are connected to there holes in the socket as shown in Fig. When the plug is inserted in the socket, proper lines get connected to the appliance.

ELECTRIC FUSE : An electric fuse is a safety device used to save the electrical appliances like electric bulbs ,electric tubes, T.V. etc. from burning when large current flows in the circuit. Electric fuse is a wire made of copper or aluminum or tin-lead alloy. The melting point of the material of which the electric fuse wire is made should be low.

Suppose a fuse is not connected in the path of a live wire of the circuit. In such case, the circuit may be over heated if the current in the circuit exceeds the safe limit. There is a change of short circuiting of the circuit which causes the fire. So to avoid short circuiting of the circuit, a fuse must be but in the path of the circuit .

FUNCTION OF EARTH WIRE : Due to the long use, some covered wires inside the appliance may become bare and may make contact with metallic body of the appliance. In such a case the appliance gives a shock if not earthed. The earth wire keeps the potential of the of appliance zero and shock is avoided

USE OF SWITCH : All electrical appliances are provided with separate switches. All switches are connected with live wire as well as with neutral wire. When we switch off an appliance, then it gets disconnected from the live wire. Now if one touches the metallic body of the appliance there is no danger of electric shock. But, if connections to the switch are in such a way that on switching off the appliance, neutral wire gets disconnected but not live wire, then is danger of electric shock.

SHORT CIRCUIT

Fig. (a) normal eclectic circuit (b) Short circuit

When the live wire and the neutral wire come into direct contact, this occurs when the insulation of wires is damaged or there is a fault in the appliance. In such a situation, the current in the circuit abruptly increases. This is called short-circuiting. When short circuiting occurs, the resistance of the circuit becomes very small and hence huge amount of current flows through it. Large amount of current in the circuit produces large amount of heat which raises the temperature of circuit to very high value. As a result of this, the circuit catches fire.

OVERLOADING: means flow of large amount of current in the circuit beyond the permissible value of current. It occurs when many electrical appliances of high power rating like geyser, heater, refrigerator, motor etc. are connected in a single socket or in a single circuit. High current flowing in the circuit due to overloading causes fire.

EXERCISE # 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

Choose the Correct One

1. The fact that magnetic field is produced around a wire carrying, was discovered by

(1) Faraday (2) Oersted (3) Maxwell (4) Joule.

2. When the current is passing through the straight wire then, the associated magnetic field is

(1) Straight (2) Elliptical (3) Circular (4) Parabolic.

3. When current is circular, the associated magnetic field is

(1) Straight (2) Elliptical (3) Circular (4) Paralytic

4. When current flows clockwise in a loop, the polarity of its face is

(1) East (2) South (3) West (4) North

5. When current flows anticlockwise in a loop the magnetic polarity of the face is

(1) East (2) South (3) West (4) North

6. For a solenoid carrying a current I and having n turns per unit length, wrapped on a core of permeability μ, the correct expression for magnetic field intensity (2) is

(1) $B = \frac{\mu_0}{\mu} nI$ (2) $B = \frac{\mu_0 \mu I}{\mu}$ (3) $B = \mu_0 \mu nI$ (4) $B = \frac{\mu_0 \mu n}{I}$

7. Magnets having temporary magnetism are called

(1) Electromagnets (2) Bar magnets (3) Circular magnets (4) Horse-shoe magnets.

8. Direction of force acting on a current carrying conductor kept in magnetic field is given by

(1) Fleming's right hand rule (2) Fleming's left hand rule

(3) Lenz's rule (4) Faraday's rule

9. The electric device which works on the phenomenon of force on a current carrying conductor in a magnetic field is

(1) Generator (2) Accelerator (3) Motor (4) Transformer.

11. Electromagnetic induction was discovered by

(1) Oersted (2) Maxwell (3) Thomson (4) Faraday.

11. Direction of induced current produced by motion of a conductor in a magnetic field is given by

(1) Fleming's right hand rule (2) Fleming's left hand rule

(3) Lenz's rule (4) Faraday's rule

12. In domestic electric circuits, fuse must be placed in series with

(1) Earth wire (2) Neutral wire

(3) Live wire (4) Any or the three wires.

13. Elective and main switch is contained in a main board fitted usually

(1) At street electric pole (2) At main gate of building
(3) In verandah or parch (4) In bed or study room.

14. High powered electrical appliances are earthed to
(1) Avoid shock (2) Avoid wastage
(3) Make the appliance look beautiful (4) Reduce the bill.

15. In a three pin socket (shoe) the bigger hole is connected to
(1) Any wire (2) Live wire (3) Neutral wire (4) Earth wire

16. Due to overloading, the current in circuit becomes
(1) Less (2) More (3) Zero (4) Not definite .

17. Coming of live and neutral wire in direct contact causes
(1) Sort-circuiting (2) Over-loading (3) No damage (4) Unknown effect.

18. Melting point of material of a fuse wire must be
(1) Moderate (2) High (3) Low (4) Infinite.

19. A high powered electric appliance has used inferior wires and is not earthed. It is source of
(1) No concern (2) Less concern (3) Moderate concern (4) Hazard.

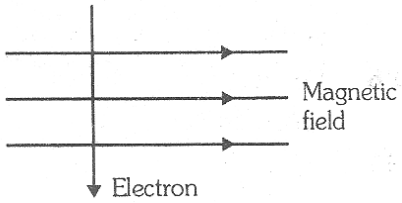
20. Which of the following correctly describes the magnetic field near a long straight wire ?
(1) The field consists of straight lines perpendicular to the wire
(2) The field consists of straight lines parallel to the wire.
(3) The field consists of radial lines originating from the wire.
(4) The field consists of concentric circles scented on the wire.

21. The phenomenon of electromagnetic induction is
(1) The process of charging a body
(2) The process of generating magnetic field due to a current passing through a coil
(3) Producing induced current in a coil by relative motion between a magnet and the coil
(4) The process of rotating a coil of an electric motor.

22. The device used for producing electric current is called a
(1) Generator (2) Galvanometer (3) Ammeter (4) Motor

23. At the time of short-circuit, the current in the circuit
(1) Reduces substantially (2) Does not change
(3) Increases heavily (4) Vary continuously

24. An electron enters a magnetic field at right angles to it as shown in figure. The direction of force action on the electron will be
(1) To the right
(2) To the left
(3) Out of the page
(4) Into the page



25. Which of the following property of a proton can change while it moves freely in a magnetic field ? (The may be more then one correct answers)
(1) Mass (2) Speed (3) Velocity (4) Momentum.

26. A positively-charged particle (alpha particle) projected towards west is deflected towards north by a magnetic filed. The direction of magnetic field is
(1) Towards south (2) Towards east (3) Downward (4) Upward.

27. A rectangular coil of copper wires is rotated in a magnetic field. The direction of the induced current changes once in each

(1) Two revolutions (2) One revolution (3) Half revolution (4) One-fourth revolution.

28. The magnetic field inside a long straight solenoid carrying current

(1) Is zero (2) Decreases as we move towards its end

(3) Increases as we move towards its end (4) Is the same at all points.

29. A magnetic field exerts no force on :

(1) An electric charge moving perpendicular to its direction (2) an unmagnified iron bar

(3) a stationary electric charge (4) a magnet

30. Which of the following instruments works by electromagnetic induction ?

(1) telephone receiver (2) simple motor

(3) dynamo (4) moving coil galvanometer.

31. The use of fuses in electric power-lines is :

(1) to keep main voltage constant

(2) to open the line permanently when there is an overload

(3) just to show an indication when there is an overload

(4) as a switch to use in an emergency

32. When the main switch of the house circuit is put off, it disconnects the :

(1) live wire (2) neutral wire (3) earth wire (4) live and neutral wires

33. When a fuse is rated at 8 A, it means :

(1) it will not work if current is less than 8 A (2) it has a resistance of 8 ohm

(3) it will work only if current is 8 A (4) it will burn if current exceeds 8 A.

34. Acceding to international Convention of colour coding of wires used in household electrical circuits :

(1) live is red, neutral is black and earth is green

(2) live is red, neutral is green and earth is black

(3) live is brown, neutral is blue and earth is black

(4) live is brown, neutral is light blue and earth is green

35. The magnetic field lines inside a current carrying solenoid are :

(1) along the axis and parallel to each other

(2) perpendicular to the axis and equidistant from each other

(3) circular and they do not intersect each other

(4) circular at the ends but they are parallel to the axis inside the solenoid

36. Potential difference between a live wire and the neutral wire is :

(1) 150 V (2) 210 V (3) 200 V (4) 220 V

37. Which of the following statements is incorrect regarding magnetic field lines :

(1) The direction of magnetic field at a point is taken to be the direction in which the north pole of a magnetic compass needle pints

(2) Magnetic field lines are closed cruces

(3) If magnetic field lines are parallel and equidistant, they represent zero strength

(4) Relative strength of magnetic field is shown by the degree of closeness of the field lines

38. Magnetic field liens determine :

(1) The shape of the magnetic field

(2) Only the direction of the magnetic field

(3) Only the relative strength of the magnetic field

(4) Both the direction and the relative strength of the magnetic field

39. The magnetic field near a long straight wire is described by :

(1) Straight field lines parallel to the wire (2) Straight field lines perpendicular to the wire

(3) Concentric circles centered on the wire (4) Radial field lines starting from the wire

40. The phenomenon of electromagnetic induction is :

(1) The process of charging a sphere.

(2) The process of producing magnetic field in a coil

(3) The process of producing induced current in a coil whenever there is a relative motion between the coil and the magnet

(4) The process of producing cooling effect.

41. Potential difference between a live wire and the neutral wire is :

(1) 200 volt (2) 150 volt (3) 210 volt (4) 220 volt

42. A rectangular coil of copper wires is rotated in a magnetic field. The direction of the induced current changes once in each :

(1) One revolution (2) One-fourth revolution (3) Half revolution (4) Two revolutions

43. Magnetic field inside a long solenoid carrying current is :

(1) Same at all pints (2) Minimum in the middle

(3) More at the ends than at the centre (4) Found to increase from one end to the other.

44. At the time of short circuit, the current in the circuit :

(1) Vary continuously (2) Reduces considerably (3) Increases heavily (4) does not change.

45. The frequency of direct current is :

(1) Zero (2) 50 Hz (3) 60 Hz (4) 100 Hz

46. The frequency of household supply of a.c. in India is :

(1) Zero (2) 50 Hz (3) 60 Hz (4) 100 Hz

47. The most important safety device method used for protecting electrical appliances from short circuiting or overloading is:

(1) Earthling (2) Use of stabilizers (3) Use of electric meter (4) Use of fuse

48. Current flowing in conductor A is 2A and current flowing in conductor B is 4A. The ratio of magnetic field produced around conductor A to the magnetic field produced around conductor B at a distance 10 cm from both the conductors is :

(1) 2 : 1 (2) 1 : 2 (3) 4 : 1 (4) $\sqrt{2}$: 1

49. Current flowing in conductors A and B is same, what is the ratio of the magnetic field produced around the conductor A at a distance of 5 cm from the conductor to the magnetic field produced around the conductor B at a distance of 2 cm from this conductor is

(1) 0.04 (2) 0.4 (3) 4.0 (4) 10

50. Same amount of current flows in the same direction . long the two parallel conductors separated by a small distance :

(1) Both conductors attract each other

(2) Both conductors repel each other

(3) Conductors neither attract each other nor repel each other

(4) Both conductors rotates about their axis

51. When an electric current flows through a long solenoid, magnetic field is set up in and around the solenoid :

(1) Magnetic field inside the solenoid is non-uniform and weak
(2) Magnetic field outside the solenoid is uniform and strong
(3) Magnetic field inside the solenoid increases as we move towards the ends of the solenoid
(4) Magnetic field of solenoid resembles the magnetic field of the bar magnet .

52. Magnetic field produced at the centre of a current carrying circular wire is :
(1) Directly proportional to the square of the radius of the circular wire
(2) Directly proportional to the radius of the circular wire
(3) Inversely proportional to the square of the radius of the circular wire
(4) Inversely proportional to the radius of the circular wire.

53. The direction of the magnetic field at a pint P above the current carrying wire is
(1) Down the page (2) Up the page (3) Into the page (4) Out of the page

54. Which of the following properties of a proton can change while t moves freely in a magnetic field ?
(1) Mass (2) Speed (3) Velocity (4) Momentum

55. A positively charged particle say an alpha particle projected towards west is deflected towards north by a magnetic field . The direction of the magnetic field is :
(1) Upward (2) Downward (3) Towards south (4) Towards east

56. Magnitude of magnetic field intensity at a point around a current carrying conductor is B. If the strength of current in the conductor becomes double, then the magnitude of magnetic field intensity at the point around the conductor is :
(1) $\frac{B}{2}$ (2) B (3) $\frac{B}{4}$ (4) 2B

57. Two parallel conductor carrying current in the same directions
(1) Repel each other
(2) Attract each other
(3) Sometimes attract and sometimes repel each other
(4) None of these

58. Two parallel conductor carrying current in the opposite directions
(1) Repel each other (2) Attract each other (3) Sometimes attract and sometimes repel each other
(4) None of these

59. Force acting on a stationary charge Q in the magnetic field B is :
(1) BQυ (2) BQ/υ (3) Bυ/Q (4) Zero

60. If the plug of the key is taken out (i.e. the circuit is made open) and magnetic field lines are drawn over the horizontal plane white sheet of paper, the lines are :
(1) Concentric circles (2) Elliptical in shape
(3) Straight lines parallel to each other (4) Parabolic in shape

61. When current passes through a long straight solenoid, N and S poles are created at the two ends of the solenoid. Which of the following statutes is incorrect ?
(1) The field lines inside the solenoid are in the form of straight lines indication that magnetic field is same at all the points inside the solenoid.
(2) The pattern of the magnetic field liens associated with the solenoid is different from the patter of the magnetic field lines around a bar magnet .
(3) N-and S-poles exchange portions when the direction of the current through the solenoid is reversed.
(4) Magnetic field produced inside the solenoid can be used to magnetize a bar of magnetic material like soft iron, when placed inside the solenoid.

In these type of Questions, two statements are given. Choose the current option by following directions given below: (Q. 80 to 83)

(1) Statement I is correct and Statement II is correct explanation of the statement I.

(2) Statement I is correct but Statement II is not the correct explanation of the statement I.

(3) Statement I is true but statement II is not true.

(4) Statement I is not true but statement II is true.

62. **Statement I :** A soft iron bar placed inside a solenoid carrying current is magnetized.
Statement II : Magnetic field inside a long solenoid carrying current is non-uniform.
(1) A (2) B (3) C (4) D

63. **Statement I :** The strength of magnetic field of a permanent magnet decreases with the increase n temperature.
Statement II : A permanent magnet can be demagnetized by heating it.
(1) A (2) B (3) C (4) D

64. **Statement I :** A charged particle moving parallel to the direction of of magnetic field experiences a force.
(1) A (2) B (3) C (4) D

65. A current carrying conductor placed perpendicular to the magnetic field experience a force. The displacement of this conductor in the magnetic field can be increased by.
(1) Decreasing the magnetic field (2) Decreasing the current in the conductor
(3) Increasing the magnetic field (4) Decreasing the length of the conductor.

66. A conductor of length 50 cm , carrying current of 0.1 A, when placed perpendicular to direction of magnetic field 0.2 T experience force :
(1) 1.0 N (2) 0.1 N (3) 0.01 N (4) 0.01 N

67. A charged particle having charge 3.2×10^{-19} C is traveling with a speed of 1.0×10^{6} ms^{-1} . When it passes perpendicular to the magnetic field 0.4 T, then the force experienced by it is :
(1) 12.8×10^{-13} N (2) 1.28×10^{-13} N
(3) 19.2×10^{-15} N (4) 1.92×10^{-15} N

68. A charged particle having charge 1.6×10^{-19} C travels with a speed of 3.2×10^{6} ms^{-1} in a direction parallel to the direction of magnetic field 0.04 T. The force experienced by the particle is :
(1) 2.0×10^{-14} N (2) 0.2×10^{-14} N
(3) Zero (4) 4.0×10^{-14} N

69. A magnetic compass is placed near a current carrying wire. The deflection of the needle of the magnetic compass increases. It shown that :
(1) Current in the wire is decreasing
(2) Current in the wire is increasing
(3) Current in the wire has nothing to do with the deflection of the needle of the magnetic compass
(4) Magnetic compass has been disturbed by some one.

70. A charged particle moving in a magnetic field experience a maximum force, when it moves at right angle to the direction of magnetic field. However, a student observes that a charged particle moving in the region of a magnetic field experiences no force and continues to move in a straight line. The observation of the student is correct only if :
(1) Charged particle moves at an angle of 30^0 with the magnetic field
(2) Charged particle moves at an angle of 60^0 with the magnetic field
(3) Charged particle either moves parallel or anti-parallel to the magnetic field
(4) Charged particle can never move in a straight line in the magnetic field

71. A soft iron rod can be made an electro-magnet. Which of the following procedures would be adopted to test that the soft iron rod had become a magnet ?

(1) Place the iron rod near a bar magnet and observe if attracts iron pins

(2) Place the iron near a current carrying wire and observe if it attracts iron pins

(3) Place the iron rod inside a current carrying solenoid and observe if it attracts iron pins

(4) None of these.

72. Current can be made to flow through a coil without connecting the coil with a battery. Which of the following procedures would be most suited to test this fact ?

(1) Connect a galvanometer across the ends of the coil and observe if galvanometer shows a deflection, when a bar magnet is placed near the one end of the coil

(2) Connect a galvanometer across the ends of the coil and observe if galvanometer shows a deflection, when a bar magnet is placed inside the coil.

(3) Connect the galvanometer across the ends of a coil and observe if galvanometer shows a deflection, when a bar magnet is moved towards or away from the coil and along the axis of the coil .

(4) Connect the galvanometer across the ends of the coil and observe if galvanometer shows a deflection, when a bar magnet is moved at right to the axis of the coil.

73. Two coil A and B placed close to each other. If current in the coil A is changed, some current will be induced in the coil B. This is because of electromagnetic induction. In this statement

(1) Inference is correct but reasoning is not correct

(2) Inference is incorrect but reason is correct

(3) Inference as well as reasoning are correct

(4) Neither inference nor reasoning is correct.

ANSWER – KEY															
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	3	1	2	4	3	1	2	3	4	1	3	3	1	4
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	2	1	3	4	4	3	1	3	4	3	4	3	4	3	3
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	2	4	4	4	1	4	3	4	3	3	4	3	1	3	1
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	4	2	2	1	4	4	4	3	4	4	2	1	4	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73		
Ans.	2	3	1	4	3	3	2	2	2	3	3	3	3		

EXERCISE # 2 (FOR SCHOOL / BORARD EXAMS)

FILL IN THE BLANKS

1. The phenomenon of production of magnetic field around a current carrying conductor is called .....effect of current.

2. The rule which relates direction of deflection of magnetic needle with direction of field is called .....rule

3. When a wire is wrapped into many close turns over a cylindrical core, it forms a ……..

4. To have north polarity at a face, the current in loop must flow in ……..direction.

5. To have south polarity at a face, the current in loop must flow in ……direction.

6. In electromagnets, magnetism is ……

7. In electromagnetic induction, motion of a……in a fixed coil produces electric current .

8. An electric current having a constant magnitude and direction, is called a …….current .

9. An electric current having a changing magnitude and direction, is called an ……current.

10. In domestic electric circuit

    (i) Fuse is connected in ……wire

    (ii) The colour of live wire is……..

    (iii) The colour of neutral wire is ……

    (iv) The colour of earth wire is ……..

    (v) Shock is avoided by ……the appliance.

11. For earthling of an appliance we use……pin plug and socket (shoe).

12. A large current flows in the circuit due to ……..and ……

13. To avoid damage due to large current we must use a……

WRITE TRUE OR FALSE IN THE FOLLOWING STATEMENTS

1. When some electric current flows through a wire, magnetic field is produced around the wire.

2. Electric current in a straight wire produces straight magnetic field .

3. A wire wrapped into many close turns over a cylindrical core forms a solenoid.

4. A coil with large number of turns forms a solenoid.

5. Direct current has a constant magnitude and direction.

6. Alternating current has a changing magnitude with no change in direction.

7. Fuse wire saves the electrical appliances from being damaged by excess current .

8. Fuse wire is used in parallel with the appliance.

9. Earthing the high powered electrical appliances saves them from electric shocks.

10. Earth wire has red colour.

11. The field at the centre of a long circular coil carrying current will be parallel straight lines.

12. A wire with a green insulation is usually the live wire.

13. Like magnetic poles attract each other and unlike magnetic poles repel each other .

14. Magnetic field lines determine only the direction of magnetic field but not the relative strength of the magnetic field.

15. Magnetic field lines are crowded in a region of strong magnetic field.

16. Two magnetic field lines can cross each other.

17. Magnetic field lines around a current carrying wires are concentric circles with their centres on the wire.

18. Two parallel wires carrying current in the same direction repel each other .

19. Two parallel wires carrying current in opposite direction attract each other.

20. The field at the centre of long circular coil carrying current will be parallel straight lines.

21. The magnetic field inside a current carrying solenoid is uniform magnetic field.

22. Direction of force experienced by a current carrying wire placed perpendicular to the magnetic field is determines by Fleming's left hand rule.

23. A static charge placed in a magnetic field experiences a force.

24. Potential difference is set up across the ends of a wire when it is placed in the uniform magnetic field.

25. The malting point of the material of which electric fuse wire is made should be low.

26. During short circuiting, the resistance of the circuit is very small.

27. A wire with a black colour is the earth wire of an electric supply.

28. A wire with a green colour is the live wire of an electric supply.

VERY SHORT ANSWER TYPE QUESTION

1. What is meant by magnetic field ?

2. How is the direction of magnetic field at a point determined ?

3. Define a magnetic field lien

4. At what place of the magnet are the magnetic field liens denser ?

5. How is the strength of the magnetic field around a wire related to the strength of the electric current flowing in the wire ?

6. How can it be shown that magnetic field exists around a wire through which direct current is passing ?

7. An alternating current has frequency of 50 Hz. How many times does it change its direction in one second ?

OR

How many times does AC used in India change direction in one second ?

8. What is the nature of magnetic field lines around a current carrying wire ?

9. State the rule used to determine the direction of magnetic field produced around a straight conductor carrying current.

10. Name the rule used to determine the direction of magnetic field due to a current carrying circular loop of a wire .

11. Write and expression for the force acting on a conductor of length (ℓ) carrying current **(I)** and placed perpendicular to the magnetic field **(B)**.

12. State the rule to determine the direction of a force experienced by a current-carrying straight conductor placed in a magnetic field which is perpendicular.

13. What is a solenoid ?

14. Write one application of magnetic field of current carrying solenoid.

15. Define electromagnetic induction.

16. When is the force experienced by a current-carrying conductor placed in a magnetic field largest ?

17. What is the frequency of direct current (D.C) ?

18. What is the meaning of the term “frequency” of an alternating current ? What is its value in India ?

19. What do you understand by the current rating of an electric fuse ?

20. Name the material generally used to make electric fuse.

21. Why is an alternating current considered to be advantageous over direct current for long range transmission of electric energy ?

22. State the rule of determine the direction of current induced in a coil due to its rotation in a magnetic field .

23. How is induced current in a secondary coil related to current in a primary coil ?

ANSWER THE FOLLOWING QUESTIONS

1. Why does a compass needle get deflected when brought near a bar magnet ?

2. Draw magnetic field lines around a bar magnet .

3. List the properties of magnetic lines of force.

4. Why don’t two magnetic lines of force intersect each other ?

5. Consider a circular loop of wire lying in the plane of the table. Let the current pass through the loop clockwise. Apply the right-hand rule to find out the direction of the magnetic field inside and outside the loop.

6. The magnetic field in a given region is uniform. Draw a diagram to represent it .

7. State Fleming’s left-hand rule.

8. Explain different ways to induce current in a coil.

9. Name some sources of direct current .

10. Which sources produce alternating current ?

11. An electric oven of 2 kW power rating is operated in a domestic electric circuit (220 V) that has a current rating of 5A. What result do you expect? Explain.

12. Name two safety measures commonly used in electric circuits and appliances.

13. What precaution should be taken to avoid the overloading of domestic electric circuits ?

14. In household circuits, is a fuse wire connected in series or in parallel ?

15. In which wire, (in A.C. circuit) , switch is introduced to operate the light ?

16. List three coerces of magnetic fields.

17. How does a solenoid behave like a magnet ? Can you determine the north and south poles of current carrying solenoid with a help of bar magnet ? Explain.

18. When is the force experienced by a current carrying conductor placed in a magnetic field is largest ?

19. Think that you are sitting in a chamber with your back to one wall. An electron beam, moving horizontally from back wall towards the front wall, is deflected by a strong magnetic field to your right side. What is the direction of magnetic field ?

20. A coil of insulated copper wire is connected to a galvanometer. What will happen if a bar magnet is
(i) pushed into the coil, (ii) withdrawn from inside the coil, (iii) held stationary inside the coil ?

21. Two circular coils A and B placed c closed to each other. If the current in the coil A is changed, will some current be induced in the coil B ? Give reason.

22. State the rule to determine the direction of a

(i) magnetic field produced around a straight conductor carrying current.

(ii) force experienced by a current-carrying straight conductor placed in a magnetic field which is perpendicular to it, and

(iii) current induced in a coil due to its rotation in a magnetic field.

23. What is the function of an earth wire ? Why is it necessary to earth metallic appliances ?

24. A current through a horizontal power line flows in east to west direction. What is the direction of magnetic field at a point directly below it and at a point directly above it ?

ANSWER KEY

EXERCISE # 2

FILL IN THE BLANKS :

1.	Magnetic	**2.**	Right hand thumb rule	**3.**	Solenoid	**4.**	Anticlockwise
5.	Clockwise	**6.**	Temporary	**7.**	Magnet	**8.**	Direct
9.	Alternating						

10.(i) Series **(ii)** Red **(iii)** Black **(iv)** Green **(v)** Earthy

11. Three **12.** Overloading, Short circuiting **13.** Fuse

TRUE & FALSE

1.	T	**2.**	F	**3.**	T	**4.**	T	**5.**	T
6.	F	**7.**	T	**8.**	F	**9.**	T	**10.**	F
11.	T	**12.**	F	**13.**	F	**14.**	F	**15.**	T
16.	F	**17.**	T	**18.**	F	**19.**	F	**20.**	T
21.	T	**22.**	T	**23.**	F	**24.**	F	**25.**	T
26.	T	**27.**	F	**28.**	F				

DEFINITION

Light is form of energy which enables up to see objects which emit or reflect light.
Light is a type of (form of) energy which can produce sensation in our eyes. So we can experience the sensation of vision.
If is travel in straight line in form of particles and waves. With the help of light we see all colours of nature. Our eyes are mostly sensitive for yellow colour and least sensitive for violet and red colour. Due to this reason commercial vehicle's are painted with yellow colour, sodium lamps are used in road lights.

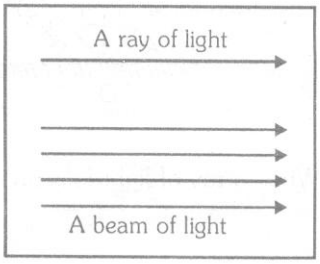
PROPERTIES OF LIGHT

Light energy propagates (travels) via two processes.

(i) The particles of the medium carry energy from one point of the medium to another.
(ii) The particles transmit energy to the neighboring particles and in this way energy propagates in the form of a wave.
(iii) It propagates in straight line.
(iv) It's velocity in vacuum is maximum whose value is 3×10^8 m/sec. (297489978 m/s)
(v) Light does not need a material medium to travel that is it can travel through a vacuum.
(vi) It exhibits the phenomena of reflection, interference, diffraction, polarization and double refection.

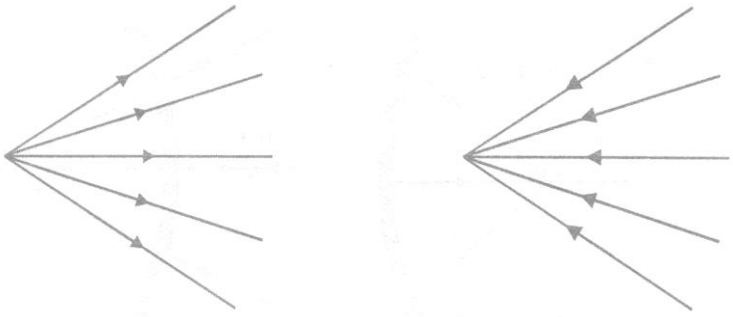
RAY OF LIGHT

A straight line show the direction moment of light is called ray of light.



BEAM OF LIGHT

A bunch of light rays or bundle of rays at a point is called beam of light,



A divergent beam of light A convergent beam of light

HOW WE SEE ?

When a light ray is falling (strike) on the surface of any object which reflect and reached to our eyes. Due to this our eyes feel a sensation then we see the object.

REFLECTING OF LIGHT

When rays of light falls on any object it return back in the same medium from the surface this phenomenon is called reflection of light. Due to reflection of light we can see all the nature.

INCIDENT RAY

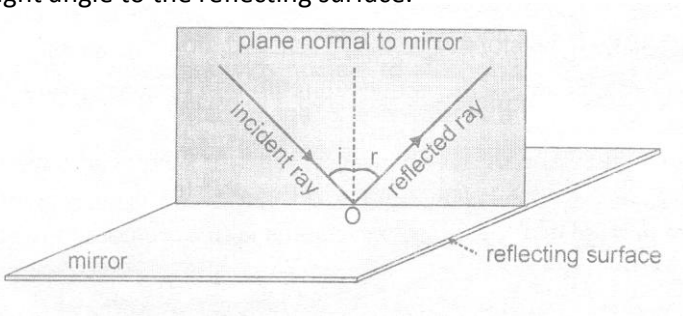
The ray light which falls on a polished surface (or a mirror) is called the incident ray of light.

REFLECTED RAY

The ray of light which gets reflected from a polished surface (or a mirror) is called the reflected ray of light.

NORMAL

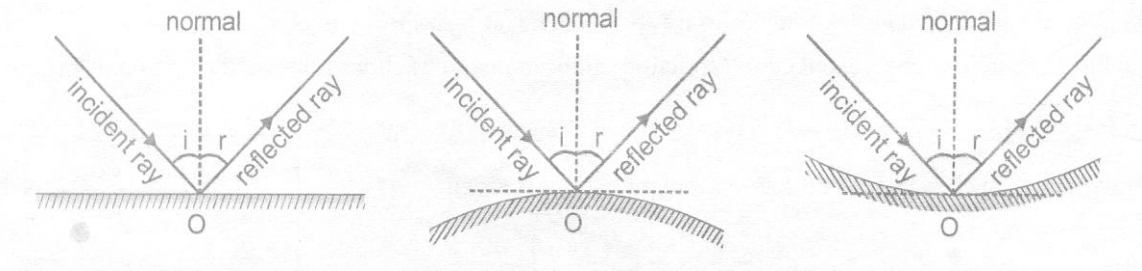
The normal is a line at right angle to the reflecting surface.



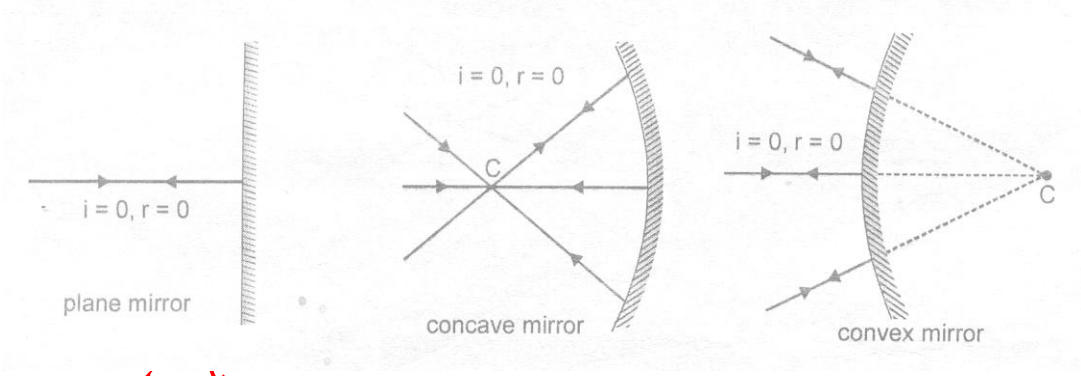
LAWS OF REFLECTION

(i) The incident ray, the reflected ray and the normal to the surface at the point of incidence all lie in the same plane.

(ii) The angle of incidence ($\angle\ i$) is always equal to the angle to reflection ($\angle\ r$) i.e. $\angle\ i = \angle\ r$



▸▸ When a ray of light falls on a mirror normally or at right angle it gets reflected back along the same path.

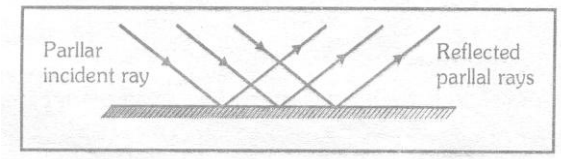


Depending on the nature of the reflecting surface there are two types of reflection :-

(i) Regular (specular) reflection (ii) Irregular (diffused) reflection

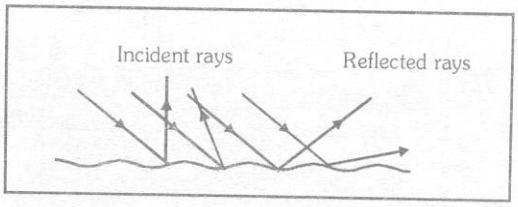
REGULAR REFECTION

When parallel light rays fall on smooth plane surface like mirror, if all rays of light are reflected parallel along a definite direction. Then this kind of reflection is called regular reflection.



IRREGULAR REFLECTION (DIFFUSED REFLECTION)

When parallel light rays fall on a rough surface all the rays of light are reflected in all possible (Different) direction this is called diffused or irregular reflection.

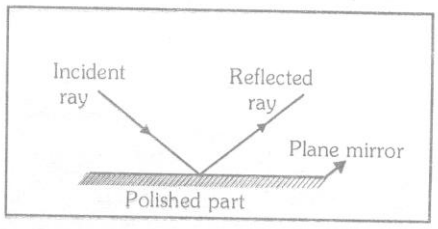


MIRROR

A smooth, highly polished reflecting surface is called a mirror.
When a glass plate is polished on one sided with reflecting material such silver or nickel then is becomes a mirror.

From the reflecting surface of mirror there are two types of mirror.
(i) Plane mirror (ii) Spherical or curved mirror

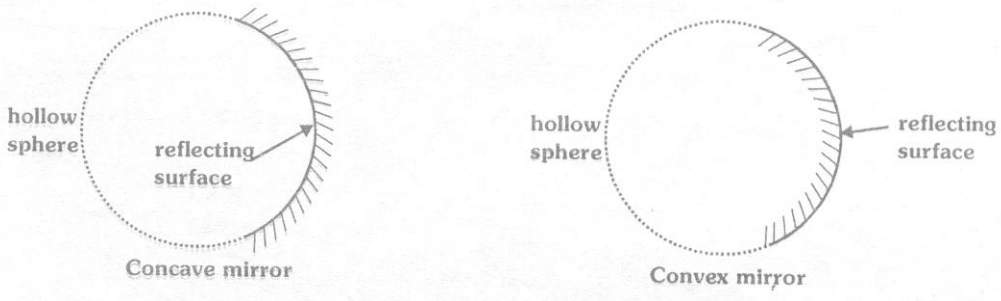
(i) **Plane mirror :** A highly polished plane surface is called a plane mirror or if a flat (totally plane) surface of a glass plate is polished one side of reflecting material is called plane mirror.



(ii) **Spherical mirror :** A mirror whose polished, reflection surface is a part of hollow sphere of glass is called a spherical mirror. For a spherical mirror, one of the two curved surfaces is coated with a thin layer of silver followed by a coating of red lead oxide paint. Thus one side of the spherical mirror is made opaque and the other side acts as a reflecting surface.

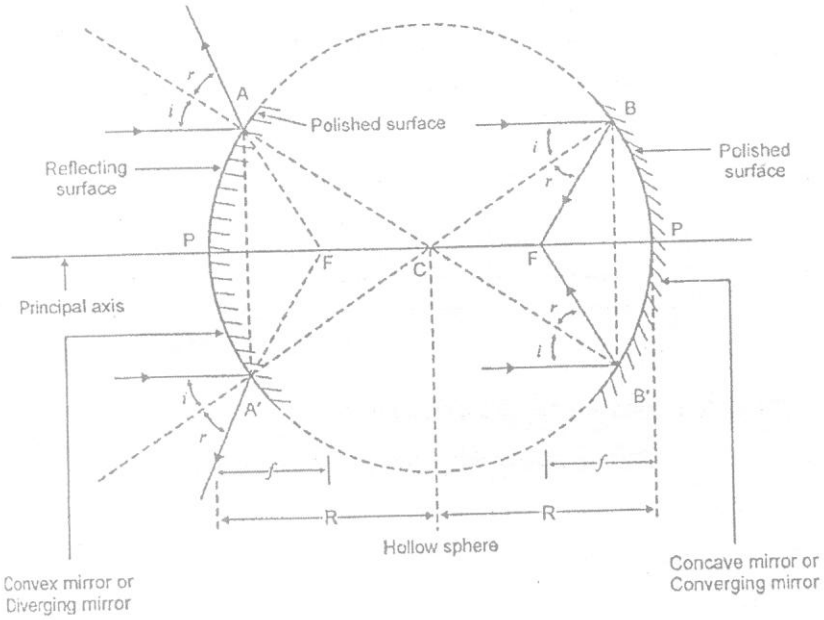
For the polishing side there are two type of spherical mirror.

(A) Convex mirror (B) Concave mirror



(A) **Concave (Converging) mirror :** A spherical mirror whose inner hollow surface is the reflecting surface .
(B) **Convex (diverging) mirror :** A spherical mirror whose outer bulging out surface is the reflecting surface.

TERMINOLOGY FOR SPHERICAL MIRRORS



(a) **Aperture :** The effective width of a spherical mirror from which reflection can take place is called its aperture AA' & BB'.

(b) **Pole (Vertex) :** The centre of a spherical mirror is called its pole it is denoted by letter P.

(c) **Centre of curvature :** The centre of the hollow sphere of which the spherical mirror is a part is called centre of curvature. It is denoted by letter C.

(d) **Radius of curvature :** The radius of the hollow sphere of which the spherical mirror is a part called the radius of curvature (R).

(e) **Principal axis :** The straight line passing through the centre of curvature C and the pole P of the spherical mirror.

(f) **Normal :** The normal at any point of the spherical mirror is the straight line obtained by joining that point with the centre of curvature C of the mirror.

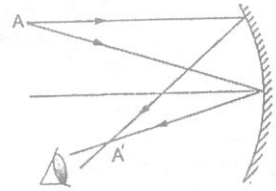
(g) **Principal focus or focus :** The point on the principal axis where all the rays coming from infinity (parallel rays) after reflection either actually meets or appears to meet is called the focus (or focal point) of the mirror. It is denoted by letter F.

(h) **Focal length :-** The distance between the pole (P) and the focus (F) is called focal length (f) and $f = \frac{R}{2}$

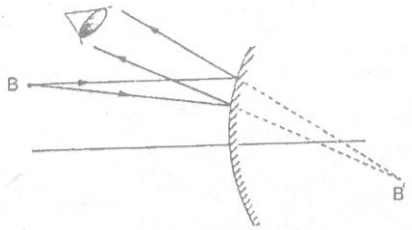
(i) **Focal plane :-** An imaginary plane passing through the focus and at right angles to the principal axis.

(j) **Real image :-** When the rays of light after getting reflected from a mirror (or after getting refracted from a lens) – actually meet at a point, a real image is formed. A real image can be obtained on a screen.

(k) **Virtual image :-** When the rays of light after getting reflected from a mirror (or after getting refracted from a lens) appear to meet at a point, a virtual image is formed. Such an image can only be been through a mirror (or a lens) but cannot be obtained on a screen.



(a) Real image

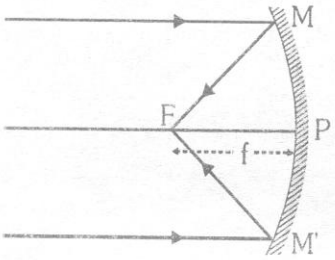


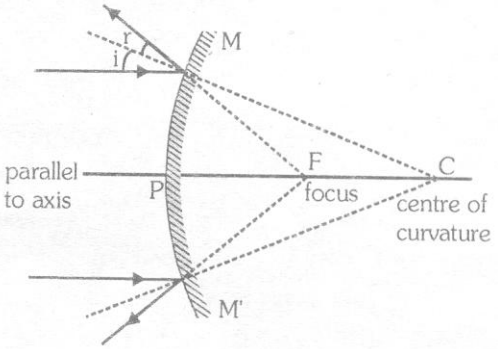
(b) Virtual image

RULES OF IMAGE FORMATION FROM THE SPHERICAL MIRROR

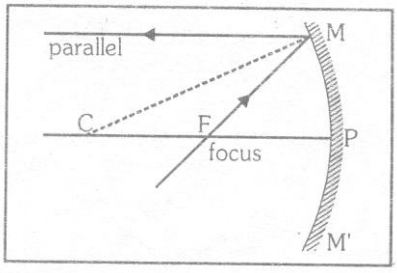
The rules of reflection from the spherical mirror are based of incident and reflection angel.

(i) A ray parallel to principal axis after reflection from the mirror passes or appears to pass through its focus by definition of focus.

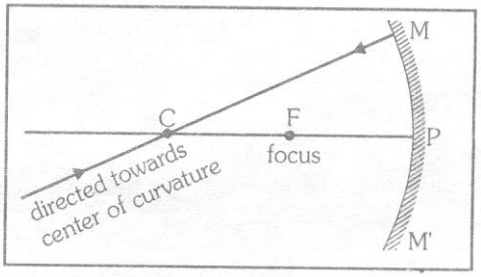




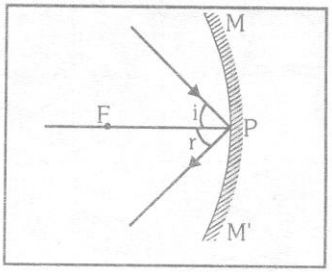
(ii) A ray passing through or directed towards focus after reflection from the mirror it will become parallel to the principal axis.



(iii) A ray passing through or directed towards centre of curvature after reflection from mirror, retraces its path. As for it $\angle$ i = 0 and so $\angle$ r = 0.



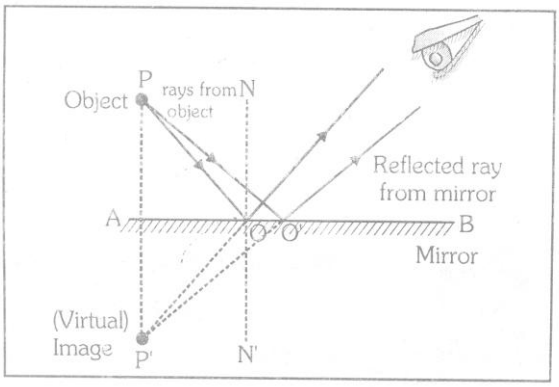
(iv) Incident and reflected rays at the pole of a mirror are symmetrical about the principal axis.



Difference between Real and Imaginary image

S.No	Real image	Virtual image
(1)	When reflected or refracted light rays actually intersect at a point.	When reflected or refracted light rays do not actually intersect at a point but appear to meet at a point
(2)	It can be obtained on a screen.	It can not be obtained on a screen.
(3)	It is always inverted.	It is always erect.
(4)	It is always formed in front of mirror.	It is always formed behind the mirror.

FORMATION OF IMAGE BY A PLANE MIRROR



Properties of image from flat (Plane) Mirror

(i) Virtual and erect.

(ii) Same in size of object.

(iii) The image is formed behind of the mirror (as far as the object from the mirror).

(iv) The image formed is laterally inverted.

LATERAL INVERSION AND INVERSION

The phenomenon due to which the image of an object turns through an angle of 180^0 through vertical axis rather than horizontal axis, such that the right side of the image appears as left or vice versa is called **lateral inversion.**

INVERSION

During inversion image turns around horizontal axis through an angle of 180^0.

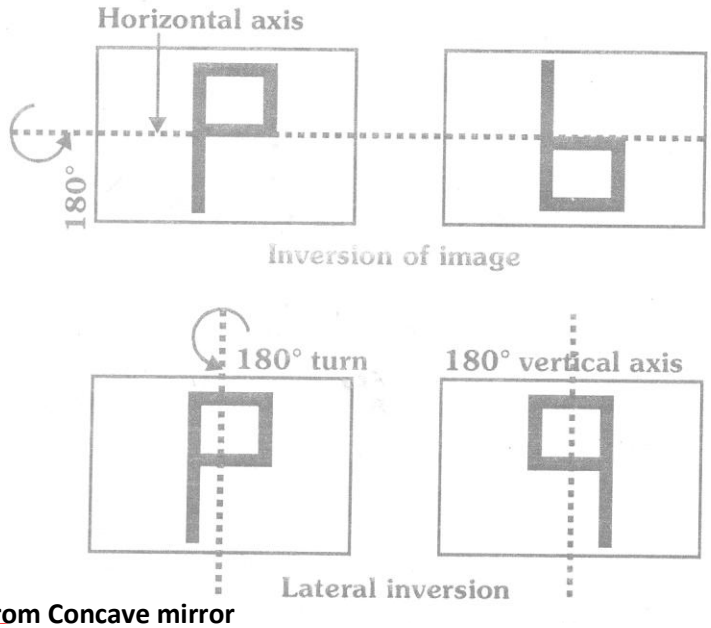
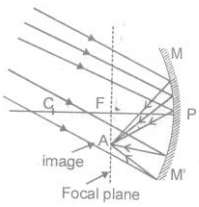
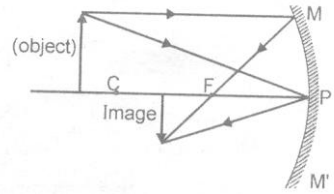
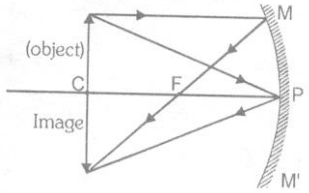
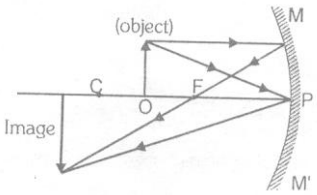
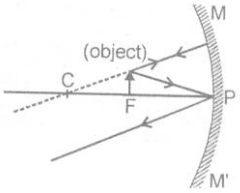
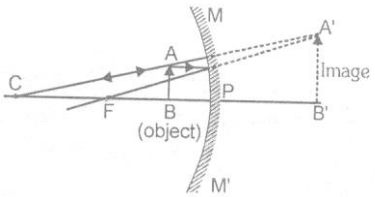


Image formation from Concave mirror

S.No.	Position of the Object	Position of the image	Nature & size of the image	Ray diagram
(1)	At infinity.	At focus F	Real. inverted and highly diminished. (point size)	

(2)	Between infinity and C	Between C & F	Real. inverted and smaller than the object	
(3)	At C	At C	Real. inverted and same size.	
(4)	Between C & F	Between C and infinite.	Real. inverted and enlarged.	
(5)	At F	At infinity.	Real. inverted and infinitely large.	
(6)	Between focus and pole	Behind the mirror.	Virtual. erect and enlarged.	

Use of Concave mirror

(i) It is used as a shaving mirror.

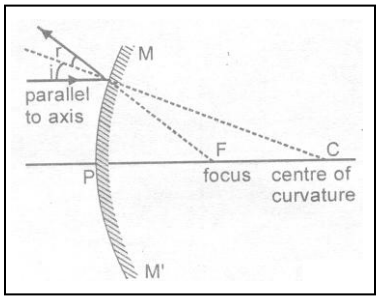
(ii) It is used as a reflector in the head light of vehicles.

(iii) It is used by doctor to focus a parallel beam of light on a small area.

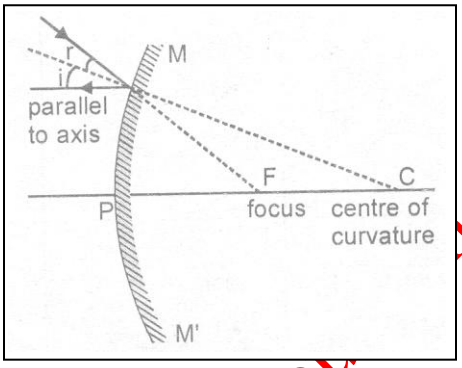
FORMATION OF IMAGE FROM A CONVEX MIRROR

There are rules of drawing images in convex mirror

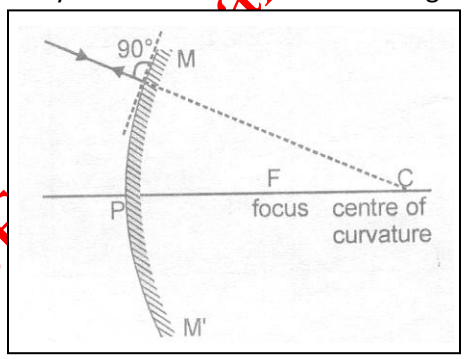
(i) Any ray of light travelling parallel to the principal axis of a convex mirror of the appears to diverge from the principal focus of the convex mirror.



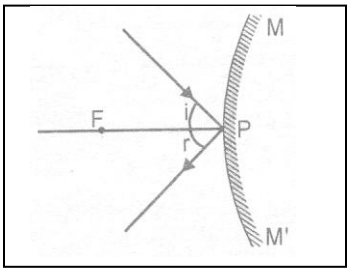
(ii) Any ray of light which travelers towards the direction of principal focus of a convex mirror, after reflection, it will travel for parallel to the principal axis of the mirror.



(iii) If ray of light which is incident along to the centre of curvature of a convex mirror after reflection it returns back on the same path. It is because the light ray strikes the convex mirror at right angle.



(iv) When the ray of light incident on the pole which is reflects or returns back on same angle from principal axis than it will reflect on the same angle of incident $\angle i = \angle r$.

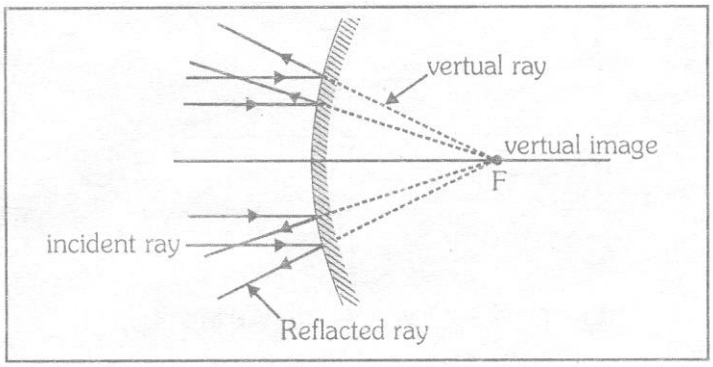


MAKING OF IMAGE FROM A CONVEX MIRROR

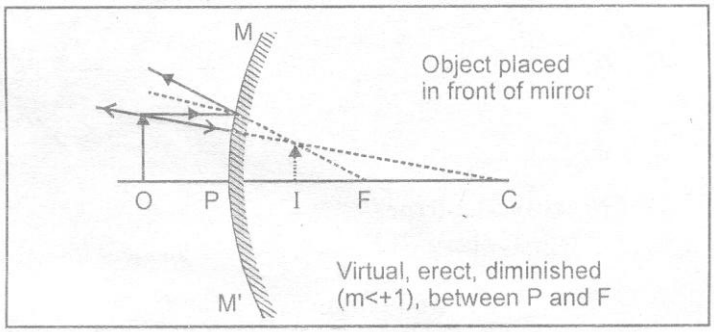
(i) When the object is at infinity

When the rays of light coming (diverging) from an object, situated at infinity are always parallel these parallel rays, strike the convex mirror, and reflected to diverge outward from convex mirror. These rays seems (appear) come from focus.

The characteristic of the image is virtual, erect, diminished to a point and formed at principal focus behind the convex mirror.



(ii) When the object is at a finite distance from the pole then the image is formed between pole and principal focus behind the convex mirror and image is virtual, erect and diminished.



Note :

There are only two position of the object for showing the image formed by a convex mirror that is –

(i) When the object is at a infinity.

(ii) When the object is at a finite distance from the pole of the convex mirror. Beside this positions are hot possible because the focus and the centre of curvature is behind the reflecting surface of the convex mirror.

Now we can study the image formation by following table

S.No.	Position of the object	Position of the image	Size of image of the image	Nature of the image
(1)	At infinity	At F, behind mirror	Highly diminished	Virtual and erect.
(2)	Between infinity and pole of mirror	Between P & F behind the mirror	Diminished	Virtual and erect.

Used of Convex mirror

(i) It is used as a rear view mirror in automobile.

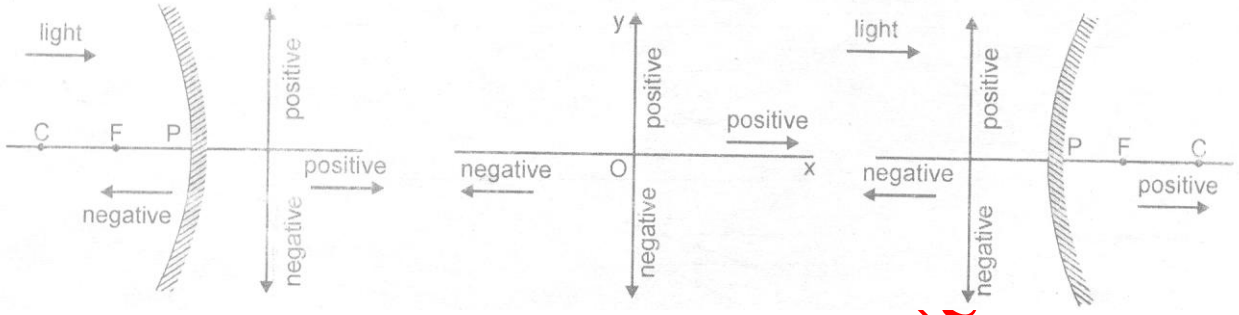
(ii) It is used as a reflector for street light.

Note : A plane mirror is not useful as a rear view mirror, because its field of view is very small.

COMPETITION WINDOW

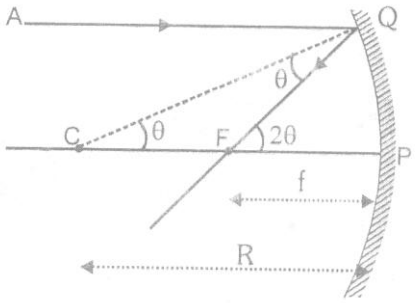
SIGN CONVENTION OF SPHERICAL MIRROR

- Whenever and wherever possible the ray of light is taken to travel from left to right.
- The distances above principal axis are taken to be positive while below it negative.
- Along principal axis, distances are measured from the pole and in the direction of light are taken to be positive while opposite to it is negative.



RELATION FROM SPHERICAL MIRROR

Relation between f and R for the spherical mirror



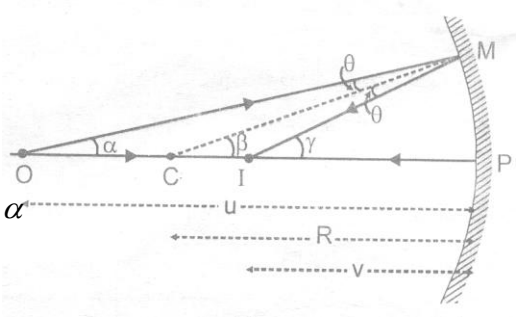
If Q is near to line P then from Δ QCP $\tan\theta \approx \theta = \frac{QP}{R}$

and from Δ QFP $\tan 2\theta \approx 2\theta = \frac{QP}{f}$

so $\frac{2QP}{R} = \frac{QP}{f} \Rightarrow f = \frac{R}{2}$

Relation between u, v and f for curved mirror

If an object is placed at a distance u from the pole of a mirror and its image is formed at a distance v (from the pole)



If angle is very small : $\alpha = \frac{MP}{u}, \beta = \frac{MP}{R}, \gamma = \frac{MP}{v}$

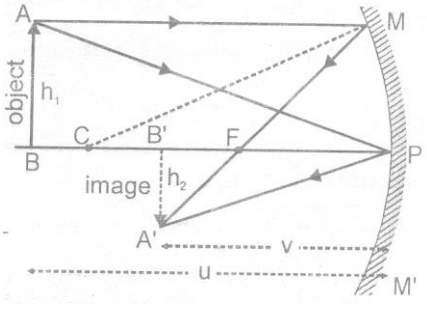
from Δ CMO, $\beta = \alpha + \theta \Rightarrow \theta = \beta - \alpha$

from Δ CMI, $\gamma = \beta + \theta \Rightarrow \theta = \gamma - \beta$

so we can write $\beta - \alpha = \gamma - \beta \Rightarrow 2\beta = \gamma + \alpha$

$\therefore \frac{2}{R} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

MAGNIFICATION



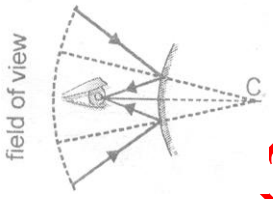
Linear magnification $m = \frac{size\ of\ image}{size\ of\ object} = \frac{I}{O}$

Δ ABP and Δ A'B'P are similar $\frac{-h_2}{h_1} = \frac{-v}{-u} \Rightarrow \frac{h_2}{h_1} = -\frac{v}{u}$

Magnification $m = -\frac{v}{u} \Rightarrow m = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f} = \frac{h_2}{h_1}$

POWER OF A MIRROR

The power of a mirror is defined as $P = -\frac{1}{f(m)} = -\frac{100}{f(cm)}$



◎ **Convex mirrors** gives erect, virtual and diminished image.
In convex mirror the field of view is increased as compared to plane mirror.
It is used as rear-view mirror in vehicles.
Concave mirrors give enlarged, erect and virtual image, so these are used by dentists for examining teeth. due to their converging property concave mirror are also used as reflectors in automobile head lights and search light
A real image can be taken on a screen, but a virtual image cannot be taken on a screen.

As focal length of a spherical mirror $f = \frac{R}{2}$ depends only on the radius of mirror and is independent of wavelength of light and refractive index of medium so the focal length of a spherical mirror in air or water and for red or blue light is same.

□ **MIRROR FORMULA**

The relation between the distance of the object from the pole of the spherical mirror (u), the distance of the image from the pole of the spherical mirror (v) and its focal length (f) is given by the mathematical formula:

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ It must be remembered that focal length (f) of a spherical mirror is half the radius of curvature (R). Thus,

(i) R = 2f, (ii) $f = \frac{R}{2}$

Important points in using the mirror formula

(i) Put the correct signs of known variables according to the sign convention.
(ii) Do not put the sign of an unknown variable. The sign will be automatically come up during calculations.
(iii) If the calculated sign turns out to be positive, then the variable calculated is behind the mirror. However,. if calculated sign turns out to be negative, then variable is to be in front of the mirror.

Linear magnification produced by spherical mirrors :

The ratio between the height of the image produced by the spherical mirror to the height of the object is called the linear magnification.

Thus, linear magnification = $\frac{\text{Height of the image}}{\text{Height of the object}}$ or $m = \frac{h_i}{h_0}$

Linear magnification when the image is real :

As we normally take object above the principal axis, therefore, h_o is always positive. The real image is always inverted and is formed below the principal axis.

Therefore, h_i is always negative. thus, Linear magnification for real images $= -\frac{h_i}{h_0}$ is always negative.

Linear magnification when the image is virtual :

In case of virtual image. It is erect and formed above the principal axis. Thus, h_o and h_i are both positive.

▸▸ The linear magnification produced by a spherical mirror is equal to the ratio of the distance of the image from the pole of the mirror (v) to the distance of the object from the pole of the mirror (u) with a minus sign.

Liner magnification, $m = -\frac{v}{u}$, Thus Linear magnification, $m = \frac{h_i}{h_o} = -\frac{v}{u}$.

Important points in using magnification formula :

(i) Put the correct signs of known variables according to the sign convention.

(ii) If 'm' is known, take the sign for virtual image positive and for real image negative.

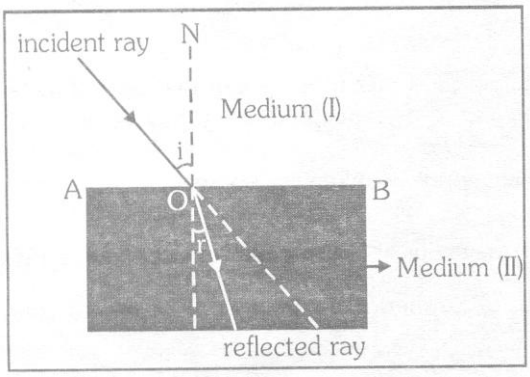
(iii) Do not put the sign of unknown variables. The sign will automatically come up during calculations.

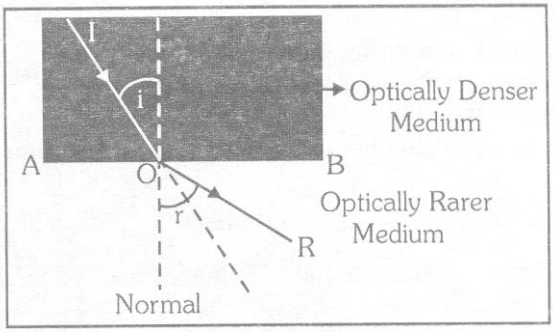
REFRACTION OF LIGHT

The bending of a ray of light as it passes from one medium to another is called refraction. It is due from change in velocity of light while travelling from one medium to another.

(i) The maximum velocity of light is 3×10^8 m/sec in vacuum or air.

(ii) The velocity is less in denser medium.





THE LAWS OF REFRACTION

(i) The ratio of sine of the angle of incidence to the sine of the angle of refraction for a particular pair of media constant. Thus if the angle of incidence is i, and that of refraction is r, then

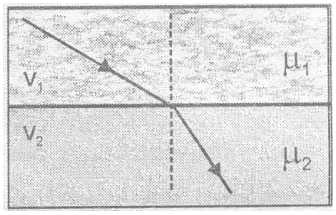
$$\frac{\sin i}{\sin r} = \text{constant} = \mu$$ it is known as Snell's law.

(ii) The incident ray, the refracted ray and the normal at the point of incidence points, all lie in the same plane.

RELATIVE REFRACTIVE INDEX

When light passes from one medium to the other, the refractive index of medium 2 relative to 1 is written as $_1\mu_2$ and is defined as

μ_{21} or $^1\mu_2$ or $_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{(c/v_2)}{(c/v_1)} = \frac{V_1}{V_2}$

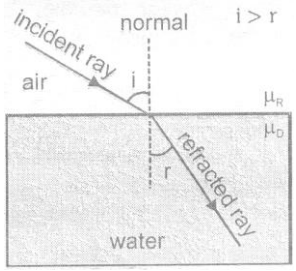


◎ **Bending of light ray**

According to Snell's law, $\mu_1 \sin I = \mu_2 \sin r$

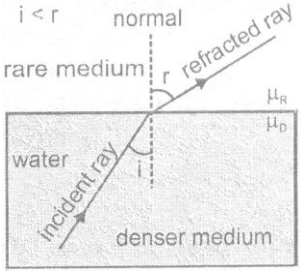
(i) If light passes from rarer to denser medium

$\mu_1 = \mu_R$ and $\mu_2 = \mu_D$



so that $\frac{\sin i}{\sin r} = \frac{\mu_D}{\mu_R} > 1 \Rightarrow \quad \angle i > \angle r$

in passing from rarer to denser medium, the ray bends towards the normal.



(ii) If light passes from denser to rarer medium $\mu_1 = \mu_D$ and $\mu_2 = \mu_R$

$$\frac{\sin i}{\sin r} = \frac{\mu_D}{\mu_R} < 1 \Rightarrow \quad \angle i < \angle r$$

In passing from denser to rarer medium, the ray bends away from the normal

Refractive index depends on nature and density of medium and colour of light refractive index is maximum for violet and minimum of red light.

COMPETITION WINDOW
APPARENT DEPTH AND NORMAL SHIFT

If a point object in denser medium is observed from rarer medium and boundary is plane, then from Snell's law we have $\mu_D \sin i = \mu_R \sin r$ .....(i)

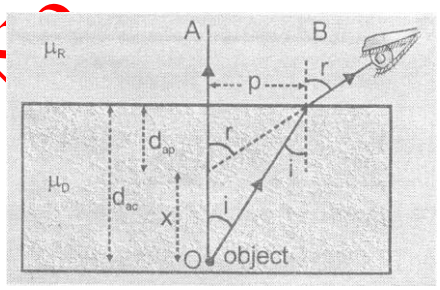
If the rays OA and OB are close enough to reach the eye.

$\sin i \approx \tan i = \frac{P}{d_{ac}}$ and $\sin r \approx \tan r = \frac{P}{d_{ap}}$

So that eqn. (i) becomes d_{ac} = actual depth d_{ap} = apparent depth

$\mu_D \frac{P}{d_{ap}} \mu_R \frac{P}{d_{ap}}$ i.e., $\frac{d_{ac}}{d_{ap}} = \frac{\mu_R}{\mu_D} = \frac{\mu_1}{\mu_2}$ (If $\mu_R = 1,\ \mu_D = \mu$)

$d_{ap} = \frac{d_{ac}}{\mu}$ so $d_{ap} < d_{ac}$ .....(ii)

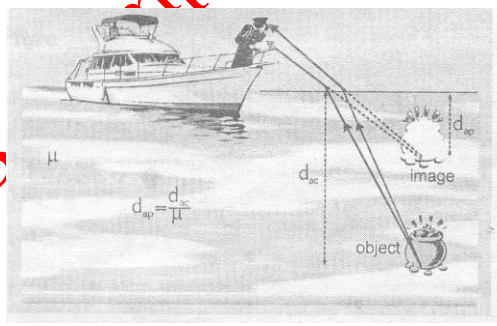


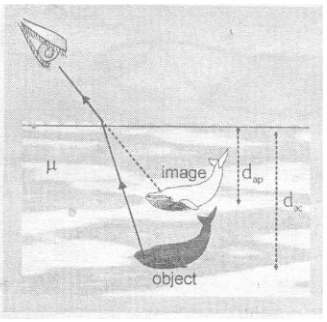
The distance between object and its image, called normal shift (x)

$x = d_{ac} - d_{ap}$ $[\because d_{ap} = \frac{d_{ac}}{\mu}]$

$x = d_{ac} - \frac{d_{ac}}{\mu} = d_{ac}\left[1 - \frac{1}{\mu}\right]$ ...(iii)

if $d_{ac} = d$, $x = d\left[1 - \frac{1}{\mu}\right]$



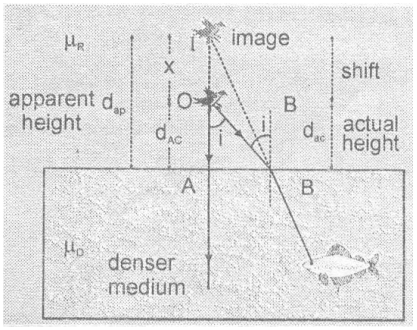


Object in a rarer medium is seen from a denser medium

$\frac{d_{ac}}{d_{ap}} = \frac{\mu_1}{\mu_2} = \frac{\mu_R}{\mu_D} = \frac{1}{\mu} (< 1)$

$d_{ap} = \mu d_{ac}$

$d_{ap} > d_{ac}$



A high flying object appears to be higher than in really

$x = d_{ap} - d_{ac}$ $\Rightarrow$ $x = [\mu - 1]d_{ac}$

Lateral Shift

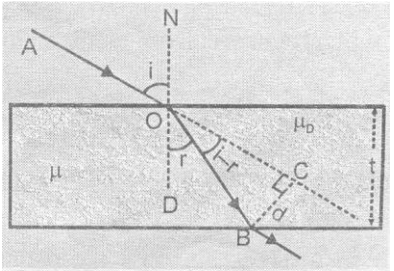
The perpendicular distance between incident and emergent ray is known as lateral shift.

Lateral Shift d = BC and t = thickness of slab

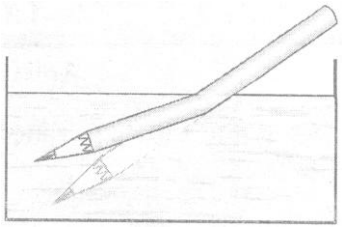
In Δ BOC

$\sin(i - r) = \frac{BC}{OB} = \frac{d}{OB}$ $\Rightarrow$ $d = OB\sin(i - r)$ ...(i)

In $\Delta OBD \cos r = \frac{OD}{OB} = \frac{t}{OB}$ $\Rightarrow$ $OB = \frac{t}{\cos r}$ ...(ii)



From (i) and (ii) $d = \frac{t}{\cos r}\sin(i-r)$



SOME ILLUSTRATIONS OF REFRACTION

◎ **Bending of an object**

When a point object in a denser medium is seen from a Rarer medium it appears to be at a depth $\frac{d}{\mu}$

◎ Twinkling of stars

Due to fluctuations in refractive index of atmosphere the refraction become irregular and the light sometimes reaches the eye and sometimes it does. This gives rise to twinkling of stars.

COMPETITION WINDOW

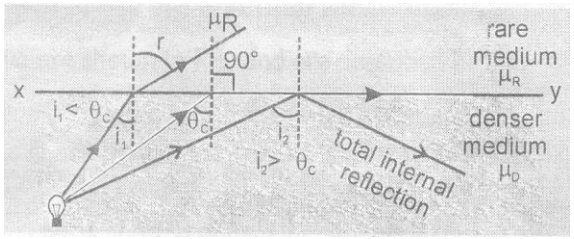
Total Internal Reflection (TIR)

When light ray travel from denser to rarer medium in bend away from the normal if the angle of incident is increased angle of refraction will also increased. At a particular value of angle of incidence the refracted ray subtend 90^0 angle with the normal, this angle of incident is known as critical angle (θ_C.) If angel of incident further increase the ray come back in the same medium this phenomenon is known as total internal reflection.

CONDITIONS

◎ Angle of incident > critical angle [i > θ_C]

◎ Light should travel from denser to rare medium $\Rightarrow$ Glass to air, water to air, Glass to water



Snell Law at boundary x – y, $\mu_D \sin\theta_C = \mu_R \sin 90^0 \Rightarrow \sin\theta_C = \frac{\mu_R}{\mu_D}$

As $\mu \propto \frac{1}{\lambda} \propto \frac{1}{v}$ $\sin\theta_C = \frac{\mu_R}{\mu_D} = \frac{V_D}{V_R} = \frac{\lambda_D}{\lambda_R}$

◎ When light ray travel from a medium of refractive index μ to air then $\mu_R = 1,\ \mu_D = \mu$

$\sin\theta_C = \frac{1}{\mu} \Rightarrow \sin\theta_C = \frac{1}{\mu} \propto \lambda \Rightarrow \theta(red) > \theta_C(violet)$

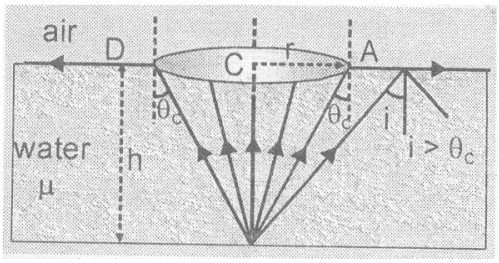
◎ For TIR i > θ_C $\Rightarrow \sin i > \sin\theta_C \Rightarrow \mu > \frac{1}{\sin i}$ $\left(\because \sin i = \frac{1}{\mu}\right)$

◎ When ray travel from glass to air $\sin\theta_C = \frac{\mu_R}{\mu_D} = \frac{1}{\frac{3}{2}} \approx 42^0 \Rightarrow$ i > 42^0(TIR), i < 42^0 (refraction)

◎ When ray travel from water to air $\sin\theta_C = \frac{\mu_R}{\mu_D} = \frac{1}{\frac{4}{3}} \approx 49^0$ i > 49^0(TIR), i < 49^0 (refraction)

◎ When rays travel from glass to water $\sin\theta_C = \frac{\mu_R}{\mu_D} = \frac{\frac{4}{3}}{\frac{3}{2}} \approx 63^0$ i > 63^0(TIR), i < 63^0 (refraction)

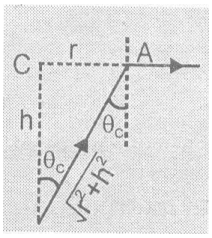
◎ A point object is situated at the bottom of tank filled with a liquid of refractive index μ up to height h. It is found light from the source come out of liquid surface through a circular portion above the object then radius and area of circle



$$\sin\theta_C = \frac{r}{\sqrt{r^2+h^2}}$$

$$\sin\theta_C = \frac{1}{\mu} \quad \Rightarrow \quad \frac{1}{\mu} = \frac{r}{\sqrt{r^2+h^2}} \Rightarrow \frac{1}{\mu^2} = \frac{r^2}{\sqrt{r^2+h^2}}$$

$$\Rightarrow \mu^2 r^2 = r^2 + h^2 \quad \Rightarrow (\mu^2 - 1)r^2 = h^2 \Rightarrow r = \frac{h}{\sqrt{\mu^2-1}}$$ and area πr^2



$$\Rightarrow \tan\theta_C = \frac{1}{h} \quad \Rightarrow r = h\tan\theta_C$$

Angle which the eye of fish make = $2\theta_C = 2\times 49^0 = 98^0$. This angle does not depend on depth of liquid

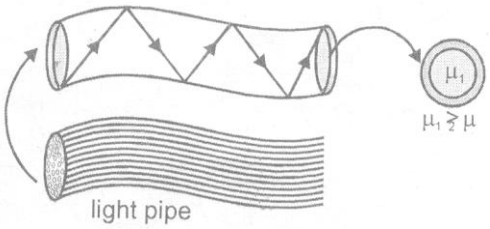
SOME ILLUSTRATIONS OF TOTAL INTERNAL REFLECTION

◎ **Sparkling of diamond**

The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so $\theta_C = 24^0$. Now the cutting of diamond are such that I > θ_C. So TIR take place again and again inside it . The light which beams out from a few places in some specific directions makes it sparkle.

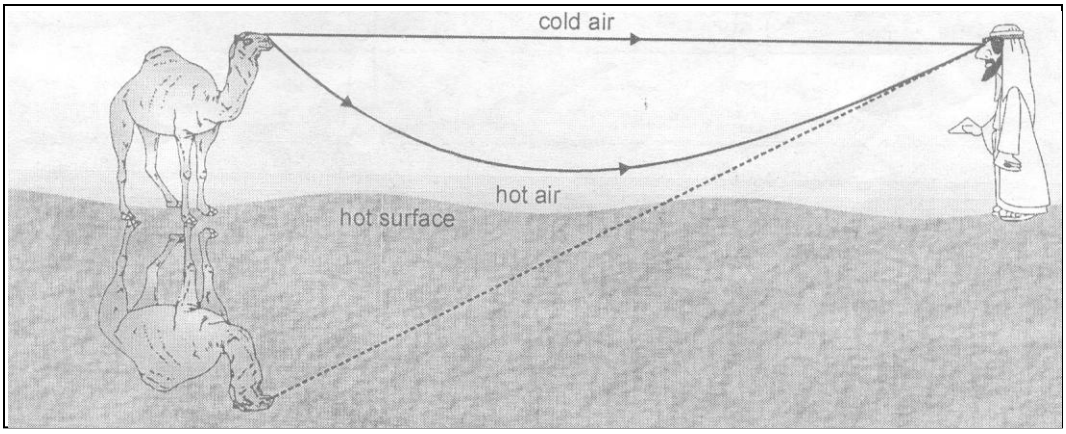
◎ **Optical Fiber**

In it light through multiple totals internal reflections is propagated along the axis of a glass fiber of radius of few microns in which index of refraction of core is greater then that of surroundings.

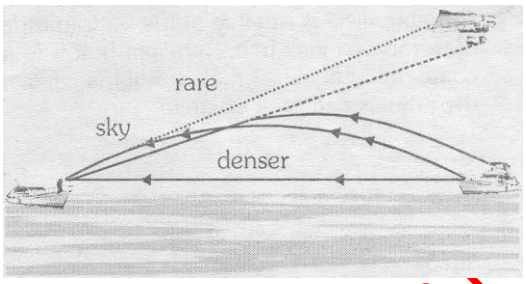


◎ **Mirage and looming**

Mirage is caused by total internal reflection is deserts where due to heating of the earth, refractive index of air near the surface earth becomes lesser than above it. Light from distant objects reaches the surface of earth with i > θ_C so that TIR will take place and we see the image of an object along with the object as shown in figure .



Similar to 'mirage' in deserts, in polar regions 'looming' takes place due to TIR. Here μ decreases with height and so the image of an object is formed in air if (i> θ_C) as shown in Fig.



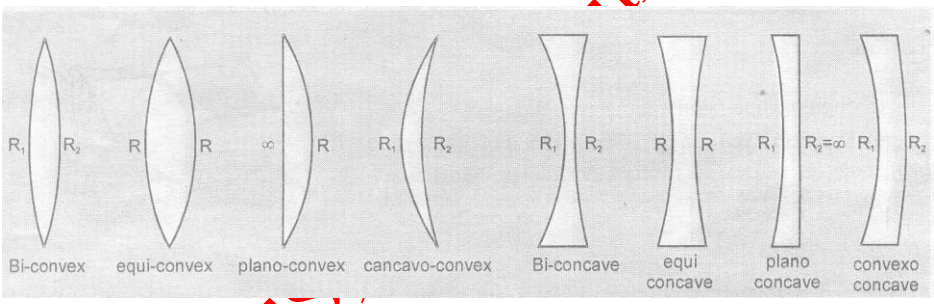
GOLDEN KEY POINTS

- A diver in water at a depth d sees the world outside through a horizontal circle of radius. r = d tan θ_C .
- For total internal reflection to take place light must be propagating from denser to rarer medium.
- In case of total internal reflection, as all (i.e. 100%) incident light is reflected back into the same medium there is no loss of intensity while in case of reflection from mirror or refraction from lenses there is some of intensity as all light can never be reflected or refracted. This is why images formed by TIR are much brighter . than formed by mirrors or lenses.

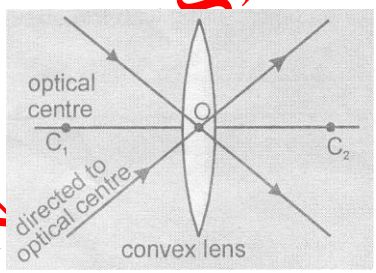
LENSES

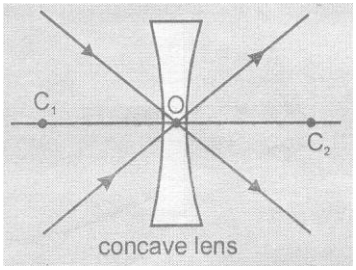
A lens is a piece of any transparent material bound by two curved surfaces or by one curved and one plane surface. Lens are of two types :

(i) Convex or convergent lens. **(ii) Concave or divergent lens.**



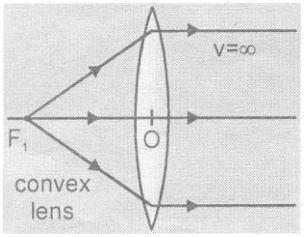
◎ **Optical Centre :** O is a point for a given lens through which any ray passes undedicated

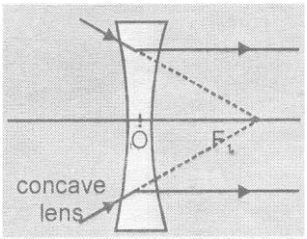




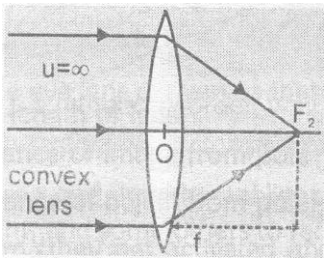
◎ **Principal Axis : C_1C_2** is line passing through optical centre and perpendicular to the lens.

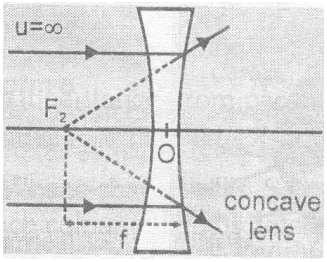
◎ **Principal Focus :** A lens has two surfaces and hence two focal points. First focal point is an object point on the principal axis for which image is formed at infinity.





While second focal point is an image point on the principal axis for which object lies at infinity





Focal Length f is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appears to converge.

Aperture : In reference to a lens, aperture means the effective diameter. Intensity of image formed by a lens which depends on the light passing through the lens will depend on the square of aperture, i.e., $I \propto (\text{Aperture})^2$

RULES FOR IMAGE FORMATION

A ray passing through optical center proceeds undeviated through the lens.

A ray passing through first focus or directed to towards it, after refraction from the lens, becomes parallel to the principal axis.

A ray passing parallel to the principal axis after refraction through the passes or appears to pass through F_2

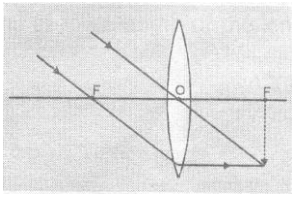
For Convergent or Convex Lens

Object	Image	Magnification
∞	**F**	**m << - 1**
∞ - 2F	**F – 2F**	**m < - 1**
2F	**2F**	**m = - 1**
F – 2F	∞ - 2F	**m > - 1**
F	∞	**m >> -1**
F – O	In front of lens	**m > + 1**

IMAGE FORMATION FOR CONVEX LENS (CONVERGENT LENS)

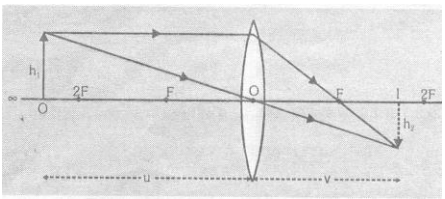
(i) Object is placed at infinity

Image : at F
real
inverted
very small in size
m << - 1



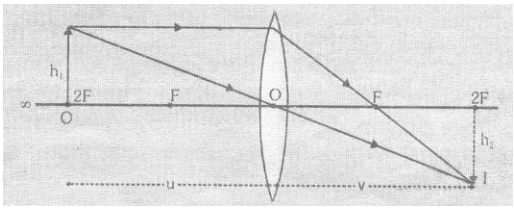
(ii) Object is placed in between ∞ - 2F

Image: real (F – 2F)
inverted
small in size
(diminished)

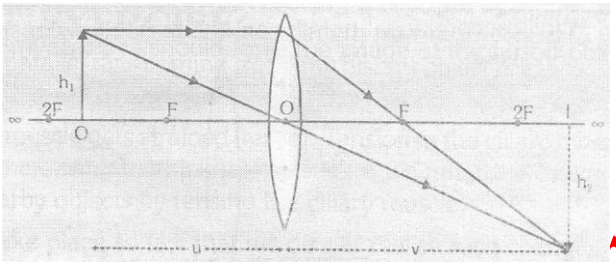


(iii) Object is placed at 2F

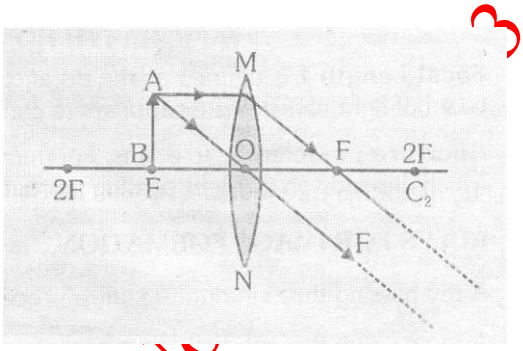
Image: real (at 2F)
inverted
equal (of same size)
(m = - 1)



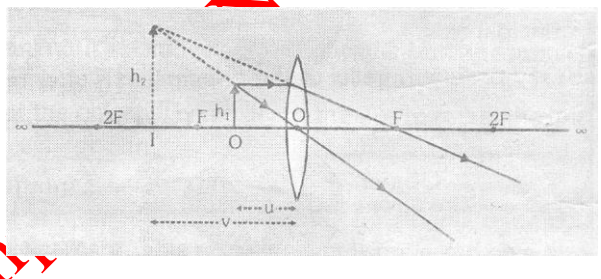
(iv) Object is placed in between
2F – F
Image: real (2F - ∞)
inverted
enlarged
m > 1



(v) Object is placed at F



(vi) Object is placed in between
F – O
Image: virtual (in front of lens)
erected
enlarge
(m > + 1)

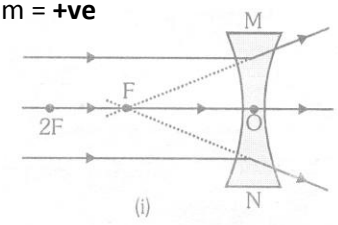


Given	Real	Virtual
u	-	-
v	+	+
h_1	+	+
h_2	-	+
m	-	+

IMAGE FORMATION FOR CONCAVE LENS (DIVERGENT LENS)

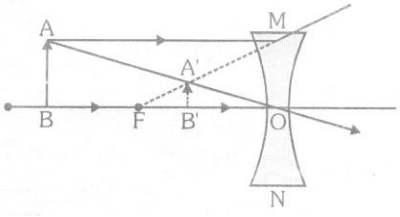
Image is virtual, diminished, erect, towards the object, m = **+ve**

(i) Object is placed at infinity
Image : At F
virtual
erected
diminished
(m << + 1)



(i)

(ii) Object is placed in front of lens
Image : between F and optical centre
virtual
erected
diminished
(m < + 1)



Formula of lens : Focal length of a lens can be find out by the following formula.

$$\frac{1}{f} = \frac{1}{v} = \frac{1}{u}$$

Where – f = Focal length of lens.

v = Distance of image from pole

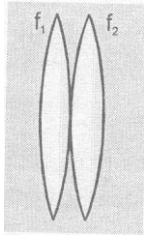
u = Distance of object from pole.

Uses of this formula

(i) Put the correct signs of known variables according to the sign conventions.

(ii) Do not put the sign of unknown variable. The sign will automatically show up during calculations.

(iii) If the calculated sign of a variable turns out positive, then the variable calculate is on the other side of the lens, i.e., on the opposite side to the object. However if calculated variable is of negative sign, then it is on the same side as the object.



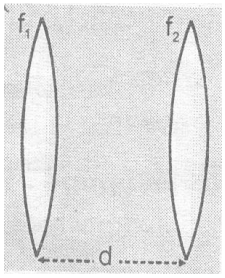
COMBINATIONS OF LENSES

Two thin lens are placed in contact to each other

power of combination.

$P = P_1 + P_2$ $\qquad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Use sign convention while solve Numericals.



Two thin lens are placed in a small distance d

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$P = P_1 + P_2 - d\ P_1 P_2$

Use sign convention when solve Numericals.

Power of lens : The power of a lens is defined as reciprocal of focal length of the lens. Focal length should always be measured in meters.

$$P = \frac{1}{f(m)}$$

$$\boxed{P = \frac{u - v}{uv}}$$

Unit of power of lens is 1/meter which is called **DIOPTER**

Magnification of lens : The magnification is defined as the ratio of the height of the image and the height of the object it is represented by M.

$$M = \frac{\text{Height of the image } (h_1)}{\text{Height of the object } (h_0)} \quad \frac{\text{Size of the image (I)}v}{\text{Size of the object (O)}u}$$

USES OF LENSES IN DAILY LIFE

Lenses are using is microscope, telescope. Prism, binoculars and slide projector etc.

RELATION BETWEEN FOCAL LENGTH (F) AND RADIUS OF CURVATURE (R)

Focal length of a lens depends on following factors :-

(i) Refractive index of the material of the lens.
(ii) Radius of curvature R_1 and R_2 of both in curved surfaces of the lens.
(iii) Colour (wave length) of the light.

Formula of making a spherical lens of specific focal length.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Where f is the average value of focal length for all the colour. μ is the refractive index of the material of the lens with respect to air. R_1 and R_2 are the radius of curvature of the curved surfaces respectively. R_1 is positive and R_2 is negative for convex lens. R_1 is negative and R_2 positive for concave lens.

THE HUMAN EYE & COLOURFUL WORLD

Our eye is the most important natural optical instrument. The eye resembles a – camera in many ways. It has "nearly a spherical shape.

What are the essential parts of the eye and their functions

The essential parts of the eye are shown in Fig. and are described below :

(a) **Cornea :** The front of the eye is covered by a transparent spherical membrane called the cornea. Light enters the eye through cornea. The space behind the cornea is filled with a clear liquid called aqueous humor.

(b) **Iris and pupil :** The iris and pupil form the variable aperture system of the eye. Iris is a dark colored muscular diaphragm which has a small circular opening in its middle. The central circular aperture of iris is called pupil. The iris regulates the amount of light entering the eye by adjusting the size of the pupil. This is explained below:
(i) When the light is very bright (as on a sunny day), the iris makes the pupil to contract. As a result, the amount of light entering the eye decreases.
(ii) When the light is dim (as in a dark room), the iris makes the pupil to expand. As a result, more light can enter the eye.

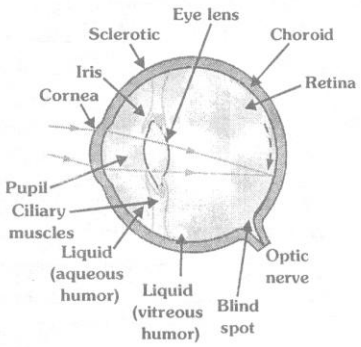
(c) **Eye lens :** The eye lens is a convex lens (converging lens) made of a transparent jelly-like pertinacious material. The eye lens is hard at the middle and gradually becomes soft towards the outer edges. The eye lens is held in position by Ciliary muscles. The curvature and therefore the eye lens may be changed by the action of the Ciliary muscles.

(d) **Retina :** The inside surface of the rear part of the eye ball where the light entering the eye is focused is called retina. The surface of retina consists: of about 125 million light sensitive receptors. The receptors are of two types – rods and cones. When the light falls on these receptors, they send electrical signals to the brain through the optic nerve.

(e) **Blind spot :** There are no rods cones at the point where optic nerves the eye ball to go to the brain. So , it any image is formed in this part of the retina, then no signal is sent to the brain. As a result the object is not seen. This part is therefore called the blind spot of the eye.

HOW DOES THE EYE WORK

The light coming from an object enters the eye through cornea and the pupil. The eye lens converges these light rays to form a real, inverted and high diminished image on the retina. The surface of retina consists of large number of light sensitive cells. When light calls on them, they get activated and generate electrical signals. These signals are then sent to the brain by the optic nerves. and the observer sees the actual-sized, erect image of the object.



Parts of the human eye

Ex. Why does it take some time to see objects in a dim room when you enter the room from bright sunlight outside ?

Sol. In bright sunlight, iris makes the pupil to become small, so that only a small amount of light is able to enter the eye. When someone enters a dim room, very little light is available to the eye. The iris then makes the pupil to gradually expand (open up) to allow more light to enter the eye. This process takes some time. That is why the pupil takes a little time to adjust itself to dim light.

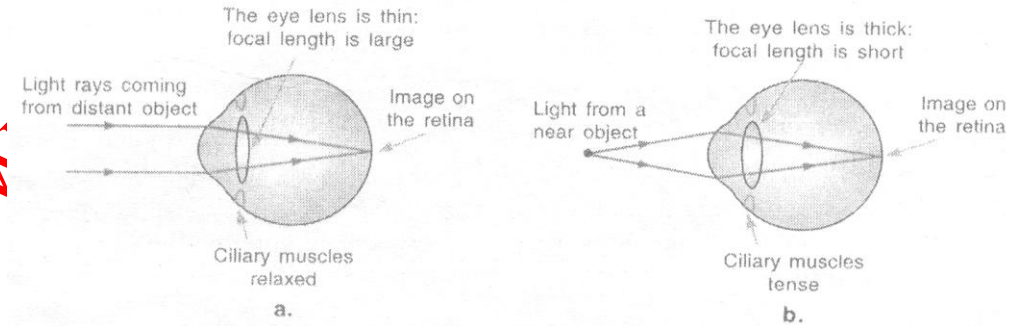
What is meant by the far point, near point , and the least distance of distinct vision ?

(a) **Far point :** The farthest point up to which an eye can see clearly is called the far point of the eye. **For a normal eye, the far point is at infinity.**

(b) **Near point :** The closest (nearest) point up to which an eye can see clearly is called the near point of the eye. The near point for a healthy normal eye of an adult lies about 25 cm from the eye.

(c) **Least distance of distinct vision :** The minimum distance up to which an eye can see clearly without any strain is called the least distance of distinct which vision (denoted by d or D). The least distance of distinct vision is thus equal to the distance between the eye and its near point For a normal eye of an adult, the least distance of distinct vision is about 25cm. This distance usually increases with age.

POWER OF ACCOMMODATION OF THE EYE



What is meant by accommodation of the eye ?

A normal eye can see both the distant and the nearby objects clearly. For a clear vision, the image of any object must fall on the retina. For a person, the distance between the retina and the eye lens is fixed. So, the distance of the

image (v) from the eye lens is fixed. For objects at different distances, the values of u are different. So, to get the image at the same v, the focal length of the eye lens should be different.

The eye can focus the images of all the objects, distant or nearby, at the same place (on the retina) by changing the focal length of its lens. The eye lens can change its focal length by changing its thickness with the help of its ciliary muscles.

The property due to which eye lens is able to change its focal length is called accommodation of the eye.

When the eye is focused on distant objects (objects at infinity), the Ciliary muscle is fully relaxed. When the Ciliary muscle is in the relaxed state, the thickness of the lens is minimum and the focal length maximum; equal to the distance of the retina from the eye lens. Therefore, the parallel rays coming from any distant object are focused on the retina, and the object is seen clearly.

When the eye is focused on a nearby object, the ciliary muscle gets strained (tense). Tension in the ciliary muscle decreases the focal length of the eye lens by slightly increasing its thickness in such a way that the image is formed on the retina. Thus, the eye focuses on the nearby objects by tensing the ciliary muscle.

These adjustments in the focal length of the eye lens take place so fast that we do not realize such changes.

What is meant by the limit of accommodation?

The eye can accommodate only up to a limit. A normal eye can accommodate up to the least distance of distinct vision: about 25 cm for a healthy adult. The objects which are very close to the eye produce blurred image. Thus, a normal eye can accommodate objects lying between infinity and the least distance of distinct vision.

What is the range of vision ?

The range of distance over which the eye can see clearly is its range of vision. A normal human eye can see objects clearly which lie between infinity and the least distance of distinct vision. So, the range of vision of a normal healthy human eye is from infinity to the least distance of distinct vision, i.e. from infinity to about 25 cm from the eye.

What is meant by the persistence of vision ?

The image formed on the retina of the eyes is not permanent. It also does not fade away instantaneously after the object is removed or we have stopped seeing the object. The image formed on the retina persists for $1/16^{th}$ of a second even after we have stopped looking at the object. The ability of the eye to retain the image (or the sensation caused by the light coming from the object) for about $1/16^{th}$ of a second even after we have stopped seeing the object is called persistence of vision.

The phenomenon of persistence of vision is made use of in motion-picture projection (or cinematography). Sequences of still pictures are recorded on a film by a movie camera. This recorded film is projected on a screen at the speed of about 24 pictures per second. Due to the persistent of vision, the successive images on the screen merge smoothly into one another giving an impression of continuity. In this way, we are able to see the pictures in motion.

COLOUR VISION

How do we see the colour

We see an object only when its image is formed at the retina.

Our retina has a large number of light-sensitive cells. These cells are of two shapes – rods and cones.

- The rod-shaped cells respond to the intensity of light, i.e. the degree of brightness or darkness is sensed by the rod-shaped cells on the retina of the eye.
- The cone-shaped cells respond to the colors. These cells are sensitive to red, blue and green colors to different extents. It is due to these cone-shaped cells that we are able to distinguish between different colors.

The cone-shaped cells become active only in bright light, That is why, we can't differentiate between colors in dim light. There are in fact three different kinds of the cone-shaped cells : one sensitive to red, second to green and the third to blue color. So, depending upon the colour of the light entering the eye, one or more kinds of the cone-shaped cells get activated.

How do certain animals including birds differ in their colour perception

The structure and the and the number of rod-shaped and cone-shade cells are different for different animals/birds. As a result, therefore, their colour perceptions are also different. For example.

- Bees have cones that are sensitive to ultraviolet light (light beyond violet colour). So, bees can see the ultraviolet light present in sunlight.

- Human beings cannot see the ultraviolet light because the cones in their retina are not sensitive to the ultraviolet light .
- The retina of chicken has mostly cone-shaped cells and only a few rod shaped cells. It is because of this reason that a chicken can see only in bright and wake up only when the sun rises and goes to sleep at sunset .

What is colour blindness

Colour blindness is a defect of vision or defect of eye. When a person is unable to distinguish between certain colors, he/she is said to be colour blind. Thus, colour blindness is that defect of the eye due to which a person is not able to distinguish between certain colors.

Colour blindness is a genetic disorder, which occurs due to inheritance (from parents to their children).

Cause of colour blindness

The retina of human eye consists of three different types of cone-shaped cells. Some are sensitive to blue light, some to green light, and some others to red light. Thus, each colour of light is detected only by the cones which are sensitive to that light. Sometimes, a person may not have a particular kind of cones on its retina. Then, such a person will not be able to distinguish between certain colors. For example. a person not having cones sensitive to the blue in his/her eye will not be sensitive to blue colour.

DEFECTS OF VISION AND THEIR CORRECTION

Abnormalities in the normal vision of the eye are called defects of vision or defects of eyes.

The most commonly observed defects of vision (or defects of eyes) are :

(i) Myopia or shortsightedness or nearsightedness

(ii) Hypermetropia or long-sightedness or hyperopic or farsightedness

(iii) Astigmatism

WHAT IS SHORTSIGHTEDNESS (OR MYOPIA)

Shortsightedness (or myopia) is the defect due to which the eye is not able to see the distant objects clearly though it can see the nearby objects clearly. So, shortsighted or myopic eye has its far point nearer than infinity.

WHAT CAUSES SHORTSIGHTEDNESS (OR MYOOPIA)

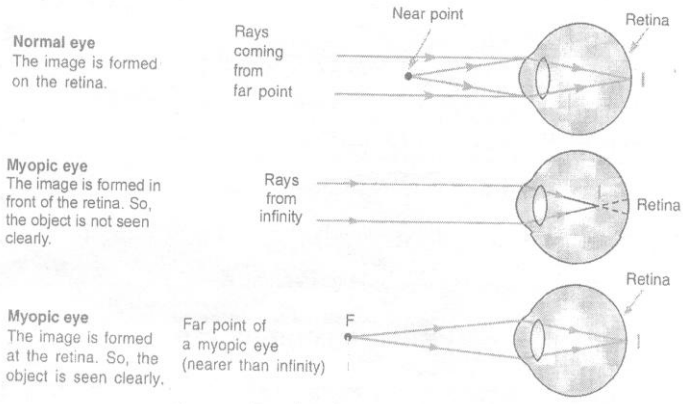
Myopia or shortsightedness is caused by the following reasons.

(a) Decrease of focal length of the eye lens,

i.e. the eye lens becomes more convergent .

(b) Elongation of the eyeball,

i.e. the increased length of the eyeball.

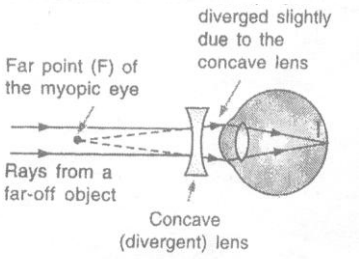


A normal and a shortsighted (or myopic) eye

How is shortsightedness (or myopia) corrected ?

The shortsightedness (myopia) can be corrected by making the eye lens less convergent. This can be done by placing a concave lens (divergent lens) of suitable focal length before the eye lens.

The rays of light coming from a distant object after passing through the concave (diverging) lens of the spectacles diverge slightly. As a result, the rays entering the eye appear to come from the far point of the myopic eye, and therefore get focused at the retina to form a clear image.



Correcting myopic (shortsighted) eye by using a concave (divergent) lens

How to calculate the focal length and power of the lens used for correcting a myopic eye

The corrective lens (concave lens) needed to correct a myopic eye should form the image of the far-off object (e.g. at infinity) at the far point (d) of the myopic person.

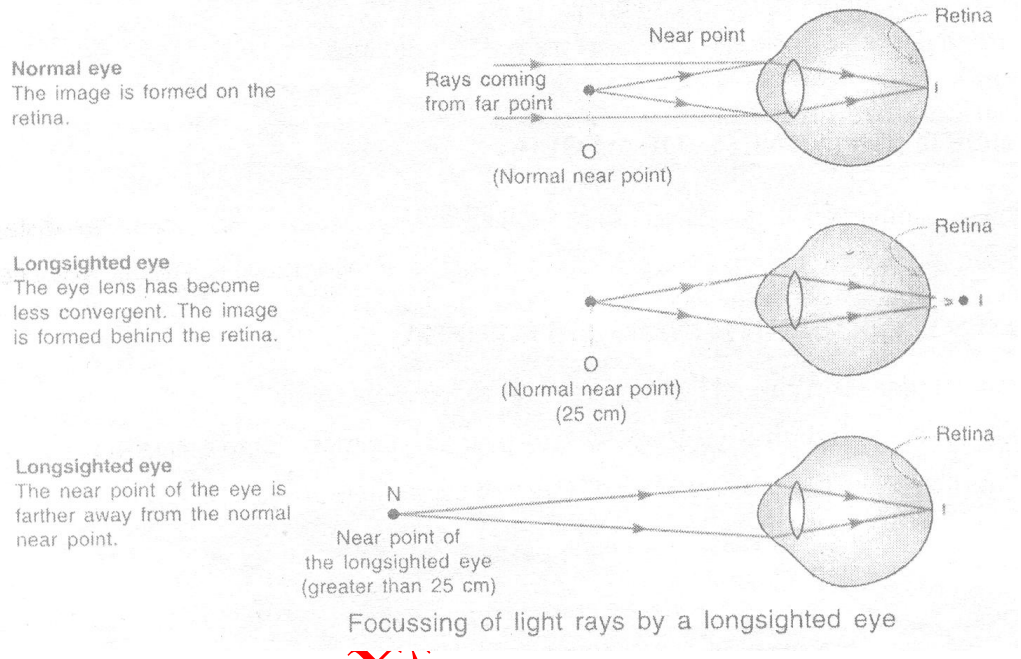
Thus, $u = -\infty,\ v = -d,\ f = ?$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = -\frac{1}{d} \qquad \Rightarrow \qquad f = -d$$

WHAT IS LONGSIGHTEDNESS (OR HYPERMETROPIA OR HYPEROPIA)

The long-sightedness (or Hypermetropia) is the defect due to which the eye is not able to see clearly the nearby objects though it can see the distant objects clearly.

So, a longsighted eye has its near point farther away from the normal near point (about 25 cm for a an adult).



Focussing of light rays by a longsighted eye

WHAT CAUSES LONGSIGHTEDNESS (OR HYPERMETROPIA)

Hypermetropia or long-sightedness is caused due to the following reasons :

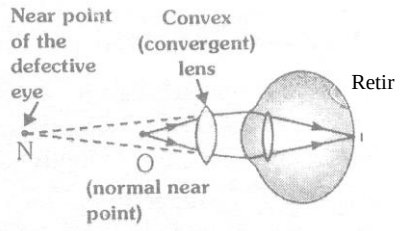
(i) Increase of the focal length of the eye lens, i.e. the eye lens becoming less convergent.

(ii) Shortening of the eye ball, i.e. the length of the eye ball has decreased.

HOW IS LONGSIGHTEDNESS (OR HYPERMETROPIA) CORRECTED

Long-sightedness (Hypermetropia) can be corrected by making the eye lens more convergent. This is generally done by placing a convex lens (converging lens) of suitable focal length before the eye lens. This is shown in Fig. in next page.

The rays from a nearby object (about 25 cm) after passing through the convex lens of the spectacles converge slightly. As a result, the rays entering the eye appear to come from the near point of the longsighted eye, and therefore get focused at the retina to form a clear image. How to calculate the focal length and power of the lens used for correcting a Hypermetropic eye



Correcting the longsightednes (hypermetropia) by using a convex (convergent) lens

The corrective lens (a convex lens) needed to correct a Hypermetropia (or longsighted) eye should form the image of the object placed at the normal near point (the least distance of distinct vision is 25 cm) at the near point of the Hypermetropia person. Thus.

v = Near point distance of the Hypermetropia eye = - d

u = Near point distance for the normal eye = - D = - 25 cm Using the lens formula,

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{-d}-\frac{1}{25cm}$$

WHAT IS ASTIGMATISM

A normal eye can focus all the light from any object at the same point. Sometimes, the light coming in the horizontal and vertical planes do not come to focus at the same point. As a result, the horizontal and vertical views of an object are not seen with the same clarity. Such a defect of eye is called astigmatism. A person suffering from astigmatism is not able to see in all directions equally well.

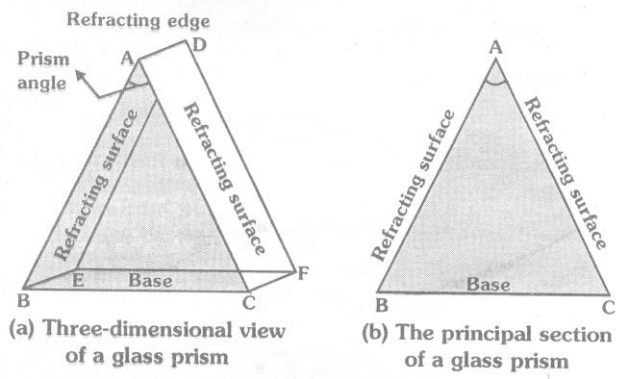
- Cause of astigmatism. Astigmatism occurs when the cornea or the eye lens or both are not perfectly spherical, i.e. the cornea or the eye lens or both are more curved in one plane than in the other.

- Correcting astigmatism. Astigmatism can be corrected by using cylindrical lenses. Cylindrical lenses have different curvature in the horizontal and vertical directions.

REFRACTION OF LIGHT THROUGH A PRISM

What is a prism

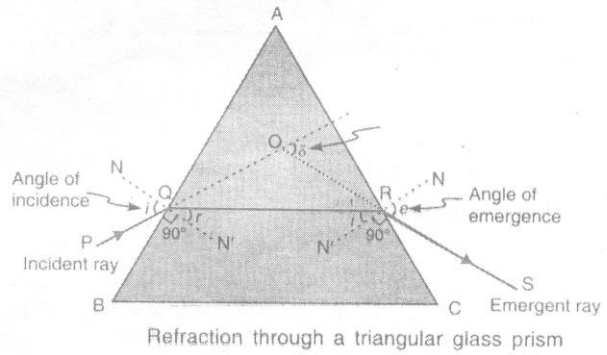
A prism is a transparent refracting medium bound by two plane surfaces inclined to each other at certain angle (commonly 60^0 or 45^0),



(a) Three-dimensional view of a glass prism

(b) The principal section of a glass prism

- The faces ABED and ACFD are refracting surfaces of the prism.
- The face BEFC is the base of the prism.
  The angle BAC is the prism angle.
- The line of intersection of the two refracting surfaces is called refracting angle (the line AD in the diagram) or the prism.
- The face ABC is the principal section of the prism.
  The principal section of a prism is perpendicular to its refracting edge.



Refraction through a triangular glass prism

HOW DOES LIGHT GET REFRACTED BY A PRISM

The incident ray suffers a deviation (or bending) through an angle δ due to refraction through the prism. The angle δ is called the angle of deviation.

Note :

- The prism is in the position a the minimum deviation when angle of emergence = angle of incidence.

  or $\angle e = \angle i$

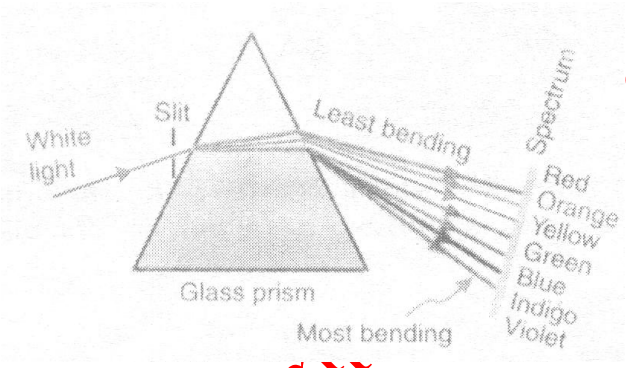
- The refractive index of the material of prism is given as

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

DISPERSION OF WHITE LIGHT BY A GLASS PRISM

What is meant by dispersion of light

The process of splitting white light into its seven constituent colors is called dispersion of white light. The band of seven colors formed on a screen due to the dispersion of white light is called spectrum of visible light or spectrum of white light.



What causes dispersion of white light

Dispersion of white light into seven colors occurs because the light of different colors has different wavelength. In this band of seven colors, red light has the longest wavelength and violet has the shortest.

Lights of all colors travel at the same speed in vacuum. But, in any transparent medium, such as glass or water, the lights of different colors travel with different speeds.

Due to difference in their speeds, the lights of different colors bend through different angles. In any transparent medium, the red light travels the fastest, and the violet light the slowest of all the seven colors. Therefore, the red light bends the least, and violet light bends the most.

Thus, dispersion of white light into seven colors occurs because the lights of different colour bend through different angle while passing through a glass prism.

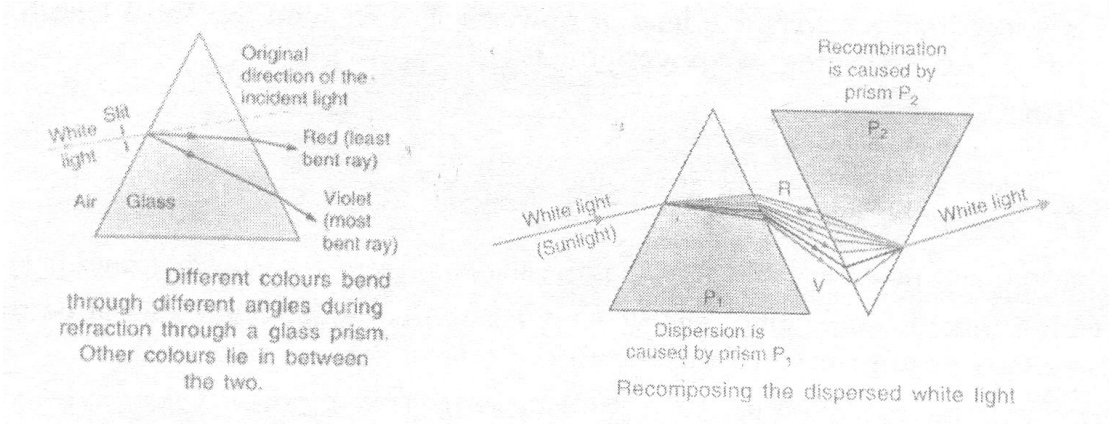
What is meant by monochromatic and polychromatic light

The light of one single colour, or one single wavelength is called monochromatic light (chrome means colour). Sodium light is golden yellow in colour. So, sodium light is monochromatic light.

The light made up of many colors, or light consisting of radiations of many wavelengths, is called polychromatic light, white light is made up of seven colors. So, white light (or sunlight) is a polychromatic light.

How is the dispersed white light recomposed

Recombination of the colors of the dispersed white light to get white light is called recomposing of the dispersed white light.

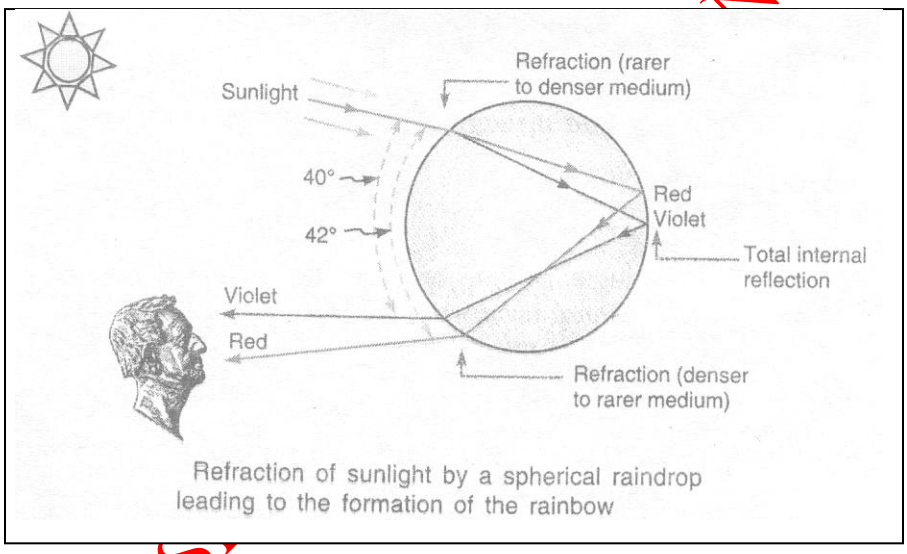


How does a rainbow form?

Rainbow is an example of the dispersion of white light.

Just after the rain, a large number of small droplets of water remain suspended in the air. Each drop acts like a small prism. When sunlight falls on these drops, the white light splits into seven colors. The dispersed light form a large number of drops forms a continuous band of seven colors. This colored band is called rainbow. Thus, rainbow is produced due to dispersion of white light by small raindrops hanging in the air after the rain.

The rainbow is seen when the sun is behind the observe.



Refraction of sunlight by a spherical raindrop leading to the formation of the rainbow

SOLVED NCERT EXERCISE

1. A lens has power of - 2.5 D. What is the focal length and nature of the lens ?

Sol. P = - 2.5 D, f = ?

From relation, $P = \frac{1}{f}$

$$f\ \frac{1}{P} = \frac{1}{-2.5} = -0.4m = -40cm$$

Negative sign indicates that it is a concave lens.

