SIMILAR TRIANGLES

★ **INTRODUCTION**

In earlier classes, you have learnt about congruence of two geometric figures, and also some basic theorems and results on the congruence of triangle. Two geometric figures having same shape and size are congruent to each other but two geometric figures having same shape are called similar. Two congruent geometric figures are always similar but the converse may or may not be true.

All regular polygons of same number of sides such as equilateral triangle, squares, etc, are similar. All circles are similar.

In some cases, we can easily notice that two geometric figures are not similar. For example, a triangle and a rectangle can never be similar. In case, we are given two triangles, they may appear to be similar but actually they may not be similar. So, we need some criteria to determine the similarity of two geometric figures. In particular, we shall discuss similar triangles.

★ **HISTORICAL FACTS**

EUCLID was a very great Greek mathematician born about 2400 years ago. He is called the father of geometry
because he was the first to establish a school of mathematics in Alexandria. He wrote a book on geometry called "The Elements" which has 13 volumes and has been used as a text book for over 2000 years. This book was further systematized by the great mathematician of Greece tike Thales, Pythagoras, Pluto and Aristotle.

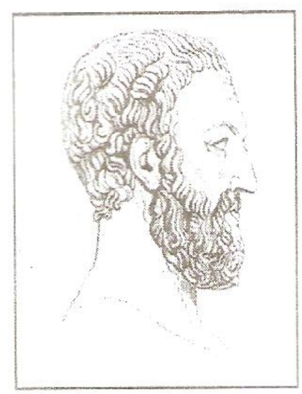
Euclid : Father of Geometry
(about 300 B.C. Greece)

Abraham Lincoln, as a young lawyer was of the view that this Greek book was a splendid sharpener of human mind and improver his power of logic and language.

A king once asked Euclid, "Isn't there an easier way to understand geometry"

Euclid replied : "There is no royal-road way to geometry. Every one has to think for himself when studying."

THALES (640-546 B.C.) a Greek mathematician was the first who initiated and formulated the theoretical study of geometry to make astronomy a more exact science. He is said to have introduced geometry in Greece. He is believed to have found the heights of the pyramids in Egypt, using shadows and the principle of similar triangles. The use of similar triangles has made possible the measurements of heights and distances. He proved the well-known and very useful theorem credited after his name : Thales Theorem.



Thales (640-546 B.C.)

★ **CONGRUENT FIGURES**

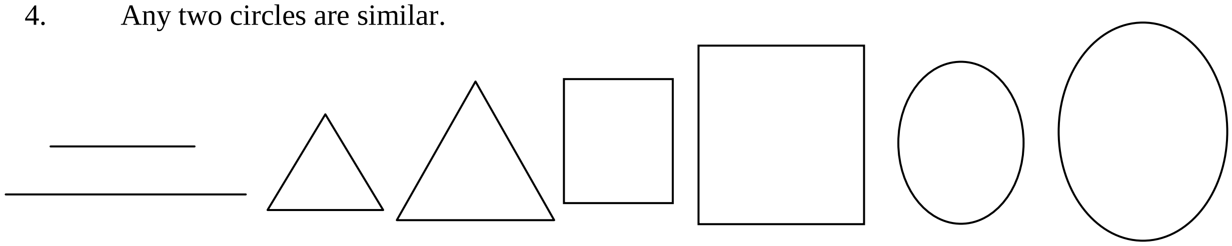
Two geometrical figures are said to be congruent, provided they must have same shape and same size. Congruent figures are alike in every respect.

EX. 1. Two squares of the same length.
2. Two circle of the same radii.
3. Two rectangles of the same dimensions.
4. Two wings of a fan.
5. Two equilateral triangles of same length.

★ **SIMILAR FIGURES**

Two figures are said to be similar, if they have the same shape. Similar figures may differ in size. Thus, two congruent figures are always similar, but two similar figures need not be congruent.

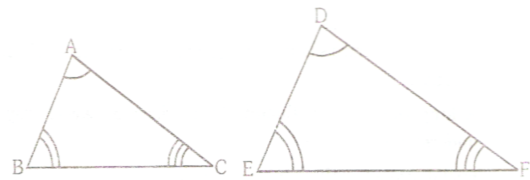
EX. 1. Any two line segments are similar.
2. Any two equilateral triangles are similar
3. Any two squares are similar.
4. Any two circles are similar.



We use the symbol '~' to indicate similarity of figures.

★ **SIMILAR TRIANGLES**

ΔABC and ΔDEF are said to be similar, if their corresponding angles are equal and the corresponding sides are proportional.



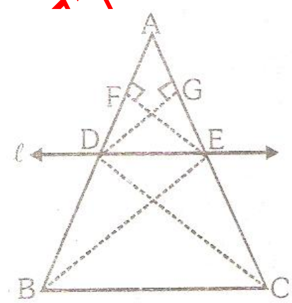
i.e., when $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$.

And, we write $\Delta ABC \sim \Delta DEF$.

The sign '~' is read as 'is similar to'.

THEOREM-1 (Thales Theorem or Basic Proportionality Theorem) : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.



Given : A ΔABC in which line ℓ parallel to BC (DE$\parallel$BC) intersecting AB at D and AC at E.

To prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

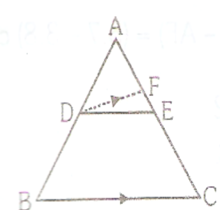
Construction : Join D to C and E to B. Through E drawn EF perpendicular to AB i.e., EF $\perp$ AB and through D draw DG $\perp$ AC.

Proof :

	STATEMENT	**REASON**
1.	Area of (ΔADE) $= \frac{1}{2}(AD \times EF)$ Area of (ΔBDE) $= \frac{1}{2}$ (BD × EF)	Area of $\Delta = \frac{1}{2}$ base × altitude
2.	$\dfrac{Area(\Delta ADE)}{Area(\Delta BDE)} = \dfrac{\frac{1}{2}AD \times EF}{\frac{1}{2}BD \times EF} = \dfrac{AD}{DB}$	By 1.
3.	$\dfrac{Area(\Delta ADE)}{Area(\Delta BDE)} = \dfrac{\frac{1}{2}AE \times DG}{\frac{1}{2}EC \times DG} = \dfrac{AE}{EC}$	Similarly
4.	Area (ΔBDE) = Area (ΔCDE)	Δs BDE and CDE are on the same base BC and between the same parallel lines DE and BC.
5.	$\dfrac{Area(\Delta ADE)}{Area(\Delta BDE)} = \dfrac{AE}{EC}$	By 3. & 4.
6.	$\dfrac{AD}{DB} = \dfrac{AE}{EC}$	By 1. & 5.

Hence proved.

THEOREM-2 (Converse of Basic Proportionality Theorem) : If a line divided any two sides of a triangle proportionally, the line is parallel to the third side.



Given : A ΔABC and DE is a line meeting AB and AC at D and E respectively such that

$\dfrac{AD}{DB} = \dfrac{AE}{EC}$

To prove : DE$\parallel$BC

Proof :

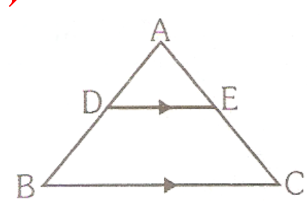
STATEMENT		REASON
1.	If possible, let DE be not parallel to BC. Then, draw DF $\parallel$ BC	
2.	$\frac{AD}{DB}=\frac{AE}{FC}$	By Basic Proportionality Theorem.
3.	$\frac{AD}{DB}=\frac{AE}{EC}$	Given
4.	$\therefore \frac{AF}{FC}=\frac{AE}{EC}$	From 2 and 3.
	$\Rightarrow \frac{AF}{FC}+1=\frac{AE}{EC}+1$	Adding 1 on both sides.
	$\Rightarrow \frac{AF+FC}{FC}=\frac{AE+EC}{EC}$	By adding.
	$\Rightarrow \frac{AC}{FC}=\frac{AC}{EC}$	AF + FC = AC and AE + EC = AC.
	$\Rightarrow FC=EC \Rightarrow$ E and F coincide. But, DF $\parallel$ BC. Hence DE $\parallel$ BC.	

Hence, proved.

Ex.1 In the adjoining figure. DE $\parallel$ BC.

(i) If AD = 3.4 cm, AB = 8.5 cm and AC = 13.5 cm, find AE

(ii) If $\frac{AD}{DB}=\frac{3}{5}$ and AC = 9.6 cm, find AE.



Sol. (i) Since DE $\parallel$ BC, we have $\frac{AD}{AB}=\frac{AE}{AC}$

$\therefore \quad \frac{3.4}{8.5}=\frac{AE}{13.5} \Rightarrow \frac{3.4\times 13.5}{8.5}=5.4$

Hence, AE = 5.4 cm.

(i) Since DE $\parallel$ BC, we have $\frac{AD}{DB}=\frac{AE}{EC}$

$\therefore \quad \frac{AE}{AC}=\frac{3}{5}\left[\because \frac{AD}{DB}=\frac{3}{5}(Given)\right]$

Let AE = x cm. Then, EC = (AC – AE) = (9.6 – x) cm.

$\therefore \quad \frac{x}{9.6-x}=\frac{3}{5} \Rightarrow 5x=3(9.6-x)$

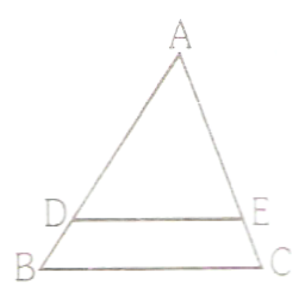
$\Rightarrow$ 5x = 28.8 – 3x $\Rightarrow$ 8x = 28.8 $\Rightarrow$ x = 3.6.

$\therefore$ AE = 3.6 cm.

Ex.2 In the adjoining figure, AD = 5.6 cm, AB = 8.4 cm, AE = 3.8 cm and AC = 5.7 cm. Show that DE $\parallel$ BC.

Sol. We have, AD = 5.6 cm, DB = (AB – AD) = (8.4 – 5.6) cm = 2.8 cm.

AE = 3.8 cm, EC = (AC – AE) = (5.7 – 3.8) cm = 1.9 cm.