2. A doctor has prescribed a corrective lens of power + 1.5 D. Find the focal length of the lens Is the prescribed lens diverging or converging ? **[NCERT]**

Sol. P = + 1.5D

f (in meters) = $\frac{1}{P} = \frac{10}{15} = \frac{2}{3} = 66.6cm$

As the focal length and power of the lens is positive therefore, lens is a convex (converging) lens).

3. A person with a myopic eye cannot see beyond 1.2 m distinctly. What should be the nature of the corrective lens used to restore proper vision ?

Sol. Corrective lens required is 'concave lens' of suitable power to restore proper vision. In this cases.

$u = -\infty$, $v = -1.2$ m

Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-1.2m} - \frac{1}{\infty} = -\frac{1}{1.2m}$

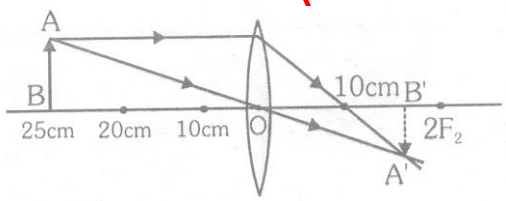
f = - 1.2 m

Power of lens, $P = -\frac{1}{1.2m} = -\frac{1}{1.2} D = -0.83D.$

4. An object 5 cm in length is held 25 cm away form a converging lens of length 10 cm. Draw the ray diagram and find the position, size and the nature of the image formed. [NCERT]

Sol. $f = -+10cm$

$u = -25cm$



Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{10} = \frac{1}{v} - \left(\frac{1}{-25}\right)$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{25}$$

$$\frac{1}{v} = \frac{3}{50} cm$$

$$\Rightarrow v = \frac{50}{3} = 16.7m$$

The image is real at a distance of 16.7 cm behind the lens

$$m = \frac{h'}{h} = \frac{v}{u}$$

$$\frac{h'}{5} = \frac{50/3}{-25} \Rightarrow h' = \frac{50/3 \times 5}{-25} = -\frac{10}{3} cm$$

Height of the image is 3.3 cm in height.

5. The refractive index of diamond is 2.47 and that of glass is 1.51. How much faster does light travel in glass than in diamond. ?

Sol. Let n_1 and n_2 be the refractive indices and v_1 and v_2 be the velocity of light in diamond and glass respectively, then

$$n_1 = \frac{c}{v_1} \quad or \quad v_1 = \frac{c}{n_1} = \frac{3 \times 10^8 m/s}{2.47} = 1.215 \times 10^8 m/s$$

$$n_2 = \frac{c}{v_2} \quad or \quad v_2 = \frac{c}{n_2} = \frac{3 \times 10^8 m/s}{2.51} = 1.987 \times 10^8 m/s$$

$$v_2 - v_1 = (1.987 - 1.215) \times 10^8 = 0.772 \times 10^8 m/s$$

$$= 7.72 \times 10^7 m/s$$

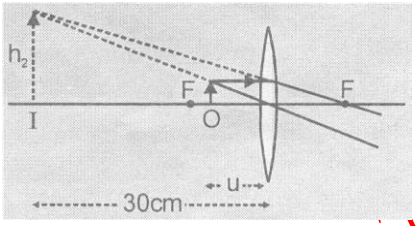
Thus light travels 7.72 ×107 m/s faster in glass than diamond.

6. A magnifying lens has a focal length of 10cm. (a) Where should the object if the image is to be 30 cm from the lens ? (b) What will be the magnification ?

Sol. (a) In case of magnifying lens, the lens is convergent and the image is erect, enlarged, virtual, between infinity

and object and on the same side of lens.

f = 10 cm and v = - 30 cm



and hence from lens-formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

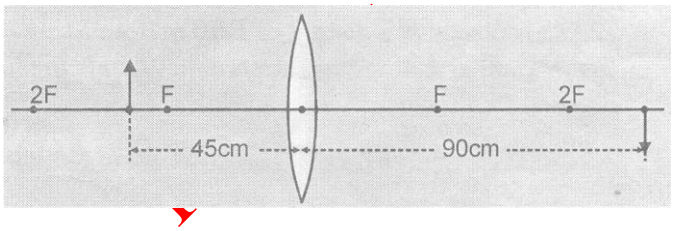
we have $\frac{1}{-30}-\frac{1}{u}=\frac{1}{10}$ i.e., u = - 7.5 cm

So the object must be placed in front of lens at a distance of 7.5 cm (which is < f) from it.

(b) $m=\left[\frac{h_2}{h_1}\right]=\frac{v}{u}=\frac{-3}{-7.5}=4$ i.e., image is erect, virtual and four times the size of object.

7. An object 25 cm high is placed in front of a convex lens of focal length 30 cm. If the height of image formed is 50 cm find the distance between the object and the image ?

Sol. As object is in front of the lens, is is real and as



$h_1=25cm,\ f=30cm,\ h_2=-50cm$

$m=\frac{h_2}{h_1}=\frac{-50}{25}=-2$

$m=\frac{f}{f+u} \quad \Rightarrow \quad -2=\frac{30}{30+u}$

$u=-45cm \Rightarrow \quad m=\frac{v}{u} \quad \Rightarrow \quad -2=\frac{v}{-45} \quad \Rightarrow \quad v=90\ cm$

As in this situation object and image are on opposite sides of lens, the distance between object and image

$d_1=u+v=45+90=135\ cm$

If the image is erect (i.e., virtual)

$m=\frac{f}{f+u} \Rightarrow 2=\frac{30}{30+u} \Rightarrow u=-15cm \Rightarrow m=-\frac{v}{u} \Rightarrow 2=\frac{-v}{-15} \Rightarrow v=30\ cm.$

As in the situation both image and object are in front of the lens, the distance between object and image

$d_2=v-u=3015=15\ cm.$

8. A needle placed 45 cm from a lens forms an image on a screen placed 90 cm on other side of the lens. Identify the type of lens and determine its focal length. What is the size of the image, it the size of the needle is 5 cm ?

Sol. Here, u = - 45 cm, v = 90 cm, f = ?, h_1 = 5 cm,

$\because \ \frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \therefore \quad \frac{1}{90}+\frac{1}{45}=\frac{1}{f} \quad \Rightarrow \quad \frac{1+2}{90}=\frac{1}{f} \quad or \quad f=30\ cm$

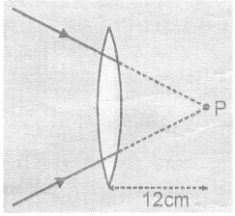
As f is positive, the lens is converging

$\because \ \frac{h_2}{h_1}=\frac{v}{u} \quad \therefore \quad \frac{h_2}{5}=\frac{90}{-45}=-2 \quad \Rightarrow \quad h_2=-10\ cm.$

Minus sign indicates that image is real and inverted

9. A beam of light converges to a point. P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm. (b) a concave lens of focal length 16 cm.

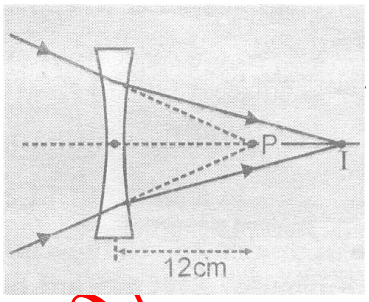
Sol. Here, the point P on the right of the lens acts as a virtual object,



$\therefore$ u = 12 cm, v = ?

(a) f = 20 cm $\quad \therefore \frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \therefore \frac{1}{v}-\frac{1}{12}=\frac{1}{20}$

$\Rightarrow \frac{1}{v}=\frac{1}{20}+\frac{1}{12}=\frac{3+5}{60}=\frac{8}{60} \quad \Rightarrow v=\frac{60}{8}=7.5\ cm$



(b) f = -16 cm, u = 12 cm

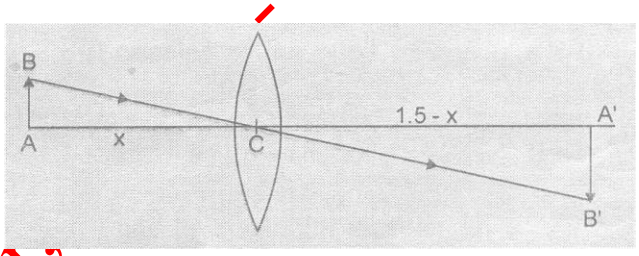
$\therefore \frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{-16}+\frac{1}{12}=\frac{3+4}{48}=\frac{1}{48}$

$\Rightarrow$ v = 48cm

Hence image is at 48 cm to the right of the lens, where the beam would converge.

10. An object is placed at a distant of 1.50 m from a screen and a convex lens placed in between produces an image magnified 4 times on the screen. What is the focal length and the position of the lens.

Sol. $m=\frac{h_2}{h_1}=-4$



Let lens is placed at a distance of x from the object.

Then u = - x, and v = (1.5 - x)

using $m=\frac{v}{u}$, we get $-4=\frac{1.5-x}{-x} \Rightarrow x=0.3$ meter

The lens is placed at a distance of 0.3 m from the object (or 1.20m from the screen)

For focal length, we may use

$m=\frac{f}{f+u}$ or $-4=\frac{f}{f+(-0.3)} \Rightarrow f=\frac{1.2}{5}=0.24\ cm$

11. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water ? If water is replaced by a liquid of refractive index 1.63 up to the same height. What will be apparent depth ?

Sol. Here, real depth = 12.5 cm: apparent depth = 9.4 cm : $\mu = ?$

$\because \quad \mu \frac{real depth}{apparent\ depth} \quad \therefore \quad \mu=\frac{125}{9.4}=1.33$

Now, in the second case, $\mu = 1.63$, real depth = 12.5 cm: apparent depth d_{ap} = ?

$\therefore \quad 1.63=\frac{12.5}{d_{ap}} \quad \Rightarrow \quad d_{ap}=\frac{12.5}{1.63}=7.67\ cm$

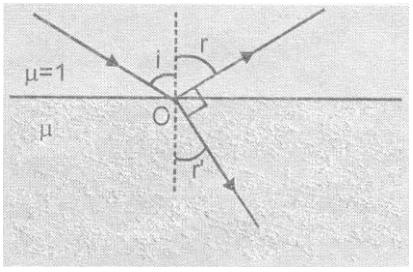
12. A ray of light is incident on a transparent glass slab of refractive index 1.62. If the reflected and refracted rays are mutually perpendicular, what is the angle of incidence ? [$\tan^{-1}(1.62) = 58.3^0$]

Sol. According to given problem, $r + 90^0 + r' = 180^0$

i.e. $r' = 90^0 - r$

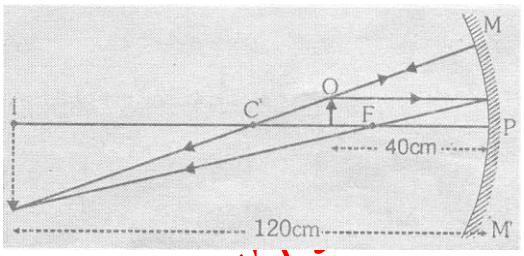
or $r' = (90^0 - i)$ $\Rightarrow$ $\sin i = \mu \cos i [\because \sin(90-i)=\cos i]$

or $\tan i = \mu$ or $i = \tan^{-1} \mu = \tan^{-1}(1.62 = 58.3^0)$



13. The focal length of a concave mirror is 30cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

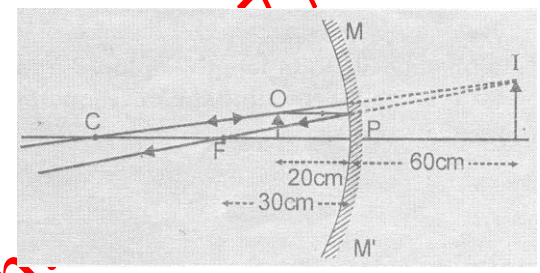
Sol. As the object is in front of the mirror it is real and for real object the magnified image formed by concave mirror can be inverted (i.e., real) or erect (i.e., virtual), so there are two possibilities.



(a) If the image is inverted (i.e., real)

$$m = \frac{f}{f-u} \Rightarrow -3 = \frac{-30}{-30-u} \Rightarrow u = -40\ cm$$

Object must be at a distance of 40 cm in front of the mirror (in between C and F).



(b) If the image is erect (i.e., virtual)

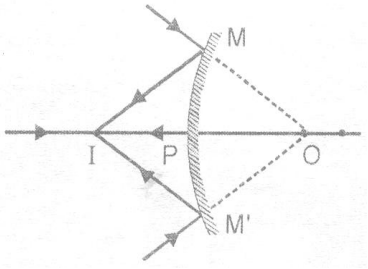
$$m = \frac{f}{f-u}$$

$$3 = \frac{-30}{-30-u} \Rightarrow u = -20cm$$

Object must be at a distance of 20 cm in front of the mirror (in between F and P).

14. A beam of light converges towards a point O, behind a convex mirror of focal length 20 cm. Find the nature and position of image if the point O is (a) 10 cm behind the mirror (b) 30 cm behind the mirror.

Sol. (a) For the situation object will be virtual as shown in figure.



Here u = + 10cm and f = + 20cm.

$$\therefore \quad \frac{1}{v} + \frac{1}{10} = \frac{1}{+20} \qquad \text{i.e., v = -20 cm}$$

i.e., the image will be at a distance of 20 cm in front of the mirror and will be real, erect and enlarged with

$$m = -\left[-\frac{20}{10}\right] = +2$$

(b) For this situation also object will be virtual as shown in Figure.

Here, u = + 30 cm and f = + 20cm

$$\therefore \quad \frac{1}{v} + \frac{1}{+30} = \frac{1}{+20} \qquad \text{i.e., v = + 60 cm}$$

i.e., the image will be a distance of 60 cm behind the mirror and will be virtual, inverted and enlarged with

$$m = -\left[+\frac{60}{30}\right] = -2$$

15. An object of size 7.0 cm is placed at 27cm in front of a concave mirror focal length 18 cm. At what distance from the mirror should a screen be placed, so that a sharp focused image can be obtained ? Find the size and the nature of the image.

Sol. Mirror : concave u=-27cm, f = -18 cm, v = ?

Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-18} + \frac{1}{27} = \frac{-3+2}{54} = -\frac{1}{54}\ cm \qquad \text{v = - 54 cm}$$

The screen should be placed at a distance of 54 cm from the mirror in front of it.

$$m = -\frac{v}{u} = -\frac{-54}{-27} = -2$$

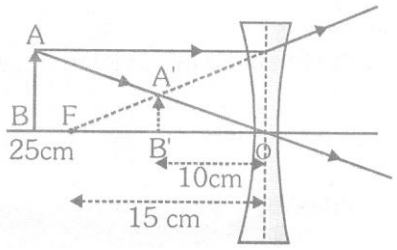
Image is real and magnified, two times the object $-2 = \frac{h'}{h}$

$$h' = -2 \times 7cm = -14cm \text{ high}$$

The image is real, inverted, enlarged and 14 cm high.

16. A concave lens of length 15 cm forms an image 10 cm from the lens. How far is the object placed from the lens ? Draw the ray diagram – [NCERT]

Sol. Using lens formula. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



$$\frac{1}{-10cm} - \frac{1}{u} = \frac{1}{-15cm}$$

$$\frac{1}{u} = \frac{1}{15} - \frac{1}{10} = \frac{2-3}{30} = -\frac{1}{30} \qquad u = -30cm$$

Thus, the object is placed at a distance of 30 cm from concave lens.

17. An object 5.0 cm in length is placed at a distance of 20 cm in front of a convex mirror of radius of curvature 30 cm. Find the position of the image, its nature and size.

Sol. Radius of curvature of convex mirror (R) = 30 cm

$\therefore$ Focal length of convex mirror $(f) = \frac{R}{2} = \frac{30cm}{2} = 15\ cm$

Using mirror formula , $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Object distance, u = - 20 cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} + \frac{1}{20} = \frac{4+3}{60} = \frac{7}{60}cm \qquad v = \frac{60}{7}cm = 8.57cm$$

The image is virtual, behind the mirror and erect.

$$m = \frac{h'}{h} = -\frac{v}{u} = \frac{h'}{5cm} = \frac{-60/7cm}{-20cm} \Rightarrow h' = \frac{-60/7cm \times 5cm}{-20cm} = \frac{3}{7} \times cm = \frac{15}{7}cm$$

h' = + 2.14 cm [It is erect and diminished].

18. The near point of a certain eye is 100cm in front of the eye. What lens should be used to see clearly an object 25 cm in front of the eye ?

Sol. u = - 25 cm v = - 100 cm

using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-100cm} - \frac{1}{-25cm} = \frac{-1+4}{100cm} = \frac{3}{100}cm$

$$f = \frac{100}{3}cm = 33.3cm$$

Hence a converging lens of focal length 33.3 cm is required.

19. Light enters from air to glass having refractive index 1.50. What is the speed of light in the glass ? The speed of light a vacuum is 3×10^8 m/s.

Sol. Refractive index $\mu_g = 1.5$

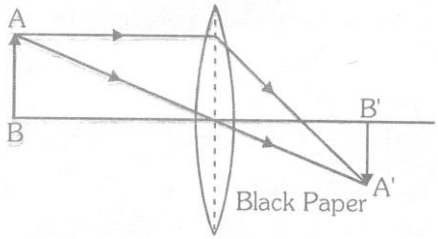
Speed of light in vacuum c = 3×10^8 m/s

Speed of light in glass, v = ?

$$\mu_g = \frac{c}{v}$$

$$v = \frac{c}{\mu_g} = \frac{3\times10^8}{1.5} = 2\times10^8 \ \ m/s$$

20. One-half of a convex lens is covered with a black paper. Will this lens produce a complete image of the object ? Verify your answer experimentally. Explain your observations. [NCERT]



Sol. Yes, it will produce a complete image of the object, as shown in fig. This can be verified experimentally by observing the image of a distance object like tree on a screen, when half of the is covered with a black paper. However, the intensity of brightness of image will reduce.

21. An object is placed at a distance of 10 cm from a convex mirror of focal length 15 cm. Find the position and nature of the image. [NCERT]

Sol. Here, object distance, u = - 10 cm focal length, f = 15 cm, image distance, v = ?

As $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\therefore \quad \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{15}+\frac{1}{10}=\frac{5}{30}=\frac{1}{6}, \qquad f = 6 \ \ cm$$

Here, + sign of v indicates image is at the back of the mirror. It must be virtual, erect and smaller in size than the object .

22. The magnification produced by a plane mirror is +1. What does this mean ?

Sol. As $m=\frac{h_2}{h_1}=+1, \quad h_2 = h_1$

i.e., size of image is equal to size of the object. Further, + sign of m indicates that the image is erect and hence virtual.

23. Find the focal length of a lens of power - 2.0 D. What type of lens is this ?

Sol. Here, focal length f = ?, power P = - 2.0 D

As $f = \frac{100}{P}$

$$\therefore \quad f = \frac{100}{-2.0} = -50 \ \ cm$$

24. A person needs a lens of power -5.5 dioptres for correcting his distant vision. For correcting his near vision he needs a lens power + 1.5 dioptre. What is the focal length of the lens required for correcting (i) distant vision, and (ii) near vision ?

25. The far point of a myopic person is 80 cm in front of the eye. What is the nature and power of the lens required to correct the problem ?

Sol. Distance of far point, x = 80 cm, P = ?

For viewing distant objects, focal length of corrective lens,

f = - x = 80 cm

$P = \frac{100}{f} - \frac{100}{-80} = -1.25 \ \ D.$ The lens is concave.

26. Make a diagram to show how Hypermetropia is corrected. The near point of a Hypermetropia eye is 1 m. What is the power of the lens required to correct this defect ? Assume that the near point of the normal eye is 25 cm.

Sol. Make diagram yourself

Here , x' = 1m = 100 cm, d = 25, f = ?

From $f = \frac{x'd}{x'-d}$

$$f = \frac{100 \times 25}{100 - 25} = 33.3 \ cm$$

$$P = \frac{100}{f} = \frac{100}{33.3} = 3D$$

27. Why is a normal eye not able to see clearly the objects placed closer than 25 cm ? **[NCERT]**

Sol. This is because the focal length of eye lens cannot be decreased below a certain minimum limit.

EXERCISE – 1 **(FOR SCHOOL/BOARD EXAMS)**

OBJECTIVE TYPE QUESTIONS

Choose the Correct One

1. A mirror forms a virtual image of a real object.

(A) It must be a convex mirror. (B) It must be a concave mirror.

(C) It must be a plane mirror. (D) It may be any of the mirror mentioned above.

2. The angle of incidence is the angle between

(A) the incident ray and the surface of the mirror (B) the reflected ray and the surface of the mirror

(C) the normal to the surface and the incident ray (D) the normal to the surface and the reflected ray

3. The angle of reflection is the angle between

(A) the incident ray and the surface of the mirror (B) the reflected ray and the surface of the mirror

(C) the normal to the surface and the incident ray (D) the normal to the surface and the reflected ray

4. An object is placed at the centre of curvature of a concave mirror. The distance between its image and the pole is

(A) equal to f (B) between f and 2f (C) equal to 2f (D) greater then 2f

5. An object of size 2.0 cm is laced perpendicular to the principal axis of a concave mirror. The distance of the object from the mirror equals the radius of curvature. The size of the image will be

(A) 0.5 cm (B) 1.5 cm (C) 1.0 cm (D) 2.0 cm

6. The magnification m of an image formed by a spherical mirror is negative. It means, the image is

(A) smaller than the object (B) larger than the object

(C) erect (D) inverted

7. A point object is placed on the principal axis of a spherical mirror. The object-distance u is

(A) definitely negative

(B) definitely positive

(C) positive if the object is to the left of the centre of curvature

(D) positive if the object is to the right of the centre of curvature

8. $f = \frac{R}{2}$ is valid

(A) for convex mirrors but not for concave mirrors

(B) for concave mirror but not for convex mirrors

(C) for both convex and concave mirrors

(D) neither for convex mirrors nor for concave mirror

9. A ray of light is incident on a concave mirror. If it is parallel to the principal axis, the reflected ray will

(A) pass through the focus

(B) pass through the centre of curvature

(C) pass through the pole

(D) retrace its path

10. If an incident ray passes through the centre of curvature of a spherical mirror, the reflected ray will

(A) pass through the pole (B) pass through the focus

(C) retrace its path (D) be parallel to the principal axis

11. To get an image larger than the object, one can use

(A) a convex mirror but not a concave mirror

(B) a concave mirror but not a convex mirror

(C) either a convex mirror or a concave mirror

(D) a plane mirror

12. A ray of light travelling in air falls obliquely on the surface of a calm pond. It will

(A) go into the water without deviating from its path (B) deviate away from the normal

(C) deviate towards the normal (D) turn back on its original path

13. A ray of light goes from a medium of refractive index μ_1 to a medium of refractive index μ_2. The angle of incidence is i and the angle of refraction is r. Then, sin i/sin r is equal to

(A) μ_1 (B) μ_2 (C) $\frac{\mu_1}{\mu_2}$ (D) $\frac{\mu_2}{\mu_1}$

14. A thin lens and a spherical mirror have a focal length of + 15 cm each.

(A) both are convex.

(B) the lens is convex and the mirror is concave.

(C) the lens is concave and the mirror is convex.

(D) both are concave.

15. A convex is

(A) is thicker at the middle than at edges (B) is thicker at the edges than at the middle

(C) has uniform thickness everywhere (D) is called a diverging lens

16. A convex lens forms a virtual image when an object is placed at a distance of 18 cm from it. The focal length must be

(A) greater than 36 cm (B) greater than 18 cm (C) less than 36 cm (D) less than 18 cm

17. An object is placed before a convex lens. The image formed

(A) is always real (B) may be real or virtual (C) is always virtual (D) is always erect

18. An object is placed before a concave lens. The image formed

(A) is always erect (B) may be erect or inverted (C) is always inverted (D) is always real

19. A lens has a power of +0.5 D. If is

(A) a concave lens of focal length 5 m (B) a convex lens of focal length 5 cm

(C) a convex lens of focal length 2 m (D) a concave lens of focal length 2 m

20. A parallel beam of light falling on the eye gets focused on the retina because of refractions at

(A) the cornea (B) the crystalline lens (C) the vitreous humor (D) various surfaces in the eye

21. The combination responsible for admitting different amounts of light into the eye is

(A) Ciliary muscles and crystalline lens (B) Ciliary muscles and pupil

(C) iris and pupil (D) rods and cones

22. The muscles of the iris control the

(A) focal length of the eye-lens (B) opening of the pupil

(C) shape of the crystalline lens (D) optic nerve

23. When the eye is focused on an object very far away, the focal length of the eye-lens is

(A) maximum (B) minimum

(C) equal to that of the crystalline lens (D) half its maximum focal length

14. Other names for myopia are

(A) hyperopia and Hypermetropia (B) long-sightedness and hyperopia

(C) near-sightedness and Presbyiopia (D) hear-sightedness and short-sightedness

25. The inability among the elderly to see nearby object clearly because of the weakening of the Ciliary muscles is called

(A) far-sightedness (B) near-sightedness (C) Presbyiopia (D) astigmatism

26. When white light passes through a prism, it splits into its component colors. This phenomenon is called

(A) spectrum (B) reflection (C) refraction (D) dispersion

27. The number of surface bounding a prism is

(A) 3 (B) 4 (C) 5 (D) 6

28. A deviation in the path of a ray of light can be produced

(A) by a glass prism but not by a rectangular glass

(B) by a rectangular glass slab but not by a glass prism

(C) by a glass prism as well as a rectangular glass slab

(D) neither by a glass prism nor by a rectangular glass slab

29. The wavelengths corresponding to violet, yellow and red lights are λ_v, λ_y and λ_r respectively.

(A) $\lambda_v > \lambda_y > \lambda_r$ (B) $\lambda_v < \lambda_y < \lambda_r$ (C) $\lambda_y < \lambda_v < \lambda_r$ (D) $\lambda_y < \lambda_r < \lambda_v$

30. If you want to see your full image then the , minimum size of the plane mirror

(A) should be of your height (B) should be half of your height.

(C) should be twice of height. (D) depends upon your distance from the mirror

31. An object A is placed at a distance d in front of a plane mirror. If one stands directly behind the object at a distance S from the mirror, then the distance of the image of A from the individual is

(A) 2 S (B) 2 d (C) S + d (D) S + 2d

32. A light bulb is placed between two plane mirrors inclined at an angle of 60^0. the number of image formed are

(A) 6 (B) 5 (C) 4 (D) 2

33. A person of height 1.8 standing at the centre of a room having equal dimensions of 10 m wishes to see the full image of the back wall in the mirror fixed on the front wall. The minimum height of the plane mirror needed for this purpose is

(A) 0.9 m (B) 1.8 m (C) $\frac{10}{3}$ m (D) 10 m

34. Two plane mirrors inclined at an angle to one another have an object placed between them. If five images of the object are observed, the maximum possible angle between the mirrors is

(A) 45^0 (B) 60^0 (C) 72^0 (D) 90^0

35. In case of a thick plane mirror multiple images arc formed. The brightest of all the images will be

(A) first (B) second (C) third (D) fourth

36. Indicate the only correct statement.

(A) The image formed by a convex mirror can be taken on the screen.

(B) A convex mirror can produce a parallel beam of light from a point source.

(C) The image of an object placed at the focus of a convex mirror will be formed at infinity.

(D) A concave mirror can never form a diminished virtual image.

37. The focal length of concave mirror is f and the distance from the object to the principal focus is x. The magnification obtained will be

(A) (f + x)/f (B) f / x (C) $\frac{\sqrt{f}}{\sqrt{x}}$ (D) f^2 / x^2

38. In a museum a child walks towards a large concave mirror. He will see that
(A) his real, erect image goes on decreasing in size.
(B) his virtual, erect image goes on increasing in size.
(C) his real, inverted image goes on diminishing in size and suddenly it becomes virtual, erect and magnified.
(D) his real, erect image goes on diminishing in size and suddenly it becomes virtual, erect and magnified.

39. The magnification of an object placed 10 cm from a convex mirror of radius of curvature 20 cm will be
(A) 0.2 (B) 0.5 (C) 1 (D) 20 cm

40. A concave mirror of focal length 10 cm produces an image five times as large as large as the object. It the image is in front of the mirror, the distance of the object from the mirror will be
(A) 10 cm (B) 12 cm (C) 16 cm (D) 20 cm

41. To form an image twice the size of the object, using a convex lens of focal length 20 cm, the object distance must be –
(A) < 20 cm (B) > 20 cm
(C) < 20 cm and between 20 cm and 40 cm (D) Cannot say

42. Refractive index of glass with respect to air is 3/2. What is the refractive index of air with respect to glass ?
(A) 2/3 (B) 1 (C) Zero (D) $(3/2)^2$

43. The mirage is formed due to
(A) reflection (B) refraction (C) total internal reflection (D) dispersion

44. A glass prism has refractive index 1.5 and the refracting angle 90^0 If a ray falls on it at an angle of incidence
(A) 60^0 (B) 45^0
(C) 30^0 (D) the ray will not emerge at all

45. When monochromatic light passes from vacuum to a material medium and vice versa : which of the following characteristics of light beam does not changes ?
(A) velocity (B) intensity (C) wavelength (D) frequency

46. A piece of glass when immersed in a transparent solution of refractive index 1.48 becomes almost invisible. The refractive index of glass used is
(A) zero (B) 1 (C) 1.48 (D) infinite

47. Which of the following conditions are necessary for total internal reflection to take place at the boundary of two optical media ?
I. Light is passing from optically denser medium to optically rarer medium.
II. Light is passing from optically rarer medium to optically denser medium.
III. Angle of incidence is greater than the critical angle.
IV. Angle of incidence is less than the critical angle.
(A) I and III only (B) II and IV only (C) III and IV only (D) I and IV only

48. The speed of light in vacuum is 3.0×10^8 m/s. If the refractive index of a transparent liquid is 4/3, then the speed of light in the liquid is
(A) 2.25×10^8 m/s (B) 3×10^8 m/s (C) 4×10^8 m/s (D) 4.33×10^8 m/s

49. The refractive indices of water and glass are 4/3 and 3/2 respectively. The refractive index of water with respect to glass is
(A) $\frac{8}{9}$ (B) 2 (C) $\frac{2}{3}$ (D) $\frac{1}{6}$

50. Which of the following can be used to form a virtual image of an object ?
I. convex lens II. concave lens III. concave mirror
(A) II only (B) II and III only (C) I and III only (D) I, II and III

ANSWER KEY															
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**D**	**C**	**D**	**C**	**D**	**D**	**A**	**C**	**A**	**C**	**B**	**C**	**D**	**A**	**A**
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	**B**	**B**	**A**	**C**	**D**	**C**	**B**	**A**	**D**	**C**	**D**	**C**	**A**	**B**	**B**
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	**C**	**B**	**C**	**C**	**B**	**D**	**B**	**C**	**B**	**B**	**C**	**A**	**C**	**D**	**D**
Que.	**46**	**47**	**48**	**49**	**50**										
Ans.	**C**	**A**	**A**	**A**	**D**										

EXERCISE - 2 **(FOR SCHOOL/BOARD EXAMS)**

1. An object is placed in front of a plane mirror. If the mirror is moved away from the object through a distance x, be how much distance will the image move

2. A ray of light on a plane mirror. Show that if the mirror is tilted through an angel θ, the reflected ray tilts through an angle 2θ.

3. A 2cm high object is placed at a distance of 32cm from a concave mirror. The image is real, inverted and 3 cm in size. Find the focal length of the mirror and the position where the image is formed.

4. A ray of light travelling in air falls on the surface of a glass slab. The ray makes an angle of 45^0 with the normal to the surface. Find the angle made by the refracted ray with the normal within the slab. Refractive index of glass = 3/2.

5. Yellow light of wavelength 590 nm travelling in air is refracted into water $\left(\mu = \frac{4}{3}\right)$. Find the wavelength of this light in water.

6. A cube of edge 6 cm is placed over a printed page. At what distance from the top surface of the cube will be letters appear when seen from above ? Refractive index of glass = 1.5.

7. An object is placed on the principal axis of a concave lens at a distance of 20 cm from it. If the focal length of the lens is also 20 cm. find the location of the image.

8. A beam of light travelling to the principal axis of a concave lens appears to diverge from a point 20 cm behind the lens after passing through the lens. Find the power of the lens.

9. A convex lens of power 4D is placed at a distance of 40 cm from a wall. At what distance from the lens should a candle of be placed so that its image is formed on the wall ?

10. A convex lens of focal length 20 cm is placed in contact with a concave lens of focal length 12.5 cm in such a way that they have the same principal axis. Find the power of the combination.

11. A convex mirror used for rear-view on an automobile has a radius of curvature of 3.00 m. If a bus is located at 5.00 m from this mirror, find the position, nature and size of the image .

12. An object, 4.0 cm in size, is placed at 25.0 cm in front of a concave mirror of focal length 15.0 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image ? Find the nature and the size of the image.

13. A concave lens has focal length of 15 cm. At what distance should the object from the lens be placed so that it forms an image at 10 cm from the lens ? Also, find the magnification produced by the lens.

14. A 2.0 cm tall object is placed perpendicular to the principal axis of a convex lens of focal length 10 cm. The distance of the object from the lens is 15 cm. Find the nature, position and size of the image. Also find its magnification.

15. An object placed in front of a diverging mirror at a distance of 30 cm, forms a virtual and erect image which is 1/5 of the size of the object. Calculate : (i) the position of the image , (ii) the focal length of the diverging mirror.

16. A light of wavelength 500 nm in air enters a glass block of refractive index 1.5. Find (a) speed; (b) frequency ; (c) wavelength of light in glass. Velocity of light in air is 3×10^8 m/s.

17. Consider a system of two plane mirror inclined to each other at a right angel. Show that a ray of light is incident on the system, the outgoing ray is parallel to the incident ray and this result is independent of the incident direction.

18. A near sighted person wears eye glass with power of -5.5D for distance vision. His doctor prescribes a correction of + 1.5D in near vision section of his bifocals, which is measured relative to main part of the lens.

(I) What is the focal length of his distant viewing port of lens ?

(II) What is the focal length of near vision section of the lens ?

19. The radius of curvature of a convex mirror used on a moving automobile is 2.0 m. A truck is coming behind it at a constant distance of 3.5 m. Calculate (i) the position, and (ii) the size of image relative to the size of the truck. What will be the nature of the image ?

20. The refractive index of dense flint is 1.65, and for alcohol, it is 1.36 with respect to air. What is the refractive index of the dense flint glass with respect to alcohol ?

21. A convex lens forms a real and inverted image of a needle at a distance of 50 cm from the lens. Where is the needle placed in front the convex lens, so that this image is of the same size as the object ? Also, find the power of the lens.

22. A person can not see objects distinctly at distances less than 1 m. Calculate the power of the spectacles lens that he should use in order to read a book at a distance of 25 cm.

23. Name the type of mirror used in the following situations:

(a) Head lights of a car.

(b) Side rear view mirror of a vehicle.

(d) Solar furnace.

Support your answer with reason.

24. What kind of lens can form a (i) Virtual, erect diminished image ? (ii) virtual, erect, magnified image ?

25. Which lens has greater power, a convex lens of focal length 10 cm or a convex lens of focal length 20 cm ?

26. A man standing in front of a special mirror finds his image having a small face, big tummy and legs of normal size. What are the shapes of three parts of the mirror ?

27. Can you change focal length of a given spherical mirror by changing the object distance from the mirror ?

28. Can you change linear magnification of a spherical mirror by changing the object distance from the mirror ?

29. What is the basic cause of refraction ?

30. What are the conditions for no refraction of light ?

31. A concave mirror is used as a head mirror by ENT specialists. The same mirror also be used as a shaving mirror. Why ?

ANSWER KEY **EXERCISE -2(X)CBS**

1. 2x **3.** f = -192 cm, v = - 48 cm **4.** 28^0 **5.** 442.5 nm **6.** 4 cm **7.** – 10 cm **8.** - 5D **9.** $-\frac{200}{3}$ cm

10. -3D **11.** 1.15 m, +0.23 **12.** - 37.5 m, - 6.0 cm **13.** – 30 cm, + 0.33 **14.**+30 cm – 4.0 cm, m = - 2

15. (i) 6 cm (ii) 7.5 cm **16.** (a) 2×10^8 m/s (b) Same (c) 333.3 nm

18. (i) -18.18 cm (ii) -25 cm **19.** (i) 0.78 m (ii) 0.22 **20.** 1.21 **21.** - 50 cm, 4 D **22.** + 3D

EXERCISE – 3 **(FORSCHOOL/BOARDEXAMS)**

PREVIOUS YEARS BOARD (CBSE) QUESTIONS :

VERY SHORT ANSWER QUESTIONS (CARRYING 1 MARKS FACH)

1. If the magnification of a body of size 1 m is 2, what is the size of the image ? **(2003)**

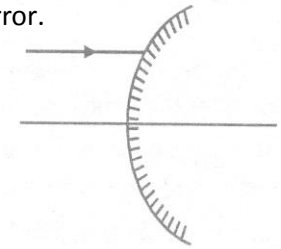
2. What is the power of a concave lens of focal length 25 cm? **(2004)**

3. What will be the focal length of a lens whose power is given as + 2.0 d ? **(2004)**

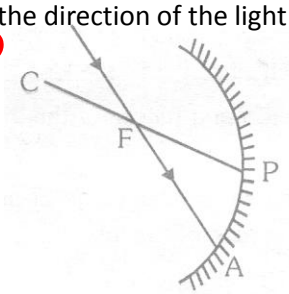
4. Where will the image be formed by a concave mirror when an object is placed between the pole and the focus point of the mirror ? **(2005)**

5. What is the value of focal length of a plane mirror ? **(2005)**

6. A ray of light is incident on a convex mirror as shown in fig. redraw the above diagram after completing the path of the light ray after reflection from the mirror. **(2006)**

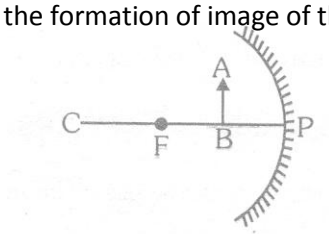


7. Copy fig. in your answer book and show the direction of the light ray after reflection . **(2008)**

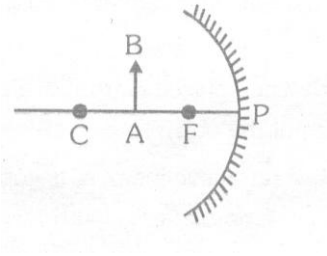


8. The refractive index of diamond is 2.42. What is the meaning of this statement in relation to speed of light ? **(2008)**

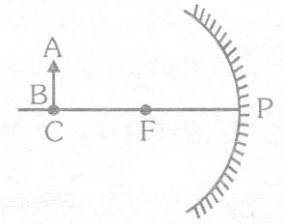
9. Draw fig. in your answer book and show the formation of image of the object AB with the help of suitable rays.



10. Draw fig. in your answer book and show the formation of image of the object AB with the help of suitable rays.



11. Draw fig. in your answer book and show the formation of image with the help of suitable rays.



SHORT ANSWER QUESTIONS (CARRYING 2 MARKS EACH)

12. Give the characteristics of image formed by a plane mirror. **(2003)**

13. An object is placed at 0.06 m from a convex lens of focal length 0.1 m. Calculate the position of the image? **(2004)**

14. An object is placed at a distance of 20 cm in front of a convex mirror of radius of curvature 30 cm. Find the position and nature of the image. **(2004)**

15. Light enters from air diamond, which has a reflective index of 2.42. Calculate has speed of light in diamond. The speed of light in air is 3×10^8 ms^{-1}. **(2005)**

16. With the help of ray diagrams, show the formation of the image of an object by a concave mirror. When it is placed (i) beyond the centre of curvature (ii) at the centre of curvature. **(2005)**

17. Draw ray diagrams to show the formation of image of an object by a concave mirror, when it is placed (i) between its centre of curvature C and focus F (ii) between pole P of mirror and its focus F. **(2005)**

18. With respect to air, the refractive index of ice is 1.31 and that of rock salt is 1.54. Calculate the refractive index of rock salt w.r.t. ice. **(2005)**

19. Draw ray diagrams to show the formation of image of an object by a concave lens when the object is placed. (i) at infinity (ii) between infinity and optical centre of the lens. **(2005)**

20. Draw a ray diagram to show the position and nature of the image formed when an object is placed between focus F and pole P of a concave mirror . **(2006)**

21. Draw ray diagrams to show the formation of the image of an object by a convex mirror, when it is placed (i) at infinity and (ii) between infinity and pole of the mirror . **(2006)**

22. Draw a labeled ray diagram to locate the image of an object formed by a convex lens of focal length 20 cm when the object is placed 20 cm away from the lens. **(2008)**

23. Explain with the help of a diagram, why a pencil partly immersed in water appears to be bent at the water surface. **(2008)**

24. Draw the ray diagrams to represent the nature, position and relative size of the image formed by a convex lens for the object placed (i) at 2 F_1 (ii) between F_1 and optical center O of the lens. **(2008)**

SHORT ANSWER QUESTIONS (CARRYING 3 MARKS EACH)

25. Calculate the distance at which an object should be placed in front of a thin convex lens of focal length 10 cm to obtain a virtual image of double it size. **(2003)**

26. A convex lens of focal length 40 cm is placed in contact with a concave lens of focal length 25 cm. What is the power is combination ? **(2003)**

27. Find the position of an object , which when placed in front of a concave mirror of focal length 20 cm produces a virtual image, which is twice the size of the object. **(2003)**

28. a concave lens made of a material of refractive index n_1 is kept in medium of refractive index n_2. A parallel beam of light is incident on the lens. Complete the path of rays of light emerging from the concave lens if
(i) $n_1 > n_2$ (ii) $n_1 = n_2$ (iii) $n_1 < n_2$.

29. Find the position, nature and size of the image formed by a convex lens of focal length 20 cm of an object 4 cm high placed at a distance of 30 cm from it. **(2004)**

30. A convex lens has focal length of 30 cm. Calculate at what distance should the object be placed from the lens so that it forms an image at 60 cm on the other side of the lens ? Find the magnification produced by the lens in this case. **(2004)**

31. Find the option, nature and size of the image of an object 3 cm high placed at a distance of 9 from a concave of focal length 18 cm. **(2004)**

32. An object 4 cm high is placed 40-0 cm in front of a concave mirror of focal length 20 cm. Find the distance from the mirror, at which a screen be placed in order to obtain a sharp image. Also find the size and nature of the image **(2005)**

33. An object is placed at a distance of 12 cm in front of a concave mirror. It forms a real image four times larger than the object. Calculate the distance of the image from the mirror.

34. A 5.0 cm tall object is placed perpendicular to the principal axis of a convex lens of focal 20 cm. The distance of the object from the lens from the lens is 30 cm. By calculation, determine (i) the position (ii) the size of the image formed.

35. An object 320 cm high is placed perpendicular to the principal axis of a concave lens of length 7.5 cm. The image s formed at a distance of 5.0 cm from the lens. Calculate (i) distance at which object is placed, and (ii) size and nature of image formed. **(2006)**

36. A concave lens has focal length of 20 cm At what distance from the lens 5 cm tall object be placed so that it forms an image at 15 cm from the lens ? Also, calculate the size of the image formed. **(Delhi, 2007)**

37. An object 50 cm tall is placed on the principal axis of a convex lens. Its 20 cm tall image is formed on the screen placed at a distance of 10 cm from the lens. Calculate the focal length of the lens. **(2007)**

LONG ANSWER QUESTIONS (CARRYING 5 MARKS EACH)

38. Draw ray diagrams to show the formation of images when the object is placed in front of a concave mirror
(i) between its pole and focus point.
(ii) between the centre of curvature and focus point.

39. (a) State the relation between object distance, image distance and focal length of a spherical mirror .
(b) Draw a ray diagram to show the image formed by a concave mirror when an object is placed between pole and focus of the mirror.
(c) A concave mirror of focal length 5 cm forms an image of an object kept at a distance of 10 cm from the mirror. Find the position, nature and size of the image formed by it.

VERY SHORT ANSWER QUESTIONS (CARRYING 1 MARKS EACH)

40. What is meant by persistence of vision ? **(2003)**

41. What kind of lens is used in the spectacles of a person suffering from myopia (near sightedness) ? **(2006)**

42. List three common defects of vision that can be corrected with the use of spectacles. **(2006)**

43. Write the function of Iris in the human eye. **(2007)**

44. To an astronaut, why does the sky appear dark instead of blur ? **(2008)**

45. Why does the sun appear reddish at sunrise ? **(2008)**

46. Why is red colour selected for danger signal lights ? **(2008)**

SHORT ANSWER QUESTIONS (CARRYING 2 MARKS EACH)

47. The far point of a myopic person is 80 cm in front of the eyes. What is the nature and power of the lens required to enable him to see very distant object distinctly ? **(2003)**

48. Explain about the colour of the sun at sunrise and sunset. **(2007)**

SHORT ANSWER QUESTIONS (CARRYING 3 MARKS EACH)

49. What are the conditions for formation of rainbow ? **(2003)**

50. Draw a labeled diagram of human eye.
What is power of accommodation of eye ? Define colour blindness. **(2005)**

51. (a) Draw a diagram to show the formation of image of a distant object by a myopic eye. How can such an eye defect be remedied ?
(b) State two reasons due to which eye defect may be caused.
(c) A person with myopic eye cannot see objects beyond a distance of 1’5 m. What would be the power of the corrective lens used to restore proper vision ? **(2008)**

LONG ANSWER QUESTIONS (CARRYING 5 MARKS EACH)

52. (a) State two main causes of a person developing near sightedness. With the help of a ray diagram, suggest how he can be helped to overcome this disability ?
(b) The far point a myopic person is 150 cm in front of the eye. Calculate the focal length and power of a lens required to enable him to see distant object clearly. **(2004)**

53. What is long sightedness ? List two causes for development of long-sightedness. Describe with a ray diagram, how this defect may be corrected by using spectacles. **(2005)**

54. (a) Explain the following terms used in relation to defects of vision and corrections provided for them : (i) Myopia
(ii) Astigmatism (iii) Bifocal lenses (iv) Far sightedness.
(b) Describe with a ray diagram how a person with myopia can be helped by spectacles. **(2005)**

55. A 14 year old student is not able to see clearly the questions written on blackboard placed at a distance of 45 m from him. (a) Name the defect of vision he is suffering from. (b) With the help of labeled ray diagrams, show how this defect can be corrected. (c) Name the type of lens used to correct the defect. **(2007)**

56. (a) What is meant by dispersion of white light ? Describe the formation of rainbow in the sky with the help of a Diagram.
(b) What is Hypermetropia ? Draw ray diagrams to show the image formation of an object by (i) Hypermetropia eye (ii) correction made with a suitable lens for Hypermetropia eye . **(2008)**

57. (a) Give reasons for the following :
(i) Colour of the clear sky is blue.

(ii) The sun can be seen about two minutes before actual sunrise.
(iii)We cannot see an object clearly if it is placed very close to the eyes.

(b) What is Presbyiopia ? Write two causes of this defect .

EXERCISE – 4 **OPTICS**

1. When does the total internal reflection take place :-
(A) Refraction from air into any denser medium
(B) Refraction of ray incident from rarer medium
(C) Ray incident from denser medium, with angle of refraction 90^0.
(D) Ray incident from denser medium with refractive index is [n > 1 /(sin of angle of incidence)]

2. When the ray of light is incident from denser medium having refractive index 2, what should be the angle of incidence for the ray to go out :-
(A) Less than 30^0 (B) Less than 45^0 (C) Less than 60^0 (D) Less than 90^0

3. A monochromatic beam of light passes from a denser medium into a rarer medium as a result :-
(A) Its velocity increase (B) Its velocity decrease
(C) Its frequency decrees (D) Its wavelength decreases

4. Immiscible transparent liquids A, B, C, D and E are placed in a rectangular container of glass with the liquids making layers according to their densities. The refractive index of the liquids are shown in the adjoining diagram. The container is alluminated from the side and a small piece of glass having refractive index 1.61 is gently dropped into the liquid laxo. The glass piece as descends downwards will not be visible in :-
(A) Liquid A and B only
(B) Liquid C only
(C) Liquid D and E only
(D) Liquid A, B D and E

$\mu = 1{\cdot}64$	A
$\mu = 1{\cdot}63$	B
$\mu = 1{\cdot}61$	C
$\mu = 1{\cdot}64$	D
$\mu = 1{\cdot}6$	E

5. Sensitivity of eye is maximum for :-
(A) 4000 $\mathring{A}$ (B) 8000 $\mathring{A}$ (C) 5550 $\mathring{A}$ (D) 6000 $\mathring{A}$

6. A bird in air looks at a fish vertically below it and inside water. x is the height of the bird above the surface of water an y is the depth of the fish below the surface of water. The distance of the fish as observed by the bird is : (Given μ = refractive index of water w.r.t. air) :-
(A) x + y (B) $x + \frac{y}{\mu}$ (C) μ x + y (D) μ x + μ y

7. In the previous question, the distance of the bird as observed by the fish is :-
(A) x + y (B) $x + \frac{y}{\mu}$ (C) μ x + y (D) μ x + μ y

8. An object is placed between two parallel plane mirror. The number of image formed is
(A) four (B) one (C) two (D) infinite

9. An object is placed between two plane mirrors inclined at some angle to each other . If the number of images formed is 7 then angle of inclination is
(A) 15^0 (B) 30^0 (C) 45^0 (D) 60^0

10. Which of the following letters do not surface lateral inversion.
(A) HGA (B) HOX (C) VET (D) YUL

11. A clock hung on a wall has marks instead of numbers on its dial. On the opposite wall there is a mirror, and the image of the clock in the mirror if read , indicates the time as 8.20. What is the time in the clock –
(A) 3.40 (B) 4.40 (C) 5.20 (D) 4.20

12. If you want to see your full image, then minimum size of the mirror

(A) Should be of your height (B) Should be of your height

(C) Should be twice of your height (D) Depends upon distance from the mirror

13. If an object is placed unsymmetrical between two plane mirrors, inclined at an angle of 72^0, then the total number of image formed is-

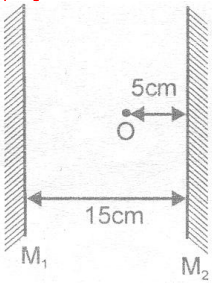
(A) 5 (B) 4 (C) 2 (D) Infinite

14. At what angle must two plane mirrors be placed so that incident and resulting reflected rays are always parallel to each other

(A) 0^0 (B) 30^0 (C) 60^0 (D) 90^0

15. Figure shows two plane mirrors parallel to each other and an object O placed between them. Then the distance of the first three images from the mirror M_2 will be : (in cm)

(A) 5, 10, 15 (B) 5, 15, 30 (C) 5, 25, 35 (D) 5, 15, 25



16. If an object is placed 10 cm in front of a concave mirror a focal length 20 cm, the image will be :-

(A) diminished, upright, virtual (B) enlarged, upright, virtual

(C) diminished, inverted, real (D) enlarged, upright, real

17. The magnification m, the image position v and focal length f are related to one another by the relation –

(A) $m = \frac{f - v}{f}$ (B) $m = \frac{f}{f - v}$ (C) $m = \frac{f + v}{f}$ (D) $m = \frac{f}{f - v}$

18. The relation between magnification m, the object position u and focal length f of the mirror is

(A) $m = \frac{f - v}{f}$ (B) $m = \frac{f}{f - u}$ (C) $m = \frac{f + u}{f}$ (D) $m = \frac{f}{f + u}$

19. v_1 is velocity of light in first medium, v_2 is velocity of light in second medium, then refractive index of second medium with respect to first medium is

(A) v_1/v_2 (B) v_2/v_1 (C) $\sqrt{v_1/v_2}$ (D) $\sqrt{v_2/v_1}$

20. The ratio of the refractive index of red light to blue light in air is

(A) Less than unity

(B) Equal to unity

(C) Greater than unity

(D) Less as well as greater than unity depending upon the experimental arrangement

21. The refractive index of glass and water with respect to air 3/2 and 4/3 respectively. The refractive index of glass with respect to water is

(A) 8/9 (B) 9/8 (C) 2 (D) 1/2

22. If ${}_i\mu_j$ represents refractive index when a light ray goes from medium i to medium j, then the product ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3$ is equal to

(A) ${}_3\mu_1$ (B) ${}_3\mu_2$ (C) $\frac{1}{{}_1\mu_4}$ (C) ${}_4\mu_2$

23. What is the basic reason for the shining of a diamond ?

(A) Reflection (B) Refraction (C) Dispersion of light (D) Total internal reflection

24. Total internal reflection of a ray of light is possible when the (i_c = critical angle, i = angle of incidence)

(A) Ray goes from denser medium to rarer medium and $i < i_c$

(B) Ray goes from denser medium to rarer medium and $i > i_c$

(C) Ray goes from rarer medium to denser medium and $i > i_c$

(D) Ray goes from rarer medium to denser medium and $i < i_c$.

25. A convex lens of focal length A and a concave lens of focal length B are placed in content. The focal length of the combination is

(A) A + B (B) (A – B) (C) $\frac{AB}{(A+B)}$ (D) $\frac{AB}{(B-A)}$

26. Near and far points of a human eye are

(A) zero and 25 cm (B) 25 cm and 50 cm (C) 50 cm and 100 cm (D) 25 cm and infinite

27. The focal length of a concave mirror is f and the distance from the object to the principal focus is x. Then the ratio of the size of the image to the size of the object is-

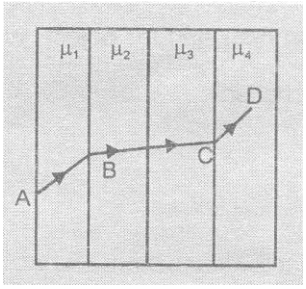
(A) $\frac{(f+x)}{f}$ (B) $\frac{f}{x}$ (C) $\sqrt{\frac{f}{x}}$ (D) $\frac{f^2}{x^2}$

28. Light travels through a glass plate of thickness t and having refractive index n. If c is the velocity of light in vacuum. The time taken by the light to travel this thickness of glass is :-

(A) $\frac{t}{nc}$ (B) tnc (C) $\frac{nt}{c}$ (D) $\frac{tc}{n}$

29. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as show in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have :

(A) $\mu_1 = \mu_2$ (B) $\mu_2 = \mu_3$ (C) $\mu_3 = \mu_4$ (D) $\mu_4 = \mu_1$



30. Which of the following is used in optical fires ?

(A) Total internal reflection (B) Scattering

(C) Diffraction (D) Refraction

31. Electromagnetic radiation of frequency n, wavelength λ, travelling with velocity v in air, enters a glass slab of refractive index μ. The frequency, wavelength and velocity of light in the glass slab will be respectively :-

(A) $\frac{n}{\mu}, \frac{\lambda}{\mu}, \frac{v}{\mu}$ (B) $n, \frac{\lambda}{\mu}, \frac{v}{\mu}$ (C) $n, \lambda, \frac{v}{\mu}$ (D) $\frac{n}{\mu}, \frac{\lambda}{\mu} v$

32. A plane glass slab is kept over various colored letters ; the letter which appears lest raised is

(A) blue (B) violet (C) green (D) red

33. A convex lens of focal length f will form a magnified real image of an object if the object is placed

(A) anywhere beyond 2f (B) anywhere beyond f

(C) between f and 2f (D) between lens and f

34. A convex lens is making full image of an object . if half of lens is covered by an opaque object, then

(A) half image is not seen (B) full image of same intensity is seen

(C) full image of decreased intensity is seen (D) half image of same intensity is seen

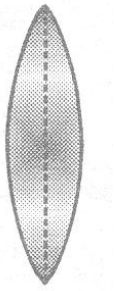
35. When a thin convex lens is put in contact with a thin concave lens of the same focal length , the resultant combination has a focal length equal to

(A) f / 2 (B) 2f (C) 0 (D) ∞

36. Focal length of a convex lens will be maximum for

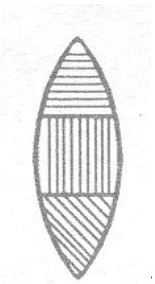
(A) blue light (B) yellow light (C) green light (D) red light

37. A convex lens has a focal length f. It is cut into two parts along the dotted line as show in the figure. The focal length of each part will be



(A) $\frac{f}{2}$ (B) f (C) $\frac{3}{2} f$ (D) 2f

38. A convex lens is made up of three different materials as shown is the figure. For a point object placed on its axis, the numbers of image formed are:



(A) 1 (B) 3 (C) 4 (D) 5

39. Myopia is the defect of vision due to which a person finds difficulty in seeing

(A) distance object (B) near object (C) object at all distances (D) colors

40. Loss of the ability of eye to focus on near and far object with advancing age is called

(A) Presbyiopia (B) Astigmatism (C) Hypermetropia (D) Myopia

41. Astigmatism can be corrected by

(A) Bifocal lenses (B) Cylindrical lenses (C) Concave lenses (D) Plano convex lenses

42. A normal eye is not able to see objects closer than 25 because

(A) The focal length of the eye is 25 cm

(B) The distance of the retina form the lens is 25 cm

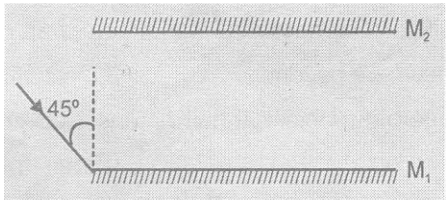
(C) The eye is not able to decrease the distance between the eye lens and the retina beyond a limit

(D) The eye is not able to decrease the focal length beyond a limit

43. Myopia can be removed by using a lenses of

(A) concave lens (B) convex lens (C) cylindrical lens (D) by surgical removal

44. Two plane mirrors M_1 and M_2 each have length 1m and separated by 1 cm. A ray of light is incident on one end of mirror M_1 at angle 45^0 . How many reflections the ray will have before going at from the other end



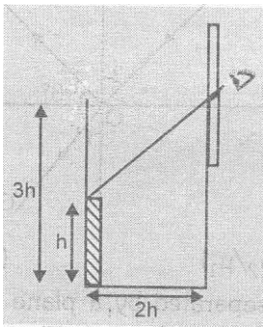
(A) 50 (B) 51 (C) 100 (D) 101

45. 'Mirage' is a phenomenon due to :-

(A) reflection of light (B) refraction of light

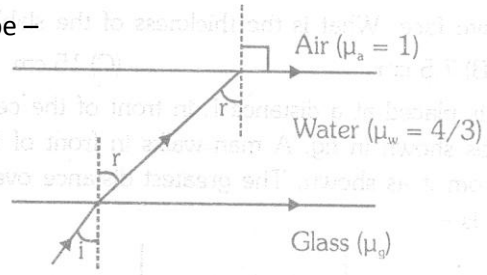
(C) total internal reflection of light (D) diffraction of light

46. An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in the figure, the beaker height is 5h and its radius h. when the beaker is filled with a liquid up to a height 2h, he can see the lower end of the rod. Then the refractive index of the liquid is –



(A) $\frac{5}{2}$ (B) $\sqrt{\frac{5}{2}}$ (C) $\sqrt{\frac{3}{2}}$ (D) $\frac{3}{2}$

47. A ray of light is incident at the glass-water interface at an angle i. It emerges finally parallel to the surface of water as shown in fig. the value of μ_g would be –



(A) $\left(\frac{4}{3}\right)\sin i$ (B) $\frac{1}{\sin i}$ (C) $\frac{2}{\sqrt{3}\sin i}$ (D) 1.5

48. When a ray of light enters a glass slab from air –

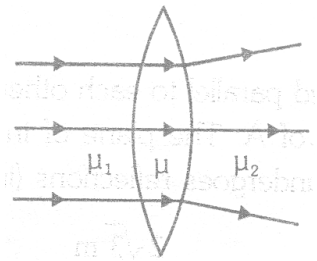
(A) Its wavelength decreases. (B) Its wavelength Increases.

(C) Its frequency increases (D) Neither wavelength nor frequency changes.

49. The distance between the object and the real image formed by a convex lens is d. if the magnification is m, the focal length of the lens is -

(A) $\frac{md}{(m+1)^2}$ (B) $\frac{md}{(m+d)}$ (C) $\frac{md}{(m-1)^2}$ (D) $\frac{md}{m-1}$

50. A parallel beam of light falls on a convex lens. The path of the rays is shown in fig. It follows that –



(A) $\mu_1 > \mu > \mu_2$ (B) $\mu_1 < \mu < \mu_2$ (C) $\mu_1 = \mu < \mu_2$ (D) $\mu_1 = \mu > \mu_2$

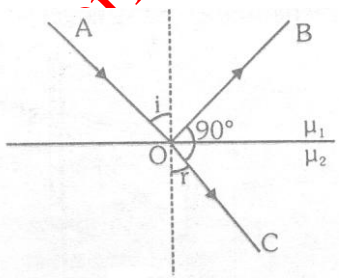
51. A person is looking at the image of his face in a mirror by holding it close to his face. The image is virtual. When he moves the mirror away from his face, the image is inverted. What type of mirror is he using ?

(A) Plane mirror (B) Convex mirror (C) Concave mirror (D) None of these

52. Two objects A and B when placed in turn in front of a concave mirror of focal length 7.5 cm, give images of equal size. If A is three times the size of B and is placed 30 cm from mirror, what is the distance of B fro the mirror –

(A) 10 cm (B) 12.5 cm (C) 15cm (D) 17.5 cm

53. A ray of light in medium of refractive index μ_1 is partly reflected and refracted at the boundary of a medium of refractive index μ_2 as shown fig. If $\angle$ BOC = 90°. The value of angle i is given by –



(A) $\tan^{-1}(\mu_1/\mu_2)$ (B) $\tan^{-1}(\mu_2/\mu_1)$ (C) $\sin^{-1}(\mu_2/\mu_1)$ (D) $\cos^{-1}(\mu_1/\mu_2)$

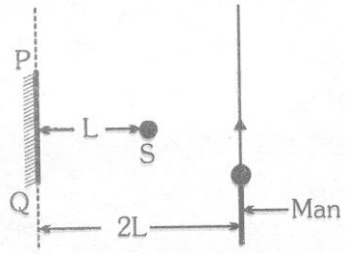
54. Two transparent media A and B separated by a plane boundary. The speed of light in medium A is 2.0×10^8 and in medium B 2.5×10^8 ms^{-1}. The critical angle for which a ray of light going from A to B it totally internally reflected is –

(A) $\sin^{-1}\left(\frac{1}{2}\right)$ (B) $\sin^{-1}\left(\frac{2}{5}\right)$ (C) $\sin^{-1}\left(\frac{4}{5}\right)$ (D) None of these

55. An air bubble in a glass slab ($\mu = 1.5$) is 6 mc deep when viewed through one face and 4 cm deep when viewed through the opposite face. What is the thickness of the slab ?

(A) 7.0 cm (B) 7.5 cm (C) 15 cm (D) 10.5 cm

56. A point source of light S is placed at a distance L in front of the centre of a plane mirror PQ. of width d hung vertically on a wall as shown in fig. A man walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown. The greatest distance over which he can see the image of the light source in the mirror is –

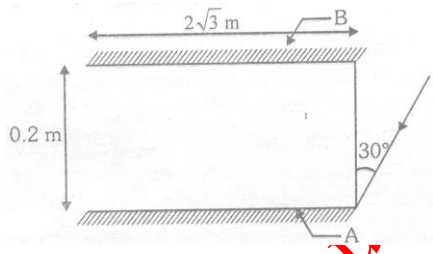


(A) $\frac{d}{2}$ (B) d (C) 2d (D) 3d

57. Two plane mirrors, each 1.6 m long, are help parallel and facing each other at a separation of $20\sqrt{3}$ cm. A ray of light is incident at a the end of one mirror at an angle of incidence of 30^0. The total number of reflections the ray suffers before emerging from the system of mirrors is –

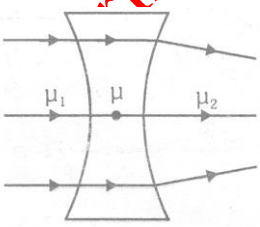
(A) 10 (B) 12 (C) 14 (D) 16

58. Two plane mirrors A and B are aligned to each other, as shown in Fig. A light ray is incident at an angle of 30^0 at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is –



(A) 28 (B) 30 (C) 32 (D) 34

59. What is the relation between refractive indies μ, μ_1 and μ_2 if the behaviour of light rays is as shown in fig.



(A) $\mu > \mu_2 > \mu_1$ (B) $\mu < \mu_2 < \mu_1$ (C) $\mu < \mu_2 : \mu = \mu_1$ (D) $\mu_2 < \mu_1 : \mu : \mu_2$

60. A lens of power +2.0D is placed in contact with another lens of power -1.0D, the combination will behave like :-

(A) A converging lens of focal length 100 cm

(B) A diverging lens of focal length 100 cm

(C) A converging lens of focal length 50 cm

(D) A diverging lens of focal length 50 cm

61. Which of the following statements is / are correct ?

(A) The laws of reflection of light hold for plane as well as curved reflecting surfaces.

(B) The size of a virtual image can be measured by receiving it on a screen.

(C) A dentist uses a convex mirror to examine a small cavity.

(D) The focal length of a spherical mirror is half the radius of curvature for all rays.

62. Choose the correct statement (s) from the following :

(A) To a fish under water looking obliquely at a man standing on the bank of lake, the man looks taller then his actual height.

(B) The apparent depth of a tank of water more for oblique viewing than for normal viewing.

(C) The focal length of a concave mirror will not change if it is immersed in water

(D) In no situation will a converging lens behave like a diverging lens.

63. An air bubble under water shines brightly because of the phenomenon of -

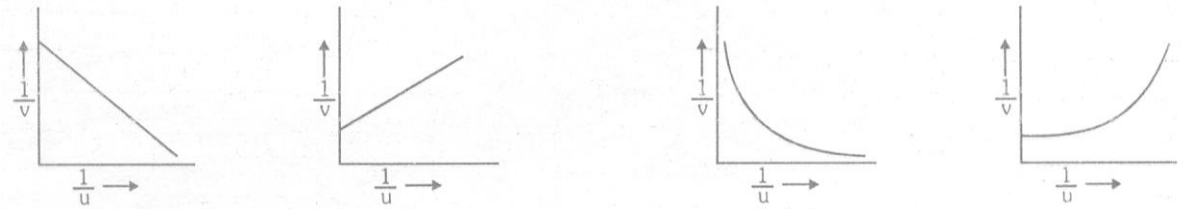
(A) Dispersion (B) Interference

(C) Diffraction (D) Total internal reflection

64. The distance v of the real image formed by a convex lens is measured for various object distances u. A graph is plotted between v and u. Which one of a the graphs shown in fig. is approximately correct ?

(A) (B) (C) (D)

65. If a graph is plotted between 1/v and 1/u, which one of the graphs shown in fig. approximately correct ?



(A) (B) (C) (D)

GENETICS

HEREDITY AND VARIATIONS

It is commonly seen that members of a species are largely alike. An elephant resembles other elephants, a rose plant looks alike other rose plant, and children resemble their parents, even grandparents or great grandparents. This resemblance among the individuals of a species has given rise to a general truth **'like begets like'** which implies continuity of life. It is, however, not absolutely true as the members f a species are seldom exactly alike. For instance, in human beings, the children often have some individual characters in which they differ from one another, and also from their parents. In fact, their differences are as marked as their resemblances. This is true about other species as well.

The similarities and differences among the members of a species are not coincidental. They are received by the young ones from their parents. The hereditary information, in fact, is present in the gametes (egg and sperm) which fuse to form the fertilized egg or zygote during sexual reproduction. The zygote then develops into an organism of a particular type. For instance, fertilized egg of sparrows hatch into sparrows only and the fertilized eggs of pigeons hatch into pigeons only. Similarly, a cow gives birth to calves only. The wheat plant gives rise to seeds which, in turn, in turn, develop into wheat plants.

Heredity :- The transmission of characters [or traits] from one generation to another generation.

Or

The transmission of characters from the parents to their off springs.

Variations :- The differences in the characters [or traits] among the individuals of a species are called variations. **e.g.**

Plant height – Tall, dwarf & middle.

Ear lobe in human being: - The lowest part of our ear is called earlobe.

◆ In most of the people, the ear lobe is hanging and it is called free earlobe.

◆ In some people, the earlobe is closely attached to the side of the head and it is called attached ear lobe.

ACCUMULATION OF VARIATIONS DURING REPRODUCTION

Heredity involves inheritance of basic body design (similarities) as well as subtle changes (variations) in it from one generation to the next generation, i.e., from parents to the offspring. When individuals of this new generation reproduces, the offsprings of second generation will have the basic body design, the differences that they inherit from first generation as well as newly developed differences.

Asexual reproduction involves single parent. When a single individual reproduces asexually, the resultant two individuals again after sometime reproduce to form four individuals. All these individuals would be similar. However, there would be only very minor differences between them. These very minor differences arise due to small inaccuracies in **DNA** copying.

Sexual reproduction, on the other hand, generates even greater diversity. This is so because sexual reproduction involves two parents (father and mother) and every offspring receives some characters of father and some characters of mother. Since, different off springs receive different combination of characters of their parents (father

and mother), they show distinct differences (variations) among themselves as well as from their parents. The variations accumulate and pass on to more and more individuals with each generation.

During sexual reproduction the variation caused by –

(i) Chance separation of chromosomes during gamete formation (Gametogenesis).

(ii) Crossing over during meiosis.

(iii) Chance coming together of chromosomes during fertilization.

(iv) Mutations, i.e., alterations in the genetic material.

All the variations in a species do not have equal chances of surviving in the environment in which they are generated. Depending upon the nature of variations, different individuals would have different kinds of advantages. For instance, bacteria that can withstand heat will survive better in a heat wave than the others. In other words, environmental factors select the variants and this and this selection forms the basis of evolution.

GREGOR JOHANN MENDEL (1822-1884)

Mendel was born on 22 July 1822 at Heinzendorf in Austria at Silesia village. Mendel had worked in Augustinian Monastery as monk at Brno city, Austria.

In 1856-57, he started his historical experiments of heredity on pea **(Pisum sativum)** plant. His experimental work continued on pea plant till 1865 (19th centaury).

The results of his experiments were published in science journal. **"Nature For Schender Verein"** in 1866.

This journal was in Germen language. This is **'**Versuche über Pflanzenhybriden**'.**

This journal was published by **'Natural History society of Bruno'.**

A paper of Mendel by the name of **Experiment in plant Hybridization** published in this journal.

Mendel were unable to got any popularity no one understand of him. He died in 1884 (due to kidney disease (Bright disease)) without getting any credit of his work.

After 16 years of Mendel's death in 1900 Mendel's postulates was rediscovered. Mendel experiment remain hidden for 34 years.

Rediscovery by three scientists independently:

1. **Carl Corens –** Germany – (Experiment on Maize)
2. **Hugo deVries** (Holland) (Experiment on Evening Primerose)
3. **Erich von Tschermak Seysenegg** – (Austria) (Experiment on different flowering plants)

CHARACTER

A recognizable feature of human beings or any other organisms are called characters.

eg. (i) Height

(ii) Complexion

(iii) Shape of hair

(iv) Colour of eyes

(v) Shape of nose

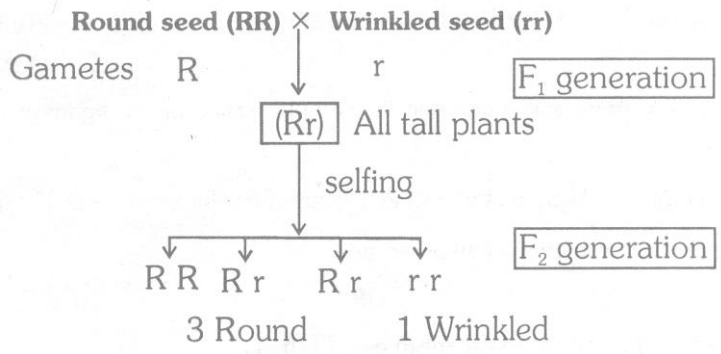
Traits :- Various forms of a character are called **traits.**

S.No.	Character	Dominant	Recessive
1.	Length of Plant	Tall	Dwarf
2.	Flower position	Axial	Terminal
3.	Shape of pod	Inflated	Constricted
4.	Colour of pod	Green	Yellow
5.	Shape of seed	Round	Wrinkled
6.	Colour of cotyledon	Yellow	Green
7.	Colour of flower	Violet	White

MENDEL'S MONOHYBRID CROSS

A breeding experiment dealing with **a angle character is called a monohybrid cross.**

Mendel first selected **'pure line' plants** (i.e. the plants that produced similar traits generation after generation). He, then, cross pollinated such plants having the contrasting traits, considering one trait at a time. For instance, in one such cross breeding experiment, he cross bred garden pea plant having round seeds with plant having wrinkled seeds. In this monohybrid cross, the pollen grains from the flower of the desired plant raised from round seeds were transferred over the previously emasculated flower of a plant raised from wrinkled seeds or vice-versa. After the transfer of pollen grains, the cross pollinated flower was properly covered and seeds produced were allowed to mature. All the seeds **F_1 generation** were carefully observed. Mendel observed that all the seeds of F_1 generation were of round type and there were no intermediate characteristics. He raised plants from F_1 seeds and allowed the flowers to self-pollinate to produce the seeds of F_2 generation. The flowers were kept covered from the beginning to avoid unwanted pollens to reach these flowers. In F_2 generation, Mendel observed the appearance of both round and wrinkled seeds in approximately 3 : 1 proportion.



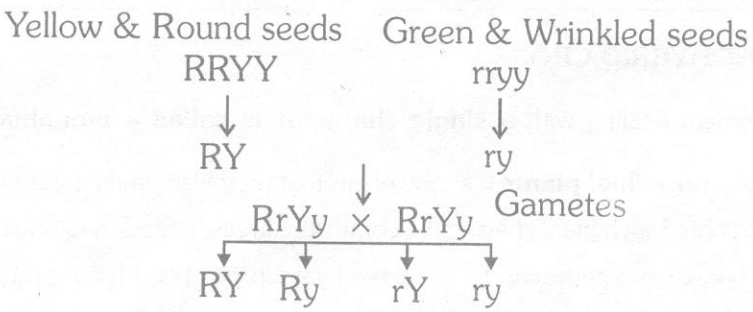
MENDELS DITHERED CROSS

A cross involving two pairs of contrasting characters.

Or

A cross in which two pair of contrasting characters are studied at a time.

In one such cross, Mendel considered **shape** as well as **colour of the seeds** simultaneously. He selected pure line plants and then cross pollinated flowers rose from seeds of round shape and yellow colour with those from wrinkled seeds and green colour. Mendel observed that in F_1 generation all seeds had the features of only one parental type, i.e., round shape and yellow colour. He raised plants from F_1 generation seeds and allowed the flowers to self pollinate to produce the seeds of F_2 generation. Theses flowers were kept covered from the beginning. In F_2 generation, Mendel observed the appearance of four types of combinations. These included two **parental types** (round shaped and yellow colored seeds, and wrinkled shaped and green colored seeds) and **two new combinations** (round shape d and green colored seeds, and wrinkled and yellow colored seeds) in approximately same proportion.



	RY	Ry	rY	ry
RY	RRYY Round Yellow	RRYy Round Yellow	RrYY Round Yellow	RrYy Round Yellow
Ry	RRYy Round Yellow	RRyy Round Green	RrYy Round Yellow	Rryy Round Green
rY	RrYY Round Yellow	RrYy Round Yellow	rrYY Wrinkled Yellow	rrYy Wrinkled Yellow
ry	RrYy Round Yellow	Rryy Round Green	rrYy Wrinkled Yellow	rryy Wrinkled Green

SOME IMPORTANT DEFINITIONS

Dominant gene :- The gene which decided the appearance of an organism even in the presence of an alternative gene.

Recessive gene :- The gene which can decide the appearance of an organism only in the presence of another identical gene.

Chromosomes :- A thread – like structure in the nucleus of a cell formed of DNA which carries the genes.

Genotype :- The genetic constitution of an organism.

Or

The description of genes present in an organism e.g. TT, tt, Tt.

Phenotype :- External and morphological appearance of an organism for a particular character.

Allele :- Alternative forms of a gene which are located on same position [loci] on the homologous chromosome.

F_1 Generation :- When two parents cross or breed to produce progeny [or off springs], then their progeny is called F_1 generation or first filial generation.

Or

The offspring produced by the parental generation.

F_2 generation :- When the first generation progeny cross or breed among themselves to produce second progeny, then progeny is called filial generation or F_2 generation.

Or

The offspring produced by the F_1 generation

Hybrid :- A new form of plant resulting from a cross or breeding of different varieties of a plant is know as a hybrid.

Pure-breeding :- Characteristics that appear unchanged generation after generation.

Dominant characters :- Any character that appears in the F_1 generation offspring from a cross between parents possessing contrasting characters such as tallness & dwarfness in pea plants.

Recessive characters :- Any character present in the parental generation that does not appear in the F_1 generation but reappears in the F_2 generation.

Homozygous :- A condition in which the 2 members of an allelic pair are similar **e.g.** TT, tt.

Heterozygous :- A condition in which the 2 members of an allelic pair are dissimilar **e.g.** Tt.

Offspring :- Organisms produced as a result of sexual reproduction.

Homologous chromosomes :- All chromosomes found in pair & chromosomes of a pair are called homologous chromosomes.

Non-homologous chromosomes :- Chromosomes of different pair are called non-homologous chromosomes.

Genes :- Unit of heredity which transfers characters from parents to their offspring during reproduction.

Gene $\rightarrow$ Protein synthesis $\rightarrow$ Enzymes [Controls phenotype of a character]

LAWS OF MENDEL

On the basis of Mendel's work, 3 basis laws of inheritance were proposed.

(i) Law of Dominance

(ii) Law of Segregation

(iii) Law of Independent Assortment

Law of dominance :- In crossing between organisms pure for contrasting characters of a pair, only one character of the pair appears in the F_1 generation. This character is termed dominant while the one which does not express itself in F_1 generation is termed recessive.

Law of segregation :- Allele or genes remain together and segregate at the time of gamete formation. This means that the alleles do not mix in the hybrids [Non-mixing of alleles]

This is also known as the **Law of Purity of Gametes.**

Law of Independent Assortment :- This law states that – when individuals differing in 2 or more than 2 pairs of contrasting characters are crossed, the inheritance of any one pair is not affected by the presence of the other. **e.g.** The inheritance of tall character is not way related to the smooth character of the seed. Rather, the 2 characters are inherited independent of each other.

REVIEW QUESTION

GIVE ANSWER OF FOLLOWING QUESTIONS

1. Give definition of heredity.
2. What do you mean by variations ?
3. Where genes present in organisms ?
4. Differentiate between Autosomes & sex chromosomes.
5. Explain the terms genotype & phenotype.
6. Differentiate between monohybrid cross & Dihybrid cross.
7. Give definition of hybridization .
8. What do you mean by pure – breeding ?
9. Differentiate character and trait.
10. Give definition of Allele.

FILL IN THE BLANKS

1. Father of genetics is known as……
2. The term genetics was coined by ……
3. A cross in which a single pair of contrasting characters is studied at a time is called ……
4. A condition in which the 2 members of an allelic pair are similar is called……..
5. Cross between two individuals having stalest one different character is called…….
6. External appearance of an organism for a particular character is called……
7. The gene which expresses himself in the presence of another or in heterozygous condition is called……
8. An unit of heredity which is responsible for a character is called……

DEOXYRIBONUCLEIC ACID (DNA)

The expanded from of DNA is deoxyribonucleic acid. It was first isolated by the scientist **Frederick Miescher** from the nucleus of the pus cells in 1869. He named it as **'Nuclein'** or nucleic acid because of its acidic nature. Later, it was experimentally proved by the scientists **Griffith (1928), Avery, McLeod and McCarty (1944)** that DNA is the carrier of the genetic information from generation to generation. It transmits the hereditary characters in a coded language from parents to off springs (i.e., from one generation to another).

DNA is a macromolecule or polymer. It is made of very large number of 'nucleotide' units and hence is termed **polynucleotide.**

Each nucleotide unit in a DNA molecule is made up of three components

1. **Deoxyribose sugar :-** It is a pentose sugar.

2. **Nitrogenous base :-** Each nucleotide unit has a nitrogen containing base. In a DNA molecule, nitrogenous bases are of two types :

(a) **Purines :-** The purines in a DNA molecule are **– Adenine (A) and Guanine (G).**

(b) **Pyrimidine :-** The pyrimidine in a DNA molecule are **– Cytosine (C) and Thymine (T).**

3. **Phosphate group :-** The phosphate group contains one phosphorus atom and four specifically linked oxygen atoms. Thus, there are four types of nucleotides in a DNA molecule depending upon the kind of nitrogenous base resent in **each** nucleotide.

Double Helical Model of DNA

J.D. Watson and **F.H.C. Crick** proposed the double helical model **of DNA** in **1953.** They were awarded the **Nobel Prize** for this discovery in **1962.** the important features of the double helical model are

(i) DNA molecule is made up of two **long polynucleotide strands** forming a **double helical structure (double helix)** just like a spiral staircase. Each helical turn of the DNA molecule is **3.4 nm** in length in which **ten nucleotide** base pairs are present.

(ii) **Deoxyribose sugar and phosphate molecules** are joined alternately to form the backbone of each polynucleotide strand. The **nitrogenous base** of each nucleotide is attached to the sugar molecule and projected towards are interior of the double helix.

(iii) In the interior of double helix, the nitrogenous base of two polynucleotide strands from a pair with the help of hydrogen bonds. **Adenine (A)** always pairs with **thymine (T) and guanine (G) always** pairs with **cytosine (C).**

Thus, the two polynucleotide strands of the DNA molecule are joined by hydrogen bonds between specific nitrogenous bases. **Such a specific of the bases of the opposite strands of the DNA molecule is called complementary pairing. Adenine (A) and thymine (T) are complementary to each other. Similarly, guanine (G) and cytosine (C) are complementary to each other.** The hydrogen bonding between the specific nitrogenous bases keeps the two strands to hold together. Therefore, all the base pairs remain stacked between the two strands.

BLOOD GROUP

Four different types of blood groups in human beings are : A, B, O, AB

O → universal donor

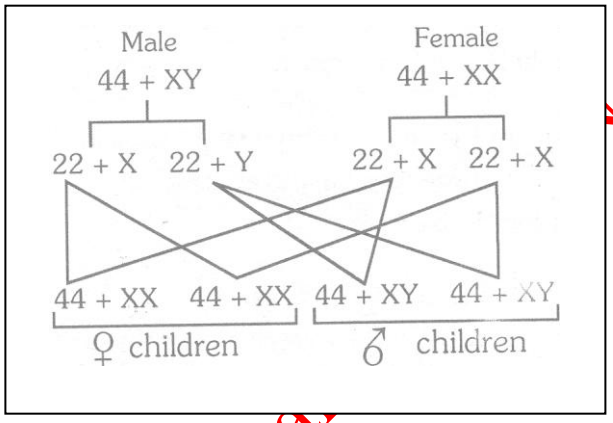
AB → universal recipient

Blood group	Genotype	Dominant	Recessive	Antigen	Antibody
A	I^AI^A I^AI^O	I^A	I^O	A	B
B	I^BI^B I^BI^O	I^B	I^O	B	A
AB	I^AI^B	I^A, I^B	-	A, B	-
O	I^OI^O	-	I^O	-	AB

SEX DETERMINATION

How is the sex of newborn individual determined? In human beings, the sex of the individual is largely genetically determined. In other words, the genes inherited from our parents have assumed that similar gene sets are inherited from both parents. If that is the case, how can genetic inheritance determine sex ?

All human beings these are 23 pairs of chromosomes are present out of these 23 pairs, are autosomes (similar in males and females) and 1 pair is called sex chromosome (different in males and females). In males sex chromosome are XY and in females' sex chromosome are XX.



So, it is clear from the above that male is responsible for producing male child.

POINTS TO REMEMBER

- The term Genetics comes from "gene" word – means "to grow".
- The term "Genetics" coined by **Bateson.**
- Study of heredity and variations is called **Genetics.**
- Father of Genetics – **Gregor Johann Mendel.**
- Father of Modern Genetics **– Bateson.**
- Father of Experimental Genetics – **Thomas Hunt Morgan.**
- "Heredity" term coined by **Spencer.**
- Bateson coined terms Genetics, Allele, Homozygous, Heterozygous, F_1 and F_2 Generation.

EXERCISE # 1 FOR SCHOOL / BOARD EXAM.

VERY SHORT ANSWER TYPE QUESTION

1. What is a gene ?
2. Write the scientific term used for 'science of heredity and variation'.
3. Define inheritance.
4. What term did Mendel use for what we now call the genes ?
5. How does the creation of variations in a species promote survival ?
6. Name the plant on which Mendel performed his experiments.
7. Define Variation.
8. What are the carriers of factors or genes ?
9. Name two human traits which show variation.
10. Write down five inherited 'characters' or traits.
11. Which of the following combinations of sex chromosomes produces a male child, XX or XY ?
12. Which type of sex chromosomes are carried by male gametes – sperms in human male ? What is their percentage ?
13. What experiment would we do to confirm that the F_2 generation did in fact have a 1 : 2 : 1 ratio of TT, Tt and tt trait combinations ?

SHORT ANSWER TYPE QUESTIONS

1. Why did Mendel choose pea plants for his experiment ?
2. What is Mendel's genetics ?
3. If a train A exists in 10% of a population of an asexually reproducing species and trait B exists in 60% of the same population, which trait is likely to have arisen earlier ?
4. A Mendelian experiment consists in breeding tall pea plants bearing violet flowers with short pea plants bearing white flowers. The progeny all bore violet flowers, but almost half of them are short. This suggests that the genetic makeup of the tall parent can be depicted as :-

   (i) TTWW (ii) TTww (iii) TtWW (iv) TtWw
5. Explain the mechanism of the determination in human beings.
6. Define genetics. What is the contribution of Mendel in this branch of Biology ?
7. Why are asexually reproducing organisms capable of showing the hereditary features more ?
8. With suitable example highlight how temperature has an effect on sex determination in the animal world.
9. Explain with an example how trait is inherited in human beings.
10. Why are traits acquired during the life-time of an individual not inherited ?

11. How does the creation of variations in a species ensure survival ?

12. A man with blood group A marries a woman with blood group O and their daughter has blood group O. Is this information enough to tell you which of the traits – blood group A or O – is dominant ? Why or why not ?

13. Why are the small numbers of surviving tigers a cause of worry from the point of view of genetics ?

14. Draw a schematic diagram to explain the independent inheritance of two separate traits, shape and colour of seeds.

15. How do Mendel's experiments show that traits are inherited independently ?

LONG ANSWER TYPE QUESTIONS

1. Explain diagrammatically a Mendel's Dihybrid cross.

2. Briefly explain contributions of Mendel in Genetics.

FILL IN THE BLANKS

1. The transmission of characters from the parents to their off springs is called……..

2. The differences in the characters [or traits] among the individuals of a species are called……..

3. A recognizable feature of human beings for any other organisms are called……

4. Various forms of a character are called……..

5. A cross in which a single pair of contrasting characters is studied at a time, called……

6. A cross involving two pairs of contrasting characters is called……

7. The gene which decided the appearance of an organism even in the presence of an alternative gene is called ……

8. The gene which can decide the appearance of an organism only in the presence of another identical gene is called……

9. Unit of heredity which transfers characters from parents to their off springs during reproduction is called ……

10. The genetic constitution of an organism is called……

ANSWERS KEY FOR SCHOOL / BOARD EXAMS.

FILL IN THE BLANKS

1. Heredity	**2.** Variations.	**3.** Character	**4.** Traits	**5.** Monohybrid cross
6. Dihybrid cross	**7.** Dominant	**8.** Recessive Gene	**9.** Genes	**10.** Genotype

EXERCISE # 2 FOR SCHOOL / BOARD EXAM.

1. Genetics is the branch of science which deals with the study of :-

(A) cell function (B) cell structure

(C) heredity and variation (D) relation between plant and environment

2. The term 'genetics' was coined by :-

(A) William Bateson (B) Gregor Mendel (C) Thomas hunt Morgan (D) W. Johannsen

3. The term 'gene' was introduced by :-

(A) Mendel (B) Bateson (C) Morgan (D) Johannsen

4. When a gene exists in more than one form, the different forms are termed :-

(A) alleles (B) heterozygotes (C) genotypes (D) complementary genes

5. The contrasting pairs of factors in Mendelian crosses are called :-

(A) alloloci (B) paramorphs (C) allelomorphs (D) multiple alleles

6. Alleles of a gene are found on :-

(A) same chromosome (B) any chromosomes

(C) homologous (D) nonhomolgous chromosomes

7. Which statement about alleles is not true :-

(A) There may be several at a locus (B) One may be dominant over another

(C) They may show incomplete dominance (D) They occupy different loci on the same chromosome

8. An organism which receives identical alleles of a particular gene from both parents is :-

(A) homozygote (B) hemizygote (C) homothallic (D) heterozygote

9. The genetic complement of an organism is known as :-

(A) genotype (B) physiotype (C) phenotype (D) morphotype

10. The physical appearance of an individual is known as :-

(A) heterotype (B) genotype (C) morphotype (D) phenotype

11. The terms 'genotype' and 'phenotype' were introduced by :-

(A) Bateson (B) Darwin (C) Johannsen (D) Mendel

12. What was Mendel's most important contribution to the modern understanding of biology:-

(A) The concept of meiosis (B) The concept of chromosome

(C) The concept the genes are ordered along chromosomes

(D) The concept that hereditary information comes in discrete units

13. Gregor Mendel was born in :-

(A) Austria (B) Russia (C) Czechoslovakia (D) United Kingdom

14. Mendelism is related with :-

(A) Heredity in living beings (B) Meiosis during sexual reproduction

(C) Mutations in living organisms (D) None of the above

15. Mendel published the results of his experiments in the year :-

(A) 1568 (B) 1773

(C) 1866 (D) 1921

16. In 1900 AD, three biologists independently rediscovered Mendel's principles. They were :-

(A) Sutton , Morgan and Bridges

(B) Bateson, Punnett and Bridges

(C) Avery, MacLeod and McCarty

(D) Hugo deVries, Corens and Tschermak

17. How many contrasting traits of pea were studied by Mendel:-

(A) Two (B) Four

(C) Seven (D) Three

18. Which one of the following traits of garden pea studied by Mendel was a recessive feature ?

(A) Green seed colour (B) Green pod colour

(C) Round seed shape (D) Axial flower position

19. Which of the following contributed to the success of Mendel :-

(A) His knowledge of biology

(B) Qualitative analysis of data

(C) Observation of distinct inherited traits

(D) consideration of one character at a time

20. The reason why pea plants were suitable than dogs for Mendel's experiments :-

(A) Dogs have many genetic traits

(B) Pea plants can be self fertilized

(C) There are no pedigree records of dogs

(D) The pea plans favor cross-fertilization

21. Mendel chose pea plants because they :-

(A) were cheap

(B) were easily available

(C) have great economic importance

(D) were having contrasting characters

22. Selection of homozygous plant is :-

(A) mass selection (B) pure line selection

(C) mixed selection (D) none of the above

23. When a cross is made between tow parents with respect to a single character, it is called :-

(A) Dihybrid (B) monohybrid

(C) Trihybrid (D) none of these

24. Which of the following is the monohybrid ratio :-

(A) 3 : 1 (B) 9 : 7

(C) 1 : 2 (D) 9 : 3 : 3 : 1

25. Mendel formulated the law of purity of gametes on the basis of :-

(A) test cross (B) Dihybrid cross

(C) back cross (D) monohybrid cross

26. How many types of gametes will be produced in F_2 generation of a monohybrid cross of Mendel :-

(A) 3 (B) 4 (C) 8 (D) 16

27. Recessive gene can express only in :-

(A) homozygous condition (B) heterozygous condition

(C) both above conditions (D) none of these conditions

28. Considering tallness and dwarfness , tallness is more wide spread among pea plants because :-

(A) Tallness is dominant over dwarfness

(B) Tallness is determined by one gene having many effects

(C) Tallness is determined by many genes having multiple effects

(D) None of these

29. A red-flowered pea plant was crossed with white-flowered plant. In F_1 generation all plants were red. It confirms that white colour is :-

(A) Recessive character (B) Dominant character

(C) non-genetic expression (D) none of the above

30. A woman with straight hair mates with a man with curly hair who is known to be heterozygous for that trait. What is the chance that their first child will have curly hair :-

(A) No chance (B) One in two (C) It is certain (D) One in four

ANSWER KEY FOR SCHOOL / BOARD EXAMS.

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	A	D	A	C	C	D	A	A	D	C	D	A	A	C
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	D	C	A	D	B	D	B	B	A	D	B	A	A	A	B

EXERCISE # 3 COMPETITIVE EXAMS.

1. The main reason of Mendel's success in discovering the principles of inheritance was :-

(A) He considered each character separately (B) He was lucky not to encounter linkage problem

(C) The plant was each pure breeding (D) All the above

2. Mendel was lucky in the choice of the material of his experiments, among the following, which contributed, to his success ?

(A) He observed distinct inherited traits (B) He qualitatively analyzed his date

(C) He liked pea plants (D) He considered only one character at one time

3. The year 1990 AD is highly significant for genetics due to :-

(A) Principle of linkage (B) Chromosome theory of heredity

(C) Rediscovery of Mendelism (D) Discovery of genes

4. Genotype means :-

(A) Genetic composition of the individual (B) Genetic composition of the germ cell

(C) Genetic composition of plastids (D) Genetic composition of an organ

5. An organism with two identical alleles of a gene is a cell is called :-

(A) Homozygous (B) Dominant (C) Heterozygous (D) Hybrid

6. When an individual is having both the alleles of contrasting characters it is said to be :-

(A) Heterozygous (B) Dioecious

(C) Monoecious (D) Linked genes

7. When an allele fails to express itself in F_1 generation in the presence of other allele, the former is said to be :-

(A) Recessive (B) Co dominant

(C) Complementary (D) Epistatic

8. If a plant is heterozygous for tallness, the F_2 generation has both tall and dwarf plants. This proves the principle of :-

(A) dominance (B) segregation

(C) independent assortment (D) incomplete dominance

9. Mendel crossed a pure white flowered recessive pea plant with a dominant pure red flowered plant. The first generation of hybrids from the cross should show :-

(A) 50% white flowers and 50% red flowers (B) all red flowered plants

(C) 75% red flowered and 25% red flowered plants (D) all white flowered plants

10. If a couple has three daughters, what are the chances that the fourth child will be a son ?

(A) 100% (B) 75% (C) 50% (D) 0%

11. If a heterozygous tall plant is crossed with a homozygous dwarf plant, the proportion of dwarf progeny will :-

(A) 50% (B) 75% (C) 100% (D) 25%

12. The crossing of a homozygous tall plant with a dwarf would yield plants in the ratio of :-

(A) two tall and two dwarf

(B) one homozygous tall, one homozygous dwarf and two heterozygous tall

(C) all homozygous dwarf

(D) all homozygous tall

13. When a true breeding tall plant is crossed with a true breeding short plant and the F_1 produced is self pollinated to produce F_2 ratio of true breeding tall and true breeding short plant in F_2 will be :-

(A) 1 : 2 (B) 1 : 1

(C) 2 : 1 (D) 1 : 3

14. Blue eye colour in human is recessive to brown eye colour . The expected children of a marriage between blue-eyed woman and brown-eyed male who had a blue-eyed mother are likely to be :-

(A) all blue-eyed (B) three blue-eyed and one brown-eyed

(C) all brown-eyed (D) one blue-eyed and one brown-eyed

15. Allosomes are :-

(A) bead like structures (B) on chromosomes

(C) sex chromosomes (D) rounded bodies

16. Mutation is :-

(A) a change that is inherited

(B) a change, which affects the parents only but never inherited

(C) a change, which affects the offspring of F_2 generation only

(D) a factor responsible for plant growth

17. The plant that was made popular by "DeVries mutation theory" :-

(A) Triticum vulgare (B) Oenothera lamarckinana

(C) Pisum sativum (D) Primula vulgaris

18. "Barr body" is derived from :-

(A) Autosomes in males (B) Autosomes in females

(C) X-chromosome in female (D) X-chromosome in males

19. The **DNA** is the genetic material was proved conclusively by :-

(A) J D Watson (B) Hershey and Chase

(C) Alfred Griffith (D) Boveri and Sutton

20. Nobel Prize for "one gene one enzyme theory' was given to :-

(A) Beadle and Tatum (B) Schleiden and Schwann

(C) Watson and Crick (D) H Harris

21. Retrovirus has the following as its genetic material :-

(A) single stranded DNA (B) double stranded duplex DNA

(C) DNA-RNA hybrid (D) RNA

22. Of the following , which sequence is present in Rous Sarcoma Virus ?

(A) DNA → RNA → proteins (B) DNA → DNA → proteins

(C) RNA → DNA → proteins (D) RNA → DNA → RNA → proteins

23. The term genome is used for :-

(A) diploid set of chromosomes (B) polyploidy set of chromosomes

(C) triploid set of chromosomes (D) haploid set of chromosomes

24. The first successfully cloned mammal that gained world-wide publicity was :-

(A) Molly, a sheep (B) Polly, a sheep (C) Chance ; a bull (D) Dolly, a sheep

25. The transgenic animals are those that have :-

(A) foreign DNA in some of its cells (B) foreign DNA in all its cells

(C) foreign RNA in all its cells (D) both (A) and (C)

ANSWERS KEY FOR COMPETITIVE EXAMS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	D	C	A	A	A	A	B	B	C	A	B	b	D	C
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	A	B	C	B	A	D	D	D	D	B					

HOW DO ORGANISMS REPRODUCE ?

INTRODUCTION

Reproduction is a process by which living organisms produce new individuals of their own kind and maintain their existence generation after generation.

Reproduction is not essential to maintain the life of an organism but it is essential to maintain life on earth and perpetuation of species from one generation to another.

Reproduction at its basic level (cellular reproduction) is involved in making similar or dissimilar body designs through the genetic material (DNA) present in the chromosomes of its nucleus.

DNA is the source of information for making proteins. Any change in the information leads to production of different proteins, which ultimately lead to altered body designs.

Basic event in reproduction is production of DNA copies in a reproducing cell the process is called DNA replication. When the cell divides into two, each cell gets a copy of each DNA of chromosome along with the whole cellular apparatus.

Complete accuracy in DNA copying leads to exactly identical cells but any error in duplication can lead to dissimilar cells or variations.

The inbuilt tendency for variations during reproduction form the basis for evolution.

Variations during reproduction enable the population of a species to get adapted easily to a particular inhabiting place/niche. Hence, reproduction is linked to the stability of populations of species.

Stronger variations are useful for the survival of species over time and enable the organisms to tide over any drastic alterations in their habitats.

IMPORTANCE OF REPRODUCTION

(i) **Maintenance of the existence :-** Organisms are maintaining the existence on the earth since their origin, million years ago only because of reproduction.

(ii) **Preservation of species :-** Species are preserved because of reproduction. It is possible because reproducing organisms produce new individuals which are very similar to themselves.

(iii) **Role in evolution :-** Some variations are produced in the new organisms during reproduction which play in important role in evolution.

TYPE OF REPRODUCTION

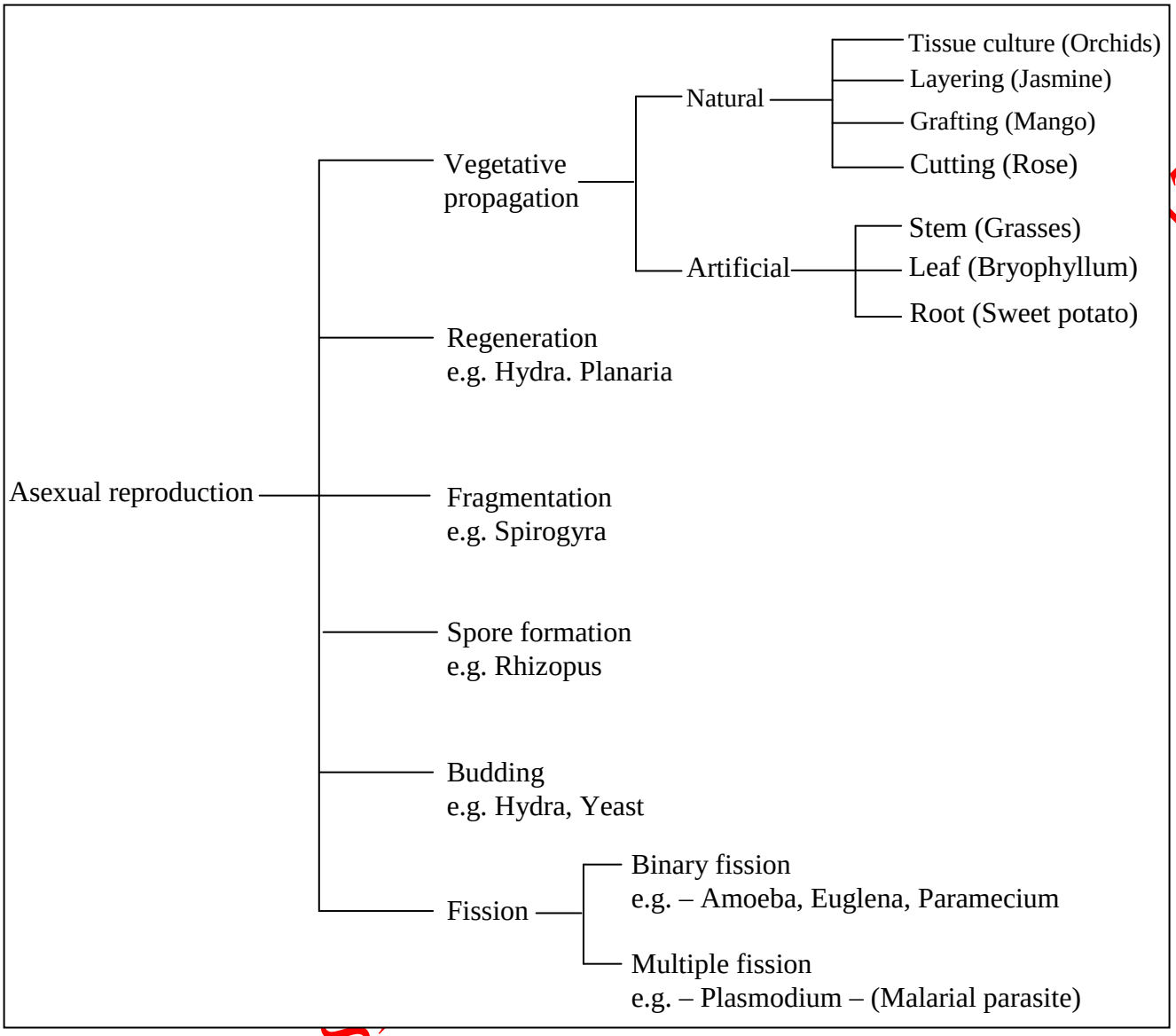
There are two main methods of reproduction in living organisms.
(1) Asexual reproduction.
(2) Sexual reproduction.

(1) Asexual Reproduction :

Production of offsprings by a single parent without the formation and fusion of gametes is called **asexual reproduction.**

It is a primitive type of reproduction in which **offspring** is produced by a cell or any vegetative organ of an organism.

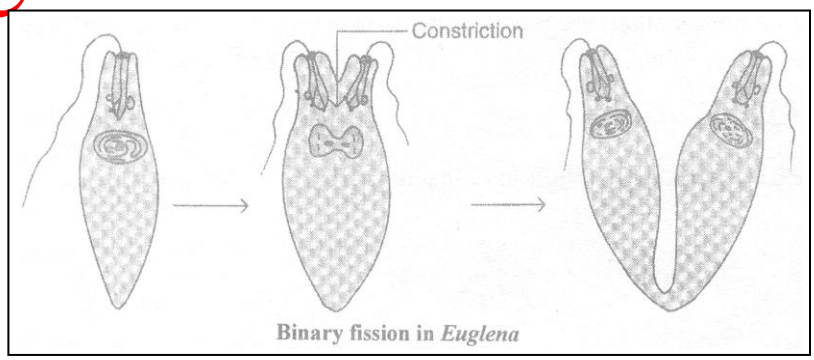
In this type of reproduction **offsprings** are generically identical to their parents.



Modes of asexual reproduction are fission, budding, spore formation, fragmentation, regenerations and vegetative propagation.

(i) Fission. It is a kind of asexual reproduction in unicellular organisms to create two new individuals. It can be of two type :

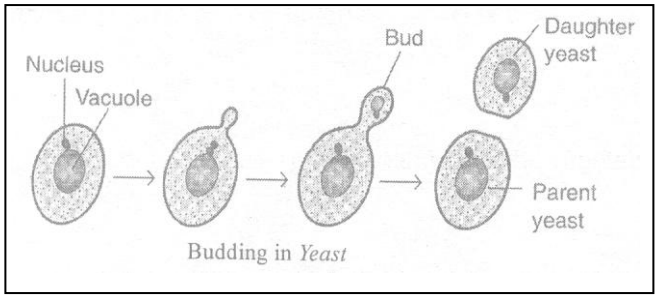
(a) Binary fission : One cell splits into two equal helve, e.g., many bacteria and protozoa like Amoeba, Paramecium and Leishmania.

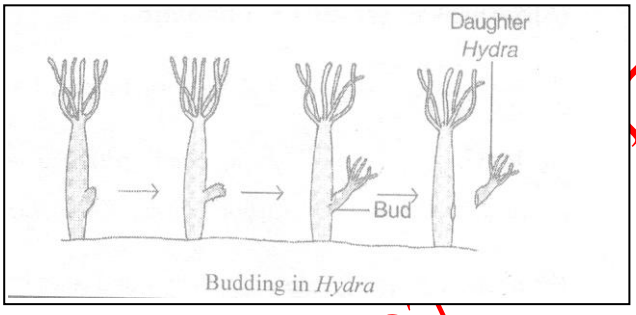


Binary fission in *Euglena*

(b) Multiple fission. One cell divides into many daughter cells simultaneously, e.g., Plasmodium (malarial parasite), Amoeba in unfavorable conditions.

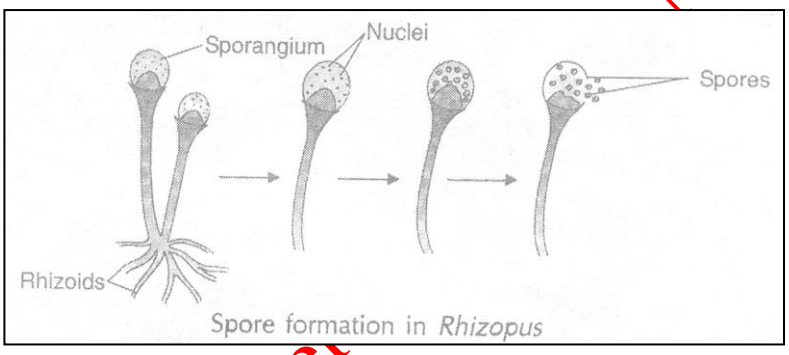
(ii) Budding : Process in which an outgrowth (bud) is formed on the body of parent organism which then detaches and become a new organism. **e.g. Yeast** and Hydra.



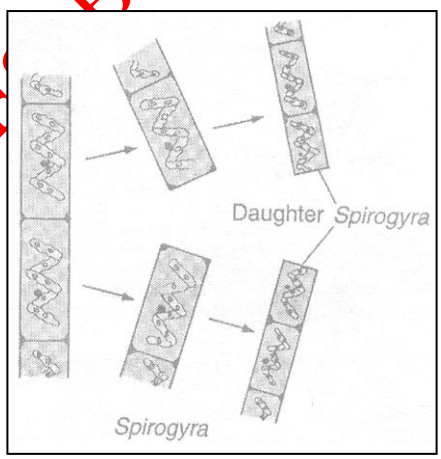


(ii) Spore formation : Spore are the microscopic asexual reproductive bodies with a thick wall. Spores are forded in **'sporangium'.**

Each spore on germination give rise to a new organism e.g.. **Rhizopus, Penicillium.**



(iv) Fragmentation: In this process an organism breaks up into two or more fragments and each fragment develops into an adult organism. e.g. **Spirogyra.**

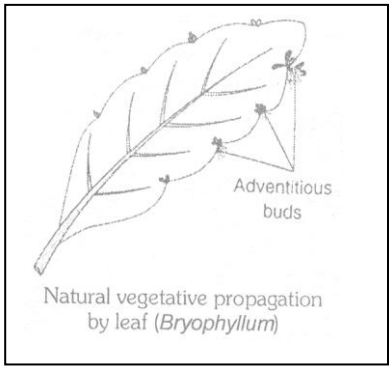


(v) Regeneration : The process of getting back a full organism from the body parts of the present individual is called **regeneration,** Regeneration in carried out by specialized cells. e.g. Hydra, Planaria.

(vi) Vegetative Propagation : This is an asexual method of reproduction in plants where vegetative parts namely root, stem and leaves give rise to new plants.

Vegetative propagation is of two types :

(A) Natural vegetative propagation



(B) Artificial vegetative propagation.

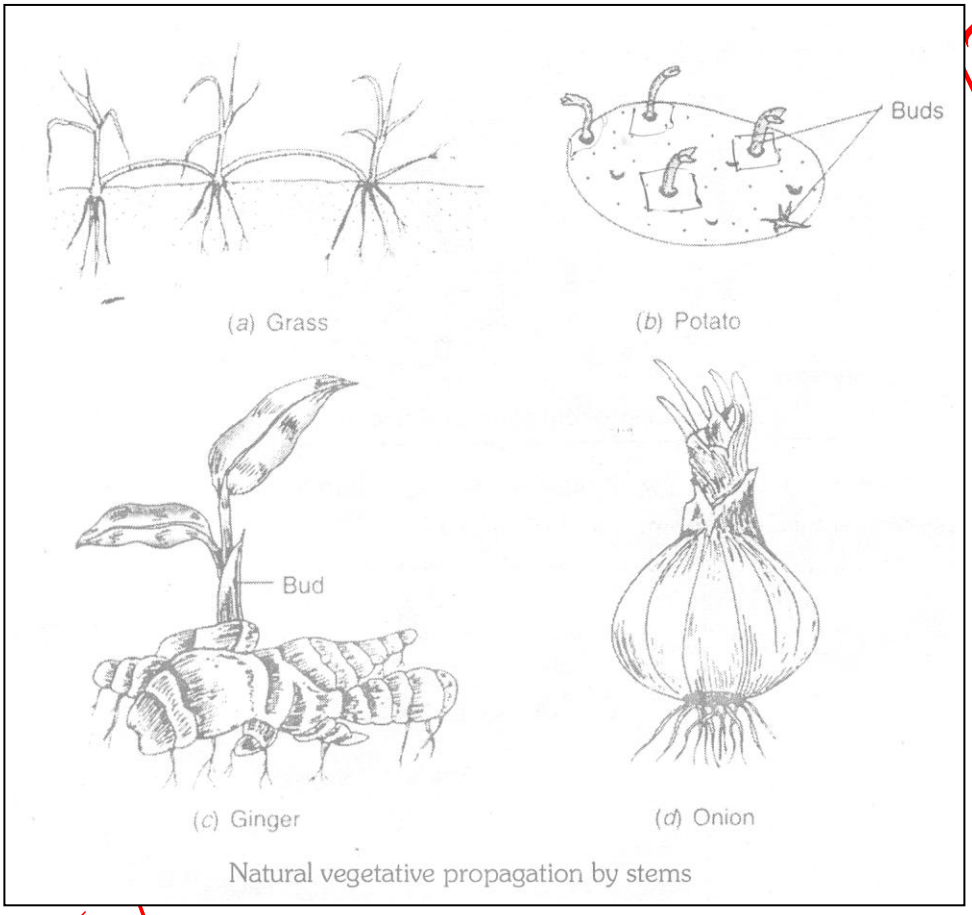
(A) Natural vegetative propagation :

Plant reproduce without the help of human being .

By leaves : Leaves of some points produce adventitious buds on their margin. Thus bubs develop into new plants e.g. Bryophyllum, Kalanchoe.

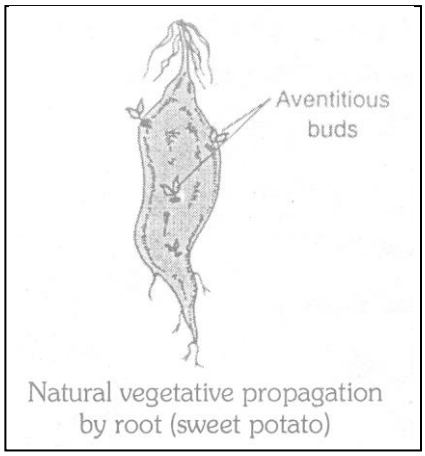
By stem : In many plant, underground stems produce aerial shoots annually under favorable conditions

e.g. Potato, Zinger, Onion, Grass.



Natural vegetative propagation by stems

● **By roots :** Roots produce adventitious buds which develops into new plants. e.g sweet potato .



Natural vegetative propagation
by root (sweet potato)

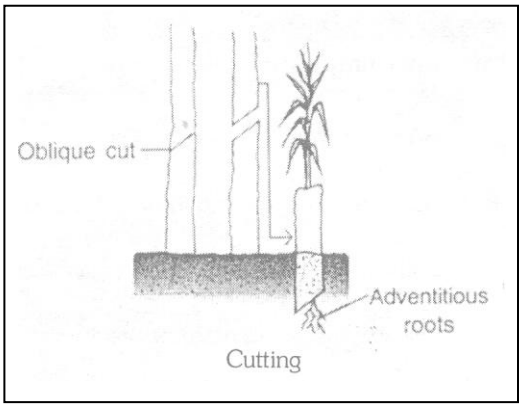
(B) Artificial vegetative propagation :

To prepare plants with desirable characters.

These are of four types.

(i) Cutting :

In this method small part of plant is cut and buried partly in the moist soil then cutting develops roots and grows into a new plant. e.g. Rose, Sugarcane, Potato, Cactus.