$\therefore \quad \frac{AD}{DB}=\frac{5.6}{2.8}=\frac{2}{1}$ and $\frac{AE}{EC}\frac{3.8}{1.9}=\frac{2}{1}$

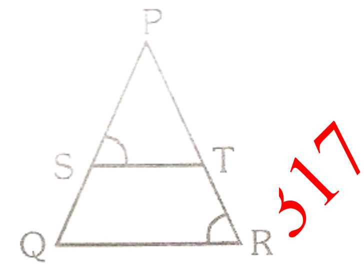
Thus, $\frac{AD}{DB}=\frac{AE}{EC}$

$\therefore$ DE divides AB and AC proportionally.

Hence, DE $\parallel$ BC

Ex.3 In fig, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.

[NCERT]



Sol. It is given that $\frac{PS}{SQ} = \frac{PT}{TR}$

So, ST ∥ QR [Theorem]

Therefore, $\angle PST = \angle PQR$ [Corresponding angles] – (1)

Also, it is given that

$\angle PST = \angle PRQ$ (2)

So, $\angle PRQ = \angle PQR$ [Form 1 and 2]

Therefore PQ = PR [Sides opposite the equal angles]

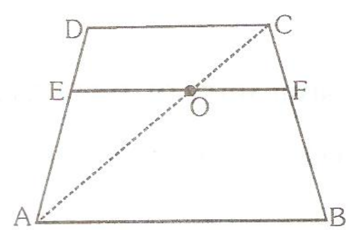
i.e., PQR is an isosceles triangle.

Ex.4 Prove that any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally (i.e., In the same ratio).

OR

ABCD is a trapezium with DE ∥ AB. E and F are points on AD and BC respectively such that EF ∥ AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$

[NCERT]



Sol. We are given trapezium ABCD.

CD ∥ BA

EF ∥ AB and CD both

We join AC.

It Mets EF at O.

In ΔACD, OE ∥ CD

$\Rightarrow \frac{AO}{OC} = \frac{AE}{ED}$ …(i)

(Basic Proportionality Theorem)

In ΔCAB, OF ∥ AB

$\Rightarrow \frac{CO}{OA} = \frac{CF}{FB}$ [B.P.T.]

$\Rightarrow \frac{AO}{OC} = \frac{BF}{FC}$ …(ii)

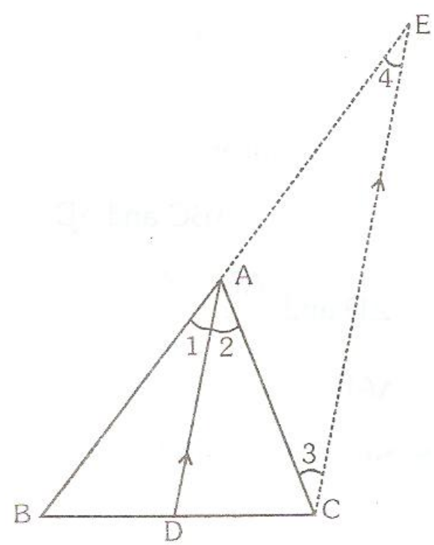
From (i) and (ii)

$\frac{AE}{ED} = \frac{BF}{FC}$

Hence, proved.

Ex.5 Prove that the internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.

(Internal Angle Bisector Theorem)



Sol. **Given :** A ΔABC in which AD is the internal bisector of $\angle A$.

To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$

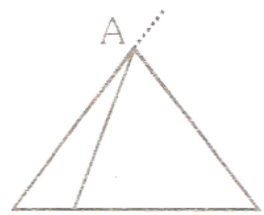
Construction : Draw CE ∥ DA, meeting BA produced at E.

Proof :

	STATEMENT	REASON
1.	$\angle 1 = \angle 2$	AD is the bisector of $\angle A$
2.	$\angle 2 = \angle 3$	Alt. $\angle$s are equal, as CE$\|$DA and AC is the transversal
3.	$\angle 1 = \angle 4$	Corres. $\angle$s are equal, as CE$\|$DA and BE is the transversal
4.	$\angle 3 = \angle 4$	From 1, 2 and 3.
5.	AE = AC	Sides opposite to equal angles are equal
6.	In ΔBCE, DA$\|$CE	
	$\Rightarrow \frac{BD}{DC} = \frac{BA}{AE}$	By B. P. T.
	$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$	Using 5

Hence, proved.

Remark : The external bisector of an angle divides the opposite side externally in the ratio of the sides containing the angle. i.e., if in a ΔABC, AD is the bisector of the exterior of angle $\angle A$ and intersect BC produced in D, $\frac{BD}{CD} = \frac{AB}{AC}$.



★ **AXIOMS OF SIMILARITY OF TRIANGLES**

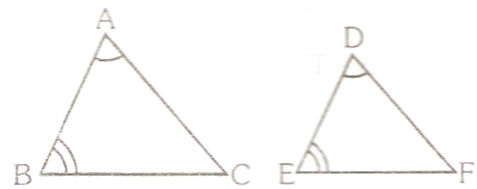
1. AA (Angle-Angle) Axiom of Similarity :

If two triangles have two pairs of corresponding angles equal, then the triangles are similar. In the given figure,

ΔABC and ΔDEF are such that

$\angle A = \angle D$ and $\angle B = \angle E$.

$\therefore \quad \Delta ABC \sim \Delta DEF$



2. SAS (Side-Angle-Side) Axiom of Similarity :

If two triangles have a pair of corresponding angles equal and the sides including them proportional, then the triangles are similar.

In the given fig, ΔABC and ΔDEF are such that

$\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$

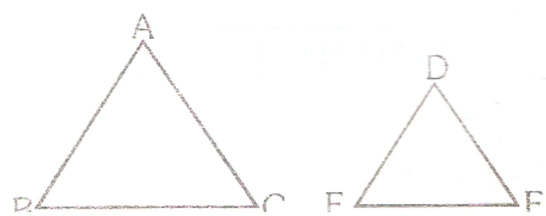
$\therefore \quad \Delta ABC \sim \Delta DEF$

3. SSS (Side- Side- Side) Axiom of Similarity :

If two triangles have three pair of corresponding sides proportional, then the triangles are similar.

If in ΔABC and ΔDEF we have :

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$, then $\Delta ABC \sim \Delta DEF$.



Ex.6. In figure, find $\angle L$.

Sol. In ΔABC and ΔLMN,

$$\frac{AB}{LM} = \frac{4.4}{11} = \frac{2}{5}$$

$$\frac{BC}{MN} = \frac{4}{10} = \frac{2}{5} \text{ and } \frac{CA}{NL} = \frac{3.6}{9} = \frac{2}{5}$$

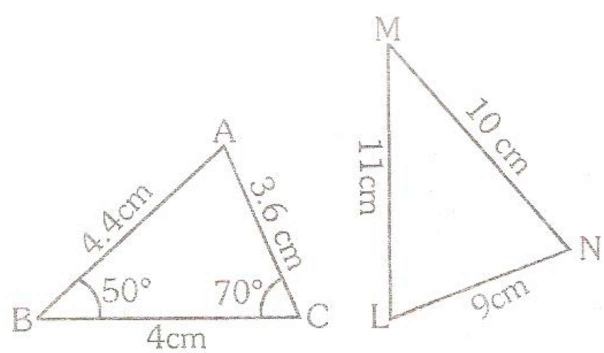
$$\Rightarrow \quad \frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{NL}$$

$\Rightarrow \quad \Delta ABC \sim \Delta LMN$ (SSS Similarity)

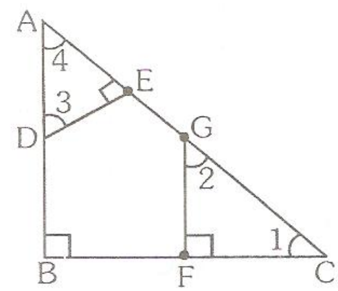
$\Rightarrow \quad \angle L = \angle A = 180^0 - \angle B - \angle C$

$= 180^0 - 50^0 - 70^0 = 60^0$

$\therefore \quad \angle L = 60^0$



Ex.7 In the figure, $AB \perp BC$, $DE \perp AC$, and $GF \perp BC$, Prove that $\Delta ADE \sim \Delta GCF$.



Sol. $\angle 1 + \angle 4 = \angle 1 + \angle 2$ (each side = 90^0)

$\Rightarrow$ $\angle 4 = \angle 2$

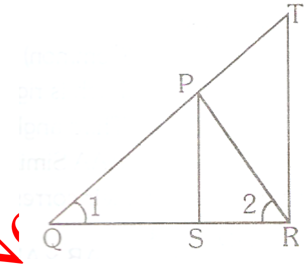
$\Rightarrow$ $\angle A = \angle G$ …(i)

Also $\angle E = \angle F$ …(ii) (each equal to 90^0)

From (i) and (ii), we get AA similarity for triangle ADE and GCF.

$\Rightarrow$ $\Delta ADE \sim \Delta GCF$

Ex.8 In fig, $\frac{QT}{PR} = \frac{QR}{QS}$ and $\angle 1 = \angle 2$. Prove that $\Delta PQS \sim \Delta TQR$.