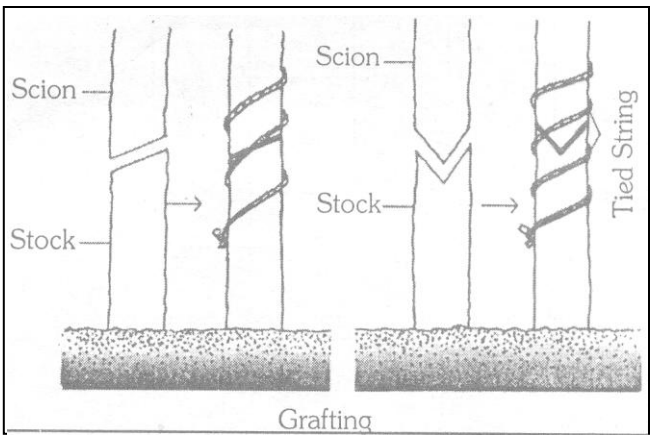
(ii) Grafting :

Two plants of closely related varieties are joined together so that they live as one plant .

The plant of which roots remain in the soil is called as **stock.**

Cutting part of a plant that is grafted on the other rooted plant is called **scion.**

e.g. Mango , Apple, Lemon.

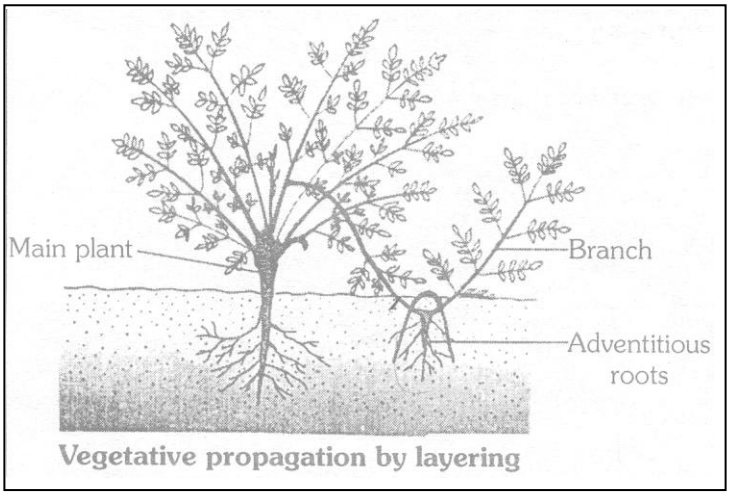


(iii) Layering :

In this method a branch of the parent plant is buried in the soil.

The portion of the branch which is contact with the soil produces roots and this rooted branch is called **layer.**

Layer is then detached from the parent plant and act as a new plant . **e.g.** Jasmine, Hibiscus.



Vegetative propagation by layering

(iv) Tissue culture or microproagation :

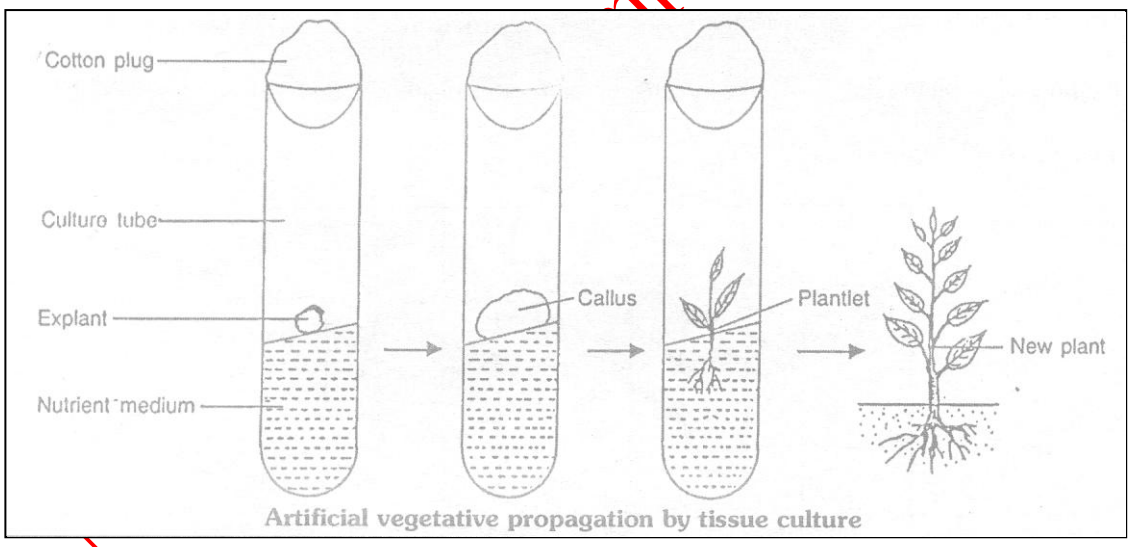
Cells or tissue which is isolated from the growing tip of plant called **explant.**

The explant develops into undifferentiated mass of cells called **callus** in the proper medium.

The callus is transferred to another medium containing hormones for growth and differentiation, that forms **plantlet.**

The plantlets are transplanted into pot or soil to form mature plant.

This technique is known as micro propagation. **e.g.** Orchids, Chrysanthemum.



Artificial vegetative propagation by tissue culture

ADVANTAGES OF VEGETATIVE PROPAGATION

It is a rapid, cheap and easy method of reproduction for the multiplication of plants.

Disease free plants can be produced.

Superior quality fruits or flowers can be produced by grafting.

Genetically identical plants are produced.

Plants raised by vegetative propagation can bear flowers and fruits than those produce from seeds.

Do you know ?

In fission the organism have lost their parental identity while in budding they have maintained their identity.

Grafting is not possible in monocot plants.

Cambium activity is essential for the union of stock and scion.

Tissue culture is also called micro-propagation because a large number of plants are formed from a small tissue.

Virus free plants are produced by micro propagation.

REVIEW QUESTIONS

1. What is meant by reproduction ?

2. What are two general methods of reproduction in organisms ?

3. Give two examples of organisms which reproduce by budding.

4. What is meant by vegetative propagation ?

5. What are the advantages of vegetative propagation ?

6. What is grafting ? Mention any two advantages of grafting.

7. In which part of Bryophyllum vegetative propagation takes place ?

8. What is micropropagation? Explain the process of micro propagation in Vitro.

(2) Sexual reproduction :-

It is a type of reproduction in which two different sexes (male and female) are involved. It involves the fusion of gametes from two different parents and results in the formation of new organism, which s genetically different from the parent.

Differences between asexual and sexual reproduction

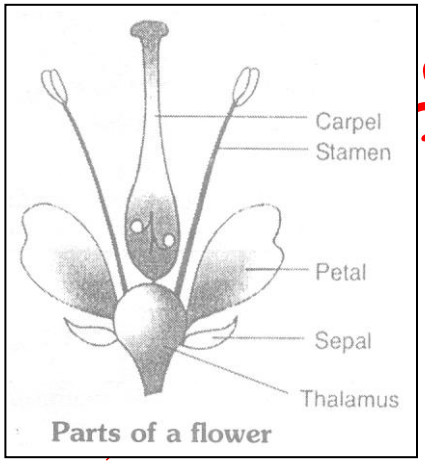
S.No.	Features	Asexual	Sexual reproduction
1	Number of parents involved	One	Two
2	Resemblance with parents	Organisms produced resemble exactly with the parent.	Organisms do not resemble exactly with the parent but resemble in certain features with both the parents.
3	Type of cell divisions	Amitotic / mitotic.	Mitotic and meiotic both are present.
4	Time duration for multiplication	Takes less time.	Takes more time.
5	Variations	Variations are absent.	Variations are present.
6	Adaptability	Organisms produced have less adaptability	Organisms produced have more adaptability
7	Examples	Fission, budding, vegetative propagation.	Human beings, higher plants.

SEXUAL REPRODUCTION IN FLOWERING PLANTS

Sexual reproduction takes place through the agency of flowers in angiosperms (flowering plants).

Flower is a specialized condensed reproductive shoot of flowering plants on which the essential reproductive parts are inserted.

A typical flower has four whorls arranged on the **thalamus.**



1. Calyx
2. Corolla

— Non essential organs

3. Androceium
4. Gynoceium

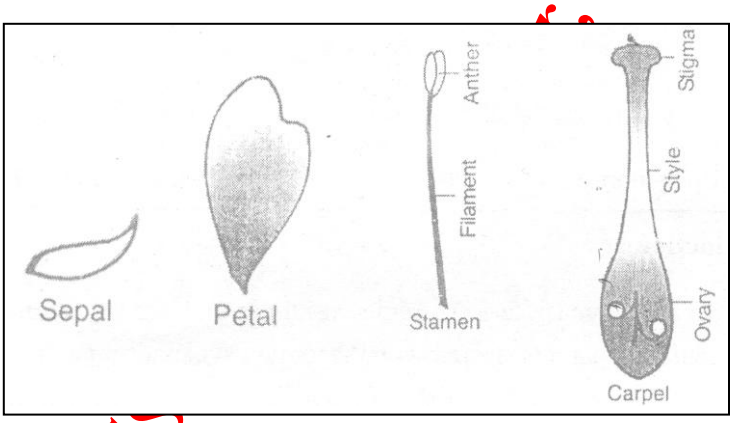
— Essential organs

1. CALYX

It is the outermost whorl consisting of **sepals.**

Sepals are green and leaf like structure.

Calyx protect the flower bud before it opens.



2. COROLLA

It is the second whorl, inner to calyx, consisting of **petals.**

Petals are generally large, colored and showy.

Corolla attract insects for pollination.

3. ANDROECIUM

It is the third whorl, inner to corolla, consisting of male reproductive parts called **stamens.**

Each stamen has two parts – Filament and anther.

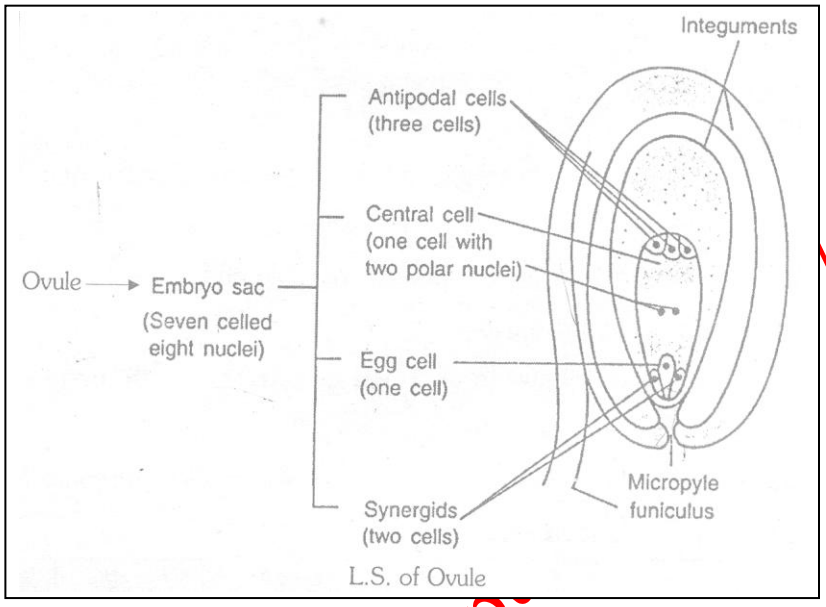
Anther is lobed structure present at the tip of filament. Each anther has pollen sacs (microsporangia) which contain pollen grains (microspores).

Each pollen grain produces two male gametes / male germ cells.

4. GYNOECIUM

It is the fourth and innermost whorl consisting of **carpels.**

Carpel is present in the centre of flower.
Each carpel has three parts – Ovary, Style and Stigma.
Ovary is a swollen basal part of carpel. It contains ovules which are attached to placenta.
Each ovule contain an embryo sac that bears a haploid egg (female gamete).
Style is the middle part of the carpel. It has stigma above it and ovary below it.
Stigma is the apical part of carpel. It receives pollen grains.



Do you know ?

Perianth :- If both sepals and petals are colored and can not be distinguished from each other, then their whorl is known as **perianth**.

Calyx and corolla are non essential parts of the flower because they are not directly involved in reproduction.

Bisexual flower :- When the male and female reproductive parts are present in the same flower are called bisexual flower e.g. **Hibiscus,** Mustard.

Unisexual flower :- When the male and female reproduction parts are present in different flowers.

e.g. : Papaya, Date palm, Mulberry, Gourd, Water melon.

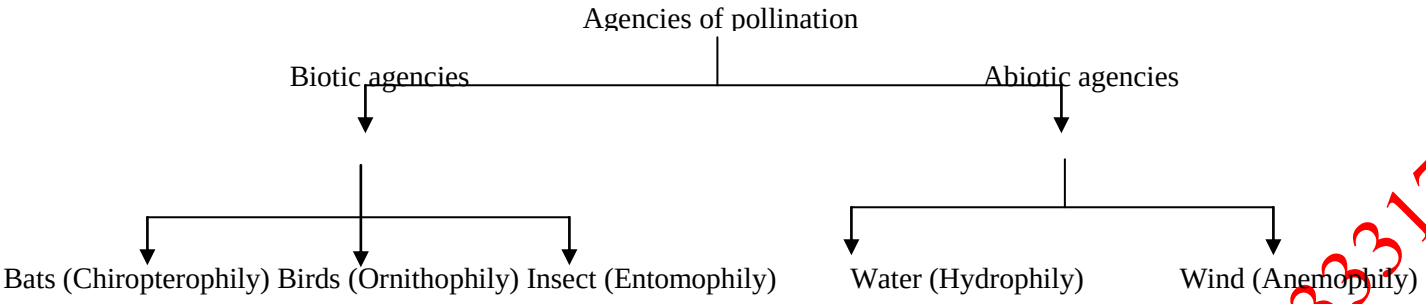
POLLINATION

Process in which pollen grain are transferred from the ripe anther to the stigma. It is of two types :

(i) Self pollination : It is the transfer of pollen grains from an anther to the stigma of the same plant. If it is in the same flower it is called **Autogamy** (e.g. **Pea**) and if it is between flower of the some plant then it is called **Geitonogamy** (e.g. **Oxalis**) .

(ii) Cross pollination :- It is the transfer of pollen grains from anther to the stigma of different plants of the same species (e.g. Mango).

Agencies of pollination :- Transfer of pollen from one flower to another is achieved by agents like wind, water, animals, insects and birds.



Significance of bright colour of flower :- The bright colour of flowers is meant to attract insects which help in pollination. White colour shine in dark which attracts insects at night. Similarly, bright colour day-blooming flowers attract insects.

FERTILIZATION

Fertilization is the process of fusion of the male and female gametes, which takes place in the **embryo sac** present in the ovule.

After pollination, pollen grains germinate on the stigma by producing pollen tube.

The nucleus in the pollen tube divides into two male gametes.

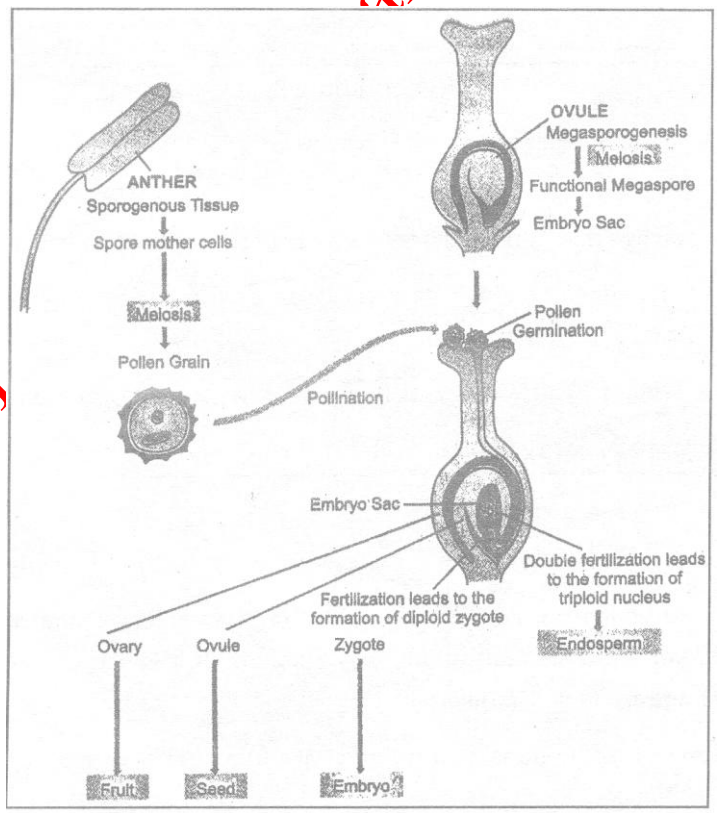
Pollen tube penetrates the stigma and passes through the style and enters the ovule through micropyle. It releases two male gametes in embryo sac.

One male gamete fuses with egg cell and second male gamete fuses with the two polar nuclei.

One male gamete + Egg cell $\xrightarrow{Syngamy}$ Zygote.

Second male gamete + Two polar nuclei $\xrightarrow{Triple\ fusion}$ Triploid nucleus (Primary Endosperm Nucleus)

Syngamy + Triple fusion = Double fertilization .



Do you known ?

POST FERTILIZATION CHANGES IN THE FLOWER

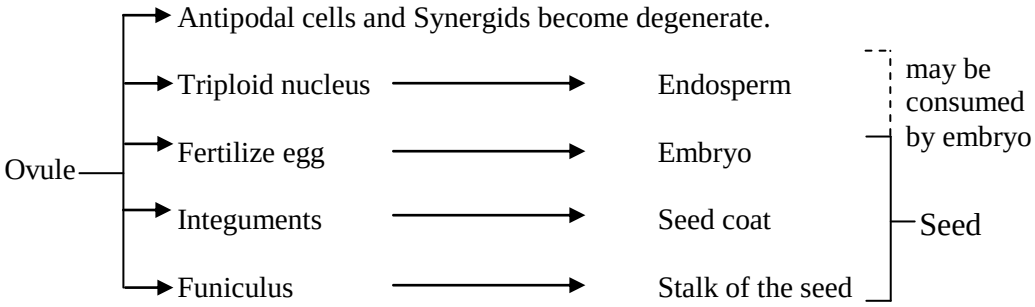
Sepals, petals and stamen withers off.

Style and stigma degenerates.

Ovary develops into fruit.

Ovule grows into seed.

The seed contains the future plant or embryo which develops into a seedling under appropriate conditions.



Endospermic seed : If endosperm is not consumed.

Non endospermic seed: Endosperm may be consumed.

REVIEW QUESTIONS

1. **Name the various parts of flower.**
2. **Write the name of non essential part of flowers.**
3. **Write the name of parts of stamen and carpel.**
4. **Name the swallow part of carpel.**
5. **Describe the unisexual and bisexual flower.**
6. **Fertilization takes place in which part of ovule.**

EXERCISE # 1 FOR SCHOOL / EXAMS.

OBJECTIVE QUESTIONS

1. Vegetative propagation is *Bryophyllum* takes place by :-
(A) stem (B) leaf (C) root (D) none of these

2. The group of petals is called :-
(A) sepals (B) calyx (C) root (D) None of these

3. During grafting, the portion of plant that is grafted is called :-
(A) stock (B) scion (C) stalk (D) stem

4. In roses, the method commonly used to produce new plants is :-
(A) tissue (B) cutting (C) layering (D) None of these

5. Pollen grains are produced by :-
(A) ovary (B) anther (C) stigma (D) petal

6. Which one is applicable to insect pollinated flowers :-
(A) Flowers are very small produced in large quantities.
(B) Flowers are not prominent and without nectar.
(C) Flowers are conspicuous and scented having nectar.
(D) None of these

7. The group of sepals is called :-
(A) gynoceium (B) calyx (C) corolla (D) androceium

8. Maize is :-
(A) self pollinated (B) cross pollinated by rain
(C) cross pollinated by insects (D) cross pollinated by wind

9. Which of the following produces male gametes in a flower :-
(A) sepals (B) petals (C) carpels (D) stamens

10. During pollination, pollen grains get carried to which part of the carpel ?
(A) Ovary (B) Stigma (C) Ovule (D) Style

11. At the time of entering into ovule, pollen tube has :-
(A) three male nuclei (B) two meal nuclei (C) one gamete nucleus (D) four male gametes

12. The transfer of pollen grains from anther to stigma is termed :-
(A) ovulation (B) double fertilization (C) pollination (D) fertilization

13. Fertilization in plants occurs in the :-
(A) embryo sac (B) style (C) pollen tube (D) stigma

14. Stem cutting are commonly used for propagation in :-
(A) mango (B) jasmine (C) cotton (D) sugarcane

15. Vegetative reproduction in plants like citrus, jasmine and grapevine is done by the process of :-
(A) stem layering (B) stem cutting (C) stem grafting (D) none of these

16. Asexual reproduction are :-

(A) Fission (B) Budding

(C) Vegetative propagation (D) All of these

17. Binary fission occurs in :-

(A) Plasmodium (B) Hydra (C) Pomegranate (D) Amoeba

18. Multiple fission occurs in :-

(A) Euglena (B) Yeast (C) Plasmodium (D) Paramecium

19. In which of the following reproduction parental identity is lost :-

(A) Budding (B) Binary (C) Multiple fission (D) Both B and C

20. Which of the following organisms shows budding :-

(A) Spirogyra (B) Hydra (C) Amoeba (D) Paramecium

FILL IN THE BLANKS :

1. In Rhizopus asexual reproduction takes place by ……..formation.

2. In Bryophyllum vegetative propagation takes place through their ……..

3. The process in which new organisms are formed by existing organism is called ……..

4. In Spirogyra, asexual reproduction, takes place by ……..

5. Natural vegetative propagation takes place in sweet potato by ……..

6. In…….method a branch of parent plant is buried in the soil.

7. In…….method a cutting part of a plant is grafted on the other plants part.

8. Unorganized mass of cells is known as……..

9. Tissue culture is also known as ……

10. Grafting is not possible in……plants.

11. All floral leaves are placed on the ……of flower.

12. Stalk of flower is known as…….

13. …….protects the flower bud before it opens.

14. If both sepals and petals are colored can not be distinguished from each other then their whorl is known as…….

15. Each stamen has two parts filament and ……

16. Each pollen grain produces ……male gametes.

17. A carpel has three parts ovary,……and stigma.

18. ……..is a swollen basal part of carpel.

19. ……..of carpel receives pollen grains.

20. Second male gamete + Two polar nuclei = ……

MATCH THE COLUMN :

	Column I		**Column II**
(a)	Calyx	(i)	Ovary
(b)	Corolla	(ii)	Sepal
(c)	Stamen	(iii)	Stigma
(d)	Carpel	(iv)	Petal
		(v)	Anther
		(vi)	Filament

VERY SHORT TYPE QUESTIONS :

1. Mention the reproductive parts of a flower.

2. Which parts of plants can grow vegetatively ?

3. What is the function of pollen gains in flowers ?

4. What is the other name of (i) Calyx (ii) Corolla (iii) Androceium (iv) Gynoceium ?

5. Give one example of each plant which propagates artificially by (a) cutting, (b) layering.

6. Which vegetative part is used in the propagation of *Bryophyllum* ?

7. What is stock in grafting ?

8. Name the agencies through which cross-pollination take place.

9. Name two abiotic agents of pollination.

10. Which group of plants shows double fertilization?

HOW DO ORGANISMS REPRODUCE — ANSWER KEY — EXERCISE # 1

● **Objective Question :**

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	B	B	B	C	B	D	D	B
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	A	D	A	D	D	C	D	B

● **Fill In The Blanks :**

1. spore **2.** leaf **3.** reproduction **4.** fragmentation **5.** root

6. layering **7.** grafting **8.** callus **9.** micro propagation **10.** monocot

11. thalamus **12.** pedicel **13.** calyx **14.** perianth **15.** anther

16. two **17.** style **18.** ovary **19.** stigma

20. primary endosperm nucleus

● **Match The Following :**

Ans. (a) –ii, (b) –iv, (c) –v, iv (d) –i, ii

● **Very Short Answer Type Questions :**

1. Stamen and carpel **2.** Root, stem and leaves

3. It produces male gametes which fertilizes the egg cell

4. (i) Sepal, (ii) Petal, (iii) Stamen, (iv) Carpel **5.** (a) Rose, (b) Jasmine

6. leaf **7.** Rooted plant is called stock **8.** Water, Wind, Insect, Bird, Man

9. Water and Wind **10.** Angiosperms

EXERCISE # 2 FOR SCHOOL EXAMS.

SHORT TYPE QUESTIONS :

1. What methods will you use for growing jasmine and rose plant ?
2. Leaves of Bryophyllum fallen on the ground produce new plants whereas the leaves of rose do not. Why ?
3. What are the two possibilities of self-pollination ?
4. What are the advantages of tissue culture ?
5. Why is cross-pollination considered to be superior than the self-pollination ?
6. List two main advantages of sexual reproduction .
7. 'Grafting is a common method of obtaining a superior plant from two different plants'. Explain.
8. Describe about the different plant of a stamen in male reproduction organ of a plant.
9. What is vegetative propagation ? Classify it along with examples.
10. How are these ornamental plats propagated ? Mention the name of the method :
(i) Jasmine (ii) Rose (iii) Bougainvillea (iv) Hibiscus

LONG ANSWER TYPE QUESTIONS :

1. Define the terms unisexual flower and bisexual flower giving one example of each.
2. Explain double fertilization in plants.
3. What is vegetative propagation ? When is it used ? Name three methods of vegetative propagation.
4. Differentiate between 'self-pollination' and 'cross-pollination'.
5. Draw a diagram of flower to show its male and female reproductive parts. Label on to :
(i) The ovary (ii) The anther (iii) The filament (iv) The stigma
6. Draw a labeled diagram of the longitudinal section of a pistil showing pollen germination .
7. Give two advantages of vegetative propagation.
8. What is pollination ? Describe cross-pollination.

NCERT QUESTIONS

1. What is the importance of DNA copying in reproduction ?
2. Why is variation beneficial to the species but not necessary for the individual ?
3. How does binary fission differ from multiple fission ?
4. How will an organism be benefited if it reproduces through spores ?
5. Can you think of reasons why more complex organisms can not give rise to new individuals through regeneration ?
6. Why is vegetative propagation practiced for growing some types of plants ?
7. Why is DNA copying an essential part of the process of reproduction ?
8. How is the process of pollination different from fertilization ?
9. What are the advantages of sexual reproduction over asexual reproduction ?
10. Draw a labeled diagram of the longitudinal section of a flower.
11. How are the modes of reproduction different in unicellular and multicellular organisms ?
12. How does reproduction help in providing stability to population of species ?

EXERCISE # 3 COMPETITIVE EXAMS.

1. If the pollen is transferred to the stigma of the same flower, it is termed :-
(A) Allogamy (B) Geitonogamy (C) Autogamy (D) all of these

2. Which part of the flower forms the fruit ?

(A) Whole flower (B) Only stamens and carpel

(C) Only ovary (D) Only carpel

3. In angiosperm after the fertilization endosperm becomes :-

(A) Haploid (B) Diploid (C) Triploid (D) Tetrapoda

4. After fertilization ovule grows into :-

(A) seed (B) fruit (C) placenta (D) None

5. Which is not a part of carpel :-

(A) Stigma (B) Ovary (C) Anther (D) Style

6. Nucleus of the bud is formed by the division of :-

(A) Meiosis (B) Amitosis (C) Mitosis (D) All of theses

7. Clones are formed as a result of :-

(A) Budding (B) Regeneration

(C) Vegetative propagation (D) All of these

8. Malarial parasite reproduces by :-

(A) Multiple fission (B) Binary fission (C) Budding (D) Regeneration

9. The outgrowth of *Hydra* is termed as :-

(A) Bulb (B) Bud (C) Daughter (D) Tentacles

10. Asexual reproduction takes place by the process of budding :-

(A) Plasmodium (B) Amoeba (C) Yeast (D) Rhizopus

11. Asexual reproduction involves :-

(A) Only one parent (B) Two parent

(C) Meiosis and syngamy (D) Fusion of two gametes

12. Which is a part of stamen :-

(A) Stigma (B) Filament (C) Style (D) None

ANSWER KEY

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	C	C	C	A	C	C	D	B	C	C	A	B

REPRODUCTION IN HUMAN BEINGS

Reproduction resulting from the fusion of male gamete and female gamete is called **sexual reproduction.**

OR

The type of reproduction in which fusion of male gamete & female gamete occur is called **sexual reproduction.**

Important features of sexual reproduction are given below :

(i) It involves two different parents i.e. one male and one female.

(ii) Each parent produces gametes.

(iii) Male gametes are called **sperms** while female gametes are called **ova** or **eggs.**

(iv) The fusion of male and female gametes is called **fertilization.** It results in to the formation of a single diploid cell **zygote.**

(v) The zygote undergoes repeated mitotic divisions to form embryo which differentiate to form full organism.

(vi) The organism produced in this type of reproduction are genetically different from both the parents and can resemble in certain features with parents.

HUMAN REPRODUCTIVE SYSTEM

Puberty :- The age at which the gametes and sex hormones to be produced and the boy and girl become sexually mature is called **puberty.**

Generally **female** pubertal age is **10-12 years, male** pubertal age is **13-14 years.**

Pubertal Changes (Secondary Sexual Characters) in Male :

Widening of shoulders.

Deepening of voice.

Growth of hairs under chest, armpits and around pubic area.

Appearance of beard and moustaches.

Growth of sex organs, [Testes & Penis].

Increased Activity of sweat and sebaceous glands.

Oily skin and appearance of pimples.

Darkening in skin colour of the genital area.

Pubertal Changes (Secondary Sexual Characters) in Female :

Widening of pelvis and hips.

High pitch voice

Growth of hairs under armpits and around pubic area.

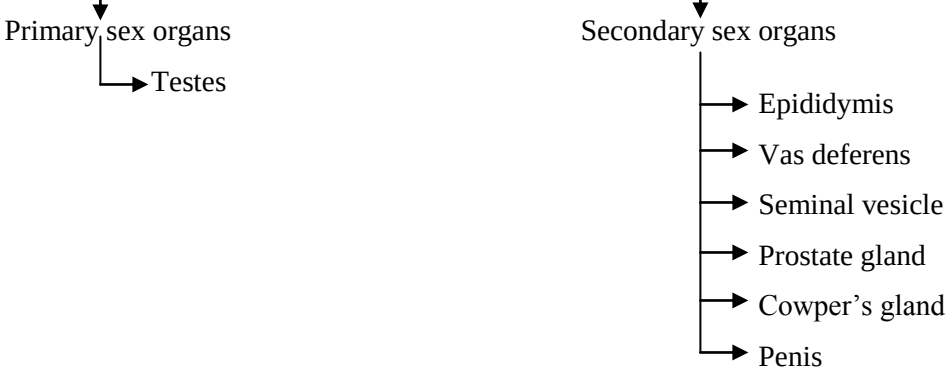
Initiation of menstrual cycle.

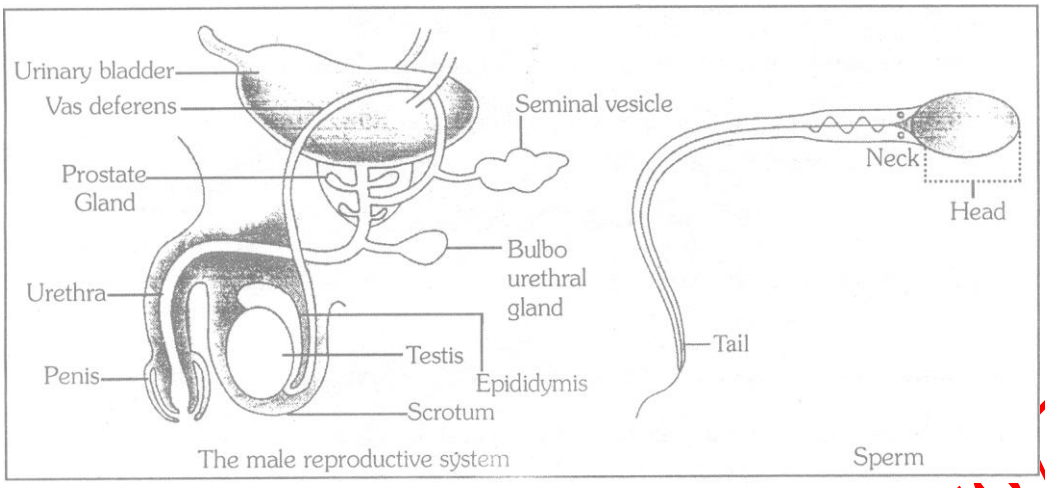
Growth of mammary glands (breasts).

Darkening in skin colour of genital area.

Maturation of secondary sex organs like fallopian tubes, uterus.

MALE REPRODUCTIVE SYSTEM





The male reproductive system consists of portions which produce the germ-cells and other portions that deliver the germ-cells to the site of fertilization.

The formation of germ-cells or sperms takes place in the **testes.** These are located outside the abdominal cavity in **scrotum** because **sperm formation requires a lower temperature [1–3^0]C than the normal body temperature.**

Testes secrete male sex hormone called **testosterone.**

In addition to regulation the formation of sperms, testosterone brings about changes in appearance seen in boys at the time of puberty. These changes are called **secondary sexual characters.**

The sperms formed are delivered through the vas deferens which unites with a tube coming from the urinary bladder. The urethra thus forms a common passage for both the sperms and urine. Hence urethra is also known as **urinogenital tract .**

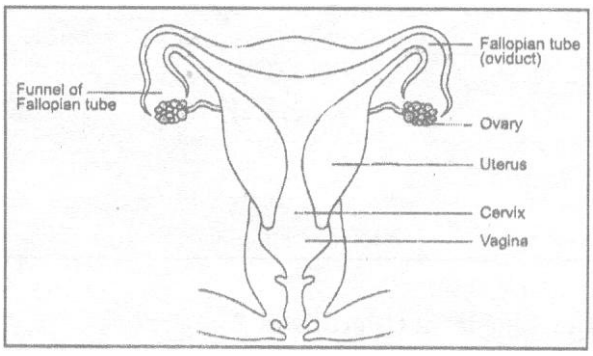
Along the path of the vas deferens, glands like the **prostate gland** and the **seminal vesicle** add their secretions so that sperms are now in a fluid which makes their transport easier and this fluid also provides nutrition.

The sperms are tiny bodies that consist of mainly genetic material and a long tail that helps them to move towards the female germ-cell (ovum).

FEMALE REPRODUCTIVE SYSTEM

- Primary sex organs
  - Ovaries
- Secondary sex organs
  - Fallopian tubes
  - Uterus
  - Vagina
  - Mammary glands

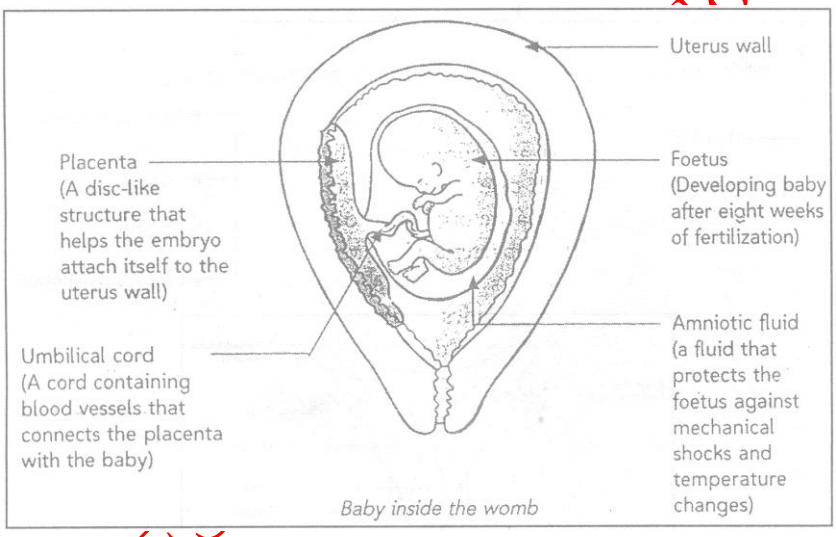
The female germ-cells or eggs are made in the ovaries. They are also responsible for the production of female sex hormones i.e., **Oestrogen** and **Progesterone.**



When a girl child is born, the ovaries already contain thousands of immature eggs. On reaching puberty, some of these start maturing.

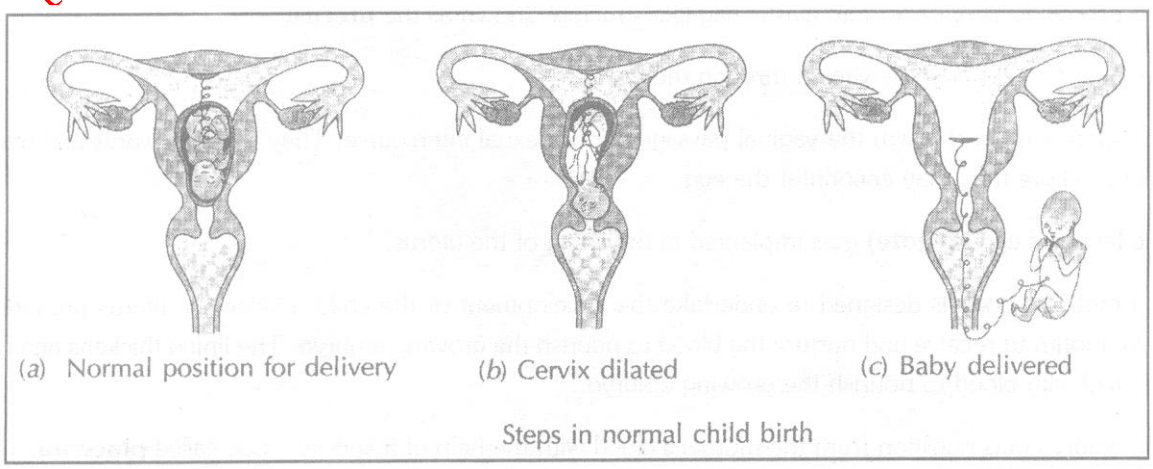
One egg is produced every month by one of the ovaries.

The egg is carried from the ovary to the womb through a thin **oviduct** or **fallopian tube.**
The two oviducts unite into an elastic bag-like structure known as the **uterus.**
The uterus opens into the vagina through the **cervix.**
The sperms enter through the vaginal passage during sexual intercourse. They travel upwards and reach the oviduct where they may encounter the egg.
The fertilized egg **(zygote)** gets implanted in the lining of the uterus.
The mother's body is designed to undertake the development of the child. Hence the uterus prepares itself every month to receive and nurture the blood to nourish the growing embryo. The lining thickens and is richly supplied with blood to nourish the growing embryo.
The embryo gets nutrition from the mother's blood with the help of a special tissue called **placenta.** This is a disc which is embedded in the uterine wall. It contains **villi.** On the mother's side are blood spaces, which surround the villi. This provides a large surface area for glucose and oxygen to pass from the mother to the embryo.
The developing embryo will also generate waster substances which can be removed by transferring them into the mother's blood through the placenta. The development of the child inside the mother's body takes approximately **nine months.** The child is born as a result of rhythmic contractions of the muscles in the uterus, called **labor pain.**



Baby inside the womb

What Happens When the Egg is not fertilized?
If the egg is not fertilized, it lives for about one day.
Since the ovary releases one egg every month, the uterus also prepares itself every month to receive a fertilized egg. Thus its lining becomes thick and spongy. This would be required for nourishing the embryo if fertilization had taken place.
This lining is not needed any longer. So, the lining slowly breaks and comes out through the vagina as blood and mucous. This cycle takes place roughly every month and is known as **menstruation.** It usually lasts for about two to eight days.



Steps in normal child birth

Reproductive Health :

Sexually Transmitted Diseases (STDs)

There are many infections diseases which are spread by sexual contact, called **Sexually Transmitted Diseases (STDs) e.g.** AIDS, Hepatitis.

STDs occur mostly in the individuals who are involved in sexual activities with many partners.

Some common sexually transmitted diseases (STDs)			
S.No.	**Name of STDs**	**Causal Organism**	**Symptoms**
1	AIDS (Acquired Immuno Deficiency Syndrome)	HIV (Human Immuno Deficiency Virus)	Destroy the immune system of body. Persistent cough and fever. Body attacked by other diseases like pneumonia, T.B. and certain cancers.
2	Syphilis	Treponema pallidum (a bacterium)	Causes sores and lesions in the genital tract. Burning sensation at urination.
3	Gonorrhea	Nesseria gonorrhoeae (a bacterium)	Infects mucous membranes of the urinogenital tact. Genital discharge, painful urination.
4	Trichomoniasis	Trichomonas vaginalis (a protozoan)	Vaginal irritation, itching and discharge.

Methods of prevention of STDs :

(i) The people should be educated about various STDs

(ii) Extra marital relations should be avoided.

(iii) No sex without proper precaution.

(iv) High standard of moral education should be given to the people

Methods adopted for population control :- The prevention of pregnancy in women is called **contraception.**

1. Planned control of population :

(i) By education people about the advantages of small family

(ii) Raising the age of marriage can help in reducing population growth.

(iii) By family planning.

2. Natural method :-

(i) Intercourse is safe for a week before and week after menstruation

(ii) Coitus interrupts involves withdrawing penis before ejaculation.

3. Mechanical methods :-

(i) It includes use of condoms which are the rubber or plastic sheets put on the penis before coital activity.

(ii) Use of **diaphragms** or **cervical caps** fitted in vagina of female to check the entry of sperms into the uterus and also help in avoiding conception.

(iii) Use of **IUCD** i.e., **I**ntra **U**terine **C**ontraceptive **D**evices like **C**opper-**T** and **loops** fitted in the uterus, help to prevent fertilization. They can cause side effects due to irritation of uterus.

4. Chemical methods :-

(i) It consists of using some chemicals which are **spermicidal.** They may be in form of tablets, jellies, paste and creams introduced in the vagina before coital activity.

(ii) Another chemical method is the use of **oral contraceptive** (OC) **pills** which inhibit the secretion of FSH and LH from the anterior lobe of pituitary gland and thus inhibiting ovulation from the ovary. These contraceptive therefore change the hormonal balance so that egg cell are not released and hence prevent fertilization.

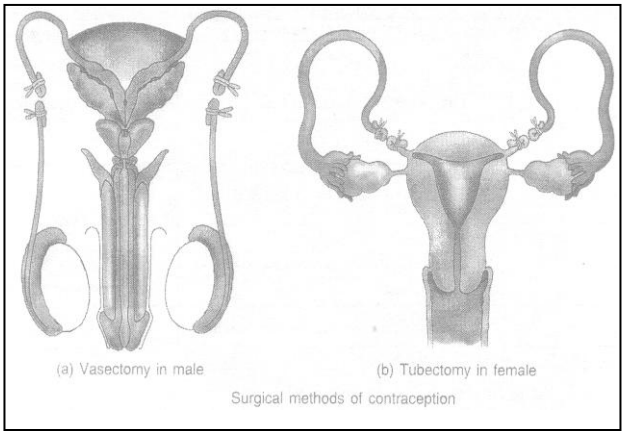
5. Surgical methods :-

(i) **Tubectomy** involves cutting of fallopian tubes in females and **Vasectomy** involves cutting of vas deferens of each side.

(ii) Removal of ovaries surgically is known as **ovariectomy** and removal of testes is known is **castration.**

(iii) Another surgical method is **MTP** i.e. Medical Termination of Pregnancy or abortion.

(iv) Other method is **tubal ligation** in which fallopian tubes are blocked by an instrument called **laparoscope.**



Surgical methods of contraception

REVIEW QUESTIONS

1. Expand the following terms :
(i) IUCD (ii) STD (iii) HIV (iv) AIDS

2. How do oral contraceptives function ?

3. What is contraception ?

4. What is family planning ?

5. What is vasectomy ?

6. What do you understand by fertilization ?

7. What is the product of fusion of a sperm and an ovum known as ?

8. What changes occur in girls in the age group of 10-13 years ?

9. Draw a well-labeled diagram depicting vasectomy and tubectomy.

10. Why is fertilization in humans possible, if copulation takes place during the middle of the menstrual cycle ?

COMPETITIVE WINDOW

India was the first country to adopt family planning as the government sponsored programme in 1951.

Menarche : The onset of menstruation in a young female at about the age of 13 years is termed as **menarche.**

Menopause : The permanent stoppage of menstruation at about the age of 45 years is termed as **menopause.**

Menstrual cycle is also called **ovarian cycle.**

In human female, ovulation starts only after puberty and occurs once in a month and midway through the menstrual cycle.

Menstrual cycle is absent before puberty and temporarily stopped during pregnancy.

Menstrual cycle occurs **only in the primates** among the mammals.

Parthenogenesis: A primitive type of sexual reproduction in which unfertilized eggs develop to form haploid adult organisms. It is found in female aphids, drones (males) of honey bees and termites, and certain lizards. In fresh water sponges **(Spongilla),** internal buds are formed within the parent body. Such buds are called **gemmules.** The gemmules, on arrival of favorable conditions, develop into a sponge colony.

After fertilization, membrane appears around the egg to prevent further entry of sperms. It is called **monospermy.** After the entry of sperm, ovum completes its maturation division.

Amniocentesis :- Amniocentesis is a prenatal diagnostic technique to determine the genetic disorders, if any, of the fetus. Unfortunately, the useful technique of amniocentesis is being misused to kill the normal female fetuses as it can help to detect the sex of fetus also. Determination of sex by amniocentesis has been banned.

EXERCISE # 1 FOR SCHOOL / BOARD EXAMS.

OBJECTIVE QUESTIONS

1. Binary fission occurs in :-
(A) Amoeba (B) Paramecium (C) Planaria (D) A & B both

2. Which one of the following is concerned with asexual reproduction ?
(A) Zygote (B) Spores (C) Gametes (D) Gonads

3. Which type of reproduction of Hydra is most common ?
(A) Budding (B) Fragmentation
(C) Sexual reproduction (D) Gametogenesis

4. The most fundamental characteristics of living being :-
(A) Locomotion (B) Regeneration (C) Fragmentation (D) Reproduction

5. Multiple fission occurs in :-
(A) Hydra (B) Plasmodium (C) Planaria (D) All of these

6. The animals consist of both male & female sex known as :-
(A) Viviparous (B) Oviparous (C) Sterile (D) Hermaphrodite

7. Animals which give birth to young ones are called :-
(A) amphibious (B) oviparous (C) Triploblastic (D) viviparous

8. Tests of man occur :-
(A) inside body (B) upper side of kidney
(C) on either side of dorsal aorta (D) in scrotal sacs

9. Fertilization of ovum takes place in :-
(A) ovary (B) fallopian tube (C) cervix (D) uterus

10. Oogenesis is a process of formation of :-
(A) Sperms (B) Ova (C) Sperms and ova (D) None of these

11. Middle piece of a mammalian sperm contains :-
(A) nucleus (B) acrosome (C) vacuole (D) mitochondria

12. Male hormone is :-
(A) Corpus luteum (B) Testosterone (C) Progesterone (D) Gonadotropin

13. Sperms move by :-
(A) head (B) acrosome (C) middle piece (D) tail

14. Binary fission is a from of :-
(A) sexual reproduction (B) asexual reproduction
(C) both of these (D) none of these

15. Fertilization of frog takes place in :-
(A) Uterus (B) Fallopian tube
(C) Water (D) Cervix

16. Which of the following can reproduce thorough regeneration ?
(A) Hydra (B) Planaria (C) Wall lizard (D) Both (A) and (B)

17. Development of an organism from an unfertilized ovum is called :-
(A) oogenesis (B) parthenogenesis
(C) vegetative propagation (D) asexual reproduction

18. Egg-producing animals such as birds are called :-

(A) unisexual (B) oviparous (C) viviparous (D) hermaphrodite

19. What is true for gametes ?

(A) They are diploid (B) They form gonads

(C) They are formed from gonads (D) They produce hormones

20. Which of these secretes seminal fluid ?

(A) Prostate gland (B) Cowper's gland (C) Seminal vesicle (D) All of these

21. The normal duration of menstrual cycle is :-

(A) 7-8 days (B) one day (C) 3-4 days (D) none of the above

22. Female sex hormones are :-

(A) Progesterone (B) Estrogen (C) Testosterone (D) A & B Both

23. The process of attachment of blastocyst in the endometrium of uterus is known as :-

(A) placentation (B) implantation (C) gestation (D) cleavage

24. Which of the following is an IUCD ?

(A) Copper-T (B) Diaphragm (C) Oral pills (D) Tubectomy

25. The Primary reason for increase in human population is :-

(A) the increase in agricultural production

(B) the increases in birth rate and decreases in death rate

(C) the improvement in medical technology

(D) all of the above

FILL IN THE BLANKS

1. The……..is also called womb.

2. The…….is primary reproductive organ in male.

3. The ……is connecting structure which helps in the transfer of substance to and from the fetus body.

4. The process of laying eggs in large number is called……..

5. ……is the primary reproductive organ in female.

6. Testes produce……

7. The cells involved in sexual reproduction are called…….

8. In animals like fish and frog……fertilization take place.

9. The human zygote gets implanted in the …….

10. IVF stands for……

CHOOSE TRUE AND FALSE STATEMENTS

1. Internal fertilization takes place outside the female's body.

2. Birds and snakes are young oviparous animals.

3. A tadpole is the young one of a frog.

4. Animals like **Amoeba** multiply by budding.

5. The fusion of male and female sex cells is called fertilization.

MATCH THE COLUMNS

	Column-A		**Column-B**
1	Tapeworm	(a)	Uterus
2	Vasectomy	(b)	STD
3	Copper-T	(c)	IUCD
4	Implantation	(d)	Vas deferens
5	AIDS	(e)	Hermaphrodite

● **Objective Question :**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	B	A	D	B	D	D	D	B	B	D	B	D	B	C
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	D	B	B	C	D	C	C	B	A	D					

● **Fill In The Blanks :**

1. Uterus **2. Testis** **3. Placenta** **4. Super ovulation**
5. Ovary **6. Testosterone** **7. Germ cells** **8. External**
9. Uterus **10. In vitro fertilization**

● **True & False :**

1. F **2.** T **3.** T **4.** F **5.** T

● **Match The Column :**

1. (1) –e, (2) –d, (3) –c, (4) –a, (5) –b

EXERCISE # 2 FOR SCHOOL / BOARD EXAMS

VERY SHORT ANSWER TYPE QUESTIONS :

1. Where does fertilization take place ?
2. What is a fetus ?
3. How is an embryo produced ?
4. What is a zygote ?
5. Write three examples of animals in which external fertilization occur.
6. Draw a labeled diagram of male reproductive system.
7. Name the parts present in human sperm. Show by labeled diagram.
8. What is Syngamy ?
9. What is the name given to primary sex organs ?
10. What are copulatory organs ?

SHORT ANSWER TYPE QUESTIONS :

1. What are the parts of a male reproductive system ?
2. Define metamorphosis .
3. How is internal fertilization differ from external fertilization ?
4. How does a sperm differ from an ovum ?
5. Draw a labeled diagram of female reproductive system.
6. What are oviparous & viviparous animals ?
7. Describe the process of development from zygote to fetus in brief.
8. What is puberty ?

9. At what age do human males and females attain puberty ?

10. What is tubectomy ?

11. Why is it important to study about reproductive health ?

12. What is ovulation ?

13. Which organ enables the developing fetus to obtain nourishment from the mother's blood ?

14. What are Graffian follicles ?

15. What is gestation ?

16. How many follicles mature every month during the reproductive phase of a human female ?

17. What is internal fertilization ? Give an example.

18. What happens if the mature ovum is not fertilized in a female ?

19. Distinguish between

(i) Egg and Embryo (ii) Male and Female Urethra

20. Why is vagina called as 'birth canal' ?

LONG ANSWER TYPE QUESTIONS :

1. Why do female frogs produce eggs in large number ?

2. "Sexual reproduction is more advance than asexual reproduction" why ?

3. How fetus is different from embryo ?

4. In which female reproductive organ does the embryo get embedded and why ?

5. How could a single cell become such a big individual ?

NCERT QUESTIONS :

1. What is the role of the seminal vesicles and the prostate gland ?

2. What are the change seen in girls at the time of puberty ?

3. How does the embryo get nourishment inside the mother's body ?

4. If a woman is using a copper-T, will it help protecting her from sexually transmitted diseases ?

5. What are the functions performed by the testis in human beings ?

6. Why does menstruation occur ?

7. What are the different methods of contraception ?

8. What could be the reason for adopting contraceptive methods ?

EXERCISE # 3 COMPETITIVE EXAMS.

1. The importance of reproduction in organisms is because of :-

(A) Formation of new individuals

(B) Production of individuals with same traits

(C) Production of individuals with different traits so as to being varieties in a population

(D) All of the above

2. Twins absolutely resembling each other in sex and external appearance result when :

(A) Two similar sperms fertilize two similar eggs

(B) Same sperm fertilizes two eggs

(C) Two halves of the same egg develop separately after it is fertilized by one sperm

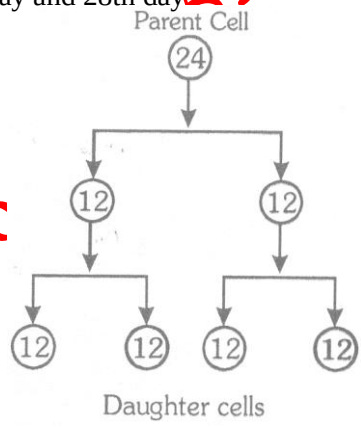
(D) Two halves of the same egg are fertilized by two separate sperms

3. Copulation in human beings may result in fertilization during (normal menstrual cycle is for 28 days)

(A) 4th day and 10th day (B) 21th day and 28th day

(C) 11th day and 21st day (D) any day between 1st day and 28th day

4. The chart given here shows a cell division. The division is :-



(A) Mitosis

(B) Meiosis

(C) Division of a zygote during development

(D) Division of an Amoeba during binary fission

5. The gametes are formed as a result of :-

(A) Vegetative propagation (B) Asexual reproduction

(C) Meiosis (D) Mitosis

6. Which of the following tests is for determining, the sex of the fetus ?

(A) Blood group test (B) Amniocentesis (C) Blood sugar test (D) pH value test

7. The structure which provides a place for attachment and exchange of materials between mother and the fetus is called :-

(A) Uterus (B) Umbilical cord (C) Oviduct (D) Placenta

8. Reproduction by budding takes place in :-

(A) Hydra and Earthworm (B) Hydra and Yeast

(C) Yeast and Bacteria (D) Bacteria and Amoeba

9. During mitosis which of the following is / are equally distributed in daughter cells ?

(A) Chloroplasts (B) Cytoplasm

(C) Chromosomes (D) Centrosomes

10. In cells after they attain a certain size, growth stops and the sell undergoes division. The probable reason for this is :-

(A) The volume increases more than the surface area and the ratio between the two becomes less

(B) Both the volume and surface area increases in equal proportion

(C) The surface area increases faster than the volume

(D) The surface area is several times more than the volume.

11. Puberty age in girls is between :-

(A) 12-18 years of age (B) 10-16 years of age

(C) 14-20 years of age (D) 15-18 years of age

12. Fertilization of an ovum with a sperm takes place in :-

(A) Uterus (B) Vagina

(C) Fallopian tube (D) Cervix

13. A human zygote has :-

(A) 46 chromosomes (B) 23 chromosomes

(C) 47 chromosomes (D) 48 chromosome

14. Regeneration is a process in which :-

(A) A tumor is produced (B) A new individual is produced

(C) Missing parts grow again (D) Old individuals are replaced by new ones

15. As compared to human egg the spermatozoa has less :-

(A) Chromosomes (B) Centrosomes

(C) Cytoplasm (D) Mobility

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	C	C	B	C	B	D	B	C	A	B	C	A	C	C

Important Notes

LIFE PROCESS – NUTRITION

WHAT IS LIFE PROCESS ?

All the living organisms including human beings perform a number of activities such as nutrition, reparation, excretion, growth and reproduction. These activities are characteristics of living organisms, and through such activities they maintain their lives these maintenance function of living organisms are known as **life processes.**

Chemical reactions which takes place within cells or organisms during various vital activities are called **biochemical reactions.**

Metabolism is a word used to describe the sum total of all the chemical and physical changes that are constantly taking place in living matter and are necessary for life. The word **metabolite** refers to a substance which undergoes various changes during metabolism. For example, carbon dioxide and water metabolites used in the process of photosynthesis.

The metabolic pathways are of two types :

(i) **Anabolic pathways** or **biosynthetic pathways** in which biosynthesis of organic compounds occurs, or in other words, complex substances are synthesized from simpler ones ; foe example photosynthesis.

(ii) **Catabolic pathways** in which the breakdown of complex organic substances into simpler ones occurs (as in respiration)

In anabolic pathways or processes of **anabolism** energy is used **(endothermic reactions),** while in catabolic pathways or **catabolism,** energy is released **(exothermic reactions).**

Criteria to define something is alive :-

1. **Nutrition :-** The processes by which the organisms obtain and utilize the nutrients (food).
2. **Respiration :-** The process that involves breakdown of respiratory substrates through oxidation and release of usable energy
3. **Transport :-** The process in which the substances absorbed or synthesized in one part of the body are carried to other parts to the body.
4. **Excretion :-** The process involved in removal of the excess or toxic waster from the body.
5. **Control and coordination :-** The process which helps the living organisms to receive information from the surroundings and behave accordingly in order to survive in the changing environment around them.
6. **Growth and development :-** Permanent increases in the size of the organisms is called growth. The whole series of changes which an organism goes through during its life cycle, is called **development.**
7. **Movement and Locomotion**

Definition :- The basic function preformed by living beings to sustain themselves are called life processes.

NUTRITION

Nutrients are inorganic as well as organic substances which the organisms obtain from their surroundings in order to synthesize their body constituents and use them as a source of energy.

The process of intake of nutrients and its utilization by an organism in various biological activities.

Or

A process to transfer a source of energy from outside the body of the organism (food), to the inside is called nutrition.

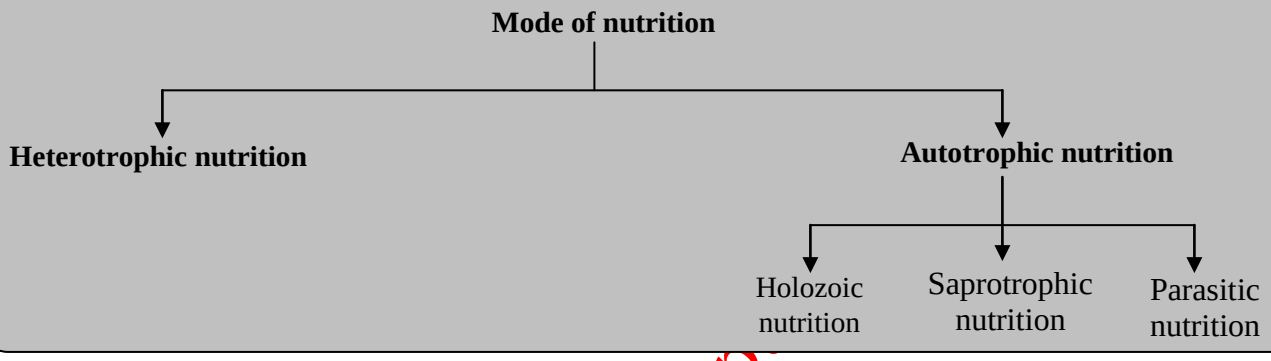
There are various type of nutrients on the basis of function they perform :-

e.g. **Energy foods :- Carbohydrates and fats.**

Body building foods :- Proteins and mineral salts.

Regulating foods :- Vitamins and minerals.

Modes of nutrition :- Method of obtaining food by the organism is called mode of nutrition.



(A) Autotrophic (Holophytic) nutrition :- The mode of nutrition in which the organisms prepare (or synthesize) their own organic food by using inorganic raw material (CO_2 & H_2O). They are also called **autotrophs.**

e.g. Plants, Photosynthetic and chemosynthetic bacteria and Cyanobacteria etc.

(B) Heterotrophic nutrition :- The mode of nutrition in which the organisms derive their nutrition from other organisms. They take ready mead organic food from other dead or living plants or animals. The living organisms showing heterotrophic nutrition, are called **heterotrophy.**

e.g. All animals, fungi, many bacterial and some non-green plants (insectivorous plants) and man.

Types of heterotrophic nutrition ;- Depending upon the mode of obtaining food, the heterotrophic nutrition is of following types.

(i) Holozoic nutrition (Holo-Complete + Zoon-animal)

The mode of nutrition in which all animals take in complex solid food material is called Holozoic nutrition.

It contains following steps :-

(i) **Ingestion :-** Taking in complex organic food through mouth opening.

(ii) **Digestion :-** Change of complex food into simple diffusible from by the action of enzymes.

(iii) **Absorption :-** Passing of simple, soluble nutrients into blood or lymph.

(iv) **Assimilation :-** Utilization of absorbed food for various metabolic processes.

(v) **Egestion ;-** Expelling out the undigested food.

e.g. All animals including vertebrates and Invertebrates.

★ Depending upon the **type of the food habit,** animals are divided into three categories :-

(a) Herbivores :- Animals that depend up on green plants are known as herbivores . **e.g.** Goat, Cow, Deer, Rabbit.

(b) Carnivores :- Animals which eat flesh of other animals as food are called carnivores. **e.g.** Lion, Tiger.

(c) Omnivores :- Animals which eat both plants and animals as food are known as omnivores. **e.g.** Rat, Pigs, Crows, Cockroaches and Humans.

(ii) Saprotrophic (Sapro-Rotten : Trophos - Feeder) Nutrition :- In this type of nutrition the organisms obtain their food from decaying organic substances. Organisms are also called **saprotrophs.**

e.g. Bacteria, Fungi.

(iii) Parasitic nutrition (Para-other) :-

The mode of nutrition in which one organism (called parasite) derive its food from other living organism (Host) is called parasitic nutrition. e.g. Tape worm, Ascaris, Plasmodium, Liver flukes, Cuscuta etc.

Differences between Autotrophic & heterotrophic nutrition :-		
Characters	**Autotrophic**	**Heterotrophic nutrition**
(1) Source of Energy	Sunlight or chemical energy	Readymade food
(2) Mode of Nutrition	Photosynthesis of Chemosynthesis	Feeding upon dead or living plants or animals
(3) Occurrence	Found in green plants, Blue-green algae, certain Bacteria	Found in Animals, fungi, Most of the bacteria
Differences between Holozoic and Saprotrophic nutrition :-		
Feature	**Holozoic nutrition**	**Saprotrophic nutrition**
1. Nature of food	Solid food (Whole plant or animal or their parts) is ingested	Liquid food (Dead and decaying organic matter) is ingested
2. Site of digestion	Inside the body	Outside the body as enzymes are released on the food material that convert solid food into simple soluble form
Examples	Most of the Animals	**Fungi –** Yeast, Slime moulds, Bacteria.

DO YOU KNOW ?

Animals which depend upon the blood of other animals known as **sanguivores.**

e.g. Bedbug, Mosquito, Leech etc

Some organisms take in predigested food through their body wall by the process of diffusion. This process of nutrition is known as **osmotrophic nutrition.**

e.g. Tapeworm, Trypanosoma.

Mutualistic nutrition :- The Mutualistic nutrition can be defined as the interdependent nutrition in which each organism is dependent mutually on the other.

e.g. The lichens share Mutualistic nutrition between a fungus and a Algae.

Review Questions

FILL IN THE BLANKS :

1. An organism which manufactures its own food from simple inorganic raw materials is known as……
2. Organisms which feed upon dead and decaying organic matter are known as ……
3. ……..and……are examples of saprophytes.
4. Yeast is ……..in nutrition.
5. Amarbel is……..in nutrition.
6. Define life process.
7. Define nutrition .
8. What is heterotrophic mode of nutrition ?
9. What criteria do we use to decide whether something is alive ?
10. What processes would you consider essential for maintaining life ?

Nutrition in unicellular organisms (Example – Amoeba) :-

⏩ **Food –** Amoeba is Holozoic and omnivorous animal. It feeds upon microscopic organisms like bacteria, paramecium, Diatoms, Algae and dead organic matter.

Mechanisms. Nutrition in Amoeba involves the following steps :

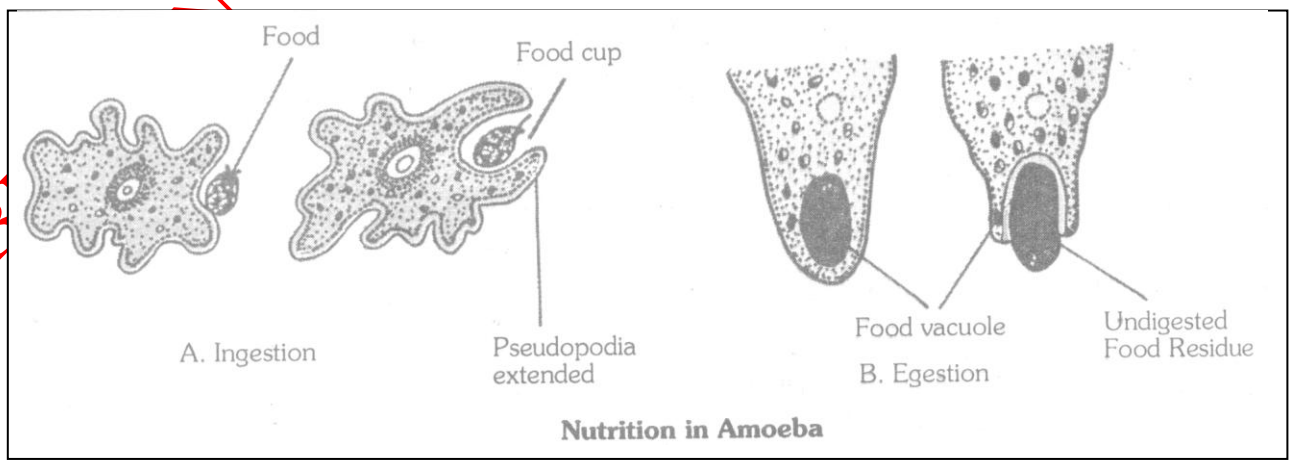
(i) Ingestions :- Amoeba has no mouth, so ingestion may occur at any point of body surface but generally it occurs at the advancing end of the body. ingestion occurs with the **help of pseudopodia.** The opening of food cup gradually becomes narrower and narrower, and finally closes. So the food is finally enveloped and taken inside a **food-vacuole** (called **phagosome**) along with a drop of water.

(ii) Digestions :- Amoeba shows **intracellular** and **vacuolar digestion.** In the cytoplasm, food vacuole fuses with Lysosome containing digestive enzymes, In this, the complex and non-diffusible nutrients are changed into simple and diffusible nutrients, Medium insider the food vacuole is **first acidic but later becomes alkaline,** (as in the alimentary canal of man).

(iii) Absorption and assimilation :- In absorption, the diffusible nutrients pass through vacuolar membrane into cytoplasm by diffusion and are then distributed to all the body parts by **streaming movements** of cytoplasm called **cyclosis.** Due to this, the size of food vacuole gradually decreases.

In the cytoplasm, a part of the absorbed food is oxidized to produce energy, most of simple nutrients are combined to synthesize complex compounds.

(iv) Egestion :- Amoeba has no anus, so Egestion may occur at any point on the body surface.



Nutrition in Amoeba

COMPETITION WINDOW

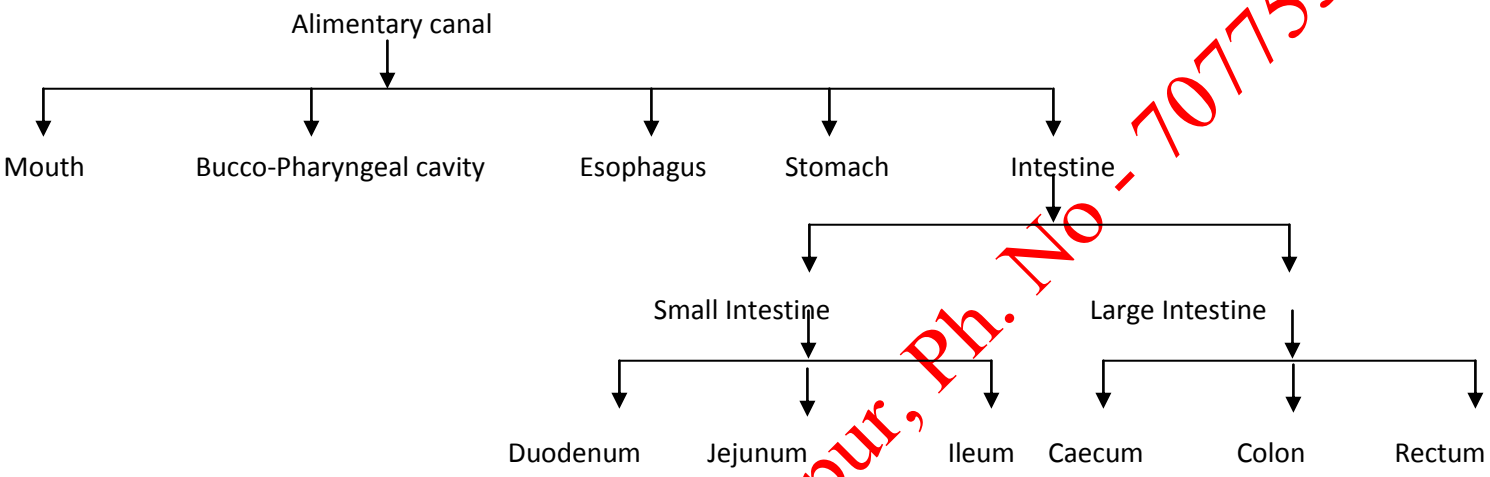
1. **Food :-** The substance which is palatable, delicious enough and energy provider is called food.
   Chemically food consists of six essential components :-
   (i) Carbohydrates (ii) Fats (iii) Proteins (iv) Minerals (v) Vitamins (vi) Water
2. **Digestion :-** Digestion is a catabolic process, in which the complex, non-diffusible and larger components of the food, are broken down into their respective simpler, diffusible and smaller form with the help of various hydrolytic enzymes in the alimentary canal of living organisms.
3. **Intracellular and Extra-cellular Digestion :-**
   Intracellular Digestion : This type of digestion occur inside the cell cytoplasm. The food inside the cell occurs as food vacuole. The digestive enzyme in this case are secreted inside the cell. They digest the contents of the food vacuole. So the entire process of digestion occurs inside the cell. e.g. Protozoans [Amoeba], Sponges
   Extracellular Digestion :- It takes place outside the cell [i.e. in the intercellular space or a cavity formed by many cells or tissue]. In all animals this cavity is found as a large canal, called **Alimentary canal.**
4. **Hydrolysis : -** It is a kind of catabolic reaction in which a compound is broken [lysis – break] down into smaller compounds, with the help [addition] of water [hydro = water].
5. **Carbohydrates :-** These are the hydrates of carbon in which the ratio among carbon, hydrogen & oxygen is 1 : 2 : 1. Carbohydrates are the quickest source of energy .
   On the basis of their composition, carbohydrates are of following types :
   (a) Monosaccharide :- The simplest sugars are called monosaccharides. These sugars cannot be further degraded to produce more sugars. e.g. Glucose, Fructose, Galactose, Ribose and Deoxyribose.
   (b) Oligosaccharides :- These are complex sugars, formed by the polymerization of a few [1 to 10] units of monosaccharide.
   Sucrose – Glucose + Fructose
   Maltose – Glucose + Glucose
   Lactose – Glucose + Galactose
   (c) Polysaccharides :- These are most complex carbohydrates, which are the polymers of thousand of units of monosaccharide. e.g. Starch – Stored food in plants, Glycogen – Stored food material in Animals. Cellulose – Constituent of cell wall.
6. **Fats :-** These are energy rich compounds. These are the esters of higher fatty acids. [Esters are formed by the addition of alcohol with acids] **Alcohol** – generally Glycerol.
7. **Proteins :-** Proteins are the polymers of amino acids. Amino acids are held together by means of peptide bond to form polypeptide chains.
8. On the basis of gross size of food, the mechanism in different animals may be of two main types :-
   (a) Microphagy – Feeding on microscopic organisms, e.g. Amoeba, Paramecium.
   (b) Macrophagy – Feeding on larger forms of organisms. e.g. Majority of non-chordates and some chordates.
9. In Paramecium, ingestion is aided by beating of cilia. It has definite food passage, mouth (cytostome) and anus (cytopyge).
10. Food vacuole is commonly called **temporary stomach** or **gastriole** as it is the site of storages of food.
11. Most common mode of ingestion in Amoeba is **circumvallation.** In this, pseudopodia extend and form a cup-like structure, called **food cup,** around the prey.

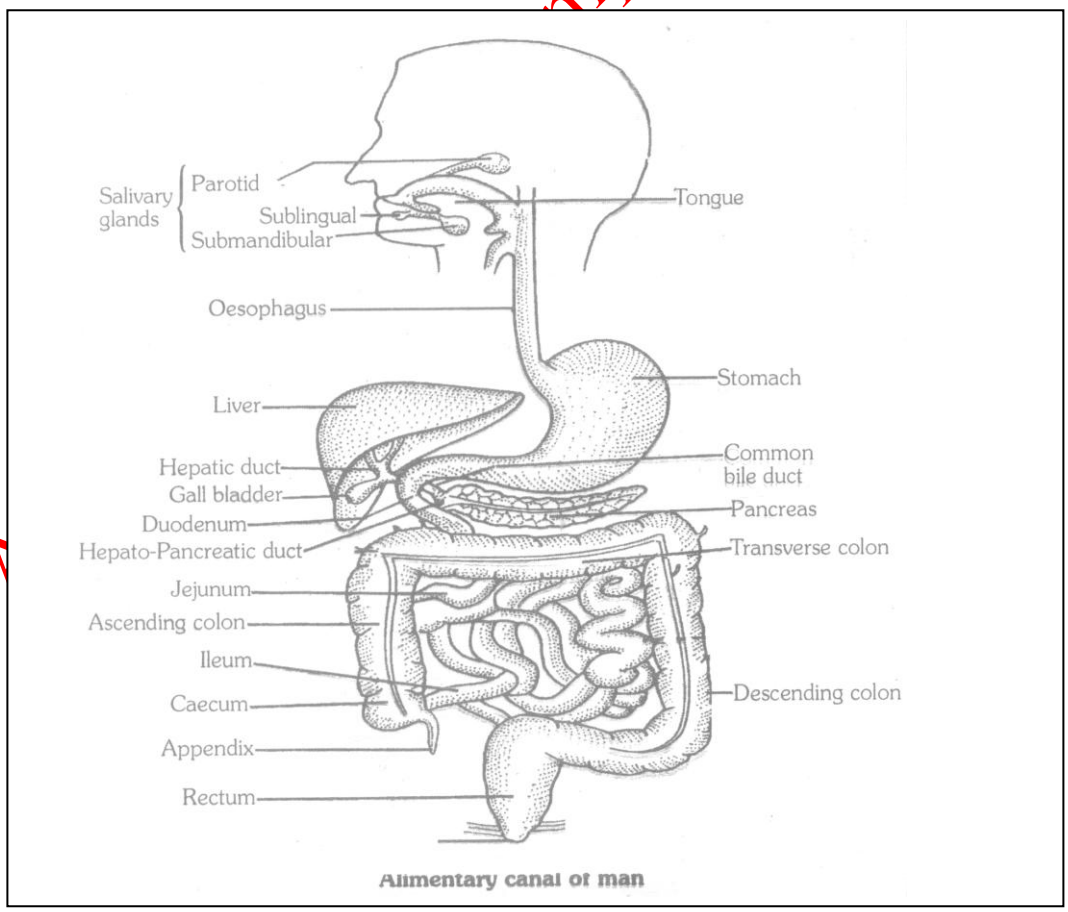
DIGESTIVE SYSTEM OF HUMAN

Human digestive system consists of the alimentary canal and digestive glands and it involves mastication, swallowing, digestion of food and elimination of undigested matter.

It consists two parts :-

[A] Alimentary canal :- The alimentary canal is basically a long tube extending from the mouth to the anus. It is differentiated into following parts.





Alimentary canal of man

(1) **Mouth :-** It is a transverse slit bounded by movable lips. The lips serve to close and open mount, holding the food in between and also help in speaking.

(2) **Buccoupharyngeal cavity / Mouth cavity :** Mouth leads into the **mouth cavity** or **oral** or **Buccal cavity.** The roof of mouth cavity is formed by palates i.e., **hard** and soft **palate,** the floor by **tongue** and the sides by the **cheeks.** The other conspicuous structures are **teeth** and **salivary glands.**

(a) **Tongue :-** The floor of the mouth cavity is occupied by muscular, large, mobile tongue. It remains attached on its under surface to the floor by fold of mucous membrane called the **lingual frenum.** The tongue is covered with mucous membrane and its upper surface is raised into l**ingual papillae** which contain microscopic taste buds.

(i) **Lingual papillae :-** Our tongue has a rough uppper surface due to three types of lingual papillae – filiform, fungiform and circumvallates.

(ii) **Taste buds :-** Tongue is an organ of taste, richly supplied with sensory heaves which end in taste buds in the papillae of the upper surface. The taste buds for **sweet taste** are located on the anterior end of the tongue, for **bitter taste** at the posterior end, for **sour taste** on its sides and for **salty taste** on a small part just behind the anterior end of the tongue.

Functions of tongue :-

(i) It acts like a spoon during ingestion of food.

(ii) It brings food under teeth for mastication

(iii) It moves food in buccal cavity for mixing of saliva.

(iv) It helps in swallowing food.

(v) It cleans teeth by removing small food particles form their surface.

(vi) It helps in speaking.

(vii) It is the main organ of taste.

(viii) It keeps the moth moist by the secretion of both mucus and serous or water like fluid .

(ix) In dogs during panting it helps in thermoregulation by quick evaporation of water of saliva.

(x) In some mammals tongue is used to clean skin by licking.

(b) **Teeth :-** Thecodont (Teeth present in bony socket), Heterodent (Teeth are of four types) and Diphyodont (Teeth that come two time in life).

Teeth are of following types :

(i) **Incisors –** Biting the food.

(ii) **Canines –** Wearing and tearing of food.

(iii) **Premolars –** Crushing and grinding the food.

(ii) **Molars –** Crushing and grinding the food .

How can the dental caries be prevented ?

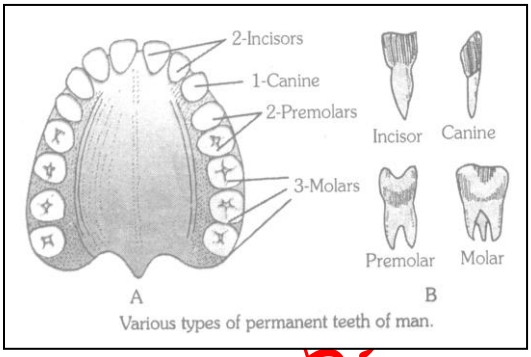
(i) Avoid sugar rich eatable .

(ii) Regular brushing of teeth after meals.

(iii) Vigorous chewing of fibrous food.

(iv) Consumption of water containing 1 ppm of fluoride.

(c) **Palate :-** The septum like structure which separates nasal path from oral path. The hard palate is the anterior part of the roof of mouth cavity. It is supported by **palatine and maxillary bones** that is why it is known as **hard palate.** There are no premaxillary bones present in our hard palate. The lower surface of hard palate covered with mucous membrane of mouth cavity is raised into transverse ridges called **palatine rugae** which serve to grip the food firmly. Soft palate is the posterior muscular part having no skeletal support. It carries a fleshy cone shaped projection called **uvula** which hangs down from it in the middle line. Uvula serve to block the internal nares during swallowing by rising up, preventing entry of food in the nasal cavities.



Various types of permanent teeth of man.

(iii) Pharynx :

The Buccal cavity opens into a short narrow chamber called **pharynx or throat.** Pharynx is incompletely divided into three parts by soft palate – the nasopharynx dorsal to the soft palate, oropharynx below the soft palate and laryngopharynx into which both these parts communicate round the freely hanging uvula. The oropharynx and laryngopharynx are associated with swallowing. The laryngopharynx communicates with esophagus through gullet and with larynx through glottis. The glottis is guarded by an elastic and muscular flap called epiglottis which closes glottis during swallowing to prevent food from entering into wind pipe. Thus pharynx serves two ways – (i) as a passage between nose and wind pipe (trachea) (ii) as food passage between mouth cavity and esophagus. Resonance of voice also occurs due to pharynx.

(iv) **Esophagus (food pipe) :-** It conducts the food by **peristalsis.**

The esophagus is a collapsible muscular tube 25-30 cm long and about 25-30 mm thick, leading from the pharynx to stomach. It runs down the neck, behind trachea and through thorax, finally piercing the diaphragm to open into stomach. The upper 1/3rd part is composed of voluntary muscles and the lower 2/3rd of voluntary muscles. The muscular coat has a peristaltic action for driving the food towards the stomach. There are no digestive glands but only mucous glands in esophagus.

(v) **Stomach :-** It is a thick, muscular and J-shaped sac present on the left side of upper part of abdomen. Beneath diaphragm lying to the left side of abdomen is sickle-shaped stomach. It is the widest part of alimentary canal, size and shape of which varies according to the contents and sex. The stomach of a woman in general is more slender and smaller than that of a man. It can be distinguished into three regions – Fundic part, Body part and Pyloric parts. Pyloric part is the lower end of stomach which narrows down like a funnel and curves to the right like – 'J' leading into duodenum. The exit is guarded by a pyloric sphincter or pylorus. The partly digested paste like food is forced into intestine through pyloric sphincter, due of peristaltic wavers of stomach.

Functions of Stomach :-

(1) Temporary storage of food.

(2) Partial digestion of food by gastric juice.

(3) Churning of food.

(4) The stomach regulates the flow of partially digested food into the small intestine.

Intestine :- It distinguished into two parts :-

(a) Small Intestine (b) Large Intestine

(a) **Small intestine :-** It is a long (about 6 meter) narrow (average diameter 4 cm), tubular and coiled part. It is differentiated into anterior duodenum, middle jejunum and posterior ileum. It is mainly concerned with completion of digestion and absorption of food.

(i) **Duodenum :** This proximal part starts after pyloric end of stomach. It is about 25 cm long lying against the posterior abdominal wall. It is served like 'C' or a horseshoe and ends behind the stomach. A common bile duct and a pancreatic duct opens in middle of 'C' of duodenum by a common aperture over a raised area called ampulla of vater.

(ii) **Jejunum ;** The Jejunum (a Latin word meaning empty) is so called because it is always found empty after death.

(iii) **Ileum :** It is the last part of small intestine.

Small intestine designed to absorb digested food as :

(i) It is lengthy about 6 meters.

(ii) Inner lining has two types of folds called villi and microvillus.

(iii) Each villus has blood capillaries and a lymph capillary.

(b) **Large intestine :-** It is shorter (about 1.5 meter) and wider (Average diameter 6 cm) than small intestine. It is differentiated into caecum, colon and rectum. It is the site of abortion of water from digested food. It helps in formation and temporary storage of faeces.

★ Caecum is vestigial in human beings.

Anus :- It lies at the base of trunk and is for Egestion.

[B] digestive glands :- They secrete digestive juices which contain digestive enzymes. These are of following types.

(i) **Salivary glands :-** In man, there are three pairs of salivary glands. these secrete saliva which contains a digestive enzyme called **ptyalin** or **Salivary amylase.**

Do you know ?

★ Saliva is an antiseptic as it kills germs and bacteria due to presence of an enzyme called **Lysozyme.** Saliva makes the food soft, slippery and helps in digestion of starch due to presence of salivary amylase enzyme.

★ Our mouth starts **watering on eating** food of our interest, This water is basically the saliva secreted by the salivary gland which get activated on eating or seeing or thinking of a food.

★ Involuntary contraction & relaxation movement is called **peristalsis.**

(ii) **Gastric glands :-** Gastric glands are present in the wall of stomach and secrete gastric juice.

(iii) **Liver :-** It is lobed and reddish-brown colored largest gland of body present in the right side of upper part of the abdomen. It synthesize and secretes bile juice. Gall bladder is present below the right lobe of liver. It stores and secretes bile.

(iv) **Pancreas :-** It is yellow – colored gland present just behind the stomach. It secretes pancreatic juice. Pancreatic duct carries pancreatic juice to small intestine.

(v) **Intestinal glands :-** These lie in the wall of small intestine and secrete intestinal juice.

[C] Physiology of nutrition :-

(i) **Ingestion :-** Man is **omnivorous** in feeding and is **Holozoic.**

Ingestion involves carrying the food to the mouth with the help of hands and cutting of food with incisors or canines depending upon the nature of food.

(ii) **Digestion :-** In man, digestion is started in **Buccal cavity** and completed in **intestine.**

(a) **In Buccal cavity :-** Here, food is chewed with the help of premolars and molars which increases the rate of action of **salivary amylase.** Food is mixed with saliva of salivary gland.

$$\text{Starch} \xrightarrow[6.8PH.CI^{-}]{Salivary\ amylase} \text{Maltose}$$

(b) **In stomach :-**

Food is mixed with gastric juice which contains mucus, hydrochloric acid, pepsin, rennin and a weak lipase enzyme .

Mucus, lubricates the food and protects the inner lining of the stomach from the action of acids.

Functions of Hydrochloric acid.

(i) Stops the action of salivary amylase in stomach.

(ii) Kills the bacteria present in the food.

(iii) Activates pepsin.

(iv) Provides acidic medium.

Pepsin hydrolyses proteins into proteases and peptones.

Lipase enzymes hydrolyses small amounts of fats into fatty acids and glycerol.

Curdling of milk is done by the enzyme **rennin.** (Rennin is not found in human beings, it found only in cattles)

(c) **In small intestine :-**

The small intestine is the site of the **complete digestion** of carbohydrates, proteins and fats.

Food is mixed with three **digestive juices,** bile juice, pancreatic juice and intestinal juice.

Bile juice provide **alkaline medium** and **emulsifies fats** [conversion of larger fat globules into smaller fat droplets] but is a non enzymatic digestive juice so has no chemical action on food.

Pancreatic juice contains **trypsin, pancreatic amylase and pancreatic lipase** enzymes which digest the peptones, starch and fact into peptides, maltose and fatty acids.

Intestinal juice contains **amino peptidase, intestinal amylase, maltase, surcease and lipase** enzymes :-

$$\text{Peptides} \xrightarrow{Aminopeptidese} \text{Amino acid}$$

$$\text{Maltose} \xrightarrow{Maltase} \text{Two glucose}$$

$$\text{Fats} \xrightarrow{Lipase} \text{Fatty acid + Glycerol}$$

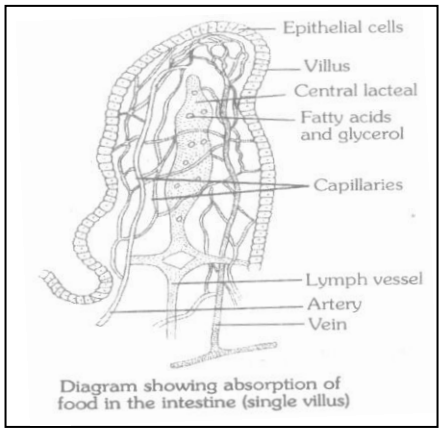


Diagram showing absorption of food in the intestine (single villus)

$$\text{Lactose} \xrightarrow{Lacase} \text{Glucose} + \text{Galactose}$$

$$\text{Sucrose} \xrightarrow{Sucrase} \text{Glucose} + \text{Fructose}$$

(iii) Absorption :-

Absorption of the digested food occurs though the epithelial surface of the villi & microvillus of small intestine.

★ Inner surface of small intestine is raised into 4 millions of finger-like folds called **villi.**

★ Each cell of villus is with electron microscopic processes called **microvillus.**

★ Each villus is with blood capillaries and a lymph capillary.

(iv) Assimilation :- It is a process by which absorbed nutrients are utilized to resynthesize complex molecules like carbohydrates, proteins and fats inside the cells.

(v) Egestion :- Removal of waste products from the body is known as **egestion**.

S.No.	Vitamin	Diseases due to deficiency	Symptoms of disease
1	Vitamin-A	Night blindness	Patient can't see in dim light or at night
2	Thiamine (Vitamin-B^1)	Beriberi	Reduced heart beat, muscles and nerves become weak.
3	Niacin (Vitamin-B^3)	Pellagra	Scaling on tongue and skin
4	Ascorbic acid (Vitamin-C)	Scurvy	Bleeding-gums, formation of spots on skin.
5	Calciferol (Vitamin-D)	Rickets in children and Osteomalacia in adults.	Bending of bone of legs.

S.No.	Name of the element	Main sources	Main functions
1	Sodium	Common salt, fish, meat, eggs, milk	Muscle contraction, neural impulse conduction.
2	Potassium	All food materials	Muscle contraction, neural impulse conduction.
4	Phosphorus	Milk, green leafy vegetables, millets, dry fruits. liver,	Strengthening of bones and teeth .
5	Iron	Liver, eggs, oat, green leafy vegetables	Formation of haemoglobin in blood
6	Iodine	Common salts, sea food, green leafy vegetable.	Goiter disease is caused due to deficiency of Thyroxin hormone.

POINTS TO BE REMEMBER

1. **Emulsification :-** Emulsification is the phenomenon of physically breaking of large sized fat globules into large number of fat droplets by the bile-salts of the bile juice. This increases the surface area for digestion of fats by lipase enzyme.

2. **Enzymes are classified into 3 groups :-**

(a)	Amylolytic enzymes	–	Carbohydrate digestion
(b)	Proteolytic enzymes	–	Protein digestion
(c)	Lipolytic enzymes	–	Lipid digestion

3. **Alimentary Canal :-** The digestive canal where the entire process of digestion a accomplished, called alimentary canal.

- It is the site for ingestion, digestion, absorption and Egestion of food material.
- In man it is about 7-8 meter long.
- The alimentary canal of herbivores is longer then the alimentary canal of carnivores, because herbivores have to digest the cellulose, which is difficult to digest.

4. **Mucus in stomach :-** Mucus is a viscous secretion. The mucus also forms a thick coating over the mucosal cells and prevents them from the harmful effects of HCl and pepsin.

5. Pepin is the chief photolytic enzyme.

6. **Castle's Intrinsic factor :-** This intrinsic factor is secreted by Fundic glands of the stomach mucosa. It combines with vitamin B_{12} [which is known as extrinsic factor].
The vitamin B_{12} from this combined complex can easily be extracted and absorbed by the intestinal mucosa. Intrinsic factor, thus, helps in the absorption of vitamin B_{12}.

7. **Colitis :-** An inflammation of the colon and rectum is called colitis. Inflammation of the mucosa reduces absorption of water and salts, producing watery, bloody faces and in severs cases, dehydration and salt depletion.

8. Digestion of proteins in man starts from stomach. In Buccal cavity there is no digestion of proteins because saliva contains no Proteolytic (protein digesting) enzyme.

EXERCISE # 1 FOR SCHOOL EXAM.

VERY SHORT ANSWER TYPE QUESTIONS :

1. Define nutrition.
2. Name the enzymes present in stomach.
3. Which part of the body secretes bile ? Where is bile stored ?
4. Define peristalsis.

5. What is emulsification of fat ?
6. Name the enzyme present in human saliva. What type of food material is digested by this enzyme ?
7. Define assimilation.
8. Name the most common method of ingestion in Amoeba.
9. Why is food vacuole of Amoeba celled temporary stomach ?
10. Name different types of teeth present in man.
11. Which part of alimentary canal is adapted for complete digestion and absorption of food ?
12. What is succus entericus ?
13. Name the largest gland of the human body.
14. Name the protein-digestion enzymes present in pancreatic juice of man.
15. What are the end-products of fat digestion ?
16. What structure prevents the entry of food particles into wind pipe ?

SHORT ANSWER TYPE QUESTIONS :

1. Describe various modes of nutrition.
2. Mention various steps of nutrition in Amoeba.
3. Draw the position of salivary glands in mouth.
4. What is the role of hydrochloric acid in our stomach ?
5. State two functions of large intestine of man.
6. What criteria do we use to decide whether something is alive ?
7. What are the differences between autotrophic nutrition and heterotrophic nutrition ?
8. How is the small intestine designed to absorb digested food ?
9. What is difference between ingestion and Egestion ?
10. How can the dental caries be prevented ?

FILL IN THE BLANKS :

1. Hard palate has transverse ridges, the ……..
2. Human teeth are Thecodont, ……..and ……..
3. Man bears 4 kinds of teeth :
   (i) …… (ii) …….. (iii) ……and (iv) ……
4. Bile and pancreatic ducts open into……
5. Villi are found in……..and……
6. Appendicitis is inflammation of …….
7. Man's three pairs of salivary glands are……..and……
8. Succus entericus is the secretion of ……
9. Enzymes enterokinase actives……
10. Kwashiorkor is the disease caused by extreme…….deficiency.

EXERCISE # 2 OBJECTIVE QUESTION

1. Human beings are -

   (A) Omnivorous (B) Herbivorous (C) Carnivorous (D) Autotrophic

2. Gastric juice is –

(A) Acidic (B) Alkaline (C) Neutral (D) Slightly alkaline

3. Bile is produced and secreted by –

(A) Gall bladder (B) Pancreas (C) Spleen (D) Liver

4. The main function of intestinal villi is –

(A) Stimulate Peristalsis (B) Prevent anitperistalsis

(C) Provide large surface area for absorption (D) Distribute digestive enzymes uniformly.

5. Major function of HCl of gastric juice is –

(A) Providing acidic medium for pepsin (B) Kill microorganisms

(C) Dissolve food (D) Facilitate absorption of food

6. The end product of fat digestion is –

(A) Glucose (B) Fatty acids and Glycerol

(C) Amino acids (D) Alkaloids

7. The action of bile can be called –

(A) Oxidation (B) Emulsification (C) Esterification (D) Dehydrogenation

8. Saliva converts –

(A) Proteins into amino acids. (B) Glycogen into glucose

(C) Starch into maltose (D) Fats into vitamins

9. Which set is mixed with the food in small intestine ?

(A) Saliva, gastric juice, bile (B) Gastric juice, bile, pancreatic juice

(C) Bile, pancreatic juice, succus entericus (D) Bile, pancreatic juice and saliva

10. A good source of lipase is –

(A) Saliva (B) Gastric juice (C) Bile (D) Pancreatic juice.

11. Ptyalin is an enzyme present in –

(A) Gastric juice (B) Pancreatic juice (C) Intestinal juice (D) Saliva

12. Which one does not produce any digestive enzyme ?

(A) Pancreas (B) Liver (C) Stomach (D) Duodenum

13. The number of salivary glands present in human beings is –

(A) 5 pairs (B) 4 pairs (C) 3 pairs (D) 2 pairs

14. Largest gland in the body is –

(A) Liver (B) pancreas (C) Gastric gland (D) Adrenal

EXERCISE # 3 FOR COMPETITIVE EXAMS

1. Prorenin occurs in

(A) Saliva (B) Gastric juice (C) Pancreatic juice (D) Intestinal juice

2. Interokinase is found in

(A) Saliva (B) Gastric juice (C) Pancreatic juice (D) Intestinal juice

3. HCl of gastric juice is produced by

(A) Chief cells (B) Oxyntic cells (C) Goblet cells (D) Columnar cells

4. Bile is released by

(A) Gastrin (B) Secretin (C) Cholecystokinin (D) Enterocrinin

5. The digestion of food starts in the mouth where starch is converted into sugar by the action of

(A) Pepsin (B) Trypsin (C) Ptyalin (D) Surcease

6. Digestion is accomplished by a chemical process called

(A) Condensation (B) Hydrolysis (C) Deamination (D) Transamination

7. Which of the following organs are not directly connected to each other ?

(A) Esophagus-stomach (B) Buccal cavity-stomach

(C) Colon-rectum (D) Stomach-duodenum

8. The major anion present in the pancreatic juice obtained through the stimulating of Secretin is

(A) Chloride (B) Bromide (C) Phosphate (D) Bicarbonate

9. Chylomicrons that enter the lacteals are composed of –

(A) Triglycerides alone (B) Triglycerides around a protein core

(C) Protein coat around triglycerides (D) Proteins alone

10. Vitamin K is required for

(A) Change of prothrombin to thrombin (B) Synthesis of prothrombin

(C) Change of fibrinogen to fibrin (D) Formation of Thromboplastin

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ANSWER KEY

EXERCISE # 1 **FOR SCHOOL EXAMS.**

Fill in the blanks

1. Rugae **2.** Heterodent, Diphyodont **3.** (i) Incisors (ii) Canines (iii) Premolars (iv) Molars

4. Duodenum **5.** Jejunum, Ileum **6.** Vermiform Appendices

7. Parotid, Sublingual and sub maxillary **8.** Intestinal glands

9. Trypsinogen **10.** Proteins

EXERCISE # 2 **OBJECTIVE QUESTIONS**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	A	A	D	C	A	B	B	C	C	D	D	B	C	A

EXERCISE # 2 **OBJECTIVE QUESTIONS**

1	2	3	4	5	6	7	8	9	10
B	D	B	C	C	B	B	D	C	B

NORTON IN PLANTS

Green plants are autotrophic. They synthesize their own food by the process of photosynthesis. Autotrophic plants are able to produce food so they are known as producers.

Photosynthesis :- Photosynthesis is a process by which green plants syntheses organic food (carbohydrate) from carbon dioxide and water using solar energy by chlorophyll pigments.

The sugar produced is strode is the form of starch in plants.

COMPETITION WINDOW

First true and oxygenic photosynthesis starts in Cyanobacteria (blue green algae).
90% of total photosynthesis is carried out by hydrophytes (mostly marine algae.)
"The process of absorption and conversion of light energy into chemical energy by green plants is called as photosynthesis".
This chemical energy is stored in the form of adenosine triphosphate (ATP) and reduced nicotinamide adenine dinucleotide phosphate ($NADH_2$).

Importance of photosynthesis :-

Photosynthesis is an anabolic process in nature for providing food supply to the living organisms .

It purifies the atmospheric air , by consuming CO_2 and evolving oxygen.

The over all equation of photosynthesis is :-

$$6CO_2 + 12H_2O \xrightarrow[Chlorophyll]{Sunlight} C_6H_{12}O_6 + 6H_2O + 6H_2O + 6O_2$$

Requirements for photosynthesis :-

(1) Sunlight **(2) Photosynthetic pigment** **(3) Carbon Dioxide** **(4) water**

CO_2 and water work as raw materials which are obtained from the atmosphere and the soil respectively.

1. **Sunlight :-** Sun is a natural source of light for photosynthesis .

Sunlight is an electromagnetic spectrum.

Photosynthetic pigments absorb only visible / white light from electromagnetic spectrum.

White light (380 nm to 160 nm) is composed of wavelength of seven different colors violet, indigo, blue, green, yellow, orange and red (VIBTGYOR).

COMPETITION WINDOW

Types of chlorophyll :-
There are six different types of chlorophyll : Chl.-a, Chl.-b, Chl.-c, Chl.-d, Chl,-e and bacteriochlorophyll .
Beside chlorophyll certain other pigments are also resent in plants like –
Carotenoid → Carotenes (orange colour) and xanthophylls (yellow colour).
Phycobilins : It is present in Blue- green algae and Red algae etc.

2. **Photosynthetic pigment :-** These are chlorophylls cartenoides (carotenes and xanthophylls) and Phycobilins. These pigments absorb only visible light.

Chl-a and b absorb only blue and red light and reflect green light .

3. **Carbon Dioxide:-**

All the plants need carbon dioxide to form carbohydrates. The carbon dioxide is obtained by the plants from the atmospheric air. In the terrestrial plants, the CO_2 enters into the cells of leaves through tiny pores called **stomata** which always remain present on the surface of leaves.

COMPETITION WINDOW

Compensation point
The intensity of light at which amount of CO_2 used during photosynthesis becomes equal to the amount of CO_2 released during respiration by plants is called as compensation point.
Compensation point occurs at low light intensity that is during morning and during evening hours.
The structure of guard cells in monocots is dumb-bell shaped.

Stomata :- These are tiny pores or microscopic aperture guarded by two kidney shaped or bean shaped guard cells.

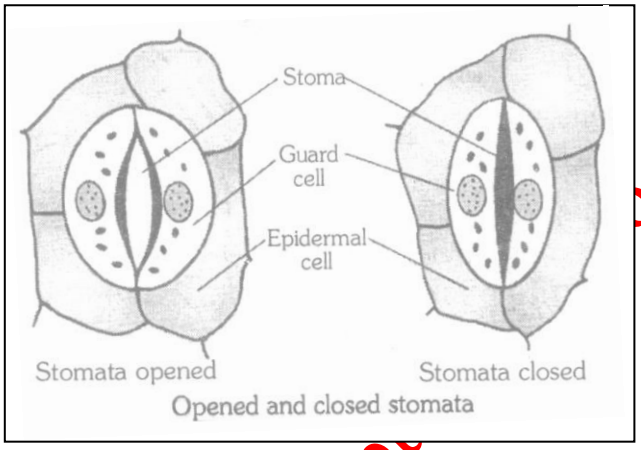
Functions :-

(i) Massive amount of a gaseous exchange take place in the leaves through stomata. Exchange of gasses also occurs across the surface of , root and leaves.

(ii) Large amount of water is lost through stomata.

Guard cells :- These are kidney shaped cells which cover single stoma. They contain chloroplast also.

Function :- They regulate the opening and closing of the stoma and also perform photosynthesis.



Opened and closed stomata

OPENING AND CLOSING OF STOMATA :-

When the guard cells swell due to the entry of water, the stomata gets opened. But when the guard cells shrink due to the loss of water, the stomata gets closed.

Do you know ?

Desert plants take up CO_2 at night and prepare an intermediate which is acted by the energy absorbed by the chlorophyll during the day and form glucose.

In aquatic plants, CO_2 is obtained from the water where it remains present in dissolved form. Such plants absorb carbon dioxide in solution form, all over their surface from the surrounding water.

4. Water :

Water is always needed by the plants for its use during photosynthesis.

Inside the chloroplasts of the leaves, the water molecules split into hydrogen and oxygen with the help of light energy of solar light.

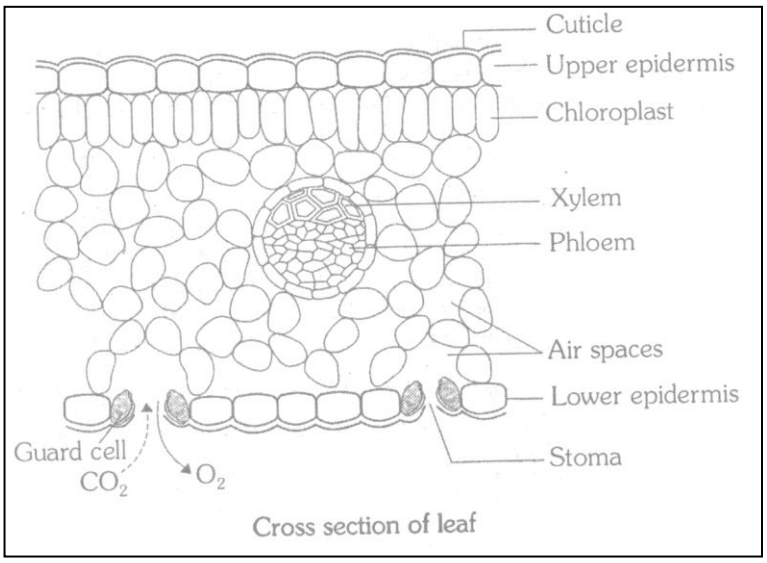
Some mineral salts like N, P, K, Fe, Mg required by the plants are also transported to different parts of the plant along with the water.

Nitrogen is an essential element used in the synthesis of proteins and other compounds (chlorophyll, DNA and RNA).

Nitrogen is taken up in the form of inorganic nitrates or nitrites which have been prepared by symbiotic bacterial from atmospheric N_2.

Site of Photosynthesis :- Green Plastid (Chloroplast or Kitchen of the cell).

When we observe the cross section of a leaf under microscope, we can see the Mesophyll cells full of green dots. These green dots are chloroplasts containing chlorophylls.



Cross section of leaf

COMPETITION WINDOW

Schimper classified plastids into 3 types or the basis of presence and absence of pigments. They are following

(a) Leucoplast :- They are colorless and generally found in those parts of the plants which are deprived of sunlight (underground parts).

(b) Chromoplasts :- They are colored plastids other than green. Chromoplast are abundantly present in the petals of flowers and abundantly present in the petals of flowers and provide them attractive colors.

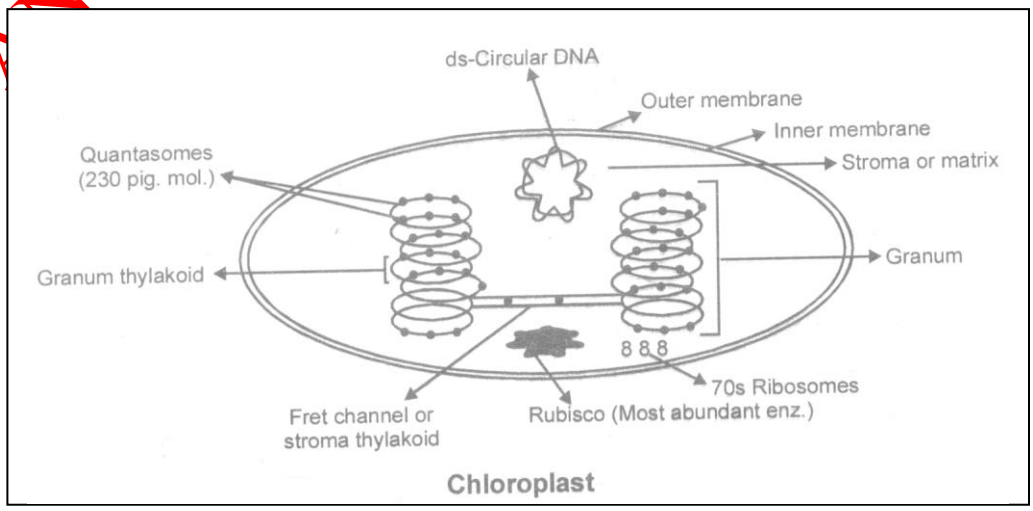
(c) Chloroplasts :- They are green colored plastids.

Plastids are found in all green cells except fungi and prokaryotes like bacterial and

Chloroplast :- They are green colored plastids. Their green colour is due to the presence of green pigments the chlorophylls. Each developed chloroplast has two distinct areas – grand and stroma.

Grana (Singular-granum) :- The light reaction of photosynthesis takes place in this part of chloroplast. In a granum large number of lamellae remain arranged like a stack of coins. These lamellae are called as **Thylakoid,** which contain chlorophyll pigments.

Stroma :- It forms the matrix of the chloroplast. The dark reactions of photosynthesis take place in stroma.



Chloroplast

MECHANISM OF PHOTOSYNTHESIS

During photosynthesis following events occur :-

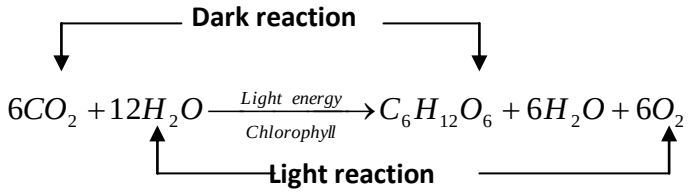
(i) Absorption of light energy by chlorophyll.

(ii) Conversion of light energy into chemical energy and splinting of water molecule into hydrogen & O_2

(iii) Reduction of CO_2 to carbohydrates.

All these events can be categorized into two main phases :-

(A) Light phase (B) Dark phase



(A) Light reaction :- This step occurs inside Thylakoid membranes of chloroplasts. In this step firstly absorption of solar energy by chlorophyll molecules takes place. During light reaction, light energy breaks up water molecules into hydrogen and oxygen and this process is called photolysis of water.

Light energy converts into chemical energy in the form of ATP and $NADPH_2$.

Oxygen is relapsed as a by product of light reaction occurring during photosynthesis.

COMPETITION WINDOW

Light reaction was discovered by 'Robert Hill'. Therefore it is also called as **Hill's reaction.** Light reaction is a photochemical process.

Photophosphorylation :-

The process of formation of ATP in the presence of sunlight is known as photophosphorylation. Oxygen released during photosynthesis comes from water instead of CO_2 as was earlier thought .

(B) Dark reaction :- In this step synthesis of carbohydrates from carbon dioxide takes place. It is not light dependent hence it is celled as dark reaction.

This reaction occurs inside stroma of chloroplasts where light energy is not captured.

During this reaction, the chemical energy formed during light reaction (ATP and $NADPH_2$) is utilized for the fixation and conversion of carbon dioxide into simple carbohydrate, that is glucose.

The glucose molecules thus formed are further converted by the cellular machinery into various chemicals required by the plants.

COMPETITION WINDOW

Dark reaction was discovered by **Melvin Calvin and Benson** therefore it also called as Calvin Cycle.

Dark reaction is a thermo chemical reaction. CO_2, $NADPH_2$, ATP, RUBP and Rubisco enzymes all are required in Dark reaction.

RUBP – Ribulose phosphate.

Rubisco – Ribulose phosphate carboxylase.

Difference between light and dark reaction

S.No.	Features	Light reaction	Dark reaction
1	Requirement of light	Required	Not required
2	Takes place inside	Thylakoid membranes of the chloroplast	The stroma region of chloroplast
3	ATP and $NADPH_2$	ATP and $NADPH_2$ are produced by the conversion of light energy into chemical energy	ATP and $NADPH_2$ formed during light reaction are used for the fixation of CO_2 into carbohydrate
4	Sugar formation	No sugar formation takes place	Sugar formation takes place
5	Release of oxygen	Oxygen is released	No oxygen is released

Factors affecting photosynthesis :-

(a) Light (b) Water (c) Temperature (d) CO_2 (e) Oxygen

(a) Light

The source of light for planet earth is sun, although some marine algae utilize the light of moon.

Out of the total solar energy , only 2% solar energy is used in photosynthesis.

The quality and intensity of light also affects photosynthesis.

Quality – Red and blue lights are most effective in photosynthesis.

But the rate of photosynthesis is maximum in red light.

There is no photosynthesis in presence of green light because green parts of plants reflect whole of the green light.

Intensity – The increase in intensity on light increases photosynthesis.

Intensity of sunlight $\propto$ Rate of photosynthesis

At very high light intensity the photosynthesis is decreased due to photo-oxidation of constituents (solarization).

(b) Water

Water is an essential raw material in photosynthesis. Only 1% of the absorbed water is utilized in photosynthesis.

Less availability of water reduces the rate of photosynthesis. Under water deficient conditions the Stomatal aperture remains closed to reduce the loss of water by transpiration. As a consequence the entry of CO_2 is also stopped into the leaves.

(c) Temperature

The rate of photosynthesis increases by increases in temperature up to 40^0C. Above the temperature, there is a decrease in the photosynthesis. Similarly, low temperature also inhibits photosynthesis. The temperature affects photosynthesis by affection the activity of enzyme. We known that the dark reaction of photosynthesis involves several enzymes. These enzymes function at a specific of temperature. Low temperature lowers the activity of enzymes and high temperature causes inactivation of enzymes.

(d) Carbon-dioxide

Atmosphere is the main source of CO_2 for terrestrial plants.

In atmosphere CO_2 is present at the tune of 0.03%.

The rate of photosynthesis increases by increasing the concentration of CO_2. But after a certain limit, the excess concentration of CO_2. proves to be toxic to the cells.

(e) Oxygen

Over concentration of oxygen stops photosynthesis .

It increases the rate of respiration manifold and disturbs the excited condition of the chloroplast. In theses conditions photosynthesis is not possible.

COMPETITION WINDOW

Bacterial Photosynthesis

It is a special kind of photosynthesis in which solar energy is utilized for the synthesis of carbohydrates and H_2S is the hydrogen donor instead of water as in normal photosynthesis. So O_2 is not liberated in bacterial photosynthesis .

e.g. *Chlorobium* (Green sulphur), *Chromatium* (Purple sulphur), *Rhodospirilium, Rhodopseudomonas* (Purple non sulphur)

REVIEW QUESTIONS

1. Name two inorganic substances required by autotrophs to carry on photosynthesis.
2. What is chlorophyll ?
3. Name the pigments which can absorb solar energy.
4. What is stroma lamella ?
5. Name the pigment other chlorophyll, which can absorb sunlight energy.
6. What are Thylakoid ?
7. Name the two stages of photosynthesis.
8. What is granum ?
9. Name the gas used in photosynthesis.
10. Name the gas produced in photosynthesis.

EXERCISE # 1 (FOR SCHOOL EXAMS)

OBJECTIVE QUESTIONS :

1. Leaf is made up of :
(A) Palisade cells (B) Mesophyll cell (C) Guard cell (D) Parenchyma cell

2. Each stoma is guard by :
(A) Guard cell (B) Palisade cell (C) Mesophyll cell (D) Parenchyma cell

3. Mesophyll consists of :
(A) Spongy parenchyma cell (B) Palisade parenchyma cell
(C) Both (A) and (B) (D) None of these

4. Xerophytes have :
(A) Stomata on both sides of leaf (B) Stomata on one side of the leaf
(C) Sunken stomata (D) No stomata

5. Stomata controls :
(A) The loss of food material from the plant (B) The loss of water from the plant
(C) The loss of air from the plant (D) The loss of energy from the plant

6. Each guard cells contain :
(A) Leucoplasts (B) Chloroplasts (C) Starch (D) Oil and protein granules

7. Grana are present inside :
(A) Mitochondria (B) Golgi bodies (C) Chloroplast (D) Ribosome

8. In dark, the guard cells are :
(A) More turgid (B) Not turgid (C) Less turgid (D) None of these

9. In dicot plants, stomata are found in :
(A) Lower and upper surface of the leaf (B) Lower surface of the leaf
(C) Tip of the leaf (D) Upper surface of the leaf

10. The shape of stomata in monocot plant is :

(A) Kidney shape (B) Dumb-bell shape (C) Bean shape (D) Heart shape

11. In aquatic plant – *Hydrilla* stomata are :

(A) Present on leaves (B) Present on stem (C) Present on roots (D) Absent

12. The following light witch is more effective in opening of stomata :

(A) Green (B) Red (C) Blue (D) (B) and (C) both

13. In light phase of photosynthesis there is formation of –

(A) ATP (B) $NADPH_2$

(C) Both ATP and $NADPH_2$ (D) Carbohydrates.