Sol. $\angle 1 = \angle 2$ (Given)

$\Rightarrow$ PR = PQ …(i)

(Sides opposite to equal angles in ΔQRP)

Also $\frac{QT}{PR} = \frac{QR}{QS}$ (Given) …(i)

From (i) and (ii), we have

$\frac{QT}{PR} = \frac{QR}{QS} \Rightarrow \frac{QP}{QT} = \frac{QS}{QR}$ …(iii)

Now, in triangles PQR and TQR, we have

$\angle PQS = \angle TQR$ (each = $\angle 1$)

and $\frac{PQ}{TQ} = \frac{QS}{QR}$ (from (3))

$\Rightarrow$ $\Delta PQS \sim \Delta TQR$ (SAS Similarity)

Ex.9 In fig, CD and GH are respectively, the medians of ΔABC and ΔFEG, If $\Delta ABC \sim \Delta FEG$, prove that

(i) $\Delta ADC \sim \Delta FHG$

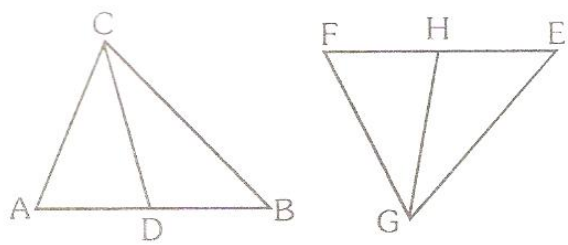
(ii) $\frac{CD}{GH} = \frac{AB}{FE}$ **(NCERT)**

Sol. $\Delta ABC \sim \Delta FEG$ (given)

$\Rightarrow$ $\angle A = \angle F$, …(i) ($\because$ the corresponding angles of the similar triangles are equal)

Also, $\frac{AC}{FG} = \frac{AB}{FE}$ (Corresponding sides are proportional)

$\Rightarrow$ $\frac{AC}{FG} = \frac{2AB}{2FH}$ $\begin{pmatrix} D \text{ is mid-point of } AB \\ H \text{ is mid-point of } FE \end{pmatrix}$



$\Rightarrow$ $\frac{AC}{AD} = \frac{FG}{FH}$ …(ii)

Now, in triangles ADC and FHG, we have

$\angle A = \angle F$ and $\frac{AC}{AD} = \frac{FG}{FH}$ (By (i) and (ii))

$\Rightarrow$ $\Delta ADC \sim \Delta FHG$ (SAS similarity)

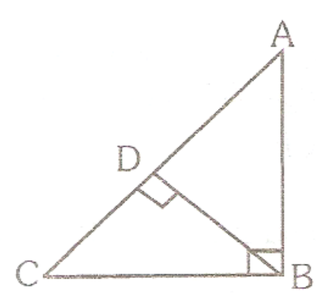
(ii) $\Delta ADC \sim \Delta FHG$

$\Rightarrow$ $\frac{CD}{GH} = \frac{AD}{FH}$ (Corresponding sides proportional)

$\Rightarrow$ $\frac{CD}{GH} = \frac{2 \times AD}{2 \times FH}$ $\Rightarrow$ $\frac{CD}{GH} = \frac{AB}{FE}$

Ex,10 ABC is a right triangle, right angled at B. If BD is the length of perpendicular drawn from B to AC. Prove that :

(i) $\Delta ADB \sim \Delta ABC$ and hence $AB^2 = AD \times AC$ (ii) $\Delta BDC \sim \Delta ABC$ and hence $BC^2 = CD \times AC$



(iii) $\Delta ADB \sim \Delta BDC$ and hence $BD^2 = AD \times DC$ (iv) $\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{BD^2}$

Sol. **Given :** ABC is right angled triangle at B and $BD \perp AC$

To prove :

(i) $\Delta ADB \sim \Delta ABC$ and hence $AB^2 = AD \times AC$

(ii) $\Delta BDC \sim \Delta ABC$ and hence $BC^2 = CD \times AC$

(iii) $\Delta ADB \sim \Delta BDC$ and hence $BD^2 = AD \times DC$

(iv) $\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{BD^2}$

Proof : (i) In two triangles ADB and ABC, we have :

$\angle BAD = \angle BAC$ (Common)

$\angle ADB = \angle ABC$ (Each is right angle)

$\angle ABD = \angle ACB$ (Third angle)

$\angle ADB = \angle ABC$ (AAA Similarity)

Triangle ADB and ABC are similar and so their corresponding sides must be proportion.

$\frac{AD}{AB} = \frac{DB}{DC} = \frac{AB}{AC} \Rightarrow \frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AB \times AB = AC \times AD \Rightarrow AB^2 = AD \times AC$. This proves (a).

(ii) Again consider two triangles BDC and ABC, we have

$\angle BCD = \angle ACB$ (Common)

$\angle BDC = \angle ABC$ (Each is right angle)

$\angle DBC = \angle BAC$ (Third angle

$\therefore$ Triangle are similar and their corresponding sides must be proportional.

i.e., $\angle BAD = \angle BAC$ $\frac{BD}{AB} = \frac{DC}{BC} = \frac{BC}{AC}$

(iii) In two triangle ADB and BDC, we have :

$\Rightarrow$ $\frac{DC}{BC} = \frac{BC}{AC} \Rightarrow BC \times BC = DC \times AC \Rightarrow BC^2 = CD \times AC$ This proves (ii)

$\angle BDA = \angle BDC = 90^0$

$\angle 3 = \angle 2 = 90^0 \angle 1$ $[\because \angle 1 + \angle 2 = 90^0, \angle 1 + \angle 3 = 90^0]$

$\angle 1 = \angle 4 = 90^0 \angle 2$ $[\because \angle 1 + \angle 2 = 90^0, \angle 2 + \angle 4 = 90^0]$

$\Delta ADB \sim \Delta BDC$ (AAA criterion of similarity)

$\Rightarrow$ Their corresponding sides must be proportional.

$\frac{AD}{BD} = \frac{DB}{DC} = \frac{AB}{BC} \Rightarrow \frac{AD}{BD} = \frac{DB}{DC} \Rightarrow BD \times BD = AD \times AC$

$\therefore$ BD is the mean proportional of AD and DC

(iv) From (i), we have : AB2 = AD$\times$AC

(ii), we have : BC2 = CD$\times$AC

(iii), we have : BD2 = ADDC

Consider $\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{AD \times AC} + \frac{1}{CD \times AC} + \frac{1}{AC}\left[\frac{1}{AD} + \frac{1}{DC}\right]$

$\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{AC}\left[\frac{DC}{AD} + \frac{AD}{DC}\right] = \frac{1}{AC}\left[\frac{AD+DC}{AD \times DC}\right] = \frac{1}{AC}\left[\frac{AC}{AD \times DC}\right]$

$= \frac{1}{AD \times DC} = \frac{1}{BD^2}$ (from (iii))

$\frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{BD^2}$

Thus we have proved the following :

If a perpendicular is drawn from the vertex containing the right angle of a right triangle to the hypotenuse then:

(a) Thu triangle on each side of the perpendicular are similar to each other and also similar to the original triangle.

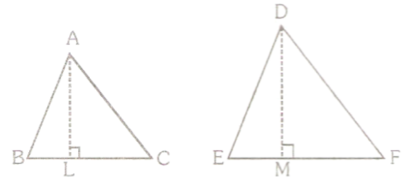
i.e., $\Delta ADB \sim \Delta BDC$, $\Delta ADB \sim \Delta ABC$, $\Delta BDC \sim \Delta ABC$

(b) **The square of the perpendicular is equal to the product of the length of two parts into which the hypotenuse is divided by the perpendicular i.e., BD2 = AD** $\times$ 0DC.

★ RESULTS ON AREA OF SIMILAR TRIANGLES

Theorem-3 : **The areas of two similar triangles are proportional to the squares on their corresponding sides.**

Given : $\Delta ABC \sim \Delta DEF$



To prove : $\frac{Area\,of\ \Delta ABC}{Area\,of\ \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Construction : Draw $AL \perp BC$ and $DM \perp EF$.

Proof:

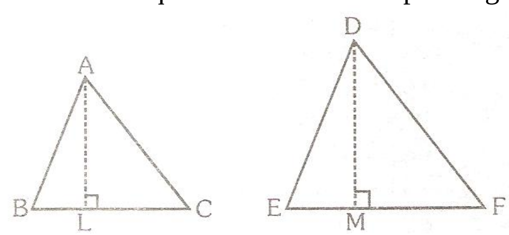
	STATEMENT	REASON
1.	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM}$ $\Rightarrow \frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{BC}{EF} \times \frac{AL}{DM}$	Area of $\Delta = \frac{1}{2} \times$ Base $\times$ Height
2.	In Δ ALB and Δ DME, we have (i) $\angle$ ALB = $\angle$ DME (ii) $\angle$ ABL = $\angle$ DEM $\therefore \Delta ALB \sim \Delta DME$ $\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$	Each equal to 90^0 $\Delta ABC \sim \Delta DEF \Rightarrow \angle B = \angle E$ AA=axiom Corresponding sides of similar Δ s are proportional.
3.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	Given. Corresponding sides of similar Δ s are proportional.
4.	$\frac{AL}{DM} = \frac{BC}{EF}$	From 2 and 3.
5.	Substituting $\frac{AL}{DM} = \frac{BC}{EF}$ in 1, we get : $\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{BC^2}{EF^2}$	
6.	Combining 3 and 5, we get : $\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$	

Corollary-1 : The areas of two similar triangles are proportional to the squares on their corresponding altitude.

Given : $\Delta ABC \sim \Delta DEF$, $AL \perp BC$ and $DM \perp EF$.

To prove : $\frac{Area\,of\ \Delta ABC}{Area\,of\ \Delta DEF} = \frac{AL^2}{DM^2}$

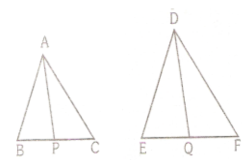
Proof :



	STATEMENT	REASON
1.	$\dfrac{Area\,\Delta ABC}{Area\,\Delta DEF} = \dfrac{\frac{1}{2}\times BC\times AL}{\frac{1}{2}\times EF\times DM}$	Area of $\Delta = \frac{1}{2}\times$ Base $\times$ Height
	$\Rightarrow \dfrac{Area\,\Delta ABC}{Area\,\Delta DEF} = \dfrac{BC}{EF} = \dfrac{AL}{DM}$	
2.	In Δ ALB and Δ DME, we have	
	(i) $\angle$ ALB = $\angle$ DME (ii) $\angle$ ABL = $\angle$ DEM	Each equal to 90^0
	$\Rightarrow \Delta$ ALB ~ Δ DME	Δ ABC ~ Δ DEF $\Rightarrow \angle$ B = $\angle$ E
		AA=axiom
	$\Rightarrow \dfrac{AL}{DM} = \dfrac{AB}{DE}$	
	Δ ABC ~ Δ DEF	Corresponding sides of similar Δ s are proportional.
3.	$\Rightarrow \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$	Given.
	$\dfrac{BC}{EF} = \dfrac{AL}{DM}$	Corresponding sides of similar Δ s are proportional.
4.	Substituting $\dfrac{BC}{EF} = \dfrac{AL}{DM}$ in 1, we get :	From 2 and 3.
		Hence proved.
5.	$\dfrac{Area\,\Delta ABC}{Area\,\Delta DEF} = \dfrac{AL^2}{DM^2}$	

Corollary-2 : The areas of two similar triangles proportional to the squares on their corresponding medians.