14. Photosynthesis proceeds in sequence of –

(A) Dark phase and light phase (B) Light phase alone

(C) Light phase and dark phase (D) Dark phase alone

15. The energy change in photosynthesis is from –

(A) Light energy to electrical energy (B) Light energy to molecular energy

(C) Light energy to chemical energy (D) Light energy to activation energy.

16. Photosynthetic pigments are located in the chloroplast in –

(A) Intrathylakoid space (B) Thylakoid membranes

(C) Intermembrane space (D) Inner membrane of envelope

17. The oxygen in photosynthesis is released from –

(A) CO_2 (B) H_2O (C) Carbohydrate (D) Chlorophyll

18. The process in which water is split during photosynthesis is –

(A) Photolysis (B) Hydrolysis (C) Plasmolysis (D) Hemolysis

19. Dark reaction of photosynthesis occurs in –

(A) Grana (B) Stroma (C) Matrix (D) Cytoplasm

20. In bacterial photosynthesis, the hydrogen donor is –

(A) H_2O (B) H_2SO_4 (C) NH_3 (D) H_2S

MATH THE COLUMN :

1. Match the contents of the columns-I, II and III (Double matching)

Column-I		Column-II		Column-IIII	
(1)	Water split into	(A)	Chemical energy	(P)	Photolysis
(2)	CO_2	(B)	Hydrogen and oxygen	(Q)	Raw materials for photosynthesis
(3)	White light	(C)	Water	(R)	Electromagnetic spectrum
(4)	Light energy	(D)	VIBGYOR	(S)	Photosynthesis

TRUE AND FALSE :

1. Rate of photosynthesis the maximum in green light.
2. Oxygen and water are the raw material for photosynthesis.
3. CO_2 is release as a by product of light reaction.
4. Dark reaction occurs inside the stroma of chloroplast.
5. In photosynthesis chemical energy converts into light energy .
6. Guard cells regulate the opening and closing of stomata.
7. White light is composed of seven different colour.
8. Light reaction takes place in the presence of light.

FILL IN THE BLANKS :

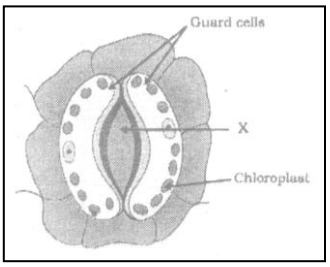
1. ……is a natural source of light for photosynthesis.
2. Photosynthetic pigments absorb only ……light from electromagnetic spectrum.
3. Chl-a and b absorb only ……light and reflect ……light .
4. "The process of absorption and conversation of ……energy into……energy by green plants is called as photosynthesis".
5. ……regulate the opening and closing of ht stoma and also perform photosynthesis.
6. When the guard cells swell due to the entry of water, the stomata gets ……
7. The water molecules split into hydrogen and oxygen with the help of……energy.
8. ……is released as a by product of light reaction occurring during photosynthesis.
9. ……reaction occurs inside stroma of chloroplasts where light energy is not captured.
10. At very high light intensity the photosynthesis is . ……

VERY SHORT ANSWER TYPE QUESTION :

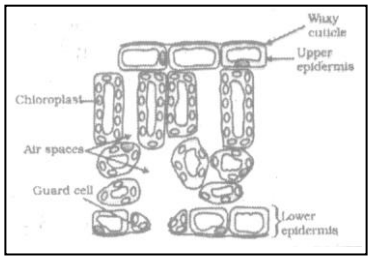
1. Name the factors which affect photosynthesis.
2. Write complete reaction of photosynthesis.
3. Name any two types of pigment present in plants which can absorb sun light energy.
4. Name the cellular organelle where photosynthesis process occurs.
5. Give the function of guard cells.

LONG ANSWERS TYPE QUESTIONS:

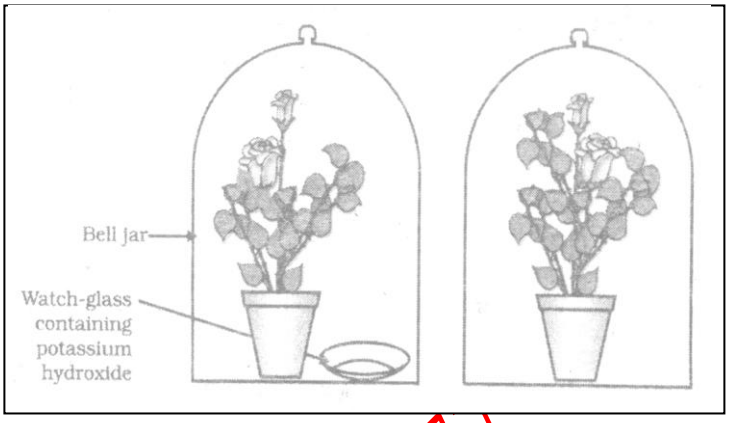
1. Look at the figure and identify X. What is its role in plants ?



2. Which is the site of Photosynthesis ? In the given figure showing a section of leaf as seen under microscope. What is role of waxy layer on upper epidermis ?



3. Look at the figure. Answer the following questions



(a) What is the need to use potassium hydroxide

(b) Why do we need tow plants ?

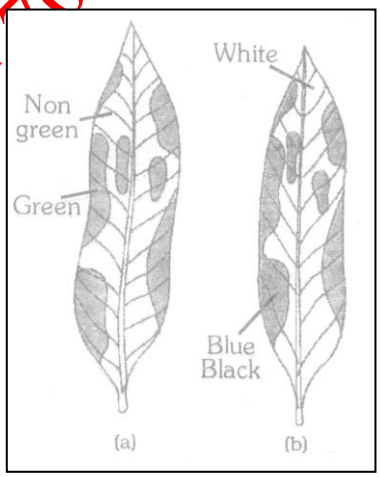
(c) Why do we cover both plants with polyanthuses sheets or bell jars ?

(d) What observation do you expect at the end of experiment and why ?

(e) What is the aim of activity ?

4. If plant is kept covered with a polythene sheet, we notice some water drops on the inner side of the sheet after some time. What are they due to ? Name and define the process. What is the significance of this process in plants and nature ? How does transpiration help in upward movement of water from roots to leaves ?

5. An activity was performed in the laboratory to show that chlorophyll is necessary for photosynthesis. Now answer the following question :



(a) What king of leaf has been used in this activity ?

(b) Wow do we de-starch a plant ?

(c) Why is the leaf heated in alcohol ?

(d) What precautions should be taken while heating the alcohol and why ?

(e) State reason for observation as shown in (ii) .

EXERCISE # 2 (FOR COMPETITIVE EXAMS)

1. The function of leaf in a plant is :

(A) Respiration (B) Transpiration (C) Photosynthesis (D) All the above

2. Leaves are destarched by keeping the plant in :

(A) 10-12 hours in night (B) 10-12 hours in day

(C) 2 hours in sun light (D) Only 2 hours in night

3. Covering with black strips should be tested for presence of :

(A) Water in leaf (B) CO_2 in leaf (C) O_2 in leaf (D) Starch in leaf

4. The type of starch produced during photosynthesis is :

(A) Polysaccharides (B) Monosaccharide (C) Oligosaccharides (D) None of these

5. If the intensity of light is increased, the rater of photosynthesis :

(A) Decreases into certain limit (B) Increases into certain limit

(C) Remain the same (D) Increase and decreases at regular at regular interval

6. The colour of light in which rate of photosynthesis is minimum :

(A) Red light (B) Blue light (C) Green light (D) Yellow light

7. Chlorophyll-a present in :

(A) Algae (B) Bacteria (C) Croton (D) Both (A) and (C)

8. A non-green cell will not have the following activity :

(A) Required water (B) Evolve carbon dioxide

(C) Requires carbohydrates (D) Evolve O_2

9. The teacher instructed a student to place a healthy potted shoe flower plant in a dark room for 24 hour prior to an experiment on photosynthesis. The purpose of placing it in a dark rook is :

(A) To increase the intake of CO_2 (B) To activity the chloroplast in the leaves

(C) To destarch the leaves (D) To denature the enzymes in the leaves

10. During light reaction photosynthesis pigments chlorophyll-a absorbs visible light energy in the form of :

(A) Quanta or photons (B) Electrons (C) Protons (D) Neutrons

NUTRITION IN PLANT — ANSWER KEY — EXERCISE # 1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	C	C	B	B	C	B	B	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	D	D	C	C	C	B	B	A	B	D

- **MATCH THE FOLLOWING :**

1-(B,P), 2-(C,Q), 3-(D,R), 4-(A,S)

- **1.** F **2.** F **3.** F **4.** T **5.** F **6.** T **7.** T **8.** T
- **FILL THE BLANKS :**

1. Sun **2.** White / Visible **3.** Blue and Red, Green **4.** Light, Chemical
5. Guard Cells **6.** Opened **7.** Light **8.** Oxygen **9.** Dark
10. Decreased

- **VERY SHORT ANSWER QUESTIONS :**

1. Light, Water, Temperature, CO_2 **2.** $6CO_2 + 12\ H_2O \xrightarrow[Chlorophyll]{Sunlight} C_6H_{12}O_6 + 6H_2O + 6O_2$

3. Chlorophyll and Carotenoid **4.** Chloroplast

5. Regulates opening and closing of stomata

EXERCISE # 2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	A	D	A	B	C	D	D	C	A

RESPIRATION

All living organisms need energy to carry out their functions. The digested food is absorbed. The absorbed food components are subsequently assimilated in the body cells. these assimilated molecules hold energy in their chemical bonds. Their bond energy is released by oxidation in the cells. the process of oxidation is carried out with the help of oxygen .

Energy is released in this process. This energy is trapped by forming bonds between ADP (Adenosine diphosphate) and inorganic phosphate (Pi) to synthesize ATP (Adenosine triphosphate) molecules.

☐ **Breathing :**

The process involving intakes of air or oxygen [inspiration] and removal of air or carbon dioxide [expiration] is called breathing. No enzymes are involved in this process.

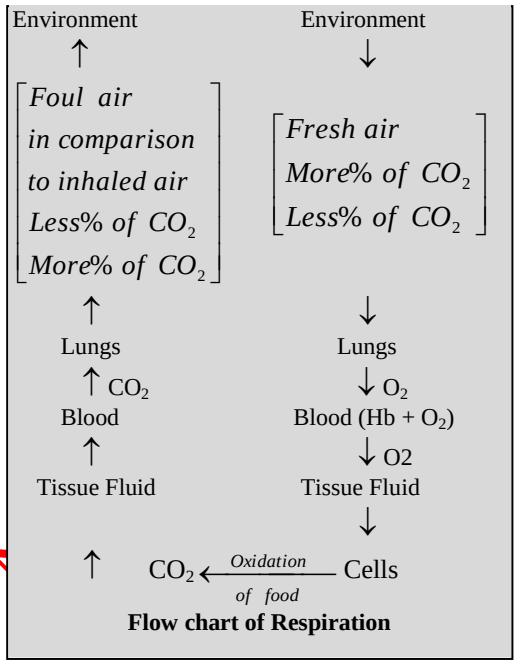
☐ **Respiration :**

The process of respiration involves taking in oxygen (or air) into the cells, using it for releasing energy by burring by food, and then eliminating the waste products (carbon dioxide and water) from the body.

OR

The process of acquiring oxygen from outside the body and to use it in the process of break-down of food sources for cellular needs, is called Respiration.

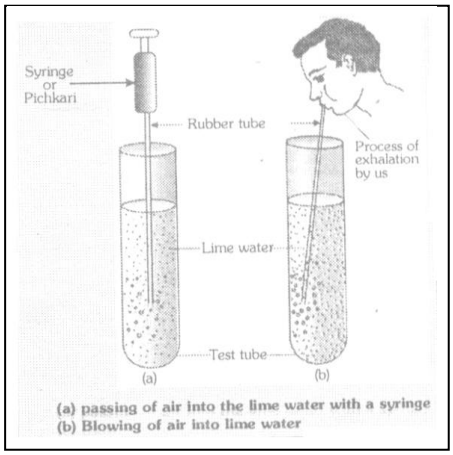
$C_6H_{12}O_6$(Glucose)+$6O_2 \longrightarrow 6H_2O+6CO_2$+Energy (in the from of ATP) Respiration is actually a biochemical process which occurs in stages and requires many enzymes.



Flow chart of Respiration

◆ **Differences Between Breathing and Respiration :**

	Breathing		Respiration
1.	It is a physical process. It involves inhalation of fresh air and exhalation of foul air	1.	It is a biochemical process. It involves exchange of respiratory gases and also oxidation of food.
2.	It is an extracellular process.	2.	It is both an extracellular as well as intracellular process.
3.	It does not involve enzyme action.	3.	It involves a number of enzymes required for oxidation of food.
4.	It does not release energy, rather it consumes energy.	4.	It releases energy.
5.	It is confined to certain organs only	5.	It occurs in all the cells of the body.



(a) passing of air into the lime water with a syringe
(b) Blowing of air into lime water

Activity : To study the release of carbon dioxide during exhalation. Take a hard glass test tube. Fill 3/4 th of this tube with freshly prepared lime water.

Close the mouth of test tube with a cark having a thin hole. Insert one thin glass through cork up to the bottom of the test tube. Make the apparatus airtight.

Now breathe out air from the mouth in the test tube through the glass tube.

We will observe that on exhaling through the tube, the limewater turns milky. This proves that exhaled air contains CO_2.

$Ca(OH)_2 + CO_2 \longrightarrow$ CaCO3 + H_2O

Lime water — Calcium carbonate (milky)

RESPIRATORY SUBSTRATES :-

In respiration many types of high energy compounds are oxidized. These are called **respiratory substrates** and may include **carbohydrates. fats and proteins.** Of these, carbohydrates such as **glucose, fructose** (Hexose) sucrose

(disaccharide) are the main substrates. Hexose are the first energy rich compounds to be oxidized during respiration. Complex carbohydrates are hydrolyzed into hexose sugars before being utilized as respiratory substrates.

Under certain conditions (mainly when carbohydrate reserves have been exhausted) fats are also oxidized. In the absence of carbohydrates and fats, proteins also serve as respiratory substrates. **Blackman** termed the reparatory oxidation of carbohydrates as **floating respiration** and that of protoplasmic protein as **protoplasmic respiration.**

Respiration and Combustion : Combustion of wood, coal , paper, etc., are also oxidation processes like respiration which also involve releases of energy in the form of heat. But respiration and combustion differ in many fundamental futures.

Oxidation of carbohydrates during respiration is not a single reaction because there is no enzyme that can catalyze the complete of glucose into carbon dioxide and water in a single step. Instead, the oxidation of glucose consists of a sequence of reactions. In such step wise oxidation energy is released slowly. It allows cells to capture more energy than would be possible if the energy is released in one big burst.

Differences Between Respiration and Combustion

S.No	Respiration	Combustion
1	It is a vital process.	It
2	Energy is released bit by bit.	All the energy is released suddenly.
3	There is very small rise in temperature	There is large increase in temperature
4	The energy released during respiration is again stored I chemical compounds. (e.g. ATP)	Most of the energy released is lost in the form of heat.
5	Respiratory reactions are regulated by enzymes.	Enzyme are not present.

Types of respiration : Process of respiration can be divided into the following two categories :

AEROBIC RESPIRATION :- When oxygen is used for respiration it is called aerobic respiration.

$$\text{Glucose} \xrightarrow[(in\ cytoplasm)]{Glycolysis} \text{Pyruvate or Pyruvic acid} \xrightarrow[(in\ Mitochondria)]{Oxygen} 6CO_2 + 6H_2O + \text{Energy [38 ATP]}$$

(1 molecules) (2 molecules)

Mechanism of Respiration :- The mechanism of respiration involves following two processes :

(i) Glycolysis – a series of reactions which does not require oxygen and by which glucose molecule is broken into Pyruvic acid.

(ii) Further breakdown of Pyruvic acid molecules by aerobic (in the presence of oxygen) or anaerobic (in the absence of oxygen) methods.

Respiration involves stepwise breakdown of glucose by a series of reactions in which the energy is released in some of the Exergonic steps. Glucose, the most common respiratory substrate, may be broken either aerobically or anaerobically. Both processes start the same way by using the anaerobic reaction pathway called **Glycolysis.** Under aerobic conditions the products of Glycolysis are completely oxidized and CO_2 and H_2O are formed as the end products, and under anaerobic conditions alcohol or lactic acid CO_2 are produced.

Breakdown of Pyruvic acid in the presence of oxygen :

In the presence of oxygen the Pyruvic and is completely oxidized and CO_2 and H_2O are formed as the end products. This oxidation is carried out by a cyclic series of reactions known as **tricarboxylic acid cycle** or **citric acid cycle or**

Krebs cycle. All reactions of Krebs cycle occur within mitochondria. Complete oxidation of each Pyruvic acid molecule produces 15 ATP molecules, therefore, **aerobic oxidation of each glucose molecule produces :** 15 × 12 = 30 ATP (from Krebs cycle) + 8 ATP (from Glycolysis) = 38 ATP

ANAEROBIC RESPIRATION :-

When food is oxidized without using molecular oxygen, the respiration is called anaerobic respiration. In this type of respiration incomplete oxidation of food takes place and in comparison to aerobic respiration, much less amount of energy is produced. It also includes Glycolysis which takes place in the cytoplasm. During this process one molecule of glucose is degraded into two molecules of Pyruvic acid (pyruvate) and little energy (2 ATP).

The Pyruvic acid is further oxidized into ethyl alcohol (ethanol) or lactic acid.

Differences between aerobic and anaerobic respiration :

S.No.	Features	Aerobic Respiration	Anaerobic respiration
1	O_2 requirement	O_2 required	Not required
2	Occurs in	Cytoplasm and mitochondria	Cytoplasm
3	Breakdown	Complete breakdown of glucose takes place	Incomplete breakdown of glucose takes place
4	End products	CO_2 and H_2O	CO_2 and ethyl alcohol or lactic acid
5	Energy produced from one glucose molecule	38 ATP	2 ATP

Fermentation : Fermentation is a kind of anaerobic respiration, carried out primarily by fungi and bacteria. A special feature of fermentation is that the substrate lies outside the cell in a liquid medium. Literally, fermentation refers to a chemical change accompanied by **effervescence.**

In fermentation anaerobic breakdown of carbohydrates and other organic compounds occur and form alcohol, organic acids, gases etc.

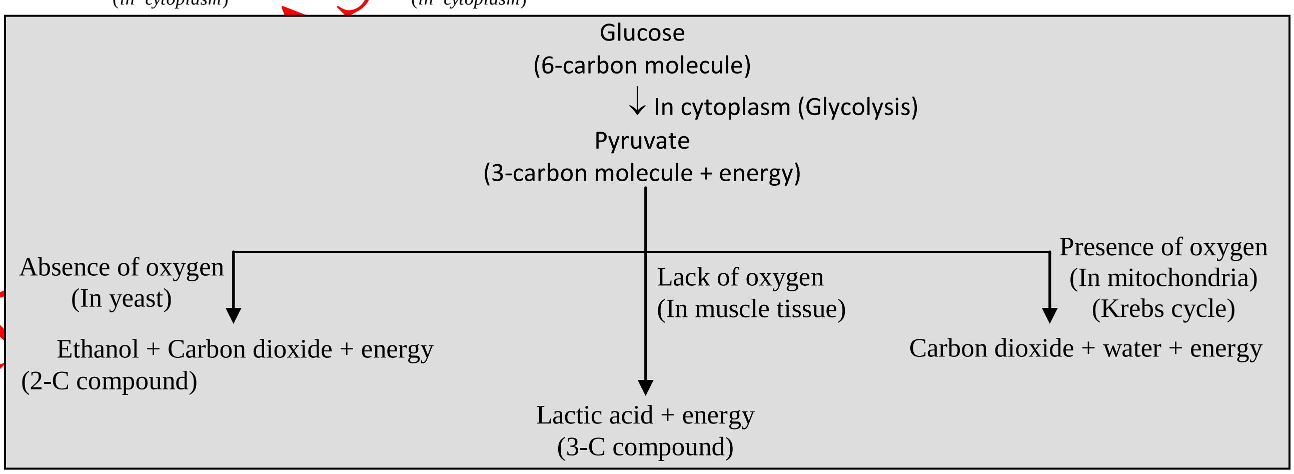
$$\text{Glucose} \xrightarrow[\text{(in cytoplasm)}]{\text{Glycolysis}} \text{Pyruvate} \xrightarrow[\text{(yeast)}]{\text{No } O_2 \text{ requrired}} \text{Ethanol} + CO_2 + \text{Energy}$$

(2 molecules) (2 ATP)

In certain bacteria and parasitic worms (Ascaris, tapeworm) glucose is metabolized to lactic acid without the use of O_2 and without the formation of CO_2.

In human beings, anaerobic respiration occurs in certain tissues such as RBC of mammals and skeletal muscles.

$$\text{Glucose} \xrightarrow[\text{(in cytoplasm)}]{\text{Glycolysis}} \text{Pyruvate} \xrightarrow[\text{(in cytoplasm)}]{\text{No } O_2 \text{ required}} \text{Lactic acid} + \text{Energy (2 molecules)}$$



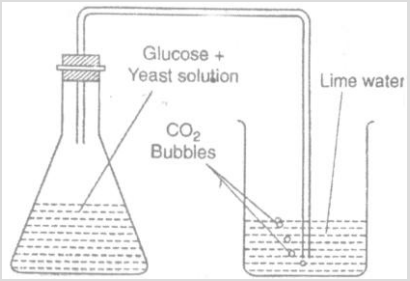
Breakdown of Glucose by Various Pathways (Aerobic and anaerobic respiration)

Differences Between Respiration and Fermentation.		
S.No.	**Respiration**	**Fermentation**
1	It may occur both in the presence and absence of oxygen.	It does not require oxygen.
2	Occurs only in living cells.	It does not occur within the living cells. It requires only enzymes and substrate.
3.	Sugar is oxidized and CO_2 and H_2O are formed as end products.	Different substrates are oxidized to form alcohol or organic acids.
4	Complete oxidation of substrates occurs, hence produces large amount of energy .	Incomplete oxidation of substrates occur and , hence less energy is produced.
5	It can occur in any living cell.	It occurs mainly with the help of yeast or bacterial cells.

Comparison Between Oxidative Phosphorylation and Photophosphorylation		
S.No.	**Oxidative photophosphorylation**	**Photophosphorylation**
1	This process occurs during respiration	It occurs during photosynthesis.
2	It occurs within mitochondria.	It occurs in the chloroplasts.
3	ATP is synthesized in the F_1 particles located on the cristae in the inner mitochondria membrane.	ATP is synthesized on the lamellae of the chloroplast.
4	This Phosphorylation requires oxygen .	It does not require oxygen but is related to photosystem-I and photosystem-II

Experiment :- To demonstrate the process of fermentation.

Method :- take some fruit juice or sugar solution in a test tube and add some yeast into it. Close the opening of test tube with a one holed cork. Insert a bent glass tube in the cork and dip the other end of tube into the other test tube containing lime water (solution of calcium hydroxide). Observe after few hours.



Observation and conclusion :- The lime water turns milky. This shown that CO_2 is liberated from the mixture of sugar solution and yeast. Fermentation of sugar results in the production of ethyl alcohol and CO_2.

Organs of Respirations :

1. **Skin** or general body surface , as in earthworm.
2. **Air tubes** or **trachea,** as in insects (grasshopper, cockroach, housefly)
3. **Gills** as in aquatic animals like fish and prawn.
4. **Lungs** as in land animals like frog, lizard, birds, rat, humans.

Frog respires through skin as well as lungs (being amphibious).

It is worth nothing that all respiratory organs whether skin, trachea, gills or lungs have three common features.

1. All the respiratory organs have a large surface area so as to get enough oxygen.
2. All have thin walls for easy diffusion and exchange of respiratory gases.

3. The respiratory organs like the skin, gills and lungs have a rich blood supply for transport of gases. In the tracheal system air reaches cells directly and blood plays no role in the transport of gases.

As a medium water is less suitable for respiration in comparison to air. Water is more dense (about 1000 times) than air and thus more energy is needed in passing it over the respiratory surface. Also since solubility of oxygen is very low in water, under similar conditions a given volume of water contains less amount of oxygen in comparison to same volume of air which contains more oxygen. Another problem faced in aquatic environments is that less oxygen is available in water as the temperature increase. Warm water contains less oxygen. However, as the temperature increases the rate of respiration increases and the animal requires more oxygen to meet with the metabolic demand.

Just as the animals in aquatic habitats have developed adaptations to tide over the problems. the terrestrial animals also have to adapt to the problems such as protection to the respiratory surface from dryness and maintaining the respiratory surface always moist. The air has an evaporative power and the air breathing land animals loose plenty of water by evaporation from the respiratory surface.

(a) Respiration through gills – Bronchial respiration

(b) Respiration through skin – Coetaneous Respiration

(c) Respiration through Lungs – Pulmonary Respiration

Respiratory organs in frogs :-

(i) Skin (iii) Bucco-pharyngeal cavity (iii) Lungs

DIFFERENCES BETWEEN RESPIRATION IN PLANTS AND ANIMALS

S.No.	Respiration in plants	S.No.	Respiration in animals
1	Respiration is carried out by all parts of the plants i.e., roots, stem, leaves.	1	Respiration occurs only in the reparatory organs.
2	It occurs at slower rate.	2	It is faster in animals .
3	In plants, there is little transport of gases to various part of the plant.	3	Transport of gases to various parts is more.
4	Products of anaerobic respiration of glucose in plants are ethanol and CO_2.	4	Products of anaerobic respiration of glucose is lactic acid and no CO_2.
5	There is no special gas transport system.	5	Blood transports oxygen .
6	Green plants have additional oxygen source from photosynthesis.	6	Animals do not have any additional source of oxygen.

DIFFERENCES BETWEEN PHOTOSYNTHESIS AND RESPIRATION.

S.No.	Characters	Photosynthesis	Respiration
1	**Site**	**It takes place in green cells of plats.**	**It takes place in all living beings.**
2	**Time**	**It occurs during day time**	**It occurs throughout the life of an organism.**
3	**Energy**	**Stored**	**Released**
4	**CO_2 and H_2O**	**Used up**	**Released**
5	**Food and oxygen**	**Produced**	**Used up**
6	**Dry weight**	**Increased**	**Decreased**
7	**Metabolism**	**Anabolic process**	**Catabolic process**

Q.1 What advantage does a terrestrial organism have over an aquatic organism with regard to obtaining oxygen for respiration ?

Ans. Terrestrial animals breathe the oxygen in the atmosphere, but animals that live in water such as fishes need to use the oxygen dissolved in water.
Since ht amount of dissolved oxygen is fairly low compared to the amount of oxygen in the air, the rate of breathing in aquatic organic is much faster than that seen in terrestrial organisms.

Q.2 What are the different ways in which glucose is oxidized to provide energy to various organisms?

Ans. In all organisms the first step in which glucose is oxidized is the breakdown of glucose, a six carbon molecule, into a three-carbon molecule pyruvate. This process takes place in cytoplasm.
The fate of pyruvate depends upon the presence or absence of oxygen.
Pyruvate may be converted into ethanol and carbon dioxide in the absence of oxygen. This process takes place in yeast during fermentation. It is called anaerobic respiration.

Q.3 Compare the functioning of alveoli in the lungs and Nephrons in the kidneys with respect to their structure and functioning.

Ans. Both alveoli in the lungs and Nephrons in the kidneys possess network of blood capillaries. The exchange of gases takes place in alveoli where impure blood (deoxygenated blood) is purified to oxygenated blood, similarly the Nephrons purify the blood by filtering out its waste products in the from of urine.

Q.4 What is difference between emphysema and asthma ?

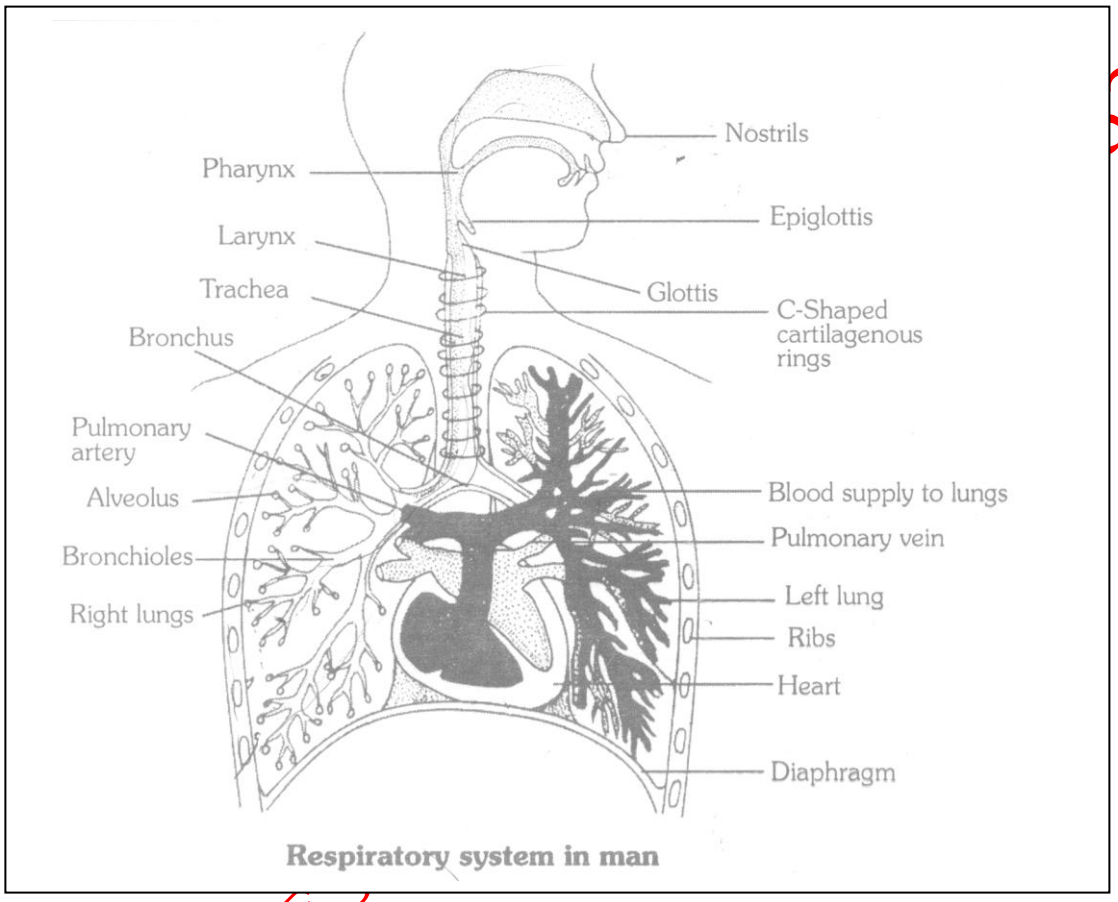
Ans. Emphysema is caused by the gradual breakdown of the thin walls of the alveoli so that the air spaces becomes larger and the total gaseous exchange diseases. Asthma is a form of difficult or heavy breathing caused by spasm f smooth muscle in the walls of bronchioles.

Respiration in human :

External nostrils ⟶ Nasal cavity ⟶ Pharynx ⟶ Larynx ⟶ Trachea ⟶ Bronchi ⟶ Bronchioles ⟶ Alveolar sacs.

1. **The respiratory tract :-** The respiratory tract of human respiration system begins from a pair of external nostrils situated at the lower end of the nose. The air enters through the nostrils and reaches into a pair of nasal cavities. The two nasal cavities are separated from each other by a nasal septum. The nasal cavities are separated from the oral cavity by a bony palate. It is due to this reason we can breathe in air while we eat. The nasal passages are linked by ciliated epithelium and mucus secreting cells, so that the inspired air get warmed, moistened and becomes dust free. The dust particles are entrapped in the mucus secreted by mucus cells. Nasal cavity is also lined with **olfactory epithelium** which acts are organ of smell. The nasal chambers open into pharynx through internal nares.
The pharynx is a short vertical tube located in the head at the back of Buccal cavity. It provides passage into which the internal and nares and Buccal cavity open to pass the air into it. The pharynx provides passage into trachea or wind pipe through a slit like aperture, called **glottis.** The glottis always remains open except during swallowing. The glottis bears a leaf like cartilaginous flap, the **epiglottis,** at its anterior margin. During swallowing, the epiglottis closes the glottis to glottis to check the entry of food into it. Entry of food into the respiratory tract can be fatal.
Trachea is about 11 cm in length and 2.5 cm in diameter. Its wall has incomplete (C-shaped) cartilaginous rings, which prevent the trachea from collapsing even if there is not much air in it. Trachea is lined internally by ciliated epithelium and mucus secreting cells. The mucus and cilia both prevent the entry of dust particles and microbes. Trachea runs down the neck and extends into thoracic cavity.
On entering the thoracic cavity, trachea divides into bronchi (singular : bronchus). On entering the lungs the right bronchus enter into right lung and left bronchus energy into left lung.

2. **The voice box :-** The voice box is also called **larynx.** It is an enlarged upper part of trachea. Before puberty, the larynx is inconspicuous and similar in both sexes. In males, if often becomes prominent and protrudes out and often called **"Adam's apple".** In side the larynx are the two vocal cords. These are folds of mucous membrane that extend into the lumen from the sides. Vibration in the vocal cords results in the production of sound which is altered and converted into speech with the help of Buccal cavity, soft palate, tongue and lips.



Respiratory system in man

3. **Lungs :-** The lungs are a pair of spongy, highly elastic, solid and bag-like organs. They are roughly cone-shaped and situated in the thoracic cavity. The lungs enclosed by a double layered membrane or covering called **pleura.** The pleural membranes are separated by a thin space filled with pleural fluid which lubricates the membrane to avoid friction.

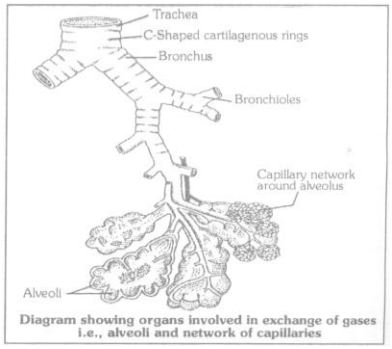


Diagram showing organs involved in exchange of gases i.e., alveoli and network of capillaries

Within the lung, each bronchus divides and redivides to form finer branches called **bronchioles.** Each bronchus with all its branches is called **bronchial tree.** After repeated divisions each bronchiole ends into a cluster of tiny air chambers called air sacs or **alveoli**.

Alveoli are functional units of as these are **actual site of respiratory exchange.**

There are about 750 million of alveoli present in lungs which have a total surface area of about 80 m^2. Alveoli are covered with a network of capillaries.

Mechanism of Breathing :- Lungs cannot expand or contract of their own. The contraction and expansion of lungs is brought about by diaphragm muscles and external intercostals muscles.

(a) **Inhalation (Inspiration) :-** Inhalation is intake of fresh air form outside into the alveoli of the lungs. It occurs by expansion of lungs which is brought about by enlargement of thoracic cavity. Inhalation involves the following steps :-

1. The diaphragm (a sheet of tissue that separates thoracic cavity from abdominal chamber) muscle contracts so that the diaphragm lowers down and becomes flat.

2. Lowering of diaphragm pushes the abdominal viscera downward resulting in the enlargement of thoracic cavity vertically.

3. External intercostals muscles contract so that the ribs and sternum are pulled upward and outward. This causes enlargement of thoracic cavity.

4. Enlargement of thoracic cavity results in the expansion of lungs.

5. Expansion of lungs reduces the pressure of air inside so that fresh air is pulled from outside into the lungs passing through nostrils, trachea and bronchi.

6. Fresh air has a rich supply of O_2 which goes into the blood passing through thin membranes of alveoli and blood capillaries. As a result the blood in the capillaries becomes loaded with oxygen and expels carbon dioxide into the alveoli for exhalation.

(b) **Exhalation [Expiration] :-** The mechanism of berating out of carbon dioxide is called exhalation.

1. During exhalation, the phrenic muscle of the diaphragm relaxes so that the abdominal viscera pushes the diaphragm upward, making it convex.

2. The external intercostals muscles also relax with the result the thoracic cavity is educed in sized and lungs also contract.

3. Contraction of lungs raises the air pressure so that foul air moves out.

An average rate of breathing in a normal adult man is 15 to 18 times per minute.

EXCHANGE OF GASES

Breathing is the first step of respiration which involves exchange of gases between the air in alveoli and the blood capillaries (around the alveoli). In this exchange, the blood takes up oxygen from the alveolar air and releases CO_2 to the alveolar air. This exchange of gases is called **external respiration** and results in the oxygenation of blood. the heart supplies the oxygenated blood to he body tissues.

The living cells perform oxidation of simple food (glucose) to release energy. This s the process of **aerobic respiration,** which utilizes O_2 and releases CO_2. The aerobic respiration occurs partly in the cytosol and partly in the mitochondria of cells. This catabolic process is called cellular respiration.

The concentration of C_2 is more in the blood and less in the tissue cells. So, the O_2 moves from blood to the tissues by the physical process of **diffusion.** Similarly, CO_2 concentration is more in tissues and less in the blood, So, the CO_2 moves from tissues to the blood. The oxygenated blood now becomes deoxygenated.

Differences between inspiration and expiration

S.No.	Characters	Inspiration [inhalation]	Expiration [exhalation]
1	Definition	Process of inhaling of fresh air in the lung alveoli	Process of exhaling of foul air out of lung alveoli

2	EICM (External inter costal muscles)	Contraction	Relaxation
3	Ribs	Moves Outward	Moves inward
4	Sternum	Moves forward	Moves backward
5	Movement of diaphragm	Towards abdominal cavity	Away from abdominal cavity
6	Shape of diaphragm	Flat	Dome shaped
7	Size of thoracic cavity	Increases	Decreases
8	Lungs	Decompression	Decreases
9	Pressure of air in lungs as compared to atmosphere	Deceases	Increases
10	Movement of air	From atmosphere to lungs	From lungs to atmosphere

Location, structure and function of respiratory system

Organ or region	Location	Structure and function
Nasal cavity	Above the mouth cavity	● Two external nostrils help in intake of oxygen
		● Lined by ciliated and sensory epithelial cells which help in filtering of air (by hair) and warming or cooling of the inhaled air.
		● Mucous secreted by them prevent dust particles for entry.
		● Two internal nostrils act as the end of this cavity from which the air enters into pharynx.
Pharynx	Behind the nasal cavity leading into the trachea	● Posteriorly it has two openings ; dorsal opening or gullet (leading to esophagus) and ventral opening or glottis (aperture of trachea).
		● A cartilaginous flap (epiglottis) guards the glottis to prevent the entry of food into the trachea.
Larynx or voice box or Adam's apple	Lies at the back of the neck	● A pair of membranes (called vocal cords) stretched in the internal cavity, partially close the air passage. Theses membranes can be relaxed or stretched. When the air passes over the vocal cords. they vibrate and produce sounds.
Trachea or Wind	Tube running through the neck in front of esophagus	● Its walls are supported by C-shaped cartilages to prevent it from collapsing. It divides into two bronchi (singular bronchus) which enter the respective right and the left lung.
Lungs	Present in the thoracic cavity	● Surrounded by two pleural membranes containing a fluid in between to reduce friction.
		● Spongy organ formed by the sub-divisions of the bronchus called bronchioles.
		● Each bronchiole ends in a structure like bunch of grapes or balloon-like structures called alveoli or air sacs. Alveoli provide the surface for the exchange of gases.
		● Each alveolus is surrounded by network of blood capillaries.

RESPIRATION IN PLANTS :

INTRODUCTION

The plants do not have any special reparatory system so they have to respire in all of their individual parts like leaf, stem and root. The plants also have to exchange gases with the atmosphere by simple diffusion process.

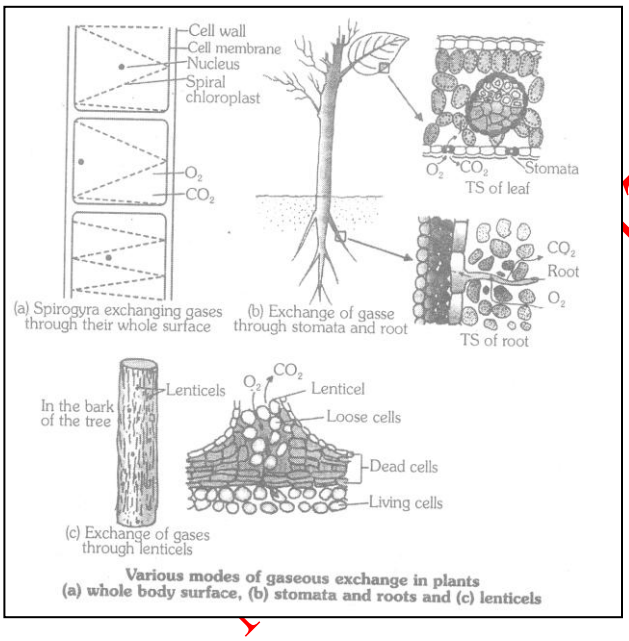
Mode of gaseous exchange (oxygen and carbon dioxide) in plats.

In terrestrial plants gaseous exchange occurs through

(a) Stomata – In leaves and green stem

(b) Lenticels – In woody stem and roots.

(c) Root hairs – In young roots.



Various modes of gaseous exchange in plants
(a) whole body surface, (b) stomata and roots and (c) lenticels

Respiration through stomata :

Stomata are small apertures found on the surface of leaf.

For the process of respiration,

oxygen enters stomata by the process of diffusion and then into other cells of the leaf.

When concentration of CO_2 increases inside the cells diffused out through stomata.

Respiration through lenticels :

Lenticels are the opening in the bark of woody stems.

They also serve as a place of gaseous exchange.

Respiration through general surface of the roots :

Ploughing or tilling of the soil creates small air spaces around soil particles which provides the sources of oxygen for the roots.

This oxygen present between the soil particles diffuses into root hairs (these are the extensions of epidermal cell of the root), by the process of diffusion.

From the root hairs, oxygen diffuse into other cells of the roof. After utilization of oxygen, CO_2 is diffused out into the soil.

In **older roots** there are no root hairs present. Instead they have layer of dead cells which is protective in nature and encloses opening (lenticels). These are used for gaseous exchange between soil and inner living cells.

Do you know ?

Aquatic plants can carry out gaseous exchange by diffusion over their whole surfaces.

Direction of diffusion depends upon the environmental condition and the requirement of the plant.

During the day time O_2 release is the major event.

Review questions

1. From which part of stem bark does gaseous exchange occur ?
2. Name the respiratory gases.

3. Name the part through which gaseous exchange takes place in plants.
4. How does diffusion occur in submerged aquatic plants ?

EXERCISE # 1 FOR SCHOOL EXAMS

VERY SHORT ANSWER TYPE QUESTIONS :

1. What are the end products of aerobic respiration & anaerobic reparation.
2. What happens to the diaphragm when diaphragm muscles contract ?
3. Name the respiratory organs of (i) fish, (ii) mosquito, (iii) earthworm, (iv) dog.
4. From where do the following take in oxygen ? (i) prawn (ii) rat.
5. Using only flow chart, write the path of C_2 from trachea to respiring tissue cell.
6. Define breathing .
7. 'Respiration is a vital function of the body'. Justify.
8. When we blow exhaled air into the solution of lime water, it turns milky. What actually happens in the solution ?
9. Arrange the following incorrect order through which the air flows up to alveoli –
   Larynx, alveoli, trachea, pharynx, external nostrils, internal nostrils, nasal cavities.
10. Explain how oxygen enters the blood in the lungs.

SHORT ANSWER TYPE QUESTIONS :

1. Why is diffusion insufficient to meet the oxygen requirements of multicellular organisms like humans ?
2. What advantage over an aquatic organism does a terrestrial organism have with regard to obtaining oxygen for respiration ?
3. How is oxygen and carbon dioxide transported in human beings ?
4. How are the lungs designed in human beings to maximize the area for exchange of gases.
5. What are the differences between aerobic and anaerobic respiration ? Name some organisms that use the anaerobic mode of respiration.
6. Compare the functioning of alveoli in the lungs an Nephrons in the kidneys with respect to their structure and functioning .
7. Name the cavity in which the lungs of man are enclosed ? What is its function ?
8. What is difference between emphysema and asthma ?
9. How is respiration different from breathing ?
10. What is photosynthesis ? How it differs form respiration ?

LONG ANSWER TYPE QUESTIONS :

1. Explain the mechanism of gaseous exchange between tissues and blood.
2. Write short notes on the following :
   (i) Exchange of gases in tissues (ii) Mechanism of inhalation and exhalation.
3. Give reasons for the following :
   (i) The glottis is guarded by epiglottis.
   (ii) The lung alveoli are covered with blood capillaries.
   (iii) The wall of trachea is supported by cartilage rings.
4. **Explain the process of respiration in plants.**

WHICH OF THE FOLLOWING ARE TRUE (T) OR FALSE (F) :-

1. Fishes respire with the lungs.
2. Lungs become empty after forceful expiration.
3. Lactic acid is produced in anaerobic respiration.
4. Maximum contraction of diaphragm causes maximum expiration.
5. Expiration is normally brought about by the relaxation of aspiratory muscles.

COMPLETE THE FOLLOWING SENTENCES :-

1. Nasal chambers are separated from the oral cavity by .....
2. Larynx communicates with the .....by glottis.
3. The.....check the entry of food into the respiratory tract.

4. Lungs lie in the thoracic cavity separated by……..from the abdominal cavity.
5. Wall of alveoli consists of simple ……Squamous epithelium.
6. Gaseous exchange in plants lake place through ……..and ……..
7. ……..are the opening in the bark of woody stem.
8. During the day time……..release is the major event.

MATCH THE COLUMN :-

1. Match the items of Column A with items of Column B.

A	B
(a) Stomata	(i) In woody part
(b) Lenticels	(ii) Release in night
(c) CO_2	(iii) In herbaceous stem
(d) O_2	(iv) Release in day

EXERCISE # 2 OBJECTIVE QUESTIONS

1. Respiration is the process in which -

(A) energy is stored in the form of ADP (B) energy is released and stored in the form of ATP

(C) energy is not released at all (D) energy is used up

2. Which of the following is the source of respiration –

(A) Stored food (B) Fats (C) Glucose (D) Proteins

3. The form of energy used in respiration is –

(A) Chemical energy (B) Electrical energy (C) Mechanical energy (D) Radiant energy

4. Respiratory structures in the insects are –

(A) Gills (B) Skin (C) Lungs (D) Trachea

5. The narrowest and most numerous tubes of lungs are termed as –

(A) Bronchus (B) Bronchioles (C) Alveoli (D) None of these

6. A normal man respires in a minute –

(A) 10-15 times (B) 14-18 times (C) 20-25 times (D) 5-30 times

7. In anaerobic respiration –

(A) O_2 is given out (B) CO_2 is given out (C) CO_2 is taken in (D) O_2 is taken in

8. The exchange of gases [O_2 and CO_2] in a mammal takes place in –

(A) Trachea (B) Bronchi (C) Bronchioles (D) Alveoli

9. During inspiration muscles of diaphragm –

(A) Contracts (B) Expands (C) No effect (D) Coiled like string

10. Expiration involves –

(A) Relaxation of diaphragm and intercostals muscles

(B) Contraction of diaphragm and intercostal muscles

(C) Contraction of diaphragm muscles

(D) Contraction of intercostals muscles

11. The structure which prevent the entry of food into respiratory tract is –

(A) Pharynx (B) Larynx (C) Glottis (D) Epiglottis

12. In fever breathing rate –

(A) Increase (B) Decrease (C) Stop (D) None

13. Mammalian lungs are –

(A) Hollow (B) Solid and spongy (C) Spongy (D) None

16. In respiration, air passes through –

(A) Pharynx, Nasal cavity, Larynx, Trachea, Bronchi, Bronchiole, Lungs

(B) Nasal cavity, Pharynx, Larynx, Trachea, Bronchi, Bronchiole, Lungs

(C) Larynx, Nasal cavity, Pharynx, Trachea, Lungs

(D) Larynx, Pharynx, Trachea, Lungs

18. Ratio of respiration is directly affected by –

(A) CO_2 concentration (B) O_2 in trachea (C) Concentration of C_2 (D) Diaphragm expansion

19. Oxygen in lungs ultimately reaches –

(A) Alveoli (B) Trachea (C) Bronchus (D) Bronchioles

20. Most of the carbon dioxide is carried in the blood as –

(A) Bicarbonates (B) Carbon monoxide (C) Carbonic acid (D) Carbonates

21. Respiration and photosynthesis are just the :

(A) Opposite process (B) Similar process (C) Burning process (D) None of these

22. In the experiment demonstrating respiration in geminating seeds, KOH is used to :

(A) Absorb carbon dioxide present in the flask (B) Absorb oxygen present in the flask

(C) Absorb water vapor released by the seeds (D) Liberate oxygen to be used by the seeds

23. In which part of the plant, respiration rate is higher :

(A) Root and stem tip (B) Buds (C) Germinating seeds (D) All of these

24. Plant cell can do :

(A) Breathing and Respiration (B) Respiration and photosynthesis

(C) Breathing and photosynthesis (D) All of these

25. Plant can respire in :

(A) Dark (B) Light

(C) Both in light and dark (D) Morning

EXERCISE # 3 FOR COMPETITIVE EXAMS

1. Exchange of gases in lung alveoli occurs thorough –

(A) Active transport (B) Osmosis (C) Simple diffusion (D) Passive transport

2. Hemoglobin is –

(A) Vitamin (B) Skin pigment (C) Blood carrier (D) Respiratory pigment

3. The maximum affinity of hemoglobin is with –

(A) Carbon monoxide (B) Carbon dioxide (C) Oxygen (D) Ammonia

4. Vocal cards occur in –

(A) Pharynx (B) Larynx (C) Glottis (D) Bronchial tube

5. Oxygen is carried by :

(A) Leucocytes (B) Erythrocytes (C) Platelets (D) None of these

6. Epiglottis guards the opening of :-

(A) Esophagus (B) Eustachian tubes (C) Larynx (D) Internal nares

7. Lungs are covered by :-

(A) Pericardium (B) Perichondrium (C) Pleura (D) Periosteum

8. Breathing is controlled by a centre in the

(A) Lungs (B) Heart (C) Cerebellum (D) Medulla oblongata

9. Oxygen passes into the blood from the alveoli by a process called :-

(A) Osmosis (B) Diffusion (C) Active transport (D) Pinocytosis

10. Exchange of gases between the blood and the tissue of the body is called :-

(A) External respiration (B) Internal respiration (C) Cellular respiration (D) counter-current exchange

11. The diffusion of CO_2 across the alveolar membrane is 20 times faster than that of O_2 because :-

(A) CO_2 is actively transferred

(B) The alveolar area available for CO_2 diffusion is larger

(C) CO_2 is more soluble in H_2O than O_2, which enables it to pass the membrane more readily

(D) The CO_2 pressure gradient in larger

12. Which of the following are respiratory organs of scorpion ?

(A) Lungs (B) Book-lungs (C) Gills (D) Ceridian

13. Carbon monoxide has greater affinity for hemoglobin as compared to oxygen by –

(A) 1000 items (B) 200 times (C) 20 times (D) 2 times

14. P_{CO_2} released from the body is about –

(A) 15 % (B) 23 % (C) 30 % (D) 70 %

15. Trachea is lined with incomplete rings of –

(A) Fibrous cartilage (B) Calcified cartilage (C) Elastic cartilage (D) Hyaline cartilage

16. Sound is produced in mammals

(A) By syrinx (B) By bronchus (C) During inhalation (D) During exhalation

ANSWER KEY

EXERCISE # 1 FOR SCHOOL EXAMS

Which of the following are True (T) or False (F) :-

1. F
2. F
3. T
4. F
5. T

Complete the following sentences :-

1. Plate
2. Laryngopharynx
3. Epiglottis
4. Diaphragm
5. Non-ciliated
6. Stomata & Lenticels
7. Lenticels
8. Oxygen

Match the column :-

1. (A) –(iii), (B) –(i), (C) –(ii), (D) –(iv),

OBJECTIVE QUESTIONS **EX # 2**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	A	D	B	B	B	D	A	A	D	A	B	B	A
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	A	A	C	D	A	A	A	D	B	C					

FOR COMPETITIVE EXAMS. **EX # 3**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	C	D	A	B	B	C	C	D	B	B, C	C	B	B	C	D	D

CIRCULATORY SYSTEM

☐ **CIRCULATION :**

The process of transporting the absorbed food, water and waster products from one place to another in body is called circulation.

COMPETITION WINDOW

Circulatory system : There are two types of circulatory system found in the animals :-

(i) **Open Circulatory System :** In this types of circulatory system, the main blood vessels arising from heart and pour the blood into tissue spaces (sinuses). **e.g. Arthropoda** (Cockroach). **Echinodermata**

(ii) **Closed Circulatory System :** In this types of circulatory system the blood remains only in the blood vessels and carried to various organ through vessels and capillaries. **e.g. human beings. Annelida** (Earthworm). **Some mollusks**

◆ In human beings, the circulatory (transport) system is divided into two system :

(i) Blood Circulatory System	**(ii) Lymphatic System**
(a) Blood	(a) Lymph
(b) Blood vessels	(b) Lymph vessels
(c) Heart	(c) Lymph muscles

BLOOD CIRCULATORY SYSTEM

☐ **BLOOD**

Blood is an important fluid conducting tissue, which transport the materials to different body parts.

◆ **Composition of Blood :**

Liquid part - (Matrix) – Blood plasma

Solid part - Blood corpuscles – (RBC, WBC and Platelets)

Plasma :

It composes 55 % of blood.

The plasma has 90-92 % water and remaining 8 %-10 % are other materials.

The plasma is a faint yellow viscous fluid.

Plasma contains some soluble proteins (serum albumin, serum globulins, prothrombin and fibrinogen), inorganic salts, food materials, waste products, dissolved gases, anticoagulants and antibodies.

Function of Plasma :

(i) Transportation of nutrients, respiratory gases, excretion of wasters and hormones of endocrine glands.

(iii) Prothrombin and fibrinogen plasma proteins help in blood clotting at injuries.

(iv) Globulins of blood plasma act as antibodies and provide immunity (disease resistance) to body.

(v) Plasma also help in transportation of minerals like iron, copper etc.

☐ **BLOOD CORPUSCLES :**

They form 45 % part of blood

◆ Erythrocytes or Red Blood Corpuscles (RBC)

◆ Leucocytes or White Blood corpuscle (WBC)

◆ Platelets or Thrombocytes

COMPARATIVE STUDY OF BLOOD CORPUSCLES

	Characters	RBCs	WBCs	Platelets
1	Shape	Circular, Biconcave	Rounded, Irregular	Rounded or Oval
2	Size (Diameter in μ m)	7-8, Smaller than WBCs	12-12 , Larger than RBCs	2-5, Smallest blood corpuscles
3	Number or Count (Per cubic millimeter)	5.5 million in male, 4.5 million in female	8000-11000	1.5-4.5 Lakhs
4	Colour	Red due to hemoglobin	Colorless	Colorless
5	Structure	At maturation they lost all	They contain are cell	Non-nucleated cells

		cell organelles	organelles	
6	Life Span	120 days	1-7 days	2-5 days
7	Functions	Transport oxygen and small amount of carbon di-oxide	Act as the soldiers, scavengers and builders of body	Help is blood clotting

Functions of Blood :

1. **Transportation of oxygen from lungs to tissues .**
2. **Transportation of carbon dioxide from the tissue to the lungs.**
3. **Transportation of excretory material from the tissues to the kidneys :** Some of the chemical activities in the body form nitrogenous end products, like urea, that are poisonous. These substances diffuse into the capillaries and are carried by plasma. When they eventually reach the kidneys, a large proportion of them is removed and excreted.
4. **Transportation of digested food from the small intestine to the tissue.**
5. **Distribution of hormones and enzymes.**
6. **Formation of clots to prevent blood loss.**
7. **Distribution of heat and temperature control :** Muscular and chemical activities release heat. The heat so produced locally is distributed all around the body by the blood and in this way an even temperature is maintained in all body regions.
8. **Presentation of infection and wound healing :** WBCs in the blood help in wound healing. Bacteria are destroyed by the WBCs before they can enter the general circulation. Also , the WBCs provide defense to the body against disease germs and foreign substances.

COMPETITION WINDOW

- **HAEMOPOIESIS :** The process of formation of blood is called hoemopoiesis. This process occurs in red bone marrow and lymphoid tissues (spleen, thymus and lymphatic nodes)
- Study of blood - Hematology
- Blood by weight - 5-6 litres in male and 4-5 litres in female
- Process of RBC formation – Erythropoiesis.
- Decrease in RBC cont – Anaemia
- Number of RBC count increases at high altitude, this condition known as **polycythemia**
- Blood red in colour due to red colored respiratory pigment **hemoglobin** is present in RBC.
- Iron (Ferrous ion Fe^{+2}) element found in **Haem** component of hemoglobin (Hb).
- **Leukemia –** Abnormal increase in TLC (Total Leukocyte Count). It is also called **blood cancer.**
- On the basis of nucleus and nature of cytoplasm WBCs are of following types :-

  (i) Agranulocytes :- (a) Monocytes (b) Lymphocytes

  (ii) Granulocytes :- (a) Acidophil (b) Basophil (c) Neutrophils

LIFE PROCESSES PART - 2

Maintenance By Platelets

When we are injured and start bleeding. Naturally the loss of blood from the system has to be minimized. In addition, leakage would lead to a loss of pressure which would reduce the efficiency of the pumping system. To avoid this, the blood has platelet cells which circulate around the body and plug these leaks by helping to clot the blood at these points of injury.

BLOOD CLOTTING

- Blood flows from cut or wound but after sometimes it stops automatically. It is called clotting of blood .
- **Mechanism of blood clotting :** (Enzyme Cascade theory)
- Proposed by **Macfarlane** and Co-workers
- According to this theory, there are 3 steps of blood clotting.
- **Step – 1 :-** Injured tissues and Damaged platelets release

$$\downarrow$$

$$\text{Thromboplastin} + Ca^{+2} \longrightarrow \text{Thrombokinase (An enzyme)}$$

- **Step – 2 :-** Prothrombin (soluble plasma protein) $\xrightarrow[Ca^{-2}]{Thrombokinase}$ Thrombin (An active enzyme)
- **Step – 3 :-** Fibrinogen (soluble plasma protein) $\xrightarrow{Thrombin}$ Fibrin fibers + blood corpuscles $\rightarrow$ Blood clot
- Serum is blood plasma from which fibrinogen, the blood clotting protein, has been removed.
- 13 clotting factors are helpful in blood clotting.
- Clotting factor I $\rightarrow$ Fibrinogen
- Clotting factor II $\rightarrow$ Prothrombin
- Clotting factor III $\rightarrow$ Thromboplastin
- Clotting factor IV $\rightarrow$ Calcium
- Vitamin-K is necessary for the synthesis of clotting factor in liver.

REVIEW QUESTIONS

1. What is meant by circulatory system ?

2. What are the components of blood circulatory system in humans ?

3. Name the mineral, Which helps in formation of RBC's.

4. Give other name for blood platelets.

FILL IN THE BLANKS

(i) The two principal fluids involved in transpiration in our body are ……..and …….

(ii) The fluid matrix of blood is called……

(iii) The RBCs are involved in transportation of ……..from lungs to tissues.

(iv) Oxygen combines with hemoglobin present in RBC and forms ……..

(v) The chief function of WBCs is to ……..

(vi) Platelets help in……

(vii) The plasma without its fibrinogen is called……

(viii) Fibrinogen is converted into fibrin by……..

BLOOD VESSELS

In human, three types of blood vessels are present.

1. **Arteries :** The vessels which carry blood from heart to various organs of the body.
2. **Veins :** They collect the blood from different parts of the body and pour it into the heart.
3. **Capillaries :** These are smallest blood vessels and one-cell thick.

The major differences between various blood vessels have been given in Table.

Comparative Study of Blood Vessels

S.No.	Features	Arteries	Veins	Capillaries
1	Direction of blood flow	Take the blood away from heart to different parts of body.	Bring the blood towards the heart from various body parts.	Blood flows from arterioles (capillaries) to venues.
2	Kind of blood	Oxygenated blood except in pulmonary artery.	Deoxygenated blood except in pulmonary vein.	Blood changes from oxygenated to deoxygenated.
3	Blood pressure	Pressure is high.	Pressure is low.	Pressure is extremely low.
4	Blood flow	Blood flows rapidly with jerks.	Blood flows smoothly without jerks.	Blood flows smoothly without jerks.
5	Lumen	Narrow	Wide	Very small
6	Semilunar valves	Absent	Present	Absent
7	Location	Mostly deep seated.	Mostly superficial .	Form a network all over the body and in the organs.

HEART

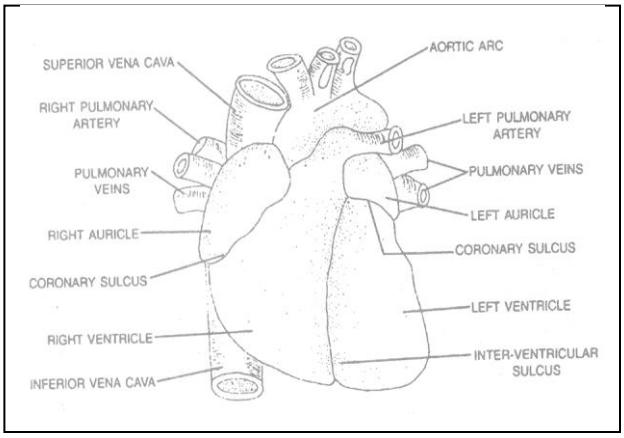
Size – 5 × 3.5 inches	**Colour** – Pink
Shape – Conical	**Weight** – 300 gm.

- **Position :** It is situated in thoracic cavity, between the lungs slightly on the ventral surface.
- Its triangular, superior-broad portion is tilted slightly towards right (dorsal) side, its lower narrow portion is tilted towards left side.
- Heart is enclosed from all the side by an envelope of two membranes called pericardial membranes (pericardium.)
- The narrow space in between these two membranes is called **pericardial cavity.** A fluid is present in this cavity, called **pericardial fluid.**
- Pericardial fluid prevents the heart from external jerks.
- It reduces the friction during contraction.
- The human heart is divisible into four chambers.
- The upper two chambers are **auricles** (atria) while the lower two chambers are called **ventricles.**
- In between the auricles and ventricles, a clear groove is present which is known as **coronary sulcus.**

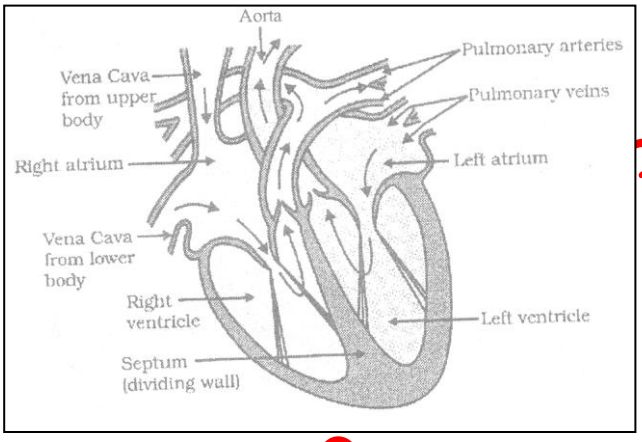
☐ **External structure of Heart :**

- Auricular part of heart is smaller and its walls are thin.
- It is divided into right and left auricles, by a groove called **inter-auricular sulcus.**

- Ventricular part is broad and muscular.
- Ventricles have thicker wall than auricles.
- The groove which divides the two ventricles is termed as **inter-ventricular sulcus.**



Heart : External Structure



Heart : Internal Structure

☐ **Internal structure of Heart :**

- Partition between right and left auricle is known as **inter-auricular septum** while partition between the two ventricles is known as **inter-ventricular septum.**
- Partition between auricles and ventricles is known as **auricular-ventricular septum.**

(A) Right Auricle (Atrium) :

- The right auricle has the openings of the **superior vena cava, inferior vena cava** and **coronary sinus.**
- Deoxygenated blood from the veins of the head, neck and upper limbs enters the right auricles by superior vena cava and from the rest of the body and lower limbs by the inferior vena cava.
- The coronary sinus, which drains deoxygenated blood from the heart muscle.
- From the right auricle blood passes into the right ventricle through a **tricuspid valve,** (so called because it has three cusps.)

(B) Right Ventricle :

- Blood leaves the right ventricle through the **pulmonary artery.** It guarded by **Semilunar valve.**
- This artery further divides into right and left pulmonary arteries entering into the lungs where they further branch into pulmonary capillaries.

(C) Left Auricle (Atrium) :

- This chamber receives four pulmonary veins, two from each lung from where they bring oxygenated blood.
- The left auricle empties its blood into the left ventricle through a **mitral** or **bicuspid valve.**

(D) Left Ventricle :

- Blood leaves the left ventricle by the large, main artery of the body called the **aorta.**
- The opening from the left ventricle into the aorta is guarded by **aortic Semilunar valve.**
- Just beyond these , a pair of **coronary arteries** are present which supply blood to heart muscles.
- This blood is brought back to heart by coronary veins which join to form coronary sinus.

COMPETITION WINDOW

Haversian valve	-	Present on the opening of superior vena cava (SVC)
Eustachian valve	-	Present on the opening of inferior vena cava (IVC)
Thebesian valve	-	Present at the opening of coronary sinus.

Columnae carneae	-	Finger like projection from ventricle inner walls.
Papillary muscles	-	Present at the tip of Columnae carneae.
Chordae tendinae	-	Arise from papillary muscles and keep the valves in proper position.
Musculi Pectinati	-	Transverse muscular ridges present on auricles' inner walls.

REVIEW QUESTIONS

1. Name the groove, which demarcates two ventricles externally.
2. Name the thickest artery.
3. Name the blood vessel that brings oxygenated blood to the heart chamber.
4. Name the arteries, which supply blood to heart itself.
5. Give other term for bicuspid valve.
6. What is function of valves in circulatory system ?
7. Which chamber of heart receives oxygenated blood ?
8. Which chamber of heart pumps blood ?
9. Name the vein which brings blood to left atrium from lungs.
10. Name the major veins which pour blood into right atrium.

FILL IN THE BLANKS

(i) Blood vessels in human are……..and ……..
(ii) Arteries carry blood from ……to …….
(iii) Veins carry blood from ……..to ……..
(iv) Except……all veins carry oxygenated blood.
(v) Except ……all veins carry deoxygenated blood.
(vi) The heart in human consists of ……chambers, comprising ……and ……..
(vii) The average pumping rater of heart in a healthy adult under rest is……..times per minute.
(viii) Lymph contains……..but lacks……and……
(ix) Lymph contains white cells called……

WORKING OF THE HEART

- The heart of the human works like a pump.
- Pure oxygenated blood enters the left auricle from lungs through pulmonary veins.
- The deoxygenated blood from various part of the body enters right auricle through veins and capillaries.
- The two auricles contract simultaneously so the oxygenated blood left auricle to left ventricle and deoxygenated blood from right auricle is pumped into right ventricle.
- Now both the ventricles contract simultaneously so the pressure is crated on the blood and the valves between auricle and ventricles close and the blood does not go back into auricle.
- Due to this pressure, aorta valve opens and the blood comes in aorta, from here, blood is sent to different parts of the body with the help of various arteries.
- By the contraction of right ventricle, blood reaches the lungs through pulmonary arteries where it gets reoxygenated.

HEART BEAT

- Rhythmic contraction and expansion of heart is called heart beat. Contraction and expansion occurs separately in atria and ventricles.
- The **Sinus-Atrial node (SA node)** found in the wall of the right atrium, is responsible to initiate and maintain the heart by generating impulses.
- SA node is also known **"Pacemaker"** of heart. In some heart patients, the heart does not beat normally. The muscle cells stop functioning. In such patients, a machine called pace maker is inserted in the patient's heart , so that heart beasts normally.

- Heart beat in human, 72 times in one minute.
- Each heart beat has two components, **systole** and **diastole.** Systole represents contraction while diastole represents expansion or distension of heart chambers.
- **Tachycardia :-** It is the condition where heart beat (rate) exceeds 90 per minute for an average adult man.
- **Bradycardia :-** It is the condition where heart beat falls below 60 per minute for an average adult man.
- **Cardiac Cycles :-** The sequence of events which takes place during the completion of one heart beat.

PULSE

- A wave of distention passes along the arteries following each ventricular systole. This waves of distension is called arterial pulse.
- It is generally felt by placing finders over the radial artery at the wrist.
- The pulse rate is same as heart beat rate.

HEART – SOUND

- **Ist Sound –** This is a contraction sound which denotes the beginning of ventricle-contraction. It arises due to closing of mitral valve and the tricuspid valve. It is weak and appears in the form of **"Lubb"** (L – U – B – B)
- **IInd Sound –** This is a diastolic sound which denotes the beginning of ventricular diastole. This arises due to the closing of the semi-lunar valves and is heard in the form of **"Dup".**
- These " Lubb" and " "Dup" sounds of the heart can be heard with the help of an instrument called **"Stethoscope.**

ELECTROCARDIOGRAM (ECG)

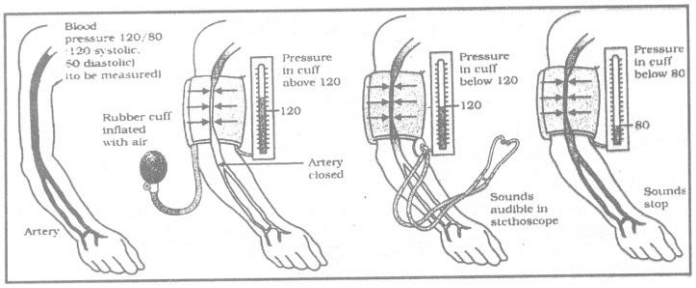
- The functioning of heart can be graphically recorded by an instrument called **electrocardiograph.**
- The heart muscles generate electric currents which bring about heart beats. The electrical changes during heart beat can be graphically recorded by placing electrodes on the chest above the heart and connecting electrodes to a sensitive galvanometer with recording device.
- The graphic recording called and electrocardiogram (ECG).
- It was first of all recorded by **"Waller"**
- **"Einthovan"** is known as the Father of Electro Cardio Graphy.

BLOOD PRESSURE

- Definition : Blood pressure is the pressure exerted by the flowing blood on the wall of blood vessel.
- **Types :** Blood pressure is of two types.
  (i) Systolic blood pressure (ii) Diastolic blood pressure

Characters	**Systolic blood pressure**	**Diastolic blood pressure**

1.	Definition	Blood pressure at the time of maximum contraction of ventricles.	Blood pressure at the time of maximum relaxation of ventricles.
2.	Value	120 mm Hg.	80 mm Hg.



- **Hypertension :-** A persistent increase in blood pressure is called **hypertension** or **high blood pressure.** It is caused by narrowing of arterial lumen and reduced elasticity of arterial walls.
- The instrument by which we can measure B.P. is called **"sphygmomanometer".**
- In man B.P. is measured in the **bronchial artery** of arm.
- Normal B.P. of a healthy person is **120/80 mm Hg.**

REVIEW QUESTIONS

1. What is systole ?
2. What is diastole ?
3. Name the instrument used to measure blood presser.
4. Name the instrument used to hear heart sounds.
5. What is the range of normal systolic and diastolic blood pressure ?
6. Name the instrument, which is inserted in the heart of the patient whose heart does not work normally?

DOUBLE CIRCULATION

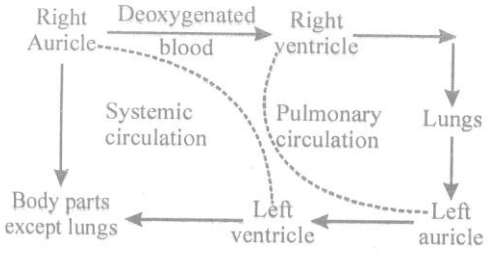
In double circulation, the blood passes twice through the heart to supply once to the body.

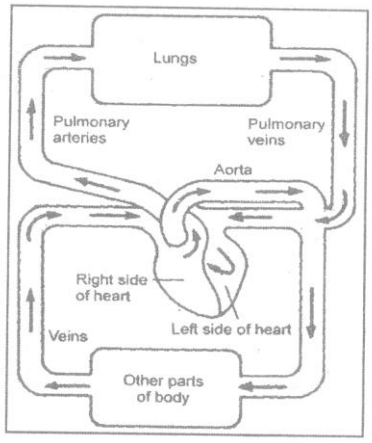
(i) Systemic circulation :

In this, blood completes its circulation form left ventricle to right auricle through the body organs. (systemic organs)

(ii) Pulmonary circulation :

In this, blood completes its circulation from right ventricle to left auricle thorough the lungs.





The right oration of heart is known as **pulmonary heart** and it have **deoxygenated blood.** The left portion of heart is known as **systemic** heart and it have **oxygenated blood.**

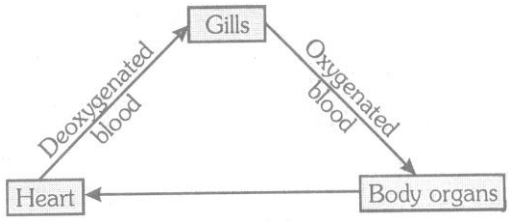
The separation of the right side and left side of the heart is useful to keep oxygenated and de-oxygenated blood from mixing.

Such separation allows a highly efficient supply of oxygen to the body. This is useful is animals that have high energy needs (e.g. birds and mammals) which constantly use energy to maintains their body temperature. In animals that doe not use energy for this purpose, the body temperature depends on the temperature of environment. (e.g. fishes, frogs)

COMPETITION WINDOW

Single circulation :

In this, the blood passes once through the heart to supply once to the body. It is found in fishes which have two-chambered heart one auricle and one ventricle. The heart receives only deoxygenated blood , which is first pumped to the gills for oxygenation and than oxygenated blood is supplied to rest of body parts by various arteries.



LYMPHATIC SYSTEM

It consists of **lymph, lymphatic capillaries, lymph vessels, lymph nodes** and **lymphoid organs (spleen thymus and tonsils).**

Lymph : When blood flows into thin capillaries some amount of plasma filters out of thin capillaries. This fluid is called **interstitial fluid** or **tissue fluid** or **lymph.** Lymph is also called **extracellular fluid .** It is colorless and contains lymphocyte cells. Unlike blood, lymph does not contain red blood corpuscles, but contains less plasma protein. Lymph flows only in one direction, that is from tissues to heart.

The functions of lymph are as under :

- It transports fatty acids and glycerol from small intestine to blood.
- Lymphocytes present in it which destroy harmful pathogens.
- It drains excess tissue fluid from intercellular spaces back into the blood.
- Lymph nodes localize the infection and prevent it from spreading to other body parts.

Blood	Lymph
It forms circulatory system.	It forms lymphatic system .
Red in colour due to presence of hemoglobin	Colorless as hemoglobin is
Contains plasma, RBC, WBC and platelets	Contains plasma land WBC
Flows inside arteries, veins and blood capillaries	Flows inside vessels and lymph capillaries
Its flow is rapid	Its flow is slow
It mainly transports materials from one organ to another in the body	It mainly conveys materials from the tissue cells into the blood.

REVIEW QUESTIONS

1. Define lymph.
2. How does lymph differ from blood?
3. What are the functions of lymph ?
4. How does lymph form ?
5. Describe double circulation.

FILL IN THE BLANKS

(i) Lymph contains……..and ……..

(ii) Lymph is also called…….

(iii) Lymph flows through ……vessels.

CIRCULATORY DISEASES

- **Thrombosis :-** Thrombosis is the formation of a small blood clot or thrombus inside a blood vessel. A blood clot, or any other solid particle floating in the blood stream is known as an embolism.
- **Hardening of the arteries :-** As people get older the elastic and muscle layers of their arteries are gradually replaced by inelastic fibrous tissue. The artery walls become stiff and hard, a condition known as hardening of the arteries, or arteriosclerosis. This condition educes the flow of blood.
- **Atheroma :-** Blood flow along an artery is sometimes slowed or stopped altogether by a layer of fatty substance called cholesterol , stuck to the artery walls. This type of blockage is called an Atheroma. Arteriosclerosis and Atheroma are very dangerous when they occur in the heart or brain.
- **Heart attacks (heart failures) :-** A heart attack, or heart failure, is the sudden slowing or stoppage of the heart beat. A heart attack occurs when a coronary artery is blocked by a thrombosis or Atheroma. Coronary arteries supply heart muscle with food and oxygen. Consequently, when a coronary artery is blocked, a section of heart muscles stops working and eventfully dies. If the whole heart is affected, death is instantaneous.
- **Angina pectoris :-** If one or both of the coronary arteries is partly blocked due to Atheroma, heart muscle is unable to work properly during exercise . This causes pains in the chest known as angina pectoris.
- **Stroke :-** A stroke, or cerebral thrombosis, is a blood clot in the brain. The blood clot suddenly blocks an artery inside the brain causing the region served by this vessel to stop working, and dies. The results of a stroke depend on the area of the area of the brain affected. Muscles may be paralyzed, and speech or memory affected. Death occurs if the brain damage is extensive.
- **Prevention of circulatory diseases :-** Studies have shown that these diseases occur far less often among people :-

1. Who never smoke cigarettes.
2. Who take regular exercise (walking, cycling, cycling, swimming, active sports , etc.)
3. Who never drink alcohol or drink only moderate amounts.
4. Who eat balanced meals.

5. Who have seven or eight hours sleep a night .

IMPORTANT POINTS

- Two chambered heart is present in fishes.
- Three chambered heart is present in amphibians.
- In reptiles, heart is almost four chambered three are two well developed auricles and two less developed unclear ventricles, but in crocodiles all the birds and all the mammals, the heart is fully developed and four chambered.
- First heart transplantation was done by **Dr. Christian Bernard** in the world.
- In India first heart transplantation was done by **Dr. P. Venugopal** on 3rd august 1994. This transplant was done on a 42 years old person named Deviram **(an AIIMS).**
- Spleen, liver and kidneys are called **filter apparatus** of blood.
- A human heart in an average life time of 70 years pumps approximately 40, 00, 00, 000 litres of blood through it.
- If all the human blood vessels (artery, veins & capillaries) are joined end to end then their total length is sufficient to go around the tropic of cancer four times.
- Study of Blood circulatory system including arteries and veins - **Angiology**
- Study of heart - **Cardiology**
- William Harvey – An English physiologist, for the first time discovered that blood flows in closed blood vessels and also known as **Father of Angiology.**
- **Mammalian erythrocytes (RBCs)**
- Have no nucleus – makes the RBCs biconcave, increasing the surface-volume ratio for more oxygen absorption.
- Have no mitochondria.
- Average life of an RBC is about 120 days.
- New born infants have a large number of erythrocytes (6-7 million RBC/microlitre).
- The person living at high altitudes (4200 m and above) RBCs are more in number by about 30 %.
- **Hemophilia :-** Hemophilia is an inherited blood disorder. The blood of a hemophiliac clots very slowly or not at all. There are two reasons why this can happen. Either hemophiliac has too few platelets, or the platelets cannot produce the chemical which causes fibrinogen to change into fibrin.

EXERCISE # 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE W\QUESTIONS

1. The smallest blood vessel in the body is ;

(1) Capillary (2) Artery (3) Vein (4) Vena cava

2. Arteries carry oxygenated blood except :

(1) Pulmonary (2) Cardiac (3) Hepatic (4) Systemic

3. Four chambered heart is found in :

(1) Cobra (2) Tortoise (3) Salamander (4) Crocodile

4. Right atrium of mammalian heart receives blood from :

(1) Sinus venosus (2) Pulmonary veins (3) Precavals (4) Pre-and postcavals

5. Mitral valve in mammals guards the opening between :

(1) Right atrium and right ventricle (2) Left atrium and left ventricle

(3) Right atrium and left ventricle (4) Left atrium and right ventricle

6. Tricuspid valve is present between :

(1) Right atrium and right ventricle (2) The two atria

(3) The two ventricles (4) Left atrium and left ventricle

7. Which of the following has the thickest walls ?

(1) Right ventricle (2) Left ventricle (3) Right auricle (4) Left auricle

8. The pacemaker of heart is :

(1) AV node (2) SA node (3) SV node (4) Tricuspid valve

9. Contraction of right ventricle pumps blood into ;

(1) Dorsal aorta (2) Pulmonary artery (3) Pulmonary vein (4) Coronary artery

10. The impulse of heartbeat originates from :

(1) SA node (2) Vagus nerve (3) AV node (4) Cardiac nerve

11. The heart of a healthy man beats normally per minute :

(1) 85-90 times (2) 80-90 times (3) 70-80 times (4) 60-70 times

12. Systole causes :

(1) Entry of blood into lungs (2) Entry of blood into heart

(3) Exit of blood from heart (4) Exit of blood from ventricles

13. Typical 'lubb-dupp' sounds heard during heartbeat are due to :

(1) Closing of bicuspid and tricuspid valves

(2) Closing of Semilunar valves

(3) Blood under pressure through aorta

(4) Closure of bicuspid-tricuspid valves followed by Semilunar valves

14. The instrument used to hear sound is ;

(1) Electrocardiograph (2) Sphygmomanometer (3) Stethoscope (4) Haemometer

15. In adult man, normal BP is :

(1) 100/80 mm Hg (2) 120/80 mm Hg (3) 100/120 mm Hg (4) 80/120 mm Hg

16. The instrument by which BP of man is determined:

(1) Ultrasound (2) BP meter (3) Stethoscope (4) Sphygmomanometer

17. William Harvey is known for discovery of:

(1) Blood circulation (2) Blood clotting (3) Respiration (4) Digestion

18. Mammals are said to have double circulation. It means:

(1) Blood vessels are paired

(2) There are two types of blood vessels attached to every organ

(3) There are two systems, one from the heart to the lungs and back to the rest of the body

(4) The blood circulates twice through the heart

19. In mammals, veins differs from arteries in having:

(1) Thicker walls (2) Deeply present

(3) Carry blood away from heart (4) Internal valves

20. Oxygenated blood returns from lungs to the heat through :

(1) Coronary vein (2) Pulmonary vein (3) Coronary artery (4) Pulmonary artery

FILL IN THE BLANKS :

1. Mineral which helps in formation of RBC's is …….

2. Blood vessels without muscular walls are…….

3. First heart sound is called ……

4. Fishes have…….chambered heart.

5. Amphibians have ……..chambered heart .

6. ……heart pumps only oxygenated blood.

7. Blood goes through the heart twice during each cycle is called……..

8. High blood pressure is called ……..

9. The force that blood exerts against the wall of a vessel is called. ……

10. Contraction of heart chambers is called……

11. Relaxation of heart chambers is called……

12. ……..help in blood clotting.

13. White colored fluid other that blood which help in transportation is called……..

14. Lymph carries digested and absorbed fat from…….

15. Lymph drains excess fluid from extra cellular space back into ……..

16. Lymph drains into……for the intercellular spaces.

17. Lymphatic capillaries join to from…….

18. Pumping organ in human body is……

19. fluid medium of blood is called……

20. Oxygen is carried by the ……

21. Blood platelets are also called……

22. Covering over heart is called……

23. ……circulation is related with lungs.

24. The series of events which occur during one complete beat of the heart is known as……..cycle.

25. The pressure wave transmitted all through the arterial system is known as…….

MATCH THE COLUMNS :

Column A		Column B	
1.	capillaries	(a)	Carries digested fat
2.	Lymph	(b)	Oxygenated blood
3.	Left ventricle	(c)	Deoxygenated blood
4.	Pulmonary artery	(d)	RBC
5.	Hemoglobin	(e)	Exchange of materials
6.	Birds	(f)	Three chambered heart
7.	Amphibians	(g)	Four chambered heart
8.	Contraction of heart chambers	(h)	Two chambered heart
9.	Relaxation of heart chambers	(i)	Diastole
10.	Second heart sound	(j)	Limb
		(k)	Systole
		(l)	Dupp

TRUE OR FALSE STATEMENTS:

1. Vessels are able to contract and expand.

2. While there is only one type of RBC, there are many white cell types.

3. All animals have a system of vessels and tubes called a circulatory system.

4. Blood is not a tissue because it is a fluid.

5. Arteries are the widest blood vessels.

6. Humans have an open circulatory system.

7. The exchange of nutrients and waste products between the blood and cells occurs with in the arteries.

8. The liquid portion of the blood is called plasma.

9. Living organisms must maintain a constant internal environment.

10. Circulatory system also performs the function of homeostasis.

CIRCULATORY SYSTEM **ANSWER KEY** **EXERCISE # 1**

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	1	4	4	2	1	2	2	2	1
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	3	4	4	3	2	4	1	4	4	2

● **FILL IN THE BLANKS**

1. Iron **2.** Capillaries **3.** Lubb **4.** Two

5. Three **6.** Fishes **7.** double circulation **8.** hypertension

9.	blood pressure	**10.**	Systole	**11.**	diastole	**12.**	Platelets
13.	Lymph	**14.**	Intestine	**15.**	Blood	**16.**	lymphatic capillaries
17.	Lymph vessels	**18.**	Heart	**19.**	Plasma	**20.**	RBC
21.	Thrombocytes	**22.**	Pericardium	**23.**	Pulmonary	**24.**	Cardiac
25.	Pulse						

● **MATCH THE FOLLOWING :**

1-(e), 2-(a), 3-(b), 4-(c), 5-(d), 6-(g), 7-(f), 8-(k), 9-(i), 10-(ℓ)

● **TRUE AND FALSE :**

True of False										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**T**	**T**	**F**

EXERCISER # 2 (FOR SCHOOL / BOARD EXAMS)

VERY SHORT ANSWER TYPE QUESTIONS :

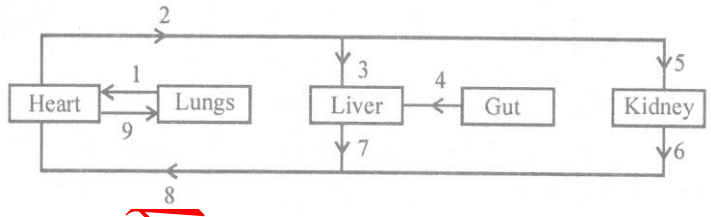
1. Name the blood-receiving chamber of heart.
2. What makes RBC's red ?
3. What is pulmonary circulation ?
4. Which chamber of heart has thickest wall ?
5. Name the larger vines that pour blood into right auricle.
6. Name the largest artery of our body.
7. Which camber of heart receives oxygenated blood from lungs ?
8. Name the valve present between the :
(a) left auricle and left ventricle (b) right auricle and right ventricle
9. What is normal blood pressure ?
10. Name the artery that carry deoxygenated blood and the vein that carry oxygenated blood .
11. What is SA node ?
12. Which instrument can record electrical changes during heart beat ?
13. Name the thickest artery .
14. Name the part of the circulatory system that acts as filter for microorganisms .
15. Name the major circulation present in our body.
16. Give the technical term for the white vascular connective tissue.
17. What is the life span of human RBC's ?
18. Why is blood called river of life ?
19. Give the position of human heart ?
20. Why there is no backflow of blood from ventricles to auricles ?

SHORT ANSWER TYPE QUESTIONS :

1. What are the components of the transport system in human beings ? What are the functions of these compound ?

2. Why is it necessary to separate oxygenated and deoxygenated blood in mammal and birds ?
3. Why is circulation in man known as “double circulation “ ?
4. Writer three differences between lymph and blood.
5. Why is heart beat rate equal to the arterial pulse rate ?
6. Is human circulatory system open or closed ? Why ?
7. In which direction does O_2 diffuse between a capillary and a cell ? Why does it diffuse in that direction ?
8. What is the primary function or RBC’s ?
9. Predict the effect on the heart if blood flow through the coronary artery is restricted or completely blocked .
10. Write two advantages of a closed circulatory system.
11. Point out three differences between artery and vein.
12. Why is SA node also called pacemaker ?
14. What is lymph ? What is its function in human body ?
15. Which part of the human heart is considered as pacemaker ? Why is it so called ?
16. What is the role of blood pressure in circulation ?
17. Give the difference between plasma and lymph .
18. What is the difference between plasma and serum ?
19. Give the difference between pulmonary artery and pulmonary vain.
20. Which blood cell in human blood carries hemoglobin ? What is its average life span ?
21. What is blood pressure ? How it is measured ?
22. Differentiate between systolic and diastolic pressure .
23. Give an account on different types of blood vessels.
24. Give the difference between pulmonary circulation and systemic circulation .

LONG ANSWER TYPE QUESTIONS :

1. What would be the consequences of a deficiency of hemoglobin in our bodies ?
2. Explain how oxygenated blood from left ventricle is sent to all parts of the body and how deoxygenated blood enter into the right auricle.
3. Describe the double circulation in human beings with the help of diagrams. Why double circulation is necessary ?
4. 

(a) Name the blood vessels labeled from 1 to 9.

(b) Which of these carry maximum oxyhaemoglobin ?

(c) Which of these contain highest concentration of amino acids and glucose soon after a meal ?

2. How does blood circulate between lungs and heart in human beings ? Give two functions of lymph in human beiges.
3. (a) Name the blood vessel that brings oxygenate blood to the human heart.

(b) Which chamber of human heart receives deoxygenated blood ?

(c) Describe how deoxygenated blood from this chamber is sent to all parts of the body .

4. Draw a diagram of the vertical section of human heart to show the internal structure. Label any one of the heart chambers and any other five parts.

5. Name the blood vessel that brings deoxygenated blood to the human heart which chamber of the human heart receives deoxygenated blood ? Describe how deoxygenated blood from this chamber is sent to lungs for oxygenation .

6. Name the constituents of blood. Why are white blood corpuscles called 'Soldiers of the body' ?

CIRCULATORY SYSTEM **ANSWER KEY** **EXERCISE # 2**

● **VERY SHORT ANSWER QUESTION :**

1. **Right auricle** **2.** **Hemoglobin**

3. In this, blood completes its circulation from right ventricle to left auricle through the lungs.

4. Left ventricle **5.** Superior and inferior vena cave **6.** Aorta

7. Left auricle **8.** a. bicuspid value b. tricuspid valve **9.** $\frac{120}{80} mmHg$

10. pulmonary artery, pulmonary vein **11.** pacemaker of heart

12. Electrocardiograph **13.** Aorta **14.** Lymph nodes

15. Systemic circulation **16.** Lymph **17.** 120 days

18. because it transports the materials and help in survival of the organism.

19. In thoracic cavity between the lungs

20. due to presence of auriculo ventricular valves.

EXERCISE # 3 (FOR COMPETITIVE EXAMS)

1. How many molecules of C_2 can associate with a molecule of hemoglobin in man ?
(A) One (B) Two (C) Three (D) Four

2. Hemoglobin is having maximum affinity with :-
(A) NH_3 (B) O_2 (C) CO (D) CO_2

3. In which from CO_2 is carried by blood :-
(A) Sodium bicarbonate (B) Sodium carbonate
(C) Potassium carbonate (D) Magnesium carbonate

4. Amount of oxygen which is transported by one gram of hemoglobin is :-
(A) 20 ml (B) 13.4 ml (C) 1.34 ml (D) None of these

5. Mountain sickness result due to :-
(A) Anemic hypoxia (B) Arterial hypoxia
(C) Lack of sufficient Hb (D) Lack of sufficient RBCs

6. Heart with single circulation is found in :-
(A) Mammals land birds (B) Reptiles
(C) Fishes and amphibians (D) Fishes only

7. The blood pressure is measured by :-
(A) Electrocardiogram (ECG) (B) Stethoscope

(C) Sphygmomanometer (D) Pulse rate

8. Chemical basis of action of nerve on heart beat was discovered by :-

(A) Otto Loewi (B) Pavlov (C) Harvey (D) Landsteiner

9. Erythrocytes of adult rabbit and other mammals are formed in :-

(A) Kidney (B) Liver (C) Spleen (D) Bone marrow

10. The blood vascular of mammals is known as double vascular system because :-

(A) A group of veins carry oxygenated blood and other group conducts deoxygenated blood

(B) Oxygenated blood funs from heart to different organs by one set of veins, while deoxygenated blood runs from heart to lungs by another set

(C) Two different blood runs never meet

(D) All of these

11. Which of the following is helpful for erythropoietin :-

(A) Fe^{++} (B) Mg^{++} (C) Ca^{++} (D) Cu^{++}

12. Heart beat originates from :-

(A) Left atrium (B) Right ventricle (C) Pacemaker (D) Cadiac muscles

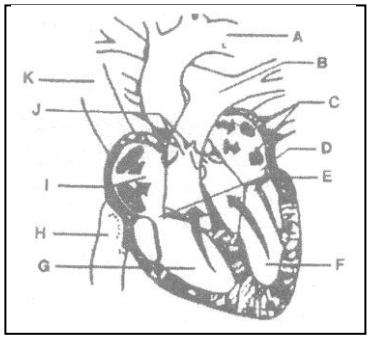
13. Match the different leucocytes given under Column I with their given under Column II. Choose the answer that gives the correct combination of alphabets of two columns :-

	Column –I (Leucocytes)		Column – II (Functions)
A	Eosinophils	p	Phagocytosis
B	Neutrophil	q	Produce antibodies
C	Lymphocytes	r	Role in allergic response
D	Monocytes	s	Prevents clotting
		t	Differentiate in macrophage

(A) A = t, B = p, C = q, D = f (B) A = r, B = p, C = q, D = t

(C) A = q, B = r, C = s, D = t (D) A = p, B = q, C = r, D = s

14. In the diagram of the vertical section of human heart given below certain parts have been indicated by alphabets. Choose the answer in which these alphabets have been correctly matched with the parts they indicate :-



(A) A = Aorta, B = Pulmonary vein, C = Pulmonary arteries, D = Left ventricle , E = Semi lunar valves, F = Left auricle, G = Right auricle, H = Superior vena cava, I = Right ventricle J = Tricuspid valves, K = Inferior vena cava

(B) A = Aorta, B = Pulmonary artery, C = Pulmonary veins, D = Left auricles, E = Tricuspid and Mitral valves, F = Left ventricles, G = Right ventricle, H = Inferior vena cava, I = Right auricle, J = Semi lunar valves K = Superior vena cava

(C) A = Aorta, B = Superior vena cava, C = Inferior vena cava, D = Right ventricles, E = Tricuspid and Mitral valves, F = Right auricle, G = Left auricles, H = Pulmonary vein, I = Left ventricle, J = Semi lunar valves, K = Pulmonary artery

(D) A = Aorta, B = Superior vena cave, C = Inferior vena cava, D = Left ventricles, E = Semi lunar valves, F = Left auricle, G = Right auricle, H = Pulmonary artery , I = Right ventricle, J = Tricuspid valves, K = Pulmonary vein

15. Cardiac cycle in man takes about :-

(A) 0.5 seconds (B) 1.0 seconds (C) 1.2 seconds (D) 0.8 seconds

16. Where are red blood cells formed ?

(A) Spleen (B) Liver

(C) Red bone marrow of short bones (D) Thyroid

17. The heart pumps only deoxygenated blood :-

(A) shark (B) frog (C) whale (D) crocodile

18. A sudden increase in the number of white blood cells in the blood is a sign of :-

(A) deficiency disease (B) better health (C) bacterial disease, infection (D) mental tension

19. Clotting of blood is achieved with the help of the following :-

(A) vitamin K, fibrinogen and calcium is ions

(B) prothrombin, fibrinogen and Thromboplastin

(C) vitamin K, fibrinogen, prothrombin and calcium ions

(D) vitamin B, fibrinogen, thrombin and potassium ions

20. The pH value of blood varies between :-

(A) 6.00-7.00 (B) 7.00-8.00 (C) 7.30-7.45 (D) 7.50-8.00

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10
Ans.	D	C	A	C	B	D	C	A	D	D
Q.No	11	12	13	14	15	16	17	18	19	20
Ans.	A	C	B	B	D	C	A	C	D	C

TRANSPORTATION IN PLANT

TRANSPORTATION : The transport of water, food, minerals , hormones and other substances from one part to another part inside the body of an organism is known as transpiration.

- Plants take in CO_2, photosynthesize and store energy in leaves. Besides CO_2 plants need other raw materials for growth. These raw materials like nitrogen, phosphorous etc., they get from the soil by the process of absorption by roots. These raw materials need to be transported to each and every part of the plant, mainly to leaves.
- If the distance between roots and leaves is small, then these raw materials can reach to the leaves easily by the process of diffusion.

COMPETITION WINDOW

Diffusion : The movement of molecules is in the direction of concentration gradient i.e., from a region of higher concentration to that of lower concentration.

In unicellular organisms like **Chlamydomona** transport of material mainly takes place by the process of diffusion. **Osmosis :** Osmosis is the flow of water molecules from the region of higher potential to the region of lower water potential through a semi permeable membrane. The osmotic entry of water into a cell, organ or system is called **endosmosis** while the osmotic withdrawal of water from the same is described as **Exosmosis.**

- In desistance between roots and leaves is more, then proper system of transpiration is required.
- In points, transportation is done by a specialized vascular system which is present in the root, stem and leaves.
- Vascular system is made up of two types of vascular tissues.

  1. Xylem 2. Phloem
- In plants the transport of materials can by divided into two parts.

(A) Transport of water and minerals through xylem tissue.

(B) Transportation of food and other substances thorough phloem tissue.

(A) TRANSPORT OF WATER AND MINERALS [ASCENT OF SAP].

- The plants require water and minerals for making food and performing other functions.
- The upward movement of water and minerals from the roots to various parts of the plant is known as **ascent of sap.**
- Ascent of sap is carried out by xylem tissue which consists of :-

  1. Xylem vessels 2. Xylem Trachieds

  3. Xylem fibre 4. Xylem parenchyma.
- In flowering plants xylem vessels and tracheas conduct water and minerals upwardly.

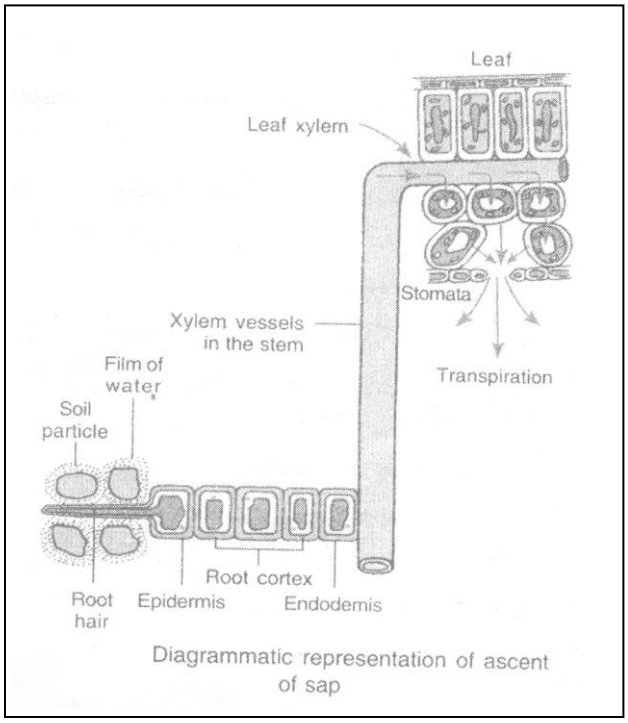
  In non flowering plants, Trachieds are the only conducting cells.

MECHANISM OF ASCENT OF SAP :-

Absorption of water and minerals :- The water and minerals are absorbed by land plants from the soil where it is present in the form of soil solution. The main water and minerals absorbing organs are root hairs. The water is absorbed actively due to the water potential difference between soil solution and root hairs.

Water molecules moves from higher water potential to lower water potential and then migrate from cell to cell passing from epidermis to cortex, from cortex to endodermis and from endodermis to xylem vessels and Trachieds. This is called lateral transportation.

The mineral slats move from higher partial pressure to lower partial pressure along the concentration gradient by passive absorption or against the concentration gradient by active absorption. Finally the water and minerals reach the vessels and Trachieds of xylem from where they move upward by the process called ascent of sap.



Diagrammatic representation of ascent of sap

Many theories have been put forward to explain the upward movement of water by distinguished scientists. Two among them are important :

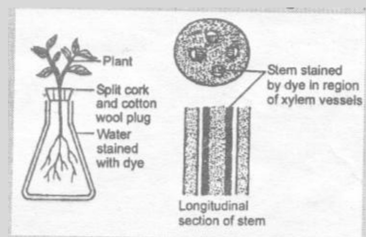
(a) Root pressure theory :- According to root pressure theory, the roots absorbs water and exert a pressure, the root pressure, which pushes the water upward. The root pressure develops in the treachery element of xylem (i.e., Trachieds and vessels) as a result of metabolic activities of roots. The root pressure theory is applicable in small herbs but not in tall trees. The effect of root pressure in transport of water is more important at night.

COMPETITION WINDOW

Experiment to show the Movement of water and minerals in the xylem.

Take a cut shoot of balsam plant and dip it in a dilute red colored dye, like eosin or safranin. Leave it for sometime. Then take sections from different parts of the stem starting from the tip region.

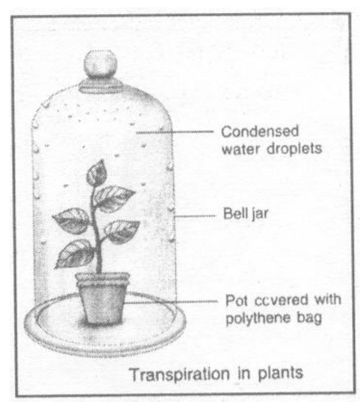
You will observe red color dye in the region of xylem .



(b) Transpiration pull and cohesion-tension theory:-

Transpiration pull and cohesion-tension theory explains the upward movement of water in tall trees. According to this theory, the main force responsible for upward movement of water is transpiration pull generated in the leaves which pulls the water column filled in the xylem Trachieds and vessels.

During day period **Cohesion force** and **Transpiration pull** help in the upward movement of sap from roots to leaves.



Transpiration in plants

- **TRANSPIRATION :** The loss of water in the form of water vapours from the aerial part of a plant is known as transpiration.
- Transpiration mainly occurs through stomata, (about 80 % to 90 %) but it may also occur through **cuticle (9 %) an lenticels (1 %).**

FUNCTION OF TRANSPIRATION :

- It helps in absorption and upward movement of water.
- It helps in temperature regulation. The plants are protected from the burring due to transpiration. Evaporation of water produces cooling effect.

Do you know ?

The minimum transportation is found in succulent xerophytes and no transportation in submerged hydrophytes.

Photometer is an instrument used to measure transpiration.

Guttation : The loss of water from the uninjured part or leaves of the plant in the form of water droplets is called as guttation Exuded liquid of guttation is not pure water. The process of guttation takes place due to the **root pressures.**

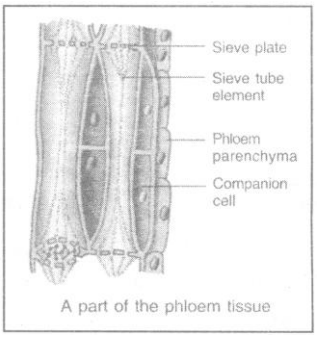
Bleeding :- Fast flowing of liquid from the injured or cutting parts of the plants is called bleeding or exudation.

Wilting :- Drooping of soft part of the plants due to loss of turgidity in their cells is called wilting. Wilting is caused due to high rate of transpiration during mid-day or deficiency water in soil and also in prolonged drought conditions.

REVIEW QUESTIONS

1. Name the conducting tissue in plant.
2. Name the components of xylem tissue.
3. Name the tissue which transports water and minerals in plants.
4. Sate the term used for loss of water in vapor form from aerial part of the plants.
5. Define the term transpiration.
6. Define the ascent of sap.

(B) TRANSPORTATION OF FOOD AND OTHER SUBSTANCES



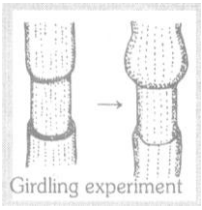
A part of the phloem tissue

- Sugar, amino acid and other substances are translocated from site of synthesize of site of storage through the phloem.
- Transport of food from leaves to different parts of plant is termed as translocation.
- Translocation may be in upward or downward direction depending on the need of the plant.
- Translocation of food takes place in the sieve tubes with the help of adjacent companion cells by phloem tissue.
- Phloem tissues consist of four components.
  1. Sieve tubes
  2. Companion cells
  3. Phloem parenchyma
  4. Phloem fibre

MECHANISM OF TRANSLOCATION

- Translocation takes place through energy in the form of ATP.
- Sugar (sucrose) made in leaves and loaded into the sieve tubes phloem by using energy from ATP.
- It increases the osmotic pressure of the sieve tubs.
- Water now enters into sieve tubes containing sugar by the process of osmosis.
- Soluble material is then transferred from phloem tissue to other tissues which have less pressure than in the phloem.
- Thus, according to plants requirement, the material is translocated from higher osmotic pressure to lower osmotic pressure areas.

COMPETITION WINDOW


Girdling experiment

Girdling experiments are also used to demonstrate that the transportation of sugar takes place through phloem tissue. Girdling of stem removes phloem tissue. If a stem is girdled, the downward and upward movement of food material gets blocked. So the trunk portion shows swelling above the girdle area due to accumulation of food material.

Do you known ?

Various theories have been proposed to explain translocation. Most widely accepted one is the **mass flow hypothesis** proposed by **Munch** in 1931.

In this way food material is translocated between the **source** of food material to the **sink** which is the site of utilization. This is **downward translocation of food.**

In spring, sugar stored in root or stem tissue would be transported to the buds which need energy t grow.

REVIEW QUESTIONS

1. Name the components of phloem tissue.
2. Which plant tissue is associated with translocation ?
3. Sate the term used for transport of food from leaves to other parts of the plant.
4. Which process in a plant is accomplished by utilizing energy from ATP. Transport of water or transport of food ?
5. Define the term translocation.

EXERCISE # 1 (FOR SCHOOL EXAMS)

OBJECTIVE TYPE QUESTION

1. Water will be absorbed by root hair when :

(A) Concentration of solutes in the cell sap is high

(B) Plant is rapidly respiring

(C) They are separated from soil by permeable membrane

(D) Concentration of salts in the soil is high

2. Which one of the following is connected with transport of water in plants ?

(A) Phloem (B) Xylem (C) Epidermis (D) Cambium

3. If the cut end of a tree's put in eosin solution :-

(A) Leaves remain fresh but ascent of sap stops

(B) Phloem gets colored because of ascent of sap

(C) Xylem elements get stained showing ascent of sap through them

(D) Ascent of sap stops

4. The principal pathways by which water is translocated in angiosperms is :-

(A) Xylem vessel system (B) Xylem and phloem

(C) Sieve tubes members of phloem (D) Sieve cells of phloem

5. The carbohydrate synthesized in the leaves are transported through sieve tubes most commonly in the form of :-

(A) Glucose (B) Triose (C) Sucrose (D) Soluble starch

6. Which of the following contributes most to transport of water form the ground to the leaves of a tall tree ?

(A) Breakdown of ATP (B) Cohesion of water and transpiration pull

(C) Root pressure (D) Capillary rise of water in xylem

7. Due to low atmospheric pressure the rate of transpiration will :-

(A) Increase (B) Decrease lowly (C) Decrease rapidly (D) Remain unaffected

8. The transpiration in plants will be lowest :-

(A) When there is high humidity in the atmosphere (B) There is excess of water in the cell

(C) Environmental conditions are very dry (D) High wind velocity

9. Rate of transpiration in a dorsiventral leaf is :-

(A) Greater at the upper surface (B) Greater at the lower surface

(C) Equal at both the surfaces (D) None of the above

10. Which of the following is not a function of transpiration ?

(A) Excretion of minerals (B) Cooling of leaves

(C) Uptake of water (D) Uptake of minerals

FILL IN THE BLANKS :

1. In plants food is transported through ……
2. In plants water is transported through ……
3. Water enter into the root hair from soil by……
4. Transport of water and minerals is called……
5. The water moving upward forms a column, which is maintained up to a certain height due to ……
6. Vascular tissues in plants are……,…..and……..
7. During day period……and ……help in the upward movement of sap from roots to leaves .
8. Transpiration mainly occurs through ……..
9. Guttation takes place through……
10. Translocation takes place through energy in the form of ……
11. Water enter into sieve tube containing sugar by the process of ……..
12. According to plants requirements, sugar is translocated from……osmotic pressure to ……osmotic pressure areas.

MATCH THE COLUMNS :

Match the items of Column A with items of Column B.

Column A		Column B	
1.	Living part of xylem	(a)	Sieve tube
2.	Dead part of phloem	(b)	Trachieds

3.	Living part of phloem	(c)	companion cell
4.	Dead part of xylem	(d)	Vessels
		(e)	Phloem fibre
		(f)	xylem parenchyma
		(g)	Phloem parenchyma
		(h)	xylem fibre

VERY SHORT ANSWER TYPE QUESTIONS :

1. What is transpiration ?
2. What is translocation ?
3. Name a plant which does not have a transport system.
4. What is the role of stomata in transpiration ?
5. Name the tissues responsible for the translocation of food in plants.
6. Name the process by which plants lose water.
7. Name that component of the vascular bundle which transports food from the leaves to different parts of a plant.
8. What is the transporting medium in higher plants ?
9. What is the upper movement of water and minerals called ?
10. Which process in plants creates suction force to help water column rise in plants ?

SHORT ANSWER TYPE QUESTIONS :

1. Write about the opening and closing of stomata.
2. Why is transportation of materials necessary ?
3. Describe transport of the following materials in plants.
4. Write the main difference between xylem and phloem.
5. How does transpiration help in ascent of sap ?

LONG ANSWER TYPE QUESTIONS :

1. Draw a sieve tube and label the various parts. Name the dead elements of the phloem.
2. Leaves of healthy potted plant were coated with Vaseline to block the stomata. Will this plant remain healthy for long ? State three reasons for your answer.
3. What is translocation ? Why is it essential for plants ? Where in plants are following synthesized.

   (a) sugar (b) Hormones

[NCERT QUESTIONS :]

1. What are the components of the transport system in highly organized plants ?

2. How is food transported in plats ?

3. How are water and minerals transported in plants ?

4. What are the differences between the transport of materials in xylem and phloem ?

EXERCISE # 2 (FOR COMPETITIVE EXAMS)

1. The rate of transpiration during :-

(A) Cactus (B) Lily (C) Hydrilla (D) All of these

2. Maximum transpiration occurs in :

(A) Mesophytic plants (B) Xerophytic plants

(C) Hydrophytic plats (D) None of these

3. Transpiration stream is continued in plants due to :

(A) Chlorophyll (B) Phloem (C) Sieve tube (D) Stomata

4. Transpiration in plants takes place through :

(A) Stomata (B) Cuticle (C) Lenticels (D) All of these

5. We feel cool near a banyan tree during summer because of :

(A) Transpiration (B) Photosynthesis (C) Green leaves (D) All of these

6. Water movement against gravity is due to :

(A) Osmosis (B) Respiration (C) Photosynthesis (D) Transpiration

7. The instrument used to measure transpiration is :

(A) Barometer (B) Pedometer (C) Thermometer (D) Potometer

8. The factors which affects the rate of transpiration is :

(A) Speed of wind (B) Temperature

(C) Surface area of leaf (D) All of these

9. The rate of transpiration increases when :

(A) Soil is dry and air is humid (B) Soil is wet and air is dry

(C) Soil is we5 and air is humid (D) Soil is dry and air is dry

10. Role of transpiration is :

(A) Conduction of water, mineral, salts (B) Cooling effect

(C) Maintenance of cell turgidity (D) All of these

TRANSPORAION IN PLANTS **ANSWER KEY** **EXERCISE # 1**

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	C	A	C	B	A	A	B	A

● **FILL IN THE BLANKS :**

1. Phloem **2.** xylem **3.** diffusion **4.** scent of sap **5.** root pressure

6. xylem, phloem **7.** cohesion force, transpiration pull **8.** Stomata

9. Hydathodes **10.** ATP **11.** Osmosis **12.** Higher, lower

● **MATCH THE FOLLOWING**

1-(f), 2-(e), 3-(a, c, g), 4-(b, d, h),

● **VERY SHORT ANSWER QUESTION :**

1. Process by which plants lose water in vapor form into the surrounding air.

2. Process of transport of food from leaves to other parts of the plant body through phloem.

3. Chlamydomona

4. Stomata takes out the extra amount of water in the form of water vapours. The rate of transpiration is directly proportional to the number of stomata.

5. Phloem **6.** Transpiration **7.** Phloem tissue **8.** Water **9.** Ascent of sap **10.** Transpiration

EXERCISE # 2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	D	D	A	D	D	D	B	D

EXCRETION IN ANIMALS

☐ **EXCRETION :**

The biological process of removal of harmful nitrogenous wastes from the body is called **excretion.** The waste products in animals include :

(i) Nitrogenous compounds like ammonia, urea and uric acid.

(ii) Carbon dioxide and water.

(iii) Excess salts and vitamins.

(iv) Unwanted medicines

COMPETITION WINDOW

Ammonotelic organisms are those which excrete ammonia. e.g. most aquatic animals.

Ureotelic organisms are those which excrete urea. e. g. sharks, frogs, mammals.

Uricotelic organisms are those which excrete uric acid e.g. birds, insects, lend snails, many reptiles

Excretory Organs / Structures in Animals :-

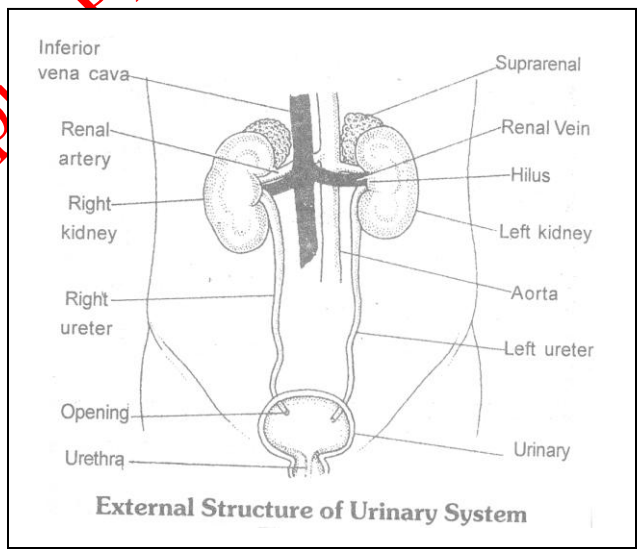
Animals	Amoeba	Hydra	Flatworm	Earthworm	Insects e.g. cockroach	All chordates
Excretory Structures	Cellular surface	Body Surface	Protonephridia (flame cells)	Nephridia	Malpighian tubules	Kidneys
Waste products	CO_2 and ammonia	CO_2 and Ammonia	Mainly ammonia	Ammonia and urea	Uric acid	Urea

REVIEW QUESTIONS :

1. Name the organ system responsible for exception.
2. Name the excretory is structure of Amoeba.
3. Name the excretory organs of vertebrates.
4. Name the major excretory product of human beings.
5. What is excretory organs of cockroach?

☐ **HUMAN EXCRETORY SYSTEM :**

- Human excretory system consists of :-
  - A pair of kidneys
  - A pair of Ureter
  - Urinary bladder
  - Urethra



External Structure of Urinary System

☐ **KIDNEY :**

- The main excretory organ of our body are **Kidneys.**

Colour	-	Dark red
Shape	-	Bean shaped
Weight	-	125-170 gms.
Size	-	10 cm length, 5 cm breadth, 3 cm thickness.
Position	-	Located laterally either sides of **vertebral column.**

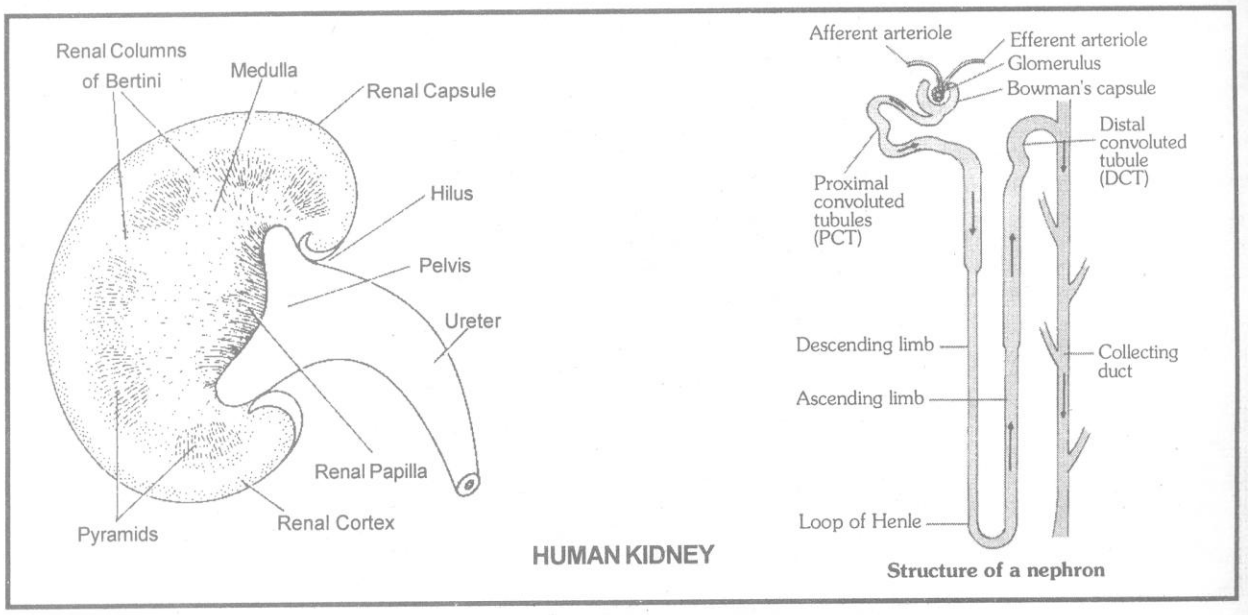
External Structure :-

- Each kidney is surrounded and covered by a tough, fibrous, capsule of connective tissue. This capsule is called **renal capsule .**
- Lateral surfaces of kidney are convex while medial surfaces are concave.
- On the inner border of each kidney is a depression called **hilum / hilus.**

★ **The human kidney are not located at similar positions due to presence of liver above right kidney so that the right kidney get slightly lower position.**

☐ **Internal Structure :**

- The internal structure of kidneys can be divided into two parts.
- Its outer part is called **cortex** and inner part is called **medulla.**
- Nephron is the structural and functional unit of excretion.
- A nephron consists of a long coiled tubule differentiated into **proximal nephron, loop of Henle** and **distal nephron.** The latter opens into the collecting tubule.
- At the proximal end of the nephron, a double walled cup shaped structure is present called **Bowman's capsule.**
- It consists of network of capillaries called **Glomerulus**.



- One end of the **Glomerulus** is attached to renal artery and the other end to the renal vein.
- In the Glomerulus, blood comes in through afferent arteriole and blood is drained out through efferent arteriole.
- Glomerulus and Bowman capsule are collectively called **Malpighian body** or **renal corpuscle.**
- The function of Glomerulus is to filter the blood passing through it. This process is called **ultra filtration.**

COMPETITION WINDOW

Structure of Nephron :-

Nephron is the structural and functional unit of kidney, which is about 3 cm long and 20-30 μm in diameter. Each kidney has about one million Nephrons in humans.

A nephron can be divided into three rations :

(I) Proximal nephron (Bowman's capsule + Proximal convoluted tubule)

(II) Loop of Henle (Ascending + Descending limb)

(III) Distal nephron (Distal convoluted tubule which opens into collecting duct)

(I) **Proximal nephron :** Nephron tubule is closed at its proximal (starting) end but its distal end is open and continues into the loop of Henle. At the proximal or closed end the nephron is expanded and curved inwardly to from a double walled cup shaped **Bowman's capsule.** Within the Bowman's capsule a network or tuft of capillaries is present, it called **Glomerulus.** Diameter of afferent arteriole is greater than efferent arteriole.

Malpighian corpuscle : Glomerulus and its surrounding Bowman's capsule together form this specialized structure.

(I) **Loop of Henle :** It stars after the proximal convoluted tubule, It ends before the distal convoluted tubule. This hairpin like loop has a descending limb, followed by an ascending limb.

(III) **Distal nephron :** The ascending limb of Henle's loop merge into distal **convoluted tubule.** The distal convoluted tubules of number of adjacent Nephrons open into a common **collecting duct** or tubule.

☐ **Ureter :**

- The collecting ducts open into the **Ureter.**
- Each Ureter originate from interior part of kidney.
- The anterior part of the Ureter is broad, like a funnel and called **pelvis** and its posterior part is in the form of long tubule.

☐ **Urinary Bladder :**

- Each Ureter opens into the **urinary bladder.**
- The structure of urinary bladder is muscular sac like and pear shaped.
- Its wall is flexible, it collect urine when necessary by the contraction of muscles, the urine is excreted through urethra.

☐ **Urethra :**

- It is a muscular and tubular structure, which extends fro the urinary bladder to the outside. It carries the urine to the outside .

☐ **Micturition :**

- Micturition is the term used for urination. (Passing out of urine)

☐ **Functions of Kidney :**

- Regulation of water and electrolyte balance. (Osmoregulation)
- Regulation of acid base balance.
- Regulation of blood pressure.
- Excretion of metabolic waste and foreign chemicals.

Physiology of Excretion :-

- The impure blood enters to each kidney through **renal artery.**
- The **afferent arterioles** which is branch of renal artery provides blood to the Glomerulus.
- Glomerulus is a group of blood capillaries formed by division of afferent arterioles located in Bowman capsule.

- The arterioles which carry blood away from Glomerulus are called **efferent arterioles**.
- The radius of afferent arterioles is greater than that of efferent arterioles so the pressure in Glomerulus increases. Which is necessary for ultra filtration.
- Due to the blood pressure, from the blood of afferent arteriole, water, glucose, urea, uric acid and some salt filter in Bowman capsule through **ultra-filtration.**
- It also contains glucose, amino acid and some useful salts along with filtrate.
- This liquid from the Bowman capsule moves through the glandular part of the nephron.
- From where glucose, useful salt and some part of water is **reabsorbed.** The amount of water reabsorbed depends on how much excess water there is in the body and on how much of dissolved water there is to be excreted.
- The remaining liquid now contain only waster material is called **urine.**
- The urine from the nephron s collected in urinary bladder through Ureter.
- Urine is stored in the urinary bladder until the pressure of the expanded bladder leads to the urge to pass it out through the urethra.
- By the contraction of muscles of urinary ladder, the urine passes out of the body when necessary.
- All the system of our body keep the internal environment stable even on the changing conditions of external environment.
- This activity is called **homeostatic** activity.
- Usually the homeostatic activities are performed by excretory organ.
- They not only excrete out salts and nitrogenous waste products but also perform important role of water balance.

◆ **The processes of maintaining the right amount of water and proper ionic balance in the body is called osmo regulation.**

◆ **Urea is always formed in liver through Ornithine cycle.**

COMPETITION WINDOW

Urine Formation (Uropoiesis) :-

It involves three processes : glomerular filtration (ultra filtration), tubular reabsorption and tubular secretion .

(i) Ultrafiltration :

Walls of Glomerulus and Bowman's capsule are thin and semi permeable membrane. In the Glomerulus there are many minute pores are present.

Afferent arteriole is wider and releases the blood into Glomerulus, whereas efferent arteriole is narrow. Thus, there is development of high blood pressure.

Due to this pressure, separation of small, selective molecules ions form the large molecules in the blood occurs and called ultra filtration. Fluid which is filtered out from the blood is called as Glomerulus filtrate / capsular / filtrate / ultra filtrate.

(ii) Tubular Reabsorption :-

The ultra filtrate contains salts, glucose, amino acids, urea, urea acid and large amount of water.

Glucose , salts, amino acids and water are reabsorbed by various parts of nephron and finally they enter into the surroundings blood capillaries.

(iii) Tubular secretion :-

It is removal of wastes from the surrounding blood capillaries into the glomerular filtrate.

Glomerular filtrate entering collecting duct is called **urine.** Urine composition is different from filtrate by the loss as well as gain of many substance during the course of Nephrons.

☐ **Chemical composition of urine :**

- Urine is slightly acidic liquid, light yellow in colour.
- The healthy human being has 95 % water, 5 % urea, uric acid and salts of phosphoric acid .
- A young and healthy person excretes 1.5-1.8 litres urine per day.
- This quantity may increase due to intake of tea, coffee, wine etc.

REVIEW QUESTIONS

1. Name three nitrogenous wastes produced during the metabolism.
2. Name various parts of excretory system of man.
3. What are the functional units of human kidneys ?
4. Give the technical term for the process of expelling of urine.
5. Where is the urine carried through Ureter ?
6. What is urethra ?
7. Name the functional and structural unit to kidney ?
8. Name the U-shaped tubule of nephron ?
9. Why is urine yellow in colour ?

☐ **Regulation of excretion :**

Following two hormones regulate the functions of kidney :

(i) **Anti Diuretic Hormone (ADH) . Vasopressin**

It is secreted by pituitary gland, it promotes the reabsorption of water through Nephrons. (DCT part)

(ii) **Aldosterone :-** It is secreted by Adrenal gland.

Aldosterone promotes the reabsorption of salts (Na^+) in the nephron (DCT part) i.e. it checks the loss of Na^+ ions through urine.

☐ **Role of lungs in excretion**

Human lungs eliminate around 18L of CO_2 per hour and about 400ml of water per day in normal resting condition . Water loss via the lungs is small in hot humid climate and large in cold dry climates.

☐ **Role of skin in excretion :**

Human possess two types of glands :

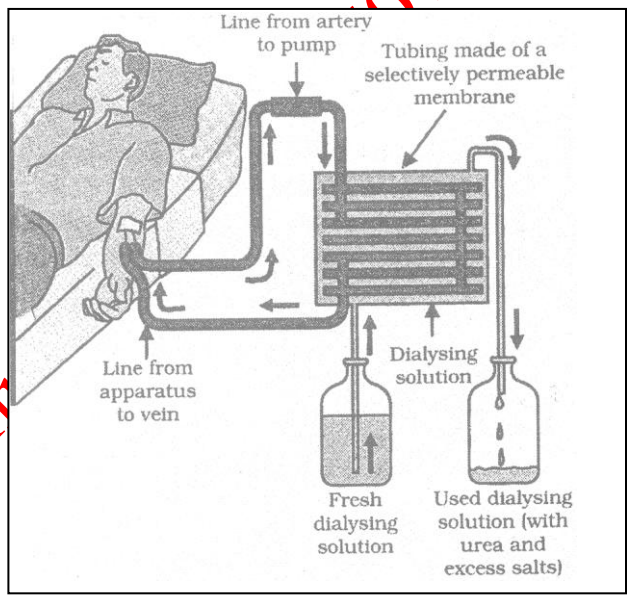
(1) Sweat glands : These excrete sweat, Sweat contain 99.5 % , Water , NaCl, Lactic acid , Urea, Amino acid and glucose.

(2) Sebaceous glands : These secrete sebum which contain waxes, sterols, other hydrocarbons and fatty acids.

☐ **Role of liver in excretion :**

Liver is the main site for elimination of cholesterol, bile pigments (billirubin & biliverdin) , inactivated products of steroid hormones, some vitamins and many drugs. Bile carries theses materials to the intestine from where they are excreted with the faeces.

ARTIFICIAL KIDNEY OR HAEMODIALYSIS



Kidney dialysis also known as **haemodialysis or renal dialysis,** is a medical treatment used to remove nitrogenous waste materials from the blood of patients lacking kidney function or kidney failure, due to infections, injury or restricted blood flow to kidneys. In this procedure, the blood is circulated through a machine known as artificial kidney or dialyses that removes wastes and excess fluid from the bloodstream.

The blood from an artery is pumped through a dialyses or artificial kidney, where it flows through a semi permeable membrane which are made up of cellophane tubes. The cellophane tubes remain suspended in a tank with dialyzing fluid which has same osmotic pressure as blood and has the same composition as that of blood plasma but it lacks nitrogenous wasters. When the blood of the patient is passed through the cellophane tubes, the dialysis fluid passing on the other side of the membrane removes unwanted elements in the blood diffusion, the blood is then retightened to body through a vein. Main difference of kidney & dialysis is that there is on reabsorption in dialysis. In kidney, initial filtrate is about 180L daily but actual excretion is only a litre or two a day

REVIEW QUESTIONS

1. Give the technical term for the dialyzing machine .

2. Name two functions of human kidneys.

3. Define uremia.

4. What is the main aim of excretion ?

5. Which part of kidney acts as dialysis long ?

COMPETITION WINDOW

☞ **Anuria :-** No production of urine

- **Polyuria ;-** Excess production of urine. More urine formation takes place due to less secretion of ADH. Due to less secretion of ADH, the amount of water increases in the urine . So, the patient feels thirsty again and again. This disease is called **Diabetes inspidus.**
- **Glyconsuria :-** Excretion of glucose through urine. This sign is present in **Diabetes mellitus.** This disease is caused mainly due to less secretion of insulin.
- **Uremia :-** Excess of urea in blood is termed as uremia.
- **Calculi and cast :-** It is also termed as **Kidney-stone.** Due to deposition of Calcium-oxalate in the kidney, stone is formed. Sometimes, calcium – phosphate and calcium-sulphate are also found. These are insoluble salts. Normally, these are not excreted by the urine.
- **Haematuria :-** Excretion of blood through urine.
- **Diuresis ;-** The process of excess formation of urine in the kidney's is termed as Diuresis.
- The urine on standing gives a pungent smell. It is due to conversion of urea into ammonia by bacteria
- The volume of urine produced per day will increase on a cold day, due to ↓ ADH secretion.
- If one kidney is removed, the remaining one enlarges and performs function of both kidneys.
- **Renal failure :** It is a syndrome characterized by renal dysfunction, oliguria, anuria, sudden rise in metabolic waste products like urea & creatinine in blood (uremia). It is either of acute (sudden onset) or chronic (slow onset) nature.
- **Diabetic nephropathy ;** It is a complication due to diabetes mellitus where the kidney progressively gets damaged leading to death ultimately due to renal failure.
- Pale yellow colour or urine is due to the **Urochrome** pigment. It is formed in the blood due to the reduction of Hemoglobin. So in the body of a healthy animal, Urochrome is found in very fewer amounts.

EXERCISE CHECK YOUR GREASE

OBJECTIVE TYPE QUESTIONS

1. Excretion is :-
(1) Removal of substances not required by body
(2) Removal of useless substances and substances resent in excess
(3) Formation of substances having some role in body
(4) All the above

2. In mammals, the urinary bladder opens into :-
(1) Uterus (2) Urethra (3) Vestibule (4) Ureter

3. Malpighian corpuscles occur in :-
(1) Medulla (2) Cortex (3) Pelvis (4) Pyramid

4. Loops of Henle occurs in ;-

(1) Cortex (2) Medulla (3) Pelvis (4) Ureter

5. "Homeostasis" term was proposed by :-

(1) Claude Bernard (2) Walter Cannon (3) Marcello Malpighi (4) Henle

6. Bile pigments are formed in :-

(1) Liver (2) Spleen (3) Every body cells (4) 1 & 2 both

7. Excretory materials are formed in :-

(1) Kidney (2) Rectum (3) Liver (4) Every cell in body

8. Diameter of renal afferent vessel is :-

(1) Same as that of efferent (2) Smaller than that of efferent

(3) Larger than that of efferent (4) There is no efferent vessel

9. Nitrogenous waster products are eliminated mainly as :-

(1) Urea in tadpole & ammonia in adult frog (2) Ammonia in tadpole and urea in adult frog

(3) Urea in both tadpole & adult frog. (4) Urea in tadpole and uric acid in adult frog.

10. Which blood vessel contains the least amount of urea ?

(1) Hepatic vein (2) Renal vein (3) Hepatic portal vein (4) Renal artery

11. Reabsorption of useful substances from glomerular filtrate occurs in :-

(1) Collecting tube (2) Loop of Henle

(3) Proximal convoluted tubule (4) Distal convoluted tubule

12. Which one is Uricotelic ?

(1) Frog and toads (2) Lizards and birds

(3) Cattle, monkey and man (4) Molluscs

13. What will happen if one kidney is removed from the body of a human being ?

(1) Death due to poisoning

(2) Uremia and death

(3) Stoppage of urination

(4) Stoppage of urination

14. In cockroach, the excretory product is :-

(1) Ammonia (2) Uric acid (3) Urea (4) Both 1 and 3

15. The mechanism of urine formation in nephron involves :-

(1) Ultrafiltration (2) Reproduction (3) Diffusion (4) Osmosis

16. In diabetes mellitus the patient drinks more water as there is urinary loss of :-

(1) Salt (2) Insulin (3) Protein (4) Glucose

17. The hormone that promotes reabsorption of water from glomerular filtrate is ;-

(1) Oxytocin (2) Vasopressin (3) Relaxin (4) Calcitonin

18. Main function of kidney is :-

(1) Passive adsorption (2) Ultrafiltration (3) Selective reabsorption (4) Both 2 and 3

19. Urea is transported by :-

(1) Plasma (2) RBC (3) WBC (4) All

20. Micturition is :-

(1) Removal of urea from blood (2) Removal of uric acid

(3) Passing out urine (4) Removal of faeces

21. Ornithine cycle performs :-

(1) ATP synthesis (2) Urea formation in spleen

(3) urea formation in liver (4) Urine formation in liver

22. The snakes living in deserts are mainly :-

(1) Ammonotelic (2) Ammonotelic (3) Ureotelic (4) Uricotelic

23. Which excretory material is less toxic :-

(1) Ammonia (2) Urea (3) Uric acid (4) All are equally toxic

24. Correct order of excretory organs in cockroach, earthworm and rabbit respectively :-

(1) Skin, Malpighian tubule, kidney (2) Malpighi tubules, Nephridia, kidney

(3) Nephridia, Malpighian tubules, kidney (4) Nephridia, kidney, green gland

25. The yellow colour of urine of the vertebrates is due to :-

(1) Cholesterol (2) Urochrome (3) Uric acid (4) Melanin

26. In the kidney, the formation of urine involve the following processes arranged as :-

(1) Glomerular filtration, reabsorption and tubular secretion

(2) Reabsorption, filtration and secretion

(3) Secretion, absorption and filtration

(4) Filtration, secretion and reabsorption

27. A condition of failure of kidney to from urine is called :-

(1) Deamination (2) Entropy (3) Anuria (4) None of these

28. Excretion is carried out by Nephridia in :-

(1) cockroach (2) amoeba (3) earthworm (4) human

29. Urea is formed in :-

(1) Liver (2) Spleen (3) Kidney (4) Lungs

FILL IN THE BLANKS :-

1. The excretory system of human beings includes a pair of.....a pair of ....., a.....and a.....

2. Kidneys are located in the .....one on either side of the .....

3. Urine is stored in .....

4. CO_2 is removed from the blood in the .....

5. Nitrogenous waster in human beings is .....

6. Functional unit of kidney is .....

7. The urinary bladder is muscular and is under .....control.

8. Passing of urine from body is called.....

9. An artificial kidney is a device to remove nitrogenous water products from the blood through ……..

10. Artificial kidney contain a number of tubes with a……lining, suspended in a tank filled with dialyzing fluid.

11. Dialyzing fluid has the……osmotic pressure as blood.

12. Glomerular filtration rate in one day in human beings is ……..L.

13. Excretory organs in fishes are……

14. Artery which carry blood into kidney is…….

15. Double walled cup shaped structure in nephron is called……..

16. Tuft of capillaries in Bowman's capsule is called……..

17. Structure which help in excretion in tapeworm is ……..

18. The structure which help in excretion in earth worm is…….

19. Urine leaves the kidney through……

20. Substance which is completely reabsorbed by nephron is ……

MATCH THE COLUMNS :

1.

Column I		Column II	
(i)	Loop of Henle	(a)	Counter current system
(ii)	Glomerulus	(b)	Hypertonic urine
(iii)	Vasa recta	(c)	Urine concentration
(iv)	ADH	(d)	Ultrafiltration
(v)	Uricotelism	(e)	Frog
		(f)	shark

TRUE AND FALSE :

1. Micturition is carried out by a reflex.
2. ADH help in water elimination making the urine hypotonic.
3. Henle's loop plays an important role in concentration the urine.
4. Glucose is completely reabsorbed in the PCT.
5. Ureter is the reservoir of urine in the body.
6. Kidney filter about 180 L urine per day.
7. Functional unit of kidney is nephron.
8. Human being is Uricotelic.
9. Collecting duct is not a part nephron.
10. Glomerulus is a tuft of capillaries around loop of Henle.

EXCRETION IN ANIMAL				ANSWER KEY						EXERCISE # 1				
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**2**	**2**	**2**	**2**	**2**	**4**	**4**	**3**	**2**	**2**	**3**	**2**	**4**	**2**	**1**
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	
Ans.	**4**	**2**	**4**	**1**	**3**	**3**	**4**	**3**	**2**	**2**	**1**	**3**	**3**	**1**	

● **FILL IN THE BLANKS :**

1. Kidneys, Ureter, bladder, urethra **2.** abdomen, backbone **3.** Urinary bladder

4. Lungs **5.** Urea **6.** Nephron **7.** Nervous

8. Micturition **9.** Dialysis **10.** Semi permeable **11.** Same

12. 180 **13.** Kidneys **14.** renal artery **15.** Bowman's cupsule

16. Glomerulus **17.** Flame cell **18.** Nephridia **19.** Urethra

20. Glucose

● **MATCH THE FOLLOWING :**

(i) – (b), (ii) – (d), (iii) – (a), (iv) – (c), (v) – (e, f)

● **TRUE AND FALSE :**

	True or False									
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	**T**	**F**	**T**	**T**	**F**	**T**	**T**	**F**	**T**	**F**

EXERCISE # 2 (FOR SCHOOL / BOARD EXAMS)

VERY SHORT ANSWER QUESTION :

1. Name the structural and functional unit of kidney .

2. Why is urine yellow in colour ?

3. Name the structure which stores urine temporarily.

4. Name the U shaped tubule of nephron.

5. What is the main drawback of artificial kidney ?

6. Name the tube which passes out from urinary bladder.

7. Name the chief nitrogenous waster materials in human beings.

8. What is the advantage of presence of two kidneys in man ?

9. Name two parts of a nephron.

10. Where does ultra filtration occur in nephron ?

11. Name three parts of nephric tubule.

12. Define Glomerulus.

13. Give the technical term for passing the useful substances from nephric filtrate back into blood in blood capillaries.

14. Give the technical term for the process of excelling of urine.

15. Which part of skeleton protects the kidney ?

SHORT ANSWER QUESTIONS :

1. Define :

(a) Osmoregulation

(b) Hemo dialysis

(c) Malpighian body

2. Name the excretory matter in :

(a) Fishes (b) Birds (c) Tadpole (d) Human

(c) Frog (f) Earthworm

3. What is the role of afferent and efferent arteriole in glomerular filtration ?

4. Explain the role of ADH is excretion.

5. Differentiate between Ureter and urethra.

6. What will happen if there is no tubular reabsorption in the Nephrons of kidney ?

7. Differentiate between excretion and Osmoregulation.

8. How is the amount of urine produced regulated ?

9. What happens to glucose which enters the nephron along with filtrate during excretion in human beings ? State two vital functions of kidney.

LONG ANSWER QUESTIONS :

1. Where and how is urea produced in areodetic ? What happens to the kidney filtrate in descending limb of loop of Henle ?

2. Describe the structure of human nephron.

3. Briefly state the mechanism of urine formation in human kidney .

4. Explain the structure of kidney with the help of a well labeled diagram.

5. Explain the following , why ?

(a) Mammals are Ureotelic, but birds are Uricotelic

(b) Skin functions as an accessory excretory organ.

(c) Frog is Ureotelic but tadpole is Ammonotelic.

(d) Urine infection is more common in women than men.

(e) Frequency of urination increases after consuming alcoholic beverages.

EXCRETION IN ANIMALS **ANSWER KEY** **EXERCISE # 2**

● **VERY SHORT ANSWER QUESTIONS :**

1. Nephron **2.** due to pigment Urochrome **3.** Urinary bladder

4. Loop of Henle **5.** There is no reabsorption of useful substances **6.** Ureter

7. Ammonia, Urea, Uric acid **8.** If one kidney fails man can live on the other kidney.

9. Bowman's capsule and nephric tubule. **10.** Bowman's capsule

11. PCT, loop of Henle, DCT

12. A tuft of capillaries present in the cavity of Bowman's capsule.

13. Selective reabsorption **14.** Micturition **15.** 11^{th} , 12^{th} ribs.

EXERCISE # 3 (FOR SCHOOL / BOARD EXAMS)

1. Contraction of right ventricle pumps blood into :-

(A) right auricle (B) pulmonary artery (C) pulmonary vein (D) dorsal aorta

2. Urine leaves the kidney through :-

(A) Urethra (B) collecting duct (C) renal vein (D) Ureter

113. Urea, a nitrogenous waste, requires a large amount of waste. Therefore it is the excretory product in :-

(A) protozoan, amphibians and reptiles (B) elasmobranchii, adult amphibians

(C) reptiles, birds and mammals (D) insects, birds ad fishes

114. In which of the three groups of the following mammals is uric acid also excreted out :-

(A) Carnivore, insectivore and marsupials (B) Elephants, chiropteran, primates

(C) Lagomorphs, man, horse (D) Man, apes, Dalmatian log

115. In Hydra and Amoeba, ammonia is the nitrogenous waster. Lizards, snakes, birds and insects excrete mostly uric but crocodiles and alligators excrete mainly ammonia through they are reptiles. So can we generalize that :-

(A) there is no uniform pattern of removal of nitrogen wastes

(B) aquatic and land animals excrete urea

(C) animals that fly excrete uric acid

(D) nitrogen waste excretion is closely related to the availability of water in the environment

116. The white matter in a bird's dropping is :-

(A) calcium carbonate (B) calcium sulphate

(C) uric acid (D) urea

118. Kidney is an excretory and regulatory organ. Which two of the following are regulated effectively by kidneys ?

(A) CO_2 and protein (B) Sugar and C_2 (C) Water and salts (D) Water and fat

119. The main function of the skin is to protect the underlying delicate tissues from environmental factors ; in addition, it performs several other functions. Some are given below. Which of the following functions is excretory in nature ?

(A) Giving out of urea and uric acid in sweat (B) Detection of changes in temperature

(C) Regulation of body temperature (D) Detection of pressure, pain or touch etc

120. Liver is an important gland of the body. in addition to function in digestion and food storage, the liver participates in excretion. Which of the following function (s) :-

(A) Deamination and urea formation (B) Elimination of hemoglobin and bile salts

(C) Inactivation of chemicals after their role is over (D) All the three mentioned above

122. Presence of a large number of mitochondria in the tubule cells of Nephrons suggests that the nephron is involved in the process of :-

(A) passive transport (B) active transport (C) formation of urea (D) diffusion

123. Which of the following parts of a kidney contains the lowest concentration of urea ?

(A) Loop of Henle (B) Branches of venal vein (C) Bowman's capsule (D) Glomerulus

124. Uriniferous tubules of a kidney are concerned with the formation of :-

(A) glucose (B) amino acids (C) hormones (D) urine

125. Removal of faeces or undigested food from the body is not an example of excretion because :-

(A) faeces is given out of the alimentary canal

(B) it has not been digested by the body

(C) undigested food is not a product of metabolism

(D) if is remains in the body, it produces foul gases

126. Dialysis is carried out in case :-

(A) both kidneys are damaged

(B) brain and spinal cord are damaged

(C) heart and lungs are damaged

(D) liver and spleen do not function

15. Excretory organs in Amoeba are :-

(A) Contractile vacuoles

(B) Cellular surface

(C) Mitochondria

(D) None of these

ANSWER KEY														
Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**B**	**D**	**B**	**D**	**D**	**C**	**C**	**A**	**D**	**B**	**B**	**D**	**C**	**A**	**D**

EXCRETION IN PLANTS

EXCRETION : The process of removal of toxic waste products from the body of an organism is known as excretion.

- The main waste products produced by plants are carbon dioxide, water vapor and oxygen
- CO_2 and water are produced as wastes during respiration by plants.
- CO_2 produced during respiration in day time is all used by the plant itself in photosynthesis. Plants excrete oxygen as a waste only during day time.
- The gaseous wastes of respiration and photosynthesis in pants are removed through the **stomata** in leaves and **lenticels** in woody stem and released to the air.
- **Oxygen** is produced as a waster during photosynthesis.
- Plants get rid of excess water by **transpiration.**
- Many plant waste products are stored in cellular vacuoles.
- Plants also store some of the waste products in their body parts (leaves, bark and fruits).
  e.g. Tannins, essential oils, latex, gums, resins.
- Tea leaves, amla, betel nut and bark of tree contain tannins.
- Leaves of Eucalyptus, lemon, tulsi, contain essential oils.
- Leaves of yellow oleander contain latex.
- Gums are found in babul tree.
- **Resins** are found in stem of conifers.

COMPETITION WINDOW
● The plants carbon dioxide produced as a waste during **respiration** in night time.
● Aquatic plants lose most of their metabolic wasters by direct diffusion into the water surrounding them.
● Terrestrial plants excrete some waste into the soil around them. The plant get rid of stored solid and liquid wastes by the shedding of leaves pealing of bark and felling of fruits.

- Quinine and morphine are medicines derived from alkaloid stored in **Cinchona bark** and **Opium** fruits respectively .
- Caffeine found in coffee seeds and nicotine in tobacco leaves are also alkaloids.
- **Calcium Oxalate crystals** accumulate in some tubers like Yam (zamikand).

EXERCISE # 1 FOR SCHOOL EXAMS

OBJECTIVE QUESTIONS :

1. Plant gets rid of excess water by :-

(A) Photosynthesis (B) Respiration (C) Transpiration (D) None of these

2. Waste product /s produced by plants is / are :-

(A) CO_2 (B) Water (C) Oxygen (D) All of these

3. Waste product produced during respiration in plant :-

(A) CO_2 (B) Water (C) Oxygen (D) A and B

4. In photosynthesis the waste product is :-

(A) CO_2 (B) Oxygen (C) Nitrogen (D) None of these

5. The gaseous wastes of respiration and photosynthesis in plants are removed through :-

(A) Stomata of leaves (B) Lenticels of stem

(C) Stomata and lenticels (D) None of these

6. Which of the following statements (s) is / are correct :-

(A) Gum are found in babul tree (B) Leaves of tulsi contain essential oils

(C) Leaves of amla and tea contain tannins (D) All of the above

FILL IN THE BLANK

1. Plants get rid of excess water by ……

2. ……and water are produced as wastes during respiration by plants.

3. ……is produced as a waste during photosynthesis.

4. The gaseous wastes of respiration and photosynthesis in plants are removed through the ……..in leaves and……in stem and released to the air.

5. The plants excrete CO_2 produced as a waste during ……process in night time.

6. Gums and resins are the ……..products of plant.

7. The phenomenon of removing of waste product from the body is known as ……..

8. Leaves of ……contain essential oils.

9. ……are found in stem of conifers as waste product.

10. Aquatic plants lose most of their metabolic wastes by ……process.

VERY SHORT ANSWER TYPE QUESTIONS :

1. Name the two part of a plant through which its gaseous waste products are released into the air.

2. Name a waste gas excreted by the plants only during the day time and only during the night time.

3. Name the process by which plants get rid of excess water.

4. Name the phenomenon of removing of waste product from the body.

5. Name the waste product which is produced by the stem of conifers.

6. Name the by products of photosynthesis.

7. Name some waste products which are stored by the plants .

8. Name the plant from which quinine is obtained .

9. Name the plant from which morphine is obtained.

10. From which part of opium morphine is obtained ?

11. From which part of cinchona quinine is obtained ?

MATCH THE COLUMN :

Match the items of Column A with items of Column B.

Column		Column	
(A)	Waste product of respiration	(i)	Resin
(B)	Waste product of photosynthesis	(ii)	CO_2
(C)	Leaves of lemon	(iii)	essential oil
(D)	Stem of conifers	(iv)	O_2

SHORT ANSWER TYPE QUESTIONS :

1. Define the excretion

2. Write about the various waste products produced by the plants.

LONG ANSWER TYPE QUESTION :

3. What are the methods used by plant to get rid of excretory process. [NCERT QUESTION]

EXCRETION IN PLANTS — ANSWER KEY — EXERCISE-1 (X) – CBSE

- **OBJECTIVE QUESTION :**

  1. (C) **2.** (D) **3.** (D) **4.** (B) **5.** (C) **6.** (D)

- **FILL IN THE BLANKS :**

  1. transpiration **2.** CO_2 **3.** Oxygen **4.** stomata ; lenticels

  5. respiration **6.** waste **7.** excretion **8.** tulsi / lemon

  9. Resins **10.** diffusion

- **VERY SHORT ANSWER TYPE QUESTIONS :**

  1. (i) Stomata (ii) Lenticels **2.** O_2 ; CO_2 **3.** Transpiration **4.** Excretion

  5. Resins **6.** Oxygen ; Water ; Starch **7.** Tanins ; Gums ; Resins ; Latex

  8. Cinchona **9.** Opium **10.** Fruits **11.** Bark

- **MATCH THE FOLLOWING :**

  1. A – (ii) **B –** (iv) **C –** (iii) **D –** (i)

ORIGIN AND EVOLUTION

EVOLUTION

The term evolution has been derived from the Latin word '**evolvere'** means unroll.

Evolution can be defined as sequence of gradual development of complex form of life from simple form of life over the course of geological time "Descent with modification."

Evolution is of two types – (i) Chemical evolution
(ii) Organic evolution.

S.No.	Features	Chemical	Organic evolution
1	**Definition**	It is the formation of the complex organic compounds from simple compound or element	It is formation of complex from of life from simple form of life.
2	**Time of occurrence**	It occurred at the time of origin of life	It is still occurring
3	**Reversibility**	Irreversible	Reversible
4	**Speed of evolution**	It is fast process	It is a slow process
5	**Index was given by**	**A.I oparin**	**Charles Robert Darwin**

Organic evolution :

After origin of a living cell the next questions that arose was how did so many different species of complex life form come into existence ? Here are various view points.

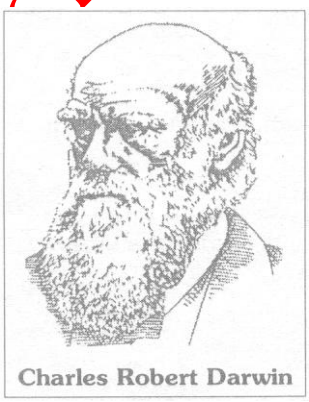
Carolus Linnaeus :- Said that no species is new i.e. each species originates from pre existing species.

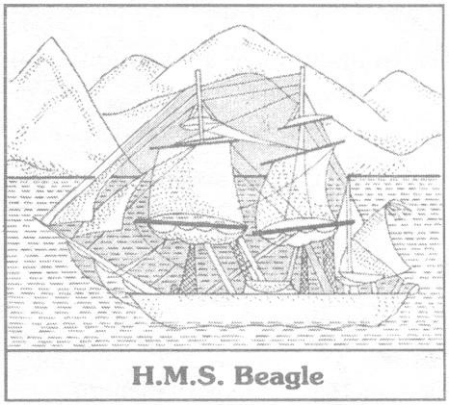
Jeans Baptist Lamarck :- Explained in his book namely **philosoophie zoologique.** The theory of inheritance of acquired characters of Lamarckism. This theory states that use and disuse of an organ leads to change in the organ. Which is inherited by the offsprings. The favorable variations which remain for longer period of time results in evolution of new species.

This theory was discarded by **August Weismann** as he experimentally proved that even after cutting tails of mice for 21 generation tailless mice was never born.

DARWINISM

Charles Robert Darwin was born on 12 Feb, 1809 in England. He travelled by **HMS Beagle** ship along with **Dr. Henslow**. He visited many islands of south America, South Africa, Australia and Galapagos Islands. Darwin was influenced by two book. **"Principal of population" of Malthus. "Principal of Geology of Charles levell.**


Charles Robert Darwin


H.M.S. Beagle

In this book **origin of species** he answered these questions. The theory presented by him is called **theory of natural selection or Darwinism.**

Alfred Russell Wallace :- He travelled south eastern Asia and south America. The idea of natural selection stroked in his mind Wallace wrote an assay and sent it to Darwin. **On the tendency of varieties to depart indefinitely from original type.** There is striking similarity between the views of Darwin and Wallace.

S.No.	Facts	Consequences (conclusions)
1.	(i) Enormous rate of reproduction among animals. (ii) Constant number of animals of a species.	Struggle for existence
2.	(i) Struggle for existence (ii) Heritable variations	Survival of the fittest or natural selection
3.	(i) Survival of the fittest (ii) Continues environment changes	Continuous natural selection leading to evolution of new species

Charles Darwin explained the mechanism of origin of new species by nature selection. But he failed to explain the mechanism of source of heritable variations. This was explained by **Hugo de Varies** a Dutch botanist. According to him heritable variations arise when there is a change in genes of the **germplasm** (protoplasm of germ cell). He called it **mutation.**

1. What do you mean by term Evolution ? Write the name of its two types.

2. Who said that "No species is new i.e. each species originates from pre existing species".

3. Name the organism on which Weismann conducted his experiment and what was his experiment ?

4. Who proposed the theory of natural selection ? Why was this theory criticized ?

5. Who first proposed the theory of inheritance of acquired characters ?

VARIATIONS

Variations are the structural, functional or behavioral changes from the normal characters developed in living organisms. There is an inbuild tendency to variation during reproduction. Both because of errors in DNA copying as a result of sexual reproduction.

Variations provide materials for evolution. These may be inheritable or non inheritable, only inheritable variation participate in evolution.

Type of variation :- On the basis of nature of calls where variations occur variations are of two types.

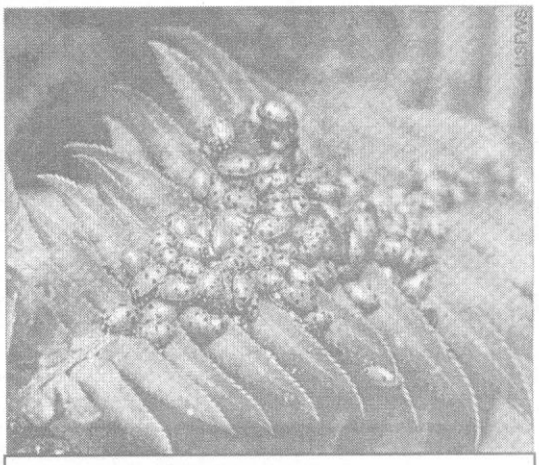
(i) Somatogenic variations or Acquired traits

(ii) Germinal (Blastomeric) variations.

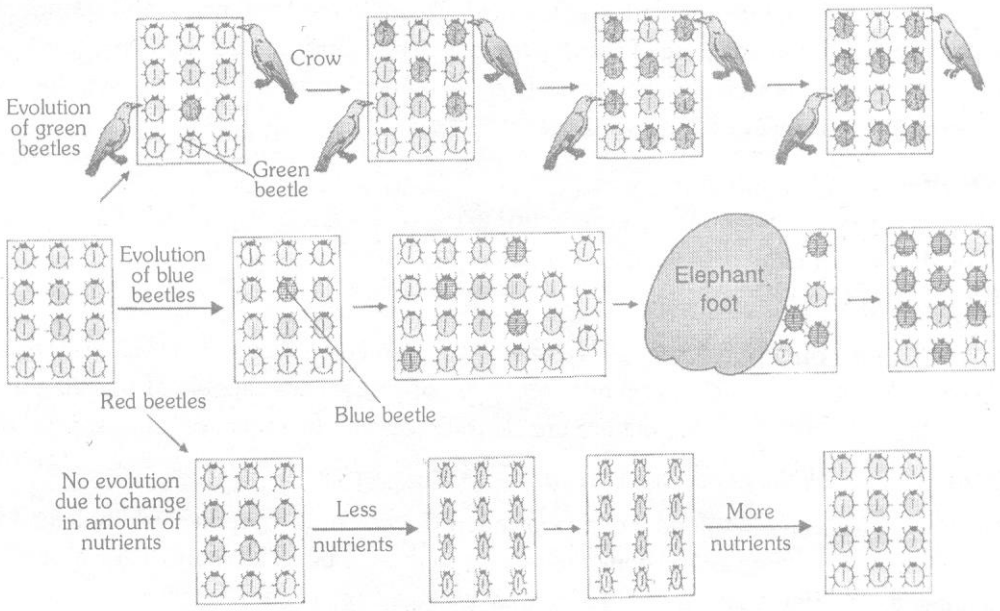
S.No.	Somatogenic variation	Germinal variations (Blastomeric variations)
1	They occur only in **somatic** (non germinal) cells of an individual.	These occur on **germinal** cells of reproductive organs of an individual
2	Theses are **acquired** during the life span of an individual.	These occur at the time of formation of gametes in reproductive organs.
3	Theses are result of **environmental factors** like changes in light, temperature, food availability etc.	These are developed either due to mutation or recombination of genes.
4	DNA of cells is not changed so it is non-inheritable	These are inheritable variations. So they are transmitted from one generation to another e.g. – Polydactyl in man.

An Illustration :- Consider the following example.

1. A group of twelve **red beetles** living in bushes with **green leaves.**
2. Beetles in the population can generate variations because these are reproducing sexually.
3. Crow can eat the beetles. The more beetles the crow eat, the fewer beetles are left for reproduction.



A population of beetles



Variation in a population inherited and otherwise

Now consider the following situations :-

1st Situation	2nd Situation	3rd Situation
● A green colour variation rises during reproduction.	● A blue colour variation arises during reproduction.	● **Condition 1 :** Appearance of plant disease in the bushes.
● One beetle.	● This blue beetle can pass the blue to its progeny	● Amount of plant (leaf) material for the beetles decreases.
● This green beetle can pass the green colour to its progeny.	● Crows can see blue as well red beetles on the green leaves of bushes so can eat them	● Beetles are poorly nourished. **Observation 1:**

●	Crows cannot see green beetles on the bushes to cannot eat them.	●	Most of the beetles are killed by elephant foot.	●	Average weight of beetles decreases.
	Observation : More population as compared to red beetles. **Conclusion :**	●	Beetles which survived are mostly blue. This is by chance.	●	**Condition 2 :** Disappearance of plant disease in the bushes.
● ●	Variations have survival advantage Rare variation came as a common characteristic in the population.		**Observation :** Population of beetles grows slowly and blue beetles are more in number. **Conclusion :** Variations do not have survival advantage .	● ●	Amount of plant (leaf) material for the beetles increases. Beetles are properly nourished.
●	In other words, frequency of certain gene traits (genes control the traits) changed over generations. This is the of the idea of evolution.	●	Frequency of certain traits / genes can be changed by accidents in small	●	**Observation 2 :** Size of the saturated beetles increases to normal.
●	The number of red beetles decreases as the number of crows increases (Natural selection is directing the evolution).	●	Both above changes provide diversity without any adaptations.	●	The change in the weight of beetles is not inherited over generations. because it is somatic variation

HERITABLE VARIATION

The reason why organisms resemble their parents lie in the precise copying of their genes. Which carry hereditary characters from one generation to the next. On the other hand no two off springs have exactly the same genes. This is because offspring of sexually reproducing organisms reproducing receive varying combination of genetic material from both parents such variation result from mutations (errors in DNA copying). Variations also result from **genetic recombination** during sexual reproduction

Genetic drift :

The random changes in the gene frequency occurring by chance alone. The effect of genetic drift is very small in large population and large in small populations.

SPECIES AND SPECIATION

Biological species concept : A species is a sexually interbreeding group of individuals separated from other species by the absence of genetic exchange. Members of species are capable of breeding with one another and produce living, fertile off spring but are unable to breed with members of other species normally.

Gene pool : Sum of all the genes of all the members of a species.

Speciation occurs when the gene pool of a population is some how reproductively isolated from other sister population of the parent species and gene flow no longer occurs between them. Then a population splits into independent species. Which become reproductively isolated from each other.

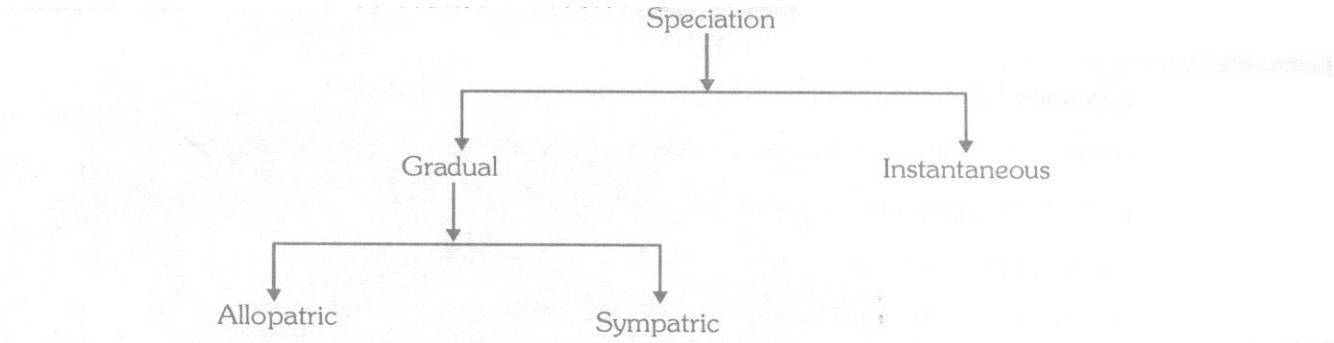
On basis of period taken in speciation there are two types of mechanism of speciation.

After years, two sub populations get reproductively isolated by natural selection and genetic drift from tow new species.

ABOVE EXAMPLE OF SPECIATION SHOWS

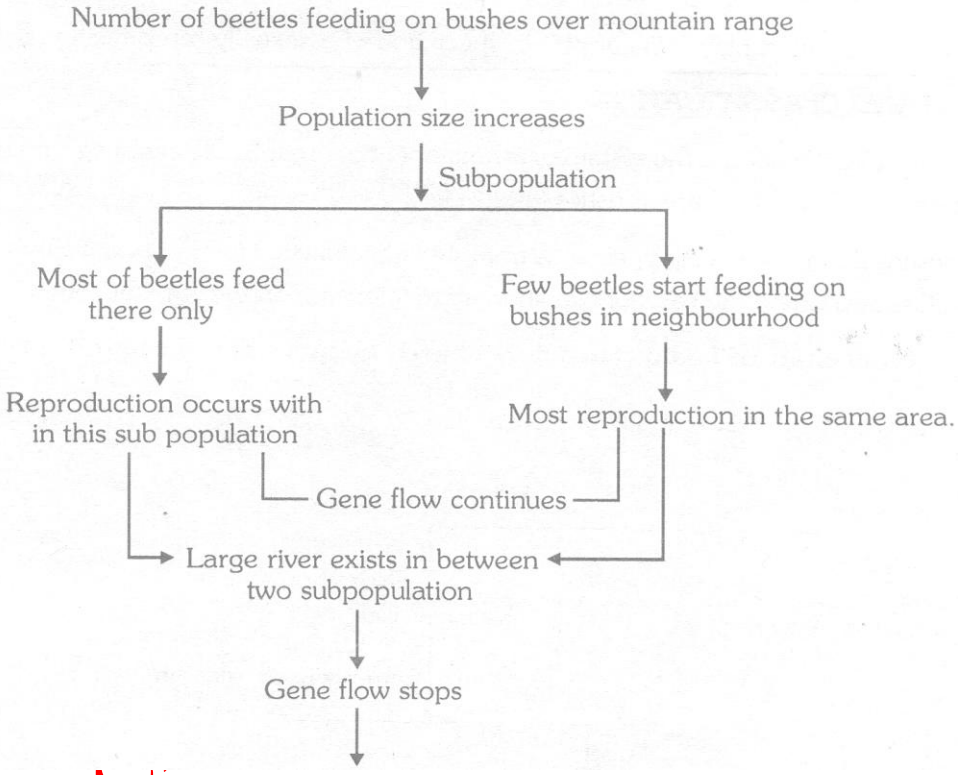
- Large population of beetles occur on a mountain range.
- Few beetles started feeding in neighborhood.
- Gene flow continued in places
- They may get isolated at larger distance because of existence of river.
- Gene flow decreases and finally stops.
- Two sub populations change with time because of genetic drifts and natural selection.
- Later they became reproductively isolated.
- Two new species came up.
- This can occur as a result of change in chromosome number.

◆ Micro evolution is very important this mean that the changes may be small but significant. Speciation due to inbreeding, genetic drift and natural selection will be applicable to all sexually reproducing animals geographical isolation does not play any role in the speciation of a sexually reproducing animals and self pollinating plants.



Speciation : The process of formation of one or more new species from an existing species is called speciation.

Let us try to understand this concept by taking the example of beetles.



Questions :

1. What are variations ? Distinguish between germinal and somatic variations ?

2. Name two types of variations on the basis of nature of cell affected.

3. Which type of variation help in evolution ?

4. Name various sources of variation.

5. Give the term of random changes in gene frequencies which occur by chance fluctuations.

6. Define species ?

7. What do you mean by reproductive isolation.

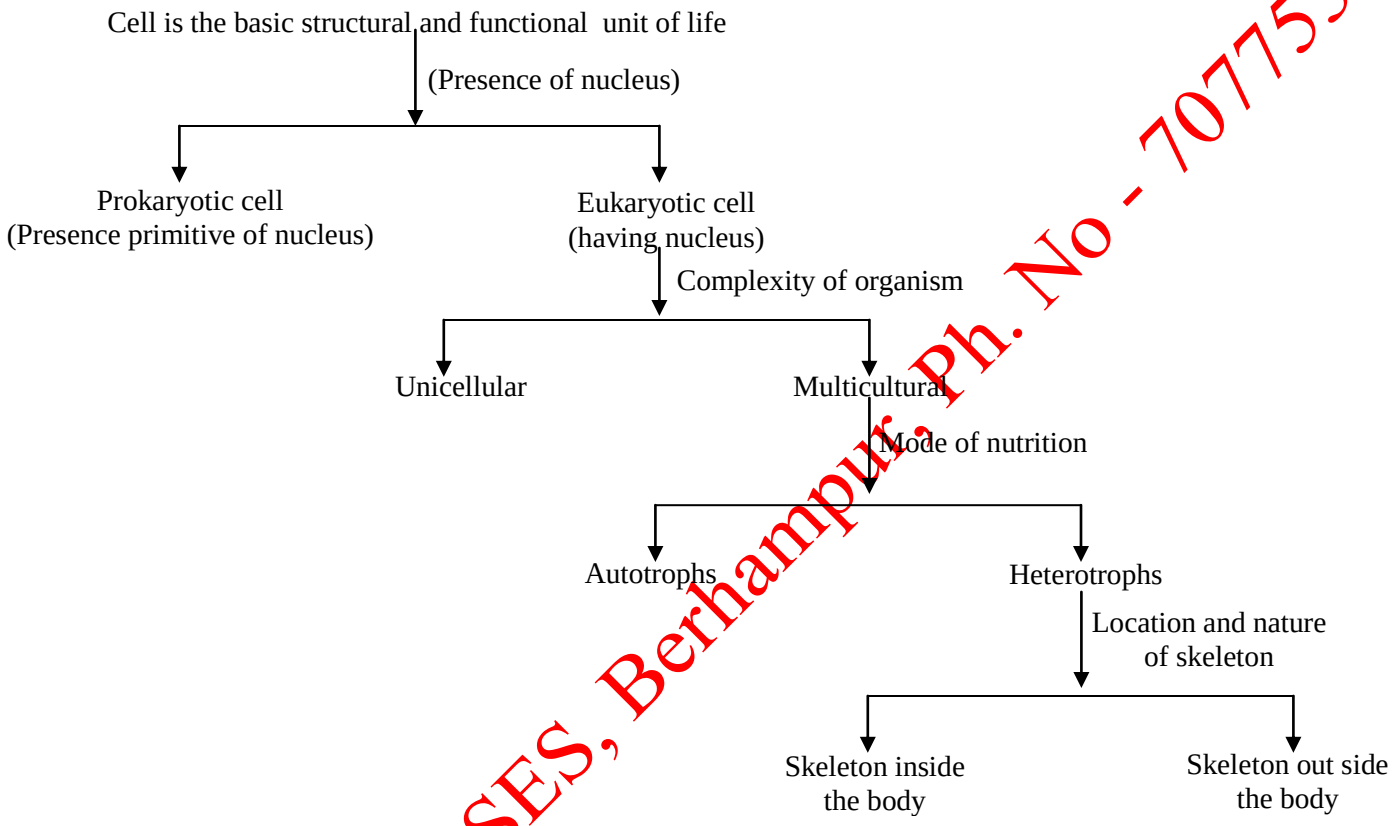
8. Does geographical isolation play a major role in speciation of a sexually reproducing organisms. Explain ?

EVOLUTION AN CLASSIFICATION

Classification : Classification is the system of arrangement of organism in certain groups or subgroups on the basis of hierarchies of certain characteristics

The characteristics are the details of appearance from structure, function and behavior **Phylogenetic taxonomy** is branch of classification on the basis of evolutionary relationship on the basis of common ancestry.

Flow chart of basic characters shared by most of he organisms.



Questions :

1. Give the term for the branch of classification on the basis of evolutionary relationship on the basis of common ancestry.
2. Name the categories in which organisms are divided on the basis of complexity of organism.

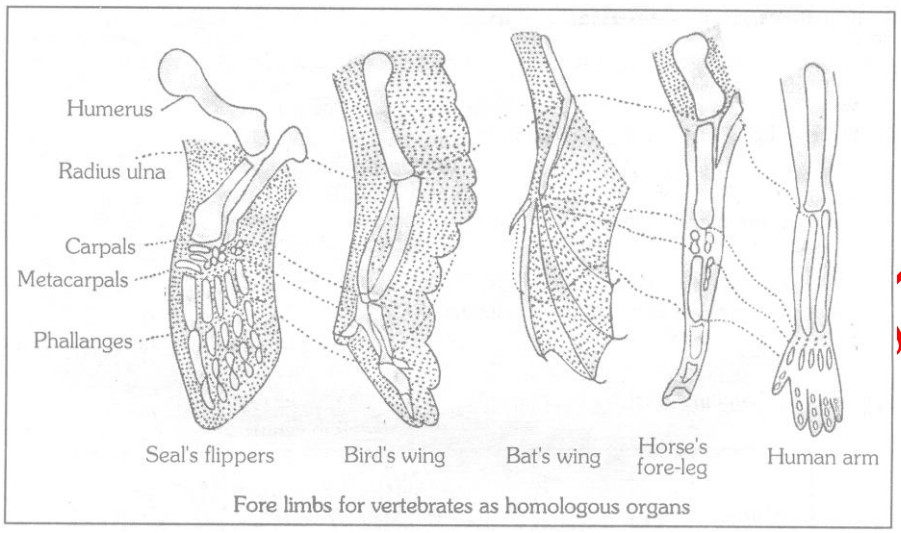
TRACING EVOLUTIONARY RELATIONSHIP

Studies on similarities in structure of different organisms suggests that present from have evolved through a process of slow and gradual change called evolution. They include the following :

1. **Homologous organs :** Homologous organs are those structures which are different in appearance and perform different functions but have similar basic structure and developmental origin. This relationship is called homology.

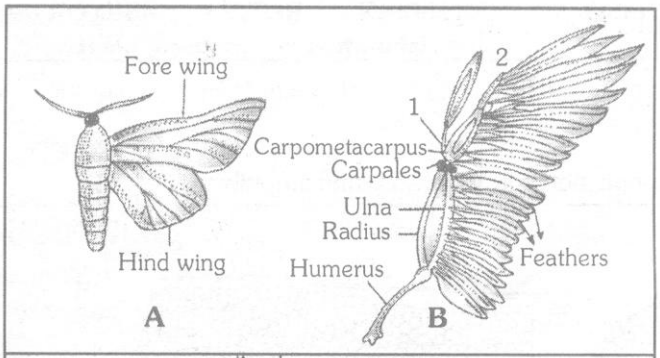
FORELIMB IN VERTEBRATES

	Seal	Bird	Bat	Horse	Man
Appearance	Flippers	Wings	Patagia	Elongated	Thumb opposability
function	Swimming	flying	Support, flying	Funning	Grasping



Fore limbs for vertebrates as homologous organs

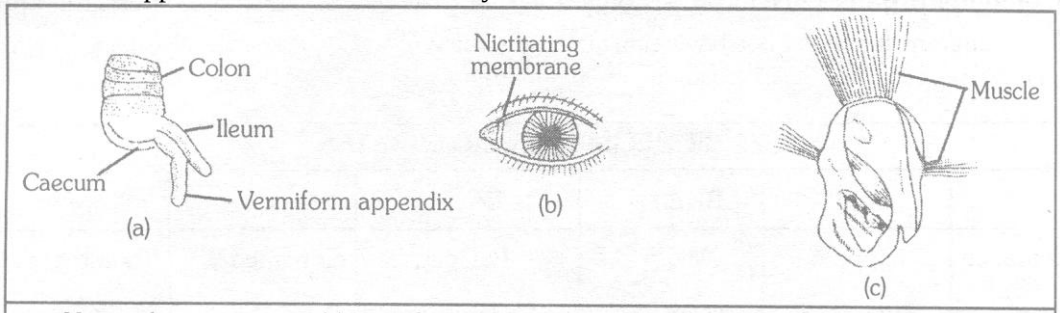
2. **Analogous :** Those organs which have different origin and structural plan but appear similar and perform similar functions are called analogous organs while this relationship is called convergent evolution or analogy.
e.g. : Wing of an insect and a bird, Hand of man & Trunk of elephant.



Analogous organs
A. Insect wing; B. Bird wing

Analogy in these organs is due to similar adaptations to perform similar functions rather than their common ancestry.

3. **Vestigial organs :** Those organs which no longer have a function are called vestigial organs. These organs have reduced structurally as well as functionally. It appears that these organs were once well developed and functional in ancestors and later on due to their less use they became reduced.
eg : Vermiform appendix, ear muscles, third eyelid in man are reduced and function less.



Vestigeal organs in man (a) Vermiform appendix (b) Nictitating membrane (c) ear muscles

Common ancestry and inter-relationship :

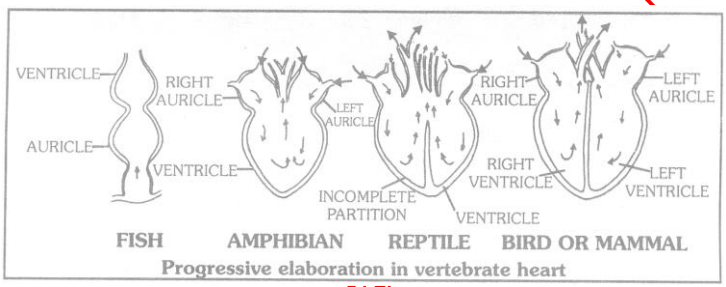
Various organisms are inter connected their resemblance suggest a common ancestry.

Eg : Heart of Fish → Only two chambers
- One auricle
- One ventricle

Heart of Amphibian → Three chambers
- Two auricle
- One ventricle

Heart of Reptilian → Three chambers and half
- Two auricle
- One incomplete partition in ventricle

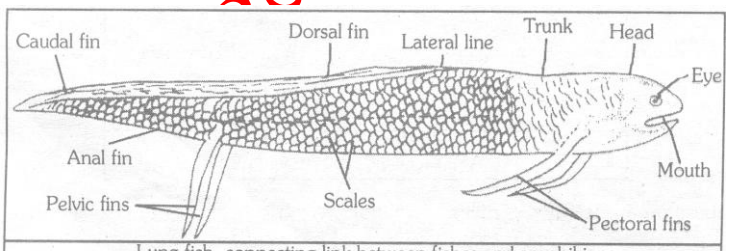
Heart of Bird and mammals → Four chambered
- Two auricle
- Two ventricles



Progressive elaboration in vertebrate heart

Connecting links : Some living organism have characteristics of two groups. They are known as connecting links.

Eg : Lung fish – show connection between fishes and amphibians.



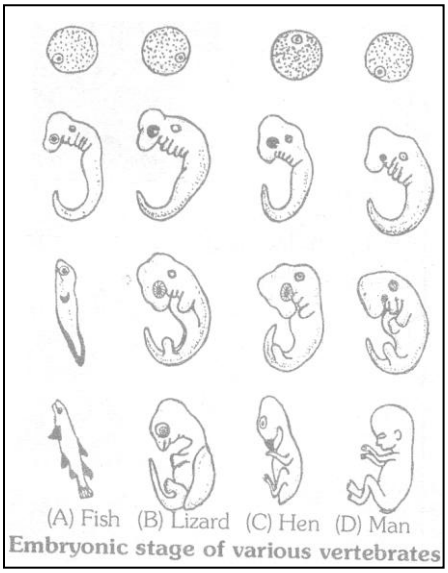
Lung fish, connecting link between fishes and amphibians.

Do you know

Various connection links

(i)	**Virus**	→	Between living and non living
(ii)	**Euglena**	→	Between plants and animals
(iii)	**Proterospongia**	→	Between protozoa and Porifera
(iv)	**Neopilina**	→	Between Annelida and Mollusca
(v)	**Peripatus**	→	Between Annelida and Arthropoda
(vi)	**Archaeopteryx**	→	Between reptiles and birds
(vii)	**Balanoglossus**	→	Between non chordates and chordates
(viii)	**Chimera**	→	Between cartilaginous fish and boney fish
(ix)	**Lung fish**	→	Between fishes and amphibian
(x)	**Platypus**	→	Between reptiles and mammals
(xi)	**Echidina**	→	Between reptiles and mammals

Jurassic period is known as golden age of reptiles.
Dinosaur of dromaesaur family had feather on body and fore limb.
Huxley called birds as glorified reptiles.
Carnivorous dinosaur called velociraptor had a wish bone like birds.



Embryonic stage of various vertebrates

Evidences from embryology :

A comparative study of the stages of embryonic development of animals reveals that in their early stages they were very similar.

These embryonic stage reflect thus ancestry. The embryological stages of an organism give us an idea about the stages of its evolution.

For example when we study the human embryo, we find that at a certain stage it has gills. This suggests that fish is one of the earliest ancestors in the evolution of mammals including human beings.

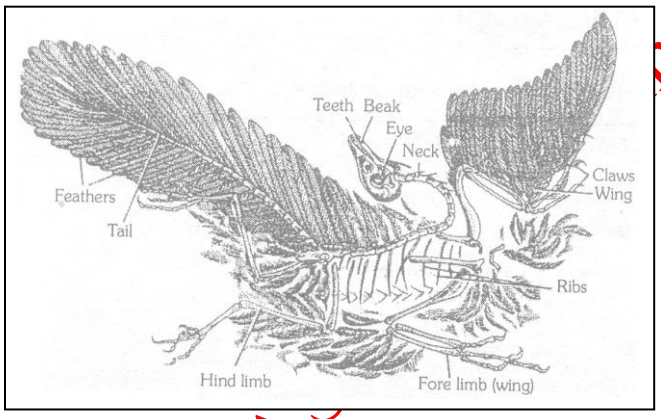
EVIDENCES FROM PALEONTOLOGY

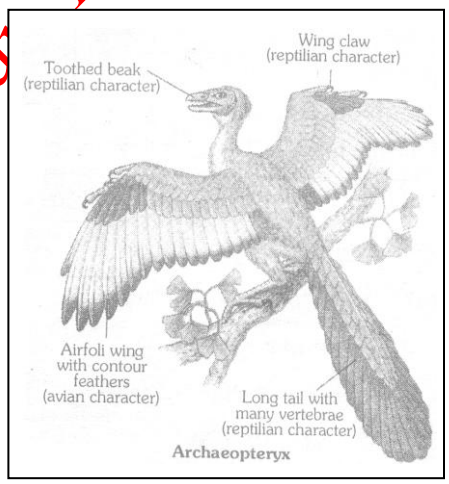
1. **Fossils :** Fossil is an organic relic of a long dead life form.

Or

Fossils are the petrified remains and for impressions of the hard parts of the extinct organism preserved in the sedimentary rock or other media.

Paleontology : Study of fossils is known as paleontology.



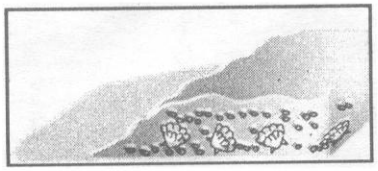


Do you know

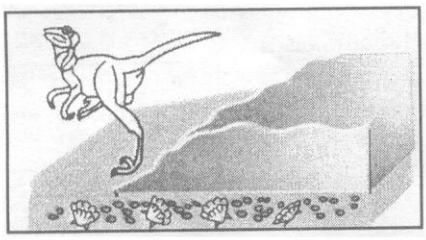
How do fossils form layer by layer ?

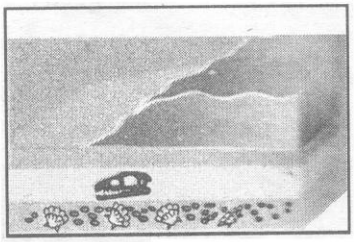
Let us start 100 million years ago. Some invertebrates on the seabed die, and are buried in the sand. More sand accumulates, and sandstone forms under pressure.

Millions of years later, dinosaurs living in the area die, and their bodies, too. are buried in mud. This mud is also compressed into rock, above the rock containing the earlier invertebrate fossils.

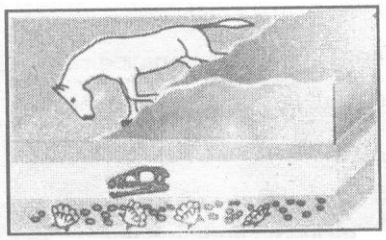


Again millions of years later, the bodies of horse-like creatures dying in the area are fossilized in rocks above these earlier rocks.

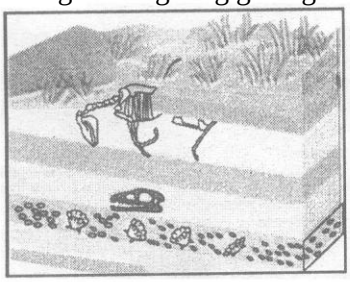




Much later. erosion by , sun, water flow, wears away some of the rock and exposes the horse like fossils. As we dig deeper, we will find older and older fossils.



Living fossils: The animals which underwent little change during long geological periods.



IMPORTANT LIVING FOSSILS

1. Peripatus, Limulus (Arthropoda)
2. Nautilus, Neopilina (Mollusca)
3. Lingula (Brachiopoda)
4. Latimeria (Coelacanth fish)
5. Spenodon (Reptila)

Determination of the age of fossil : There are three ways of determining age of the fossils.

1. **Relative method :** If we dig into the earth and starts finding fossils it is reasonable to suppose that the fossils we find closer to the surface are more recent than the fossils we find in deeper layer.

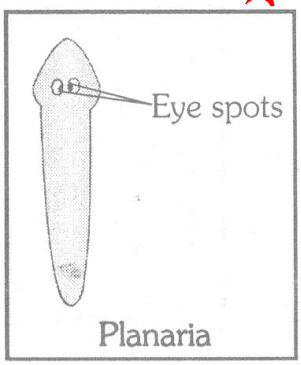
2. **Using Radioactive Elements :** The age of a fossil is determined with the help of certain radioactive elements such as uranium present in the rock .

3. **Carbon dating :** Carbon dioxide of air contains a small proportion of radioactive carbon (C^{14}). CO_2 is used during photosynthesis and there is equal proportion of C^{14} among carbon atoms of all organisms.

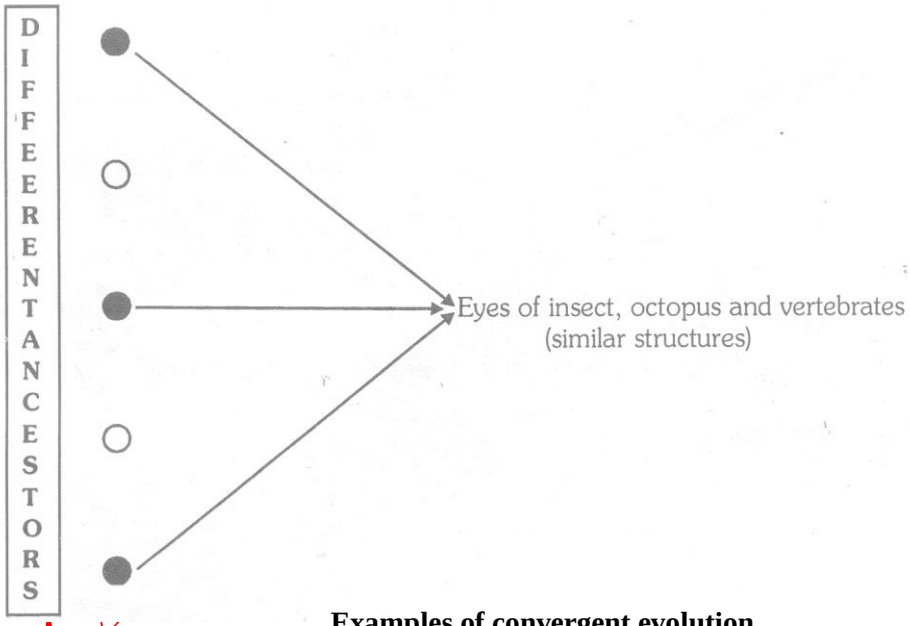
The radioactivity of C^{14} is lost at a precise rate half life of C^{14} is about 5,600 years.

If a fossil shows radioactivity one fourth of that found in the living organisms the organisms died about 11,200 years ago. (Two half lives.)

Evolution by stages : Evolutionary changes are fundamental characteristics of living organisms such changes may be convergent. It means that the similar looking structures may have different ancestral designs. This can be explained by example of eye.



Eye : Eyes of insects, octopus and vertebrates have similar looks but different structure and must have separate evolutionary origin or different ancestral designs. Rudimentary eye can be useful to some extend.



Examples of convergent evolution

Question :

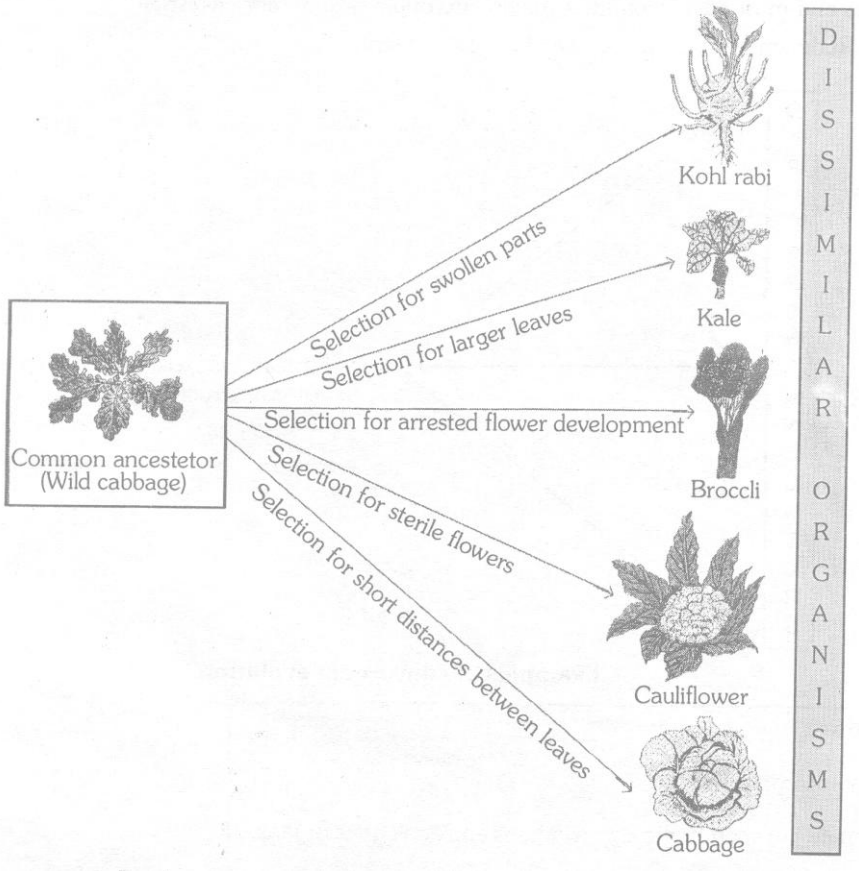
1. **Define homologous**
2. Differentiate between homologous organs and analogous organs.
3. What are vestigial organs ? Give an example.
4. How do embryological studies provide an evidence. for evolution ?

Feathers : The function of feathers was insulation cold weather later feature might have proved to be useful for the purpose of light.

Dinosaurs is example depicting presence of feathers in them but these were not used for flying later birds seem to have adaptation of flying using feathers. This shows close relationship of biers of reptiles and proves that characters appearing as an variation can be useful later to perform different functions.

Cabbage : Humans have developed different types of vegetables from the wild cabbage by artificial selection.

S.No.	Vegetable evolved	Edible part
1.	Cabbage	Selection of fleshy terminal buds and short distance between the leaves.
2.	Brussels' sprouts	Selection for fleshy lateral
3.	Kohl rabi	Selection for fleshy stem.
4.	Kale	Selection for large sized leaves so it is a leafy vegetable.
5	Broccoli	Selection for leafy sized and arrested flower developments.
6.	Cauliflower	Selection for fleshy sterile flower.



Evolution of wild cabbage [Divergent evolution]

MORE TO KNOW

MOLECULAR PHYLOGENY

Ancestors of different organism including humans can be traced by studying the change in their DNA.

A change in DNA means a change in its protein sequences. The ancestry or phylogeny determined by comparative study DNA sequences is called molecular phylogeny.

Studies in molecular phylogeny help in the classification of organisms.

Haeckel propounded **"The theory of recapitulation or 'Biogenetic law'.** Which states that an individual organism in its development (ontogeny) tends to repeat the stages passed through by its ancestors (phylogeny means ontogeny repeats phylogeny.)

EVOLUTION SHROUDS NOT BE EQUATED WITH PROGRESS

Though organic evolution involves descent with modification in which there is a progressive trend of emergence of more or more complex body designs from earlier similar body designs by gradual changes but evolution should not equated with progress because of following reasons.

1. In evolution older species are not eliminated during formation of new species and most of older and simple species still survive.

   eg : Earliest organisms like bacteria are found even in many hospitable habitats like hot springs, deep-sea, thermal vents, Antarctic ice. etc.

2. The evolved species are not always better than the parental species evolution depends upon natural selection and genetic drift which is together result in population which is reproductively isolated from the parental species.

HUMAN EVOLUTION

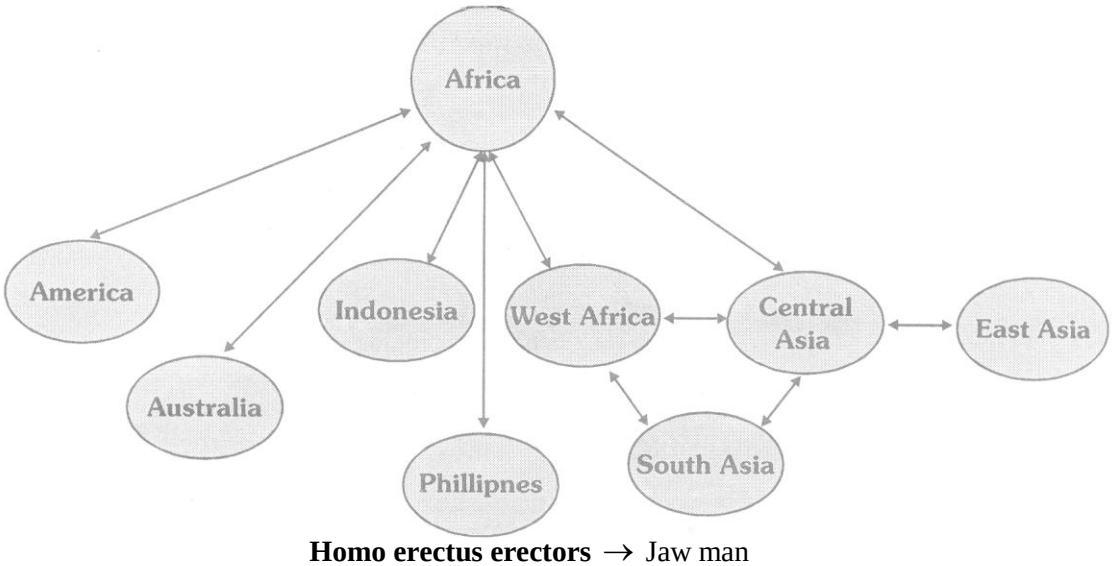
Evolutionary history of man has been guilt from the paleontology (fossil studies) and molecular biology (especially DNA changes).

For example :- It is not true that human beings have evolved from chimpanzees. Rather both human beings and chimpanzees have common ancestors a long time ago. That common ancestors is likely to have been nether human or chimpanzee. The two resultant species have probably evolved in their separate ways to give rise to the current forms.

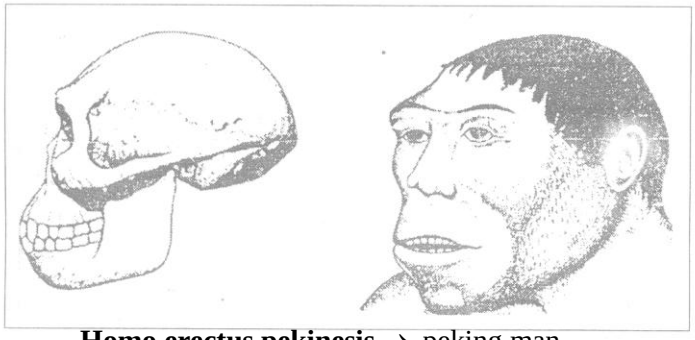
Anthropology : The scientific study of tracing of human evolution is called anthropology. Scientists involved in studying human evolution are called **anthropologists.**

Studies have revealed that human evolution started in Africa and earliest human type was Australopithecus Africans. African ape man fossil was discovered by **Prof. Raymond Dart** fossil of skull of 5-6 years old baby from old Pliocene rock to Tuang region (S. Africa). He name it Tuang baby. It had many ape like characters but had a bipedal locomotion like man. The cradle of human evolution is East Africa where genetic foot prints of earliest members of human species **Homosapiens** can be traced. A couple of hundred thousand years ago some of own ancestors left across the planet from Africa.

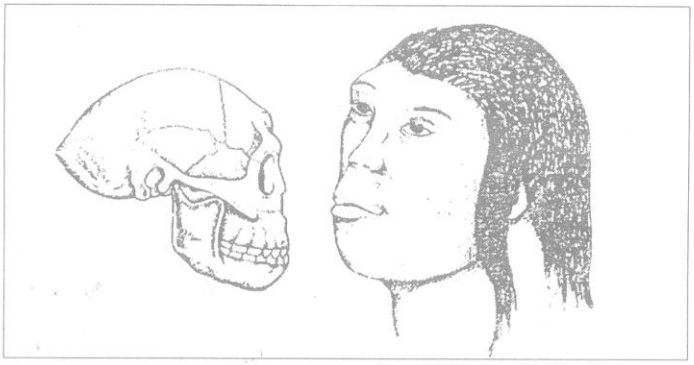
The first human types, evolved into modern man Homosapiens thorough a number of intermediate human types.



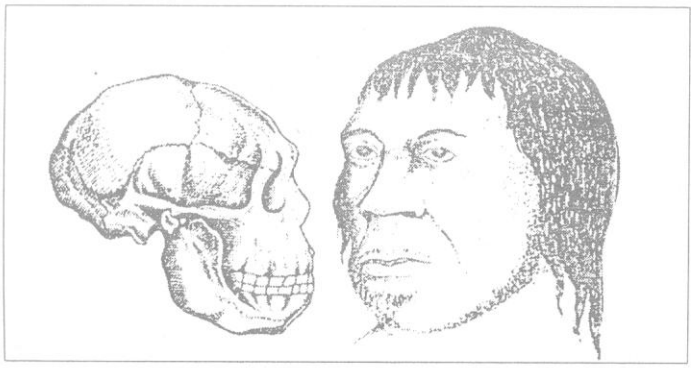
Homo erectus erectors → Jaw man



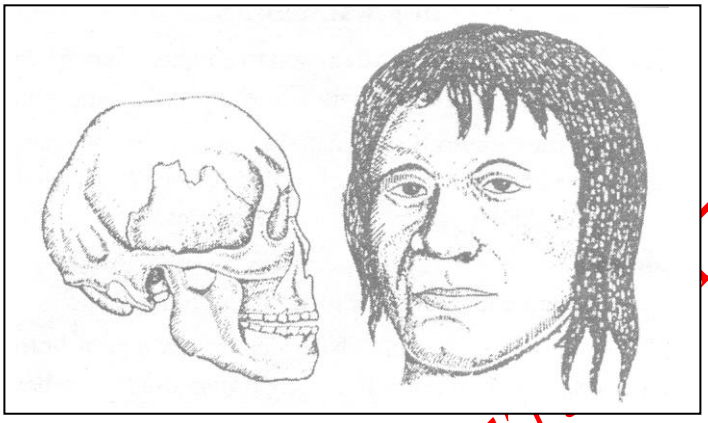
Homo erectus pekinesis → peking man



Homo sapiens neanderthalensis → Neanderthal man



In the course of their evolution theses migrant human types went forward and backwards and moved in and out of south Africa. Modern man evolved from Cro-Magnon man about 25000 years ago and spread all over the world about 10,000 years.



Modern man is divided into four ethnic groups :

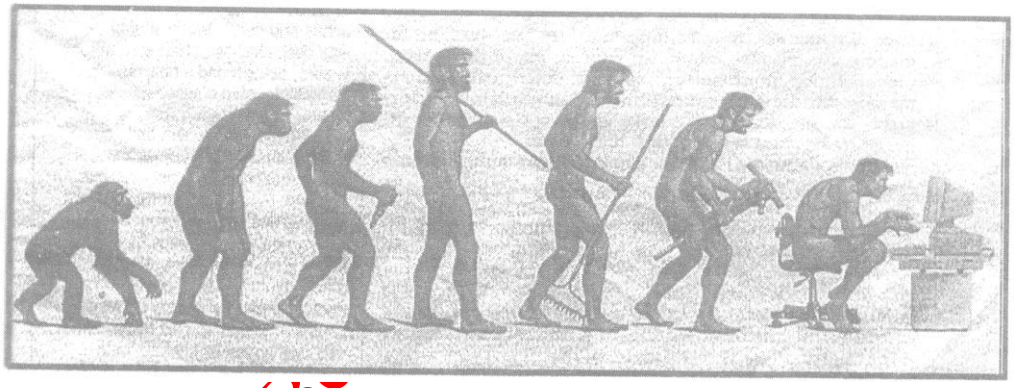
Negroid : African Pygmies and bushman

Caucasian : Italian English

Eastern : Chinese Japanese Eskimos

Homo sapiens fossils → Cro-Magnon man

Mongoloid : These ethnic group differ from one another in their skin coloration lips and hair pattern but all of these belong to same species because these are not reproductively isolated from one another. All human races have same chromosomes number and similar grass morphology of chromosomes.



Man of future :

Homo sapiens futuralis.

(A prediction by American anthropologist **Dr. Sapiro.**)

Questions :

1. Give one term for scientific study of human evolution.
2. Name some human type.
3. In evolutionary terms can we say which among bacteria, spiders fish and chimpanzees have a better body design why or why not ?
4. How human evolution took place over the years.
5. Define Anthropology ?

QUICK REVISION

1. **Heredity** is the continuity of features from one generation to another. Heredity information is present in fertilized egg or zygote. The zygote develops into an organism of a particular type only.
2. **Variation** can be define as the occurrence of differences among the individuals. No two individuals are exactly alike. Variations arising during the process of reproduction can be inherited. These variations may lead to increased survival of the individuals. Variations in species may confer survival advantages or merely contribute to the genetic drift.

3. Genetics is the branch of biology which deals with heredity and variation.

4. Accumulation of variation during reproduction : Nest generation inherit both a common basic body design and variations from the previous generation. The second generation will have differences that they inherit from the first generation, as well newly created difference. These variations may lead to increased survival of individuals.

5. The original organisms will give rise to two individuals B and C with similar body design but with subtle differences.

6. Each of the two will form two more individuals (B_1 and B_2) and (C_1 and C_2) in the next generation.

7. Each of the four individuals in the second generation will be different from each other. While some of the differences will be unique, others will be inherited from their respective parents, who were different from each other.
In asexual reproduction, there would be minor differences generated due to errors in DNA copying but in sexual reproduction greater diversity will be generated.

8. Evolution : Evolution can be defined as a naturally occurring slow, continues and irreversible process of change. The gradual change of living organisms from pre-existing organisms since the beginning of life is called organic evolution. Whereas gradual change in elements from one form to another with time is termed as inorganic evolution.

9. Variations occur during reproduction both because of error in DNA copying and as a result of sexual re-production. Variations contribute to evolution.

10. Natural selection results in adaptations in population to fit their environment better. Thus, natural selection direct evolution in the population of a particular species.

11. Inherited traits are those which are passed from one generation to another through specific genes. Any change in DNA of the germ cells be passed.

12. Acquired traits are those traits which are acquired by the organism in its life time. e.g. removal of tail cannot change the genes of the germ calls of the mice thus cannot be passed to next generation.

13. Origin of life on earth : Darwin's theory of evolution tells us how life evolved from simple to more complex forms and Mendel's experiments give us the mechanism for the inheritance of traits from one generation to the next. But neither tells us anything bout how life began on earth in the first place.
(i) **J.B.S. Haldane** suggested in 1929 that life must have developed from the simple inorganic molecules which were present on earth soon after it was formed. He speculated that the conditions on earth at that time could have given rise to more complex organic molecules that were necessary for life.
The first primitive organism would arise from further chemical synthesis.
(ii) **How did organic molecules arise ?**
An answer was suggested by the experiment conducted by **Stanley L. Miller** and **Harold C. Urey.** They assembled an atmosphere similar to that which was thought to exist on early. They assembled molecules like methane, ammonia, hydrogen sulphide and water (no oxygen). This mixture was maintained at a temperature just below 100^0C and speaks ware passed through the mixture of gases to simulate lightning. At the end of a wee, 15% of the carbon had been converted to simple compound of carbon including amino acids which make up protein molecules. These simple organic compounds further form complex, organic compounds which later organized into tho living matter.

14. **SPECIATION** means origin of new species from the existing one.

To explain the mechanism of speciation, consider a situation that the bushes on which beetles feed on are widely spread over a mountain rang. The beetle population becomes very large as a result of reproduction. Individual beetles do not travel far. So, in this huge population there will be sub-populations in neighborhoods. Mostly reproduction will be within these sub-populations. Occasionally adventurous beetle might go from one site to another or beetle is dropped to another site by crow. In either case, the migrant beetle reproduce with the local population. As a result

genes of the migrant beetle enter a new population. This kind of gene flow occur between population that are partly but not completely separated.

If between two such sub-populations a large river comes into existence, the two populations will be further isolated. The level of gene flow between them will decrease.

Over generations, genetic drift will accumulate different changes in each sub-population. Natural selection may also operate simultaneously in different way in these geographically isolated sub-populations. Together, the processes of genetic drift and natural selection will result in these two isolated sub-populations of beetles becomes more and more deferent from-each other. Ultimately, members of these two groups will be incapable of reproducing with each other. [Thus reproductive isolation occur between these two groups]. Effectively, new species of beetles are being generated.

15. Evolution and classification : Characteristics are details if appearance or behaviour, in other words, a particular form or a particular function. e.g. we have four limbs is thus a characteristic. The more characteristics two species will have in common, the more closely they are related, the more recently they will have had a common ancestor. Classification of species is infect a reflection of their evolutionary relationship. We can thus build up small groups of species with recent common ancestors, then super groups of these groups with more distant common ancestors and so on. We can keep going backwards like this until we come to the notion of a single species at the very beginning of evolutionary time.

(A) **Tracing evolutionary relationships :** When we try to follow evolutionary relationship, we identify char-ataractics in common. These characteristics in different organism would be similar because they are inherited form a common ancestor.

(i) **Homologous organs** are those having similar basic structure but has been modified to perform different functions. e.g., forelimb of reptiles, frog, lizard bird and human (amphibians and mammals) are homologous organs. Such homologous characteristic helps to identify an evolutionary relationship between apparently different species.

(ii) Analogous organs are those oranges which are different in basic structure but perform same function. e.g., wings of bat and wings of birds.

(B) **Fossils :** Evolution can be worked out by the study of not just living species, but also fossils.

Fossils are the impression or remains of ancient of life found preserved in the sedimentary rocks. There are two ways to know the age of fossils :

(i) **One is relative.** If we dig into the earth and start finding fossil, the fossils we find closer the surface are more recent than the fossils we find in deeper layers.

(ii) **Second way is to date the fossils.** Dating fossils is done by detecting the ratios of different isotopes of the same element in the fossil material.

(C) **Evolution by stages**

(i) Complex organs may have evolved because of the survival advantage of even the intermediate stage e.g., In evolution of eye, an intermediate stage, such as rudimentary eye in Planarian give a fitness advantages to it.

(ii) A change that is useful for one property to start with, can become useful later four quite a different function. During course of evolution, features or organs may be adopted to new functions, e.g., feathers are thought to have been initially evolved for warmth and later adapted for flight.

(iii) Another way of tracing evolutionary relationships is to compare the DNA of different species. Comparing the DNA of different species give us direct estimates of how much the DNA has changed during formation of theses species.

Evolution cannot be said to prepress from lower forms to higher forms. There are multiple branches possible at each and every stage of this process. A new species has emerged but that does not man that the old species will disappear. In fact, there is no real 'progress' in the idea of evolution. Evolution is to simplify the generation of diversity and the shaping of the diversity by environmental selection. Emergence of more and more complex designs is the only progressive trend in evolution.

Human evolution : There is great diversity of human forms and features across the planet. Evolutionary studies showed that all humans are a single species. The earliest members of the human species, homo sapiens, can be traced to Africa. Like all other species, they developed genetic variations and gave rise to present forms of human beings.

16. **Dominant and recessive trait :** Sexually reproducing individuals have two copies of genes for the same trait. If the copies are not identical, the trait that gets expressed is called the dominant trait and the other is called the **recessive trait.**

Traits in one individual may be inherited separately, giving rise to new combinations of traits in the offspring of sexual reproduction.

EXERCISE # 1 FOR SUMMATIVE ASSESSMENT

1. Who introduced the idea of a spontaneous generation ?

(A) Anaximander (B) Empedocles (C) Anaximus (D) Aristotle

2. Spontaneous generation of bacteria from decomposing broth was disproved in 1860 by :

(A) Joseph Lister (B) Louis Pasteur (C) Francesco Redi (D) Lazzaro Spallanzani

3. The principle of sterilization is based upon experiments carried by :

(A) Oparin (B) S.L Miller (C) L. Pasteur (D) V. Helmont

4. Life was created by some supernatural power. This theory is :

(A) Spontaneous generation (B) Spore theory

(C) Special creation (D) All of these

5. About how long ago was the earth formed ?

(A) 4.6 billion years ago (B) 10 billion years ago

(C) 3.0 billion years ago (D) 20 billion years ago

6. Modern theory of origin of life was propounded by :

(A) Miller (B) Darwin (C) Khoranaq (D) Oparin

7. Which of the following gas was absent on primitive earth :

(A) O_2 (B) CH_4 (C) CO_2 (D) NH_3

8. Which English scientist worked on origin of life and finally settled in India:

(A) A.I. Oparin (B) Louis Pasteur (C) J.B.S. Haldane (D) archbishop Ussher

9. Nucleoproteins most probably gave the first sign of :

(A) Proteins (B) Mimicry (C) Evolution (D) Life

10. Life originated :

(A) In water (B) On land (C) In air (D) In all of these

11. There is no life in moon due to the absence of :

(A) water (B) light (C) oxygen (D) temperature

12. Homologous organ are :

(A) similar in origin with similar or dissimilar functions

(B) similar in origin which are dissimilar functions

(C) dissimilar in origin and dissimilar in structures

(D) dissimilar in origin but similar in functions

13. Which of the following are homologous structures :

(A) ginger and sweet potato (B) trunk of elephant and hand of chimpanzee

(C) nail of man and claw of cat (D) wing of bird and butterfly

14. Evolution o diversified species due to environmental changes is called :

(A) Divergent evolution (B) Convergent evolution (C) Evolutionary inertia (D) None of these

15. Wings of birds and butterflies are :

(A) homologous organs (B) analogous organs (C) vestigial organs (D) grafted organs

16. Which of the following is a vestigial structure in man ?

(A) Muscle of glottis (B) Wisdom tooth

(C) Intestine (D) Ear Pinna

17. Which of the following is not a vestigial organ in man :

(A) wisdom tooth (B) vermiform appendix (C) ileum (D) muscles of ear Pinna

18. Peripatus is a connecting link between :

(A) annelids and mollusks (B) annelids and helminthes

(C) annelids and arthropods (D) reptiles and mammals

19. A connecting link between Protozoa and Porifera is :

(A) Chlamydomona (B) Euglena (C) Protopterus (D) Proterospongia

20. Which one represents a connecting link as an evidence from comparative anatomy in favor of organic evolution ? (A) Whale between fishes and mammals

(B) Duck-billed platypus between reptiles and mammals

(C) Java ape-man between modern man and Peking man

(D) Archaeopteryx between birds and mammals

21. Fossil remains of Archaeopteryx indicate that :

(A) reptiles gave rise to birds during Jurassic

(B) reptiles gave rise to birds during Permian

(C) it was a flying reptile from Permian

(D) it was a flying reptile from Triassic

22. "Ontogeny repeats phylogeny" was coined by :

(A) Darwin (B) Lamarck (C) Morgan (D) Haeckel

23. Study of human evolution is under :

(A) Paleontology (B) Anthropology (C) Arthrology (D) Mammology

24. Evolution of man is believed to have taken place in :

(A) Central America (B) Africa (C) Asia (D) Australia

25. The main advantage to bipedal locomotion is :

(A) Availability of hands for other uses (B) Increased speed

(C) Better body balance (D) Reduced weight

26. At the time of origin of life which was not existed on free stage :

(A) NH_3 (B) CH_4 (C) O_2 (D) H_2

27. At the time of formulation of theory of natural selection, Darwin was influenced by :-

(A) Essay of Malthus on population (B) Cell theory

(C) Mendel's laws of inheritance (D) None

28. The cause of mutation is :-

(A) Changes in DNA (B) Changes in chromosome

(C) Changes in gene (D) All

29. Galapagos island is associated with :-

(A) Lamarck (B) Darwin (C) Mendel (D) Wallace

30. If an organ is used , it gets developed and if an organ is not used , it gets reduced. It was given by :-

(A) Darwin (B) De Vries (C) Lamarck (D) Miller

31. The search of life on other planets or in the space is celled as :-

(A) Space biology (B) Exobiology (C) Planet biology (D) None

32. The colleague of Darwin on his ship Beagle was :-

(A) Dr. Hensley (B) Packard (C) Gado (D) None

33. The book written by Weisman:-

(A) Philosphie zoologic (B) Principles of geology (C) Germ fluid (D) None

34. The Spark-discharge apparatus to test chemical evolution of life was designed by :-

(A) Urey and Miller (B) Oparin and Haldane (C) Dixon and Jolley (D) Jacob and Monad

35. Swan-necked flask experiment was done by :-

(A) Francisco Redi (B) Aristotle (C) Robert Koch (D) Louis Pasteur

36. The presence of gill slits in the embryos of all vertebrates supports the theory of :-

(A) biogenesis (B) recapitulation (C) metamorphosis (D) organic evolution

MULTIPLE CHOICE OBJECTIVE QUESTIONS

37. Wings of locust, pigeon & bats are the example of :-

(A) Vestigial organs (B) Analogous organs (C) Homologous organs (D) Exoskeleton

38. Age of fossils can be detected by :-

(A) Residual quantity of calcium (B) Quantity of radioactive carbon compound

(C) Phylum of other mammals (D) Structure of bones

39. Term "evolution" was first used by :-

(A) Spancer (B) Mendel (C) Weismann (D) Kellogg

40. The factors which causes genetic drift :-

(A) Mutations & variations

(B) Natural selection & small size of population

(C) Random copulation

(D) All the above

41. Connecting link between Annelida & Mollusca is :-

(A) Cuttle fish (B) Octopus (C) Neopilina (D) Nautilus

42. "Struggle for existence" & survival of fittest are associated with :-

(A) Lamarckism (B) Oparin hypothesis (C) Mendelism (D) Darwinism

FILL IN THE BLANKS

1. The life arose from nonliving molecules was suggested by……..

2. Experimental evidence in favor of chemical origin of life provided by ……and ……

3. The process by which new species arise is called……

4. The biogenetic law was proposed by……

5. The fossils of birds ……..show that biers have evolved from reptiles.

6. Darwin made an extensive study of the flora and fauna of the …….Islands in South America.

7. The origin of species was written by……..

8. Organs similar in structure and origin but different in function are called …….

9. Vermiform appendix in human is a……organ.

10. The evolution of eye is an example of evolution by ……..

ASSERTION AND REASON TYPE

Each question contains STATEMENT -1(Assertion) and STATEMENT – 2 (Reason). Each questions has 5 choices (A), (B), (C), (D) and (E) out of which only one is correct.

(A) Statement-1 is True, Statement – 2 is True ; Statement – 2 is a correct explanation for Statement – 1

(B) Statement – 1 is True, Statement – 2 is True ; Statement – 2 is NOT correct explanation for statement – 1

(C) Statement – 1 is true, Statement – 2 is false.

(D) Statement – 1 is false, Statement – 2 is false.

1. **Statement – 1 :** Ear muscles of external ear in man are poorly developed.
Statement – 2 : These muscles are useful which move external ear freely to detect sound efficiently.

2. **Statement – 1 :** Although living organism always arise from other living organisms, life should certainly have had a beginning.
Statement – 2 : The study of the condition and the mechanism involved in the creation of most primitive living structures on earth is actually the problem of origin of life.

3. **Statement – 1 :** The establishment of reproductive isolation is an event of biological significance
Statement – 2 : In the absence of reproductive isolation species can merge back into single population.

4. **Statement – 1 :** The birds have large, light spongy bones with air sacs.
Statement – 2 : These adaptations help them during flight.

ANSWERS

OBJECTIVE QUESTIONS :

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	C	C	A	D	A	C	D	A	A	A	C	A	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	C	D	B	A	D	B	B	A	C	A	D	B	C
Que.	31	32	33	34	35	36	37	38	39	40	41	42			
Ans.	B	A	C	A	D	B	B	B	A	D	C	A			

FILL IN THE BLANKS :

1. Anaximander **2.** A.I. Oparin, J.B.S. Haldane **3.** Speciation **4.** Haeckel
5. Archaeopteryx **6.** Galapagos islands **7.** Charles Robert Darwin
8. Homologous organ **9.** Vestigial **10.** Stages

ASSERTION AND REASON TYPE :

1. C **2.** A **3.** A **4.** A

EXERCISE # 2 NCERT SOLVED EXERCISE

1. What factors could lead to the rise of a new species ?

Ans. Following factors could lead to rise of a new species :

(i) Natural selection (ii) Processes of genetic drift.

(iii) Mutation (sudden change in genetic makeup) (iv) Environmental factors.

(v) Local factors

(vi) The differences or variations from one generation to the next.

2. Give an example of characteristic being used to determine how close two species are in cautionary terms.

Ans. Forelimbs of human and birds show the closeness between theses species. The basic structure of the limbs is similar, though it has been modified to perform different functions.

3. Can the wing of a butterfly and the wing of a bat be considered homologous organs ? Why or why not ?

Ans. The wing of a butterfly and the wing of a bat cannot be considered homologous because they have a common use for flying but their origins and structure area not common. Rather, they are analogous organs.

4. What are fossils ? What do they tell us about the process of evolution ?

Ans. The remains of dead plants and animals which were buried under the rocks millions of years ago are called fossils. Fossils tells us about the process of evolution. The fossils of different organisms have some features similar to the other species. In this way, they show the link between two species. They tell us that one species evolves from the other species.

5. Why are human beings who look so different from each other in terms of size, colour and looks, said to belong to the same species ?

Ans. The human beings are different from each other in terms of size, colour and looks but they are said to belong to the same species. It is due to their fundamental characteristics. The fundamental characteristics of one species are more

closer among these organisms than the other organisms of other species. All human have same fundamental characteristics. So they belong to the same species.

6. In evolutionary terms, can we say which among bacteria, spiders, fish and chimpanzees have a better body design ? Why or why not ?

And. Chimpanzees have a better body design because they a well-developed body system, well-developed brain, and the thumb opposite to the finger to catch very small and minute things.

7. How are the areas of study – evolution and classification – interlinked ?

Ans. Evolution and classification are interlinked with each other. Classification is the most important term to explain evolution. Classification is based on the similarities and differences between two species or among two organisms. More closer the characteristics, are the more closer is the evolution and more chances are to remain in the same group of classification. We can appreciate that classification of species is in fact a reflection of their evolutionary relationship.

8. Explain the importance of fossils in deciding evolutionary relationship.

Ans. Fossils are the remains of dead plants or animals which died millions of years ago. The study of fossils helps us to know about the evolution or the link between two species. Fossils tells us how new species are developed from the old. So fossils have an important deciding evolutionary relationship.

9. How does the creation of variations in a species promoter survival ?

Ans. Natural selection the individuals having useful variations which ensure their survival in the prevailing conditions of environment. Variant individuals that can withstand or cope with prevailing environment will survive better and will increase in number through differential reproduction.

10. What are the different ways in which individuals with a particular trait may increase in a population ?

Ans. Different ways are : variation, natural selection and genetic drift (isolation).

11. Why are traits acquired during the lifetime of an individual not inherited ?

Ans. Because acquired characters bring changes only in non reproductive tissue and cannot change the genes of the germ cells. Thus acquired traits cannot be passed to next generation.

12. Why are the small numbers of surviving tigers a cause of worry from the point of view of genetics ?

Ans. **(i)** If any natural calamity occurs and kills this small number of surviving tigers, they can become extinct resulting in the loss of some genes forever.

(ii) Small number will lead to little recombination and therefore lesser variations. These both are very important for giving better survival chances to the species.

(iii) Less number of species means lesser extent of diversity and lesser number of traits which reduces the chances of adaptability with respect to the change in the environment.

13. What factors could lead to the rise of a new species ?

Ans. Genetic variations, natural selection and reproductive isolation could lead to the rise of a new species.

14. Will geographical isolation be a major factor in the speciation of a self-pollinating plant species ? Why or why not ?

Ans. No, because pollination occur on the same plant in self pollination plant species.

15. Will geographical isolation be a major factor in the speciation of an organism that reproduces asexually ? Why or why not ?

Ans. No, because asexual reproduction involves single parent or organism.

16. Give an example of characteristics being used to determine how close two species are evolutionary terms ?

Ans. Homologous organs, analogous organs and vestigial organs helps to identify evolutionary relationships amongst the species.

17. Can the wing of butterfly and the wing of a bat e considered homologous organs ? Why or why not ?

Ans. No, wing of a bat and wing of a bird cannot be considered as monologues organs because they have different basic structure.

18. What are fossils ? What do they tell us about the process of evolution ?

Ans. Fossils are the impression or remains of ancient life found preserved in the sedimentary rocks. Fossils are directs evidences of evolution. Fossils also help to identify evolutionary relationship between apparently different species. They also tell about the extent of evolution that has taken place.

19. The human hand, cat paw and horse foot, when studied in detail show the same structure of bones and point towards a common origin. What do you conclude from this ? What is the term given to such structures ?

Ans. **(i)** It shows an evolutionary relationship between apparently different species, i.e., one form has evolved from the other.

(ii) These are called as homologous organs.

20. What is molecular phylogeny ?

Ans. The approach of molecular phylogeny is based on the idea the organisms which are more distantly related will accumulate a greater number of differences in their DNA. Such studies traces the evolutionary relationships among different organisms and thus help in their classification.

21. How does the creation of variations in a species promote survival ?

Ans. Natural selection selects the individuals having useful variations which ensure their survival in the prevailing conditions of environment. Variant individuals that can withstand or cope with prevailing environment will survive better and will increase in number through differential reproduction.

22. Define heredity.

Ans. Heredity can be defined as, "resemblances among individuals related by descent" or transmission of traits from parents to the offsprings".

23. Define variation.

Ans. Variation can be defined as "the occurrence of differences among the individuals".

24. Give example of analogous organs.

Ans. Wings of bat and wings of insects.

25. Define speciation.

Ans. Speciation means origin of new species from the existing one.

26. Who gave the idea of evolution of species by natural selection ?

Ans. Charles Darwin gave theory of natural selection.

27. What are the basic events in evolution ?

Ans. The changes in DNA during reproduction are the basic events is evolution.

28. What is genetic drift ?

Ans. The change in the frequency of certain genes in population over generations.

29. What do you mean by natural selection ?

Ans. Natural selection is the phenomenon in the nature the selects traits favorable to species in adjusting, to its environment.

30. Define fossils .

Ans. Fossils are the impression or remains of ancient life found preserved in the sedimentary rocks.

31. What is meant by 'characteristics' ?

Ans. Characteristics are details of appearance or behaviour ; in other words a particular form or a particular function.

32. What are the tools used for studying human evolution ?

Ans. Excavating, time-dating and studying fossils.

33. How does the mechanism of heredity work ?

Ans. Cellular DNA is the information source for making protein. A section DNA that provides information for one protein is called the gene for that protein. Proteins control the characteristics.

34. Describe briefly four ways in which individuals with a particular trait may increase in a population.

Ans. Traits arise due to variations which occur due to sexual reproduction or inaccuracies during DNA copying or environmental factors. Now, the individuals with a particular trait may increase in a population due to the following factors :

(i) Natural selection : Those variations which give survival advantage to an organism are selected nature and such traits increase in population.

(ii) Genetic drift : It occurs due to change in gene frequency due to accumulation of particular type of genes.

(iii) Geographical isolation : It leads to change in gene frequency leading to expression of one type of traits in a geographically isolated population.

(iv) Migration : It leads to flow of a particular type of gene in a specific population.

35. Distinguish between homologous and analogous organs. To which of theses categories the spine of cactus and thorn of Bougainvillea belong and why ?

Or

Where are genes located ?

What is the chemical nature of genes.

Ans. (a)

Homologous organs	**Analogous organs**
The orgs which show structural similarity in their origin but have different functions are called homologous organs.	The organs which are structurally different but perform the same function are called analogous organs.
e.g. the forelimbs of a frog , a bird and a human being ar homologous organs.	e.g., the wings of birds and insects are analogous organs.

(b) The spine of CACTUS and thorn of BOUGAINVILLEA are structurally different but they perform the same function and therefore, they will be called analogous organs.

Remember : Spine of cactus, is the modification of a leaf while thorn of bougainvillea is humidification of stem.

Or

Genes are located on a chromosome and occupy specific positions. Genes are the segments of DNA having specific sequence of nucleotides which determines the functional property of a gene.

36. Define acquired and inherited traits with examples.

Ans. Inherited traits are those traits which are passed from one generation to another through specific genes. Any change in DNA of the germ of cells will be passed to next generation resulting in variations. Acquired traits are those traits which are acquired by the organism in its life time. e.g., removal of tail cannot change the genes of the germ cells of the mice thus cannot be passed to next generation.

ENVIRONMENT

Environ – surrounding **– Everything which surrounds and influences the organism directly or indirectly** is included in the environment

OR

The environment is the sum total of all living and non-living factor that surround and an organism. It has two components :-

(a) Biotic components	(b) Abiotic components
⇓	⇓
All Animals, Plant, Micro-organisms	Air, Water, Temperature, Humidity, Topography etc.

★ The study of environment includes the study of its various components, and their protection, preservation & management.

Ecology :- Greek word Oikos – home + logos – to study

Ecology means study of habitat or living or living place of organisms or study of organisms at home.

OR

"The study of inter relationships of organisms between the biotic and abiotic components of their environments. The 'Ecology' term was coined by **Ernst Haeckel (1868).**

- **Ecosystem :**
  - ◈ Ecosystem is the structural & functional unit of biosphere.
  - ◈ It has a property of self perpetuation.
  - ◈ It is an open system and depend on sun light.
  - ◈ It may be small and also very large.
  - ◈ Ecosystem term coined by Sir Arthur Tansley (1935).
- **Definition :** According to **Eugene P. Odum-Ecosystem is the basic unit of ecology and the biotic and abiotic components present in it influence each other. Both the components are essential for the sustenance of life.**

OR

A self-contained unit of living things [Plants, animals & decomposers] and their non-living environment [soil, air & water].

TYPES OF ECOSYSTEM

In the biosphere, ecosystem may be classified on the basis of their nature, duration and size :

(A) Nature : On the basis of nature, ecosystem may be classified as :

(1) Natural ecosystem

(2) Artificial ecosystems

(1) Nature ecosystem : These ecosystem operate in the nature by themselves without any human interference. Common examples of natural ecosystem are : a pond , a lake, a meadow , a desert, a grassland , a forest, and ocean etc.

(2) Artificial ecosystem : These are maintained by man and hence are also termed **man-made** or **man-engineered** ecosystem. In these ecosystem, man maintains / disturbs the natural balance by the addition of energy and planned manipulations Common examples of artificial ecosystem are croplands, orchards, gardens, aquarium etc.

(B) Duration : On the basis of duration, ecosystem may be classified as :

(1) Temporary ecosystem : e.g. – short lived ecosystem, rain fed pond

(2) Permanent ecosystem : ex. – a lake, a forest, a desert

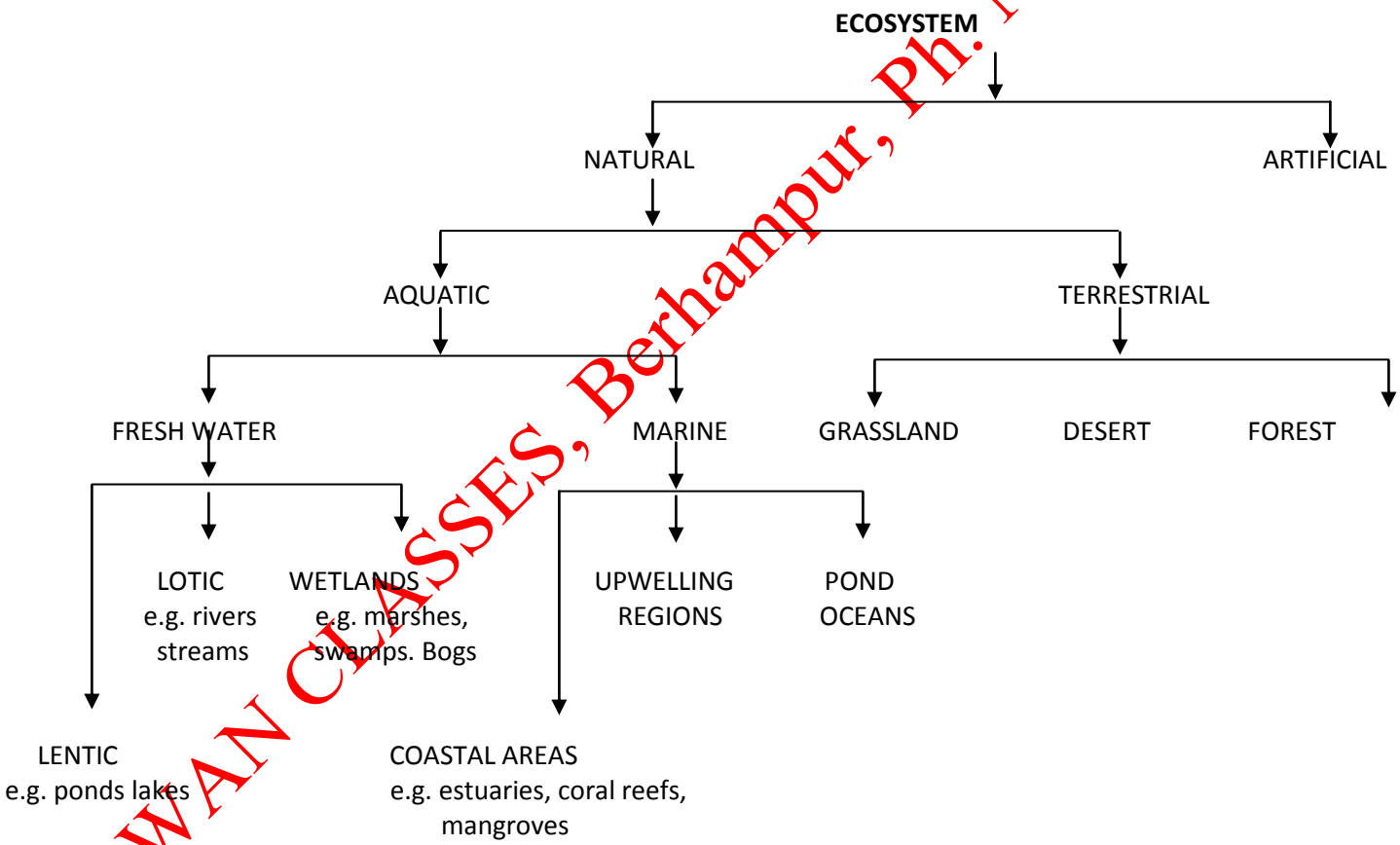
(C)Size :

(1) Small ecosystem : eg :- lake, a forest, a desert

(2) Large ecosystem : eg :- forest, desert, ocean.