Given : Δ ABC ~ Δ DEF and AP, PQ are their medians. **To prove :** $\dfrac{Area of\ \Delta ABC}{Area of\ \Delta DEF} = \dfrac{AP^2}{DQ^2}$



Proof :

STATEMENT	REASON

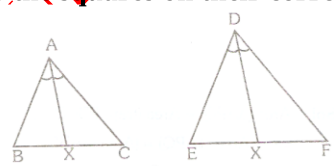
1.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AB^2}{DE^2}$ ……I.	Given Area of two similar Δs are proportional to the squares on their corresponding sides.
2.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{2BP}{2EQ} = \frac{BP}{EQ}$ ……II.	Corresponding sides of similar Δs are proportional
3.	$\frac{AB}{DE} = \frac{BP}{EQ}$ and $\angle A = \angle D$ $\Rightarrow \Delta APB \sim \Delta DQE$ $\Rightarrow \frac{BP}{EQ} = \frac{AP}{DQ}$ …….III $\Rightarrow \frac{AB}{DE} = \frac{AP}{DQ}$ $\Rightarrow \frac{AB^2}{DE^2} = \frac{AP^2}{DQ^2}$ ……..IV	From II and the fact the $\Delta ABC \sim \Delta DEF$ By SAS-similarity axiom From II and III.
4.	$\Rightarrow \frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AP^2}{DQ^2}$	From I and IV.

Hence, proved

Corollary-3 : The areas of two similar triangles proportional to the squares on their corresponding angle bisector segments.

Given : $\Delta ABC \sim \Delta DEF$ and AX, DY are their

To prove : $\frac{Area\,of\,\Delta ABC}{Area\,of\,\Delta DEF} = \frac{AX^2}{DY^2}$

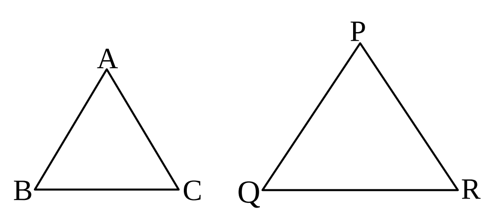


Proof :

	STATEMENT	**REASON**
1.	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AB^2}{DE^2}$	Area of two similar Δs are proportional to the squares on their corresponding sides.
2.	$\Delta ABC \sim \Delta DEF$ $\Rightarrow \angle A = \angle D$ $\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle D$ $\Rightarrow \angle BAX = \angle EDY$	Given $\Rightarrow \angle BAX = \frac{1}{2}\angle A$ and $\angle EDY = \frac{1}{2}\angle D$
3.	**In** ΔABX and ΔEDY, we have $\angle BAX = \angle EDY$ $\angle B = \angle E$ $\therefore \Delta ABX \sim \Delta DEY$ $\Rightarrow \frac{AB}{DE} = \frac{AX}{DY} \Rightarrow \frac{AB^2}{DE^2} = \frac{AX^2}{DY^2}$	Given From 2. $\Delta ABC \sim \Delta DEF$ By AA similarity axiom
4.	$\frac{Area\,\Delta ABC}{Area\,\Delta DEF} = \frac{AX^2}{DY^2}$	From 1 and 3.

Ex.11 It is given that $\Delta ABC \sim \Delta PQR$, area ($\Delta ABC$) = 36 cm^2 and area (ΔPQR) = 25 cm^2. If QR = 6 cm, find length of BC.

Sol. We know that the areas of similar triangles are proportional to the squares of their corresponding sides.



$$\therefore \quad \frac{Area\,of\,(\Delta ABC)}{Area\,of\,(\Delta PQR)} = \frac{BC^2}{QR^2}$$

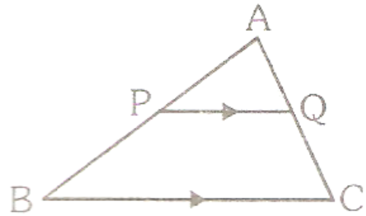
Let BC = x cm. Then.

$$\frac{36}{25} = \frac{x^2}{6^2} \Leftrightarrow \frac{36}{25} = \frac{x^2}{36} \Leftrightarrow x^2 = \frac{36\times 36}{25} \Leftrightarrow x = \left(\frac{6\times 6}{5}\right) = \frac{36}{5} = 7.2$$

Hence BC = 7.2 cn

Ex.12 P and Q are points on the sides AB and AC respectively of Δ ABC such that PQ $\|$ BC and divides Δ ABC into parts, equal in area. Find PB : AB.

Sol. Area (Δ APQ) = Area (trap. PBCQ) [Given]



$\Rightarrow$ Area (Δ APQ) = [Area (Δ ABC) – Area (Δ APQ)

$\Rightarrow$ 2 Area (Δ APQ) = Area (Δ ABC)

$$\Rightarrow \quad \frac{Area\,of\,(\Delta APQ)}{Area\,of\,(\Delta ABC)} = \frac{1}{2} \quad \ldots(i)$$

Now, in Δ APQ and Δ ABC, we have

$\angle$PAQ = $\angle$BAC [Common $\angle$A]

$\angle$APQ = $\angle$ABC [PQ $\|$ BC, corresponding $\angle$s are equal]

$\therefore$ Δ APQ ~ Δ ABC.

We known that the areas of similar Δs are proportional to the squares of their corresponding sides.

$$\therefore \quad \frac{Area\,of\,(\Delta APQ)}{Area\,of\,(\Delta ABC)} = \frac{AP^2}{AB^2} \Rightarrow \frac{AP^2}{AB^2} = \frac{1}{2} \qquad \text{[Using (i)]}$$

$\Rightarrow \quad \frac{AP}{AB} = \frac{1}{\sqrt{2}}$ i.e., AB = $\sqrt{2}$. AP

$\Rightarrow$ AB = $\sqrt{2}$ (AB – PB) $\Rightarrow \sqrt{2}$ PB = ($\sqrt{2}$ – 1) AB

$$\Rightarrow \quad \frac{PB}{AB} = \frac{(\sqrt{2}-1)}{\sqrt{2}}.$$

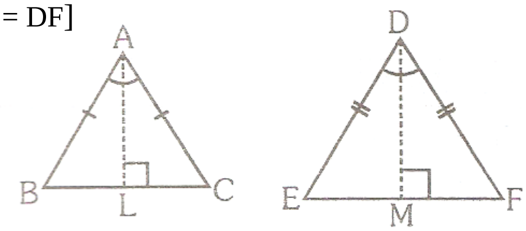
$\therefore$ PB : AB = ($\sqrt{2}$ – 1) : $\sqrt{2}$

Ex.13 Two isosceles triangles have equal vertical angles and there areas are in the ratio 16 : 25. Find the ratio of their corresponding heights.

Sol. Let Δ ABC and Δ DEF be the given triangles in which AB = AC, DE = DF, $\angle$A = $\angle$D and

$\frac{Area\,of\,(\Delta ABC)}{Area\,of\,(\Delta DEF)} = \frac{16}{25}$ Draw AL $\perp$ BC and DM $\perp$ EF

Now, $\frac{AB}{AC} = 1$ and $\frac{DE}{DF} = 1$ [$\because$ AB = AC and DE = DF]



$\Rightarrow \quad \frac{AB}{DE} = \frac{AC}{DF}$.

$\therefore$ In Δ ABC and Δ DEF, we have

$\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle$A = $\angle$D

$\Rightarrow$ Δ ABC ~ Δ DEF [By SAS similarity axiom]

But, the ratio of the areas of two similar Δs is the same as the ratio of the square of their corresponding heights.

$$\frac{Area\,of\,(\Delta ABC)}{Area\,of\,(\Delta DEF)} = \frac{AL^2}{DM^2} \quad \Rightarrow \quad \frac{16}{25} = \left(\frac{AL^2}{DM^2}\right) \quad \Rightarrow \quad \frac{AL}{DM} = \frac{4}{5}$$

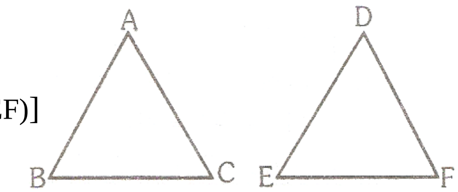
$\therefore$ AL : DM = 4 : 5, i.e., the ratio of their corresponding heights = 4 : 5,

Ex.14 If the areas of two similar triangles are equal, prove that they are congruent.

Sol. Let Δ ABC ~ Δ DEF and area (Δ ABC) = area (Δ DEF).

Since the ratio of the areas of two similar Δs is equal to the ratio of the squares on their corresponding sides, we have

$$\frac{Area\,of\,(\Delta ABC)}{Area\,of\,(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$$



$\Rightarrow \quad \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2} = 1 \qquad$ [$\because$ Area (ΔABC) = Area (ΔDEF)]

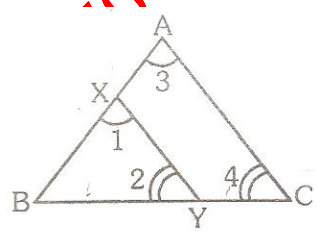
$\Rightarrow \quad AB^2 = DE^2,\ AC^2 = DF^2$ and $BC^2 = EF^2$

$\Rightarrow \quad$ AB = DE, AC = DF and BC = EF

$\therefore \quad \Delta ABC \cong \Delta DEF \qquad$ [By SSS congruence]

Ex.15 In fig, the line segment XY is parallel to side AC of ΔABC and it divides the triangle into two parts of equal areas. Find the ratio $\frac{AX}{AB}$. **[NCERT]**

Sol, We are given that XY $\parallel$ AC.