Components of Ecosystem :

The various components of any ecosystem may be grouped into two main types :



(A) Abiotic (non-living) components

These include the non-living physio-chemical factors of the environment

ABIOTIC FACTORS INCLUDE :

(1) Inorganic substance **(2) Organic compounds** **(3) Climatic factors**

(1) Inorganic substance : Inorganic substances, e.g., carbon, nitrogen, oxygen, calcium, phosphorus etc. and their compounds (water, carbon, dioxide, etc.) constitute the main abiotic component.

(2) Organic compounds : These include carbohydrates, proteins, lipids, nucleic acids etc.

(3) Climate factors : These include light, temperature, humidity, wind , rainfall, water etc. and also **edaphic factors** (e.g., soil and substrate, topography, minerals, minerals, pH etc.)

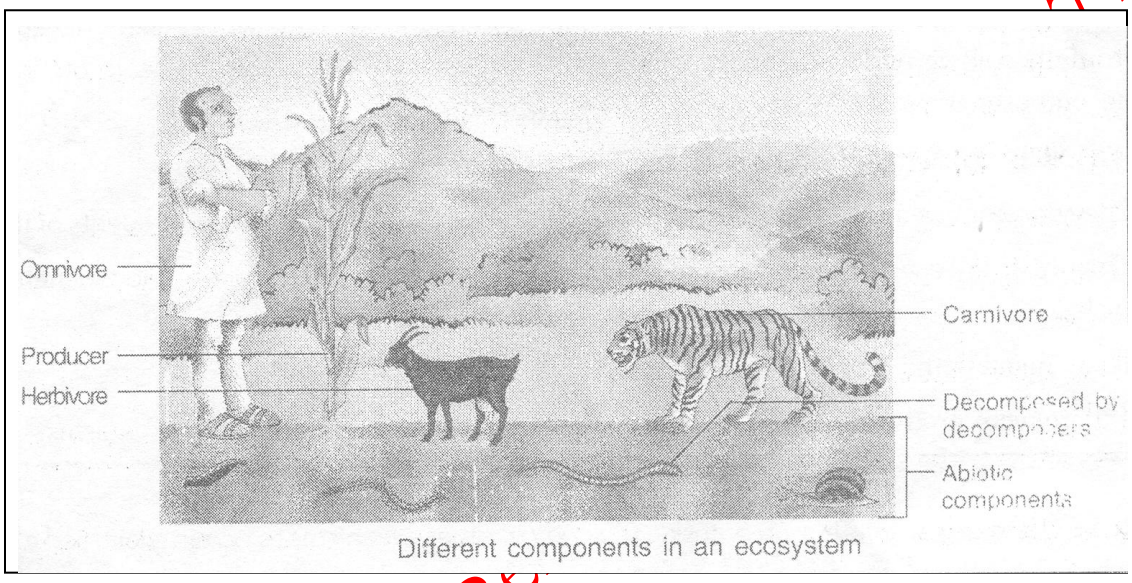
Example of Ecosystems – (i) Grass land [meadow] ecosystem, (ii) Forest ecosystem, (iii) Desert ecosystem (iv) Mountain ecosystem (v) Pound ecosystem

BIOTIC COMPONENTS :

The biotic community of an ecosystem includes 3 types of organisms :-

1. **Producers :** The organisms which can prepare their own food from simple inorganic substances like carbon dioxide and water by using sunlight energy in the presence of chlorophyll.

e.g. Green plants & Certain blue green algae.



Different components in an ecosystem

2. **Consumers :** Those organisms which consume food or eat food prepared by producers are called consumers. The consumers depends on producers for food, directly or indirectly. All the animals are consumers

Consumers can be further divided into three groups

(a) **herbivores –** Those animals which eat only plants are called herbivores.

e.g. Buffalo, Goat, Sheep, Horse, Deer, Camel.

Since herbivores obtain their food directly from plants or producers, therefore herbivores are primary consumers.

(b) **Carnivores :** Those animals which eat other animals as food are called carnivores.

Or

Those animals which eat the meat [or flesh] of other animals are called carnivores.

e.g. Lion, Tiger, Frog, Vulture, Kingfisher.

The small carnivores which feed on herbivores [primary consumers] are called secondary consumers.

e.g. Grasshopper. Rat, Frog.

The large carnivores [or top carnivores] which feed upon the small carnivores [secondary consumers] are called tertiary consumers.

e.g. Lion, Tiger, Hawk

(c) **Omnivores :** Those animals which eat both, plants & animals, are called omnivores.

Or

Those animals which eat plant food as well as the meat (or flash) of other animals.

e.g. Man, Dog, Crow, Sparrow, Bear & Ant.

(3) **Decomposers :** The micro-organisms which break down the complex organic compounds present in dead organisms like dead plants and animals and their products like faeces, urine into simple substances are called decomposes .

e.g. Bacteria & Fungi.

The decomposes help in decomposing the dead bodies of plants and animals and hence act as cleansing agents of environment.

Due to the presence of decomposers the various nutrient elements which were initially taken by plants from the soil, air and water are retuned to the soil, air and water after the death of plants animals thus decomposer organisms help in recycling the materials.

IMPORTANCE OF DECOMPOSERS

◇ They decompose the dead bodies of animals and plants, thus acts as cleansing gents of the environments.

◇ They help in recycling the material in biosphere thus play a vital role in biogeochemical or nutrient cycles.

◇ They maintain the futility of soil.

◇ If there were no decomposes, the earth would have been a heap of dead organisms.

COMPETITION WINDOW

Parasites : These organisms live on or inside the body of other organisms to obtain their food e.g. – Escherichia coli (bacteria) tapeworm.

Detritivores : These are organisms which feed on detritus (dead remains of plants & animals) They are also called as scavengers.

REVIEW QUESTIONS :

1. Name two non-biodegradable wastes ?

2. What are non-biodegradable wastes ?

3. Give the technical term for the sum of biotic and abiotic components of a specific area.

4. Give the importance of producers in an ecosystem.

5. Name some natural primary consumers of an aquatic (pond) ecosystem .

6. Give technical term for the group of organisms deriving from same source.

FOOD CHAIN

It is a sequence of organisms through which energy is transferred in the from of food by the process of one organisms consuming the other ?

Or

List of organisms [living beings] showing "who eats whom" is called a food chain.

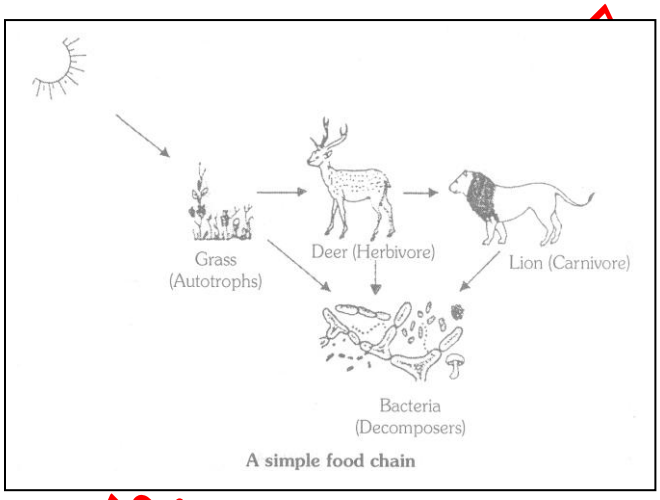
→ A food chain represents unidirectional transfer of energy

→ All the food chains begin with a green plants.

(i) Grass → Deer → Lion
(Producer) (Herbivore) (Carnivore)

(ii) Grass → Insects → Frog → Birds
(Producer) (Herbivore) (Carnivore) (Top carnivores)

CHARACTERISTICS OF FOOD CHAIN



A simple food chain

1. A food chain involves a nutritive interaction between the living organisms (biotic components) of an ecosystem in a food chain, there occurs **repeated eating ,** i.e., each group eats the other group and subsequently is taken by some other group organisms .
2. A food chain in always straight and proceeds in a progressive straight line.
3. In a food chain, there is unidirectional flow of energy from sum to producer and subsequently to series of different types of consumers.
4. Usually, there are 3 or 4 trophic levels in the food chain, In few chains, there may be maximum of 5 tropic levels.
5. Some organisms are omnivores. These occupy different trophic positions in different food chains.
6. At each transfer, generally 80-90% of energy is lost as heat in accordance with second law of thermodynamics. There are two types of food chains.

(a) Grazing food chains

Grazing type food chain					
Type of Ecosystem	**Producers**	**Herbivores**	**Primary Carnivores**	**Secondary carnivores**	**Tertiary carnivores**
(A) Grassland ecosystem	1. Grass 2. Grass 3. Grass	Insects (Grasshopper) Rats and mice Rabbits	Frog Snakes Foxes	Snakes Predatory birds Wolves	Predatory birds Lions
(B) Pond ecosystem	Phytoplanktons	Zooplanktons	Small fish	Large fish	Predatory birds

(b) Detritus food chain

Detritus food chain				
Detritus	**Detritivores**	**Detritivores consumers**	**Small carnivores**	**Large carnivore**
Mangrove fallen leaves and dead bodies of animals	**Fung, Bacteria and Protozoans**	**Insects larvae, certain crustaceans, mollusks and fish**	**Minnows, small game fish, etc.**	**Large fish, fish-eating birds**

SIGNIFICANCE OF FOOD CHAINS

The significance of food chains can be seen with the help of following functions :-

(i) It is a means of transfer of food from one trophic level to another.

(ii) It provides information about the living components of an ecosystem.

(iii) It helps us in understanding the interactions and interdependence amongst different organisms in an ecosystem .

(iv) It is pathway for the flow of energy in any ecosystem.

- Plants ⟶ Men (Two trophic levels)
- Plants ⟶ Goat ⟶ Men (Three trophic levels)
- Plants ⟶ Mice ⟶ Snakes ⟶ Peacocks (Four trophic levels)
- Plants ⟶ Insects ⟶ Frogs ⟶ Snakes ⟶ Eagles (Five trophic levels)
- Plants ⟶ Grasshoppers ⟶ Frogs ⟶ Snakes ⟶ Hawks (Five trophic levels)

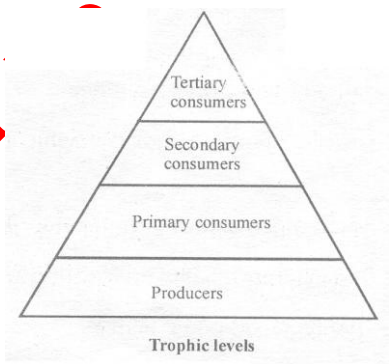
Trophic levels :- The distinct sequential steps in the food chain where transfer of energy occurs are referred to as different trophic levels. For example, green plants **(Producers)** form the **first trophic level –** the producer level ; the plant eaters **(herbivores)**, also called primary consumers, belong to **second trophic level –** the primary consumer level : and the flesh eaters **(carnivores),** also called secondary consumers, represent the **third consumer level –** the secondary consumer level and so on.

Producers – T_1 - Firs trophic level

Herbivores – T_2

Primary carnivores – T_3

Secondary carnivores – T_4

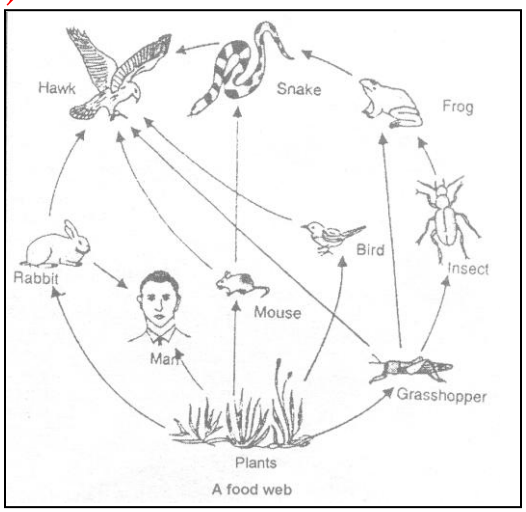


FOOD WEB

The inter – connected chains operating in an ecosystem which establish a network of relationships between various species is called a food web.

Or

The network of a large number of food chains existing in an ecosystem is called a food web.



DIFFERENT BETWEEN FOOD CHAIN AND FOOD WEB

S.No.	Food Chain	Food Web
1	**The sequence of eating and being eaten among the living organisms to transfer food energy is called food chain**	**It is a system of interconnected food chains. The network of food chains develop a relationship between various organisms**
2.	**It is having 4-5 populations (trophic levels) of different species.**	**It is having numerous populations of different species.**
3.	**It is part of food web.**	**It contains many food chains**
4.	**Food chains do not help in increasing population of endangered species.**	**Food web helps in increasing the population of endangered species .**

TEN PERCENT LEW

It was put forth by Lindeman (1942). It is also termed as second law of thermodynamics or law of entropy . According to this law, transfer of energy from one trophic level to other trophic leave is never 100 percent. It is so because most of energy gets lost as heat in the environment during each transfer. **On an average, about 10% energy is actually available to the next trophic level .**

FLOW (TRANSFER) OF ENERGY IN AN ECOSYSTEM

Each organism needs energy to carry on vital activities, and for building up and repairing the body tissues.

- The ultimate source of entire energy, used by organisms, is the **sun .**
- In a community, each food chain, in fact, represents stepwise transfer of food (energy).
- Of the total solar radiations falling on the earth, only about 10% are captured by green plants in a terrestrial ecosystem and construed into food energy by photosynthesis. This energy is stored as **chemical energy of food.**
- When green plants are eaten by herbivores (primary consumers), a great of energy is lost as hest to the environment in accordance with the second law of thermodynamics. On an average, 10% of the food (energy) is turned into body of herbivores. In the same way. 10% of total energy available to the herbivores is made available for the next level of consumers. In other words, on an average, 10% of the amount of organic matter that is present at each step reaches the next consumer level.
- Since, amount of available energy goes on decreasing at each trophic level, food chains usually consist of only 3 or 4 steps and rarely maximum of 5 steps.
- In an ecosystem, generally the producers are maximum in number. As we move along the chain, the number of individuals at each trophic level decreases .

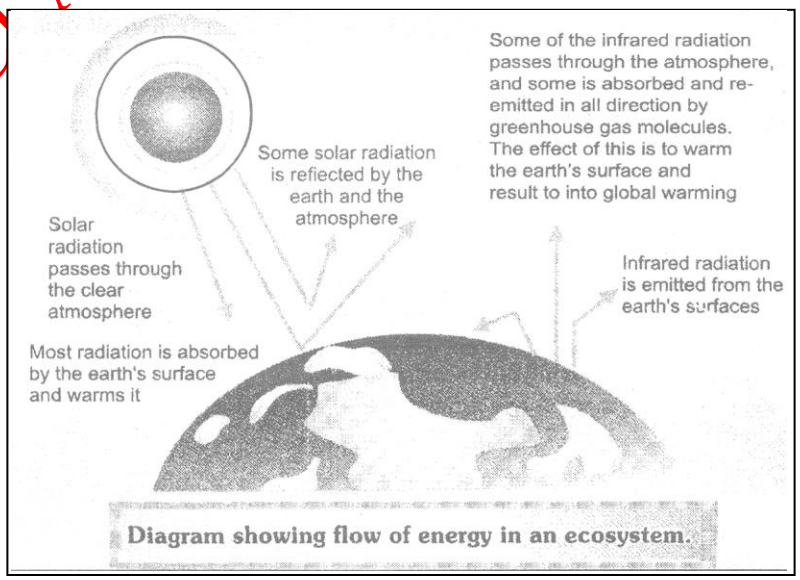


Diagram showing flow of energy in an ecosystem.

Do you know ?

- In an ecosystem, energy comes from the sunlight. It is converted from one form to another.
- Energy gets continuously transferred from one trophic level to other by repeated eating in a food chain.
- There occurs loss of energy as heat at each step of energy transfer (trophic level) in the environment.
- On an average, about 10% of available energy actually gets transferred from one trophic level to other in a food chain. This limits the steps to maximum of 4 or 5 in food chain.
- The phenomenon that involves progressive increase in concentration of harmful non-biodegradable chemical at different trophic levels in a food chain called **biomagnifications.**

REVIEW QUESTIONS

1. What is detritus ?
2. What is the branch of biology dealing with energy transfer ?
3. Define ecological pyramid .
4. Who proposed 10% laq ?
5. Which type of organisms belong to T_1 and T_2 trophic levels of a food chain ?
6. Which crop has maximum energy conserving efficiency ?
7. Give the technical term for the increase in the amount of a non-biodegradable substance in successive trophic levels of a food chain.

HOW DO ONE ACTIVITIES AFFECT THE ENVIRONMENTS?

There are two major environmental problems:

(1) Solid waste and their disposal

(2) Depletion of ozone layer.

(1) Solid waste and their disposal : Solid water generally comes from residences, cattle sheds, industries, agricultural fields, any many other places. It includes peelings of fruits and vegetables, other kitchen waste, ash, paper, cow dung, human excreta, glass, plastics, leather and rubber articles, brick, worn out clothes and metal objects etc. When accumulated, these heaps of solid waste make the surroundings dirty, and pollute the soil.

GARBAGE

The sight of a dustbin overflowing and the unpleasant smell rising from it are the familiar sights and smells of a crowded city. You look away from it and hold your nose as you cross it. Have you ever thought that you also have a role to play in the creation of this unpleasant odour ? Whether we can also play any role in reducing this smell and making this waste bin book a little more attractive if we follow proper methods of disposal of the waste generated in our houses ? What happens to the materials thrown away?

Type of solid wasted :- 2 Types

- **Biodegradable Wastes :** They are those waste materials which can be broken down into simpler, non-poisonous by the action of micro-organisms, for example, Cattle dung, Waste vegetables, fruits, and Compost etc.

- **Non-biodegradable Wastes :** The waste materials which cannot be broken down into simpler substances easily in nature, for example, Aluminum Cans, Coal, DDT, Plastics and Radioactive wastes.

S.No.	Biodegradable Waste	Non-biodegradable Waste
1.	They can be broken down into simper substances by the activity of biological catalysts called enzymes (present in surrounding bacteria or other saprophytes). Physical processes like heat and temperature help in the functioning of enzymes.	They can't be broken down into simpler and harmless products because the biological catalysts called enzymes can't act upon them. They can be acted upon only by some physical processes like heat and pressure.
2.	They can enter the biogeochemical cycles.	They cannot enter the biogeochemical cycles.
3.	They become pollutants only when they accumulate in large quantities and not degraded at the right time	They always act as pollutants whether present in small or large quantity.
4.	All the biodegradable wastes should be treated properly be before discharging them into water or soil.	They can't be treated properly before discharging them into water or soil. Instead, they can be either recycled or reused.
5.	They do not persist in the environment for a long time.	They persist in the environment for a long time .
6.	**Examples :** Urine and faecal matter, Sewage, Paper, Vegetable and fruit peels, Agricultural resides, Wood and cloth.	**Examples :** Heavy metals like Mercury, Lead, Arsenic, Radioactive wastes like Uranium. Plutonium, insecticides and Pesticides like DDT and BHC.

MODES OF WASTE DISPOSAL

Some prominent methods of waste disposal are :

1. **Land fills :** In urban areas, majority of the solid wastes are buried in low lying areas to level the uneven surface of land. This method of waste disposal is commonly called land fills.
2. **Recycling of wastes :** Number of solid wastes (paper, plastics, metal, etc.) can be recycled by sending them to respective recycling. For instance, paper is sent for recycling into special paper mills : brock plastic (e.g., plastic bags, buckets, bowls, dishes, mugs, disks, etc.) are sent to plastic processing factories.
3. **Preparation of compost :** Household waste such as peeling of fruits and vegetables. Left-over food, fallen dead leaves of kitchen garden plants and potted plants etc. can be converted into compost and used as manure.
4. **Incineration or burning at high temperature :** Incineration is the process of burning of substance at high temperature (usually more than 1000^0C) and ultimately converting them into ashes. This ash can be diposed of by land fills. In cities, municipal committees / corporations generally do large scale disposal of waste by incineration .
5. **Production of biogas and manure :** Biodegradable , waste can also be used in biogas plants to generated biogas and manure. Biogas is a cheap source of fuel, and manure, a cheap fertilizer.

DEPLETION OF OZONE LAYER

OZONE GAS

Ozone is poisonous in nature

- ◈ Ozone is formed in atmosphere by the action of ultraviolet radiation on oxygen gas.
- ◈ The high energy ultraviolet radiation (UV radiation) coming from the sun splits oxygen gas into free oxygen atoms

$$O_2 \xrightarrow[\text{radiations from sun}]{\text{Ultraviolet}} 2O \quad \text{(oxygen atom)}$$

- ◈ The free oxygen atoms are highly reactive. One oxygen atom reacts with an oxygen molecule to form an ozone molecule .

$$O_2 + O \longrightarrow O_3 \text{ (Ozone molecule)}$$

- ◈ **OZONE LAYER**

It is a layer of the earth's atmosphere, where ozone is concentrated. The Ozone layer is very important for the existence of life on earth because it absorbs most of the harmful ultraviolet radiation coming from the sun and prevents them from reaching the earth.

The thinning of ozone layer is commonly called ozone depletion. Ozone is being depleted by air pollutants. Chlorofluorocarbons (CFCs) are air pollutants that are mainly responsible for the depletion of ozone layer in the stratosphere. Besides, methane (CH_4) and oxides of nitrogen (NO_x) also cause destruction of ozone.

EFFECT OF OZONE DEPLETION

(i) Ultra violet radiations cause skin cancer.

(ii) These cause damage to eyes and also can cause increased incidence of cataract disease in eyes.

(iii) These cause damage to immune system by lowering the body's resistance to diseases.

POINTS TO BE REMEMBER

(i) The depletion of ozone layer is due to the use of chemicals called CFC – chlorofluorocarbons. CFC used in Refrigerators, in fire extinguishers and in aerosol sprayers.

(ii) In 1987 in an attempt to protect ozone layer, the United Nations Environment Programmed (UNEP) forged an agreement among its member countries to freeze CFC production at 1986 levels.

(iii) Incineration – Reducing to ashes

The burning of substance at high temperature [more than 1000^0C] to form ash is called incineration.

(iv) The use of a lot of clay for making millions of kulhads daily led to the loss of fertile top soil from fields. So, the practice of using kulhads has been discontinued .

(v) Effects of lowering of ozone depletion are :-

(i) skin cancer

(ii) damaging for eyes – cataract

(iii) UV rays damage immune system by lowering the body's resistance to diseases.

(vi) Montreal Protocol (1987) – Identified eight chemicals as ozone layer depletors.

Vienna Convention for protection of ozone layer (1932) identified at total of 19 ozone depletors.

(vii) Ozone hole : Decline in thickness of ozone layer a restricted area is called ozone hole. Ozone hole was first discovered over Antarctica in 1985. Amount of atmospheric ozone is measure by Dobson spectrometer and is expressed in Dobson units (DU).

REVIEW QUESTIONS

1. What do you understand by environment ?

2. Define ecosystem.

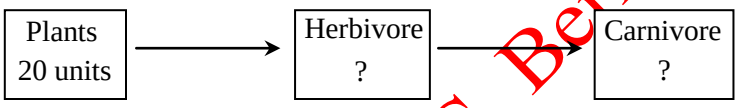
3. Define trophic levels.

4. Give two examples each of (a) natural ecosystem, (b) terrestrial ecosystem.
5. Explain the statement that "the flow of energy in an ecosystem is always unidirectional".
6. Pick out biodegradable wastes from the following : excreta, pesticides, sewage, agricultural wastes, plastic , **lead** arsenic.
7. List two harmful effects of non-biodegradable wastes.
8. What is ozone ? How is it formed ?
9. Define consumers ?
10. List any three consequences of ozone layer depletion.

EXERCISE – 1

SHORT ANSWER TYPE QUESTIONS

1. Differentiate between biodegradable and non-biodegradable wastes.
2. Describe a forest food chain.
3. Distinguish between producers and consumers.
4. Mention the importance of decomposers in an ecosystem.
5. The number of malaria patients in a village increased tremendously when a large number of frogs a exported from the village. What could be the cause for it ? Explain with the help of food chain.
6. What is food chain ? Give an example of four step food operating in a large lake or grassland.
7. Study the following food chain. How much energy (in units) will pass from producers to herbivores and from herbivores to carnivores ?



OBJECTIVE QUESTIONS

1. **Components of environment are :**
   (A) Biotic (B) Abiotic
   (C) Resources and regulatory factor (D) All the above
2. **Which one is nearly absent in ozonosphere ?**
   (A) Dust particles (B) Water vapors
   (C) CO_2 (D) All the above
3. **How many ozone molecules can be destroyed by one chlorine fragment before other chemical processes remove the chlorine from the atmosphere :-**
   (A) 10 (B) 100 (C) 1000 (D) 100, 000
4. **Ozone of ozonosphere is formed from :-**
   (A) Nitrogen oxides and Oxygen (B) Chlorine, Water and Oxygen
   (C) Oxygen and Oxygen (D) All the above
5. **Pedology is a study of :-**
   (A) Locomotion of animals (B) Rocks
   (C) Soil (D) Crop diseases
6. **Ecology (oekologic) was coined by :-**

(A) Linnaeus (B) Haeckel
(C) Harvey (D) Odum

7. Ecology is study of relationships between :-
(A) Members of a family (B) Man and environment
(C) Organisms and environment (D) Soil and water

8. Ecosystem may be defined as :-
(A) A species along with environment
(B) Plant found in water
(C) Plants found on land
(D) All plants and animal species along with environment

9. A food chain is :-
(A) Group of organisms which eat the same type of food
(B) Animals eating the plants
(C) Series of plants / animals which are interrelated in the form of organism being eaten as food by the other
(D) None of these

10. Lion is :-
(A) Producer (B) Primary consumers (C) Secondary consumers (D) Top carnivore

11. A food chain always starts with :-
(A) Photosynthesis (B) Respiration (C) Nitrogen fixation (D) Decay

12. Secondary consumers are :-
(A) Green plants (B) Herbivores (C) Carnivores (D) All the above

13. Producers in any grazing food chain :-
(A) Feed the herbivores (B) Feed the carnivores
(C) Clean the atmosphere (D) Capture solar energy

14. Ultimate source of energy for living beings is :-
(A) Carbohydrates (B) Fats (C) Sunlight (D) ATP

15. Food web is constituted by :-
(A) Various interlinked food chains in a community
(B) Relationship between animals and plants
(C) Relationship between animals plants and microbes
(D) Relationship between animals, plants and microbes

16. In pyramid of food, the producers occupy :-
(A) The base (B) Position near the base
(C) Apex (D) Position near apex

17. Energy flow in an ecosystem is :-
(A) Unidirectional (B) Bidirectional (C) Multidirectional (D) All the above

18. Which forests have maximum diversity ?
(A) Subtropical ran forests (B) Coniferous forests
(C) Tropical forests (D) Deciduous forests

19. Which is correct food chain ?
(A) Phytoplankton ⟶ Fishes ⟶ Zooplankton (B) Zooplankton ⟶ Phytoplankton ⟶ Fishes
(C) Phytoplankton ⟶ Zooplankton ⟶ Fishes (D) Fishes ⟶ Zooplankton ⟶ Phytoplankton

20. Who proposed the term ecosystem ?
(A) Odum (B) Gardner (C) Warming (D) Tansley

21. Which one is the correct food chain ?
(A) Eagle ⟶ Snake ⟶ Grasshopper ⟶ Grass ⟶ Frog

(B) Frog ⟶ Snake ⟶ Eagle ⟶ Grasshopper ⟶ Grass

(C) Grasshopper ⟶ Grass ⟶ Snake ⟶ Frog ⟶ Eagle

(D) Grass ⟶ Grasshopper ⟶ Frog ⟶ Snake ⟶ Eagle

22. Pick up the correct food chain :-

(A) Grass ⟶ Chameleon ⟶ Insect ⟶ Bird (B) Grass ⟶ Fox ⟶ Rabbit ⟶ Bird

(C) Phytoplankton ⟶ Zooplankton ⟶ Fish (D) Fallen leaves ⟶ Bacteria ⟶ Insect ⟶ larvae

23. Path of energy flow in an ecosystem is :-

(A) Herbivores ⟶ Producers ⟶ Carnivores ⟶ Decomposers

(B) Herbivores ⟶ Carnivores ⟶ Producers ⟶ Decomposers

(C) Producers ⟶ Carnivores ⟶ Herbivores ⟶ Decomposers

(D) Producers ⟶ Herbivores ⟶ Carnivores ⟶ Decomposers

24. Energy enters a food chain through :-

(A) Producers (B) Decomposers (C) Herbivores (D) Carnivores

25. The most stable ecosystem is :-

(A) Forest (B) Mountain (C) Desert (D) Ocean

ANSWER KEY																			
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**110**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	**D**	**D**	**D**	**C**	**C**	**B**	**C**	**D**	**C**	**D**	**A**	**C**	**D**	**C**	**A**	**B**	**A**	**C**	**C**	**D**
Que.	**21**	**22**	**23**	**24**	**25**															
Ans.	**D**	**C**	**D**	**A**	**D**															

EXERCISE – 2

VERY SHORT ANSWERS TYPE QUESTIONS

1. Name the protective carver in the atmosphere which absorbs harmful ultra-violet (UV) radiations.

2. What do you mean by ozone hole ?

3. Arrange the following in a food chain :

Mouse, snake, owl, plant

4. Name the various steps in the food chain where food energy is transferred.

5. Name any two step food chain in forest ecosystem.

6. Name any artificial ecosystem

7. Expand the abbreviation CFCs.

8. What kind of cups are being used in railway trains etc. for serving tea, coffee etc, now a days ?

9. Why suggestion regarding use of kulhads (disposable cups made of clay) at very large scale has been discontinued ?

10. Name three biotic components of an ecosystem.

SHORT ANSWERS TYPE QUESTIONS

1. Write the importance of decomposers in any ecosystem.

2. Write characteristics of food chains.

3. Differentiate between biodegradable and non-biodegradable substances.

4. In which part of atmosphere ozone is concentrated ? Why is it important ?

5. What do you mean by 'waste disposal' ? Explain incineration method in detail.

LONG ANSWERS TYPE QUESTIONS

1. Explain biological magnification phenomenon by taking any suitable example.

2. What do you mean by ozone depletion ? List chemicals that cause it. How can we check ozone depletion .

3. How our life styles and attitude affect the environment ? Explain with suitable examples.

4. Give examples of following :-

(i) Two-step food chain in a forest ecosystem.

(ii) Three-step food chain in a forest ecosystem.

(iii) Four-step food chain in a pond ecosystem

EXERCISE – 3

REASONING QUESTIONS WITH SOLUTIONS

1. In which layer of atmosphere is present the ozone blanket ? How is ozone being maintained ?

Ans. Ozone blanket is present in the stratosphere layer of atmosphere. In the stratosphere, ozone is being photo dissociated and generated by the absorption of harmful, short wavelength ultraviolet (uv) radiations coming from the sun.

$$O_3 \underset{}{\overset{\text{uv radiation from sun}}{\rightleftharpoons}} O_2 + [O]$$

Ozone oxygen molecule Oxygen atom

The two reactions (photo dissociation of O_3 and its generation) are in equilibrium there by maintaining steady concentration of ozone in the stratosphere.

2. Why was use of 'kulhads' (disposable cups made of clay) at a mass scale in the railways discouraged?

Ans. Use of 'kulhads' (disposable cups made of clay) at a mass scale in the railways was discouraged at making these kulhads on a large scale would have resulted in the loss of the fertile top soil.

3. Give reason why :

(i) Forest ecosystem is more stable than crop land ecosystem.

(ii) Available energy goes on decreasing at each trophic level in a food chain.

Ans. (i) Greater are the number of alternatives available at each trophic system, more stable is that ecosystem . In forest ecosystem. More alternative in the form of variety of organisms are available at each trophic level. This is not the case in cropland ecosystem. Hence, forest ecosystem is more stable.

(ii) According to second law of thermodynamics, at each step of energy transfer, majority of the energy is lost as unavailable heat. This is the reason why available energy goes on decreasing at each trophic level in a food chain.

4. Due to uncontrolled excessive hunting, the population of tigers in a forest becomes zero. Discuss the long term effects of this situation on the population of deer in that forest.

Ans. Tiger is the top carnivore of a number of food chains and is the symbol of richness of biodiversity of an area Uncontrolled hunting of tigers will lead to rapid increase in deer population, which in turn may cause ecological imbalance due to rapid consumption of vegetation of the area.

5. Why is energy flow a “one-way street” ?

Ans. Because the energy always flows in one direction in a food chain i.e. from sun to the producers (autographs) and then to various categories of consumers (heterotrophy) and not vice versa. So the energy trapped by the producers does not revert to the sun and the energy passed on to the herbivores cannot be returned to the producers.

6. What is the role of decomposers in the ecosystem ?

Ans. Decomposers are essential of ecosystem. They decompose dead remains of plants and animals and their waster organic products into simpler, inorganic substance. The latter are released into the environment for their reuse as raw materials by producers. These , therefore , provide space for new life to settle in the biosphere.

SOCIAL STUDIES

(HISTORY, CIVICS, GEOGRAPHY, ECONOMICS)

HISTORY

THE AGE OF INDUSTRIALISATION

◙ IMPORTANT TERMS

- **Innovation:** Newly produced or introduced; a novelty.
- **Guild:** An association of craft man or merchants following same craft to protect the member's interest and supervise the standard of the work.
- **Stapler:** A person who staples or sorts wool according to, its fibre.
- **Fuller:** A person who full - gathers cloth by pleating.
- **Carding:** A process of preparing fibres such as cotton or wool, prior to spinning.
- **Efficacy:** Effectiveness; power to produce certain effect.
- **Mill:** Building fitted with machinery for manufacturing processes/factory.
- **Tanning:** Convert raw hide into leather by soaking in liquid containing tannic acid.
- **Food Processing:** Technique of cropping and mixing food for making jam, juices etc.
- **Victorian Britain:** Britain during the reign of Queen Victoria.
- **Vagrant:** A person without a settled home or regular work.
- **Brewery:** A place where beer etc. in brewed commercially Brewing is a process of infusion, boiling and fermentation.
- **Night Refuge:** Night shelter for homeless people.
- **Company officials:** Officials of the East India Company.
- **Bourgeoisie:** The upper middle class.
- **Monopoly:** Exclusive right of trade in an article or good granted bgy some authority or license authorizing this.
- **Sepoy:** Indian soldier of the British army.
- **Fly shuttle:** A shuttle worked by machinery (shuttle is a small instrument or soldering thread holder that carries the under thread in a sewing machine)
- Spinning Jenny-devised James Hardgrave's in 1764.
- Building activities intensified in the cities after 1840.
- Place good accounted for 33% of Indian's export in 1811-12.
- India's export reduced by 1950-51.
- Cotton piece goods constituted over 31% by 1850.
- The first cotton mill in Bombay come up in 1854.
- Four mills were at work by 1862.
- The first Jute. Mill set up in Bengal in 1855.
- The Elgin Mill was started in Kanpur in 1860.
- The first cotton mill of Ahmadabad was set up by 1874.
- There were 2.436,000 workers in Indian factories by 1946.
- Cotton piece goods production in India doubled in between 1910-12.
- Handloom production almost trebling between 1900 and 1940.

◙ BEFORE THE INDUSTRIAL REVOLUTION

i. **The coming up of the Factory:** The earliest factories in England come up by the 1730s. But it was only in the late eighteenth century that the number of factories multiplied.

The first symbol of the new era was cotton. Its production boomed in the late nineteenth century. In 1760 Britain was importing 2.5 million pounds of raw cotton to feed its cotton industry. By 1787 this import soared to 22 million pounds. This increase was linked to a number of changes within the process of production. In the early nineteenth

century factories increasingly became an intimate part of the English landscape. So visible were the imposing new mills, so magical seemed to be the power of new technology, threat contemporaries were dazzled. They concentrated their attention on the mills, almost forgetting the bylines and the workshops where production still continued.

ii. **The pace of industrial change:** The most dynamic industries in Britain were clearly cotton and metals. Growing at a rapid pace. Cotton was the leading sector in the first phase of industrializations up to the 1840s. After that the iron and steel industry led the way. With the expansion of railways in England From the 1840s and iron and steel worth about E77 million, double the value of its cotton export.

(a) **The new industries could not easily displace traditional industries:** Even at the end of the nineteenth century, less than 20 per cent of the total workforce was employed in technologically advanced industrial sectors. Textiles were a dynamic sector, but a large portion of the output was produced not within factories outside, within domestic units.

(b) **The pace of change in the 'traditional' industries** was not set by steam powered cotton or metal industries, but they did not remain entirely stagnant either. Seemingly ordinary and small innovations were the basis of growth in many non-mechanized sectors such as food processing, building, pottery, glass work, tanning, furniture making and production of implements.

(c) **Technological changes occurred slowly:** They did not spread dramatically across the industrial landscape. New technology was expensive and merchants and industrialists were cautions about using it. The machines often broke down and repair was costly. They were not their investors and manufactures claimed.

James Watt improved the steam engine produced by Newcomer and patented the new engine in 1781. His industrialist friend Mathew Bolton manufactured the new mode. But for yeas he could find no buyers. At eh beginning of the nineteenth century, there were no more than 321 steam engines all over England, of these, 80 were in cotton industries, nine n wool industries, and the rest in mining, canal works and iron works. Steam engines were not used in any of the other industries till much later in the century. So, even the most powerful new technology that enhanced the productivity of labor manifold was slow to be accepted by industrialist.

◙ **HAND LABOUR AND STEAM POWER**

In many industries the demand for labor was seasonal. Gas works and breweries were especially busy through the cold months. So they needed more workers to meet their peak demand. Book binders and printers, catering to Christmas demand, to needed extra land before December. At the waterfront, winter was the time that ships were repaired and spruced up. In all such industries where production fluctuated with the season, industrialists usually preferred hand labor, employing workers for the season.

A range of products could be produced only with hand labor. Machines were oriented to producing uniforms standardizes goods for a mass market. But the demand in the market was often for goods with intricate designs and specific shapes. In mid-nineteenth century Britain, for instance, 500 varieties of hammers were produced and 45 kinds of axes. These required human skill, not mechanical technology.

In Victorian Britain, the upper classes- the aristocrats and the bourgeoisie-preferred things produced by hand. Handmade products came to symbolize referent and class. They were better finished. Individually produced and carefully designed. Machine made goods were for export to the colonies.

- **Life of the workers:** If you had a relative or a friend in a factory, you were more likely to get a job quickly but not everyone had social connections. Many job seekers had to wait weeks, spending nights under bridges or in night shelters. Some stayed in Night Refuges that were set up by private individuals; others went to the casual wards maintained by the poor Law authorities.

Seasonality of work in many industries meant prolonged periods without work**.** After the busy season was over, the poor were on the streets again. Some returned to the countryside after the winter, when the demand for labor in

the rural areas opened up n places, but most looked for odd jobs, which were difficult to find in the Mid-Nineteenth Century.

Wages increased somewhat in the early nineteenth century. But they tell us little about the welfare of the workers. The average figures hide the variations between trades and the fluctuations form year to year. FOI instance, when prices rose sharply during the prolonged Napoleonic war the real value of what the workers earned fell significantly, since the same wages could not buy fewer things. Moreover the income of workers depended not on the wages rate alone. What was also critical was the period of employment; the number of days of work determined the average daily income of the workers. At the best of time till the mid-nineteenth century, about 10 percent of the urban population was extremely poor. In periods of economic slump like the 1830s, the proportion of unemployed went up to anything between 335 and 75 parent in different regions.

After the 1840s, building activity intensified in the cities, opening up greater opportunism of employment. Roads were winder end new railway stations came up railway lines were extended, tunnels dug, drainage and sewers laid, rivers embanked. The number of workers employed in the transport industry doubled in the 1840s and doubled again in the subsequent 30 years.

◙ **INDUSTRIALISATION IN THE COLONIES**

(i) **The Age of Indian Textiles:** Before the age of machine industries, silk and cotton goods from India dominated often came from India, America and Persian merchants took the goods form Punjab to Afghanistan; eastern Persia and Central Asia. Bales of fine textiles were carried on camel back via the north-west frontier, through mountain passes and across deserts. A vibrant sea trade operated through the main pre-colonial rots. Surat on the Gujarat coast connected India to the Gulf and Red sea Ports. Masulipatam on the commanded coast and Hooley in Bengal had trade links with Southeast Asian ports.

However, once the East India Company established political power, it could assert a monopoly right to trade. It processed to develop a system of management and control that would eliminate competition, control costs, and ensure regular supplies of cotton and silk goods. This it did through a series of steps.

(a) **The company tried to eliminate the existing traders and brokers** connected with the trade and establish a more direct control over the weaver. It appointed a pain servant called the gomastha to weavers, collect supplies, and examine the quality of cloth.

(b) It prevented Company weavers from dealing with other buyers. One way of doing this was through the system of advances. Once an order was placed, the weavers were given loans to purchase the raw material for their production. Those who took loans had to hand over the cloth they produced to the gomastha. They could not take it many other traders.

Soon, however, in many weaving villages there were reports of clash between weavers and gomasthas. Earlier supply merchants had very often lived within the weaving villages and had a close relationship with weavers, looking after their needs and helping them in times of crisis. The new gomasthas were outsiders, with no long term social link with the village. They acted arrogantly, marched into villages with spays and peons, and punished weavers for delays in supply-often beating and flogging them. The weavers lost the space to a bargain of reprises and self to different buyers; the price they received from the company was miserably low and the loans they had accepted tied them to the Company.

In many places in Caryatid and Bengal, weavers deserted villages and migrated, setting up looms in other villages where they had some family elsewhere, weavers along with the village traders revolted, opposing the company and its official. Over time many weavers began refusing loans, closing down their workshops and taking to agricultural labor.

(ii) **Manchester Comes to India:** As cotton industries developed in England, industries groups began worrying about imports from other countries. They pressurized the government to impose import duties on cotton exiles so that Manchester goods could sell in Britain without facing any competition from outside. At the same time industrialists persuaded the East. India Company to sell British manufactures in Indian markets as well. Exports of British cotton goods increased dramatically in the early nineteenth century. At the end of the eighteenth century there had been virtually on import of cotton piece-goods into India. But by 1850 cotton piece-goods constituted over 31 percent of the value of Indian imports; and by the 1870s this figure was over 50 percent.

Cotton weavers in India thus faced two problems at the same time: their export market collapses and the local market shrank being glutted with Manchester imports. Produced by machines at lower costs the imported cotton goods were so cheap that weavers could not easily compete with them. By the 1850s reports from most weaving regions of India narrates stores of decline and desolation.

By the 1860s, weavers faced a new ruble. They could not get sufficient supply of raw cotton of goods quality. When the American civil war broke out and cotton supplies form the US were cut off. Britain turned to India. As raw cotton exports from India increased the price of raw cotton shot up. Weavers in India were starved of supplied and forced to by raw cotton at higher prices. In this, situation weaving could not pay.

◙ **FACTORIES COME UP**

First cotton mill in Bombay came up in 1854 and it went into production two years later. By 1862 four mills were at work with 94,000 spindles and 2,150 looms. Around the same time jute mills came up in Bengal, the first being set up in 1855 and another one seven years later. In 1862 in north India, the Elgin Mill was started in Kanpur I the 1860s and a year later the first cotton mill of Ahmadabad was set up. By 1874, the first spinning and weaving mill of Madras began production.

(i) **The Early Entrepreneurs:** many Indians became junior players in this trade, providing finance, producing supplies, and shipping consignments. Having earned through trade, some of these businessman had visions of developing industrial enterprises in India. In Bengal. Dwarkanath Tagore made his fortune in the China trade before he turned to industrial investment, setting up six joint-stock companies in the 1830s and 1840s. Tagore's enterprises sank along with those of others in the wider business crises of the 1840s, but later in the nineteenth century many of the china traders became successful industrialists. In Bombay, Parsis like Dinshaw Petit and Jamsetji Nusserwanji Tata who built huge industrial empires in India accumulated their initial wealth earth form exports to china and party form raw cotton shipments to England, Seth Hukam Chand, a Marwari businessman who set up the first Indian jute mill in Calcutta in 1917, also traded with china. So did the father as well as grandfather of the famous industrialist G.D. Birla.

As colonial control over Indian trade tightened, the space within Indian merchants could function became increasingly limited. They were barred from trading with Europe in manufactured good, and had to export mostly raw materials and food grains raw cotton, opium, wheat and indigo required by the British. They were also gradually edged out of the shipping business.

(ii) **Where did the workers come from? :** In most industrial regions workers came from the districts around. Peasants and artisans who found no work in the village went to the industrial centers in search of work. Over 50 per cent workers in the Bombay cotton industries in 1911 came from the neighboring district of Rangier, while the mills of Kanpur got most of their textile hands from the villages within the district of Kanpur. Most often mill workers moved between the village and the city, returning to their village homes during harvests and festivals. Over time, as news of employment spread, workers traveled great distances in the hope of work in the mills. From the United Provinces, for instance, they went to work in the textile mills of Bombay and in the jute mill of Calcutta.

◙ **THE PECULIARITIES OF INDUSTRIAL GROWTH**

When Indian businessmen began setting up industries in the late nineteenth century, they avoided competing with Manchester goods in the Indian market. Since yarn was not an important part of British imports into India, the early cotton mills in India produced coarse cotton yarn thread rather than fabric. When yarn was imported it was only of the superior variety. The yarn produced in Indian spinning mills was used by hand loom weavers in India or exported to Chain.

By the first decade of the twentieth century a series of changes affected the pattern of industrialization. As the Swadeshi movement gathered momentum nationalists mobilized people to boycott foreign cloth. Industrial groups organized themselves to protect their collective interests, pressurizing the government to increase tariff protection and grant other concessions from 1906, moreover the export of Indian yarn to China declined since produce from Chinese and Japanese mills flooded the Chinese market. So industrialists in India began shifting from yarn to cloth production. Cotton piece goods production in India doubled between 1900 and 1912.

Yet, till the First World War, industrial growth was slow. The war created a dramatically new situation. With British mills busy with war production to meet the needs of the army. Manchester imports into India defined:

Suddenly, Indian mills had a vast home market to supply, As the war prolonged, Indian factories were called upon to supply was needs : jute bags, cloth for army uniforms, texts and leather boots, horse and mule saddles and a host of other items. New factories were set up and old ones ran multiple shifts. Many new workers were employed and everyone was made to work longer hours. Over the war years industrial production boomed.

(i) **Small-Scale Industries Predominate:** While factory industries grew steadily after the war, large industries formed only a small segment of the economy. Most of them-about 6 per cent in 1911-were located in Bengal and Bombay. Over the rest of the country, small-scale production continued to predominate. Only a small proportion of the total industrial labors force worked in registered factories 5 per cent in 1, 911 and 10 per cent in 1931. The rest worked in small workshops and household units often located in alleys and balances, invisible to the passed by.

In fact in some instance, handicrafts production actually expanded in the twentieth century. This is true even in the case of the hand loom sector of we have discussed. While cheap machine-made thread wiped out the spinning industry in the nineteenth century, the weavers survived, despite problems. In the twentieth century handloom cloth production expanded steadily almost trembling between 1900 and 1940.

Certain groups of weavers were in better position than others to survive the competition with mill industries. Amongst weavers some produced coarse cloth while others more finer varieties. The coarse cloth was bought by the poor and its demand fluctuated violently. In time of bad harvests and famines, when the rural poor had little to eat, and their cash income disappeared, they could not possibly buy cloth, the demand for the finer varieties bought by the well-to-do was more stable. The rich could buy these even when the poor starved. Famines did not affect the sale of Banarasi or Baluchari saris. Moreover, as you have seen, mills could not imitate specialized weaves, Saris with woven borders, or the famous lungs and handkerchiefs of Madras, could not be easily displaced by mill production.

◙ **MARKET FOR GOODS**

When Manchester industrialists began selling cloth in India, they put labels on the cloth bundles. The label was needed to make the place of manufacture and the name of the company familiar to the buyer; the label was also to be a mark of quality. When buyers saw 'MADE IN MANCHESTER' written in bold on the label, they were expected to feel confident about buying the cloth.

But labels did not only carry words and texts. They also carried images and were very often beautifully illustrated. If we took at these old labels, we can have some idea of the mind of the manufacturers, their calculation, and the way they appealed to the people.

Images of Indian gods and goddesses regularly appeared on these labels, it was as if the association with gods gave divine approval to the goods being sold. The imprinted image of Krishna or Saraswati was also intended to make the

manufacture from a foreign land appear somewhat familiar to Indian people. By the late nineteenth century, manufactures were printing calendars were used by people who could not read. They were hung in tea shop and in poor people's homes just as much as in offices and middle class apartments. And those who hung the calendars had to see the advertisements day after day, through the year. In these calendars once again we see the figure gods being used to sell new products.

EXERCISE

◙ VERY SHORT ANSWER TYPE QUESTIONS

1. Who invented the spinning Jenny? What was its function?
2. Name any two inventions that helped to speed up textile industry.
3. What is meant by proto-industrialization?
4. Mention four steps required in the production process in textile industry.
5. Name the first two flourishing industries of England.
6. Name any four industries I which the industrialists preferred manual labor to machine.
7. Name two type of shelters setup for homeless labors.
8. How did Armenian and Persian merchants carry goods from India in Pre-colonial period?
9. Name two Indian seaports form which a vibrant trade was carried out from India with south East Asian parts.
10. Why were there clashes between the Indian weavers and company Gomasthas?
11. When and where was the first jute mill set in India?
12. Where and by whom was the first Jute mill setup in India?
13. Name the Indian entrepreneur who set up six joint stock companies in India in 1830s and 1840s. How did he make fortune for investments?
14. Mention the restrictions imposed upon the Indian merchants in 19^{th} century.
15. What is Flying Shuttle? What was its function?
16. How did Indian advertisement become a vehicle o nationalist aspiration?
17. Why were the images of Indian Gods and Goddesses imprinted on British manufacturers in 19^{th} century?

◙ SHORT ANSWER TYPE QUESTIONS

18. Explain the role of a guild in the production process of crafts.
19. How did factory production begin in England?
20. Explain two reasons for the fast growth of cotton and steel industry in 18^{th} century England.
21. Why were some industrialists reluctant to introduce machines?
22. Why did the bourgeoisie prefer handmade products in 19the century?
23. After 1840s the life of workers improved in England Explain.
24. Why is it necessary to use advertisement for various products?
25. Examine the nature of the network of Indian. Export trade before 1750.
26. Examine how the British companies gradually asserted monopoly rights in India.
27. What role did a Jobber play for an industrialist?
28. How did Indian entrepreneurs accumulate capital for investment?
29. What is meant by Enclosure movement?
30. Mention the role of middle class as shown time to time.
31. Write down some of the miserable conditions of workers.
32. Mention the social effects of unplanned industrial towns.

◙ LONG ANSWER TYPE QUESTIONS

1. How was the life of Indian weavers affected by Indian trade under company rule?
2. Explain the reasons for decline of Indian textile industry by the end of 19th century.
3. Explain the various problems faced by the Indian weavers in 19th century.
4. Name any three managing agencies in India before the First World War. What were their main functions?
5. How did the pattern of Indian industry change in early 20th century?
6. Why there was need for economic reform? Explain.
7. What do you know regarding the awakening among workers?
8. Explain the causes of India's industrial backwardness under the British rule.
   How the British rule hampered Indian industries.

THE MAKING OF A GLOBAL WORLD

- **IMPORTANT TERMS**

- **Global (worldwide):** Involving all countries of the globe.
- **Globalization:** Integration economy of a country with the economies of other countries under condition of free flows of trade, capital and labors.
- **Cowries:** A Hindi world meaning sea shells. These are used in ancient world as form of currency.
- **Silk Route:** Route taken by traders to carry silk cargos from china to the west.
- **Spaghetti:** A type of noodle popular in loyally.
- **El Dorado;** Imaginary land of great wealth fabled land of gold.
- **Plantation:** Estate for cultivation of cash crops like tea coffee, cotton, tobacco, sugarcane etc.
- **Corn Laws:** British laws which improvement to one another and to the import of corn.
- **Ecology:** Study of organisms in relation to one another and to their surroundings.
- **The canal colonies:** the iconologies/areas irrigated by new canals, where peasants from parts of Punjab sated.
- **Rudderpost:** Castle plague- A fast spreading among cattle.
- **Chutney Music (Popular music in Trinidad):** Supposed to be the result of cultural fusion.
- **Entrepreneur:** one who undertakes commercial enterprise with chance of profited or loss. Start enterprise by him/her at his/her risk.
- **Hose:** A riotous carnival in Trinidad when workers of all races and religion join to celebrate.
- **Coolie:** Unskilled native laborers. Indentured Indian laborers were often referred to as coolie in Trinidad.
- **Indigo:** Deep violet blue color.
- **Allies:** Before the First World War, Britain, France and Russia formed an alliance and fought together in the First World War Together they were known as Allies.
- **Central Power:** An alliance formed by Germany, Austria-Hungary, Italy, Later Turkey, Fought together in the First World War. Together they were known as Allies.
- **Axis-power:** Germany, Italy, Austria, Turkey were known as Axis powers during the Second World War.
- **IMF:** International monetary fund.
- **NIEO:** New International Economic order.

- **THE PRE-MODERN WORLD**

All through history human societies have become steadily more interlinked. From ancient times, travelers, Traders, priests and pilgrims traveled vast distances for knowledge, opportunity and spiritual fulfillment, or to escape persecution. They carried goods, money, values, skills, ideas, inventions and even germs and diseases.

(i) **Silk Routes Link the World:** The name 'Silk routes' points to the importance Chinese Silk cargoes along this route. Historians have identified several silk Routes, over land and by sea, knitting together vast regions of Asia, and linking Asia with Europe and North Africa. Chinese Pottery also traveled the same route, as did textiles and spices from India and South East Asia. In return precious metals flowed from Europe to Asia. Early Christian missionaries certainly traveled this route to Asia, as did early Muslim preachers a few centuries later. Buddhism emerged from eastern India and spread in several directions.

(ii) **Food Travels:** Spaghetti and Potato: Traders and travelers introduced new crops to the lands they traveled. Even 'read' food stuff in distant parts of the world Might share common origins. Take spaghetti or noodles. It is believed that noodles traveled west from china to become spaghetti. Or perhaps Arab traders took Pasta to Sicily. Many of our common foods, such as potatoes, Soya, groundnuts, maize, tomatoes, chilies, sweet potatoes, and so on were not known to our ancestors until about five centuries ago. These foods were only introduced in Europe and Asia after Christopher Columbus accidentally discovered the vast continent that would later become known as the Americas.

(iii) **Conquest, Disease and Trade:** In the 16th Century after European sailors found a Sea route of Asia and also successfully crossed the western ocean to America. For centuries before, the Indian Ocean had known a bustling trade, with goods, People, knowledge, customs etc. Before its 'discovery', America had been cut off from regular contact with the Rest of the world for millions of years. But from the 16th century, its vast lands and abundant crops and mineral began to transform trade and lives everywhere. The Portuguese d Spanish conquest and colonization of America was decisively Under way by the mid 16th century. In fact, the most powerful weapon of the Spanish conquerors was not a conventional military weapon; It was the germs such as small pox that they carried on their person. Small pox in particular proved a deadly killer. It spread deep into the continent, ahead even of any Europeans reaching there; It killed and decimated whole countries, paving the way for conquest. Until the nineteenth century, poverty and hunger were common in Europe. Cities were crowded a deadly diseases were widespread. Religious conflicts were common and religious dissenters were persecuted. Thousand therefore fled Europe from America. In the 15th century, China and India were among the world's richest countries. However, from the 15th century, China is said to have restricted overseas contacts and retreated in to isolation. The rising importance of the Americas gradually moved the centre of world trade westwards. Europe now emerged as the centre of world trade.

◙ **THE NINTENTH CENTURY (1815-1914)**

In the 19th century, the world changed due to economic, political, social, culture Technological factors to transform societies and reshape external relation. Economic identity has three types of movement or flow with in international economic exchanges.

(a) The flow of trade (b) The flow of the labor (c) the movement of capital

All three flows were closely interwoven and affected people's lives more deeply now than ever before. The interconnection could sometimes be broken. For example, lab our migration was often more restricted than goods or capital flows.

(i) A world Economy takes shape: Traditionally, countries liked to be self sufficient in food. But in 19th century Britain, self sufficiency in food meant lower standards and social conflict. As urban centers expanded and industry grew the demand for agriculture products up, pushing up agriculture prices. Unhappy with high food prices, industrialists and urban dwellers forced the abolition of the com laws. As food prices fell, consumption in Britain rose. Form the mid

nineteenth century, faster industrial growth in Britain also led to higher incomes, and therefore more food improves. Around the world in industrial Eastern Europe, Russia, America and Australia. The demand for lab our in places where was in short supply-as in America and Australia led to more migration. By 1890, a global economy had taken shape, accompanied by complex changes in labor movement patterns, capital flow, ecologies and technology.

(ii) **Role of Technology:** Railways, steam shipping and telegraph were important inventions. Technological advance was often the result of larger social, political and economic factors railways lighter wagons and larger ships helped move food more cheaply and quickly from far away farms to markets. Now animal were slaughtered for food over at the starting point in America, Australia or New Zealand then transported to Europe as. Frozen meat. This reduced shipping costs and lowered meant prices in Europe. Better living conditions promoted social peace within the country and support for imperialism abroad.

(iii) **Late 19th century colonialism:** It is important to realize the darker side this process. In many parts of the world, the expansion of trade and a closer relationship with the world economy also meant a loss of freedoms and livelihoods. Late 19th century European conquest produced many painful economic social and ecological changes. Britain and France made vast additions to their overeats territories in the late 19th

Century Belgium and Germany became new colonial powers. The US also became a colonial power in the late 1890s by taking over some colonies earlier held by Spain.

(iv) **Reinterprets or the Cattle Plague:** In Africa, in the 1890s fast spreading disease of cattle plague had a terrifying impact on people's livelihoods and the local economy. Africa had abundant land and a relatively small population. For centuries, land and livestock sustained African livelihood and working for a wage was not a wide spread. In late 19th century Africa there was few consumer goods that wages could buy. In the late 19th century, Europeans were attracted to African by its sources of land minerals. Europeans come to Africa hoping to establish plantations and mines to produce crops and minerals for export to Europe. But there was an unexpected problem a shortage of labors willing to work for wages.

(v) **Indentured labors Migration from India:** In the 19the century, hundreds of thousands of Indian are Chinese laborers went to work on plantations, in mines and in road and railway construction projects around the world. Most Indian indentured workers came from the eastern Uttar Pradesh, Bihar, Central India and the dry districts of Tamil Nadu. I the mid 19th century these regions experiences many changes cottage industries declined, land rent rose, lands were cleared for mines annotations. All this affected the lives of poor; they fail to pay their rents, got deep in debt and were fore to migrate in search of work. 19the century indenture has been described as a new system of slavery. On arrival at the plantations, laborers found conditions of different form what they had imagined. Livid and working conditions were harsh and there were few legal rights. But workers discovered their own ways of surviving. Many of them escaped into the worlds through if caught they faced severe punishments. From the 1990s, India nationalist leaders oppose the system of indentured labor migration as abusive and cruel. It was abolished in 1921. Yet for a number of decades afterwards, descendants of India indentured workers, after thought of a coolest remained an uneasy-minority in the Caribbean islands some of Naipaul's early novels capture their sense of loss and alienation.

(vi) **Indian Entrepreneurs Abroad:** Indian trades and money lenders also followed European colonizers into Africa. Hyderabadi Sindhi traders, however, ventured beyond European colonies. From the 1860s they established flourishing emporia at busy ports worldwide, selling local and imported curious to tourists.

(vii) **Indian Trade, Colonialism and the Global system:** Fine cottons produced in India were exported to Europe. British cotton manufacture began to expand and an industrialist pressurizes the government to restrict cotton imports and protect local industries. Tariffs were imposed on cloth imports into Britain. Consequently the inflow of me Indian cotton began to decline. From the early 19the century British manufactures also began to seek overseas markets for their cloth. Excluded from the British market by tariff barriers,. Indian textile now faced still competition in other

international markets; While exports if manufactures declined rapidly, export of raw materials increase equally fast .indigo used for the dyeing cloth was another important export for many secedes Britain grew opium in India and export it to china, and with the money earned through this sale, it financed its tea and other imports from china.

During 19th century British manufactures flooded the Indian market. Food grain and raw material export from India to Britain and the rest of the word increased, but the value of British exports of India was much higher than value of British imports from Indian. Thus Britain had a 'trade' surplus with India. Britain used this surplus to balance its trade defects with other countries.

Britain's trade surplus in India also helped to pay the so called 'home charges that included private remittances home by British official and traders, interested payment on India's external debt, and pensions of British official in India.

◙ **THE INTER WAR ECONOMY**

The First World War (1914-18) was mainly fought in Europe. During this period the world experienced wide spread economic and political instability and another catastrophic war.

(i) **Wartime Transformation:** The First World War was fought between two powers blocks. On the one side were the Allies-Britain, France and Russia (later joined by the) US, and on the other side were the central powers- Germany, Australia, Hungary and ottoman turkey. It was the first modern industrial war. It saw the use of machine guns, tanks, aircraft, chemical etc. to the frontlines on large ships and trains. The scale of death and destruction nine million dead and 20 million injured was unthinkable before the industrial age. During the war industries were restructure to produce war related goods entire societies were also reorganized for war as men went to battle, women in to undertake jobs that earlier only men were expected to do.

(ii) **Post War recovery:** Britain which the world's leading economy in the power period, in particular faced prolonged crisis while Britain was preoccupied with war, industries had developed in India and Japan. After the war Britain found it difficult to recapture its earlier position of dominance in the Indian market, and to compete with Japan internationally. Moreover war expenditures Britain had borrowed liberally from the US. The war had led to an economic boom, i.e. to a large increase in demand and employment. When the war boom ended production contracted and unemployment led to huge job losses in 1921 one on every five British workers was out of job. Before the war, Eastern Europe was major was supplier of wheat in the world market. When this supply was disrupted during the war, wheat production in Canada, America and Australia in Canada, America and Australia expanded dramatically. But once the war was over, production in Eastern Europe revived and create li glut in wheat output. Grain prices fell, rural incomes declined and farmers fell deeper in to dept.

(iii) **Rise of mass production and consumption:** The move towards mass production had begun in the late 19th century, but in the 1920s it became a characteristic feature of industrial production in the US. A well known pioneer of mass production was the car manufacturer Henry ford. He realized that the 'Assembly line method would allow a faster and cheaper way of producing vehicles. As result, Henry Ford's car came to the assembly line at three-minute intervals, a speed much faster than achieved by previous methods. The T Model ford was the world's first mass produced car.

Mass production lowered costs and prices of engineered goods such as cars, refrigerators, washing machine, radios, gramophone player, all through a system of 'hire purchase'.

The housing and consumer boom of the 1920s create the basic of prosperity in the US large investment of housing and hold goods seemed to create a various of higher employment and incomes, rising consummating demand, more investment and yet more employment and incomes.

In 1923, the US resumes exporting capital to the rest of the world and become the largest overseas lender. US important and capital export also boosted recovery.

(iv) **The Great Depression:** The great Deprecation began around 1929 and lasted till mid 1930s. During this period most parts of the world experienced catastrophic declines in production, employment, incomes and trade, Agriculture regions and communities were the worst affected.

Causes: Agriculture over production remained a problem. This made worse by falling agriculture prices. Farmers tried to expand production and bring a larger volume of market to maintain their over income. This worsened the glint in the market, pushing down prices even further. Farm produce rotted for a lack of buyers. In the mid 1920s, many countries financed their investments through loans from the US. In the first half of 1938 US overseas loans amounted to over $1 billion. A year later it was one quarter of that amount country that depended crucially on US loans one faced an acute crisis.

In Europe the withdrawal of us loans led to the failure of some major banks and collapse of currencies such as the British pound sterling. In Latin America and elsewhere it intensified the slump in agriculture and raw material prices. US banks also slashed domestic lending and called back loans. Farms could not sell their harvest; households were ruined and business collapsed. Due to falling incomes, many households in the US could not repay what they had borrowed, and were forced to give up their homes, cars and other consumer durables. The US banking system itself collapsed. Unable to recover investment, collect loans and repay depositors, thousands of banks went bankrupt and were forced to close.

(v) **India and great Depression:** India's exports and imports nearly halve between 1928 and 1934. During this period wheat prices in India fell by 50% though agriculture prices fell sharply, the colonial government refused to reduce revenue demands.

The price of jute in Bengal crashed to more than 60%. Parent's indebtedness increased. They used up their saving, mortgaged lands, and sold whatever jewellery and metals they had to meet their expense. In these depression years, India became an exporter of precious metal gold.

The depression proved less him for urban India. Because of falling prices those with fixed incomes say town dwelling landowners who received rents and middle class salaried employers-now found themselves better off.

◙ **REBUILDING A WORLD ECONOMY: THE POST WAR ERA**

The Second World War was fought between the Axis powers (Germany, Japan and Italy) and the Allies (Britain, France; the Soviet Union and the US). It was fought over land, on sea and in air. In this war least 60 million people are believed to have been killed, and millions more were injured. Vast parts of Europe and Asia were devastated and several cities were destroyed by aerial bombardment or artillery attacks. The atomic bombing of Hiroshima and Nagasaki alone is estimated to have killed between 1, 50, 000 and 2, 50, 000 men, women and children. The war causes an immense amount of economic devastation and social disruption.

The after affects of the war were –the US's emergence as the dominated economic, political and military Powel in the western world. On the other hand the Soviet Union defeated: Germany and became a world power during the very years when the capital world was trapped in the great depression.

(I) **Post –war Settlement and the Britton-Woods Institutions:** In brief, the main aim of the post-war international economic system was to preserve economic stability and full employed in the industrial world. The Britton-Woods conference established the international Monetary Fund (IMF) to deal with external surpluses and deficits of its member nations. The International Bank (World Bank) for Reconstruction and development was set up to fiancé Post War reconstruction. The IMF and the World Bank are referred to as the Bretton-Woods institutions or sometimes the Bretton Woods twins.

The US has an effective right of veto over key IMF and World Bank decisions. The Bretton-Woods system was based on fixed exchange rates. In this system, National currencies, such as for example the Indian rupee, were pegged to the dollar at a fixed exchange rate.

(II) The Early Post –War years: The Bretton-Woods System inaugurated an era of unprecedented growth of trade and incomes for the western industrial nations and Japan.

(III) Decolonization and Independence: After the end of Second World War most colonies in Asia and Africa emerged as free independent nations. They were over burdened by poverty and a lack of resources and their economies and societies were handicapped by long period of colonial rule.

The IMF and the World Bank were designed to meet the facials needs' of the industrial countries. But from the late 1950s the Bretton-Woods institutions began to shift their attention more towards developing countries.

Now newly independent countries facing urgent pressures to lift their populations out of poverty came under the guidance of international agencies dominated by the former colonial power. British and French business still controlled vital resources such as minerals and land in many of their former colonies or in other parts of the world where they had earlier wielded political influence.

Most of the developing countries did not benefit from the fast growth the western economies experienced in the 1950s and 1960s. So they organized themselves as a group-the group o0f 77 (G-77) to demand a new international economic order (NIEO). By the NIEO they meant a system that would give them real control over their natural resources, more development, assistance, and fairer prices for raw materials and better access for their manufactured goods in developed countries markets.

(IV) End of Britton-Woods and the Beginnings of 'Globalization': From the

1960s the rising costs of its overseas involvements weakened the US's finances and competitive strength. The dollar could not maintain its value in relation to gold.

Earlier, developing countries could turn to international institutions for loans and development assistance. But now they were forced to borrow from western commercial banks and private leading institutions. This led to periodic debt crises especially in Africa and Latin America.

The industrial world was also hit by unemployment. From the late 1970s, MNCs also began to shift production operation to low wage Asian countries.

New Economic policy in China and th4e Collapse of the Soviet Union and Soviet style communism in Eastern Europe brought many countries back in to the fold if the world economy.

China and other countries, where the wages were low, became attractive destinations for investments by foreign MNCs competing to capture world market

In the last two decades the world's economic geography has been transformed as countries such as India, china & Brazil have under gone taped economic transformation.

EXERCISE

MULTIPLE CHOICE QUESTIONS

1. Name one people who did not contribute in making the human societies more interlinked.

(A) Travelers' (B) Traders (C) Imperialists (D) Prints

2. Who were the first Europeans to conquer America?

(A) The French (B) The Portuguese (C) the Spanish (D) The Germans

3. Who worked in the plantations set up in America by the European nations?

(A) The Indians (B) The Chinese (C) the Slaves from Africa (D) The Arabs

4. What were the Corn Laws?

(A) They were passed by the British Government to restrict the import of corn

(B) They were passed by the British Government to restrict the export of corn

(C) They were passed by the French Government to export corn to Canada

(D) They were passed by the America to import corn from other countries

5. Which one of the following countries did not try to expand food production to meet the British demand?

(A) Eastern Europe (B) Russia (C) America (D) Japan

6. Up to the 18th century, which two countries among the following were the countries of the world?

(A) India (B) Turkey (C) South Africa (D) China

7. Which new invention made it possible to transport perishable foods over long distances?

(A) Gun Powder (B) Refrigerated Ships (C) Compass (D) Bombs

8. Where was the Chutney music popular?

(A) China (B) Japan (C) North America (D) South America

9. What is Rudderpost?

(A) A cattle disease in Africa (B) A cattle disease in China

(C) A cattle disease in U.S.A. (D) A cattle disease in Russia

10. Which one of the following countries was a part of the Allies in the First World War (1914-1916)?

(A) Britain (B) France (C) United States of America (D) Japan

11. Which one country was not a part of Central Powers in the First World War?

(A) Germany (B) Austria- Hungary (C) Turkey (D) United States of America

12. When did the Great Depression begin?

(A) In 1929 (B) In 1930 (C) In 1931 (D) In 1932

13. Which one country was not a part of the Axis Powers in the Second World War (1939 – 1945)?

(A) Russia (B) Germany (C) Japan (D) Italy

14. Which one country was not a part of the Allied Powers in the Second World War?

(A) Great Britain (B) France (C) Germany (D) United States of America

15. What was the human loss in the Second World War?

(A) About 30 million people were killed in this War

(B) About 40 million people were killed in this War

(C) About 50 million people were killed in this War

(D) About 60 million people were killed in this War

16. What do you mean by G -77?

(A) It is a group of rich countries (B) It is a group African country

(C) It was a Group of 77 developing countries which did not benefit from the fast growth which the Western economics experienced in 1950 (D) It is a group of Asian countries

ANSWER KEY					EXERCISE					
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	B,C	C	A	D	A,D	B	D	A	D
Que.	11	12	13	14	15	16				
Ans.	D	A	A	C	A	C				

EXERCISE

◙ VERY SHORT ANSWER TYPE QUESTIONS

1. Mention any two modes of exchange of ideas and goods in ancient world.
2. What is meant by 'Cowries'? For what purpose were these used?
3. What is 'silk route '? What role did this play in linking other countries?
4. What is referred to as EI Dorado which attracted traveler?
5. Name two European countries which attracted traveler?
6. Name the explorer who discovered the sea routes to the Americas.
7. Which two countries were the richest prior t9 18th century?
8. What is com law? Why was it abolished?
9. Give the main reason for the migrating of European people to America in 19th century?
10. Name any four important inventions which transformed 20th century world?
11. Why is the year 1885 significant in the history of colonization?
12. What is meant by indentured labor?
13. What is hussy carnival of Trinidad?
14. Give two examples which reflect social-cultural fusion between Caribbean people with indentured Indians?
15. What is referred as 'chutney' music?
16. Who is V.S. Nepal? What is his main achievement?
17. Name the groups in which the world was divided before the First World War?
18. Why was the Britton Woods conference held in New Hampshire in US significant
19. What is referred to as Breton wood twins?
20. Mention any two economic effects of Second World War?

◙ SHORT ANSWER TYPE QUESTIONS

1. How did the silk route help in trade & cultural exchange?
2. How did food habits travel from one place to another in the process of cultural exchange?
3. Identify various types of flows or movements within international economic exchange? How do these affect economy?
4. Which part of India had the first experience of now agricultural economy? How did this happen? What were the canal colonies of Punjab?
5. Explain how with new technology people of one part of the world could have import varieties of food from other parts of the world at a lower cad?
6. How did the Reinterprets reach Africa? How did this create havoc for the African people?
7. What steps did Hennery ford take in 1914 to retain labor?
8. Examine the condition of the 19th century which compelled the Indians to become indentured lab our and migrate to other countries in search of work?
9. Why was the 19th century indenture system referred to as the new 'system of silvery'?
10. Why did the British impose tariff on the import of cotton textiles after 19th century
11. How did the Britain's trade surplus from India help her to balance its other trade deflects?
12. How did the First World War transform U.S. economy to make it an international creditor from international lab our?
13. What was the effect of the First World War on the economy of Britain?
14. In the context of industrial production what is meant by assembly –line method?

15. What was the major economic effect of Second World War?

◙ **LONG ANSWER TYPE QUESTION**

1. Explain how travels and trade help in establishing links among various countries.
2. Explain the main features of global agriculture economy that emerged around 1890.
3. In many parts of the world the expansion of trade and closer relationship with world economy also meant a loss of freedom & livelihood. "Justify the statement by giving examples.
4. What key lesson did the politicians and economist learn from the interwar economy experience?
5. What is the great Depression? Mention some of its causes.
6. What was the effect of the great Deprecation on the Indian economy?
7. How past war economic recovery proved difficult for Britain? Explain.

NATIONALISM IN INDIA

Nation:

The people who reside in a particular state having definite boundaries or fixed territories sharing common culture, history and common economics system.

Nationalism: is love for one's own country or a sense of pride for one's own country.

Factor responsible for the rise of nationalism

1. The political unification of the country due to revolt of 1857, which market a turning point in the history of India.
2. British policy of drain of wealth theory.
3. Western education and means of transport and communication.
4. Growth of press
5. Racial discrimination policy of British in India.
6. Indifferent attitude of British towards famines and floods and development activities.
7. This sense of being oppressed under colonialism provided a shared bond that tied many different groups together.

British dragged India into 1st world war indirectly.

Thus British had ruined India socially, economically, politically which made Indian realize this and gradually there was rise of nationalism in the country.

Br. declared India as partner or associate (Ally) (engaged in war).

Fight in First World War.

The First World War (1914 – 1918)

1. The 1st world war led to a heavy war expenditure on India in purchase of weapons through financed war loans.
2. Taxes were increased, custom duties were raised and income-tax was introduced.
3. During 1st world war there was considerable increase in price rise –Inflation on all goods.
4. Village were forced to supply soldiers & there was forced recruitments, which led to wide spread anger.
5. There was acute shortage of food. As crops failed due to famines & 12 to 13 million people died due to epidemic.
6. People unheartedly supported British thinking that the war would end their hardship and British. Would Grant Independence to India.

But after the war British failed to keep up its promise.

Entrance of Gandhi into Indian polities:

Gandhi**:** Returned from South Africa to India in January 1915 and started the nationalist movement. Gandhi believed in truth, and believed that the dharma of non violence could unit all Indians.

In 1916 he fought for the case of Champaran peasants in Bihar. He fought for rights of indigo planters & compelled British government to give them justice.

In 1917 – He organized Satyagraha & supported the peasants of Kheda district of Gujarat the farmers who could not pay revenue. In 1918, He went to Ahmadabad to organize Satyagraha movement amongst cotton mill workers.

The idea of Satyagraha

1. It was a non violent method of mass agitation against the oppressors.
2. It emphasized the power of truth & the need to search truth.
3. it suggested that if the case was true & if the struggle was against injustice than there is no need for physical force to fight the oppressor. In 1919, Gandhi decided to launch a massive Satyagraha against Rowlatt act 1919.

Rowlatt Act 1919: This act empowered the British government to repress political activities allowed detention of political prisoners without trial for 2 years. So on 6th April 1919 hartal was declared against it.

- Rallies were organized
- Railway workers went on strike
- Shops were closed down
- Means of transport & communication were disrupted.

British Repressive policies

On 10th April local leaders

Like Dr. Saifuddin Kitchlew & dr. Satyapal Singh were picked up from Amritsar.

On 10th April police in Amritsar fired upon a peaceful procession

Martial law was imposed & general dyer took command.

JALLIANWALLA BAGH MASSACRE

On 13th April, 1919 thousands of people had gathered in Jallianwala Bagh, being outside the city People in the Bagh were unaware of martial law, among the crowd were women children & old People. It was a peaceful gathering when general dyer with his troop entered the park, closed the only Exit and without any prior warning ordered the troops of fire the firing lasted for about 10 minutes 100's of people died of around 1000 people were wounded. General Dyer had no regrets to this massacre. His aim was to produce of a feeling terror and awe in the minds of Satyagrahis.

Effect of Jallianwala Bagh massacre on Indians

There were strikes in protest against the massacre.

Police and the government building were attacked.

British suppressive policies

The Satyagrahis were brutally repressed.

They were humiliated & terrorized & were forced to crawl & rub their nose on the ground and salute the British authorities. Villages were bombed

Khilafat movement: After the First World War, British dismembered turkey and turkey was defeated and declared as the mandate of British. There were rumors that harsh peace treaty was going to be imposed on the ottoman emperor, the spiritual head of Islamic world.

Gandhiji in order to promote Hindu Muslim unity, he took up the Khilafat issue and a Khilafat committee was formed in march 1919 in Mumbai with the help of two Ali brothers, Mohammad Ali & Shaukat Ali. On 17th October, 1919, nationwide hartals were observed and slogans such as Hindu Muslim Ek hai were given. In September 1920, at the congress session it was decided to launch the non cooperation movement.

The non cooperation movement causes:

CAUSES

1. To redress the wrong done to the people of Punjab.
2. To redress the wrong done to the people of turkey.
3. For attainment of Swaraj

Programme or methods of NCM

- Renunciation of titles given by British Government (Gandhi gave back Kesar-i-Hind title)
- Boycott of civil services, Army police, courts Legislative councils, schools.
- To promote Swadeshi & start the programme of nation education.
- No tax campaign.

Events of NCM

- Began in January 1921.
- Many social groups participated.
- Each had their specific aspirations.
- All answered the call of Swaraj.
- But Swaraj did not mean same to all.
- Gandhi toured whole country to whip up enthusiasm of people.

The movement in the town:

- 1, 50,000 volunteers came forward to carry in the campaign.
- In towns, the middle class took up the fight for Swaraj.
- Students boycott British school & colleges.
- In elections of legislatures 2/3rd of the voters did not vote.
- People resigned from govt. jobs.
- Foreign cloth was burnt in bonfires.
- Women picketed liquor shops.

ECONOMIC FRONT

- Traders refused to trade in foreign goods.
- People started making use of Swadeshi goods.
- Indians textile mills & handloom increased production to meet the demand.
- Import of foreign cloth halved between 1921 & 1922 its value dropped from 102 Crores to 57 Crores.
- Khadi become the symbol of freedom.

Slowing down of the movement:-

Khadi was more expensive than the mass produced mill cloth. The poor could not afford it.

The Educational institution were slow to come up, so the teachers began trickling back to govt. school & lawyers joined back to work in government courts.

The NCM in the country side:

From the cities, the NCM spread to the country side. It drew into its fold the struggles of peasants & Tribals.

Each struggle was against a different target.

Some struggles were against a different Gandhi's principles as they became violent.

All action was taken in the name of mahatma.

In Awadh, the peasant's movement demanded reduction of revenue, Abolition of beggar & social boycott of Oppressive landlords. Nai dhobi Bandhs were organized by Panchayat to deprive landlords of the services of barbers and washer mans.

Oudh Kishan Sabha was set up Jawaharlal Nehru, Baba Ramchandra within a month over 300 branches were set up in village around the region.

Tribal peasants: Andhra Pradesh, Gudem hills:

The peasant led a guerrilla movement in early 1920, became the colonial govt. had closed large forest areas preventing people from entering the forest to graze their cattle or collect fuel wood & fruits. This arranged the hill people as they were deprived of their traditional rights and livelihood.

They revolted when govt. forced them to do beggar for building roads.

Their reader was Alluri Sitaram Raju, these tribal believed that has got special spiritual powers & that he was incarnation of god he too was inspired by Gandhi's NCM but he believed Swaraj could be achieved by force and not by non violence. So the Gudem Tribals rebels attacked police station, attempted to kill British of official & carried on gorilla warfare to achieve Swaraj finally, he was captured & executed in 1924.

SWARAJ IN THE PLANTATION

OR

The movement in the plantation

- Workers had their own idea Swaraj.
- They believed Gandhi raj was coming and everyone would be given land in their own village.
- The plantation workers of Assam waited for the right to move freely and not be confined to the gardens they were working in.
- The British had passed an island Emigration act 1859 which prevented them from leaving the plantations without permission.
- When they heard of NCM, 1000's of workers defied the authorities, left the plantations & headed home. They however reached their destination. Stranded on the way by a railway & streamer strike, they were caught by the police and brutally beaten up.

British Attitude towards NCM

- British tried to crush the movement.
- 30,000 people were jailed.
- British fired the demonstrations.
- Revolutionaries were brutally killed.
- Gandhi was arrested.

Withdrawal of NCM

In February, Gandhi decided to launch the final programme of no tax campaign. When on 5th February 1922, in chauri-chaura, U.P. people turned violent to set fire & a police stations causing death of 22 policemen. When news reached Gandhi, he called off the movement.

CONCLUSION:

It was the 1st attempt of all India mass struggles against British.

It clearly demonstrated that thousands of poor Indians were capable undergoing sacrifices.

People realized unity councils.

FORMATION OF THE SWARAJ PARTY

C.R. Das and Moti Lal Nehru founded Swaraj. Party within the congress to fight elections for the councils and demanded reforms, decided to participate in election & wreck the British government from within Young leaders like Jawaharlal Nehru & Subash Chandra Bose pressed for full Independence.

Two factor that influenced Indians polities in the late 20th century were there was worldwide economic depression, Agricultural prices fell down b/w 1926 and 1930.Farmers could not sell their products.

THE SIMON COMMISION-1928

In 1927, British Government appointed a commission to look in the working of the Government act 1919 or Montague Chelmsford reforms & to suggest changes.

Drawback:

All Englishman were the members of this commissions and not a single member was Indians.

Event: The commissions arrived in India is February 1928 and was welcomed with nationwide HARTAL & BLACK FLAG and was written on it 'Go back Simon'

Anti Simon committees were formed to organize hartal and reject the commission.

Result: Lala Lajpat Rai received severe Lathi blows and died and Punjab.

Even Govind ballabh pant and Jawaharlal Nehru received Lathi blow and became disabled for life long. People got annoyed with British Barbaric treatment towards Indians. They made several conspiracies & fought with British Government to overthrow its rule.

Lord Irwin, indoor to win the confidence of Indians offered "Dominion status" for India in an unspecified future & a round table conference to discuss a future constitution.

PURANA SWARAJ / COMPLETE INDIPENDENCE

In the Lahore congress session under the presidency of Pandit Jawaharlal Nehru, In December 1929, the demand for Poorna Swaraj or full independence was made.

It was declared that 26th January 1930 would be celebrate as the Independence Day on this day, people were to take pledge to struggle for complete Independence. But the celebration attracted very little attention. So Mahatma Gandhi on 31st January 1930 sent a letter to Viceroy Irwin stating 11 demands including specific demands of different classes, from Industrialist & peasant. This letter was the ultimatum to British that if the demand were not fulfilled by 11th march 1930, CDM Would be launched. The target of this demand was to fulfill the desires or every Indian & it aimed at bringing together a united campaign. Some demands like (Reduction of land revenues, abolition of salt tax, impose protective tariff on foreign cloth)

The most important of all the demand was:-

"To abolish salt tax"

Why Salt: Because salt is consumed by both poor and rich, very essential commodity of food (Mahatma Gandhi declared that tax on salt by British government is the most oppressive & Repressive act of British on India.

THE SALT MARCH

On 12th march 1930, Gandhi along with 78 follows (trusted volunteers) marched from Sabarmati ashram to Dandi coast in Ahmadabad.

He walked 240 miles to illegally manufacture salt.

Volunteers walked for 24 days, about 10 miles a day.

There was a gathering of thousands mob at every stoppage of Mahatma Gandhi to hear is speeches. Wherever he stopped, he urged people to peacefully defy British without any violence.

Women paid tribute by spinning yarn on the charkhas.

Newspaper carried day to day report of the effect of Gandhiji speeches in the village he passed through.

Around 300 Gujarat village officials resigned their post in answers to Gandhiji appeal.

On 11th April 1930, Gandhi ji reached Dandi, a small village in the Gujarat sea coast and ceremonially broke the salt law by boiling sea water and manufacturing salt and civil disobedience Movement began.

THE CIVIL DISOBEDIENCE MOVEMENT 1930 - 1931

CAUSES:-

1. **Failure of Simon commission:** The Simon commissions was constituted in response to the nationalist movement but the commission failed to satisfy the Indian people.
2. **Poorna Swaraj :** was goal declared by pt J.L. Nehru in Lahore congress session on 26 jan. 1930 people took pledge to struggle for complete Independence day.

FAILURE OF GANDHI'S 11 DEMAND:-

3. **Economic cause:** The great depression of 1929 had a deep impact on Indian economy especially on agriculture. Prices of agricultural produce began to fall from 1926 and collapsed after 1930. As the demand for agricultural goods fill & export declined, peasants found of difficult to sell their harvest & to pay their revenue. The government refused to lower the taxes by 1930, when the farmers were in poor condition.

Events under CDM

CDM began on 6th April 1930; When Gandhi broke the salt law by manufacturing the salt.

1000's of people demonstrated in front of salt factories & also manufacturing salt to break salt laws. Boycott of foreign cloth & goods.

Women picketed liquor shops.

Gandhi now asked people not only to refuse cooperation to the British but also to break colonial laws. Peasants refused to pay revenues & Chaukidari taxes.

Village officials resigned, & many people broke forest laws & entered into reserved forest to collect wood & graze cattle.

Many social groups participated in CDM.

British Attitude towards CDM

British government began arresting one popular leader after another.

Jawaharlal Nehru was arrested on 14th April 1930.

Abdul Gaffar khan, a disciple of Gandhi was arrested on 23rd April 1930 (he started khudai khidmatgar & was against the creation of Pakistan) when people learnt about his arrestment, the angry crowd demonstrated in the streets of Peshawar, forcing armored cars & police firing. Many were killed.

On 4th may 1930, Gandhi was arrested as a result of which, the industrial workers in Sholapur attacked police station, government building, law courts & railway station.

Peaceful Satyagrahis were brutally beaten up.

Around 1 lakh people were arrested.

The Government tried to diffuse the situation by making the viceroy, Lord Irwin to sign a pact; called the Gandhi-Irwin pact on 5th march 1931. Gandhi agreed to attend the round table conference in London & govt. allowed to release all political prisoners.

ROUND TABLE CONFERENCE

Gandhi went to London in December 1931 to attend the R.T.C., but the negotiations broke down. There was not a single mention for Swaraj. At this Gandhi got disappointed and came back to India and revived the CDM. The movement continued over a year and lost its momentum by 1934.

Result:

Congress was declared illegal.

Many leaders including Jawaharlal Nehru & Gaffar khan were arrested.

All boycott movement was stopped by British government.

How Participants saw the movement

Swaraj meant different to different social groups who joined CDM.

In the countryside:

Many activists the rich peasant's communities, like patidars of Gujarat and Jats of Uttar Pradesh. They were mostly the producers of Commercial crops; they were very hard hit by trade depression and following prices.

Hence, they could not pay the revenue demands of the government.

Government refused to reduce the revenue tax, which resulted in the peasants join the national movement and support for CDM.

The fight for independence was related to their immediate grievances demand for reduction of revenues. Greatly disappointed when the movement was called off in 1931, without the reduction of revenues.

The Poor: They were the peasant who had rented the land from landlords and cultivated it. The depression made than unable to pay the rents to the landlords. Their own incomes dwindled. They wanted the congress to help them in getting the unpaid rent remitted but the congress did not give any support for no rent campaign. So their relations with the congress become uncertain.

The Business Classes

The Indian industrialist and merchants had made huge profits and became powerful.

They were now very keen in expanding their business and hence reacted against colonial policies that restricted business activities. They wanted protection against import of foreign goods & a rupee sterling foreign exchange ratio that would discourage import.

To organized business interest, they formed the Indian industrial and commercial congress in 1920 & the federation of Indian chamber of commerce & industries (FICCI) in 1927. Some prominent industrialist like Purshottamdas Thakurdas & G. D. Birla, the industrialist attacked colonial control over the Indian economy and supported the CDM. These industrialists refused to buy or sell foreign goods & also gave financial assistance to the movement.

But after the failure of RTC, business class were no longer interested, it curbed their enthusiasm. they were worried about socialism among the youth as it would prolong disruption of business, were apprehensive of the militant activities and the growing influence of socialism.

The Industrial Class (Worker laborers)

Workers did not participate in CDM in large number expect in Nagpur.

They adopted a few Gandhian programmes like boycott of foreign goods as part of their own movement against low wages and poor working conditions.

There were strikes by railway workers in 1930 & dock- worker in 1932.

Thousands of workers in Chota Nagpur took part in rallies and boycott campaign. But the congress was reluctant to include workers demand in their struggle for independence, as it would alienate the industrialist.

Women's and the CDM

Women entered in the very beginning of the movement.

Thousands of women had picketed liquor shops, foreign cloth & goods. Many went to jail most of the women were from high caste in urban areas & rich women from rural areas.

They served national movement as their sacred duty, women who had never stepped out of their house participated in the movement.

THE LIMITS OF CIVIL DISOBEDIENCE

In CDM many social groups did not participate.

The untouchables or Dalits as Gandhi called them as Harijans. Gandhi declared that India would not achieve Swaraj without the active participation of Dalits and if untouchability was not totally removed.

He launched satyagraha to secure them entry to temples & access to public wells.

Dalits wanted political right & solutions to their problem. They demanded reservation of seats in educational institutions, separate electorate to choose their representatives to the legislative council. They wanted to solve their social problems.

Dr. B .R.. Ambedkar became their leader; they formed associations in 1930. He called the association as the depressed classes association. He demanded separate electorate for Dalits. British Government accepted his demand but Gandhi did not agree as he believed that the Dalits would never be integrated into society if they got separate electorates.

Gandhi began to fast unto death finally in September 1932, Dr. Ambedkar signed a pact with Gandhi called Poona pact in September 1932. It gave reserved seats in provincial & central legislative council to the depressed classes. They were to be voted by the general electorate.

Muslims: Muslims did not participate in CDM for following reasons:-

Muslims felt alienated from congress seemed to be associated with Hindus & was termed as Hindu Mahasabha. There were Hindu-Muslim clashes and riots in many areas which worsened the relations between Hindus & Muslim.

Attempt was made in 1927 by the congress & Muslim league to from an alliance. It seemed possible as Mohd. Ali Jinnah, the leader of Muslim league agreed to give up the demand for separate electorates if. Muslims were assured reserved seats central assembly.

Representation in proportion to population in Muslim dominated provinces (Bengal & Punjab)

Result:

But negotiation failed in 1928, when M.R. Jayakar of Hindu Mahasabha strongly opposed efforts at compromise.

So under CDM, Muslim feared that their culture & identity would be submerged and majority domination of Hindus.

THE SENSE OF COLLECTIVE BELOGING

In the topic we will try to find out why nationalism captured the imagination of the people and how did sense of belonging develop in the minds of people who belong to different communities regions & speak different languages:-

1. A strong belief that we all are the part of same nation this binds them together.
2. We all were together in struggle against British like the revolt of 1857, NCM, CDM.
3. We all had common history.
4. We all have common folklore & songs popular prints & symbols are the factors that gave rise to nationalism.
5. **India developed symbols like**

   Bharat Mata: When nationalism in India reached zenith the symbol of Bharat Mata aroused the sense of pride that 'India is the goddess, the land of religion.
6. Bankim Chandra Chattopadhyay created the image in his song Vande Mataram – a hymn in praise of mother India 1870 in his novel. Anand Math – this sung was sang during Swadeshi movement.

   Rabindranath Tagore – painted Bharat Mata as a calm, composed divine & spiritual figure.

   Bharat Mata became the symbol of nation for non people were ready to sacrifice their lives.
7. **Folk Songs & Folklore**

   These also become the symbol of nation & revived people's pride in their culture.

   Nationalist like Rabindranath Tagore of Bengal, Natesa Sastri of madras compiled songs, ballad myths & folklore &also published a 4 volume collection of 'The folklore of southern India.
8. The third rallying part was reinterpretation of history. The British painted Indian as backward, primitive in capable of governing themselves.
9. Indians delved into the past history and discovered India's great achievement, India's glorious development in mathematics literature religion culture philosophy craft and trade instilled a sense of pride among the Indians. People realized that all this declined due to the advent of British & colonial rule. All the miseries were caused by

British rule. Not only this, British also sowed the seed of communalism by announcing separate electorates for the Muslims.

CONCLUSION:

Gandhiji channelized the anger against the colonial government into a common struggle for freedom. He forged Unity between Hindus & Muslim.

He also tried to remove differences tried not to alienate any group. Gandhi saw the emergence of a nation waiting to liberate itself from colonial rule.

EXERCISE

Answer the following question:-

1. When was Gandhi-Irwin pact signed?
2. What is meant by Gandhi's idea of Satyagraha?
3. Why were Indians outraged by the Rowlatt act?
4. Write short note on Simon commission.
5. Why did Gandhi ji decide to start Non-cooperation?
6. Why did Gandhiji decide to withdraw Non-cooperation?
7. Discuss Gandhiji attitude towards Dalits?
8. Discuss the salt march to make clear why was it an effective symbol of resistance against colonialism?
9. Which richest peasant communities were active in civil Disobedience movement?
10. When were Indian industrial and commercial congress and FICCI founded and why?
11. Who was B.R. Ambedkar? What was Poona act?
12. Who created the first image of Bharat Mata?
13. Discuss the course of Non-cooperation in towns.
14. What was the reaction of British Indian colonial government towards civil disobedience movement?
15. What was island emigration act of 1859?
16. What was the nation of Swaraj for the plantation workers in Assam?
17. Who were the founders of Swaraj party?
18. When and which city did the Jallianwala Bagh incident occur?
19. Name any two industrialists who actively participated in the civil disobedience movement.
20. What was the impact of First World War on India?
21. How was the Non-cooperation movement converted into a national movement by Gandhiji?
22. Explain the role of women in civil disobedience movement?
23. How flag was used to promote the spirit of nationalism among Indians?
24. How was the civil disobedience movement different from Non-cooperation movement?
25. What were the limits of civil disobedience movement?

THE RISE OF NATIONALISM IN EUROPE

✱ **IMPORTANT TERMS**

☛ **National:** A sense of belonging to one nation. Feeling or pride and patriotism towards the country on, belongs.

☛ **Absolutist:** Unrestricted, despotic and authoritarian often refers to a centralized repressive monarchial government.

☛ **Utopian:** An ideal situation a vision too good to be realized in practice.

- **Nation State:** A state having a cornrnon and contiguous boundary with inhabitants, people sharing common language, race and religion. Majority of its citizen develop a sense of cornrnon identity and share a common history.
- **Plebiscite:** A direct vote by which the people of a region, themselves decide of accept or reject a proposal.
- **Sovereignty:** Supreme Power.
- **Monarchy:** Form a government headed by a monarch or a hereditary or dynamics ruler.
- The estates General: Referred to the French Parliament an elected body which was renamed as the national Assembly after the revolution of 1789.
- **Civil Code:** A systematic set of laws for the citizen.
- **Liberalism:** Derived from the word 'liber' meaning free. The idea of liberalism stands for freedom of individual and equality of all before law. Politically it refers to representative government.
- **Suffrage:** the right to vote.
- **Elle:** Elle was used to measure cloth, prevalent in German states.
- **Zollverein:** A custom union, formed in 1034 in Prussia to remove barriers of trade.
- **Considervatism:** A spirit or philosophy which believes in maintaining and preserving traditional values and institutions. It prefers gradual change to quick and drastic revolutionaries.
- **Carbonari:** A secret society of Italy consisting of young revolutionaries.
- **Young Italy:** A secret society founded by Mazzini at Masseles for organizing revolutionary activities.
- **Ottoman Empire:** Turkish Empire ruled by the caliph. He is the spiritual and temporal head of the Muslim.
- **Romanticism:** A cultural movement which aimed at developing a particular form of national sentiment and promote a feeling of collective heritage as the basis of motion.
- **Das yolk:** A German word meaning common people.
- **Republic:** A state where the head of the state is elected and does not hold a hereditary position.
- **Feminist:** People who advocate women are right on the basis of equality of sexes.
- **Ideology:** System of ideas reflecting a particular social and political vision.
- **Allegory:** Symbol representing an abstract idea; an idea indentified through a person or a thing.
- **Balkan region:** A region in Europe with geographical and ethnic variation. The region covers the states of modem day Romania, Bulgaria, Albania "Greece, 'Macedonia, Croatia, Bosnia, Herzegovina, Slovenia, Serbia, Montenegro. The inhabitants of the region are known as the slaves.

✱ IMPORTANT DATES AND LANDMARKS

1780 - French Revolution.

1804 - Napoleonic code was introduced abolishing privileges based on birth. He upheld equality before law.

1815 - Congress of Vienna. Representatives of Britain, Russia, Prussia and Austria met at Vienna to Draw up a settlement for Europe.

1830 - Revolution in France.

1834 - Zollverein or the customs union was formed in Prussia (largest state in Germany) abolishing Tariff barriers.

1848 - Great revolution in France and Formation of French republic and abdication of French monarch Louis Philippe.

1861 - Frankfurt Parliament in Germany was conveyed to set up an all-German national assembly to Draft a German constitution.

- Victor Emmanuel II of piedmont was proclaimed the king of united Italy. An assembly comprising princes of German states, representatives of army and Prussian ministers headed by Otto von Bismarck gathered in the Palace of Versailles to proclaim the German empire (United Germany), headed by Kaiser William I of Prussia.

1914 - Beginning of First World War.

The Concept and Parties of a modern state in which a centralized power exercised sovereign Control over a clearly defined territory had been developing over a long period of time in Europe. But a nation state was one in which the majority of its citizens and not only its rules, came to develop a sense of common identity and shared history or descent. This commonness did not exist from time memorial it was forget through struggles, through the actions of leaders and the common people.

✱ THE FRENCH REVOLUTION AND THE IDEA OF THE NATION

France was a fully fledged territorial state in 1789 under the rule of an absolute monarchy. The political and constitutional changes that came in the wake of French Revolution led to the transfer of sovereignty from the monarchy to a body of French citizens. The revolution proclaimed that it was the people who would hence forth constitute the nation and shape its destiny.

A centralized administrative system was put in place and it formulated uniform laws for all citizens within its territory. Internal customs, duties and dues were abolished and a uniform system of weights and measures was adopted. Regional dialects were discouraged and French as it was spoken and written in paris, became the common language of the nation.

When the news of the events in France reached the different cities of Europe, Students and other members of educated middle classes began setting up Jacobin Clubs. Their activities and campaigns prepared the way for the French armies which moved into Holland, Belgium, Switzerland and much of Italy in the 1790s. With the outbreak of the revolutionary wars, the ***French armies began to carry the idea of nationalism abroad.*** Though a return to monarchy napoleon had, no doubt, destroyed democracy in France, but in the administrative field he had incorporated revolutionary principles in order to make the whole system more rational and efficient.

The civil code of 1804 usually known as the Napoleonic code did away with all privileges based or birth, established equality before the law and secured the right to property. This code was exported to the regions under French Control in the Dutch Republic in Switzerland, in Italy and Germany. Napoleon abolished the feudal system and freed peasants from serfdom and memorial dues. In the towns too guild restriction were removed. Transport and communication systems were improved Peasants, artisans, workers and new businessmen enjoyed a new-found freedom.

✱ THE MAKING OF NATIONALISM IN EUROPE

Germany, Italy and Switzerland were divided into kingdoms, duchies and cantons whose rules had their autonomous territories. Eastern and Central Europe were under autocratic monarchies within the territories of which lived diverse peoples. They did not see themselves as sharing a Collective identity or a common culture. Such differences did not easily promote a sense of political unity. The only tie binding these diverse groups together was common allegiance to the emperor.

(i) **The Aristocracy and the new middle Class:** The members of this class were united by a common way of life that cut across regional diversions. They owned castes in the countryside and also town houses. This powerful autocracy was however, numerically a. small group. The majority of the population was made up of the peasantry. Industrialization began in England in the second half of the 18th century, but in France and Parts of the German states it occurred only during the 19th century. In its wake new social groups came in to being a working class population and middle classes made up to industrialists, businessmen, and professionals. It was among the educated, liberal middle classes that ideas of national unity following the abolition of aristocratic privileges gained popularity.

(ii) **What did liberal nationalism stand for?**

The term 'Liberalism' derives from the Latin root "liber' means free. For the new middle classes liberalism stood for freedom for the individual and equality of all before the law. Politically it emphasized the concept of government. Sine the French Revolution, liberalism had stood for the end of autocracy and clerical privileges a constitutional and representative government through parliament.

In the economic sphere, liberalism stood for the freedom of markets and the abolition of state imposed restrictions on the movement of goods and capital. During the 19th century this was a strong demand of the emerging middle classes.

(iii) **A New Conservation after 1815**: After the death of Napoleon in 1815 European governments were driven in a spirit of conservatism. It means a political philosophy that stressed the importance of tradition established institutions and customs and preferred gradual development to quick change.

In 1815 representatives of the European powers, Britain, Russia, Prussia and Austria Who had collectively detected napoleon met at Vienna to draw up a settlement for Europe.

The Bourbon dynasty, which had been deposed, during the French revolution, was restored to power, and France lost the territories it had annexed under, Napoleon. A series of states were set up on the boundaries of France to prevent French expansion in future. Thus the kingdom of the Netherlands, which included Belgium, was set up in the north and Geneva was added to Piedmont in the south Prussia was given important territories on its western frontiers, while Austria was given control of northern Italy. But the German confederation of 39 states that had been set up by Napoleon was left untouched. Russia was given part of Poland while Prussia was given a portion of Saxony. The main intention was to restore the monarchies that had been over thrown by napoleon and create a new conservative order in Europe.

(iv) The Revolutionaries: Matter 1815, the fear of repression drove many liberal nationalists underground. Secret societies sprang up in many European states to train revolutionaries and spread their ideas. To be revolutionary at this time meant a commitment to oppose monarchical forms that had been established after the Vienna congress and to fight for liberty and freedom. Most of these revolutionaries also saw the creation of nation states as a necessary part of this struggle for freedom:

One such was the Italian revolutionary Giuseppe Mazzini. He became a member of the secret society of the Carbonari. He subsequently founded two more underground societies first, young Italy and Young Europe, whose members were like minded. Mazzini believed that god had interned to be the natural interest of mankind.

✱ **THE AGE OF REVOLUTIONS: 1830-1848**

The first revolt took place in France in July 1830. The kings who had been restore to power during the conservative's reaction after 1815, were now over thrown by liberal revolutionaries who installed a constitutional monarchy with Louis Philippe. 'When France Sneezes' Metternich once remarked 'the rest of Europe catches cold'.

(i) **The Romantic Imagination and National Feeling:** Romanticism, a cultural movement which sought to develop a particular form of nationalist sentiment. Romantic artists and poet's generally criticized the glorification of reason and

science and focused instead on emotions, intuition and mystical feelings. Their efforts were to create a sense of shared collective heritage of common cultural past.

The other Romans as the German philosopher to hear G. Herder claimed that true German culture was to be discovered among the common people. It was through folk songs, folk poetry and folk dances that the true spirit of the nation was popularized. The Emphasis on Vernacular language and the collection of local folklore was not just to recover an ancient national spirit, but also to carry the modern nationalist message to large audiences who were mostly illiterate. Karol Kurpinski, for example, celebrated the national struggle through his operas and music, turning folk dances like the polonaise and mazurka into nationalist symbols.

Language too played an important role in developing nationalist sentiments. After Russian occupation, the polish language was forced out of schools and the Russian language was imposed everywhere.

(ii) **Hunger, hardship and Popular Revolt:** The 1830s were years of great economic hardship in Europe. Population from rural areas migrated to the cities to live in overcrowded slums. Europe where the aristocracy still enjoyed power, peasants struggled under the burden of feudal dues and obligations.

The year 1848 was one such year. Food shortage and widespread unemployment brought the population of Paris out on the roads. Barricades were erected and Louis Philippe was forced to leave National Assembly proclaimed a Republic, granted suffrage to all adult males above 21 and granted the right to work. National workshops to provide employment were set up.

On 4 June at 2 P.M. a large crowd of weavers emerged from their homes and in Paris up to the intention of their contractor demanding higher wages. They were treated with scorn and threats alternately Following this a group of them forced their way into the house, smashed its window panes; furniture proclaimed The contractor fled with his family to a neighboring village, which, however refused to shelter such a person. He returned 24 hours later having requested the army. In the exchange that followed eleven weavers were shot.

(iii) **1848: The Revolution of the Liberals:** parallel to revolts of the poor, unemployed and starving peasants and workers in many European countries in the year 1848, a revolution led by the educated middle classes was under way. Events of February 1848 in France had brought about the abdication of the monarch and a republic based on universal male suffrage had been proclaimed. In other parts of Europe where independent nation. States did not yet exist such as Germany, Italy, Poland, the Austro Hungarian Empire men and women of the liberal middle classes combined their demands for constitutional with national unification. They took advantage of the growing popular unrest to push their demands for the creation of a nation-state on parliamentary principles a constitution, freedom of the press and freedom of association.

Though conservation forces were able to suppress liberal movements in 1848, they could not restore the old order. Monarchs were beginning to realize that the cycles of revolution and repression could only be ended by granting concessions to the liberal-nationalist revolutionaries. Hence in the years after 1848, the automatic monarchies of central and Eastern Europe began to introduce the changes that had already taken place in Western Europe before 1815. Thus serfdom and bonded labor were abolished both in the Habsburg and in Russia. The Habsburg rulers granted more autonomy to the Hungarians in 1867.

✱ UNIFICATION OF ITALY

Unification of Italy is the result of the dedication, contribution and effort of three great leaders – Mazzini, the idealist, Cavour, the diplomat and Garibaldi. The patriot soldier, However but for the support of King Victor Emmanuel II of piedmont and Sardinia, a unification would not have been possible.

Italy has a long history of political fragmentation. Italians were scattered over several dynastic states were ruled by the rulers of Bourbon dynasty of Spain and France.

During the 1930's., Giuseppe Mazzini sought to unite these regions into an Italian republic. He organized the youth through a secret society called 'Young Italy' and inspired them with revolutionary ideas. However their effort was crushed in 1830 and 1848.

It was clear that the responsibility of uniting Italian states has to be shouldered by Victor Emmanuel II and to be achieved through war. In this task Emmanuel II got the whole hearted support of his chief minister Cavour who led the movement of unification. Through a tactful diplomatic alliance with France, Cavour succeeded in defeating the Austrian forces, in 1859. Now he was able to secure the support of Garibaldi, another great patriot. Earlier Garibaldi Joined Young Italy movement and participated in the republican uprising in Piedmont in 1834. He had to leave the country because of his activities. After his return he organized a volunteer army called Red Shirts and used them for Italian unification.

In 1860, Garibaldi led the famous expedition to south Italy and freed the states from bourbon rulers. In 1867, he led his army to Papal States but could not match the strength of the French troops who were guarding the Papal States.

In 1870, when during a war with Austria French withdrew their troops, Papal States were occupied by Italian army and Italian unification was completed.

Unification of Germany

The nationalist feelings were widespread among middle-class Germans, who in 1848 tried to unite the different regions of the Germany confederation into a nation-state governed by an elected parliament. This liberal initiative to nation-building was however, represented by the combined forces of the monarchy and the military, supported by the large land owners (called Junkers) of Prussia. From then on, Prussia took on the leadership of the movement for national unification. Its chief minister, Otto von Bismarck, was the architect of this process carried out with help of the Prussian army and bureaucracy. Three was over seven-years-with Austria, Denmark and France-ended in Prussian Victory and completed the process of unification. In January 1871, the Prussian king, William I, was produced German Emperor in a ceremony held at Versailles.

✱ VISUALISING THE NATION

Artists in the 18th and 19th centuries found a way out by personifying nation. In other words they represented a country as if it were a person. Nations were then portrayed as female figures. The nation did not stand for any particular woman in real life; rather it sought to give the abstract idea of the nation a concrete form. That is the female figure became an allegory of the nation. After this so many countries used the same symbol (female).

✱ NATIONALISM AND IMPERIALISM

The most serious source of nationalist tension in Europe after 1871 was the area called the Balkans. The Balkans was region of geographical and ethnic variation comprising modern day Romania, Bulgaria, Albania, Greece, Macedonia, Croatia, Bosnia, Herzegovina, Slovenia, Serbia and Montenegro whose inhabitants were broadly known as the slaves. A large parts of the Balkans was under the control of the Ottoman Empire.

The spread of the ideas of romantic nationalism in the Balkans together with the disintegration of the Ottoman Empire made this region very explosive. All through the 19th century the Ottoman Empire had Sought to strengthen itself through modernization and internal reforms but with very little success. One by one its European subject nationalities broke away from its control and declared independence. The Balkan peoples asked their claims for independence or political rights on nationalistic and used history to prove that they had once been independent but had subsequently been subrogated by foreign powers. Hence the rebellions nationalities in the Balkans though of their singles as attempts to win back their long lost independence.

Short note on (a) Giuseppe Mazzini

The Italian revolutionary Giuseppe Mazzini, Born in Genoa in 1807, he became a member of the secret society of the Carbonari. As a young man of 24, he was sent into exile in 1831 for attempting a revolution in Liguria. He subsequently founded two more underground societies, first, young Italy in Marseilles, and then, Young Europe in Berne, whose members were like-minded young men from Poland, France, Italy and the German states. Mazzini believed that god had intended nations to be the natural units of mankind. So Italy could not continue to be a patchwork of small states and kingdoms. It had to be forgetting into a single unified republic within a wider alliance nations. This unification alone could be the basis of Italian liberty. Following his model, secret societies were set up in Germany, France, Switzerland and Poland Mazzini's relentless opposition to monarchy and has vision of democratic republics frightened the conservatives. Metternich described him as 'the most dangerous enemy of our social order.

Count Carnillo-de Cavour

Chief minister Cavour who led the movement to unify the regions of Italy was neither a revolutionary nor a democrat. Like many other wealthy and educated members of the Italian elite, he spoke French much better than he did Italian. Through a tactful diplomatic alliance with France engineered by Cavour, Sardinia-Piedmont succeeded in defeating the Austrian forces in 1859. Apart from regular troops, a large number of armed volunteers under the leadership of Giuseppe Garibaldi joined the fray. In 1860, they marched into South Italy and the kingdom of the Two Sicily and succeeded in winning the support of the local peasants in order to drive out the Spanish rulers. In 1861 Victor Emmanuel II was proclaimed king of united Italy. However, much of the Italian population, among whom rates of illiteracy were very high, remained blissfully unaware of liberal nationalist ideology. The peasant masses who had supported Garibaldi in southern Italy had never heard of –la Talia, and believed that 'la Talia' was Victor Emmanuel's wife!

The Greek war of independence

An event that mobilized nationalist feelings among the educated across Europe was the Greek war of independence. Greek has been part of the Ottoman Empire since the fifteenth century. The growth of revolutionary nationalism in Europe sparked off a struggle for independence amongst the Greeks which began in 1821. Nationalists in Greece got support from other Greeks living many and also from many. West Europeans who had symptoms for ancient Greek culture. Poets and artists lauded Greece as the cradle of European civilization and mobilized public opinion to support of struggle against a Muslim empire. The English poet Lord Byron organized funds and later went to fight in the war, where he died on fever in 1824. Finally, the Treaty of constitutional of 1832 recognized Greece as an independent nation.

Frankfurt parliament

In the German regions a large number of political associations whose members were middle-class professionals, businessmen and prosperous artisans came together in the city of Frankfurt and decided to vote for an all German national Assembly. On 18 may, 1848,831 elected representatives marched in a festive procession to take their places in the Frankfurt parliament convened in the church of S. Paul. They drafted a constitution for a German nation to be headed by a monarchy subject to a parliament. When the deputies offered the crown on these terms to Friedrich Wilhelm IV, King of Prussia, he rejected and joined other monarchs to oppose the elected assembly. While the opposition of the aristocracy and military became stronger, the social basis to parliament eroded. The parliament was dominated by the middle classes who resisted the demands of workers and artisans and consequently lost

EXERCISE

VERY SHORT ANSER TYPE QUESTIONS

1. What is “State of liberty”? What does the statue signify?
2. What is a nation – state? What are its main features?
3. In the context of pre-revolutionary France’s what is referred to as the Estates general.
4. Why was the Napoleonic code significant?
5. What does liberalism stand for?
6. Why was Zollverein set up in Prussia in 1834? What was its significance?
7. Name four European powers that collectively defeated Napoleon.
8. Who was Metternich?
9. With what aim was the Treaty of Vienna (1815) signed?
10. Name two secret societies set up in Italy in early 19th century.
11. When and under whom was the constitutional monarchy setup in France?
12. Name the treaty which recognized Greece as an independent nation.
13. What was the main feature of European Romanticism?
14. Why did the weavers in Silesia rise In revolt against the contractor?
15. Who constituted the European middle class in 19th century?
16. State any two steps taken by the conservative European ruler to prevent the spread of nationalistic and revolutionary ideas.
17. Name two European states which were unified into nation states in the latter half of 19th century name one leader of each of these two countries.
18. Who was Garibaldi?
19. Name the four ethnic groups which inhabit England.
20. What is the importance of Act of union, 1707?
21. What are the national symbols of the New Britain?
22. What does the word “Germania” stand for?
23. How is Germania depicted?
24. Usually how is a nation personified?

SHORT ANSWER TYPE QUESTIONS

1. What is the main symbol of an independent nation?
2. Examine any four reasons for the nationalistic upsurge in 19th century Europe?
3. Explain the concept of a nation state.
4. What are the most important achievements of the French Revolution of 1789?
5. How did the French revolutionary ideas spread to other countries?

6. What was the significance of the Napoleonic code?
7. Why was the Napoleonic rule over other region unpopular with some section of people?
8. Examine the reasons for the political disunity in Hobsburg Empire of Austria and Hungry.
9. How did the growth of industrialization change the social and political equation of Europe?
10. Examine the liberal ideology imbibed by the revolutionary leader after French revolutions.
11. How did liberalism give rise to economic nationalism?
12. What was the impact of treaty of Vienna (1815) on European people?
13. Describe the contribution of Mazzini In the unification of Italy?
14. Who was Metternich? What was his role in congress of Vienna?
15. Examine the effects of revolutionary upheaval in France in 1830.
16. What was indicated by Metternich's can most 'If France sneezer, rest of Europe catches cold".
17. How do folklore and vernacular language help in developing nationalist sentiments?
18. Examine the events leading to the formation of the French Republic in 1848.
19. What was the significance of the Frankfurt parliaments (1848)? Why did it fail?
20. How was Cavour? Examine any two of his contributions.
21. How was Ireland incorporated in the United Kingdom?
22. Who was Garibaldi? Examine his contribution in the unification of Italy.
23. Why are allegories used to generate nationalism? Give two examples of allegories used in France and Germany.

LONG ANSWER TYPE QUESTIONS

1. Mention some of the factors of comparison of Cavour and Bismarck.
2. Mention the main factors responsible for the rise of nation state.
3. What do you know about the 'black ten years'? Explain events related to this.
4. What was the compromise between Austria and Hungary?
5. How for it was correct that the necessity of both Italian and German unification was not only political but also economic?
6. How was the history of the development of nationalism in Britain unlike the rest of Europe?
7. Mention the powers that had collectively defeated Napoleon?
8. Discuss the importance of language and popular traditions in the creation of national Identity.

   Examine the process of unification of Italy?

POLITICAL SCIENCE (CIVICS)

POWER SHARING

IMPORTANT TERMS AND CONCEPTS:-

1. **Ethnic:** A social division based on shared culture. People belonging to the same ethnic in their common decent because of similarities of physical type or both they need not always have the same religion or nationality.
2. **Majoritarianism:** A belief that the majority community should be to able to rule a country in which ever way it wants by disregarding the wishes and need of the minority.
3. **Civil war:** A violent conflict between opposing group with in a country that becomes so intense that it appears like a war.
4. **Prudential:** it is a set of reason which favors power sharing. It is based on prudence or on careful calculation of gain and losses. Prudential reason stress beneficial consequences.

5. **Moral:** it is also a set of reason which emphasis the worth of power sharing. It is based on moral considerations.
6. **Community government:** A type of Government which is elected by people belonging to one language community is called "Community Government: e.g. Dutch French and German Speaking no matter where they live. This is a very specific type of Government in Belgium.
7. **Federal Government:** Federal Government is a type of Government in which power are shared among the different level. Like union level. In this the government for the entire country at the union level is called "federal Government"
8. **Power sharing: "**power sharing" is the distribution of power/ subject among of the government like legislature executive and judiciary. It is an intelligent step to ensure the stability of political order. Besides power sharing also includes, sharing of power among different level like union, state and local bodies.
9. **Checks and Balances:** A system in which each organ of the government checks the other which result in a balance of power among various institution. It ensures that none of the organ can exercise unlimited power.
10. **Vertical distribution of power:** It is a type of distribution of power which involves the higher and lower levels of government such as central, provincial or regional levels.
11. **Coalition Government:** When the alliance of two or more parties gets elected and forms a government it is known as the "Coalition government". This is another form of power sharing.
12. **Sri Lankan Tamil:** Tamil native of Sri Lanka are called. "Sri Lankan Tamils". They constitute 13 percent of the population.
13. **Indian Tamil:** The Tamilians whose forefather came from. India as plantation worker during colonial period and settle these (Sri Lanka) are called "Indian Tamil" They constitute 5 percent of the population.
14. **Horizontal Distribution of power:** A type of distribution in which power is shared among different organ of government such as the legislation legislative and judiciary. They are placed at the same level.

Belgium

It is small country in Europe with a population of a little more than a crore has an ethnic problem that is a highly complex. It has border with Netherlands France & Germany.

- Of the country's total population, 59% live in the Flemish region and speak Dutch language.
- 40% people live in the Walloon region and speak French.
- Remaining 1% of the Belgians speak German.
- In the capital city Brussels 80% French while 20% are Dutch speaking.
- The minority French speaking community was relatively rich and powerful.
- Dutch speaking people received benefits of economic development & Education much later & this led to tension between the two groups.
- The conflicts between the two communities was more sever in Brussels because the Dutch speaking people constituted a majority in the country, but a minority in the capital.

SRI LANKA:

- It is island nation. Just a few kilometers of the southern coast of Tamil Nadu.
- It has the population of about 2 crore.
- It has the diverse population. The major social group in Sri Lanka are:
  - Sinhala speaking which are 74%
  - Tamil speakers which are 18% Tamil are also divided in two groups.

● Indian Tamil's whose forefather comes from India as plantation worker during colonial period are 50%

- Sri Lanka Tamil are concentrated in north and east of the country.
- Sinhala speaking people are Buddhist.
- Tamil are either Hindus or Muslim
- About 7% are Christian, who is both Tamil & Sinhalas.

In Belgium & Sri Lanka the majority community is Dutch speaker & Sinhalas could take advantages of their numeric majority & implode their will on the entire country.

MAJORITARIANISM IN SRI LANKAS:

The leader of Sinhala community dominated the government & adopted a serious of Majoritarian measure.

- Sri Lanka becomes independence in 1948.
- In 1956 an Act was passed to recognize Sinhala as only language, disregarding Tamil.
- Sinhala applicants were favored for university and government job.
- The constitution gives special protection to Buddhism.
- The result of these policies make the Sri Lanka Tamils feel that

● Leaders were indigestive to their language and culture.

● Government polities deprived them of equal political right.

- They were discriminated against in getting jabs & other opportunities.

● Recognition of Tamil as an official language

● Regional autonomy

● Equality of opportunity in securing education & job

● Demand was made for an independent Tamil Elam in northern part of Sri Lanka. The distrust between the two communities turned in to widespread conflicts. It soon turned into a civil war.

Accommodation in Belgium

The Belgium leader took a different path. They recognized to existence and cultural diversities between 1970 and 1993 they amended their constitution four times so as to work out an arrangement that would enable everyone to together with in the same country.

1. Constitution prescribes that the number of Dutch and French- speaking minister shall be equal in the central government.
2. Many power of the central government have been given to state government of the two region of the country the state government are not subordinate to the central Government.
3. Brussels has a separate government in which both the communities have equal representation. The French speaking people accepted equal representation in Brussels because the Dutch-speaking community has accepted equal representation in the central Government.
4. Apart from the central and state Government there is a third kind of government the community government. This community government is elected by people belonging to one language community Dutch. French German speaking no matter where they live. This government has the power regarding cultural educational and language related tissue.
5. Thus the Belgium model very complicated. It indeed is very complicated even for people living in Belgium. But these arrangements have worked well so far. They helped to avoid civic strife between the two major communities and a possible division of the country on linguistic line. When many countries of Europe came together to form the European Union, Brussels was chosen as it head quarters.

What do we learn from these two stories of Belgium and Sri Lanka?

Both are democracies. Yet they diet with the question of power sharing differently. In Belgium the leaders have realized that the unity of the country is possible only by respecting the feeling and interest of different community and region.

Sri Lanka shows us a contrasting example. It shows us that if a majority community wants to force its dominance over other and refuses. To share power, it can undermine the unity of the country.

Why power sharing is desirable?

Comparison: In Belgium leader realized that by mutual arrangement of sharing power unity in the country was possible.

In Sri Lanka the majority county wanted to dominate & refused to share power. This hindered the unity of the country.

Power sharing is good because:

(i) It helps to reduce the conflict between social groups.

(ii) It is a good way to ensure the stability of political order.

(iii) It is good for democracy. It is the spirit of democracy because people have the right to be consulted on how they are to be governed.

(iv) The first point is called prudential because it state that power sharing will bring out better out comes.

(v) Moral reason emphasizes that very act to power sharing as valuable.

Forms of Power sharing:-

(i) M any people felt that if the power to decide is dispersed, it would not be possible to take quick decision and to enforce them.

One basic principle of democracy is that people are the source of all political power, & people rule themselves, in a good democratic government, due respect is given to diverse groups that exist in a society. Very one has a voice in the shaping of public policies.

POWER SHARING ARRANGEMENT IN MODERN DEMOCRACY

1. Power is shared among different oranges of the government

Horizontal distribution of power

- In this form of distribution power is shared among different oranges of government such as the legislature, executive and judiciary.
- This type of distribution advocates separation of powers in which different organs of government at the same lever exercise different powers.
- Separation of power ensures a check over the unlimited powers of each organ. This result in a balance of power among various institutions.
- It is also called as system of “checks and balances”.

2. Vertical Distribution of powers:-

- Under this form of power sharing arrangement power is shared among government at different level. E.g. a general government at different levels. E.g. a general government for the entire country and government at the provincial level.
- A general government of the entire country is usually called Federal government. In our country it is called central government,
- In some countries like India & Belgium the constitution clearly lays down the powers of the government at different levels. This is called the “federal division powers”

All such division of powers involving higher and lower level of government is called vertical division of powers.

3. **Power sharing among different social groups:**

- In another way power may also be shared among different social groups, such as the religious and linguistic groups.
- This type of arrangements is meant to give space in the government and administration to diverse social group who otherwise would feel alienated from the government.
- This method is used to give minority communities a fair share in power.
- **Community Government** in Belgium and **Reserved constituencies is assemblies** and **Parliament** in India is examples of this type of arrangement.

4. **Power sharing among policies parties and pressure groups:**

Sometimes power sharing arrangement can also be seen in the way political parties, pressure groups and movements control or influence power because in a democracy the citizens must have a choice among various contenders for powers.

SOLVED EXERCISE

1. When did Sri Lanka gain independence?

Ans. Sri Lanka gained independence from the United Kingdom on February 4, 1948.

2. List all the language spoken in Sri Lanka.

Ans. **S**inhalas and Tamil

3. Which language was recognized as the official langue after independence in Sri Lanka?

Ans. **S**inhala was recognized as the official language in an Act passed in 1956.

4. Trace out the ethnic composition of Sri Lanka has a diverse population?

Ans. With a complex ethnic composition like:-

(a) Sinhalas speaking people 74% a major social group.

(b) Tamil 18% among Tamils there are two sub groups.

(i) Sri Lankan Tamil- 13%

(ii) Indian Tamil- 05%

5. Trace out the ethnic composition of Belgium has a diverse population.

Ans.

- In Belgium (Europe), 59% Dutch speaking lives in the Flemish, 40% French speaking in Wallonia region and 1% is German speaking.
- The capital city of Belgium is Brussels and in Brussels 80% people speaks French and 20% people speak Dutch language.
- There was a tension between the Dutch and French speaking communities during the 1950s and 1960s over the issue of power sharing.

6. What kinds of measures were adopted in Sri Lanka after independence?

Ans. **A** Majoritarianism were adopted to establish Sinhala supremacy in the government.

7. When did Belgium gain independence?

Ans. Belgium independence was declared on October 4, 1830 but it was recognized on April 19, 1839.

8. List all the languages spoken in Belgium.

Ans. Dutch, French, German.

9. How much time was Belgium constitution amended?

Ans. It was amended four times between 1970 and 1993.

10. What is civil war?

Ans. A civil war is a violent conflict between various groups in a country itself often due to undermining of a particular groups interest or sentiments by the other group.

11. What is meant by prudential reason?

Ans. Prudential reason are the one advanced by sensible and careful though and due calculation of the resulted gain and lasses.

12. What is meant by moral reasons?

Ans. Moral reasons are based on the right and wrong in an event. These decisions are physiological rather than practical and are concerned with the principles of behave our in according with standard of right and wrong.

13. What is community government?

Ans. A community government is one in which different social groups are given the power to handle the affairs related to their communities. They are expanded to work joined for the benefits of the common masses without undermining any one community.

14. What is the relationship between democracy and power sharing?

Ans. Democracy means giving every citizens the tight and power to make the decisions though their elected repetitive. Power sharing is dividing power among various social groups to give them equal representation in the in the governance. Power sharing is the essence of democracy where every individual irrespective of cultural and language differences feels involved in the political sector.

15. What is a coalition government?

Ans. A collation government is a cabinet of parliamentary government in which several parties cooperate. It generally happens when no parties gets majority in the parliament and several parties join together to prove.

16. Give reasons why the Sri Lanka government adopted Majoritarian policy.

Ans. When Sri Lankan got independence in 1948, the leaders of Sinhalas community wanted to secure dominance of government by virtue of their. Therefore to established Sinhala supremacy they adopted Majoritarian policy.

17. Which Act favored Sinhala community and disregarded Tamil community in Sri Lankan?

Ans. The Sri Lankan government passed an act in 1956 which recognized Sinhalas as the only official language. By the preference policy, the government favored Sinhalas applicant for government job and university position.

18. "Belgium has three types of governments" explain with examples.

Ans. Belgium has three types of government namely:-

(i) The central government

(ii) The state government

(iii) The community government

Belgium has control any state government which is common with government in other countries but is unique in having a third kind government called 'Community Government' community government is elected by the people belonging to one language community i.e. Dutch French and German speaking communalities.

19. Is power sharing desirable? Support your answer.

Ans. Yes power sharing is desirable because of the following reason:-

(i) Power sharing help to reduce the possibility of conflict/war between social groups.

(ii) It ensure political economic; social and culture stability of a nation.

(iii) Power sharing is the basic spirit of democracy.

20. Explain how power sharing consists of both prudential and moral reasons?

Ans. Prudential reasons are based on careful calculation of gains and losses. For e.g.- power sharing is good because it help to prevent the conflict between social groups. It lays streets on outcome of democracy. Moral reasons are based on why power sharing leads to lenitive and responsible, government it emphasizes value of democracy.

21. What steps were taken by the Sri Lankan Tamils in demanding power in Sri Lanka?

Ans. (i) The Sri Lankan Government adopted the policy of Majoritarianism and discouraged the demands of the Tamils.

(ii) Against the dominance of Sinhalas, the Tamil language political parties, organization and military group (like LTTE)

(iii) They launched struggle for the recognized of Tail as an official language.

(iv) The Tamil demand regional autonomy, aqua of opportunity in education and government jobs.

(v) Several political parties were formed in 1980s and demanded an independent state called Tamil Elam.

22. What do you know about "Belgium model"? Do you think it is good for democracy?

Ans. (i**)** Belgium model of government refers to community Government which is Different from that of other countries.

(ii) The main idea behind the Belgian model is to accommodate different Religious and cultural group into a single community and to share quality power.

(iii) The constitution clearly prescribes that the number of Dutch and French Speaking ministers shall be equal in the central government.

(iv) No single community can take decisions on their own and some special laws were to be followed.

(v) The state government is not subordinate to the central government because many Power of the central government have been given to the state.

(vi) The French speaking people accepted equal representation and the Dutch speaking community in the central government.

EXERCISE

VERY SHORT ANSWER TYPE QUESTIONS:-

1. What does the term power sharing signify?

2. What is ethnic composition?

3. Why was the Act of 1956 passed?

4. What do you mean by community government?

5. Define the term civil war?

6. What do you mean by the term 'prudential'?

7. Give the difference between 'prudential and moral :

8. What is horizontal distribution of power?

9. What is the system of check and balances?

10. What is vertical division of power?

11. Define the term Federal Government?

12. What were the series of Majoritarian measure?

13. Why the ethnic composition of Belgium is very complex.

14. Give an account of ethnic composition of Sri Lanka.

15. What made Sri Lankan Tamil feel alienated?

16. Differentiate between horizontal division of power and vertical division of power?

17. How power are share in different social group and why?

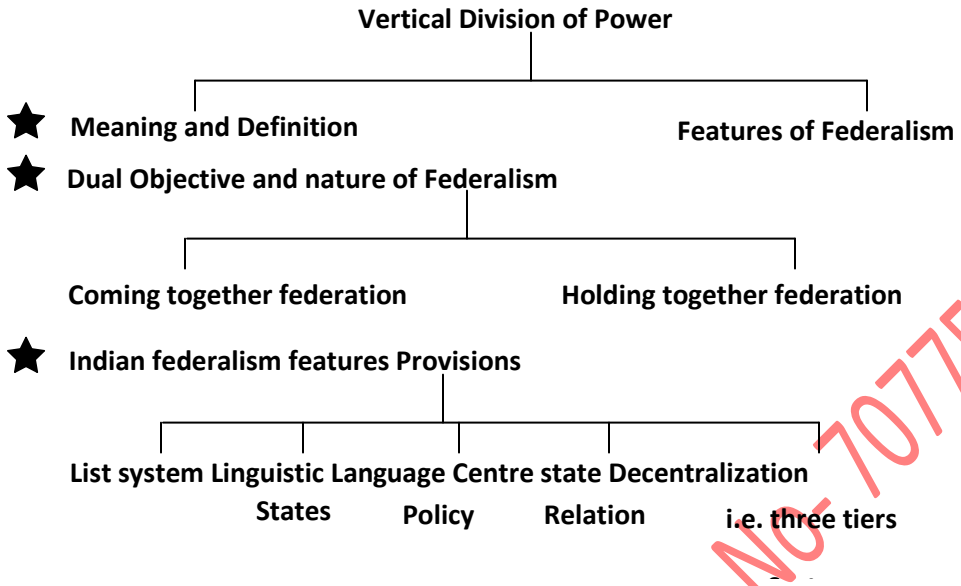
LONG ANSWER TYPE QUESTIONS:-

1. Why is power sharing desirable?

2. Explain the major form of power sharing in modern democracies.

3. Explain the power sharing arrangement among the political and pressure group.

FEDERALISM



INTRODUCTION OF FEDERALISM

- **Federalism:**

It is a system of government is which the power is divided between a central authority and various constituent units of the country. Usually, a federation has two level of government one is the government for entire country that is usually responsible for a few subject of common national interest. The other are government at the level of states that look after much of the day to day administering of their state both these levels of governments enjoy their power independent of the other.

- **Unitary system:**

There is only one level of government or the sub units are subordinate to the central government. The central government can pass an order to the provincial or the local government. But in a federal system, the central government cannot order the state government to do something. State government has powers of its own for which it is not answerable to the central government.

- **Features of Federalism:**

1. **Two level of Government:** In the federalism there are two or more levels at the provincial central or state level.
2. **Distribution of power:** In federalism different tier of government govern the same citizen, but each tier has its own jurisdiction in specific matters of legislating, taxation and administration.
3. **Constitutional Status:**_The jurisdictions are specified in the constitution. Authority of each tier of government is constitutionally guaranteed.
4. **Equal Status:**_The fundamental provisions of the constitution cannot be unilaterally changed by. One level of government, such changes require the consent of both the levels of government.
5. **Independent Judiciary: -** Court has the power to interpret the constitutions and power of different level of government.
6. **Financial Autonomy:**_Federalism provides specified sources of revenue for each level of government. Every level free in its own way of imposing taxes and raising funds through remunerative enterprises.
7. **Dual Objective:**_Federal system has dual objective. (i) To safe guard and promote unity of the country. (ii) to accommodate regional diversity.

The exact balance of power between the central and the state government depends mainly on the historical context in which the federation was formed. There are two kinds of routes.

1. **Coming together federation:**

A. It is the first route of the formation of federalism. It involves independent states coming together on their own to form a bigger unit.

B. It tries to increase, their security by pooling sovereignty and retaining identity.

C. All the constituent units usually have equal power.

D. Example- U.S.A., Switzerland, Australia.

E. States have equal power.

2. **Holding Together Federalist:**

A. It is the second route of the formation of federalism where a large power or country decides to divide its power between the constituent state and national government.

B. The central government tends to be more powerful vis-à-vis the states.

C. Example- India, Belgium, Spain

The constitution declared India as a union of states. Although it did not use the word federation, The Indian union is based on the principle of federalism.

The union government or the central government represents the union of India and the state government. Later a third tier of federation was added in the form of Panchayats and Municipalities. The constitution contains three lists.

1. **Union list:**

A. It includes subjects of national importance e.g. defense of the country, foreign affair, communication and currency.

B. There is a uniform policy on these mettles throughout the country.

C. The union government alone can make laws relating to the subjects mentioned in the union list.

2. **State List:**

A. It contains subject of state and local importance e.g. Police, Trade, commerce.

B. Agriculture and irrigation etc.

C. The state government alone can make laws relating to the subject mentioned in the state list.

3. **Concurrent list:**

A. It includes subject of common interest to both the union government as well as the state government, education, forest, trade unions, marriage, adoption and succession.

B. Both the union as well as the state government can make laws on the subject mentioned in the list.

C. If their laws conflict with each other, the law made by the union government will prevail.

All states in the Indian union do not have identical powers. Some states like Jammu & Kashmir enjoy a special status. Many provision of the Indian constitution are not applicable to this state without the approval of the state Assemble. Indians who are not permanent residents of this state cannot buy land or house here.

Areas like Chandigarh or Lakshadweep or the Capital city of Delhi are called union Territory. These territories don not have the powers of a state. Central government has special powers in running these areas.

Sharing of power between the union government and the state government is basic to the structure of the constitution. It is not easy to make changes to this poor sharing arrangement. The parliament cannot on its own change arrangement. Any change to it has to be first passed by both the house of parliament with at least two thirds majority. Then it has to be ratified by the legislature of at least half of the total state.

In case of any dispute about the division of power, the high court and the Supreme Court make a decision.

◙ **HOW IS FEDERALISM PRACTICED**

The real success of federalism in India can be attributed to the nature of democratic politics in our country. This ensured that the sprite of federalism, respect for diversity and desire for living together become a shared ideal in our country.

- **Linguistic States**

Regarding federalism that first and major test for democratic politics in our country was the creation of linguistic states. In 1947, the boundaries of several old states of India were changed to ensure that people who spoke the same langue lived in the same state.

The formation of linguistic states has made the country, more united and also make the administration easier.

- **Language Policy:**

A second test for Indian federation is the langue policy. In our constitution Hindi is identified as the office langue. These are also 21 language recognized as scheduled langue by the constitution. A candidate in an examination conducted for the central government position may opt to take the examination in any of this language. Much of the government work takes place in the official language of the concerned states.

- **Controversy over the Hindi & English:**

1. According to the constitution the issue of English for official purposes was to stop in 1965.
2. Many non Hindi speaking states demanded that the use of English should continue e.g. Tamil Nadu.
3. So the central government responded by agreeing to continue the use of English along with Hindi for official purposes.
4. Promotion of Hindi continue to be the office policy of the government of India but it does not mean that the central government can imposed Hindi on state where people speak a different language.

◙ **CENTRE STATE RELATION**

Improving the centre state relation is one more way in which federalism has been strengthened in practice Though Indian constitution has divided by the powers of union and state government but still the union government can have influence over the state in many ways.

In the past, central government has often misused the constitution to dismiss the state governments that were controlled by rival parties. This undermined the spirit of federalism and that of democracy.

The judiciary has played a major role in improving the autonomy of the state governments because many a times, it has reduced state government which was dismissed in an arbitrary manner.

◙ **DECENTRALISATION IN INDIA**

Federal government has two tiers of government. Vast country like India cannot be run only through these two tiers. Federal power sharing in India needs another tier of government, below that of the state government. This is the rationale for decentralization of power. Thus results a third tier of government called local government.

- **Definition:** When power is taken away from central and state government and given to local government, it is called decentralization.

A. The basic idea behind decentralization is that there are a large number of problems and issues which are best settled at the local level.

B. People have better knowledge of problems in their localities.

C. People have between ideas on where to spend money and how to manage things more efficiently.

D. At the local level it is possible for the people to directly participate in decision making

E. The helps to inculcate a habit of democratic participation.

F. Local government is the best way to relies the important piratical of democracy, namely local self govt., Panchayats in villages and municipalities in Urban areas were setup in all the states.

- **A major step towards decentralization was taken in 1992.**

1. It is constitutionally mandatory to hold regular election to local government bodies.

2. Seats are reserved for the SC, ST and OBC.

3. At list one third of all position are reserved for women.

4. An independent institution called state Election commission has been created in each state to conduct Panchayat and municipal election.

5. The sate governments are required to share some power and revenue with local government bodies.

6. Local government is the best way to realize the important principle of democracy, namely local self govt. Panchayats in villagers and municipalities in urban areas were setup in all the states.

◙ **PANCHAYATI RAJ**

A. (i) Each village, or a group of village in some state, has Gram Panchayat.

(ii) This is a council consisting Panch and Sarpanch.

(iii) They are directly elected by all the adult population living in that village.

(iv) It is the decision making body for the entire village.

(v) The Panchayat works under the overall supervision of the gram Sabha, All the voters in the village are its member.

(vi) It has to meet at least twice or thrice in a year to approve the annual budget and to review the performance of the Gram Panchayat.

B. (i) At the district level a few Gram Panchayats are, grouped together to form is called Panchayat Samiti.

(ii) The member of this representative body elected by al the Panchayat member in that area.

C. (i) All the Panchayat Samiti in a district together constitutes the Zilla (district) Parishad. Most members of Zilla Parishad are elected.

(ii) Member of the Lok Sabha and MLAs of that district and some other officials of other district level bodies are also its members.

(iii) Zilla Parishad Chair person is the political head of the Zilla Parishad.

D. (i) Local Government bodies exist for urban area as well. Municipalities are step up in the towns. Big cities are constituted into municipal corporations.

(ii) Both municipalities and municipal corporations are controlled by elected bodies consisting of people's representatives.

(iii) Municipal chairperson is the political head of the municipality.

(iv) In a municipal corporations such an officer is called the Mayor.

◙ **IMPORTANT TERMS**

1. **Union list:** Subject of national importance like defense, foreign affairs, atomic energy, banking, post and telegraph are included in the union list. The central government can pass law on the subjects mentioned in the union list.

2. **State List:** It comprises those important subjects on which the state government can pass law. Subjects like police, local government, trade and commerce within the state and agriculture are included in the state.

3. **Concurrent List:** It comprises that subjects which are of common concern both to the centre, and the state government. Both the government can from laws on these subjects.

4. **Residuary Powers:** Matters which are not included in the division of powers are known as residuary powers.

5. **Tier system:** It is the system which signifies levels of government. It may be of two levels and three level.

6. **List system:** It is the distribution of powers thought the list like union list for central level or state list for state level.

7. **Linguistic State:** The creation of stats on the basis of language is known s Linguistic state.

8. **Unitary system:** It is a system of government in which either there is only one level, of government or sub units are subordinate to the central government.

9. **Scheduled Language:** Language which is listed in the list or schedule of the constitution is known as scheduled language.
10. **Decentralization:** When power is taken away from central and state government and given to Local Government it is called Decentralization.

EXERCISE

◙ VERT SHORT ANSWER TYPE QUESTIONS

1. What do you mean by the term 'Federalism'?
2. What is unitary government?
3. The federal system has a dual objective. Mention the dual objectives.
4. There are two kinds of routers through which federations have been formed. Name the two routes.
5. What is a union list?
6. What is a state list?
7. What is a concurrent list?
8. What are residuary powers?
9. What is decentralization?
10. What is Gram Panchayat?
11. What is Panchayat Samiti?
12. What is Zilla Parishad?
13. Who is a Mayor?
14. \What is coming together federation?
15. What is the basic idea behind the power sharing of Indian Federalism?
16. List out the significance of the formation of linguistic states.
17. Examine the concept of Linguistic states?

◙ SHORT ANSWER TYPE QUESTIONS

1. List out the basic traits of Federalism.
2. Discuss the dual objectives of federalism.
3. Examine the general nature of federalism.
4. Distinguish between coming together federations and holding together federations.
5. What are the power sharing arrangements in our country?
6. "Judiciary plays an important role I in dial's Federation". Prove the statement.
7. Discuss the language policy of Indian Federation.
8. Examine the controversy over Hindi & English as the official language.
9. Discuss the centre state relations in Indian Federalism.
10. Examine the significance of Decentralization.
11. Discuss the structure and composition of rural local self government.
12. Examine the composition and structure of urban local self government.

◙ LONG ANSWER TYPE QUESTIONS

1. Examine the basic features of federalism.
2. Discuss the list system of Indian federalism.
3. How is federalism précised in India?
4. Examine the concept of three tiers government of Indian federalism?

5. Give a Critical analysis of new experiment on the local self government.

6. List out the attempts of the Indian constitution in order to decentralize power.

7. "India is country of diverse language". Examine the statement under the linguistic diversity of India.

SOLVED EXERCISE

1. What is the meaning of federalism?

Ans. Federalism means the sharing of power between a central governing authority and state authority by the constitution so that both can work independently in them respective field. Both the government is answerable to the citizen of the country who elects them. Few countries that have federal system are Canada, Argentina, Australia, Austria, Belgium, Brazil, Germany, India, Mexico and Switzerland.

2. How are the powers divided between the centre and the state in India?

Ans. There is clear division of powers between the central and state government by the constitution under three list (i) Union list (ii) State list (iii) Concurrent list

(i) Union list: - The union list includes 97 matters of national importance upon which the central government would take the decision. It includes subjects like defense of the country, foreign affairs, banking currency and communication.

(ii) State list:- The state list includes 66 matters of the state-level and local level importance on which the decision would be taken by the state governments. It includes subjects like police, trade, commerce, agriculture and irrigation.

(iii) Concurrent list: - The concurrent list includes 47 subjects which are o common interest to both the central as well as state government. It includes matters such as education, forest, marriage and trade unions.

The subjects that do not fall in any of the union state or concurrent lists are decides upon by the union government. Example- computer software, mobile phones etc.

3. What are union territories? Why union territories are not states?

Ans. Some units of the Indian union are smaller in size in comparison to other states and cannot be merged with other states. Such units are called union territories. They are not given the static of states since they are too small in size to be considered as independent units. There are seven union territories. They are:-

(1) Andaman and Nicobar Island
(2) Chandigarh
(3) Dadra and Nagar Haveli
(4) Daman and Dice
(5) The capital city Delhi
(6) Lakshadweep
(7) Pondicherry

4. "All states in the Indian union do not have identical powers" Do you agree? Support your answer with examples.

Ans. Yes, all states in the Indian do not have equal powers. Some states enjoy a special static's

- ❖ For example- Jammu and Kashmir has its own constitution. Article 370 of the constitution ensures the special powers.
- ❖ Many Provisions of the India constitution are not applicable to Jammu and Kashmir state and it has to get the approval of the state assemble.
- ❖ Only the permanent residents of Jammu and Kashmir can purchase land or a house in the state.

5. What is the role of judiciary in implementing the constitutional provisions?

Ans. The judiciary plays an important role in the implementing of constitutional procedure and provision.

- ❖ In case of any dispute about the division of powers (between the union and state) the Supreme Court makes a decision. Between the state and other lower units, the high court makes decision.

6. Write down the importance of 73rd and 74th (Amendment) Act.

OR

Write the main provision f 1992 Act related to decentralization.

Ans. A major step towards decentralization was taken in 1992 and it became third tier of democracy.

Main provisions are:-

(1) To hold regular election to local bodies is constitutionally mandatory.

(2) At least one third of all position is reserved for women.

(3) To conduct Panchayat and municipal elections a state Election. Commission has been created in each state.

(4) The state governments are required to share some powers and revenue with local bodies.

(5) Seats should be reserved for the weaker sections like SCs, STs and OBCs.

7. Write short note on the structure of local government.

Ans.

- ❖ The government at the village level is known as Gram Panchayat and the head is called Sarpanch.
- ❖ A few gram Panchayats are grouped together and called Panchayat Samiti at Taluk/Mandal/block level.
- ❖ All Panchayat Samitis in a district are called as Zilla Parishad.
- ❖ Most o the members of the Zilla Parishad are elected and members of Lok Sabha and MLAs of that district and some officials are also its members.

8. Write a short note on local government at the urban level.

Ans.

- ❖ Local government bodies at the urban level (towns) are called Municipalities.
- ❖ Big cities have a Municipal Corporation.
- ❖ Head o a municipal corporation is called the Mayor.
- ❖ Municipal Chairperson is the head of the municipality.
- ❖ Both municipalities the municipal corporations are controlled by elected bodies consisting of representatives of the people.

9. Explain briefly the language policy of India.

Ans. Language policy of India is one of the tasks for Indian federation. India is known for multi-lingual due to its diversity. Out Constitution does not give the status of national language to any particular language. Hindi was declared as the official language (which is the mother tongue of only about 40 per cent of Indians.)

There are 21 languages (by sides Hindi) recognized as scheduled languages by the constitution.

According to the Constitution, the use of English for official purposes was to stop in 1965. However mainly the non-Hindi speaking states demanded that the use of English be continued. Thus, the central government agreed to continue the use of English along with Hindi.

Promotion of Hindi continues to the official policy of the government of India.

In general, India's language policy can be described as three language formula, i.e., Hindi as official language, English as link language and langue of the respective state of region (like Gujarati, Marathi, and Telugu, etc.)

10. Point out feature in the proactive of federalism in India that is similar to and one feature that is different from that of Belgium.

Ans. One similar Feature of Federalism between India and Belgium. In Belgium there are three kinds of government at the centre, government at the state level and third kind of government is the community government. The community government has the power regarding Cultural, educational and language related issues.

But in India, there is no third kind of government, there is no community government.

Here there are only two types of governments. The first kind go government is at the centre and the second kind of government is at the state level.

11. Fill in the blanks:

Since the United States is a _____ type of federation, all the constituent states have equal powers and states are _____ vis-à-vis the federal government. But India is a _____ type of federation and some states have more power than other. In India the _____ government has more powers.

Ans. Since the United States is a coming together type of federation, all the constituent states have equal powers and states are stronger vis-à-vis the federal government. But India is a holding together type of federation and some states have more power than other. In India the Central government has more powers.

POLITICAL PARTIES

❒ POLITICAL PARTIES

- A political party is a group of people who come together to contest election and hold power in the government.
- It mobilizes voters to support common sets of interests. Concerns and goals.
- A political party fixes the political agenda and policies and tries to persuade people by claiming their policies are better than those of other parties.
- To prove this they put up candidates to fight elections.
- A political party is the means through which people can speak to the government and have say in the governance of any country.
- A party is about a part of the society and thus involves partisanship. It means it is known by the part is stands for, the policies it supports.
- A political party has three components:- the leaders, the active members and the followers.

▸▸ FUNCTIONS:-

- Parties contest elections by putting up candidates.
- In some countries candidates are selected by members and supporters of a party (e.g. USA).
- In other countries candidates are chosen by top party leaders. (E.g. India).
- Parties have different policies and programmes voters choose from them. In a democracy a large number of people with similar opinions group together and form a party and then give a direction to the policies followed by the government.
- A government bases its policies on the time taken by the ruling party.
- Parties play a definite role in making the laws of a country, they form and run governments. They get the laws passed by the legislature, recruit leaders and train them and make them ministers to run the government.
- The parties that lose election form the opposition and voice different views and criticize government for their failures. They mobilize opposition to the government.
- They shape public opinion. Parties with the help of pressure groups launch movements for solving problems faced by the people.

- They provide people access to government machinery and welfare schemes. The local party leader acts as a link between the citizen and the government officers.

➤➤ **NEED FOR POLITICAL PARTIES:-**

- That democracies cannot exist without political parties is clear from the function they perform. If there were no political parties then:

(i) All candidates in an election would become independent candidates. They cannot promise any major policy change to the people. A government formed in this way will have no unity. No one will be responsible for how the country is run; elected leaders would be responsible or accountable to their constituencies only.

(ii) Political parties are a must in representative democracies. In large societies, only representative agency to gather different views on various issues and present them to government. They bring various representatives together to form a responsible government and that is why they are a must in a democracy. In India, 750 parties are registered with the Elections Commission of India.

❒ PARTY SYSTEM:-

➤➤ There are three types of party systems:-

(i) **One-Party system: -** There is no competition in this system. The lone party nominates the candidates and the voters have only two choices- (i) Not to vote at all or (ii) write 'yes' or 'no' against the name of the candidates nominated by the party. This system has been popular in Communist countries and other authoritarian regimes. The countries following this system are china, North Korea and Cuba. This system was Single- party system; however, limit the choices and freedoms of the citizens.

(ii) In a two-party system power shifts between two major, dominant parties. In this system to win elections, the winner has to get maximum number of votes, but not necessarily a majority of votes. The smaller parties usually merge with the bigger parties or they drop out of elections. This parliamentary system prevails in Great Britain and Canada, in which only two parties hold significant numbers of seats. Supporters of this system believe that this prevents dangers of fragmentation.

(iii) Multi-Party system is the most common type of party systems.

In this system three or more parties have the capacity to gain control of the government separately or in coalition.

In this system, the general constituencies form multiple, distinct, officially recognized groups called political parties.

- Each competes and contests for votes. This system is essential for a representative democracy, because it prevents a single party leadership from making policies and controlling the government without challenge.
- Parliamentary government or Congresses based on proportional representation support and develop multi-party systems. In this system, the number of seats held by any party in the legislature depends on the proportion of votes it received in the most recent election. When no party gains a majority of the legislative seats in multi-party Parliamentary system. Then several parties join forces and form a coalition government.
- Supporters of this system point out that it allows more points of views to be represented in the government.

➤➤ An Alliance:-

- When several parties in a multi-party system join hands for the purpose of contesting elections and winning power it is called an alliance or a front. India in 2004 had the such Alliances of parliamentary elections:

(i) National Democratic Alliance (ii) The United Progressive alliance and (iii) Left front.

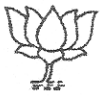
❒ **POLITICAL PARTIES IN INDIA**

National Parties: - If a political party fulfils the criteria under Election symbols (Reservation) Order 1968 in more than four states, it is deemed to be a National Party.

State Parties or Federalists Parties:- A political party recognized in less than states is a state party in the state or states in which is recognized.

▶▶ **Difference between a National Party and a Regional Party**

- A national party has influence all over the country or in many states of India. The influence of a state party is in a state or a few regions.
- National parties care for national interests, whereas regional parties promote mainly regional interests. For example, the DMK or AIDMK promote Dravidian culture.
- Regional parties stand for greater autonomy for the states. The national parties. On the other hand, have to harmonize various conflicting regional interests. They try to end regional imbalances.
- An exclusive symbol such as (lotus or hand) is reserved for a national party throughout India. But in case of a regional party a symbol for it is reserved for it in the state which it is recognized.
- Every party in India has to register with the election commission. The commission treats every party as equal to the others, but it offers special facilities to large and established parties. They are given a unique symbol and are called, "recognized political parties." They must get a certain proportion of votes in order to be a recognized party- 6% of votes in an election to Legislative Assembly of a state and win at least two seats to be recognized as a state party. If a party wins 6% of total votes in the Lok Sabha and at least 4 seats in a State Assembly, it is recognized as a national party.

S.N.	Symbol	Party
1.		Bharatiya Janata Party
		Bahujan Samaj Party
2.		Communist Party of India

S.N.	Symbol	Party
4.		Communist Party of India (Marxist). CP(M)
		Indian National Congress (INC)
5.		Nationalist Congress Party (NCP)

INTRODUCTION TO MAJOR POLITICAL PARTIES IN INDIA

1. INDIAN NATIONAL CONGRESS (INC)

- It is India's oldest political party founded in 1885. It led the movement for independence against the British rule, and after independence became free India's premier political party. It maintained this position under Jawaharlal Nehru, the first Prime Minister of free India. In the General Elections held in 1952, 1957, 1962, 1967 and 1971, the Congress virtually controlled the politics of the country. It lost the elections in 1977 following the Emergency rule.

- They formed a present coalition government called the United Progressive Alliance (UPA). The party identifies itself with six basics of governance.
- The six basics are Samajik Sadbhavana (Social harmony), Yuva Rozgar (employment for the youth), Grameen Vikas (rural development), Arthik Punarutthan (economic resurgence), Mahila Sashaktikaran (women's empowerment), and Saman Avsar (equal opportunity). It supports secularism.

2. BHARTIYA JANATA PARTY (BJP)

- It was created in 1980. It champions the socio-religious values of the Hindu majority of India. Conservative social policies and strong national defense. It is strongly supported by Hindu organizations, informally known as the Sangh Parivar [league of RSS-led organisations] in which the Rashtriya Swayamsevak Sangha (RSS) Plays a big role.
- It wants full territorial integration of Jammu and Kashmir with India and a uniform civil code. The BJP, in alliance with several other parties, led the Government of India between 1998 and 2004.
- It is now the recognized opposition, and the leading party within the National Democratic Alliance (NDA).

3. BAHUJAN SAMAJ PARTY (BSP)

- The Bahujan Samaj party is a party formed to represent the OBC, SC, ST and religious minorities, those at the bottom of India's caste system. The BSP was formed in 1984 by two leaders, Kanshiram and Mayawati. The main base of the party is in Uttar Pradesh. It draws inspiration from the teachings of Shahu Maharaj, Mahatma Phule, and Periyar Ramaswami Nayakar.
- The BSP also draws inspiration from the teachings of Dr. B.R. Ambedkar who was of the opinion that abolition of the caste system was indispensable for the abolition of untouchability. In now wants to uplift the "Bahujan Samaj" (majority castes) and promotes "Sarva Samaj" (the whole society).

4. COMMUNIST PARTY OF INDIA (MARXIST) (CPI - M)

- The Communist party of India (Marxist), usually known as CPI (M), split from the communist party of India in 1964. It is strongest in the states of Kerala, West Bengal and Tripura as of 2006, and leads the government in all these states.
- In supports socialism, secularism and democracy. It opposes imperialism and communalism. It believes in democratic elections and supports them as means of securing socio-economic justice in India. They criticize free flow of foreign capital and goods into the country.

5. COMMUNIST PARTY OF INDIA (CPI)

- It was formed in 1925, believes in Marxism-Leninism, secularism and democracy. It is opposed to the forces of communalism and split the party in 1964 and the formation of CPI (M) the position weak.

6. NATIONAL CONGRESS PARTY (NCP)

- It was formed on May 25, 1999, by Sharad Pawar, P.A. Sangma and Tariq Anwar after they were thrown out of the Congress party. They had objected to a person of foreign origin becoming the Prime minister of India. It was Maharashtra who had first supported Sharad Pawar. In January, 2004, Sangma quit the NCP because he thought Sharad Pawar was getting closer to the congress. The NCP claims that it supports democracy, Gandhian secularism, equity, social justice and federalism. Since 2004, NCP is a member of the united progressive Alliance, which governs the country.

CHALLENGES TO POLITICAL PARTIES

1. Political parties are crucial for the working of democracies.
2. People blame political parties for whatever is wrong with the government.

3. The main points of dissatisfaction and criticism are four in number.

4. Political parties will have to face these challenges and overcome them, if they want to be effective.

▶▶ **Challenges faced by political parties:-**

- Political parties have to face the challenges and to improve, if they want to be effective in performing their functions.

1. Lack of Internal Democracy:-
   - **(i)** Every member of the party does not have a chance to take part in decision making process.
   - **(ii)** Every member is not consulted before taking a decision.
   - **(iii)** There is no proper organization or registration of members.
   - **(iv)** Power remains in the hand of a few top leaders, who do not consult ordinary members.
   - **(v)** Ordinary members have no information about the internal workings of the party.
2. **Dynastic succession: -** With power in the hands of a few top leaders. All party position to their family members. These members may not be qualified or have the ability to hold their positions. People who come from weaker sections, or have no contact with the top leaders, cannot rise to the top positions.
3. **Money and Muscle Power: -** various political parties contest elections, there is competition and a lot of money is needed. Money is needed to organize demonstrations, public meetings and speeches to publicize the image of the party. Parties choose those candidates who can raise money for the party and win election with their money.
4. **Meaningful choice to others: -** In India, like many countries, the fundamental issues raised by parties are the same. On the economic front, all parties have the same agenda- help the weaker sections, equity and jobs for all etc. The only difference is the priority they will give to any problem. Voters do not have a meaningful choice.

▶▶ **Defection and Affidavit**

- When an elected MP or MLA changes his party it is called Defection.
- An affidavit is a signed document submitted to an officer, where a person makes a sworn statement regarding her/his personal information. In the case of politics, a candidate has to file an affidavit giving details of his property and criminal cases pending against him.

▶▶ **Some of the suggestions made to reform political parties:-**

- Laws made to regulate internal politics.
- It should be compulsory to maintain a register of its members.
- Hold independent inquiry in case of a party dispute.
- Hold open elections to top party posts.
- It should be mandatory to give 1/3 party tickets to women candidates. There should be a quota for women in decision making bodies.
- The government or the state should help parties during elections by funding them.
- Expenses for patrol, paper, telephone etc. given.
- Cash can be given on the basis of votes secured by the party in the last elections.

▶▶ **How can an ordinary citizen help in reforming political parties?**

- Pressure should be put on the political parties. This can be done through petitions, publicity agitations. Ordinary citizens. Pressure groups and the modern can bring about report if they join bands. If political parties feel that they would lose popular support, they will have to reform.

- Political parties can improve if they allow more and more people to join their organization. The quality of democracy improves if there is maximum participation by the public. If the public only criticizes from the outside it has no effect. To improve bad politics has to take charge.

EXERCISE

ANSWER THE FOLLOWING QUESTION BRIEFLY:-

1. Who is the ruling party?

2. Who is the opposition party?

3. Whom do political parties have to register themselves?

4. How many parties are registered with Election Commission of India?

5. What are the two kinds of political parties?

6. What is meant by partisanship?

7. What are political parties?

8. Name the three components of a political party.

9. What is meant by one-party system? Give an example.

10. What is meant by two-party system? Given and examples.

11. What is meant by multi-party system?

12. What do you mean by alliance?

13. Define National Party.

14. Define State Party.

15. Name the six major national parties of India.

16. What do you mean by the term 'defection'?

17. What is an affidavit?

18. What are the characteristics of a political party?

19. How do the political parties form the public opinion?

20. What is the importance of opposition?

21. What is the importance of regional parties in India?

22. What is meant by dynastic succession in the political system?

23. What role does money and muscle power play in political system?

24. What is meant by offering a meaningful choice of political parties to the voters?

25. What is the need of political parties?

26. What are the various challenges faced by a political party?

27. Write the full form of (i) BJP (ii) INC (iii) BSP (iv) NCP (v) CPI (vi) CPI (M)

28. Who is the founder of BSP party?

29. How has multiparty system strengthened democracy?

OUTCOMES OF DEMOCRACY

IMPORTANT TERMS AND CONCEPTS

- **Outcome:** Result, consequences or output.
- **Characteristics of Democracy:** Promotes equality, enhances the dignity of the individual, improves the quality of decision making, provides methods to resolve conflicts, and accommodates correct mistakes.
- **Alternative forms of government than democracy:** Monarchy military rule by religious leaders.
- **Elements? Constituents of democracy**: Formal constitution, election, political parties, constitutional rights.
- **Monarchy:** the government ruled by the monarch or king of the country i.e., monarch or king is the head of the country.
- **Dignity:** The word denotes privileged position, honorable rank or importance given to any particular post or personating.
- **Outcome of democracy:** Political equality and political justice social equality and social justice, economic equality and economic justice. Dignity to the individual.
- **Measures for democratic outcome:** Regular free and fair elections; open public debate on major policies, right to information given to the people.
- **Economic Development:** It refers to the growth advancement of the country in the way of providing better facilities and services to the individual.
- **Transparency:** Right or means to examine the process of decision making.
- **Accountable:** Responsible, answerable, and legitimate to the individual/post and their needs and expectations.
- **Poverty:** It is a socio economic phenomenon which in general terms is the denial of opportunities to lead a long, healthy, creative life and to enjoy a decent standard of living.
- **Inequality:** Absence of equal opportunities, equal treatment and equal status at any level, social economic or political is known as inequality.
- **Measures to sustain democracy:**
  - Unity in diversity
  - Historical background of the country
  - Tolerance power among people
  - Feeling of nationalism and patriotism
  - Peaceful co-existence
  - Non-alignment
  - International peace and understanding
- Civil Liberties: These are privileges or rights thought to be valuable in them and important for the functioning of democratic.

HOW DO WE ASSESS DEMOCRACY'S OUTCOMES?

Democracy is a better form of government when compared with dictatorship or any other alternative. It was said that democracy was better because it:

(i) Promotes equality among citizens

(ii) Enhances the dignity of the individual

(iii) Improves the quality of decision-making

(iv) Provides a method to resolve conflicts, and

(v) Allows a room to correct mistakes

Over a hundred countries of the world today claim and practice some kind of democratic politics: They have format constitutions, they hold elections, they have parties and they guarantee rights of citizen. While these democratic elements are common to most of them, these democracies are very much different from each other in terms of their social situation, their economic achievements and their cultures. Clearly, what may be achieved or not achieved under each of these democracies will be very different.

Sometimes we expect everything and anything from democracy. Our interest in and fascination for democracy often pushed us in taking a position that democracy can address all socio-economic and political problems. If some of our expectations are not met, we start blaming the idea of democracy or we start doubting if we are living in a democracy. It can only create conditions for achieving some things. The citizens have to take advantage of those conditions and achieve those goals. Moreover, democracy is just not related to many other things that we value. Democracy is not a magical remedy for all our social ailments.

ACCOUNTABLE, RESPONSIVE AND LEGITIMATE GOVERNMENT

They are some things that democracy must provide. In a democracy, we are most concerned with ensuring that people will have the right to choose their rules and people will have control over the rulers. Whenever possible and necessary, citizens should be able to participate in decision making that affects them all. Therefore, the most basic outcome of democracy should be that it produces a government that is accountable to the citizen, and responsive to the needs and expectations of the citizens.

Is the democratic government efficient? Is it effective: Some people think that democracy produces less effective government? It is of course true that non-democratic rulers do not have to deliberate in assemblies and worry about majorities and public opinion. So, they can be very quick and efficient in decision making and implementation.

Democracy is based on the idea of deliberation and negotiation. So, some delay is bound to take place.

The democratic government will take more time to follow procedures before arriving at a decision. But because it has followed procedures, its decisions always be both more acceptable to the people and more effective So, the cost of time that democracy pays is perhaps worth it.

Democracy ensures that decision making will be based on norms and procedures. So a citizen, who wants to know if a decision was taken through the correct procedures, can find this out. She has the right and the means to examine the process of decision making. This is known as transparency. This factor would often be missing from a non-democratic government. We can expect that the democratic government develops mechanism for citizens to hold the government accountable and mechanisms for citizens to take part in decision making whenever they think fit. If we wanted to measure democracies on the basis of this expected outcome. We would look for the following practices and institutions; regular free and fair election; open public debate on major policies and legislations and citizens' right to information about the government and its functioning. The actual performance of democracies shows a mixed record on this. Democracies have had great success in setting up regular and free elections and in setting up regular and free elections and in setting up conditions for open public debate. But most democracies fall short of elections that provide a fair chance to everyone and in subjecting every decision to public debate. All one can say in favor of democratic regimes is that they are much better than any nondemocratic regime in these respects.

In substantive terms it may be reasonable to expect from democracy a government that is attentive to the needs and demands of the people and is largely free of corruption. Democracies often frustrate the needs of the people and often ignore the demand of majority of its population. The routine tales of corruption are enough to convince us that

democracy is not free of this evil. At the same time there is nothing to show that-non democracies are less corrupt or move sensitive to the people. There is one respect in which democratic government is certainly better than its alternatives: democratic government is legitimate government. It may be slow, less efficient not always very responsive or clean. But a democratic government is people's own government. This is why there is an overwhelming support for the ideal democracy all over the world.

ECONOMIC GROWTH AND DEVELOPMENT

If we consider all democracies and all dictatorship for the fifty years between 1950 and 2000, dictatorship have slightly higher rate of economic growth.

Evidence shows that in practice many democracies did not fulfill this expectation. The inability of democracy to achieve higher economic development worries us. But this alone cannot be reason to reject democracy. The difference between less developed countries with dictatorships and democracies is negligible. Overall, we cannot say that democracy is a guarantee of economic development. But we can expect democracy not to lag behind dictatorship in the respect.

ECONOMIC OUTCOME OF DEMOCRACY

Within democracies there can be very high degrees of, inequalities. In democratic countries like South Africa and Brazil the top 20 per cent people take away more than 60 percent of the nations income, leaving less than 3 percent only at the bottom. Countries like Denmark and Hungary are much better in this respect.

Rates of economic growth for different countries , 1950-2000

Type of regimes and countries	Growth rate
All democratic regimes	3.95
All dictatorial regimes	4.42
Poor countries under dictatorship	4.32
Poor countries under democracy	4.28

Inequality of income in selected countries

		% share of national income
	Top 20%	Bottom 20%
South Africa	64.8	2.9
Brazil	63.0	2.6
Russia	53.7	4.4
USA	50.0	4.0
United Kingdom	45.0	6.0
Denmark	34.5	9.6
Hungary	34.4	10.0

REDUCTION OF INQUALITY AND POVERTY

Democracies are based on political on political equality. All individuals have equal weight in electing representatives. A small number of ultra-rich enjoy a highly disproportionate share of wealth and incomes. Not only that, their share in the total income of the country has been increasing. Those at the bottom of the society have very little to depend upon. Their incomes have been declining. Sometime they find it difficult to meet their basic needs of life, such as food, clothing, house education and health.

In actual life democracies do not appear to be very successful in reducing economic inequalities. The poor constitute a large proportion of our voters and no party will like to lose their votes. The situation is much worse in some other countries. In Bangladesh more than half of its population lives in poverty. People in several poor countries are now dependent on the rich countries even for the food supplies.

ACCOMMODATION OF SOCIAL DIVERSITY

Democracies usually develop a procedure to conduct their competition. This reduces the possibility of these tensions becoming explosive or violent. Not society can fully and permanently resolve conflicts among different groups. But we can certainly learn to respect these differences and we can also evolve mechanisms to negotiate the differences. Democracy is best suited to produce this outcome. Non democratic regimes often turn a blind eye to or suppress internal social differences. Ability to handle social difference, divisions and conflicts is thus a definite plus point of democratic regimes. But the examples of Sri Lanka remind us that a democracy must fulfill two conditions in order to achieve this outcome:

(i) It is necessary to understand that democracy is not simply rule by majority opinion. The majority always needs to work with the minority. So that governments function to represent the general view. Majority and minority opinion are not permanent.

(ii) It is also necessary that rule by majority does not become rule by majority community in terms of religion or race or linguistic group, etc. Rule by majority means that in case of every decision or in case of every election, different persons and groups may be can form a majority. Democracy remains democracy. Only as long as every citizen has chance of being in majority at some point of time. If someone is barred from being in majority on the basis of birth, then the democratic rule ceases to be accommodative for that person or group.

DEGNITY AND FREEDOM OF THE CITIZENS

Democracy stands much superior to any other form of government in promoting dignity and freedom of the individual. Every individual wants to receive respect from fellow beings. Often conflicts arise among individuals because some fell that they are not treated with due respect. The passion for respect and freedom are the basis of democracy. Democracies throughout the world have recognized this, at least in principal. This has been achieved in various degrees in various democracies. Democratic regimes do not always value the liberty of its citizens in practice. Yet, for societies, which have been built for long on the basis of subordination and domination, it is not a simple matter to recognize that all individuals are equal. Take the case of dignity of women. Most societies across world were historically male dominated societies.

Long struggle by women have created some sensitivity today that respect and equal treatment of women are necessary ingredients of a democratic society. That does not mean that women are actually always treated with respect. But once the principle is recognized, it becomes easier for women to wage a. struggle against what is now unacceptable legally and morally. In a non-democratic set up, this unacceptably would not have legal basis because the principle of individual freedom and dignity would not have the legal and normal force there. The same is true of caste inequalities. Democracy in India has strengthened the claims of the disadvantaged and discriminated caste for equal status and equal opportunity. There are still instances of caste-based inequalities and atrocities, but this lack the moral and legal foundation. Perhaps it is this recognition that makes ordinary citizen values their democratic rights. People believe that their vote makes a difference to the ways the government is run and to their own self-respect. Expectation from democracy also functions as the criteria for judging any democratic country. Democracy examination never gets over. As it passes one test, it produces another test. As people get some benefits of democracy they ask for more and want to make democracy even better. The fact that people are complaining is itself a testimony to the success of democracy.

EXERCISE

VERY SHORT ANSWER TYPE QUESTIONS

1. Why is democracy preferred as the better form of government than dictatorship?
2. What is the dilemma regarding the practical aspect of democracy?
3. What are the basic elements of democracy in practical sense?
4. In what ways are democracies different from each other?
5. What thought should be put in to assess the outcome of democracy?
6. What should be the basic outcome of democracy?
7. What is the importance of castes of time in decision making?

8. What do democracies ensure regarding decision making? What can we expect from democracy regarding its outcome?
9. It the democratic government efficient and effective?
10. List out the factors for the economic development. Do democracies produce economic development?
11. Examine the forms of economic inequality in democracy.
12. Do democracies appear to be successful in reducing economic inequalities?
13. What do you mean by civil liberties?
14. How has dignity of women been ensured in a democracy?
15. List out the merits and demerits of democracy?

SHORT ANSWER TYPE QUESTIONS

1. How can we measure democracy on the basis of its expected outcome?
2. In what ways a democratic government is better than its alternatives?
3. Examine the political outcome of democracy.
4. Write a note on economic outcome.
5. Discuss the social outcome of democracy.
6. The ideals, principles and norms of democracy require certain conditions to be implemented. Explain the statement.
7. When was democracy introduced in India? Examine its relevance?
8. Discuss the factors that denote the successful working of democracy in India.
9. Explain any four outcomes on which democracy has failed.
10. "Democratic government is a legitimate government." Explain?
11. "To accommodate social diversities democracy must fulfill some basic conditions" Explain the basic conditions.

LONGE ANSWER TYPE QUESTIONS

1. How do we assess democracy's outcome?
2. What outcome can one reasonably expect of democracies?
3. Can or should democracy be judged by its outcome?
4. Does democracy in India meet these expectations?
5. What sustains democracy in India?

CHALLENGES TO DEMOCRACY

IMPORTANT TERMS AND CONCEPTS

- **Challenges:** Difficulties which are significant and which can be overcome are known as challenges.
- **Political Reforms:** Suggestion or proposals about overcoming various challenges to democracy are called "Democracy Reforms" or "Political Reforms".

- **Legal constitutional changes:** Legal ways of reforming politics and to think of new laws to ban undesirable things are known as the legal constitutional changes.
- **Right to Information Act:** It is an act that empowers the people to find out-what is happening in government and act as watchdogs of democracy.
- **Bureaucracy:** It refers to the rule by the office or rule by the officials. The term denotes the organization, officials and set procedures associated with any large administrative set up.
- **Challenges to democracy:** It means various problems that come while ensuring a democratic set up in a country. A challenge is not just any problem.
- **Forms of democracy:** There are two forms of democracy direct and indirect.
- **Direct Democracy:** In the Direct democracy people govern themselves.
- **Indirect Democracy:** People keep control on the government through their representatives.
- **By Elections:** Due to the certain causes, if election held out of the normal schedule, is called by elections.

THINKING ABOUT CHALLENGES

Different countries face different kinds of challenges. At least one fourth of the globe was still not under democratic government. The challenge for democracy in these parts of the world is very stark. These countries, Face the foundational challenge of making transition of democracy and then instituting democratic government. This involves, keeping military away from controlling government and establishing a sovereign and functional state.

Most of the established democracies face the challenge of expansion. This involves applying the basic principal of democratic government across all the regions, different social groups and various institutions. Ensuring greater power to local government across all the regions, different social groups and various institutions, Extension of federal principle to all the units of the federation, inclusion of women and minority groups etc. falls under this challenge. This also means that less and less decisions should remain outside the arena of democratic control. Most countries including India and old democracies like the US face this challenge. The third challenge, of deepening of democracy, faced by every democracy in the one form or another. This involves strengthening of the institutions and practices of democracy. This should happen in such a way that people can realize their expectations of democracy. But ordinary people have different expectations from democracy in different societies. Therefore this challenge takes different meaning and paths in different parts of the world. In concrete terms it usually means strengthening those institutions that help people's participation and control. This requires an attempt to bring down the control and influence of the rich and powerful people on decision making.

DIFFERENT CONTEXTS, DIFFERENT CHALLENGES

Case and context:

(i) **Chile:** General Pincochet's government defeated, but military still in control of many institutions.

(ii) **Poland:** After the first success of solidarity, the government imposed martial law and banned solidarity.

(iii) **Ghana:** Just attained independence Nkrumah elected president.

(iv) **Myanmar:** Suu Kyi under house arrest for more than 15 years army rulers getting global acceptance.

(v) **International organization:** USA as the only super power disregards the UN and takes unilateral action.

(vi) **Mexico:** Second free election after the defeat or PRI in 2000; defeated candidate alleges rigging.

(vii) **China:** Communist party adopts economic reforms but maintain monopoly over political power.

(viii) **Pakistan:** General Musharraf holds referendum allegations of fraud in voters list.

(ix) **Iraq:** Widespread sectarian Violence as the new government fails of establishes its authority.

(x) **South Africa:** Mandela retires from active politics, pressure on this successor Mbeki to withdraw some concessions given to White minority.

(xi) **US, Guantanamo Bay :** UN Secretary General calls this a violation of international law, US refused to respond.

(xii) **Saudi Arabia**: Women not allowed to take part in public activities no freedom of religion for minorities.

(xiii) **Yugoslavia:** Ethnic tension between Serbians and Albanians on the rise in the province of Kosovo Yugoslavia disintegrated.

(xiv) **Belgium:** One round of constitutional change taken place, but the Dutch speakers not satisfied; they want more autonomy.

(xv) **Sri Lanka**: The peace talks between the government and the LTTE breaks down renewed violence.

(xvi) US, Civil Right: Blacks have won equal rights, but are still poor, less educated and marginalized.

(xvii) **Ireland:** The civil war has ended but Catholics and Protestants yet to develop trust.

(xviii) **Nepal**: Constituent Assembly about to be elected, unrest in Terrain areas Maoists has not surrendered arms.

(xix) **Bolivia**: Morales, as supporter of water struggle, becomes prime minister, MNCs threaten to leave the country.

◙ **DIFFERENT TYPES OF CHALLENGES**

(i)	Democratic rights	(ii) Working of institutions	(iii) Elections
(iv)	Federalism, decentralization	(v) Accommodation of diversity	(vi) Political organization
(vi)	Foundational challenge	(viii) Challenge of expansion	(ix) challenge of deepening

Inspite of being the largest democracy in the world Indian democracy also suffers from some challenges of its own nature. E.g., challenge of expansion challenge of deepening democracy challenges of criminalization and challenge of corruption.

(i) **Challenge of expansion:** This challenge of Indian democracy involves the practical aspects of

▶▶ Ensuring greater power to local governments.
▶▶ Extension of federal principles to all the units of the federation
▶▶ Inclusion of women and minority groups.

Although India has the federal nature of democracy based on decentralization of power, the local self government is not implemented in a fair and just manner at every local or regional level because India has quasi-Federal nature of democratic setup which is unitary basis.

Women and minority groups still have not got a proper representation in the electoral politics of India. Hence, electoral reforms and political reforms have to be introduced in India.

(ii) Presently, India is suffering from the challenges like casteism, communalism, political violence etc. casteism plays a dominant role in the Indian polity. There are many caste pressure groups like scheduled caste Federation, etc., which influence those who are in power by different methods to get their demands met. For example political parties like the AIDMK, DMK and BSP have come up to protect the interest of the backward classes communalism has also its negative implications in Indian politics like:

(a) It leads to the formation and growth of "Pressure groups to protect and promote specific interest of the community e.g. VHP etc.

(b) It also leads to intolerance, suspicion and fear towards members of the other communities. Besides, there are various types of violence like caste, communal electoral in our country.

(a) The most serious manifestation of political and social violence is the growth of "Terrorism"

(b) Rising state violence is assuming serious dimensions and harms the very foundation of democratic polity. Thus, this challenge requires an attempt of bring down the control and influence of the rich and powerful people in governmental decision making.

(iii) Challenge of criminalization: This challenge of Indian democracy reflects:

- Money and muscle power, violence,
- Multiplicity of candidates,
- Lack of electoral ethics, insufficient representation of various sections of society like minorities and women etc have polluted the electoral process.

Hence, meaningful and comprehensive electoral reforms are urgently needed.

(iv) **Challenge of corruption:** This challenge of Indian democracy is related to the criminal record and personal possession of politicians. Now it is mandatory for every candidate who contests election to the file on "Affidavit" giving details of his property and criminal cases pending against him. The new system has made a lot of information available to the public. But there is no system to check if the information given by candidates is true. As yet we do not know if it has led to decline in the influence of the rich and the criminals.

THINKING ABOUT POLITICAL REFORMS

(i) No doubt, law has an important but limited role to play in political reform. Carefully devised changes in law can help to discourage wrong political practices and encourage good ones, but legal constitutional changes by themselves cannot overcome challenge to democracy.

(ii) Any legal change must carefully look at what results it will have on politics. Sometimes the result it will have on politics sometimes the results may be counter-productive.

For example many states have banned people who have more than two children from contesting Panchayats elections. This has results in denial do democratic opportunity to many poor and women, which was not intended. Generally, laws that seek to ban something are not very successful in politics. Laws that give political actors incentives to do good things have more chances of working. The best laws are those which empower the people to find out and act as watchdogs of democracy. Such a law helps to control corruption and supplements the existing laws that banned corruption and imposed strict penalties.

(iii) Democratic reforms are to be brought about principally through political practice. Therefore the main focus of political reforms should be on ways to strengthen democratic practice. As we discussed in the chapter on political parties, the most important concern should be increase and improve the quality of political participation by ordinary citizens.

(iv) Any proposal for political reforms should think not only about what is a good solution but also who will implement it and why. It is not very wise to think that the parliaments will pass legislation that go against the interest of all the political parties and MPs. But measures that rely on democratic movements, citizen's organizations and the media are likely to succeed.

Proposed reforms regarding doctors:

(i) The government should make it compulsory for the doctors to live in the village where they are posted, otherwise their service should be terminated.

(ii) District administration and police should carry out surprise raids to check the attendance of the doctors.

(iii) Village Panchayat should be given the power to write the annual report of the doctor which should be read out in the Gram Sabha meeting.

(iv) Problems like this can be solved only if Uttar Pradesh is split into several smaller states which can be administered more efficiently.

Proposed reforms regarding political founding:

(i) The limit of the maximum expenditure allowed for any candidate (between 20-25 Lakhs in most parts of the country in a Lok Sabha election) Should be reduced further and strictly observed.

(ii) The financial accounts of every political party should be made public. These accounts should be examined by government auditors.

(iii) There should be state funding of elections. Parties should be given some money by the government to meet their election expenditure.

Citizens should be encouraged to give more donations to parties and to political workers. Such donations should be exempt from income tax.

◙ **REDEFINING DEMOCRACY**

For the expansion the definition slightly to add some qualifications

(i) The rules elected by the people must take all the major decisions.

(ii) Election must offer a choice and fair opportunity to the people to change the current rulers.

(iii) This choice and opportunity should be available to all the people on an equal basis.

(iv) The exercise of this choice must lead to a government limited by basic rule of the constitution and citizens rights you may have felt disappointed that the definition did not refer to any high deals that we associate with democracy.

Various aspects of democratic government and politics

(i) We discussed democratic rights at length and noted that these rights are not limited to the right to vote, stand in election and form political organization.

(ii) We have taken up power sharing as the spirit of democracy and discussed how power sharing between government and social groups is necessary in a democracy.

(iii) We saw how democracy can not be the brute rule of majority and how a respect for minority voice is necessary for democracy.

(iv) Our discussion of democracy has gone beyond the government and its activities.

EXERCISE

◙ **VERY SHORT ANSWER TYPE QUESTIONS**

1. What do you mean by the term challenge?

2. What type of challenge is faced by nondemocratic country for democratic setup?

3. What type of challenge do established democracies face?
4. Which is the very common challenge faced by a democracy?
5. What do you mean by political reform?
6. What are the legal ways of reforming politics?
7. Which type of law is best for political reform? Give an example/
8. How can the principles of democracy be applied to all spheres of life?
9. Which is the most common form of democracy in today's world and why?
10. How do we define a good democracy?
11. What distinguishes democracy from other forms of government?
12. What is the use of the right to information Act?
13. Mention some important qualification for a democracy?
14. What do you mean by Bureaucracy?
15. Explain the "Right to Information act".

SHORT ANSWER TYPE QUESTIONS

1. Is the idea of democracy shrinking?
2. Discuss briefly foundational challenge of making transition to democracy.
3. "Challenge of expansion is very common which an established democracy faces". Comment.
4. Write a note on the challenge of "deepening democracy".
5. Write a note on the expanded definition of Democracy.
6. Examine the features of democracy.
7. Discuss the expanded scope of democracy in the modern world.
8. List out the demerits of democracy.
9. Why is democracy considered the best form of government?
10. What is the role and importance of mass media in a democracy?
11. Mention some of the suggestion for improvement.
12. What role do the political parties plan in representative democracy?

LONG ANSWER TYPE QUESTIONS

1. Describe briefly the challenges faced by modern democracies of the world.
2. How can democracy be reformed and deepened? Suggest some guide lines.
3. What role can an ordinary citizen play in deepening democracy?
4. What are the major challenges to democracy in India?
5. What is the significance of election in a democracy?

GEOGRAPHY

RESOURCE AND DEVELOPMENT

GLOSSARY

1. **Abiotic Resources:** All those resources which are composed of non living eg. Land, water, soil.
2. Afforestation: It means planting trees.
3. **Biotic Resources:** All those resources which have life. E.g. Forest and their product, agricultural crop, animals and birds, marine life etc.
4. **Conservation:** It means judicious and planned use of natural resource so that we can get the greatest possible benefit from these resources over the longest possible period of time.
5. **Fallow land:** It is a cultivable land which land which is being allowed to rest, uncropped or partially cropped for one or more reason so that it can regain its fertility.
6. **Gully Erosion:** It makes place when running water cuts deep ravines in the absence of vegetation. It makes soil unfit for cultivation.
7. **Land degradation:** Rendering the land unfit for cultivation is called degradation of land.
8. **Renewable Resources:** These are the resources which can be renewed or reproduced by physical, mechanical, chemical processes in a given of time. E.g. solar energy, forest, agricultural product etc.
9. **Man made Resources:** These are those resources which are created by human being with the help of machine.
10. Natural Resource: Resources which are endorsement/gift of nature. E.g. River, mountain
11. **Non-renewable resources:** These are those resources which are once used, cannot be replenished. E.g. coal, Petroleum, minerals etc.
12. **Ravine:** This is a land which is unsuitable for cultivation due to soil erosion.
13. **Resource planning:** It is a technique or skill for proper utilization of resources.
14. **Soil:** The upper most layer of the earth's crust, which is loose fragmented and useful for growing plants is called soil.
15. **Soil Erosion:** The removal of soil by the forces of nature, particularly wind and water.
16. **Resources:** Object or things in the environment that fulfill the basic needs of man. There are technologically accessible, economically feasible and culturally acceptable.
17. **Resource Development:** An exercise that makes it possible to utilize the available natural resources for human satisfaction.
18. **Contour ploughing:** Ploughing along the contour lines instead of up and down the slope.
19. **Sheet Erosion:** Erosion of the top soil along gentle slopes caused by rain water.
20. **Bad land:** Land consisting of numerous gullies and ravines.
21. **Leaching:** Process of carrying away of soil nutrients by rain water.
22. **Sustainable economic development:** It means development should take place without damaging the environment and development in the present should not compromise with the needs of the future generation.

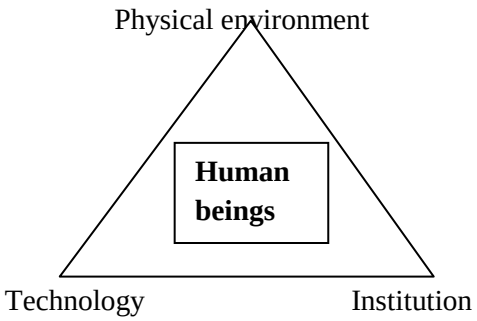
RESOURCE

Everything available in our environment which is used to satisfy our needs is termed as Resource. It should be

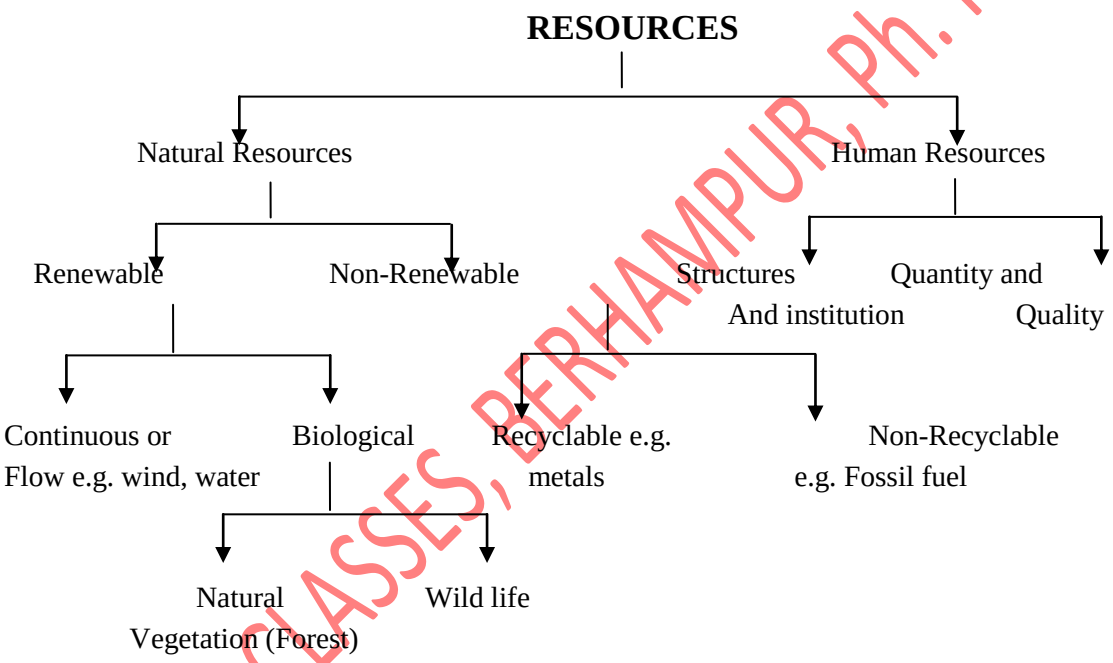
- Technologically accessible.
- Economically feasible/approachable.

- Culturally acceptable.

There is an interdependent relationship between nature, technology and institution. The human beings are linked, in ‘this interaction. They interact with nature through technology and create institutions for economic development.

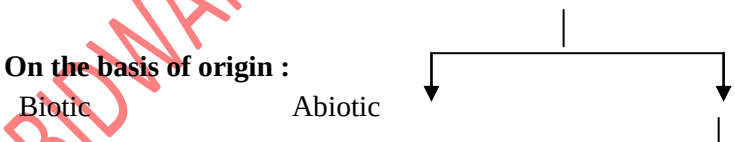


Human, beings are very important as they transform materials available In our environment into resources and use the.

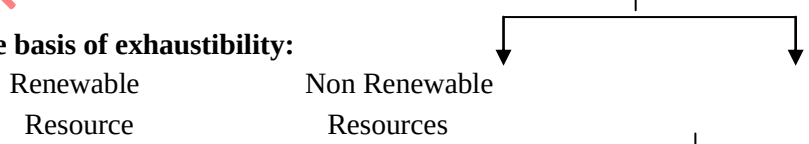


TYPES OF RESOURCES OR CLASSIFICATION OF RESOURCES

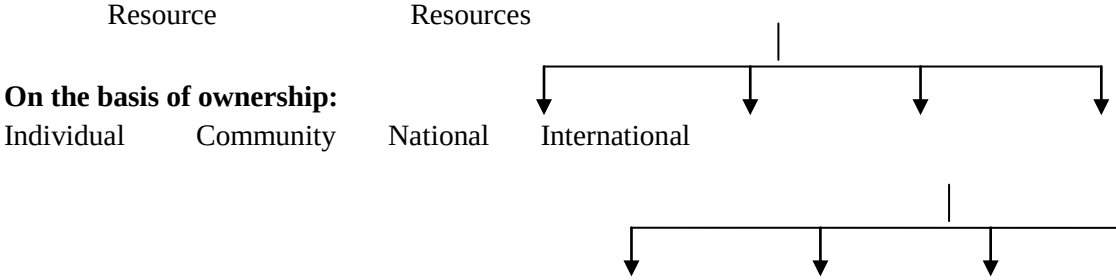
1. **On the basis of origin :**



2. **On the basis of exhaustibility:**



3. **On the basis of ownership:**



Resources Resources Resources Resources

4. On the basis of state of development:

Potential Resources Developed Resources Stock Reserves

1. ON THE BASIS OF ORIGIN

(a) **Biotic:** All those resources which are obtained-from biosphere and have life are known as biotic resources. E.g. Human Beings, flora, fauna etc. are example of renewable biotic resources.

(b) **Abiotic:** All those resources which are composed of non living things are called abiotic resources. They can be renewable as well as non renewable resources. Example: Land, water, Iron, Bauxite.

2. ON THE BASIS OF EXHAUSTIBILITY

(A) Renewable Resources; those resources that can be used again-and again or can be reproduced by physical, mechanical and chemical processes are known as renewable resource etc.
Ex: Solar energy, air, water, soil, forest & wild life.

(B) Not-Renewable Resources: Those resources that cannot be replaced' at all.

1. They take million of years to form.
2. They occur over a long geological time.
3. It is divided into recyclable and Non Recyclable Resources.
4. Metals and' Fossil fuel are non renewable resource. However metals are Recyclable but fossil fuels are non recyclable.

3. ON THE BASIS OF OWNERSHIP

(A) Individual Resources: Resources which are owned by private individuals are known as individual resources.
Ex: Plot, Fields, House, Car, Book, well.

(B) Community Resources: Resources which are owned by all members of the community.
Ex: Village pond, public parks, plays ground etc.

(C) National Resources: All the resources which are under the control of state or union government.
Ex: All mineral resources, water, land, forest and wild life.

(D) International Resources: These are international institutions that control all areas which do not belong to any individual country.
Ex: Area beyond the exclusive economic zone belongs to the open ocean & no individual country can utilize this area without the permission of international institutions.

4. ON THE BASIS OF DEVELOPMENT

(a) **Potential Resources:** Resources which are found in a region but have not been utilized due to lack of capital technology, skill etc.
Ex: Rajasthan and Gujarat have enormous potential for the development of wind and solar energy,
But so far these have not been developed properly.

(b) **Developed Resources:** Resources which have been surveyed and their quality and quantity have been determined for utilization. Example: Coal resource of India.

(c) **Stock:** These are the material in the environment which have the potential to satisfy the human needs but could not be used due to lack of appropriate technology. Example: Water (H_2O) is a compound of two gases. But we do not have the technology to trap the energy of Hydrogen (H_2) & Oxygen (O_2).

(d) **Reserves:** Resources which can be put into use with the existing technology but their uses have been postponed keeping in mind the needs of the future generations. Example: we are using river water for generating hydroelectricity only in a limited quantity.

►► DEVELOPMENT OF RESOURCES

Resources are vital for human survival, development and for maintaining the quality of life. Over utilization of resources has led to many problems.

1. Depletion of Resources: over utilization has led to depletion of resources for meeting the greed of few individuals.
2. Concentration of Resources: This has divided the society into have and 'have-not's and rich and poor.
3. Global Ecological Crisis: It has led to global ecological crisis such as global warming, depletion of ozone layer, pollution and land degradation.

►► RESOURCE PLANNING

- **Definition:** Resource planning: It is a technique or skill of proper or judicious use of resources.
- **Need for resource planning**:

1. Most of the resources are limited in supply.
2. Most of the resources are unevenly distributed over the country.
3. Over utilization of resources may lead to pollution of environment.
4. Planning of resources is also required to become self dependent.
5. Under utilization of resources will lead to under development of the economy.

►► RESOURCE PLANNING IN INDIA

This involves 3 processes or stage:

1. Identification & Inventory of resources: This in includes surveying, mapping, qualitative & quantitative estimation & measurement of resources.
2. Making a plan structure using appropriate technology skill and institutional setup so that all resources developmental plans can be implemented.
3. Matching the resources development plans with the national development plans.

- Conservation of Resources; Conservation of natural resources means judicious and planned use of natural resources so that we can get the maximum possible benefit from these resources over the longest possible period of time.

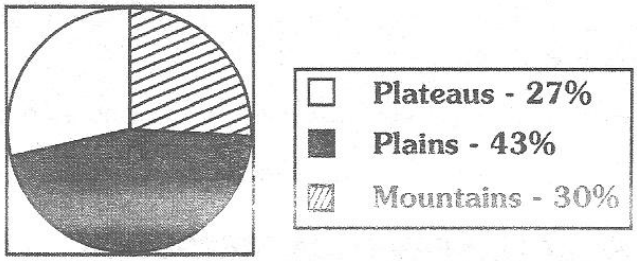
►► LAND RESOURCES

Land is an important resource because.

1. It supports natural vegetation.
2. It supports wild life & human Life.
3. It supports human economic activities and transport and communication system.

India has land under a variety of relief features

(a) 43% of plains which is important for agriculture & industries.
(b) 30% is mountains. They help to provide rivers facilities for tourism & ecological aspects.
(c) 27% is plateau region which is rich in minerals fossils fuels & forest.



►► LAND UTILIZATION

The land resources are used for the following purposes:

1. Forest
2. Land not available for cultivation

(a) Barren and waste lands

(b) Land' used for non agricultural activities such as building roads, factories.

3. Uncultivated land:

(a) Permanent pasture & grazing land

(b) Land under the crop graves.

(c) Cultivable waste land

4. Fallow land which is left uncultivated for one to five years.
5. Net sown area: Area sowed more than once in an agricultural year plus net sown area is know –as gross cropped area.

►► LAND USE PATTERN IN INDIA

Land use depends upon the following factors:

1. Physical factors: It includes topography, climate, soil.
2. Human factors: Population density, Technological Capability, culture, Traditions etc.
3. Total area of India is 3.28 million km^2. Land use data accounts for only 93% because for most of the north –east states land use reporting is not done.

S. No.	Categories	11960-61	2002-03
1	Forest	18.11%	22.57%
2	Barren & Waste Land	12.01%	06.29%
3	Area under nonagricultural uses	04.95%	07.92%
4	Permanent pasture & grazing land'	04.71%	03.45%
5	Tree crops and grasses-	01.5%	01.1%
6	Cultivable, land waste	06.23%	04.41%
7	Fallow other than current fallow	03.5%	03.82%
8	Current fallow	3.73%	07.03%
9	Net sown area	45.26%	43.41%

(a) Net sown area differs greatly from one state to the other.

(b) In Punjab & Haryana it is over 80% and, in Arunachal Pradesh, Mizoram, Manipur, Andaman and Nicobar Island it is less than 10%.

(c) Waste land includes rocky arid and desert area, land used for housing, roads, railways etc.

(d) According to National forest Policy 33% should be under forest cover but in India it is only 22%.

►► LAND DEGRADATION & CONSERVATION MEASURES

- Land Degradation: It is a process through which land becomes unfit for cultivation.

Factors Responsible for land Degradations

1. Mining: It is most important factor for land degradation. The mining sites are abandoned after excavation work is over. The excavation work leaves deep scars and other material which degrades the soil. This is common in state like Jharkhand, Chhattisgarh, Madhya Pradesh and Orissa.
2. Overgrazing: Overgrazing of land by animals results in removal of grass over a large are making it easy for wind and water to remove the soil. Example Gujarat, Rajasthan. Madhya Pradesh, Maharashtra etc.
3. Water Logging: Over irrigation of land is also responsible for land degradation water logging increases salinity and alkalinity in soil making it unfit for cultivation.
4. Industrialization: Industrial waste leads to water and land degradation.

►► CONSERVATION OF RESOURCES

1. Afforestation should be encouraged.
2. Proper management of grazing.
3. Control on overgrazing.
4. Planting of shelter belts of plants
5. Stabilization of sand dunes by growing thorny bushes.
6. Proper management of waste land.
7. Control of mining activities.
8. Proper discharge & disposal of industrial waste.

►► SOIL

Soil is a living system which takes millions of years to form.

- **Formation of soil:** Soil is the thin upper surface layer on the earth, comprising mineral particles formed by the breaking down of rocks, weathered mineral particles, decaying organic matter, living organism, water and air.
- **Importance of Soil:**

(a) Soil is the medium in which plants grow. All living things depends directly or indirectly on soil for food.
(b) Agricultural production is basically dependent on the fertility of the soil.
(c) The rich deep fertile soils supports high density of population through agricultural prosperity.

- Important factors of soil formation

1. **Parent material:** The source of the rock fragments that make up the soil is parent material which may either be bedrock or loose sediments transported elsewhere by water wind or ice.
2. **Relief:** The most important being the slope of land steep slope encourage the swift flow the water, so steep slopes usually have a thin soil layer.
3. **Climate**: Temperature and moisture (rainfall) are the climatic variables of greatest significance in soil formation.
4. **Natural Vegetation:** Vegetation of various kinds growing is soil" performs certain vital function.

(A) The decayed leaf material adds much needed humus to soil there by increasing its fertility.
(B) Roots hold the soil and so prevent erosion.

►► CLASSIFICATION OF SOIL

Soil can be classified on the basis of colour, thickness texture & physical properties. Indian soil are classified as :

1. Alluvial soil 2 black soil 2. Red & yellow soil 3. Laterite soil
4. Arid soil 5. Forest soil

►► ALLUVIAL SOIL

- Formation: Most of the soil is derived from the sediments deposited by rivers as in the Indo-Gangetic plain. It consists of sand silt and clay.
- Features of alluvial soil:

1. This is the most wide spread soil in India.
2. It is the most fertile soil.
3. Due to high fertility they are intensively cultivated and are densely populated.
4. They are very fine grained, rich in potash and phosphoric acid but deficient in nitrogen and humus.
5. According to age alluvial soil is classified into.

(A) Old Alluvial or Bhangar
(B) New Alluvial or Khaddar

	Khaddar		Bhangar
1	The Khaddar soils are found in low areas of valley bottom which are flooded every year.	1	The Bhangar soils are found in the higher reaches about 30 m above the flood level.

2	These soils are fine in texture.	2	These are coarse in texture.
3	The Khaddar soils are more fertile.	3	The Bhangar soils are less fertile.

- Crops: Rice, Wheat, Sugarcane, Tobacco, Jute Fruits and Vegetables.
- Regions: Extended over Northern plains, small parts of Rajasthan & Gujarat, Eastern Coastal plain particularly deltas of Mahanadi Godavari, Krishna & Kaveri.

►► BLACK SOIL

- Formation: These soils have been formed due to the weathering of the lava spread over large areas during volcanic activity is the Deccan Plateau and different climatic conditions.
- **Features:**

1. The black soils are made of extremely fine material i.e., clayey material.
2. These soils are rich in nutrients such as calcium carbonate, magnesium carbonate, Potash and lime.
3. The soil is known for its capacity to hold moisture.
4. During dry condition deep cracks develop in the soil, this help in proper aeration of the soil.
5. When wet, the soil become sticky and is difficult to work with.
6. It is also known as Regur soil.

- Regions: There soils are mainly found in Maharashtra, Western Madhya Pradesh part of Karnataka, Andhra Pradesh, Gujarat and Tamil Nadu.
- Crops: cotton, sugarcane, wheat, tobacco and oil seeds.

►► RED AND YELLOW SOIL

- Formation: It is formed from igneous rock. Red colour is due to the presence of iron in the crystalline, igneous & metamorphic rock.
- Features:

1. Soils are loamy in deep depression and in uplands they consist of loose gravels highly coarse material.
2. Soils are deficient in phosphoric acid, organic matter and nitrogenous material but are fairly rich in potash.
3. Crops are cultivated with the use of fertilizers.

- Crops: Cotton, wheat, rice, pulses, millets etc. can be produced by using fertilizers and irrigations.
- Regions: Tamil Nadu, Parts of Karnataka, South east of Maharashtra, Madhya Pradesh, Orissa and on the Chota Nagpur, Plateau of Jharkhand.

►► LATERITE SOIL

- Formation: It develops in areas with high temperature and heavy rainfall. It is a result of intense leaching owing to heavy tropical rains.
- Features:

1. The soils are acidic in nature, coarse and crumbly ‘in texture.
2. Due to lack of nitrogen, potassium and organic elements laterite soil lack fertility and are not suitable for cultivation.
3. When the soils are manured and irrigated some crops can be cultivated.
4. They provide valuable building material.

- Crops: Tea, coffee, Rubber, cinchona, Coconut cashew nut and Tapioca.
- Regions: Karnataka, Kerala, Tamil Nadu, Madhya Pradesh and Hilly areas of Orissa.

►► FOREST SOIL

- Formation: The soils are formed due to mechanical weathering caused by snow, rain, temperature variation etc.
- Features:

1. These soils are heterogeneous in nature and their character changes with mountain environment and their character changes with mountain environment and altitude
2. The soils are very rich in humus but are deficient in potash, phosphorus and lime.

3. The soils are especially suitable for plantation of tea, coffee, spices and tropical fruits.
4. Regions: Hilly region of Jammu & Kashmir, Sikkim, Assam and Arunachal Pradesh.

►► ARID SOIL

- Feature:

1. Colour ranges from red to brown.
2. Sandy in texture" Saline- in nature. Common salt is obtained by evaporating the water.
3. Soil lacks humus & moisture due to dry climate and high temperature.
4. Lower layer of the soil contain Kankar due to calcium content.
5. With irrigational facilities soil becomes cultivable.

- Regions: Western Rajasthan, Northern Gujarat and Southern Punjab.

►► SOIL EROSION AND CONSERVATION

- Soil Erosion: It is the removal of soil by the forces of nature like wind and water, more rapidly than the various soils forming process can replace it.

►► HUMAN FACTOR CAUSING SOIL EROSION

1. Deforestation: This has resulted in reckless cutting of forest which has led to the problem of soil erosion
2. Overgrazing: This leads to lose structure of soil and the soil is easily washed away by rains.
3. Faulty Methods of Agriculture: Ploughing field along the slope make it easier for running water and wind to cause erosion.

►► VARIOUS METHODS OF SOIL EROSION ARE:

Gully erosion: When the running water cuts through clayey soil & makes deep channels which are called gullies. It is called gully erosion & the land is called land, as it becomes unsuitable for cultivation. Example: The ravines of the Chambal basin

Sheet erosion: When water flows as a sheet over a large area over a slope washing away the top soil. This is known as sheet erosion.

Wind erosion: Loose soil is blown off flat or sloping land by the action of wind & thus known as wind erosion.

Soil erosion due to defective methods of farming:

Ploughing done up & down on the slope, forming channels results in quick flows of water leading to soil erosion.

►► SOIL CONSERVATION:

- Contour ploughing: Ploughing along the contour lines reduces the flow of water on the slope.
- Terrace farming: Steps are cut on the slopes making terraces to check soil erosion. This is found in western & central Himalayas.
- Strip farming/cropping: Large fields are divided into strips and grass is allowed to grow b/w the crop. This is done to break the force of wind.
- Planting of shelter belts: Lines of trees are grown, stabilization of sand dunes & deserts in western India. These shelter belts check wind erosion.

SOLVED EXERCISE

1. What is a resource?

Ans. Everything available in our environment which can be used to satisfy our needs, provided it is technologically accessible, economically feasible and culturally acceptable can be termed as 'Resource'.

Example: Coal, water, air

In other words –any material which can be transformed in such a way that it becomes more valuable and resourceful can be called a resource. These are those products and objects which have value for human beings.

Its characteristics are:-

(i) They have utility – i.e. the power to satisfy our needs.
(ii) They help us to create goods and provide services.
(iii) With an improvement in science and technology, the utility of a resource changes.

2. Differentiate between natural resources and the man made resources.

Natural Resources:-

Natural Resources are gifts of nature which can be extracted from nature with the help of human labour and used to produce more goods.

E.g. minerals, soil, water crops etc.

Human Made Resources:-

Human made Resources include all those products that human beings have created by using the natural resources.

E.g.; roads, machines, buildings, factories, etc

3. Difference between biotic and Abiotic Resources

	Biotic		Abiotic
1	Resources obtained from biosphere and have life	1	Resources composed of non-living things
2	They go on reproducing and regenerating till favorable conditions are available in the environment	2	As they are non-living things they do not reproduce or regenerate
3	As these resources have life. They cannot be called exhaustible or inexhaustible	3	Abiotic resources are of both types exhaustible and inexhaustible
4	Biotic Resources are dependent on Abiotic for their sustenance E.g.: animals plants etc.	4	Abiotic Resources are independent of biotic resources E.g.: water, soil, minerals etc.

4. Explain the classification of resources on the basis of exhaustibility

Ans. (i) Renewable Resources

Natural Resources which can be used again and again i.e. renewed or can be reproduce by physical, mechanical and chemical processes (Also called Replenish able resources)

Eg. : Solar & wind energy, forests, wildlife, air, water etc.

They can further be divided into continuous (or flow) and Biological-

(a) Continuous (or flow) – eg- wind, water
(b) Biological- Which get reproduced biologically can be divided into

1. Natural Vegetation (forests)
2. Wild life

(ii) Non-Renewable Resources

Natural resources cannot be replaced at all. They have accumulated over millions of years. Once used, these resources cannot be got back again; they will be finished forever e.g.: minerals, fossil fuels like oil, gas, coal etc.

* They can further be divided into Recyclable and non-Recyclable.

(a) Recyclable – e.g. Metals
(b) Non- Recyclable- e.g. fossil fuels (These will exhaust with their uses).

5. How can resources be classified on the basis of ownership?

Ans. On the basis of ownership, resources can be classified as follows

(i) Individual Resources:-

Resources which are owned by private individuals are called individual resources. E.g. Plots, fields, pasture lands, houses cars etc.

(ii) Community Owned Resources:-

Resources which are accessible to all the member of the community are called as Community owned resources.
E.g.: Public parks, playgrounds, burial grounds.

(iii) National Resources :-

All the resources of the nation (Union or state Government) are National Resources. All resources within the political boundaries of the nation and the government have legal power to acquire even individual private property for public good.

E.g.: All minerals water resources, forests, wildlife resources land within political boundaries and resources in the oceanic area up to 12 miles from the coast.

(iv) International Boundaries:-

The resources owned and regulated by international institutions. No individual country can use oceanic resources beyond 200 km of the exclusive Economic Zone as they belong to open-Explain Ocean.

6. Explain the classification of resources on the basis of status of development.

(i) Potential Resources:-

Resources which are found in a region but have not been utilized due to lack of capital or other reasons are called as Potential resources.

E.g.:- In Rajasthan and Gujarat there is enormous potential for the development of wind and solar energy, but so far these have not been developed properly.

(ii) Developed Resources:-

These are resources which have been surveyed and their quality and quantity have been determined for utilization and are being utilized. (The development of these resources depends on technology and the level of their feasibility).

E.g.:- India's coal reserves.

(iii) Stock:-

The materials in the environment which have the potential to satisfy human needs but have not been used as the human beings do not have the appropriate technology to convert them into usable form. They are at present lying unutilized because of the lack of technical know-how.

E.g.:- Water is a compound of two inflammable gases- Hydrogen & Oxygen, human beings do not have the required technology to use them as a source of energy.

(iv) Reserves:-

Reserves are those resources, which can be put into use with the existing technology but their use has been postponed keeping in mind the needs of the future generations.

e.g.: India has sufficient amount of forests and river water, but they are being kept unutilized for future generations.

7. What are the major problems regarding resources?

Ans. (i) Resources are being depleted for satisfying the greed of few individuals.

(ii) Resources have accumulated in few hands, dividing the society into rich and poor.

(iii) Indiscriminate exploitation of resources has led to global ecological crises such as Global warming, ozone layer depletion, environmental pollution, land degradation etc.

8. What is Resource Planning? Why is there a need for Resources planning?

Ans. Resources Planning:-

A resource planning is strategy for judicious use of resources. There is a need for Resource planning because :-

(i) Resources are limited in supply

(ii) Resources are unevenly distributed over the country.

(iii) Over-utilization of resources is leading to ecological crisis.

(iv) Accumulation of resources in a few hands has led to a division in the society into rich & poor.

9. Why is Resource planning important in India?

Ans. (i) There is diversity in the available resources.

(ii) There are regions which are rich in certain types of resources- but are deficient in some other resources.

(iii) There are some regions which can be considered self sufficient it terms of the availability of resources and there are some regions which have acute shortage of some vital resources.

E.g.

State/Region	Sufficient	Deficient
Arunachal Pradesh Rajasthan, Ladakh- (Cold desert region)	Water resources, solar and wind Energy, Rich cultural heritage	Infrastructural Development, water Resources, water resources Minerals, infrastructure

10. Mention steps or stages involved in resource planning

Or

How is Resource planning carried on?

Ans. Resource Planning is a complex process which involves:-

(i) Identification and inventory of resources. This involves surveying, mapping and qualitative and quantitative estimation and measurement of resources.

(ii) Evolving a planning structure endowed with appropriate technology; skill and institutional, setup for implementing resource development plans.

(iii) Matching the resource development plant with overall national development plans.

11. What is conservation of resources? Why is it necessary?

Ans. Conservation of resources means a judicious and planned use of natural resources so that sufficient resources are left over to meet the needs of the future generation.

It is necessary because:-

(i) Resources are in limited quantity.

(ii) Over utilization of resources can lead to environmental problems.

(iii) Over utilization and indiscriminate use of resources can lead of social economic problems.

12. How have technical and economic development led to more consumption of resources?

Ans. (i) Technology and economic development leads to industrialization-requiring more natural resources.

(ii) Industrialization requires a lot of raw material especially different types of minerals which leads to excessive use of resources.

(iii) Technological development has converted the subsistence agriculture to commercial agriculture and this has lead to over utilization of land.

(iv) Economic development leads to urbanization and modernization which demands more resources.

13. What is meant by land degradation?

Ans. Rendering the land unfit for cultivation is called land degradation. Land is rendered unfit for cultivation due to many reasons.

(i) Humans Activities:-

(a) Indiscriminate cutting of trees by man has turned land into wasteland

(b) Construction of buildings roads etc.

(c) Dirty and poisonous effluents of the factories collect in the surrounding areas and degrade the land.

(d) Dumping of city waste on any land also leads to land degradation.

(e) Overgrazing.

(f) Mining and quarrying activities.

(g) Over-irrigation.

(ii) Natural forces:-

(a) Heavy rains take away the upper layer of land (soil) rendering the land unfit for cultivation. This is erosion.

(b) Winds also take away soft grains of the earth and make the fertile land into wasteland.

14. What is net sown area? What percentage of total area is under net sown area?

Ans. Area sown once in a year is known as net sown area. In 2002-03, 43.41% of India's area was under net sown area

15. What is gross sown area?

Ans. Area sown more than once in an agricultural year plus net sown area is known as gross sown area.

16. Name any two states with high net sown area and two with low net sown area.

Ans. (i) High net sown area: Punjab and Haryana.

(ii) Low net sown area: Arunanchal Pradesh and Mizoram.

17. How does the soil of the Ganga-Yamuna plain differ from that of central Maharashtra?

Ans. The Ganga-Yamuna plain has alluvial soils whereas Central Maharashtra has black soils. The alluvial soils are formed by the depositional work of rivers in the river valleys, flood plains and deltas. The black soils develop from volcanic rocks of lava flows.

18. What are the causes of soil erosion in :

(i) Shiwaliks or the Outer Himalayas? (ii) North- Eastern parts of India? (iii) Arid regions of India?

ns. (i) Destruction of vegetation is the main cause of soil erosion in hilly areas because when vegetable is removed, the soil surface becomes loose and is more easily removed by running water.

(ii) Heavy rainfall which leads to frequent floods is responsible for soil erosion.

(iii) In deserts and dry regions, where there is little or no vegetation, wind is the most powerful agent of soil erosion, blowing away fine particles of sand, depositing them in other areas making both area unproductive.

19. Mention any four characteristics of alluvial soils.

Ans. Formation: Alluvial soils are transported soils.

Features: Most of the soils are derived from the sediments deposited by rivers as in the Indo-Gangetic plain. Thus the parent material of these soils is of transported origin. These soils consist of varying proportion of sand, silt and clay- alluvium soils as a whole are very fertile are intensively cultivated and densely populated. In the upper course of the river, the soil is coarse. In the middle course, it is medium and fine grained in the lower course.

Soils are described according to their age as well. They are old alluvium and new alluvium.

Locally the old alluvium is called 'Bangar' and the new alluvium is called 'Khaddar'. The old alluvium often contains Kankar nodules, with calcium carbonates in sub-soil. The new alluvium is more fertile than the old alluvium.

20. Distinguish between Khaddar and Bhangar.

	Khaddar (New)		**Bhangar (Old)**
1	The Khaddar soils are found in the low areas of valley bottom which are flooded every year.	1	The Bhangar soils are found in the higher reaches about 30 m above the flood level.
2	These soils are finer in texture	2	These are coarse in texture.
3	The khaddar soils are more fertile in the low areas of valley bottom which are flooded almost every year.	3	These soils are less fertile as these are found on the higher reaches about 30m above the flood level.

21. Write short note on soil erosion.

Ans. The removal of soil by the forces of nature, particularly wind and water is known as soil erosion –wind and water are powerful agents of soil erosion because of their ability to remove sort and transport it- soil erosion makes land unsuitable for cultivation and the land so developed is called bad land. Soil erosion in certain areas has become a menace. Large areas of agricultural land in the states of Uttar Pradesh, Madhya Pradesh, Rajasthan and Gujarat have been transformed into ravines.

Gully- erosion is the most spectacular types of erosion. This has already degraded about 40 lakh hectares of land in the country. Reducing flow of water by constructing Bandhs, by planting more trees, by restricting grazing of animals and applying proper farming techniques are some of the methods to check soil erosion.

22. How is red soil formed? Mention its four characteristics and the crops grown on this type of soil.

Formation: Most of the red soils have some into existence due to weathering of ancient crystalline igneous rocks.

Features:-

(i) Soils are loamy in deep depression and in uplands they consist of loose-gravels, highly coarse material.

(ii) The colour of these soils is generally red, often grading into brown, chocolate or yellow. The red colour is due to wide diffusion rather than high percentage of iron content. It looks yellow when it occurs in a hydrated form.

(iii) Soils are deficient in phosphoric acid, organic matter and nitrogenous material but are fairly rich in potash. But crops are cultivated with the use of fertilizers.

Crops: Crops like cotton, wheat rice, pulses, millets tobacco, etc can be produced by using fertilizers and irrigation.

23. How is mountain soil formed? Mention any four characteristics of forest soil.

Ans. Forest soils are mainly found on the hill slopes where sufficient rainforests are available. These soils occupy about 8% of the total land area of India. Formation: The soils are formed due to mechanical weathering caused by snow, rain, temperature variation etc.

Features:

(i) These soils are heterogeneous in nature and their character changes with mountain environment and altitude

(ii) The soils are very rich in humus but are deficient in potash, phosphorus and lime.

(iii) The soils are especially suitable for plantation of tea, coffee, spices and tropical fruits.

(iv) The soil is loamy and silty in valley sides and coarse grained in the upper slopes. It is acidic with low humus content in the snow covered areas. It is fertile in the lower parts of the valleys.

24. Mention any four features of arid soils.

Ans. Arid soils cover 4% of the total geographical area of India. These soils are found in those regions which receive less than 50 cm of annual rainfall.

Features:

(i) The soil range red to brown in colour.

(ii) The soils contain considerable amount of soluble salts.

(iii) The soils contain low percentage of organic matter due to dry climate and absence of vegetation.

(iv) The soil is alkaline in nature as there is no rainfall to wash soluble salts.

(v) The lower horizon of the soil is occupied by 'Kankar' because of the increasing calcium content downwards. The Kankar' layer formations in the bottom horizons restrict the infiltration of water.

(vi) The soils are infertile but with irrigation and fertilizers the drought resistant and salt tolerant dry crops such as barley, cotton, wheat, millets, maize, pulses are grown.

25. Explain the formation and important features of laterite soil.

Ans. Formation: It develops in areas with high temperature and heavy rainfall. The laterite soil is a result of intense leaching owing to heavy tropical rains.

Features:-

(i) The soils are acidic in nature, coarse and crumbly in texture.

(ii) Due to lack of nitrogen, potassium and organic elements laterite soils lack fertility and are not suitable for cultivation.

(iii) As the soils are indefinitely durable so they provide valuable building material.

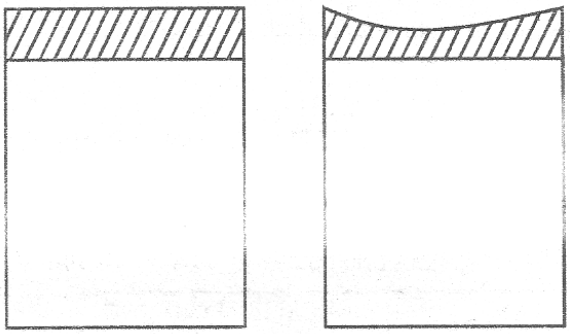
26. What is soil erosion? Explain the major types of soil erosions prevailing in India.

Ans. "Soil erosion is the removal of soil by the forces of nature like wind and water, more rapidly than the various soils forming process can replace it. Generally there is a balance between soil forming process and erosion process. The balance can be disturbed by natural or human factors.

Types of soil Erosion:-

(a) Water erosion: Water is a powerful agent of soil erosion. Following are the major types of erosion caused by water.

(i) Sheet Erosion: - When the top layer of the soil is removed over a large area by running water, it is called sheet erosion.



A : Surface layer of soil before eros

B : Surface layer of soil after sheet erosion.

(ii) Gully – Erosion: - This is the third stage of sheet erosion. With further erosion of the soil, the soils may deepen and become enlarged and are ultimately turned into gullies. The main cause of gully-erosion is the removal of vegetation, particularly of tress with their widespread binding roots. Gullies cut up agricultural land and the entire area may be turned into bad land topography. Gully-erosion is also responsible for the formation of ravines.

27. Explain the land use pattern of India.

Ans. (i) According, to given figure net sown area has decreased from 45.26% to 43.21% this means that more and more agricultural land is being shifted to other activities. This is not a healthy trend and must be checked.

(ii) The pattern of net sown area varies greatly from one state to another. It is over 80 percent of the total area in Punjab and Haryana and less than 10 percent in Arunachal Pradesh, Mizoram, Manipur and Andaman Nicobar Islands.

(iii) Area under forests:- Though the area under forests has increased from 18.11 % in 1960-60 to 22.57% in 2000-03 but still is far below than the scientific norms.

(iv) The land under permanent pasture is very low i.e. only 3.45%. This shows the tremendous pressure of livestock population on agricultural land. Cattle are reared mainly on farm wastes, grain chaff and few fodder crops.

(v) Area under fallow land has also decreased, which shows, that subsistence agriculture is being replaced by commercial agriculture.

(vi) Apart of the land is termed as waste land and land put to another non-agricultureal uses. Water land includes rocky, and desert areas and land put to other non-agricultural uses includes settlements, roads railways, industries etc.

28. How is soil formed?

Or

Analyze any four factors which help in the formation of soils.

Ans. The formation of soil is a very slow process. It may take thousands of years of make one to two centimeters thick stable soil cover. In the formation of soils, various factors and processes play a major role.

(i) The parent rock is the first factor which provides the basic material for the formation of soils.

(ii) The second factor is the climate which breaks the parent rock into smaller pieces in so many ways.

(iii) The third factor is the-plant and animal organisms which enter the rocks through many fissures and holes in the rocks and help in the weathering of the rocks slowly but continuously.

(iv) The fourth factor is topography or elevation and the relief which in their own way help in the weathering (or breaking up) of the rocks. Last but not the least is the time factor which plays a mighty role in changing everything not to speak of land- forms alone.

29. How soil erosion is caused?

Or

What is soil Erosion? Mention two main causes of it.

Ans. Removal of soil, especially of top soil, either naturally or as a result of human activity is called soil erosion. Soil erosion is most significant in dry and desert areas where they carry with them top soils and deposit them in other areas. Sometimes human action like over-grazing also leads to soil erosion and the removal of upper layer of the soil. Soil erosion renders the land infertile and creates lot problems for the farmers. They are not able to grow crops and are rendered poorer day by day. The country also suffers because of the infertility, caused by soil erosion.

30. What steps can be taken to control soil erosion in the hilly areas?

Ans. The following steps can be taken to control soil erosion in hilly areas:-

1. Contour ploughing:- By this method, the field are ploughed, harrowed and sown along the natural contour of the hills instead of up and down the slopes. This prevents the rain water from down the hill. It stands in the level furrows and soaks into the ground, so the plants receive more water. This is suitable for Afforestation and grassland development work.
2. Terracing:- By this method, a series of wide steps are made along the slope following the contours. This method in very common in Asian countries in regions of rice cultivation.
3. Strip-Cropping:- In the method, cover crops, such as grasses and small grains are planted alternatively with cultivated crops. These cover, absorb the moisture and hold the surface soil together.
4. Plugging of Gullies:- This is done by building dams of stones or fixing wire-netting or planted plugging building planting trees across gullies. These measures check the flood waters, and so cause filling of silt in the gullies.

31. How is nature responsible for the degradation of land?

Ans. As for as nature is concerned, torrential rains and storms play havoc with land especially by taking, away the upper layer of the land called soil by their ferociousness. In a more or less, leveled land, the rains take away the soil water with its water and render the land waste. This is called the sheet erosion. But in mountainous area, the rain water with its great speed cuts the land deep and make gullies it is called the gully erosion. Sometimes, winds also play havoc when it taken away soft grains of the earth with it thereby rendering the land waste sometimes when they take a away particles of sand to far off places, they render the fertile lands into wastelands. Such a degradation of land due to expansion of deserts is posing a great danger to those states which are situated close to deserts.

32. How is man responsible for the degradation of land?

Ans. But man's responsible, in rendering the land waste or his part in the degradation of the land is far more than the nature.

1. Firstly, by cutting the trees indiscriminately, he turns the land into wasteland. If there are no trees then there are no rains and if there are no rains the land is automatically degraded.
2. Secondly, his indiscriminate building activities not only swallow the land but they also render the surrounding land degraded.
3. Thirdly, the dirty and poisonous effluents of his factories which go on collection in the surrounding areas completely degrade lands and render them waste.
4. Not only this, the dumping of city waste on any land completely leads to its degradation.

33. Distinguish between:- Sheet Erosion and Gully Erosion.

Ans. Sheet Erosion: - When water flows as a sheet down a slope and as a result, the top par of the land is washed away, such erosion is called sheet erosion.

Gully Erosion:- When the running water cuts through the clayey soils and makes deep channels or ravines as gullies in the absence of vegetation. This type of erosion makes soil unfit for cultivation, and then such erosion is called gully erosion. The Chambal ravines are the result of such gully erosion.

34. How can we conserve or plan our resources?

Ans. Ways of planning or conserving our resources:- The different ways of conserving or planning our resources are the following:-

1. Water and marshy lands should be reclaimed and the available land should be put to optimum use.
2. Rivers may be linked to each other and if desired their directions be changed so that not a drop of water goes waste into the sea. The waste water from the factories be recycled to be used over and over again.
3. The non-renewable resources like different minerals require a special attention. Wastage during mining and processing should be reduced to minimum.
4. Where possible, alternative products such as wood and plastic should be used.
5. Scrap should be recycled so as to prolong the life of our limited metallic mineral resources. For example, iron scrap should be processed into steel.
6. New methods can be devised for utilizing ores of lower grade.
7. We should use the available stock of mineral wealth most judiciously so that the coming generations are not deprived of the benefits of the mineral resources.

35. Give a brief account of the soils found in India.

Ans. There are different types of soils are available and so are their characteristics. The main soil types of India are the following:-

Laterite Soils:- Laterite soils are commonly found in the hilly or upland areas. Such soils are fond in hilly or upland parts of the Deccan, Karnataka, Kerala, Orissa and some parts of Assam and Meghalaya. These soils have a surface layer of a red hard crust. These, being unsuitable for crop cultivation are, however, suitable for tree. Plantation. So the cashew, eucalyptus and other trees have been planted in these areas.

Red and Yellow Soils:- Red and yellow soils cover a vast area of the Peninsular plateau. These soils are red in colour because of the presence of lime in them. Such soils are found in the plateau covering Karnataka, Tamil Nadu, Chhattisgarh, Orissa and Jharkhand. These soils are fertile and useful for cultivation. These soils vary in texture, colour and thickness due to topography, different amount of rainfall etc. Low lands have red loamy soils.