$\Rightarrow \quad \angle 1 = \angle 3$ and $\angle 2 = \angle 4 \qquad$ [Corresponding angles]

$\Rightarrow \quad \Delta BXY \sim \Delta BAC \qquad$ [AA similarity]

$\Rightarrow \quad \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{(BY)^2}{(BA)^2} \qquad$ [By theorem] …(i)

Also, we are given that

ar (ΔBXY) = $\frac{1}{2} \times ar(\Delta BAC) \Rightarrow \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{1}{2}$ …(ii)

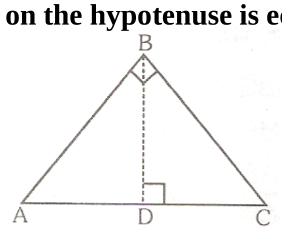
From (i) and (ii), we have $\left(\frac{BX}{BA}\right)^2 = \frac{1}{2} \Rightarrow \frac{BX}{BA} = \frac{1}{\sqrt{2}}$ …(iii)

Now, $\frac{AX}{AB} = \frac{AB - BX}{AB} = 1 - \frac{BX}{AB} = 1 - \frac{BX}{BA} = 1 - \frac{1}{\sqrt{2}}$ [By (iii)]

$= \frac{\sqrt{2} - 1}{\sqrt{2}} = \frac{2 - \sqrt{2}}{2}$

Hence, $= \frac{AX}{AB} = \frac{2 - \sqrt{2}}{2}$

Theorem-4 [Pythagoras Theorem] : In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



Given : A ΔABC in which $\angle B = 90^0$.

To prove : $AC^2 = BA^2 + BC^2$.

Construction : From B, Draw BD $\perp$ AC

Proof :

STATEMENT	REASON

1.	In Δ ADB and Δ ABC, we have : $\angle$BAD = $\angle$CAB = $\angle$A $\angle$ADB = $\angle$ABC $\therefore \Delta ADB \sim \Delta ABC$ $\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ $\Rightarrow AB^2 = AD \times AC$ ..(i)	Common Each = 90^0 By AA axiom of similarity Corr. sides of similar Δ s are proportional
2.	In Δ CDB and Δ CBA, we have : Δ CDB = Δ CBA $\angle$BCD = $\angle$ACB $\therefore \Delta CDB \sim \Delta CBA$ $\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$ $\Rightarrow BC^2 = DC \times AC$ ..(ii)	Each = 90^0 Common By AA axiom of similarity Corr. sides of similar Δ s are proportional
3.	Adding (i) and (ii), we get $AB^2 + BC^2 = AD \times AC + DC \times AC$ = (AD + DC) $\times AC = AC^2$	$\because AD + DC = AC$

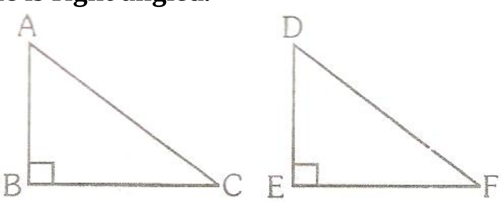
Hence, $AB^2 + BC^2 = AC^2$

Theorem-5 [Converse of pythagoras Theorem] : In a triangle if the square of one sides is equal to the sum of the squares of the squares of the other two sides, then the triangle is right angled.

Given : A Δ ABC in which $AB^2 + BC^2 = AC^2$

To prove : $\angle B = 90^0$.

Construction : Draw a Δ DEF in which CE = AB, EF = BC and $\angle E = 90^0$



Proof :

	STATEMENT	REASON
1.	In Δ DEF, we have : $\angle = 90^0$ $\therefore DE^2 + EF^2 = DF^2$ $\Rightarrow AB^2 + BC^2 = DF^2$ $\Rightarrow AC^2 = DF^2$	By Pythagoras Theorem $\because$ DE = AB and EF = BC $\because$ $AB^2 + BC^2 = AC^2$ (Given)
2.	In Δ ABC and Δ DEF, we have : AB = DE BC = DF $\therefore \Delta ABC \cong \Delta DEF$ $\Rightarrow \angle B = \angle E$ $\Rightarrow \angle E = 90^0$	By construction By construction Proved above By SSS congruence c.p.c.t $\because \angle E = 90^0$

Hence, $\angle B = 90^0$

Ex.16 If ABC is an equilateral triangle of side a, prove that its altitude $= \frac{\sqrt{3}}{2}$ a.

Sol. ΔABC is an equilateral triangle.

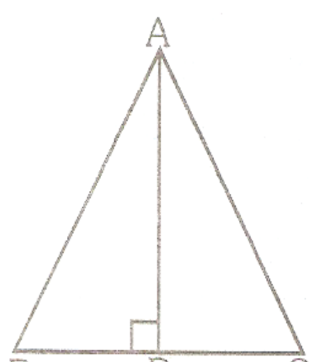
We are given that AB = BC = CA = a. AD is the altitude, i.e., AD $\perp$ BC.

Now, in right angled triangles triangles ABD and ACD, we have

AB = AC [Given]

and AD = AD [Common side]

$\Rightarrow$ Δ ABD = Δ ACD [By RHS congruence]



$\Rightarrow \quad BD = CD \Rightarrow BD = DC = \frac{1}{2}BC = \frac{a}{2}$

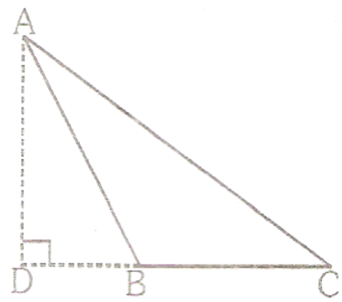
From right triangle ABD,

$AB^2 = AD^2 + BD^2 \qquad \Rightarrow a^2 - AD^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow \quad AD^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$

$\Rightarrow \quad AD = \frac{\sqrt{3}}{2}a.$

Ex.17 In a ΔABC, obtuse angled at B, if AD is perpendicular to CB produced, prove that :
$AC^2 = AB^2 + BC^2 + 2BC \times BD$



Sol. In ΔADB, $\angle D = 90^0$.

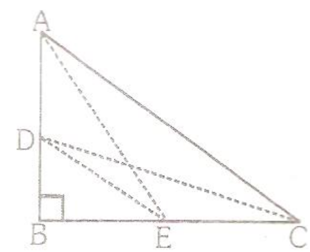
$\therefore \quad AD^2 + DB^2 = AB^2 \quad \ldots(i) \quad$ [By Pythagoras Theorem]

In ΔADC, $\angle D = 90^0$.

$\therefore \quad AC^2 = AD^2 + DC^2 \quad$ [By Pythagoras Theorem]

$= AD^2 + (DB + BC)^2$
$= AD^2 + DB^2 + BC^2 + 2DB \times BC$
$= AB^2 + BC^2 + 2BC \times BD \quad$ [Using (i)]
Hence, $AC^2 = AB^2 + BC^2 + 2BC \times BD$.

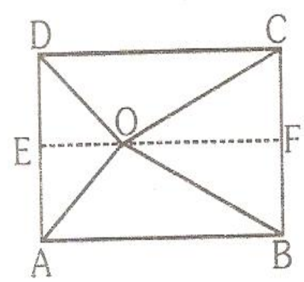
Ex.18 In the given figure, $\angle B = 90^0$. D and E are any points on AB and BC respectively. Prove that :
$AE^2 + CD^2 = AC^2 + DE^2$.



Sol. In ΔABE, $\angle B = 90^0$
$\therefore \quad AE^2 = AB^2 + BE^2 \ldots(i)$
In ΔDBC, $\angle B = 90^0$.
$\therefore \quad CD^2 = BD^2 + BC^2 \ldots(ii)$
Adding (i) and (ii), we get :
$AE^2 + CD^2 = (AB^2 + BC^2) + (BE^2 + BD^2)$
$= AC^2 + DE^2 \quad$ [By Pythagoras Theorem]
Hence, $AE^2 + CD^2 = AC^2 + DE^2$.

Ex.19 A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D. Prove that : $OA^2 + OC^2 = OB^2 + OD^2$

Sol. Through O, draw $EOF \parallel AB$. Then, ABFE is a rectangle.
In right triangles OEA and OFC, we have :
$OA^2 = OE^2 + AE^2$
$OC^2 = OF^2 + OF^2$
$\therefore \quad OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2$
Again, in right triangles OFB and OED, we have :
$OB^2 = OF^2 + BF^2$
$OD^2 = OE^2 + DE^2$
$\therefore \quad OB^2 + OD^2 = OF^2 + OE^2 + BF^2 + DE^2$
$= OE^2 + OF^2 + AE^2 + CF^2 \quad \ldots(i) \quad [\because BF = AE \ \& \ DE = CF]$



From (i) and (ii), we get
$OA^2 + OC^2 = OB^2 + OD^2$.

Ex.20 In the given figure, ΔABC is right-angled at C.
Let BC = a, CA = b, AB = c and CD = p, where $CD \perp AB$.

Prove that : (i) cp = ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Sol. (i) Area of ΔABC = $\frac{1}{2}$ BC $\times$ CD $\frac{1}{2}$ cp.

Also, area of ΔABC = $\frac{1}{2}$ BC $\times$ AC $\frac{1}{2}$ ab.

$\therefore \frac{1}{2}$ cp = $\frac{1}{2}$ ab. $\Rightarrow$ cp = ab

(ii) cp = ab $\Rightarrow$ p = $\frac{ab}{c}$

$$\Rightarrow \quad p^2 = \frac{a^2b^2}{c^2}$$

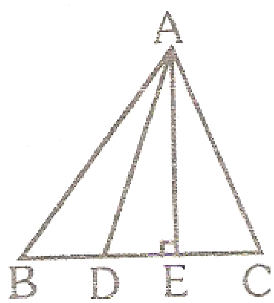
$$\Rightarrow \quad \frac{1}{p^2} = \frac{c^2}{a^2b^2} = \frac{a^2+b^2}{a^2b^2} \qquad [\because c^2 = a^2 + b^2]$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{a^2} = \frac{1}{b^2}$$

Ex.21 Prove that in any triangle, the sum of the square of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side. **(Appollonius Theorem)**

Sol. Given : A ΔABC in which AD is a median.

To prove : $AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$ or $AB^2 + AC^2 = 2(AD^2 + BD^2)$



Construction : Draw AE $\perp$ BC.

Proof : $\because$ AD id median

$\therefore$ BD = DC

Now, $AB^2 + AC^2 = (AE^2 + BE^2) + (AE^2 + CE^2) = 2AE^2 + BE^2 + CE^2$

$= 2[AD^2 - DE^2] + BE^2 + CE^2$

$= 2AD^2 - 2DE^2 + (BD + DE)^2 + (DC - DE)^2$

$= 2AD^2 - 2DE^2 + (BD + DE)^2 + (DC - DE)^2$

$= 2(AD^2 + BD^2) = = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$

Hence, Proved.

★ **SYNOPSIS**

- **SIMILAR TRIANGLES.** Two triangles are said to be similar if

  (i) Their corresponding angles are equal and (ii) Their corresponding sides are proportional.

- All congruent triangles are similar but the similar triangles need not be congruent.

- Two polygons of the same numbers of sides are similar, if

  (i) their corresponding angels are equal and

  (ii) their corresponding sides are in the same ratio.

- **BASIC PROPORTIONALITY THEOREM.** In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the third side.

- **CONVERSE OF BASIC PROPORTIONALITY THEOREM.** If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.

- **AAA-SIMILARITY.** If in two triangles, corresponding angles are equal, i.e., the two corresponding angles are equal, then the triangles are similar.
- **SSS-SIMILARITY.** If the corresponding sides of two triangles are proportional, then they are similar.
- **SSS-SIMILARITY.** If in triangles one pair of corresponding sides proportional and the included angles are equal then the two triangles are similar.
- The ratio of the areas of similar triangles is equal to the ratio of the squares of their to the sum of the squares.
- **PYTHAGORAS THEOREM.** In a right triangle, if the square of one side is equal to the sum of the squares of the other two sides.
- **CONVERSE OF PYTHAGORAS THEOREM.** In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is a right angle.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. Triangle ABC is such that AB = 3 cm, BC = 2 cm and CA = 2.5 cm. Triangle DEF is similar to ΔABC. If EP 4 cm, then the perimeter of ΔDEF is :
(A) 7.5 cm (B) 15 cm (C) 22.5 cm (D) 30 cm

2. In ΔABC, AB = 3 cm, AC = 4 cm and AD is the bisector of $\angle$A. Then, BD : DC is :
(A) 9 : 16 (B) 16 : 9 (C) 3 : 4 (D) 4 : 3

3. In a equilateral triangle ABC, if AD $\perp$ BC, then :
(A) $2AB^2 = 3AD^2$ (B) $4AB^2 = 3AD^2$ (C) $3AB^2 = 4AD^2$ (D) $3AB^2 = 2AD^2$

4. ABC is a triangles and DE is drawn parallel to BC cutting the other sides at D and E. If AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm, then AE is equal to :
(A) 1.4 cm (B) 1.8 cm (C) 1.2 cm (D) 1.05 cm

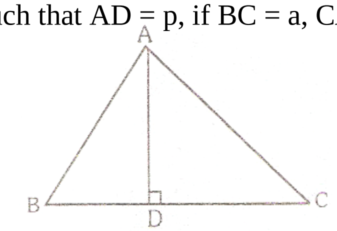
5. The line segments joining the mid points of the sides of a triangle from four triangles each of which is :
(A) similar to the original triangle (B) congruent to the original triangle.
(C) an equilateral triangle (D) an isosceles triangle.

6. In ΔABC and ΔDEF, $\angle A = 50^0, \angle B = 70^0, \angle C = 60^0, \angle D = 60^0, \angle E = 70^0, \angle F = 50^0$, then ΔABC is similar to :
(A) ΔDEF (B) ΔEDF (C) ΔDFE (D) ΔFED

7. D, E, F are the mid points of the sides BC, CA and AB respectively of ΔABC. Then ΔDEF is congruent to triangle
(A) ABC (B) AEF (C) BFD, CDE (D) AFE, BFD, CDE

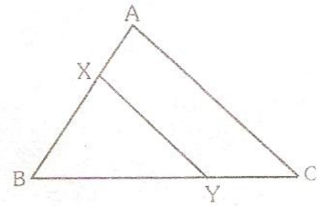
8. If in the triangles ABC and DEF, angle A is equal to angle E, both are equal to 40^0, AB : ED = AC : EF and angle F is 65^0, then angel B is :-
(A) 35^0 (B) 65^0 (C) 75^0 (D) 85^0

9. In a right angled ΔABC, right angled at A, if AD $\perp$ BC such that AD = p, if BC = a, CA = b and AB = c, then :
(A) $p^2 = b^2 + c^2$ (B) $\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{c^2}$



(C) $\frac{p}{a} = \frac{p}{b}$ (D) $p^2 = b^2 c^2$

10. In the adjoining figure, XY is parallel to AC. If XY divides the triangle into equal parts, then the value of $\frac{AX}{AB}$ =



(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

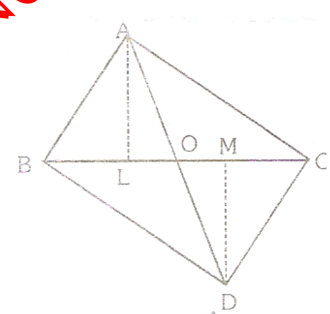
(C) $\frac{\sqrt{2}+1}{\sqrt{2}}$ (D) $\frac{\sqrt{2}-1}{\sqrt{2}}$

11. The ratio of the corresponding sides of two similar triangles is 1 : 3. The ratio of their corresponding heights is :
(A) 1 : 3 (B) 3 : 1 (C) 1 : 9 (D) 9 : 1

12. The areas of two similar triangles are 49 cm^2 and 64 cm^2 respectively. The ratio of their corresponding sides is :
(A) 49 : 64 (B) 7 : 8 (C) 64 : 49 (D) None of these

13. The areas of two similar triangles are 12 cm^2 and 48 cm^2. If the height of the similar one is 2.1 cm, then the corresponding height of the bigger one is :
(A) 4.41 cm (B) 8.4 cm (C) 4.2 cm (D) 0.525 cm

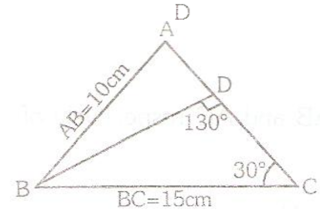
14. In the adjoining figure, ABC and DBC are two triangles on the same base BC, AL $\perp$ BC and DM $\perp$ BC. Then, $\frac{area(\Delta ABC)}{area(\Delta DBC)}$ is equal to ;



(A) $\frac{AO}{OD}$ (B) $\frac{AO^2}{OD^2}$

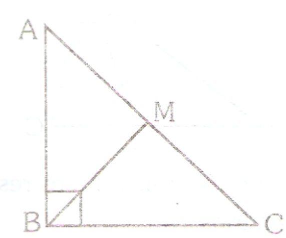
(C) $\frac{AO}{AD}$ (D) $\frac{OD^2}{AO^2}$

15. In the adjoining figure, AD : DC = 2 : 3, then $\angle$ABC is equal to :
(A) 30^0 (B) 40^0 (C) 45^0 (D) 110^0



16. In ΔABC, D and E are points on AB and AC respectively such that DE $\parallel$ BC. If AE = 2 cm, EC = 3 cm and BC = 10 cm, then DE is equal to ;
(A) 5 cm (B) 4 cm (C) 15 cm (D) $\frac{20}{3}$ cm

17. In the given figure, $\angle$ABC = 90^0 and BM is a median, AB = 8 cm and BC = 6 cm. Then, length BM is equal to :
(A) 3 cm (B) 4 cm (C) 5 cm (D) 7 cm



18. If D, E, F are respectively the mid points of the sides BC, CA and AB of ΔABC and the area of ΔABC is 24 sq. cm, then the area of ΔDFE is :-
(A) 24 cm^2 (B) 12 cm^2 (C) 8 cm^2 (D) 6 cm^2

19. In a right angled triangle, if the square of the hypotenuse is twice the product of the other two sides, then one of the angles of the triangle is :-
(A) 15^0 (B) 30^0 (C) 45^0 (D) 60^0

20. Consider the following statements :
1. If three sides of a triangles are equal to three sides of another triangle, then the triangles are congruent.
2. If three angles of a triangles are respectively equal to three angles of another triangle, then the two triangles are congruent.

Of these statements,
(A) 1 is correct and 2 is false (B) both 1 and 2 are false
(C) both 1 and 2 are correct (D) 1 is false and 2 is correct

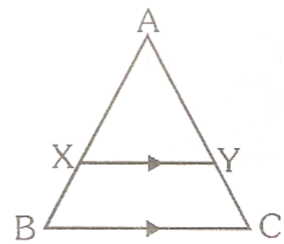
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	C	A	A	D	D	C	B	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	C	A	B	B	C	D	C	A

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

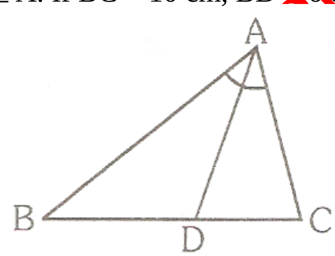
VERY SHORT ANSWER TYPE QUESTIONS

1. In the given figure, $XY \parallel BC$.
Given that AX = 3 cm, XB = 1.5 cm and BC = 6 cm.
Calculate :
(i) $\frac{AY}{YC}$ (ii) XY

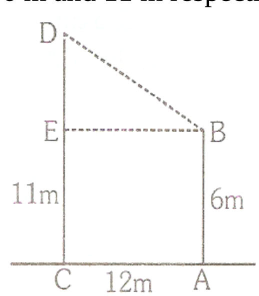


2. D and E are points on the sides AB and AC respectively of ΔABC. For each of the following cases, state whether $DE \parallel BC$:
(i) AD = 5.7 cm, BD = 9.5 cm, AE = 3.6 cm, and EC = 6 cm
(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 9.6 cm, and EC = 2.4 cm.
(iii) AB = 11.7 cm, BD = 5.2 cm, AE = 4.4 cm, and AC = 9.9 cm.
(iv) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm, and AE = 2.8 cm.

3. In ΔABC, AD is the bisector of $\angle A$. If BC = 10 cm, BD = 6 cm and AC = 6 cm, find AB.



4. AB and CD are two vertical poles height 6 m and 11 m respectively. If the distance between their feet is 12 m, find the distance between their tops.



5. ΔABC and ΔPQR are similar triangles such that are (ΔABC) = 49 cm^2 and area (ΔPQR) = 25 cm^2. If AB = 5.6 cm, find the length of PQ.

6. ΔABC and ΔPQR are similar triangles such that are (ΔABC) = 28 cm^2 and area (ΔPQR) = 63 cm^2. If PR = 8.4 cm, find the length of AC.