Alluvial Soils: - Alluvial soils are of the most widespread category. They are spread over a vast area of the Northern Plains beginning from the Punjab in the west to the Brahmaputra Valley in the east. These soils being fertile are very suitable for cultivation. Irrigational facilities are an added advantage.

However, these soils are deficient in organic and nitrogenous content. Soils in the dry areas are found to be more alkaline. The old alluvium contains in some degrees Kankar nodules along with calcium carbonate in its sub-soil.

Black Soils:- These soils were formed by the lava sheets and hence they are black in colour. These black soils are found in certain part of Maharashtra, Madhya, Pradesh, Gujarat, Karnataka, Andhra Pradesh and Tamil Nadu. These soils are also called Regur soils and they are generally shallow in depth. These soils, however, are highly fertile and are suitable for cotton cultivation. Besides cotton, millets are also cultivates in these soils.

Mountain or forest Soils:- These soils have different characteristics from place to place. They are sandy, gravelly, devoid of humus and porous. These soils are found in the mountainous regions of Eastern ranges, Arunachal Pradesh, Meghalaya, Uttaranchal, Himachal Pradesh and Jammu and Kashmir.

Desert or Arid Soils: - Such soils are largely found in and areas of Rajasthan, Punjab and Haryana. These soils are mostly covered with sand-particles and as such they the infertile. But if irrigation facilities are available, they can prove quite useful as is the case with Shri Ganganagar area of Rajasthan.

36. Write a short note on- land- Conservation measures.

Ans. Conservation measures to check land degradation – Some of the conservation measure to check degradation are the following:-

1. **Contour Ploughing:-** By this method, the fields are ploughed, harrowed and sown along the natural contour of the hills instead of up and down the slopes. This prevents the rain water from flowing down the hill. It stands in the level

furrows and soaks into the ground, so the plants receive more water- this suitable for Afforestation and grassland development work.

2. **Terracing:-** By this method, a series of wide steps are made along the slope following the contours – This method is very common in Asian countries in regions of rice cultivation.
3. **Strip- Cropping**: - In this method, cover crops, such as grasses and small grains are planted alternatively with cultivated crops. These cover crops, absorb the moisture and hold the surface soil together.
4. **Plugging of gullies**: - This is done by building dame of stones of fixing wire netting or plating trees across gullies.
5. Planting of shelter Belt's:- This measure is adopted in the case of wind erosion. Belts of trees and shrubs are planted to check the velocity of the wind and this stop soil movement.
6. **Fallowing**: - this is allowing the much used land to rest or lie fallow so that the natural forces can act on the soil.
7. **Crop rotation: -** to grow different crops on the same land. This prevents the crops from exhausting. One kind of mineral nutrient in the soil. For eg. Potatoes require much potash but wheat requires nitrates. This it is best to alternate crops in the fields
8. **Use of Fertilizers: -** Use of fertilizer is another way of soil conservation. Farmers all over the world are now using chemical fertilizers along with organic manures. These fertilizers (chemical) are easier to use and enormously help in growth of agriculture.

EXERCISE

VERY SHORT ANSWER TYPE QUESTIONS

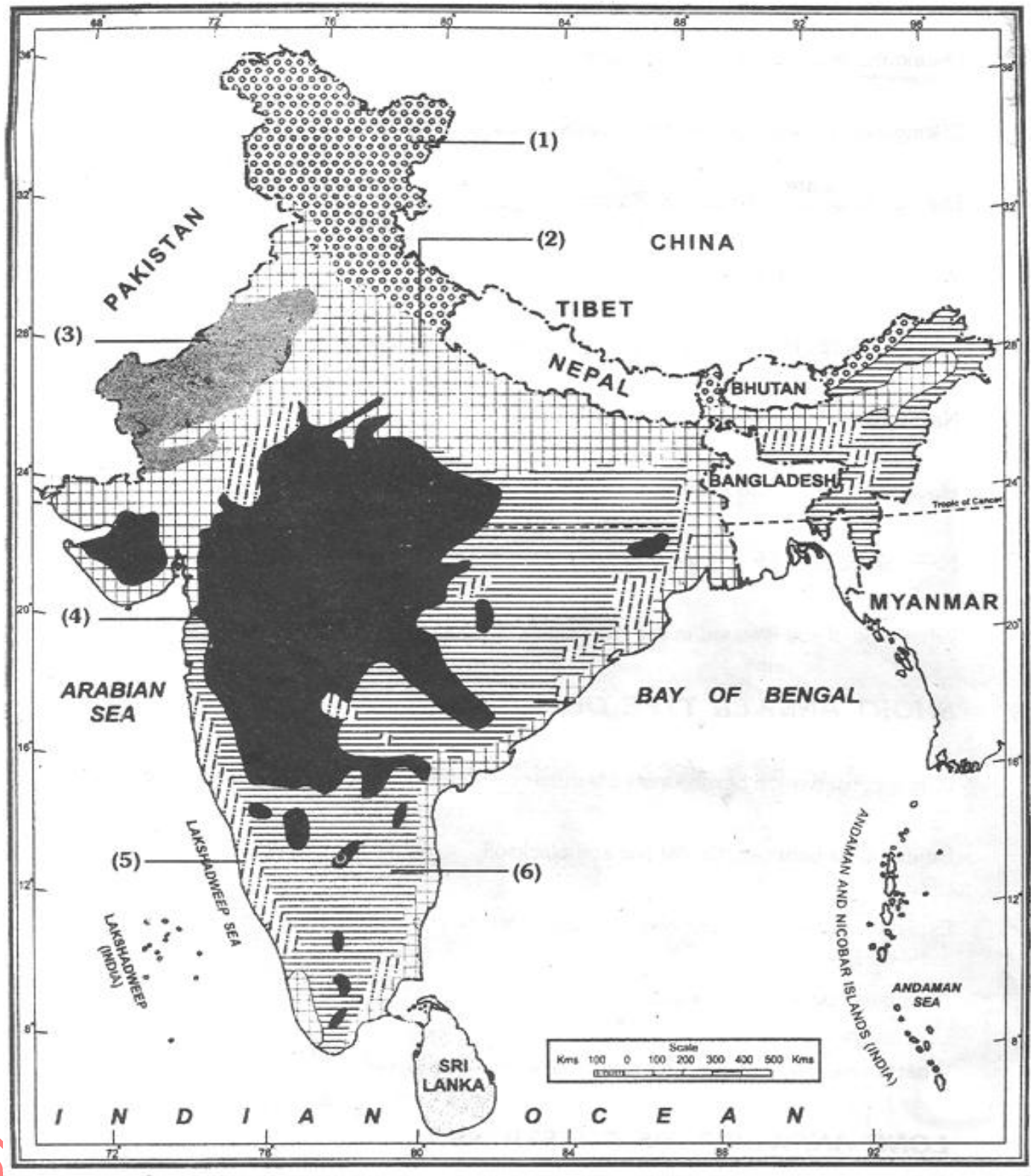
1. Distinguish between renewable and non renewable resources.
2. Distinguish between Human made resources and Natural resources.
3. Distinguish between khaddar & Bangar.
4. What is grass sown area?
5. What is leaching? Name the soils which develop the leaching.
6. Name any two crops associated with laterite soil.
7. How does contour ploughing help in soil conservation?
8. Name three state having black soil and the crop which is mainly grown in it.
9. What type of soil is found in the river deltas of the eastern coast?

SHORT ANSWER TYPE QUESTIONS

1. Why is conservation of resources essential? List out three methods of soil conservation?
2. Differentiate between alluvial soil and black soil.
3. Explain how resources are classified based on the state of development.
4. Distinguish between stock and reserve.
5. What is resource planning? Mention the step which is involved in resource planning.

LONG ANSWER TYPE QUESTIONS

1. Describe the classification of the resource on the basis of different types.
2. Write short notes on soil erosion.
3. What is land degradation? Describe any five measures of controlling land degradation.
4. How have technical and economic development led to more consumption of resources
5. Identify the type of soils found in each of the shaded region and write their names on the lines market in the map.



AGRICULTURE

◙ **IMPORTANT TERMS**

Production in certain cereals, specially wheat due to the use of new seeds, manures and chemical fertilizer and use of machinery.

- **Agriculture:** The art and science of a cultivating soil, raising crops and rearing live stock including fishing and forest.
- **Commercial agriculture:** farming in which farmer grow the crop with the aim of selling it. In the market,:
- **Dry farming:** Dry farming is adopted. To such types of crops are grown which require less irrigation facilities.
- **Extensive agriculture:** agriculture in which the farmer greatest output by tries to get the greatest output by bringing more and more new land areas under cultivation.
- **Green revolution:** A term coined in late 1960's. recent development in agriculture in our country which has led to considerable increase in agriculture.
- **Horticulture:** intensive cultivation of vegetables, fruits and flowers.
- **Intensive Agriculture:** increase in the agriculture product5ion by using scientific methods and better agricultural inputs.
- **Kharif season:** It is an agricultural cropping season from early June to October.
  e.g.: Rice, millets act.
- **Plantation Agriculture:** A large scale farming of one crop resembling factor production based on capital investment and application of modem science and technology in cultivating, processing and marketing and final products.
- **Rabi season:** It is an agricultural cropping season from November to May e.g. wheat, gram oilseeds etc.
- **Minimum support Price:** It is the minimum reasonable price fixed by the government at which the farmer can sell his product either in the open market or to the government agencies.
- **Shifting Agriculture:** It is that type of agriculture in which farmers clear forest land and use it for cropping when the fertility of the soil decreases the farmer shifts to new land.
- **Subsistence Agriculture:** Farming in which the main production is consumed by the farmers house hold.
- **Blue Revolution:** A package programmed introduced to increase the production of fish and fish product.
- **Sericulture:** Rearing of silk worms to produce raw silk.
- **Zaid:** It is short season summer crop where fruits like watermelon and vegetables like cucumber are grown

◙ **AGRICULTURE**

The term agriculture is derived from two Latin words, anger meaning land and culture meaning cultivation modern day's agriculture also includes animal husbandry, Forestry and Pisciculture.

Definition:

The art and science of cultivating soil, raising crops and rearing livestock including fishing and forest is called as Agriculture.

Important of Agriculture:

1. Two thirds of the population is dependent on agriculture.
2. It generates large scale employment.
3. It provides raw material to many agro based industries.
4. Export of agricultural products earns valuable foreign exchange. It contributes 26% of GDP.

◙ **TYPES OF AGRICULTURE IN INDIA**

- **Primitive subsistence Farming or Shifting Agriculture**

(a) This agriculture is practiced on small patches of land with the help of primitive tools like hoe, and digging stick and family/community labor.
(b) Farming depends upon monsoons, natural fertilely of the soil and suitability of the environmental conditions.

(c) It is a slash and bum agriculture, Farmed shifts and clear a fresh patch of land cultivation.

(d) When the soil fertility decreases, the farmers shift and clear a fresh patch of land cultivation.

(e) Nature replenishes the fertility of the soil through natural process.

(f) Farmers do not use manure, fertilizer or other modern inputs.

(g) It is known by different names in different parts of the country.

(1) Jhumming –Assam, Meghalaya, Mizoram, Nagaland.

(ii) Pamlou- Manipur

(iii) Dipa: Bastar (Chhattisgarh) and Andaman and Nicobar Island.

- **Intensive Substance Farming**

(a) Fields are very small.

(b) There is intensive use of land due to high pressure of population on the agriculture land

(c) Cropping pattern is dominated by food crops.

(d) More than one crop is grown in the same field.

(e) Farmers apply modem inputs to obtain high yield.

(f) It is labor intensive farming.

- **Commercial Farming**

(a) Use of higher dose of modem inputs e.g. .HYV seeds, chemical fertilizers, insecticides and pesticides.

(b) The degree of commercialization of agriculture varies from to. e.g. Rice is commercial crop in Haryana and Punjab but in Orissa , it is subsistence

- **Plantation Farming**

1. It is also a type of commercial farming.

2. A single crop is grown on a large area.

3. It has an interface of agriculture and industry.

4. It is done over large tracts of land using capital intensive inputs.

5. All the products are used as raw material in respective industries.

6. The production is mainly for market.

7. A well developed network of transport and communication connecting the plantation areas, processing industry and market is important.

8. Examples of plantation crops are tea, coffee, rubber, sugarcane, banana etc.

- **AGRICULTURE SEASONS**
- **Rabi crops**

(a) Rabi crops are sown in winter from October to December.

(b) Harvesting in summer from April to June.

(c) Example of Rabi crops are wheat, barley, pea's gram and mustard etc.

(d) Region: Punjab, Haryana, Himachal Pradesh, Jammu Kashmir, Uttar Pradesh and Uttaranchal.

Factor responsible for growth of rabbi crops:

1. Availability of precipitation due to western disturbances.

2. Fertile alluvial traits deposited due to western disturbances.

3. Success of green revolution.

- **Kharif crops**

(a) The crops which are sown with the onset of monsoon and harvesting in September- October.

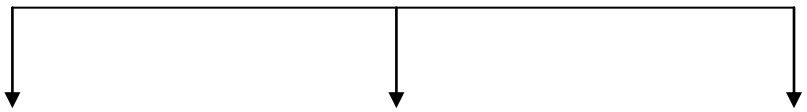
(b) Rice, maize, Jowar, bajra, moong, cotton, jute groundnut are some of the important Kharif crops.

(c) Important rice growing regions are Assam, west Bengal, Costal region of Orissa, Andhra Pradesh, Tamil Nadu, Kerala and Maharashtra.

(d) In states like Assam, West Bengal and Orissa three crops of paddy grown in the year. These are Aus, Aman and Boro.

- **Zaid**

1. The crops which are grown in between rabbi and the Kharif crops are known as Zaid crops.
2. Watermelon, cucumber, vegetables and the Kharif crops are known as Zaid crops.



Food crops	Cash Crops	Plantation crops
(Rice, wheat millets)	(Cotton, Jute etc.)	(Coffee, tea etc)

◙ **FOOD CROPS OF INDIA**

- **Rice**

Feature

1. It is the staple food crop of the people living in eastern and southern par of India.
2. In the north it is a Kharif crop and in the south it grows with the Help of irrigation throughout the year.
3. India is the second largest producer of rice in the world after Chin.

Geographical Requirements

Temperature: Above 25^0C (Sowing, growing and harvesting)

Rainfall

1. Annual rainfall above 100 cm.
2. The field must be flooded with water at the time of transplantation and during early stage of growth.
3. Frequent showers before ripening ensure larger grain size.

Soil: it can grow in variety of soil including stills, loam and But it grow best in alluvial soil.

Area of production: West Bengal, Andhra Pradesh, Uttar Pradesh, Bihar, Punjab, Orissa, and Karnataka.

- **Wheat**

Features

1. Second most important cereal crop.
2. Main food in north and north-western part of the country.
3. India is the fourth largest producer of wheat in the world.
4. It is a Rabi crop. (Winter crop).
5. It grows well in a coal and moist climate, fertile soil moderate rain fall.

Geographical Requirements

Temperature: 24^0C monthly.

1. 10^0C to 15^0C during growing season.
2. 25^0C to 28^0C at the time of ripening.

Rainfall

1. 50 cm to 75cm
2. The western disturbances cause light rainfall is highly beneficial to the wheat crop.

Soil

1. Well drained fertile soil, heavy textured soil with some amount of lime.
2. Clayey, loamy soil of the Ganga plain.
3. Black soil of Deccan Plateau.

Area of Production: Uttar Pradesh, Punjab and Haryana are major producer of wheat. Rajasthan, Madhya Pradesh, Gujarat and Maharashtra etc

- **Millets**

Features

1. Jowar, bajra and ragi are important millets grown in India.
2. It is the common name for several species of the grass family.
3. It is known as coarse grain.
4. They have very high nutritional value.

Jowar

(a) It is third important food crop of India.
(b) It is a rain fed crop which can be grown in the arid areas.
(c) It needs less irrigation.
(d) Maharashtra, Karnataka, Andhra Pradesh are the leading producer of Jowar.

Bajra

(a) It is a dry crop.
(b) It grows well in sandy and shallow black soil.
(c) Rajasthan is the largest producer of Bajra.
(d) Uttar Pradesh, Maharashtra, Gujarat and Haryana are other state.

Ragi

(a) It is a crop of dry regions.
(b) It grows well on red, black, sandy, loamy and shallow black soil.
(c) It is very rich in iron, calcium and roughage.
(d) Karnataka is the largest producer of ragi followed by Tamil Nadu, Himachal Pradesh, Uttaranchal, Sikkim, Jharkhand and Arunachal Pradesh.

Maize:

1. It is a coarse grain.
2. It is used both as food and fodder crop. It is a Kharif crop.
3. Temperature: 21^0C to 27^0C.
4. Soil: old alluvial soil. Major producing state: Karnataka, U.P., Bihar, Andhra Pradesh, M.P.
5. In Bihar it is grown in rabbi-season.

- **Pulses**

Features

1. India is the largest producer as well as the consumer of pulses in the world
2. Major source of protein in it vegetarian diet.
3. Being leguminous crops, all these crops except Arhar help in restoring soil fertility by fixing nitrogen from the air.
4. Tuar, Urad and Moong are grown as Kharif crops.
5. Masur, peas and gram are grown as Rabi crops.

Geographical Requirements

Temperature: 20^0C to 30^0C

Soil:

(i) Grown in all types of soil.
(ii) Dry soil is most suitable.

Rainfall

1. Low to moderate rainfall
2. 25 cm to 50 cm
3. Too much rain fall after sowing and during flowering is damaging.
4. **Area of Production:** Punjab, Haryana, U.P., M.P., Rajasthan, Maharashtra and Bihar.

◙ **FOOD CROPS OTHER THAN GRAINS**

- **Sugarcane**

Features

1. It is a tropical as well as sub tropical crop.
2. It belongs to the grass family.
3. India is the second largest producer of sugarcane only after Brazil,
4. It is the main source of sugar, gur, khandsari and molasses.

Geographical Requirements

Temperature

1. Hot and humid climate
2. 21^0c to 27^0c
3. Coal temperature is needed at the time of ripening.

Rainfall

1. 75 cm to 100 cm
2. 100 heavy rain fall results in low sugar content.

Soil

1. It can grow variety of soils like black alluvial, loamy and reddish loam.
2. The best soil is alluvial soil of the Ganga plain and black soil of Southern India.

Areas of Production:

1. Uttar Pradesh is largest producer of sugar cane. Bihar, Punjab and Haryana are other state.
2. Maharashtra, Karnataka, Tamil Nadu and Andhra Pradesh are the state of Peninsular India.

- **Oil seeds**

Features

1. India is the largest producer of oil seeds in the world.
2. Different oilseeds are grown crowing approximately 12% of the total cropped area of the country.
3. Main oil seeds are ground nut, mustard, and coconut, sesame (till), soybean, castor seeds, cotton seeds linseeds and sunflower.
4. Groundnut is a Kharif crop and is produced in Andhra Pradesh, Tamil Nadu, Karnataka, Gujarat and Maharashtra.
5. Linseed and Mustard are rabbi crops.
6. Sesame is a Kharif crop in north and rabbi crop in south India.
7. Costar seed is grown birth as Rabi& Kharif crop.

◙ **ECONOMIMIC IMPORTANCE OF SEEDS**

1. Most of these are edible and as cooking medium.
2. Extracted oil is also used as raw material for manufacturing large number of items
3. Oil cake which is the by-product obtained after the extraction of oil from oil seeds is excellent cattle feed.
4. Oil cake is also used as fertilizer.

◙ **BEVARAGES-CROPS**

- **Tea**

Features

1. it is an important beverage crop introduced in India initially by the British.
2. Tea is a labor intensive industry.
3. It requires abundant, “cheap and skills labor”.
4. Tea is processed within the tea garden to restore its freshness.
5. India is the leading producer as well as exporter of tea in the word.

Geographical Requirements

Temperature

1. Tropical and sub tropical climate.
2. Warm and moist frost free climate.
3. 20^0c to 30^0c but 25^oc is ideal temperature

Rainfall

1. Heavy rainfall.
2. 150 cm-250 cm
3. The rainfall should be distributed thought the year.

Soil

1. Light loamy soil.
2. Rich in humus and iron content.
3. Frequent use of chemical fertilizers and manure is essential.

Area of production

Assam, hills of Darjeeling and Jalpaiguri, west Bengal, Tamil Nadu, Kerala, Himachal Pradesh, Uttaranchal Meghalaya, Andhra Pradesh and Tripura.

- **Coffee**

Features

1. Indian coffee is known in the world for its good quantity.
2. Arabica quality of coffee is grown in India.
3. It is one the oldest plantation crop in India.

Geographical Requirements

Temperature

1. Monthly temperature from 14^0c-26^0c.
2. Bright sunlight and warm weather.
3. Frost free climate.

Rainfall

1. 125cm-250cm
2. It should be well distributed thought the year.
3. Stagnant water is harmful.

Soil

1. It grows be well distributed thought the year.
2. In India coffee is grown on red and literate.

Area of production

Nilgiri is the area of production in Karnataka, Kerala and Tamil Nadu.

- **HORTICULTURE CROPS**

1. India is largest producer of fruits and vegetables in the world.

2. India is the producer of tropical as well as temperate fruits
3. Mangoes are found in Andhra Pradesh, West Bengal and V.P.
4. Oranges in Nagpur and Cherrapunji 9meghalaya).
5. Bananas in Kerala, Mizoram, Tamil Nadu and Maharashtra.
6. Litchi and Guava in V.P. and Bihar.
7. Pineapples in Meghalaya.
8. Grapes in Andhra Pradesh and Maharashtra.
9. Apples, pears, Apricots and walnuts in Jammu& Kashmir and Himachal Pradesh.
10. India produces 13% of the world's vegetable. It is an important producer of pea, onion, cabbage and cauliflower.

◙ **NON-FOOD CROPS**

- **Rubber**

Features

1. It is an equatorial crop.
2. It is also grown in tropical or sub tropical areas.
3. It is a kind of natural plastic with many invaluable qualities such as elasticity, non conduction of electricity.
4. It is an important industrial raw material.
5. India ranks fifth among the world's natural rubber production.

Geographical Requirements

Temperature: Above 25^0c

Rainfall

1. It needs well and heavy distributed rainfall.
2. 200 cm-400 cm.

Soil

Area of Production

1. Kerala is the largest production of rubber.
2. Kerala account for about 91% of total area under rubber plantation.
3. Tamil Nadu, Karnataka and Adman and Nicobar Island and Agro Hills of Himalayas are the other producers.

- **Fiber crops**

Features

1. Four major crops of India are
   1. Cotton 2.Jute 3. Hemp 4. Natural silk
2. Cotton, jute and Hemp are derived from the crops grown in soil.
3. Natural silk is obtained from cocoons of the silk worms, fed on green leaves especially mulberry.
4. Rearing of silk worm for the production of silk is known as sericulture.

- **Cotton**

Features

1. It is the most important fiber crop of India.
2. It is the one of the basic raw material-s for the coupon textile industry.
3. India is the third largest producer of cotton in the world after china, U.S.A. and Russia.
4. It is the crop of tropical and sub-tropical areas.
5. It is cultivated as a Kharif crop and requires 6 to 8 months to mature.

Geographical Requirements:

Temperature

1. Needs warm climate.
2. 21^0c-27^0c
3. Abundant sun shine is necessary during the growth of the plant.
4. 210 frost free days.

Rainfall

1. It requires light rainfall.
2. 50 cm to 80 cm is adequate.
3. With the help of irrigation crop can be grow.

Soil

1. It grows well in Black cotton soil which is found in drier Plots of the Decant Plateau.
2. It also grows well in alluvial soils of Sutlej Ganga Basin.

Area of Production: Gujarat, Maharashtra, Andhra Pradesh, Punjab, Haryana, Karnataka, Tamil Nadu and Madhya Pradesh. **Jute**

Features

1. It is known as the golden fibre.
2. It is a tropical fibre crop.
3. India earns valuable foreign exchange by exporting jute.
4. Jute is used in making gunny bags, nuts, ropes, yam, carpet and other ornamental artifacts.
5. Due to high cost, it is losing to synthetic fibers, and packing materials, particularly Nylon.

Geographical Requirements:

Temperature: high temperature is required during the time of growth..

Soil: Well drained fertile soil.

Area of production: West Bengal, Bihar, Assam Orissa and Mahayana are major jute producing state.

◙ **FACTORS RESPONSIBLE FOR LOW PRODUCTIVITY IN INDIA**

1. **Over Crowding in agriculture:** There are too many people who depend on agriculture. Since 1901 the proportion of people dependent on agriculture has almost remained constant i.e. 70 %.
2. **Problems of Inputs:** Indian agriculture has suffered because of the inadequacy of finance, seeds, fertilizers, marketing and transportation etc.
3. **Size of land Holding:** The average size of holding in India is very low, less than 2 Hectors or 5 acres.
4. **Pattern of land Tenure:** Under the Zamindari system the cultivator was a tenant. Who could be turned out of the land. The cultivator has to pay high rents of the land he cultivates and he has no security of tenancy and may be tanned out of his landlords' desire.
5. **Poor Techniques of Production:** The Indian farmers have been using old and inefficient methods and techniques of production. There is also a shortage of various kinds of inputs HYV seeds. Fertilizer, pesticides etc.

◙ **TECHNICAL & INSTITUTIONAL REFORMS**

1. **Land Reforms:** In order to encourage agriculture government has abolished Zamindari system. Consolidation of fields has also been done. To help the landless workers government has granted them fields.
2. **Lunching of irrigation schemes:** Many multipurpose projects have been constructed to provide power and irrigation facilities to the farmer.
3. **Subsidies:** Government is providing subsidies on the agriculture inputs .At present maximum subsidy is being given on fertilizers.
4. **Free Power:** certain states are providing free electricity to the farmers to enhance the agriculture production.

5. **Public procurements System & Agriculture price commission:** In this system government declares the price of agriculture product in advance to check the exploitation of farmers by middle men.
6. **HYV Seeds & Agriculture Finance:** Government is providing HYV seeds to the farmers' .Special seminars are of also being held. Many new Agriculture Universities have been set up.
7. **Crops Insurances & Agriculture Finance:** Indian agriculture is mainly dependent on nature; crops are also being insured due to risk. Government has established special bank to provide agriculture loans to farmers.

◙ **CONTRIBUTION OF AGRICULTURE TO THE NATIONAL ECONOMY, EMPLOYMENT AND OUTPUT**

Agriculture has been the back bone of Indian economy thought its share in the GDP is declining. Yet its share is providing employment and livelihood continuous to be as high as 63% in 2011.

Steps taken by Indian government to modernize agriculture efforts:

1. Establishment of Indian council of Agriculture. Research (ICAR).
2. Agriculture Universities.
3. Veterinary services and animal breeding centers.
4. Horticulture development.
5. Research, Weather forecast.
6. Improvement of the rural infrastructure.

Draw back of Indian government

1. Government is reducing investment in agriculture sector mainly in irrigation, power, rural roads, market an mechanization.
2. Subsidy on fertilizers has decreased.
3. Reduction in import duties on agriculture products have proved harmful to agriculture in the country.

EXERCISE

◙ **VERY SHORT ANSWER TYPE QUESTION**

1. Which is the most important cereal crop of India? Name the two major producing state of this crop.
2. Which are the two major fibre crops of India?
3. Write two main features of dry land farming?
4. Write two main features of wet land farming?
5. What are Kharif crops? Give two examples.
6. What are Rabi crops? Give two examples
7. What is horticulture?
8. What is sericulture?
9. Name the leading rubber producing states in India.
10. Which crop is called as 'Golden Fibre' of India?
11. What is Agriculture?
12. Name any four agriculture products exported by India.
13. What is intensive subsistence farming? Mention any four plantation crops produced in India.
14. What is the period of Kharif crop?
15. What is the period of Rabi crop?
16. Name the two most important wheat producing state of India?
17. Why should the production of pulses be increased?
18. Name some major oilseeds of India.
19. "High humidity" is good for the cultivation of tea why?
20. Why is shifting agriculture discouraged?

21. Name two cotton producing states of India?
22. Which region is ideal for the cultivation of cotton?
23. Name some three cash craps?
24. Give two features of the green Revolution?

◙ **SHORT ANSWER TYPE QUESTION**

1. Describe any steps taken by the government to improve Indian agriculture after independence.
2. Distinguish between dry land farming and wet land farming?
3. Describe favorable climatic condition for growing wheat and maize crops in India?
4. What are millets? Why are mallets very important food crops in India?
5. Describe any four features of Indian agriculture.
6. Agriculture is the main stay of India economy. Discuss.
7. What is slashing and burn agriculture? Give four silent features of this agriculture.
8. Mention any four features of the primitive subsistence farming
9. Name any four factors the have distorted the cropping pattern in India
10. What is importance of rubber for the Indian economy?
11. Write two differences between intensive an extensive farming.
12. Where is tea grown in India? Describe the suitable condition for its growth.
13. Differentiate between commercial agriculture and subsistence agriculture.
14. Which are the two steeple food crops of India? Compare and contrast the climate and soil requirements of the two.
15. What suggest you that Indian agriculture has transferring from subsistence to commercial farming?

◙ **LONG ANSWER TYPE QUESTIONS**

1. Mention any four steps which have been taken by the government to modems the Indian agriculture.
2. Explain any four factors which have hindered the pace of agriculture development in India?
3. How is the government helping the Indian farmers in increasing their agricultural production? Explained any four points.
4. What is plantation agriculture? Write some features of type of farming?
5. Discuss some new trends in Indian agriculture?
6. Why has Indian agriculture started a deckling in food production? How can we overcome this problem?
7. How have the technological and institutional reforms been to improve the condition of Indian agriculture?

MANUFACTURING INDUSTRIES

◙ **IMPORTANT TERMS**

1. **Agro based Industries:** The industry which converts the agricultural products into industrial products.
2. Basic Industry: It is heavy industry which is fundamental to other industries e.g. iron & steel.
3. **Co-operative Industries:** These industries are organized by a group of people who are also the produce, of raw material and help in running industries by Co-operating each other.
4. **Consumer Industries:** This industry provides goods primarily for the consumption of people.
5. **Cottage Industry:** These industries are the ones which are practiced on a small scale in homes villages.
6. **Heavy Industries:** These industries use heavy raw material and manufacture of finished products. E.g. Iron steel industries.
7. **Industry:** Industry in the enhancement of the value of the raw material through the manufacturing process of factory.
8. **Joint sector Industries:** These industries are owned by both the state and some private industrialists or firms.
9. **Light Industries:** These industries use light raw material and make light finished product in weight. E.g. electronics and fans.

10. **Large scale Industry:** The industries employing large number of persons in each unit and having large production level e.g. jute textile.
11. **Mineral based Industries:** the industry in which mineral products are processed and turned into finished goods.
12. **Public Sector Industries:** These industries are owned by the central or the state government. E.g., BHEL and HEC.
13. **Primary Industry:** It is an industry producing raw material.
14. **Private sector Industries:** These industries are owned and controlled by individuals, firm or component.
15. **Secondary Industry:** These industries manufacture goods by using processed material of primary industry.
16. **Small scale Industry:** The industry employing small number of persons and invest capital of about 1 crore e.g. Readymade garments.

◙ **MANFACTURING**

The conversion of primary products into more refined and useable form is known as manufacturing.

◙ **IMPORTANCE OF INDUSTRIAL DEVELOPMENT**

Industrialization play a vital role in the economic development of a country.

▸▸ **Utilization of Natural Resources:**

Utilization of huge volume of natural resources has become possible with the development of industries in the country.

▸▸ **Balanced Sectional Development:**

Growth of industrialization in the country can attain balanced sectional development and it can reduce the too much dependence of the economy on the agricultural sector.

▸▸ **Enhanced Capital Formation:**

Increasing volume of investment in industries has led to enhancement in the rate of capital formation in the country.

▸▸ **Increase in National Income& Foreign Exchange:**

Organized and unorganized industries are jointly contributing ‘a good portion of the total national income of the country.

▸▸ **Increase in Job Opportunities:**

In increase the job opportunities for a large section of the population of the country.

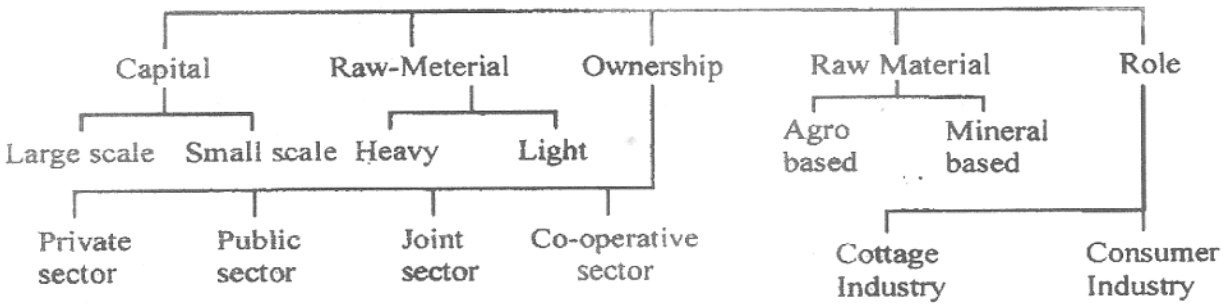
- **Contribution of Agriculture to Industry:**

1. Agriculture provides raw material to industry such as Jute, Cotton etc.
2. It also acts as a source of capital formation which can be utilized in industry.
3. It provides food to the industrial worker.
4. It provides good market to the industrial products.
5. It also decreases pressure on industry.

- **Contribution of Industry to Agriculture:**

1. Industry provides inputs to the agriculture such as fertilizers, pesticides.
2. It provides infrastructural facilities to the industry.
3. It reduces pressure on agriculture.
4. Industry provide processed product to the agriculture.
5. It also increases the market value of agriculture product.

◙ **CLASSIFICATION OF THE INDUSTRY ON THE BASIS OF**



ON THE BASIS OF CAPITAL INVESTMENT

- **Large scale Industry:**

1. Employ a large number of laborers.
2. Huge investment i.e. more than crore is involved in large scale industry.

- **Small Scale Industry:**

1. Industries which are owned by individuals.
2. Employ a small number of laborers.
3. Small investment i.e. less than 1 crore Rs. Are invested.

◙ **ON THE BASIS OF RAW MATERIAL**

- **Heavy Industries:**

Industries which use heavy and bulky raw materials and produce products which are heavy and bulky are called heavy industry. E.g. Iron & steel industry

- **Light industries:**

The light industry uses light raw material and produce light finished product. E.g., Electric fans, Sewing machines

◙ **ON THE BASIS OF OWNERSHIP**

1. Private sector Industries: Industries which owned by individual or firms such as Bajaj, TISCO are called private
2. Public sector In
3. Joint sector Industries: Industries owned jointly by the state and private firms such as Gujarat Alkalis Ltd. Or Oil India Ltd.
4. Co- Operative sector Industries: Industries owned and run Co-operatively by a group of people who are generally producers of raw materials of the given industry. E.g. sugar mill.

◙ **ON THE 'BASIS OF SOURCE OF RAW MATERIAL**

- **Agro Based industries:**

Those industries which obtain raw materials from agriculture is called as Agro based Industries. E.g. Cotton textile, Jute textile, sugar etc

- **Mineral Based Industries;**

The industry that receives raw material from minerals is called as Mineral based Industries. E.g. Iron and steel industry, Aluminium and cement industry.

◙ **ACCORDING TO THEIR ROLE**

- **Cottage Industry:**

Industries which artisans setup in their own houses, work with wood, cane, brass, stone are called cottage industries e.g. Handloom, Khadi and leather.

- **Consumer Industry :**

Consumer industries convert raw material or primary products into commodities which are directly used by the people. E.g. cotton textile, sugar industry, vegetable oil etc.

◙ **FACTORS IN THE LOCATION OF INDUSTRIES**

1. **Availability of Raw Material:**

It is the major factor affecting the location of the industry. Agro based industry will be located in agriculture dominating areas where as mineral based –industry will be located in the mineral dominating areas.

2. **Power:**

Power can be transmitted but those industries which consume large quantities of power are located near the source of power.

3. Labor: Labor intensive industries mostly concentrate in densely populated areas.

4. **Transport:** Transport system helps in the movement of goods and raw material.

5. **Market:** Nearness to market is essential for quick disposal of manufactured goods and for purchasing raw material. It reduces the cost of transportation.

6. **Government Policies:**

Government activity in planning the future distribution of industries, for reducing regional disparities, elimination of pollution of air and water and for avoiding their heavy clustering in big cities has become an important location factor.

◙ **AGRO BASED INDUSTRIES**

- **Textile Industry:**

1. It adds about 14 % to the industrial production and about 4% to the GDP.
2. It provides employment to about 35 million people. Together with allied agriculture sector, it provides employment to over 90 mil join people.
3. It is the only industry which is self-reliant from raw material to the highest value added product.

- **Cotton Textiles:**

1. In ancient India, cotton textiles were produced with hand spinning and hand loom weaving techniques.
2. Today there are nearly 1600 cotton and human made fibre textile mills in the country. There are several thousand small factories with 4 to 10 looms.

◙ **FACTORS RESPONSIBLE FOR THE HIGH CONCENTRATION OF COTTON MILLS IN MAHARASTRA & GUJARAT.**

1. **Climate:** Humid climate is essential for this industry because thread does not break so frequently.
2. Transportation: This help in the import of machinery and long staple cotton.
3. Power: Cheap hydroelectricity is readily available from the nearby areas.

4. Raw material: The black cotton soil provided cotton as the basic raw material.
5. Cheap Labor: They have high density of population so both skilled and unskilled labor is available in large number
6. Market: There is ready market for Mumbai Product- both in India and abroad.
7. Capital: Mumbai is a great commercial and financial centre of India. So capital which is required is easily available.

◙ **PROBLEMS OF INDIAN COTTON TEXTILE INDUSTRY**

1. **Problems of Raw Material:**

The industry faces the problems of building up a regular supply of its material.

2. **Problem of Power:**

Frequent power cuts and load shedding had affected the industry badly.

3. **Obsolete machinery and need for modernization:**

The mill sector has been working with obsolete machinery. The problem of replacement of obsolete machinery and modernization has become really acute.

4. **High cost and competition in for-ign market**

It has been facing increasing competition in world market. This is largely due to low productivity and high cost and high prices of Indian cotton textile.

◙ **JUTE INDUSTRY**

- **Importance of the Industry**

1. It is a labor intensive industry so it provides employment to the people.
2. Jute products are the major items of exports.
3. India is the second largest exporter of jute products.
4. It accounts for more than 20% of the total export earnings.

- **Factors responsible for their location in Hugli basin are**

1. Proximity of the Jute producing areas.
2. Inexpensive water transport.
3. Supported by a good network of Railways, Road ways and water ways to facilitate movement of raw material to the mills.
4. Abundant water for processing raw jute.
5. Cheap labor is available there.
6. Kolkata as a large urban centre provides banking, insurance and port facilitate for export of Jute goods.

- **Problems of the India Jute Industry:**

1. Problem of raw material.
2. International competition from synthetic substitutes and other competitions like Bangladesh, Brazil, Egypt, Thailand.
3. Less Demand
4. High Prices.

- **Steps taken by Indian government. To solve the problems:**

1. Internal demand has been increased due to. The government policy.
2. In 2005 National Jute policy was formulated with the objective of increasing productivity, improving quality, ensuring good prices to the jute farmers and enhancing the yield per hectare.

- **Market:**

1. The main markets are U.S.A., Canada, Russia, U.K. United Arab Republic and Australia.
2. The growing global concern for environment friendly, biodegradable material has once again opened the opportunity for jute products.

◙ **SUGAR INDUSTRY**

- **Importance of the sugar Industry**

1. India stands second as a world producer of sugar but occupies the. First place in the production of gur and khandsari.
2. It ranks as the third largest industry in terms of its contribution to the net value added by manufacture.
3. It is also an important source of excise duty for the central government.

- **Distribution:**

The sugar industry is established in areas of sugar cultivation because:

1. Its raw material is heavy and perishable.
2. Sugar, cane cannot be stored for long as the loss of sucrose is inevitable.
3. It cannot be transported over long distances because it may dry on the way.

◙ **DIFFERENCE BETWEEN THE SUGAR INDUSTRY' Q. FNORTHERN AND PENINSULAR INDIA**

1. **Climate:** It is a tropical crop Peninsular India has tropical climate which gives higher yield per unit area as compared to north India.
2. **Higher sucrose:** The sucrose content is also higher in tropical variety of sugarcane in the south.
3. **Long Crushing period:** It is also much longer in the south than in the north.
4. **Better management:** The co-operative sugar mills are better managed in south than in the North.

- **Problems of sugar Industry**

1. Low yield of sugarcane
2. Short crushing season.
3. High cost of production.
4. Old and obsolete machinery.
5. Under utilization of by products.

◙ **MINERALS BASED INDUSTRY**

- **Minerals based Industries**

Those industries which use minerals as the basic raw material!.

- **Iron and Steel Industry:**

1. It is the basic industry since the entire industries-heavy, medium and light depends on it for their machinery.

2. Steel is needed to manufacture a variety of engineering goods, construction material, defense material, telephonic, scientific equipment and variety of consumer goods.
3. It is a heavy industry because all the raw material-as well as finished goods is heavy and bulky entailing heavy transport ion cost.
4. Iron ore, cooking coal and lime stone are required in the ratio of approximately 4:2:1. Some quantities of manganese are also required to harden the steel.
5. India ranks ninth among the world crude. Ste. El. Producer. It is the largest producer of sponge iron.
6. There are 10 primary integrated and many steel plants in India.
7. All Public sector undertakings market their steel through, Steel Authority of India Ltd. (SAIL) while TISCO markets its produce through Tata Steel.
8. Chota Nagpur plateau region has the maximum concentration of iron and steel Industries because

(A) Low cost of iron ore
(B) High grade raw material in proximity
(C) Cheap labor and vast growth potential in the home market.

- **Problems of Iron and Steel Industry**

1. Shortage of raw material.
2. Shortage of power.
3. Lower productivity of Labor.
4. Poor infrastructure.
5. Lower investment in Research and Development.

- **To improve the production government of India had taken following steps:**

1. Government has adopted a new economic policy of Liberalization, privatization and globalization.
2. Government is promoting Foreign Direct Investment in the sector.
3. Government has launched Duty Exemption Scheme.

◙ **ALUMINIUM SMELTING**

1. It is the second most important metallurgical industry in India.
2. It is light, resistance to corrosion, a good conductor of heat, malleable and becomes strong when it is mixed with other metals.
3. It has gained popularity as a substitute of – steel, copper zinc and lead.
4. There are 8 aluminum smelting plants in the country located in Orissa, West Bengal, Kerala, Uttar Pradesh, Chhattisgarh, Maharashtra and Tamil Nadu.
5. Bauxite, the raw material used in smelters in a very bulky, dark reddish colored rock.

- **Prime factors of Location of the Industry**

1. Regular supply of electricity at minimum cost.
2. Assured source of raw material.

◙ CHEMICALS INDUSTRY

The industry has two components.

1. Inorganic chemicals.
2. Organic chemicals.

- **Inorganic chemicals**

It includes Sulphuric acid (used to manufacture fertilizers, synthetic fibres, plastics, adhesives paints, dyes stuffs), Nitric acid alkalis, soda ash (used to make glass, soap and detergents, paper) and caustic soda.

- **Organic chemicals:**

It includes petrochemicals, which are used for manufacturing of synthetic fibres, synthetic rubber, plastics, and dyestuffs.

◙ IMPORTANCE OF CHEMICAL INDUSTRY

1. **Employment:** It is major source of employment for large number of people.
2. **Foreign exchange:** Export of chemicals and chemical products brings foreign exchange to India.
3. **Reduction of pressure on Land:** It reduces pressure on land by providing employment to workers.
4. **Development of Agriculture:** It supplies pesticides and weedicides to agriculture to control harmful insects and weds.
5. **Contribution to G.D.P. and national Income:** It contributes 3% of the G.D.P. It also contributes 20% of the excise duty to the government.

◙ FERTILISER INDUSTRY

1. The fertilizer industry is centered on the production of nitrogenous fertilizers phosphatic fertilizers and ammonium phosphate and the complex of fertilizers which have a combination of nitrogen (N) and Potash (K).
2. Potash is entirely imported.
3. India is the third largest producer of nitrogenous fertilizers.
4. There are 57 fertilizer units manufacturing nitrogenous and complex nitrogenous fertilizers 29 for urea and 9 for producing ammonium sulphate as a by producing ammonium sulphate as a byproduct and 68 other small units produce single super phosphate.
5. There are 10 public sector undertakings and one is cooperative sector at Hazira in Gujarat under the fertilizer
6. Gujarat, Tamil Nadu, Uttar Pradesh, Punjab and Kerala contribute half the fertilizer production. Other is Andhra Pradesh, Orissa, Rajasthan, Bihar, Maharashtra, Assam west Bengal Goa, Delhi, Madhya Pradesh and Karnataka.

◙ CEMENT INDUSTRY

1. It is essential for all construction activities.
2. It earns valuable foreign exchange.
3. Indian cement export in Bangladesh, Indonesia, Malaysia, Nepal, Middle East and Africa.
4. It requires bulky and heavy raw materials like lime stone, silica, alumina and gypsum, coal and electric power.

5. The first cement plant was setup in Chennai in 1904. Decontrol of price and distribution since 1989 and other policy reforms let to the cement Industry to made rapid strides in capacity. Process, technology and production.

6. There are 128 large plants and 332 mini cement plants in the country.

7. Efforts are being made to generate adequate domestic demand and supply in order to sustain this industry.

◙ **AUTOMOBILE INDUSTRY**

1. Automobiles provide vehicle for quick transport of good services and passengers. E.g. Trucks, Bus, car etc.

2. After the liberalization, the coming in of new and contemporary models stimulated the demand for vehicles which led to a healthy growth.

3. Foreign direct investment brought in new technology. And aligned the industry with global development.

4. At present there are 15 manufactures of passenger cars and multi utility vehicles. 9 of commercial vehicles. 14 of two and three wheelers.

5. The industry is located around Delhi, Gurgaon, Mumbai, Pune, Chennai, Kolkata, Lucknow, Indore, Hyderabad, Jamshedpur and Bangalore.

◙ **INFORMATION TECHNOLOGY & ELECTRONIC INDUSTRY**

1. The electronic industry covers a wide range of products including T.V. sets, transistor sets, telephone exchange, cellular telecom, compute and many other types of equipment required by the telecommunication industry.

2. Bangalore has emerged as the electronic capital of India. Other are Mumbai, Delhi, Hyderabad, Pune Chennai, Kolkata, Lucknow and Coimbatore.

3. 18 software technology parks provide single window service and high data communication facility to software experts.

4. It has provided employment over million people.

5. This industry is major foreign exchange earner.

6. It has helped the growth of service centre.

◙ **INDUSTRIAL POLUTION AND ENVIRONMENTAL DEGRADATION**

Industries are responsible for four types of pollution

(a) Air, (b) Water, (c) Land, (d) Noise

- **Air pollution :**

1. It caused by presence of high proportion of undesirable gases such as sulphur dioxide and crbon monoxide.
2. The smoke emitted by the industries pollutes air and water very badly.
3. Toxic gas leaks can be very hazardous with long term effects.
4. It adversely affects human health, animals, plants and the atmosphere as whole.

- **Water Pollution :**

1. It is caused by organic and inorganic industrial wastes and effluents discharged into rivers. Paper, pulp, chemical, textile and dyeing, petroleum refineries, tanneries and electro plating industries let out dyes detergents acids. Salt, soap and heavy metals like lead and mercury etc into the water bodies.
2. Fly ash phospo-gypsum and iron and steel slogs are the major solid wastes in India.

- **Thermal:** It occurs when hot water from factories and thermal plant is drained into rivers and ponds before cooling.
- **Nuclear Pollution:**

  Wastes from nuclear power plants, nuclear weapon production causes cancer birth defects and miscarriages.
- **Noise Pollution:**

  Unwanted loud noise is also pollution. It can cause hearing impairment, increase h-art rate irritation and anger etc.

◙ CONTROL OF ENVIRONMENTAL DEGRADATION

1. Minimizing the use of water of processing by reusing and recycling it in two or more successive stages.
2. Harvesting of rainwater to meet water requirement.
3. Overdrawing of ground water reserves needs to be regulated legally.
4. Smoke can be reduced by using oil or gas instead, of coal in factories.
5. Machinery and equipment and generators should be fitted with silencers.
6. All machinery can be redesigned to increase energy efficiency and reduce noise.
7. Particulate matter in the air can be reduced by fitting smoke stake to factories with electrostatic precipitators, fabric filters, scrubbers and inertial separators.
8. Treating hot water and effluents before releasing them in rivers and ponds.

EXERCISE

◙ VERY SHORT ANSER TYPE QUESTIONS

1. What are the two major problem faced by the Jute industry in India?
2. Name any two buyers of our jute products.
3. How does industries causes water pollution?
4. How does industries pollution degraded the environment?
5. What is the importance of manufacturing?
6. What are agro based industries? Give two examples.
7. Mention the raw material of agro based industry.
8. List out the Problems faced by cotton textile industry.
9. Sugar mill are shifting towards southern and western states. Why?

10. Why is iron & steel industry called a basic industry?

11. Name the two factors that determine the location of aluminum industry.

12. Which states lead in the production of fertilizers?

13. Which city is known as electronic capital of India?

14. Why has the government set up the National Manufacturing competitiveness council?

15. What are agglomeration economies?

16. Name any four countries which import cotton goods from India.

17. What are software technology parks?

18. Name any two states which are the leading producer of fertilizers.

19. Mention the basic input of the iron and steel industry.

20. Name any four industries which are responsible for water pollution.

SHORT ANSWER TYPE QUESTIONS

1. Describe any four major problems faced by cotton textile industry in India.

2. Explain any four factors which are responsible for decentralization of cotton industry in India.

3. Explain any four measures to control air pollution created by industry in India.

4. Explain any four reasons for the rapid growth of the chemical industry in India.

5. Explain how industrial pollution degrades the environment.

6. Describe four physical factors that affect the location of an industry.

7. What factors have led to the localization of cotton textile industry in Mumbai and Ahmadabad?

8. State the various measures taken by the government to boost the production of jute goods.

9. Explain how industry and agriculture go hand in hand with each other.

10. Bring out the importance of Information technology and electronic industry in India.

11. Distinguish between private sector and public sector industries.

12. Explain four facts about the development of sugar industry in India.

13. What were the major objectives of the national Jute policy, 2005? Why the internal demand for jute has been on the increase.

14. "Treatment of industrial wastes can be done in three phases". Discuss.

◙ LONG ANSWER TYPE QUESTIONS

1. What is the importance of sugar industry? Why does sugar industry show the tendency to migrate towards the southern states of India? Give four reasons?

2. How has chemical industry gained an important position in the Indian economy? Explain any six points in this regards?

3. Why is jute mills concentrated along the Hugli River? Explain any six reasons.

4. Explain aluminum smelting and copper smelting industries with reference to the following for each of them.

(i) Two major uses.

(ii) Names of two smelting states.

(iii) Annual Production.

5. Attempt a classification of industries based on various factors.

6. Bring out the importance and development of cotton textile industry in India.

LIFE LINES OF NATIONAL ECONOMY

◙ IMPORTANT TERMS

1. **Balance of trade:** Relation between Nation's exports and imports.
2. **Balance of Payment:** It is a comprehensive record of economic transactions of the residents of a country with the rest of world during a given period of time.
3. **Exports:** When a nation sends its home manufactured products to outside countries these are called exports.
4. **Foreign exchange**: It is the exchange of currency of one country with the currency of another country.
5. **Favorable balance of Trade:** It is a situation in which value of export is more than imports.
6. **Harbor:** It is a deep cost of which provides shelters to the sea going vessels.
7. **Golden quadrilateral:** It is a network of roads which connects Delhi, Mumbai, Chennai and Kolkata.
8. **Imports**: When citizen of a country consume the products of other countries these are known as imports.
9. **International Trade**: Trade carried between various nations to exchange their goods.

10. **Mass communication**: The means of communication through which one can communicate with several people at the same time.
11. **National Highway:** These are broad roads connecting many states and capitals.
12. **Transport:** It is an act of carrying goods or passengers from one place to another.
13. **Unfavorable balance of trade:** It is a situation in which value of imports is more than exports.

◙ **IMPORTANCE OF MEANS OF TRANSPORT AND COMMUNICATION**

The world is developing very fast and means of communication play a vital role in the development of the world.

1. These help to transfer better technology from one country to other or from one place to other.
2. Means of communication help the traders to have links with other traders and means of transport provide essential product.
3. Means of communication act like nervous system, in the human body.
4. Means of transportation carry common people from one place to another.
5. They maintain the balance of demand and supply in the economy.
6. This plays a very vital role in the defense of a country.
7. They have shortened the distances between various places.
8.

◙ **ROADWAYS**

Roadways have an edge over railways due to following features.

1. Road require less investment as compared to railway.
2. These can be built at higher altitudes and at any place.
3. Maintenance cost is also low.
4. Road transportation has the merit of offering Personal services.
5. Extension of agriculture is possible only through road transportation.
6. It is easy and is within the reach of common man.

- **Problems of Indian roads:**

1. The road network is inadequate in India as compared to demand and volume of traffic.
2. The condition of most of the roads is very poor these become muddy during the rainy season.
3. They are highly congested in. Cities.
4. Most of the highways lack side amenities like telephone booth, police station etc.

▸▸ **Golden Quadrilateral Super Highways:**

The super highways have been planned to meet the requirement of fast movement of traffic. The government has launched' a major road development project linking Delhi-Kolkata-Chennai-Mumbai and Delhi by six lane super high ways. The North-South corridors linking Sri-Nagar (Jammu & Kashmir) and Kanyakumari (Tamil-Nadu), and East-West corridor connecting Silcher (Assam) and Porbander (Gujarat) are part of this project.

- **Major Objective:**

To reduce the time and distance between the mega cities and is "Implemented by- National Highway Authority of India".

▸▸ **National Highways:**

The Main road which are constructed and maintained by the central public works department (C.P.W.D.) are known as National Highways.

Main Features:

1. The National Highways maintain the availability of essential product.
2. The state of state trade is possible because of National Highways.
3. All the essential products are transported through these roads.

▸▸ **State Highways:**

1. These care constructed and maintained by the state government.
2. State highways join the state capital with district head quarters and other important towns.
3. The total length of state highways in India is 4 lakh km approx.
4. These also contribute in the development of states.

▸▸ **District Road:**

1. These roads link the district centres with the major roads.
2. Their total length is nearly 6 lakh km.
3. These roads are maintained by Zilla Parishad.

▸▸ **Village Roads:**

1. These connect the village with the neighboring towns and cities.
2. These play very vital role in the development of villages.
3. These help the farmers to take their products to cities and district head quarters.

▸▸ **Border Roads:**

The Border roads organization was established in 1960 for the development of roads of strategic importance in the northern and north eastern border areas.

◙ **Railways**

- **Major Problems of Indian Railways**

1. Most of the roads are very old.
2. Many passengers travel without tickets.
3. People pull chains unnecessarily which cause heavy loss to the railways.
4. Frequent accidents also cause great loss to the railway.
5. Airways are also giving it a tough competition.

- **Main factors which influence the Railways Network**

1. Physiographic 2. Economic and Administrative

- **Physiographic Factors:**

1. The level lands of the Great Plains have favored the development of Railways.

2. The flood plain of Bihar and Assam and the rugged topography of the Himalaya region have let to very few Railways lines.
3. The sandy desert of Rajasthan and hilly tracts of Sahayadri are unfavorable for the development? Railways.

- **Economic Factors:**

1. Rich agriculture and greater industrial activity on the Great Plains have favored the development of Railways.
2. Capital of India New Delhi is also located in Northern plain.

- **Importance of Railways:**

1. It carries most of the long distance passengers' traffic and 80 % of the total freight of the country.
2. It provides employment to a large number of people in the country.
3. Iron and steel industry solely depends upon the Railways.
4. It plays a significant role in the integration of the country by bringing different people and regions together.
5. They carry big chunks of goods from the place of Production to the part and vice versa.

◙ **PIPELINES**

- **Advantages of pipeline transportation:**

1. Transportation through pipe lines rules out delay and transportation.
2. Though initial cost of laying pipelines is high but running cost is very low.
3. It maintains continuous supply of gas and oil.
4. The pipes can be laid through difficult terrains as well as under sea.

- **Three Important network of pipeline:**

1. From oil field in upper Assam to Kanpur (U.P.) Via Guwahati, Barauni and Allahabad. It has branches from Barauni to Haldia, via Rajbandh, Rajbandh to Maurigram and Guawahati to Siliguri.
2. From Salaya in Gujarat to Jalandhar in Punjab via Viramgam, Mathura, Delhi and Sonipat. It has branches 'to connect Koyali.
3. Gas pipeline from Hazira in Gujarat connect Jagdishpur in Uttar Pradesh via Bijapur in Madhya Pradesh. It has branches to Kota in Rajasthan, Shahajahanpur, Sabrala and other places in Uttar Pradesh.

◙ **WATER WAYS**

- **Importance of Water ways**

1. It is most suitable for carrying heavy and bulky goods."
2. It is a fuel efficient and environment friendly mode of transport.
3. The element of wear and tear is very less.
4. More than 95% of the country's trade volume "is moved by the sea.
5. With the development of National waterways it has become main source of transportation for the natural trade.

- **Inland water ways:**
- **National water ways:**

1. The Ganga river, between Allahabad and Haldia (1620 km)
2. The Brahmaputra River, between Sadiya and Dhubri. (891km).

3. The West-Coast canal, between kollam and Kottayam in Kerala. (168 km)
4. The Champakara canal (14 km) and Udyogmandal canal in Kerala (22km)

- **SEA PORTS**

1. **Kandla:** It is Kuchchh district of Gujarat. This was the first port developed soon after the independence of India. It caters to the convenient handling of exports and imports of highly productive granary and industrial belt stretching across the states of Jammu and Kashmir, Himachal Pradesh, Punjab, Haryana, Rajasthan and Gujarat.
2. **Mumbai:** It is the biggest port with a spacious natural and well sheltered harbor.
3. **Jawaharlal Nehru Port:** The Port at Nhava Sheva was planned with a view to decongest Mumbai port and serve as hub port for this region.
4. **Mormugao Port (Goa):** It is the premier iron ore exporting port of the country. This ports accounts for about 50% of India's iron ore export.
5. **New Mangalore:** The port is located in Karnataka. It caters to the export of iron ore concentrates from Kudremukh mines.
6. **Kochi:** It is the extreme south-western port, located at the entrance of lagoon with a natural harbor.
7. **Tuticorin:** The port is in Tamil Nadu. This port has a natural harbor and rich hinterland. It has a flourishing-trade handling of a large variety of cargoes to Sri Lanka & Maldives.
8. **Chennai:** It is one of the oldest artificial ports of the country. It is ranked next to Mumbai in terms of the volume of trade and cargo.
9. **Vishakhapatnam**: It is the deepest land locked and well protected port. This port was originally, conceived as an outlet for iron ore exports.
10. **Paradeep:** The ports is located in Orissa, specializes in the export of iron ore.
11. **Kolkata**: It is an inland riverine port. This port serves a very large and rich hinterland of Ganga Brahmaputra basin. It is a tidal port. So it requires constant dredging of Hooghly.
12. **Haldia:** It is in West Bengal. It was development as a subsidiary port, in order to relieve growing pressure on the Kolkata port.

◙ **AIRWAYS**

- **Advantages of air ways:**

1. It is the fastest means of transport and save time.
2. It can cover very difficult terrains like high mountains, desert, thick forests and long stretches of seas with great ease.
3. It also helps in increasing the trade especially that of perishable product.
4. It reduces pressure on roads and railways.
5. It was National in 1953.

◙ **COMMUNICATION**

There are basically two types of means of communication:

1. Personal Communication.
2. Mass communication.

- **Personal communication**

1. Personal messages are exchanged.
2. These can be used by individual for his personal activity.
3. Telephone, Mobile, postal services, post cards etc are the main sources of personal communication.

- **Mass communication:**

1. One person can communicate with several people at the same time.
2. They can be used by the government to create awareness among the people about various national programmes.
3. Radio, T.V., Newspaper etc are the main source of mass communication.

◙ **INDIAN POSTAL SYSTEM**

1. It is the largest in the world.
2. In handles parcels as well as personal written communication.
3. Cards and envelopes are considered first class mail and are airlifted between stations covering both land and air.
4. The second class mail includes the book packets, registered newspaper and periodicals.
5. Six mail channels have been introduced recently. They are called Rajdhani channel, Metro channel, Green channel business channel, Bulk mail channel and periodical channel.

◙ **TELECOMMUNICATION**

1. India has one of the largest Telecom network in Asia.
2. With the opening of Telecom sector to private investment this becomes fast growing sector of Indian economy.
3. By the end of 2004-05, India was the 10th largest telecom network in the world measured in terms of number of phones.

◙ **INTERNATIONAL TRADE**

- **Trade:** The exchange of goods and services among people states and countries is referred as trade.
- **Market**: It is the place where exchange of goods takes place.
- International Trade: Trade between two countries is called international trade. Trade takes place through sea, air or land.

- **Importance of Trade:**

1. No country can survive without international trade because resources are space bound.
2. Advancement of international trade of a country leads to its economic prosperity.
3. We earn much of our foreign exchange which is required for importing many essential goods.
4. Exports and imports are the two compounds of trade.

- **Favorable balance of trade:**

If the value of exports of any country is more than the value of imports, It is called favorable balance of trade.

- **Unfavorable balance of trade**

The value of imports is more than the value of exports, it is called an unfavorable balance of trade.

◙ TOURISM

- **Importance of tourism:**

1. It helps in earning foreign exchange.
2. It provides direct as well as indirect employment to people.
3. It promotes national integration.
4. It supports cottage and handicraft industry because foreigners like to purchase these products.
5. It helps in developing friendly relation with other countries and promotes international understandings.

EXERCISE

◙ VERY SHORT ANSWER TYPE QUESTIONS

1. Name any four means of mass communication.
2. What is a transport?
3. What are express ways?
4. What is National Highway?
5. Why do the Great Plains of India have dense network of railway?
6. Name any four water ways of India.
7. What do you mean by favorable balance of trade? Is India's trade favorable?
8. Name a tidal port?
9. Name the extreme stations which are connected by the East-West corridor.
10. Name the institution which implements the super Highways.
11. Name the extreme cities which are connected by the NHI.
12. What is the density of a road?
13. Name any two inland waterways.
14. Name any four ports of east coast.
15. Define the term 'port'.
16. What are considered the first class mail?
17. What are considered the second class mail?
18. Why mail channels have been introduced?
19. What do you mean by balance of trade?
20. **Define the following:**
    (a) Market (b) Trade (c) International Trade
21. What are the means of mass communication?

◙ SHORT ANSWER TYPE QUESTIONS

1. What are super Highways? Name any two roads which have been constructed under this project.
2. What is the main objective behind the super Highways? Which agency is responsible for building these raods?
3. What are national Highways? What is their importance?
4. Mention any four features of the border Roads.
5. Distinguish between metaled road and un-metaled roads.

6. What are major problems of Indian roads?
7. What is the importance of ports for the Indian economy?
8. Mention any four features of the telecom Network of India.
9. "Transport routes are called the basic arteries of our economy". Support the statement with four examples.
10. How do physiographic and economic factors influence the distribution pattern of Indian railways network?

LONG ANSWER TYPE QUESTIONS

1. What is the importance of roads in India? Explain any four major problems faced by road transportation.
2. Explain the importance of means communication in modern times. How are personal communications different from mass communication?
3. Compare and contrast roads and railways as means of transport?
4. Why are means of transport and communication essential requirements of human life?
5. Explain the importance of air transport in India.
6. Explain any six qualitative improvements made in Indian Railways.
7. Explain any four advantages of pipeline transportation. Name any two important Networks of pipe line transportation in India.
8. What is importance of tourism?

ECONOMICS

DEVELOPMENT

Development is a process of change: Change for betterment is development.

For a landless rural laborer more days of work and better wages means development.

- On the other hand, for a body form a rich urban family, pursuing studies abroad may be development.

Thus we can say that:

- DIFFERENT PERSONS CAN HAVE DIFFERENT DEVELOPMENTAL GOALS
- WHAT MAY BE DEVELOPMETN FOR ONE MAY NOT BE DEVELOPMENT FOR THE OTERH, IT MAY BE DESTRUCTIVE FOR OTHERS

Example: To get more electricity an industrialist may want more dams but the community whose land gets submerged due to building of dam may oppose it.

What are the general goals for development?

- INCOME- The most common goal for people is to desire for regular work, better wages decent price of their crop or products, in other rewords they want more **income.**
- NON MATERILA THINGS- which sometimes improve the quality of life are sometime more important than income, for example_ Equal treatment, freedom, security, respect are some of the many things that cannot be easily measured but mean a lot in our lives.
- MIX OF GOALS- for development people look for a mix of goals. A woman who is engaged in paid work enjoys more dignity in the household and society and if there is respect for woman there will be more sharing of work and acceptance of women working outside, this will encourage more women to take up jobs.

Some examples where factors other tan income are important aspects of our lives?

The factors other than income which are important in our lives are freedom, security, equal treatment, facilities for family, working atmosphere, quality, respect etc.

Example:

1. A job may give you less pay but may give you regular employment i.e. A senses of security
2. A job may give at a far off place but it may give you facilities for your family and a working atmosphere, this would give you more freedom.
3. A woman who is engaged in paid work earns more respect in her household and society.

National development-

Different people may have different as well as confliction ideas of a country's development. But form national development we have to think of goals which are fair and just for all. The ideas should benefit a large number of people.

Comparisons between different nations

Some countries are considered more developed than others. This comparison can be done on the basis of some attributes. One of the most important attributes is income. Countries with higher income are more developed as such countries are able to provide all the things man needs with the help of that income.

National Income

National income is the sum total of income from goods and services produced within the country and income coming form abroad.

For comparing countries national income is not a very useful measure as countries have different population and by comparing national income we cannot tell what an average person earns in that country. Therefore were have to compare the **average income/per capita income.**

Average / Per capita income

The average income of a country is the total income of the country divided by its total population.

Average income = $\frac{National.income}{Total.population}$

Disadvantage of comparing countries on the basis of Average income

The major disadvantage is that it hides disparities, i.e.. It does not tell us how income is distributed among the people. E.g. Country A and B may have identical average income but country A may be preferred as it has more equitable distribution of wealth on the other hand country B may e having mostly poor and very few rich people.

World Bank's classification of countries:

According to would development report 2006 brought out by the World Bank countries are classified on the basis of their per capita income. The classification is thus.

1. Countries with per capita income above Rs. 4, 53, 000 per annum are rich countries or developed countries e.g. U.S.A., U.K., Japan, Germany, Singapore-etc.
2. Countries with per capita income between Rs. 4, 53, 000 to 37,000 are called developing, or middle income countries. E.g. Sri Lanka, China, Brazil etc.
3. Countries with per capita income less than Rs. 37,000 are called low income countries, e.g. India, Bangladesh, Pakistan, Nepal etc. Per capita income in India is Rs. 28000 per annum.

Other attributes to be considered besides per Capita income are:

(a) Infant mortality rate- The number of children that die before that age of one year as a proportion of 1000 live children born in that particular year.

(b) Literacy rate- measures the proportion of literate population in the 7 and above age group.

(c) Net attendance ratio-is the total number of children of age group 6-10 attending "school as a percentage of total number of children of the same age group. Therefore we can say that money alone is not an indicator of the facilities, goods and services that citizens can use, e.g. In Kerala the per capita income is lower than Punjab but the infant mortality rate is low and the health, education and PDS works better there. **Criterion of UNDP for measuring development of countries** is different form world bank it compares countries based on a) Education level b) health status c) per capita income."

Sustainability of development

Sustainability is the capacity to use the resources judiciously and maintain their balance. Every generation wants to take maximum benefic form the resources but this would lead to disastrous consequences because the resources would get exhausted and our future generations would be deprived of these resources.

Non renewable resources

These resources cannot be replenished. New sources may be added but they too may get exhausted. E.g. Crude oil is being constantly explored so that we can add to the resource. These resources must be used carefully and we must try to find methods of recycling and reusing them.

Renewable resources

The resolves like wind, water, solar energy are inexhaustible but we must see that they are not put to wring use otherwise they too would soon be degraded.

"The earth has enough resources to meet the needs of all but not enough to satisfy the greed of even one person". This is a relevant statement in today's times as we may have abundance of resources if we use them judiciously and save for out future generations. If we use them greedily then there will not be any resource left for our children.

Environmental degradation around us

The consequences of environmental degradation can be felt everywhere and it is not just a national issue it can be felt internationally. If due to overuse or misuse of natural resources one part of the world gets degraded, it has effects on other parts of the world also e.g.

1. Testing of atom bomb can have bad effects on other parts of the world.
2. Dumping of industrial waste into rives have led to their degradation.
3. The effect of oil spill in the Persian Gulf during the gulf war was also seen in the snowfall in Kashmir during winters.

Reasons for 40% people being undernourished even though there is enough food in the country

1. A large number of people are so poor that they cannot afford nutrition's food.
2. Lack of edition and health facilities is also a cause of low nutritional level.
3. In several states of India the public Distribution system is not functioning properly.

Body Mass Index (BMI)

This index is used to calculate if the adults are undernourished. Take the weight of a person in kg and his height in meters. Divide the weight by square of the height. If it is less than 18.5 then the person is undernourished and if it is more than 25 then the person is overweight.

EXERCISE

SOLVED EXERCISE

Q.1 What is development?

Ans. Development refers to the process as a result of which along with increase in real per capita income there is a reduction in inequality, Proverb, Illiteracy and diseases. In other worlds, this is improvement in the economic welfare of the people and their standard of living rises.

Q.2 Mention any four characteristics of development.

Ans. (i) Different people have different developmental goals.

(ii) What may be development for one may not be development for the other. It may be destructive for the other.

(iii) Income is the most important component of development but along with income people also seek equal treatment, good health, peace, literacy etc.

Q.3 What is national income?

Ans. National income is defined as the total value of all the good and services produced within a country plus income coming form aboard.

Q.4 What is capital income?

Ans. When the total National income is divided by the Total population, it is called per capital income.

Q.5 What is infant mortality Rate?

Ans. It indicates the number of children that die before the age of one year as a proportion of 1000 live children born in that particular year.

Q.6 What is Literacy rate?

Ans. It measures that proportion of literate population in the 7 and above age group.

Q.7 What is Net attendance Ration?

Ans. It is the total number of children of age group6-10 attending school as a percentage to total number of children in the same age group.

Q.8 What contributes to the human development?

Ans. There are many economic as well as non-economic factors which contribute to the human development.

(i) Living a long and healthy life.

(ii) To have education, information and knowledge,.

(iii) Enjoying decent standard of living.

(iv) Enjoying basic fundamental rights like freedom, security, education ect.

(v) To have equality and enjoyment of human rights.

Q.9 What is the main criterion used by the World Bank in classifying different countries? What are the limitations of the criterion?

Ans. **1. Rich or High Income countries:** Countries with per capita income of Rs. 4, 53,000 per annum and above in 2004 are called rich countries.

2. Poor or Low Income countries: The countries with per capita income of Rs. 37,000 or less are called low-income counties.

India comes in the category of low-income counties because its per capital income in 2004 was just Rs. 28,000 per annum. The rich countries, excluding, countess of Middle East and certain other small counties are generally called development counties.

Limitation

(i) It covers only economic aspect ignoring peace, health, environment, education, longevity etc.

(ii) The method cannot provide us distribution of income.

Q.10 Distinguish between human and economic development.

Ans.

Human Development	Economic Development
1. It is broader aspect of development as it includes monetary as well as. 2. it is the process of both quantitative and qualitative growth. 3. Human development is end or the final goal of all developments.	1. It is a narrow concept as it includes only monetary aspects. 2. It includes only quantitative growth. 3. It is means to achieve human development.

Q.11 What are major features of sustainable development?

Ans. Main features of sustainable development are as under

1. Efficient use of natural resources: Sustainable development does not mean that natural resources should not be used a all. It simply means that natural resources and environment should be used in an efficient manner as to achieve along term net objectives like increases in income and employment abolition of poverty, Important in sandal of lining etc.

2. No reduction in the quality of life of the future generation: Sustainable development descries those activities which in order to maintain existing high stand of living prove detrimental to natural recourses and environment. According to this concept, one should desist from undertaking such activities as increase in pollution and decrease n quality of life of future generations.

3. No increase I pollution: Sustainable development decries those activities which, in order to maintain existing. High standard of living proves determinable to natural resources and environment. According to this concept, one should desist from undertaking, such activities as increase in pollution and decrease in quality of life of future generations.

Q.12 "The Earth has enough resources to meet the needs of all but not enough to satisfy the greed of even one person". How is this statement relevant to the discussion of development? Discuss.

Ans. Sustainability is the capacity to use the resources judiciously and maintain their balance. Every generation wants to get the retest benefits from the available resources but such dung would-be quite disastrous because the available resources shall be exhausted at a result and especially our coining generations will be deprived of such resources. So we should use our resources most judiciously so that a certain part of them is left for our coming generation to use it in fuse. We should protect our animal wealth, both animals and birds form human exploitation otherwise some species sill be lost forever. In such case our corning generations would be deprived of much charm and beauty which we enjoy today corning be seeing different varieties of animals and birds. Certain resources like water, wind energy and solar energy etc. are inexhaustible but we must see that we are not put wrong use- There is no harm in using the land for cultivating 6 we should see that its fertile outhouse sooner or later it will turn into a waste land.

Most of the mineral wealth and iron, cooper, gold, silver etc. is exhaustible and as such must be used gold carefully. However, we must devise methods of recycling these resources and reusing them.

No doubt, we should utilize our resources for our benefit but we should sustain them and maintain them of our future as well as for the welfare of our coming generations. That is why, it has been said, Earth ash enough sources to meet the demands of all but enough to satisfy the greed **of even one** person.

SECTORS OF THE INDIAN ECONOMY

◙ IMPORTANT TERMS

- **Primacy Activity:** those occupations which are closely related to man's natural environment e.g. gathering, hunting, fishing etc.
- **Secondary activity:** The occupation which produces finished goods by using the products of primary activities as raw material e.g. sugar from sugarcane, manufacturing of cloth from cotton etc.
- **Tertiary activity:** This activity consists of all service occupations e.g. transport & communication, trade health, education and administration etc.
- **Intermediate goods.** Those goods which are sold by one firm to another of resale or for further processing.
- **Final goods.** They have crossed the boundary line of production and are ready for use by the final users e.g. cloth, coolers, T.V. etc.
- **Double counting:** The counting of the value of a product more than once is called double counting.
- **Gross Domestic product:** It is the market value of the final goods and services produced with in the domestic territory of a country during one year.
- **Disguised Unemployment:** It is the situation in which more workers are working in an activity than required.
- **Public sector:** It is the sector which is controlled, managed and owned by the government.
- **Private sector:** It is owned, controlled and managed by individual or by a group of individuals.

◙ **SECTORS**

People in society are engaged in various economic activities. Some produce goods where as some produce services in order to understand these activities people are divided into groups. These groups are called sector or activity

- **Primary sector:**

(i) Those occupations which are closely related to man's natural environment. Hunting, fishing, lumbering, animal rearing, farming and mining are important example of primary activity.

(ii) This sector is also called agricultural and related sector.

(iii) Primary products are the base for all other products that we subsequently make.

- **Secondary Sector:**

(i) The occupation which produces finished goods by using the product of primary activity as raw materials are included in secondary activity. E.g. Manufacturing of cloth from cotton, sugar from sugarcane and steel from iron ore etc.

(ii) This sector is related to industries and hence can be called industrial sector.

(iii) The product is not produced by nature.

- **Tertiary Sector:**

(i) These, are activities that help in the development of primary and secondary sector.

(ii) These activities by themselves do not produce a good by they are an aid or a support for the production process.

(iii) Since these activities generate services rather than goods. They are called service sector e.g. Transport, storage communication, Banking etc.

◙ **COMPARING THE THREE SECTORS:**

- **Intermediate Goods:**

Those goods which are sold by one firm to another either for resale or for further processing is called as Intermediate Goods. E.g. paper sold by a paper mill to a publisher is intermediate goods.

In other words these goods which are within the boundary line of production. The value of these goods in not include in the national income of a country. Their value is reflected in the value of final goods.

- **Final Goods:**

Those goods which are used either for [mal consumption or for capital formation. These are not resold. In other words final goods have crossed the boundary line of production and are ready for use by the [mal users.

- **Double counting:**

The counting of the value of a product more than once is called double counting. Gross Domestic Product is the market value of the final goods and services produced with in the domestic territory of a country during one year.

◙ **IMPORTANCE OF THE TERTIARY SECTOR**

- **Factors responsible for the growth of tertiary sector.**

1. **Basic services:**

In any country several services such as hospitals, school, post and telegraph police station, courts, village defense, transport, banks etc are required. These can be considered as basic services. The government has to take responsibility for the provision of these services.

2. **Development of means of transport and communication**
The development of agriculture and industry leads to the development of services such as transport communication trade etc. All these come under tertiary sector.
3. **More income more service:** The per capita income in our country is rising. As income level risers people demand are also rising.
4. **New services:** With modification and globalization some new services based on information and communication technology have become important and essential.

◙ **IMPORTANCE OF PRIMARY SECTOR**

1. The secondary and tertiary sectors have failed to create enough job.
2. Even though industrial output or the production of goods went up by eight times during the period, employers in the industry went up by only 2.5 times.
3. More than half of the workers in the country are still working in the primary sector mainly in agriculture producing only quarter of the GDP.

◙ **DISGUISED UNEMPLOYENT**

It is saturation in which more workers a working in an activity than required.

1. **Diversification of Agriculture:** More than 60% our workers are employed in agriculture but our farmers are producing only limited crops. There is need to diversify agriculture. Farmers should be encouraged to adopt Pisciculture, horticulture animal rearing etc. along with cultivation of crop.
2. **Cheap Credit:** Most of the farmers depend on informal sources of credit. Government should encourage she commercial banks t provide loans to the farmers at cheaper rates.
3. **Provision of basic facilities:** Our rural areas lock basic facilities like roads, transportation, banking, market etc. The government should invest some money in these sectors so that Indian village can be linked to other market. The activity can provide productive employment to not just for farmers but also other such as those in services like transport or trade.
4. **Promotion of local industries:** Promote and locate industries especially the cottage and small scale industries in semi rural areas where a large number of people may be employed.
5. **Improvement in Education & Health:** Most of the Indian villages lack education and health facilities. A study conducted by he planning commission estimates that nearly 20 lakh job can be create in education sector one. Similarly, we need many more doctors, nurses, health workers in rural area.

◙ **NATIONAL RURAL EMPLOMENT GUARANTEE ACT (NREGA)-2005**

- **Main feature of the scheme are:**

1. The scheme targets scheduled Castes (SC) and scheduled Tribes (ST) and poor women who suffer from poverty.
2. To provide livelihood to the people below the poverty line, guarantees 100 day of wage employment in a year to every household in the county.
3. Under the scheme, the Gram Panchayat after proper verification will register household and issue job card to registered household.

◙ **DIVISTION OF SECTOR AS ORGANISED AND UNORGANISED**

1. **Organized sector:** Organized sector covers those enterprises or places of work where the terms of employment are regular and therefore people have assured work. They are registered by the government and have to follow its rules and regulation which are given in various laws such as Factories Act. Minimum wages Act. Payment of Gratuity Act, shop and Establishment Act. Etc.

- **Advantages of organized sector**

1. Workers in this sector enjoy security of employment.
2. They work only for fixed numbered of house. If they work more, they have to be paid overtime by the employer.
3. They also get several other benefits from the employers like paid leave, payment during holidays, provident fund etc.

4. They also get several other benefits under the laws.
5. The workers working in organized sector also get pension after retirement.

- **Unorganized sector:** It is a sector which is not registered by the government. It is characterized by small and scattered units, which are largely outside the control of the government. These are rules and regulations but these are not followed.
- **Disadvantages of unorganized sector:**

1. Workers working in unorganized get fewer wages.
2. There is no provision for overtime, paid leaves, holidays, leaves due to sickness etc.
3. Employment in subject to high degree of insecurity people can be asked to leave without any reason.
4. A large number of people doing small jobs come under unorganized sector.
5. Land-less and causal workers also comes under unorganized sector.
6. People working in unorganized sector are exploited.

◙ **UNORGANISED SECTOR OF RURAL AREAS**

In the rural areas, the unorganized sector comprises mainly of workers in small scale industry, casual workers in construction, trade and transport etc. and those who work as street venders, head load workers garment makers, rag pickers etc.

◙ **HOW TO PROTECT WORKERS IN THE UNORGANISED SECTOR**

1. **Alternative sources of employment:** this is the best policy, on long term basics, to create ample opportunities of employment in the areas of other than agriculture..
2. **Public works programmers:** Majority of the people get employment during sowing and harvesting periods. For the remaining part of the year they usually remain unemployed. It is necessary to organize the public works programmers to the rural workers thought the year.
3. **Social security:** The workers of unorganized sector have no social security of job. So there is sincere efforts are needed.

◙ **SECTOR IN TERMS OF OWNERSHIP**

On the ownership of resources economic activities can be classified as-

1. **Private sector:** it is sector which is controlled, managed and owned by the individual e.g. Tata iron & steel co., Baja Auto etc.
2. **Public sector:** It is the sector which is controlled, managed and owned by the government e.g. Indian Railways, BHEL, Sindri Fertilizer ltd. Etc.

- **Motivation:** The private sector activities are centered on profit motives and the contest of service is of secondary consideration.

Public sector activities are determined by the social welfare consideration with the movie of providing basic needs of people of the country and the profit motives are secondary consideration.

◙ **ROLE OF GOVERNMENT OR PUBLIC SECTOR:**

1. **Development of infrastructure;** Development of infrastructure comprising of transport, power communication, basic industries etc. is a precondition of growth. Expenditure on the developing of infrastructure is known as social over head costs.
2. **Development of backward areas:** the government finds it necessary to start industrial production in these areas on its own.
3. **Basic facilities:** running proper school and providing education in the duty of the government.
4. **Other problems:** there are many other problems like malnourishment, high infant motility rate, unsafe drinking water etc. which need special attention. These problems can be solved only with the help o government.

EXERCISE

◙ **VERY SHORT ANSWER TYPE QUESTIONS**

1. What are primary activities? Give two examples.
2. What are intermediate goods? Give two examples.
3. What is double counting?
4. What is GDP?
5. What is disguised employment?
6. What are final goods?
7. What are basic services?
8. What is under employment?
9. What is public sector?
10. What is private sector?
11. What is organized sector?
12. What is an unorganized sector?
13. State an example each of activity in primary secondary in tertiary sector.
14. Name some occupations where problem of under employment prevails.
15. Mention any two long term measures to remove the problem of underemployment in rural area.

◙ **SHORT ANSWER TYPE QUESTIONS**

1. What do you understand by disguised unemployment? How can it be removed?
2. In India, the development process has led an increase in the share of the tertiary; sector in G.D.P do you agree with this statement? Discuss.
3. State measures that your feel will help to protect the laborers in the unorganized sector.
4. How is the tertiary sector different from other sector? Illustrate with few example.
5. Is the service sector actually growing? Substantiate your answer.
6. Differentiate between public sector and private sector citing examples.
7. How these are activates in the economy classified on the basic of employment conditions.
8. Compare the employment condition prevailing in the organized and unorganized sector.
9. Distinguish between, final goods and intermediate goods.
10. Distinguish between, final goods and unorganized sector.
11. Who are the people that work in unorganized sector?
12. What are the objectives of NREGA 2005?

◙ **LONG ANSWER TYPE QUESTION**

1. Why is the tertiary sector becoming so important in India? Give at least for reasons.
2. Suggest some ways which can be helpful in creating employment in rural areas.
3. What are the advantages of working organized sector?
4. What are the disadvantages of working in unorganized sector?
5. Explain the role f government is public sector.
6. How to protect workers in an unorganized sector?
7. Primary sector is more important than tertiary sector. Discuss.

EXERCISE

OBJECTIVE QUESTIONS

1. The sectors are classified into public and private sector on the basis of (choose one):

(a) Employment conditions (b) The nature of economic activity

(c) Ownership of enterprises (d) Number of workers employed in the enterprises

2. Production of commodity, mostly through the natural process, is an activity in …….sector:

(a) Primary (b) Secondary

(c) Tertiary (d) Information technology

3. GDP is the sum total of the value of …….. goods produced during a particular year:

(a) All goods and services (b) All final goods and services

(c) All intermediate goods and services (d) All intermediate and final goods and services

4. In terms of GDP the share of tertiary sector in 2003 is ……:

(a) Between 20 percent to 30 percent (b) Between 30 percent to 40 percent

(c) Between 50 percent to 60 percent (d) 70 Percent

5. Which one of the following is not an activity of the primary sector?

(a) Fishing (b) Mining

(c) Manufacturing (d) Lumbering

6. Which one of the following is not an activity of the secondary sector?

(a) Brick-making (b) sugar-making

(c) Manufacturing (d) Forestry

7. Which one of the following activity does not belong to the Tertiary sector?

(a) Fishing (b) Banking

(c) Trade (d) Transport

8. Which activity continued to prove largest employer in between 1973 to 2000?

(a) Sugar industry (b) Agriculture

(c) Mining (d) Banking

9. Which is disguised unemployment?

(a) When more people are made to work less than their potential

(b) When people are ready to work but they have not jobs or work to do

(c) When less people are made to work more than their potential

(d) When everyone is made to work according to his potential

10. Which one of the following is not an example of the public sector?

(a) Railways (b) Post office

(c) Reliance Industries (d) Army

ER KEY	EXERCISE									
Que.	1	2	3	4	5	6	7	8	9	10

Ans.	C	A	B	C	C	D	A	B	A	C

MONEY AND CREDIT

IMPORTANT TERMS

- **Barter System**: The system of exchanging goods is known as Barter system.
- Double coincidence of wants: It means both parties i.e. buyer a seller have to agree to sell and buy each other commodities
- **Cheque:** It is a paper instructing the bank to pay a specific amount from the person's account to the person in whose name to cheque has been made.
- **Cash Reserve Ratio:** The part of the total deposit which the bank keeps with itself in cash form in return for the promise of future payment.
- Formal lender Resources: There include those resources which are controlled by the government. E.g. Banks & Cooperatives.
- Informal lender resources: These include money lenders, traders, friends and relatives.
- **Debt trap:** It is a situation which pushes the borrower into a situation from which recovery is very painful.
- **Money:** Any things which is generally accepted by the people in exchange of goods or services or in repayment of debts.
- Demand Deposits: Deposits in the bank which are payable on demand are called demand deposits.

BARTER SYSTEM

The system of exchanging goods is known as barter system.

- **Limitation of Barter system**

(i) Two people with different types of needs and goods must be there to satisfy each other's needs.
(ii) There are many products which cannot be divided.
(iii) Valuation of goods is very difficult.
(iv) It was time consuming.

MONEY AS MEDIUM OF EXCHANGE

It is an important function of money. It means that money act as an intermediary for the goods and services in an exchange of transaction. Use of money as a medium of exchange has removed the major difficulty of double coincidence of wants in the barter system.

- **Double coincidence of want**

It means both parties i.e. buyer and seller have to agree to sell and buy each other commodities.

- **Advantage of Money**

(i) It is used as a medium of exchange. (ii) It offers economic freedom to the people.
(iii) It is used for purchasing goods and services. (iv) It I s easy to store money.

- **Measure of value**

Money serves as measure of value in terms of unit of account. Unit of account means that the value of each good or services is measured in the monetary unit.

- **Modern forms of money**

It includes currency, paper notes, coins and deposits with the bank.

- Money is used as a medium of exchange because:

(i) It is authorized by the government of the country.

(ii) Its demand and supply can be controlled by the Central Bank of the country. In case of India the Reserve Bank of India issues currency notes on behalf of the central government.

(iii) In India the law legalizes the use of rupee as a medium of payment that cannot be refused in settling transaction in India. No Individual can legally refuse a payment made in rupees.

(iv) Value of each good or services is measured in the monetary unit.

◙ DEPOSITS WITH BANK

- **Demand deposit:** The deposits in the bank accounts which can be withdrawn on demand are known as demand deposit.
- Advantages of depositing money in the bank:

(i) It is safer place to keep money as compared to the house or a working place.

(ii) People can earn interest on the deposited money.

(iii) People have the provisions to withdraw the money as and when they require.

(iv) People can also make payments through cheque.

- **Cheque:** It is a paper instructing the bank to pay a specific amount from the person's account to the person in whose name the cheque has been made.
- **Advantages:**

(i) It is the safest mode of transaction.

(ii) It is easy to carry a cheque as compared to money

◙ LOAN ACTIVITES OF BANKS

(i) Bank keeps only a small proportion of their deposits as case with themselves. (About 15% for their provision)

(ii) Bank use the major portion of the deposits to extend loans. Bank makes use of deposits to meet the loan requirements of the people.

(iii) In this way bank mediate between those who have surplus funds and those who are in needs of funds.

(iv) Bank charge a higher interest rate of on loans than what they offer in deposits. The difference between what is charged from the borrowers and what are paid to the deposition are their main sources of income.

- **Credit:** It refers to an agreement in which the lender supplies the borrower with money goods or services in return for the promise of future payment.
- **Importance of Credit**

(i) It helps the people to purchase houses.

(ii) It helps the businessman to expand their business.

(iii) The difference between the lending rate and borrowing rate is the source of income for the banks.

(iv) In rural areas, the main demand for credit is for crop production. Farmers usually take crop loans at the beginning of the season and repay the loan after harvest. Repayment of the loan is crucially dependent on the income from farming.

- **Disadvantage of Credit**

(i) Banks charge a very high rate of interest which means a large part of earning of the borrowers is used to repay the loan.

(ii) If the borrowers fail to repay the loan, the bank has the right to sell the assets of the borrowers.

(iii) If loan is used for unproductive activities the borrower can be pushed into a debt trap.

(iv) Banks don't provide credits to the poor people as they don't have any approved security.

◙ TERMS OF CREDIT

- **Interest Rate:** Every loan agreement specifies an interest rate which the borrower must pay to the bank along with the repayment of the principal amount.

There are two types of interest rate:

- **Fixed:** This rate is fixed and do not change throughout the loan period,
- **Floating:** This rate vary according to the bank's policy and policy of the R.B.I.
- **Debt trap:** It is a situation which pushes the borrower into a situation from which recovery is very painful.
- **Collateral:** It is an asset that the borrower owns (such as lands, building, vehicles, live stocks, deposits with bank) and uses this as a guarantee to a lender until the loan is repaid. If the borrower fails to repay the loan, the lender has the right to sell the collateral to obtain payment.

◙ **LOAN FROM COOPERATIVES**

(i) **Aims:** The main aim of these societies is to given short period and medium term loan to its member. These societies encourage the habit of thrift among its member.

(ii) **Finance**: To run their affairs, these societies procure credit from various sources. These sources may be divided into two parts.

(a) **Internal Sources:** There are many sources such as entry fee, share capital, deposits of member & reserves.

(b) **External sources:** The government, central financial institutes, Reserve Bank of India
And other is external sources.

(iii) **Loans:** These societies give short period loans to their members for productive purposes. The society can also give, up to certain limit, medium period loans.

- Central Co-operative Bank: These banks were established according to the Co-operative societies Act- 1912.

Functions:

(a) These banks give interest-free loan to the primary Agricultural societies, but from others interest is charged.

(b) These banks also perform general banking function such as to accept deposit from the people, transfer of money etc.

(c) These banks help the primary societies in solving their problems.

◙ **FORMAL SECTOR IN CREDIT**

(i) **Formal lender resources:** These include those resources which are controlled by the government. Banks and cooperative banks fall in to the formal category.

(ii) **In formal lender resources:** These include money lenders, traders, relatives and friends.

Difference between formal and informal credit

	Formal	Informal
1	These resources work under the supervision of the R.B.I.	These do not work under any government organization.
2	The rate of interest is very low.	The rate of interest is very high.
3	Commercial banks, cooperative bank Societies are the main sources of Formal Credit.	Relatives money lender and land lord are the main sources of informal

- **Unequal distribution of formal sector loans:**

Even after more than 50 year of independence most of rural and poor people still depends on non formal resources for their loan requirements 85% of the loan taken by poor households in the urban areas are from informal sources. The poor households have to pay a heavy price for borrowing.

◙ **SELF HELP GROUPS (SHG) FOR THE POOR:**

It helps in pooling the saving of the members, who are poor. Members can get timely loans for a variety of purposes and at a reasonable rate of interest.

- **Major feature of SHGs:**

1. Typical SHGs can have 15-20 members usually belonging to the same village.

2. The main motive of SHGs to pool the savings of the poor people.

3. Saving per member can vary from Rs. 25 to Rs. 100 or more depending on the ability of the people and the strength of the group.
4. It provides loans to their members at a reasonable rate.
5. After a year or two, if the group is regular in savings, it becomes eligible for bank loans.
6. Loan is sanctioned in the name of the group with the main motive to create self employment opportunities for the members.
7. Most of the SHGs work in a democratic way.

EXERCISE

◙ VERY SHORT ANSWER TYPE QUESTIONS

1. Why are banks willing to lend to women organized in SHGs?
2. Define the term money.
3. Explain Barter exchange system with the help of an example.
4. Problem of lack of double coincidence of wants exists in barter system. Explain?
5. How money serves as a medium of exchange?
6. Why everybody is bound to accept money against any transaction?
7. What does terms of credit include?
8. Explain loans from cooperatives?
9. What is debt trap? Mention any two factors responsible for the dept trap.
10. What are formal sources of Credit?
11. What are formal sources of Credit?
12. What is the source of income for the banks?
13. What is the importance of credit?
14. What are the two limitations of Barter system?
15. What is Cash Reserve Ratio?

◙ SHORT ANSWER TYPE QUESTION

1. Why do we need to expand formal sources of credit in India?
2. What are the reasons that the banks are not willing to lend to certain borrowers?
3. Differentiate between money and barter system.
4. In what way does RBI supervise the functioning of the banks?
5. "The modern currency is without any use of its own". Why is it accepted as money?
6. What are the limitations of barter system?
7. What are the importance/ uses of money?
8. "The rupee is widely accepted as a medium of exchange". Explain.
9. What are the advantages of depositing money in the Bank?
10. What is a cheque? What is its importance?
11. What is collateral?
12. Explain money as a medium of exchange?
13. Explain the loan activities of banks?
14. Why should credit at reasonable rates be available for all?
15. Distinguish between formal and informal sources of credit?

◙ LONG ANSWER TYPE QUESTIONS

1. Explain the major features of SHGs in rural area.
2. Analyze the role of credit for development.
3. Explain the loan activities of banks.
4. Most of the poor household is still dependent an informal sources of credit'. Explain.
5. Why should the banks and cooperative societies provide more loan facilities to the rural households in India?
6. Explain the inconvenience of barter system.
7. Write a note on formal and informal credit in India.

GLOBALISATION & THE INDIAN ECONOMY

◙ IMPORTANT TERMS

- **Globalization:** It means free interaction of an economy with the economies of the rest of the world.
- **Liberalization:** Liberalization of the economy means to free it from direct or physical controls imposed by the government.
- **Privatization:** It means allowing the private sector to set up industries which were earlier reserved for the public sector.
- **MNC:** It is a company that owns or controls production in more than one country.
- **Investment:** The money that is spent to buy assets such as land, building, machines and other equipment is called investment.
- **Foreign investment:** Investment made by MNCs is called foreign investment.
- **Fair Globalization:** It refers to globalization which creates opportunities for all and ensures that its benefits are better shared.

◙ PRODUCTION ACROSS COUNTRIES

Until the middle of the 20th century, production was organized with in countries. Even in our country most of the production was in the hands of the government. But with the entry of MNCs (Multi National corporations) the whole world market has changed.

- **MNC:** A MNC is a company that owns or controls production in more than one country for e.g. Pepsi, Samsung, Onida, Glaxo, Ponds, and LG etc.

◙ FEATURES OF MULTINATIONAL CORPORATIONS

1. **International Operations:**

Activities of MNCs are spread over many countries their parent corporation is located in located in one country and subsidiaries are scattered in many countries. For e.g. ITT, a very large American MNC designs its products in research centres in the U.S.A. and then the components are manufactured in China. These components are then shipped to Mexico and Eastern European countries where these are assembled and the finished products are sold all over the world.

2. **Giant Size:** Most of the MNC are giant size their assets, sales and profits run into multi-Crores.

◙ ADVANTAGES OF MULTINATIONAL CORPORATIONS

1. Availability of capital and foreign investment: MNC help to solve the problem of capital and foreign investment by under developed and developing countries. MNCs set up factories, and offices for production in these developing and

under developing countries and make huge investment. The money that is spent to buy asset in called investment. Investment made by these MNCs is called foreign investment.

2. Availability of foreign exchange: MNCs can be helpful in solving the problem of foreign exchange of the underdeveloped and developing countries. In 1905 India faced a huge shortage of foreign exchange but with the entry of MNCs it has surplus foreign exchange reserves.
3. Promotion of small scale industries: Most of the MNCs take help from small scale and local industries in manufacturing. Garment, Footwear, sports items etc are carried out by a large number of small producers around the world. The products are supplied to the MNCs which then sell these under their own brand names to the customers.
4. Foreign Trade and Integration of Market: With the entry of MNCs even the small countries have depend upon their domestic markets for other countries.
   - ▸▸ It increases foreign trade.
   - ▸▸ It creates an opportunity for the local producers to reach beyond the domestic market.
   - ▸▸ Producers can sell their products not only in local market but also in foreign market.
   - ▸▸ Buyers and consumer are getting variety of goods of their choice.
   - ▸▸ Prices of most of the consumer goods have fallen down due to the competition.

- Availability of modern techniques and management:

Modern technology and managerial services are made available to the local companies. As a result the productivity of the local enterprises increases and resources are optimally utilized.

◙ DESADVANTAGES OF MULTINATIONAL CORPORATIONS

1. Harmful for host country: The main objective of the MNCs is to earn maximum profit. They over exploit the natural resources of the host country.
2. Harmful for the local producers: Most of the local producers have failed to compete with the MNCs so, either they have sold their units to MNCs or have been wiped off.
3. Harmful for Economic Equality: MNCs are interested in setting up industries in particular region and hence those regions develop very rapidly and other regions remain under developed.
4. Harmful for freedom: The Corporation makes all efforts to bring to power in the host country a political party that is favorably inclined to them.

◙ GLOBALISATION

- Globalization: It is the process of integration or inter-connection between countries. It allows free flow of trade, capital and human resources across borders.
- Factors responsible for Globalization:

1. Rapid improvement in technology has been one of the major factors that have stimulated the globalization process. Due to major improvement in transportation technology, goods can be transported throughout the world in a short period of time and a lower cost.
2. Development telecommunications, computers, internet has given a big boost to the process of globalization. It is very easy to access information instantly and to communicate from remote areas.

- **Use of IT in Globalization:**

1. Helps in communication across the world at negligible cost.
2. Transfer of data and other information.
3. Transfer of money across the countries.
4. To link the market

5. To setup customer care centres.

◙ LIBERALISATION

- Trade Barriers: It refers to the various restrictions which are used by the government to increase or decrease foreign trade. E.g. tax on imports.

The Indian government, after independence has put many such barriers to foreign trade and foreign investment. This was considered due to the following reasons.

1. All the basic industries required huge investment which was beyond the reach of private sector. So all these industries were developed under public sector.
2. Government wanted to control basic industries because it was felt that the government control of critical industries would provide necessary resources to undertake development activities in different segment of the economy.
3. The private sector was allowed to establish industries and business enterprises, but was subject to controls and regulations that came in the form of laws. This was considered necessary so that resources and wealth would not get concentrated in a few hands.
4. The main purpose behind the mixed economy strategy was to eliminate poverty, inequalities in the distribution of income and wealth, unemployment and to achieve economic growth and social justice.

◙ LIBERALISATION:

Removing barriers or restrictions set up by the government is known as liberalization.

In 1991 it was decided to life all the unwanted restriction such as industrial licensing system, price control, import license etc.

For this following steps were taken

1. All the industries except three industries were exempted from any kind of industrial licensing.
2. Under the policy of liberalization industries are free to expand and produce according to the need of market,
3. Now the producers are free to import the machinery and raw material from abroad.
4. Now the industries are. Also free to import modern technology from other countries.

◙ PRIVATISATION

Privatization was another component of liberalization.

It means allowing the private sector to setup industries which were earlier reserved for the public sector.

The following steps have been taken:

1. The number of industries reserved for public sector. Has been reduced from 71 to 3.
2. Now public sector can enter in core industries like iron and steel, electricity, transportation communication, ship building etc.
3. Process of disinvestment has been initiated by the government in respect or those public sector industries which have been running in loss.
4. The private sector has been free from many restrictions such as licensing, permission to import raw materials, regulation of prices and restriction on investment.

◙ POSITIVE IMPACT OF GLOBALISATION IN INDIA

1. **Variety of products:** Many MNCs have invested their capital in India. So the Indian consumers are getting variety and quality products at cheaper rates.
2. **Development in infrastructure:** Due to this policy the condition of infrastructure has improved considerably. The government is building Golden Quadrilateral which will connect all major cities. Much progress can be seen in the communication sector.

3. **Boost to Indian Companies:** Due to this policy the private sector has gained a big push. Now the private sector is free to import raw material and technology from other countries. Many restrictions on the import and exports have been lifted. Globalization has enabled some large Indian companies to emerge as multinational companies themselves. Tata motors, Ranbaxy, Asian Paints are some companies which are spreading their operation worldwide.
4. **Boost to service sector:** Globalization has also created new opportunities for companies providing services, particular those involving information and communication technology.
5. **Foreign currency and foreign direct investment:** The foreign currency reserves have multiplied to a great extent due to new economic policy. Foreign direct investment which was just Rs. 174 crore in 1991 has risen to Rs. 9, 338 in 2000.
6. **Global form of modern business:** Due to globalization the business has now become global. Now India exports and imports of goods. Our industries have also entered in the all kind foreign market.
7. **Increase in competition:** The process of globalization and liberalization has increased the competition among the different industries. The competition has increased the efficiency and productivity levels of the private as well as public sector.

◙ **NEGATIVE ASPECT OF GLOBALISATION**

1. **Exploitation of workers:** Large MNCs with worldwide network look for the cheapest goods in order to maximize their profits. Workers are forced to work for long hours and work night shifts on a regular basis during the peak season. Workers are denied their fair share of benefits brought by globalization.
2. **Less importance to agriculture:** New economic policy of globalization has ignored the significance of agriculture sector in the Indian economy.
3. **Failure in poverty alleviation:** It has failed to solve the problem of poverty which is a major economic problem of India. The process of globalization has widened the gap between the rich and poor.
4. **Problem for small scale Industries:** The MNCs have entered in the production of such items which were earlier reserved for small scale industries. The small scale industries have failed to compete with the MNCs.
5. **Competition and uncertain employment:** Globalization and the pressure of competition have substantially changed the lives of workers. Faced with growing competition, most employers these days prefer to employ workers flexibly'. This means that workers jobs are no longer secure.

◙ **THE STRUGGLE FOR OR A FAIR GLOBALISATION**

The above evidences indicate that not everyone has been benefited from globalization. People with education, skill and wealth have made the best use of the new opportunities on the other hand, there are many who have not shared the benefits. So no there is need for fair globalization i.e. where all get equal opportunities and development takes place but not at the cost of poor people and environment.

- **Role of Government:**

1. Government should prepare such policies that must protect the interest not only of the rich and the powerful, but of all the people in the country.
2. Government can ensure that labor laws are properly. Implemented and the workers at their rights.
3. Government can reserve some items exclusively for small scale and local producers.
4. If necessary, the government can use trade and investment barriers like quota system, imports duties etc.
5. It can negotiate at the WTO for fair rules.

EXERCISE

◙ VERY SHORT ANSWER TYPE QUESTIONS

1. Define HNCs.
2. Write any two features of a MNC.
3. "Immediately after independence the government of India gave more thrust to the public sector" Why?
4. Why is the production process of MNCs spread out across the globe?
5. Why MNCs are setting their customer care centres in India?
6. What is investment?
7. What is foreign investment?
8. What is (a) Globalization (b) Privatization?
9. Why people usually move from one country to another?
10. What are trade barriers?
11. What is the importance of trade barrier for the government?
12. What is the process of globalization?
13. How has competition benefited people in India?
14. What is fair globalization?

◙ SHORT ANSWER TYPE QUESTIONS

1. List the factors that have promoted globalization. List some benefits of globalization.
2. What are the fears of globalization?
3. What are the characteristics of MNCs?
4. What was the development strategy prior to 1991 adopted by India?
5. New economic policies were required in 1991. Why?
6. How do the MNCs help in the growth of local companies?
7. How has the MNCs changed the world economy?
8. What is the importance of global trade?
9. Mention any four vocational factors of the MNCs.
10. Explain the role of government in fair globalizations.

◙ LONG ANSWER TYPE QUESTIONS

1. "MNCs have international operations and giant size". Explain it.
2. Discuss the major features of new economic policy of liberalization and globalization.
3. What are the advantages & disadvantages of MNCs?
4. "Rapid improvement in technology has been one major factor that has stimulated the globalization process" Explain.
5. What is liberalization? What steps were taken by the government to liberate the Indian economy?
6. What changes have occurred in India due to the adaptation of the policy of liberalization and globalization?
7. What are the positive and negative impacts of globalization?

CONSUMER RIGHTS

◙ IMPORTANT TERMS

1. **Adulteration:** It is a process by which the quality of food substance is lowered through the addition of other inferior substances to make higher profits:
2. **Consumer Awareness:** Consumer's consciousness towards their right and social and legal obligation of the business and the government towards consumers are known as consumer awareness.
3. **Consumer Exploitation:** It is a situation in which consumer is cheated by the producer.
4. **Consumer courts:** These are courts which have been established under the consumer protection Act, 1986 at different levels for the protection and promotion of consumer interest.
5. **Consumer:** Any person who buys anything in the market and uses it.
6. **Consumer protection Act, 1986:** It is one of the most important legal measures in protecting the rights of the consumer.
7. **Agmark:** It is meant for the agricultural marking for agricultural product.

◙ CONSUMER

Any person who buys anything in the market or pays for any service is a consumer.

◙ CONSUMER EXPLOITAION

When a consumer is cheated in any way, either by the shopkeeper or the producer, by giving him poor quality or adulterated goods or by charging more prices for a commodity or a service, it is called consumer's exploitation.

- **Consumers are cheated by the manufacture by the following ways:**

1. **High Prices:** Traders can charge a price higher than the price prevailing in the market because of the ignorance and urgency of the customer.
2. **Underweight and under measurement:** By their cleverness, some traders scoop so low that they cheat the consumer by resorting to underweight and under measurement tactics.
3. **Sub standard quality:** Some traders sell substandard quality products to the consumer. Nowadays markets are full of duplicate products.
4. **Adulterated and impure products:** In edible items such as oil, ghee and spices adulterated is made in order to earn higher profits.
5. **Improper information:** Companies spend a considerable amount on advertisement alone to attract consumers and feed information that they want the consumers to know but not the information the consumers need about the products.
6. **Lack of safety devices:** Some producers try to produce things of poor quality without caring for the standard safeguard norms.
7. **Poor after sale service:** Many things need adequate after sale service. But most of the sellers do not provide it.

◙ MAJOR FACTORS RESPONSIBLE FOR THE EXPLOITATION OF THE CONSUMER

1. Limited information: In the absence of information about different aspects of the products namely price, quality, composition, condition of use etc., the consumers are liable to make a wrong choice and loss money.
2. Wrong information: In the absence of full and correct information a consumer may get exploited.

3. Shortage of supply: Due to under development of industry there is a shortage of supply. This gives rise to hoarding and price rise.

4. Limited competition: Due to under development of industry there is a lack of competition in the market. This may lead to exploitation of the consumers.

5. Illiteracy and ignorance of the consumers: In most of the developing and under developed economies illiteracy rate is very higher consumers can be easily cheated by the producer.

◙ **NEED FOR RULES AND REGULATIONS TO SAVE THE CONSUMERS**

1. Producers are spending a lot of money to influence the consumers which makes difficult for the consumers to make a' correct choice so there is, need for consumer awareness.

2. Producers do not provide sufficient information to the consumers and even some times harass them.

3. Consumer awareness is also must because at times greedy traders begin to play with, the health of the people by indulging in adulteration of edible oils, milk, butter etc.

4. There is need for rules and regulations because most of goods and services are being produced by private sector with profits as main motive.

◙ **CONSUMER MOVEMENTS**

The consumer movement arose out of dissatisfaction of the consumer as money unfair practices were being indulged in by the sellers. Before 80's there was no legal system available to consumers to protect them from exploitation in the market place for a long time.

In India, the consumer movement as a "social force" originated with the necessity of protecting and promoting the interest of consumer against unfair trade practices. Food shortage, hoarding, black marketing high prices gave birth to consumer movement in an organized form in the 1960s. More recently, India witnessed an upsurge in the number of consumer groups. This is because more & more cases of the consumer exploitation by the private sector. The activities of various consumer forms forced the government to enact the consumer Protection Act 1986, popularly known as COPRA.

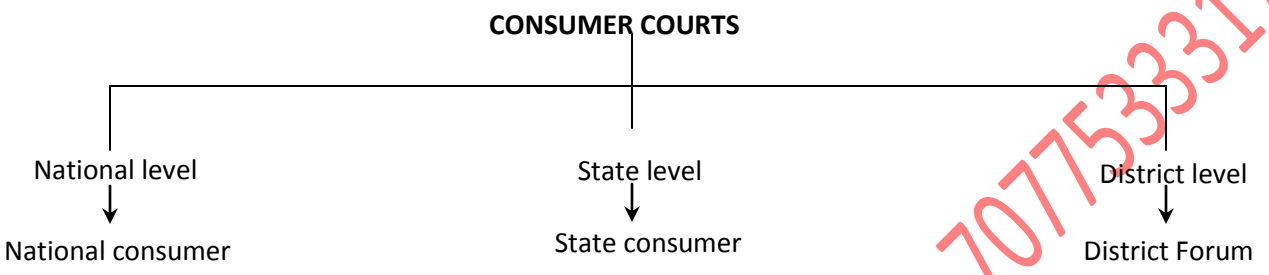
- **Main features of the Act.**

1. The act, applies to all goods and services.
2. It covers all the sectors whether public, private or Co-operative.

◙ **RIGHTS OF THE CONSUMERS**

- **Right to safety:** The consumers have right to be protected against marketing of goods and services which are hazardous to life and property.
- **Right to be informed:** The consumers have the right to know about the quality, quantity and price of goods, date of manufacture, expiry dates etc. This right has been provided to the consumer so that consumer can approach the courts if cheated.
- **Right to choose:** Consumers have the right to check the variety of goods and services at competitive prices. This right gives the consumer assuring that no producer can force him/her to purchase a particular brand.

- **Right to seek Redressal:** The consumer has the right to seek Redressal against unfair trade practices and exploitation. If any damage has been done to the consumer. He/she has the right to get compensation depending upon degree of damage.
- **Consumer Courts:** Most important feature of the consumer protection Act is the provision of three 'e' tier system popularly known as consumer courts.



1. These courts look into the grievances and complaints of the consumers against the traders and manufactures and provide the necessary relief and compensation.
2. These courts are required to dispose of each complaint within three months.
3. These courts reduce burden on other courts.

- **STANDARDISATION**

For maintaining the minimum standard government has created various institutions:

1. Through standardization of product the government tries to protect the consumers from lack of quality and varying standards of goods.
2. Government of India has established two agencies to check the quality and standard of the products.

(a) Bureau of Indian standards: It has the responsibility of laying down the standards for industrial and consumer goods on a scientific basis and certifying the goods that meet the standard quality.

(b) Agmark: It is implemented under the Agricultural Produce Act, 1937, as amended in 1986. This scheme is run by DMI in the ministry of Agriculture Government of India. Products such as honey, Masala and spices carry such marks.

3. If any consumer is dissatisfied with the quality of a certified product, he/she can complain to the nearest office of the BIS.

◙ **DUTIES OF A WELL INFORMED CONSUMER**

1. While purchasing the goods, consumer should look at the quality of the product, the marked price, guarantee or warrantee card/period.
2. Consumer should preferably purchase standardize products which contain seal of ISI or Agmark.
3. Consumer should ask for a cash-memo and warrantee card.
4. Consumer must be aware of his rights and duties.

5. Consumer should form consumer awareness organization, which can be given representation in various committees formed by the government.

6. He should know the method how to proceed if he is cheated.

EXERCISE

◙ VERY SHORT ANSWER TYPE QUESTIONS

1. What do you mean by COPRA?
2. What is meant by consumer exploitation?
3. Define term consumer protection?
4. In which month world consumer day is celebrated?
5. When was RTI enacted?
6. What is the full form of RTI and COPRA?
7. Where is headquarter of BIS is situated?
8. What do you mean by hoarding?
9. Name the consumer court at state, District and national level.
10. Explain two ways by which consumers are exploited by producer or wholesalers.
11. Mention the rights of consumers.
12. What are the duties of a well aware consumer?
13. What is a consumer?
14. When COPRA was enacted?
15. What is consumer awareness?
16. When did consumer movement begin in India?
17. What is adulteration?
18. What is the full form of: MRP, ISO, and FAO?
19. What is the role of government to protect consumer?
20. How do duplicate articles and adulteration cause heavy loss to the consumer?
21. What do you know about the right to be informed?
22. Write a note on Agmark.
23. What do you mean by consumer movement?
24. What is right to choose?
25. Define standardization.

◙ WRITE THE ANSWER IN DETAIL

1. Describe the consumer court set up in India under COPRA.
2. Analyze the functions of the consumer protection council.
3. What do you know about consumer protection? Why?

4. Explain COPRA.

5. Explain the following consumer rights:

(i) Right to representation (ii) Right to Redressal (iii) Right to safety

6. What is the standardization of the product?

7. Discuss the salient features of consumer protection Act 1986.

8. What are the various ways in which a consumer can be exploited?

9. What is the need for consumer awareness?

10. What are consumer forums? What is their importance?