7. $\Delta ABC \sim \Delta DEF$. If BC = 4 cm, EF = 5 cm and area (ΔABC) = 32 cm^2, determine the area of ΔDEF.

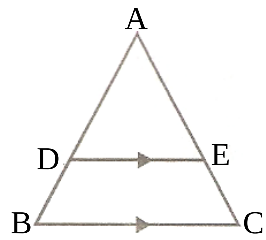
8. The areas of two similar triangles are 48 cm^2 and 75 cm^2 respectively. If the altitude of the first triangle be 3.6 cm, find the corresponding altitude of the other.

9. A rectangular field is 40 m long and 30 m broad. Find the length of its diagonal.

10. A man goes 15 m due west and then 8 m due north. How far is he from the starting point?

11. A ladder 17 m long reaches the window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

SHORT ANSWER TYPE QUESTIONS



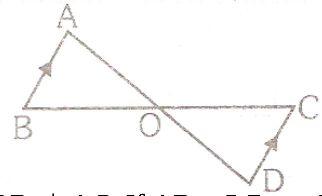
1. In the given fig, DE $\parallel$ BC.

(i) If AD = 3.6 cm, AB = 9 cm and AE = 2.4 cm, find EC.

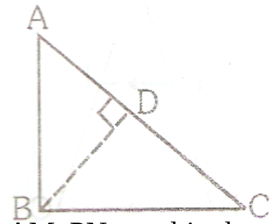
(ii) If $\frac{AD}{DB} = \frac{3}{5}$ and AC = 5.6 cm, find AE.

(iii) If AD = x cm, DB = (x – 2) cm, AE = (x + 2) cm and EC = (x – 1) cm, find the value of x.

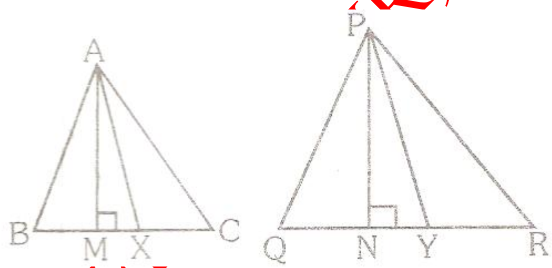
2. In the given figure, BA $\parallel$ DC. Show that ΔOAB ~ ΔODC. If AB = 4 cm, CD = 3 cm, OC = 5.7 cm and OD = 3.6 cm, find OA and OB.



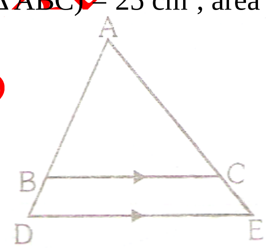
3. In the given figure, $\angle ABC = 90^0$ and BD $\perp$ AC. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.



4. In the given figure, ΔABC ~ ΔPQR and AM, PN are altitude, whereas AX and PY are medians. Prove that $\frac{AM}{PN} = \frac{AX}{PY}$



5. In the given figure, BC $\parallel$ DE, area (ΔABC) = 25 cm^2, area (trap. BCED) = 24 cm^2 and DE = 1 4 cm. Calculate the length of BC.



6. In ΔABC, $\angle C = 90^0$. If BC = a, AC = b and AB = c, find :

(i) c when a = 8 cm and b = 6 cm.

(ii) a when c = 25 cm and b = 7 cm.

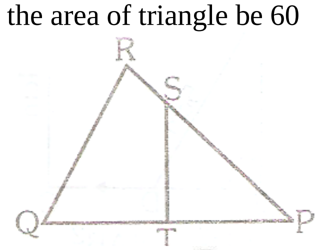
(iii) b when c = 13 cm and a = 5 cm.

7. The sides of a right triangle containing the right angle are (5x) cm and (3x – 1) cm. If the area of triangle be 60 cm^2, calculate the length of the sides of the triangle.

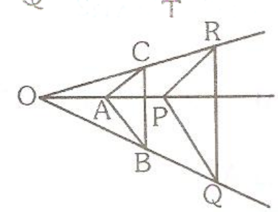
8. Find the altitude of an equilateral triangle of side $5\sqrt{3}$ cm.

9. In the adjoining figure (not drawn to scale), PS – 4 cm, SR = 2 cm, PT = 3 cm and QT = 5 cm.

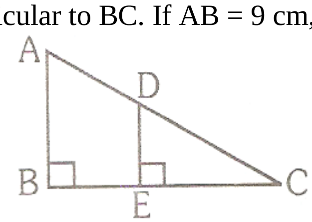
(i) Show that ΔPQR ~ ΔPST. (ii) Calculate ST, if QR = 5.8 cm.



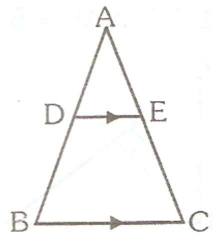
10. In the given figure, AB $\parallel$ PQ and AC $\parallel$ PR. Prove that BC $\parallel$ QR.



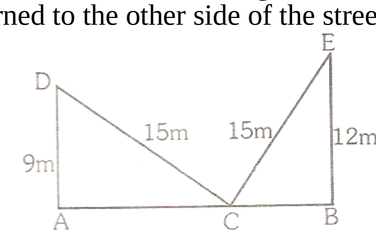
11. In the given figure, AB and DE are perpendicular to BC. If AB = 9 cm, DE = 3 cm and AC = 24 cm, calculate AD.



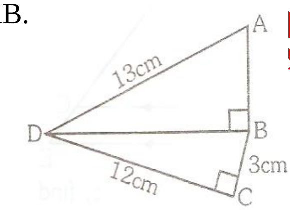
12. In the given figure, DE ‖ BC. If DE = 4 cm, BC = 6 cm and area (ΔADE) = 20 cm^2, find the area of ΔABC.



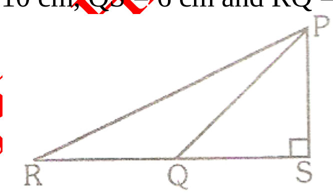
13. A ladder 15 m long reaches a window which is 9 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned is turned to the other side of the street to reach a window 12 m high. Find the width of the street.



14. In the given figure, ABCD is a quadrilateral in which BC = 3 cm, AD = 13 cm, DC = 12 cm and $\angle$ABD = $\angle$BCD = 90^0. Calculate the length of AB.

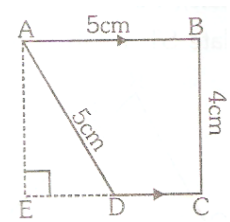


15. In the given figure, $\angle$PSR = 90^0, PQ = 10 cm, QS = 6 cm and RQ = 9 cm, calculate the length of PR.



16. In a rhombus PQRS, side PQ = 17 cm and diagonal PR = 16 cm. Calculate the area of the rhombus.

17. From the given figure, find the area of trapezium ABCD.

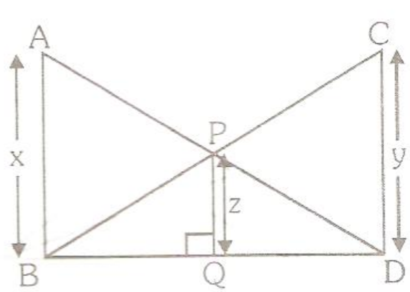


18. In a rhombus ABCD, prove that AC^2 + BD^2 = 4AB^2.

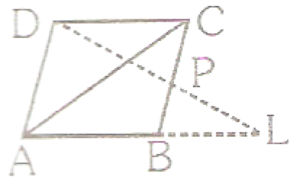
19. A ladder 13 m long rests against a vertical wall. If the foot of the ladder is 5 m from the foot of the wall, find the distance of the other end of the ladder from the ground.

Long Answer Type Questions

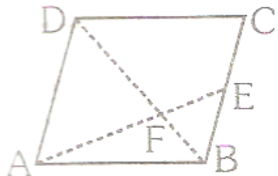
1. In the given figure, it is given that $\angle$ABD = $\angle$CDB = $\angle$PQB = 90^0. If AB = x units, CD = y units and PQ = z units, prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$



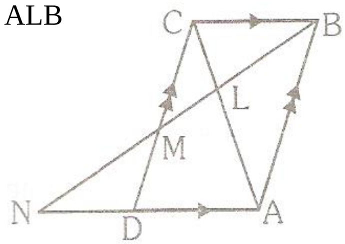
2. In the adjoining figures, ABCD is a parallelogram, P is a point on side BC and DP when produced meets AB produced at L. Prove that : (i) DP : PL = DC : BL (ii) DL : DP = AL : DC.



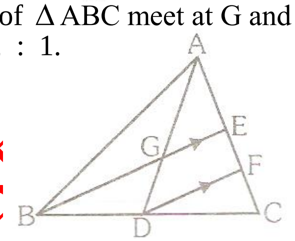
3. In the given figure, ABCD is a parallelogram, E is a point on BC and the diagonal BD intersects AE at F. Prove that DF $\times$ FE = FB $\times$ FA.



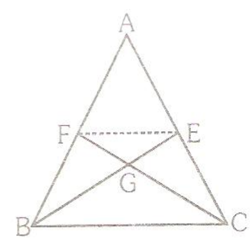
4. In the adjoining figure, ABCD is a parallelogram in which AB = 16 cm BC = 10 cm and L is a point on AC such that CL : LA = 2 : 3. If BL produced meets CD at M and AD produced at N, prove that :
(i) ΔCLB ~ ΔALN (ii) ΔCLM ~ ΔALB



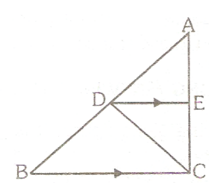
5. In the given figure, medians AD and BE of ΔABC meet at G and DF $\|$ BE. Prove that
(i) EF = FC (ii) AG : GD = 2 : 1.



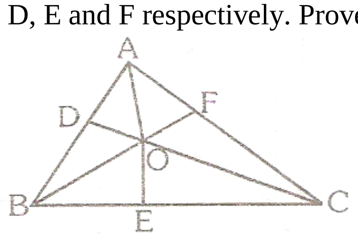
6. In the given figure, the medians BE and CF of ΔABC meet at G. Prove that :
(i) ΔGEF ~ ΔGBC and therefore, BG = 2 GE. (ii) AB$\times$AF = AE $\times$ AC.



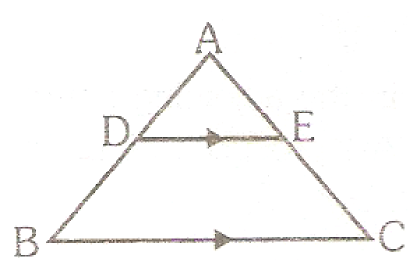
7. In the given figure, DE $\|$ BC and BD – DC.
(i) Prove that DE bisects $\angle$ADC.
(ii) If AD = 4.5 cm, AE = 3.9 cm and DC = 7.5 cm, find CE.
(iii) Find the ratio AD : DB.



8. O is point inside a ΔABC. The bisectors of $\angle$AOB. $\angle$BOC and $\angle$COA meet the sides AB, BC and in points D, E and F respectively. Prove that AD-BE-CF = DB.EC.FA



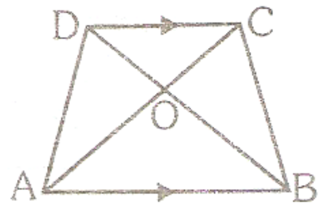
9. In the figure, DE $\|$ BC.



(i) Prove that ΔADE and ΔABC are similar.

(ii) Given that AD = $\frac{1}{2}$ BD. Calculate DE, if BC = 4.5 cm.

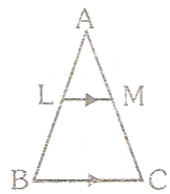
10. In the adjoining figure, ABCD is a trapezium in which AB $\|$ DC and AB = 2 DC. Determine the ratio of areas of ΔAOB and ΔCOD



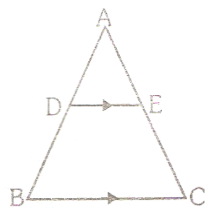
11. In the adjoining figure, LM is parallel to BC. AB = 6 cm, AL = 2 cm and AC = 9 cm. Calculate :

(i) the length of CM.

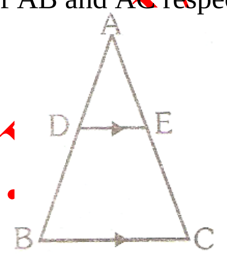
(ii) the value of $\frac{Area(\Delta ALM)}{Area(trap.LBCM)}$



12. In the given figure, DE $\|$ BC. and DE : BC = 3 : 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.

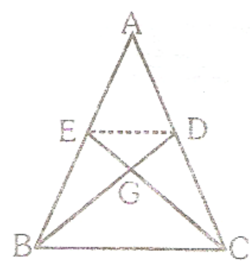


13. In ΔABC, D and E are mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC.

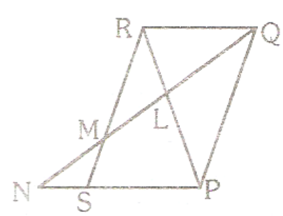


14. In a ΔPQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$. Prove that (i) $\Delta PQL - \Delta RPM$ (ii) QL.RM = PL.PM (iii) $PQ^2 = QL.QR$

15. In the adjoining figures, the medians BD and CE of a ΔABC meet at G.
Prove that:
(i) $\Delta EGD \sim \Delta CGB$
(ii) BG = 2 GD from (i) above.



16. In the adjoining figure, PQRS is a rarallelog'am with PQ = 15 cm and RQ = 10 cm. L is a point on RP such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N. Find the length of PN and RM.

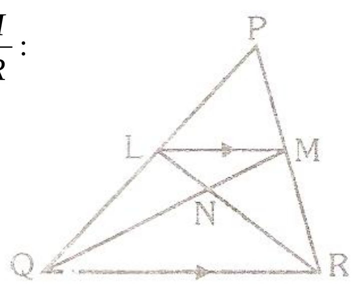


17. In ΔPQR, LM $\|$ QR and PM : MR = 3 : 4. Calculate:

(i) $\frac{PL}{PQ}$ and then $\frac{LM}{QR}$:

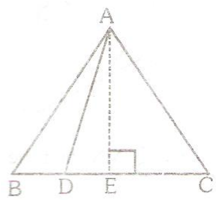
(ii) $\frac{Area(\Delta ALM)}{Area(\Delta MNR)}$

(iii) $\frac{Area(\Delta LQM)}{Area(\Delta LQN)}$



18. In ΔABC, $\angle B = 90^0$ and D is the mid point of BC.
Prove that :
(i) $AC^2 = AD^2 + 3CD^2$
(ii) $BC^2 = 4(AD^2 - AB^2)$

19. In ΔABC, if AB = AC and D is a point on BC. Prove that $BC^2 - AD^2 = BD \times CD$.



VELY SHORT ANSWER TYPE QUESTIONS

1. (i) $\frac{1}{2}$ (ii) 4 cm **2.** (i) Yes, (ii) No, (iii) No, (iv) Yes **3.** 9 cm **4.** 13 m **5.** PQ = 4 cm **6.** AC = 5.6 cm
7. 50 cm^2 **8.** 4.5 cm **9.** 50 m **10.** 17 m **11.** 8m

SHORT ANSWER TYPE QUESTIONS

1. (i) 3, 6 cm, (ii) 2.1 cm, (iii) x = 4 **2.** OA = 4.8 cm, OB = 7.6 cm **3.** 8.1 cm **5.** 10 cm **6.** (i) 10 cm, (ii) 24 cm, (iii) 12 cm **7.** 15 cm,8 cm, 17cm **8.** 7.5 cm **9.** 2.9 cm **11.** 16 cm **12.** 45 cm^2 **13.** 21m **14.** 4 cm **15.** 17 cm **16.** 240 cm^2 **17.** 14 cm^2 **19.** 12 m

LONG ANSWER TYPE QUESTIONS

7. (ii) 6.5 cm, (ii) 3 : 8 **9.** DE = 1.5 cm **10.** 4 : 1 **11.** (i) 6 cm, (ii) $\frac{1}{8}$ **12.** 9 : 16 **13.** 1 : 4

16. PN = 15 cm, RM = 10 cm **17.** (i) $\frac{PL}{PQ} = \frac{LM}{QR} = \frac{3}{7}$ (ii) 3 : 7 (iii) 10 : 7

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PERVIOUS YEARS BOARD QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. ΔABC and ΔDEF are similar, BC = 3 cm, EF = 4 cm and area of $\Delta ABC = 54\ cm^2$. Determine the area of ΔDEF. **Delhi-1996**

2. In ΔABC, CE $\perp$ AB, BD $\perp$ AC and & BD intersect at P, considering triangles BEP and CPD. Prove that BP $\times$ PD = EP $\times$ PC. **Delhi-1996C**

3. A right triangle has hypotenuse of length q cm and one side of length p cm. if (q – p) = 2, express the length of third side of the right triangle in terms of q. **Al-1996C**

4. In the given figure, ABC is a triangle in which AB = AC. D and E are points on the sides AB and AC respectively, such that AD = AE. Show that the points B, C, E and D are concyclic. **Al-1996C**

5. In a ΔABC, AB = AC and D is a point on side AC, such that BC2 = AC $\times$ CD. Prove that BD = BC. **Al-1997**

6. ΔABC is right angled at B. On side AC, a point D is taken such that AD = DC and AB = BD. Find the measure of $\angle CAB$ **Delhi-1998**

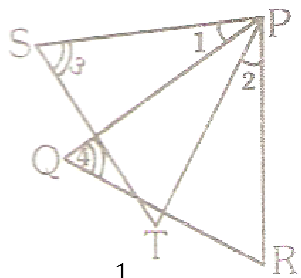
7. In a ΔABC, P and Q are points on the sides AB and AC respectively such that PQ is parallel to BC. Prove that median AD, drawn from A to BC, bisects PQ. **Al-1998**

8. Two poles of height 7 m and 12 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tips. **Al-1998C**

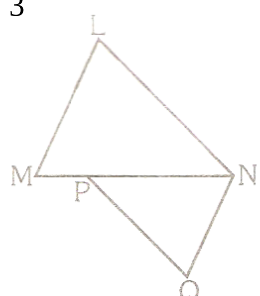
9. In a ΔABC, D and E are points on AB & AC respectively such that DE is parallel to BC and AD : DB = 2 : 3. Determine Area (ΔADE) : Area (ΔABC). **Foreign-1999**

10. In the given figure, $\angle A = \angle B$ and D & E are points on AC and BC respectively such that AD = BE, show that DE $\parallel$ AB. **Delhi-1999**

11. In figure, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Show that PT. QR = PR. ST. **Foreign-2000**



12. In figure, LM ∥ NQ and LN ∥ PQ. If $MP = \frac{1}{3}$ MN, find the ratio of the areas of ΔLMN and ΔQNP.
Foreign-2000



13. ABC is an isosceles triangle right angled at B. Two equilateral triangles BDC and AEC are constructed with side BC and AC. Prove that area of $\Delta BCD = \frac{1}{2}$ area of ΔACE. **Delhi-2001**

14. The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. If the altitude of the first triangle 6.3 cm, find the corresponding altitude of the other. **Al-2001**

15. L and M are the mid-points of AB and BC respectively of ΔABC, right-angled at B. prove that $4LC^2 = AB^2 + 4BC^2$. **Al-2001;Foreign-2001**

16. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm. find the corresponding median of the other. **Al-2001**

17. In an equilateral triangle ABC, AD is the altitude drawn from A on side BC. Prove that $3AB^2 = 4AD^2$.
Delhi-2002

18. (i) Prove that the equilateral triangle described on the two sides of a right angled triangle are together equal to the equilateral triangle on the hypotenuse in terms of their areas. **Al-2002**

(ii) P is a point in the interior of ΔABC, X, Y and Z are point on lines PA.PB and PC respectively such that XY ∥ AB and XZ ∥ BC. Prove that YZ ∥ BC. **Al-2002 : Delhi-2003 [NCERT]**

(iii) D and E are points on the sides AB and AC respectively of ΔABC such that DE is parallel to BC and AD : DB = 4 : 5. CD and BE intersect each other at F. Find the ratio of the areas of ΔDEF and ΔBCE
Al-2000 : Al-2003

(iv) P. Q are respectively points on sides AB and AC of triangle ABC. If AP = 2 cm. PB = 4 cm. AQ = 3 cm and QC = 6 cm. prove that BC = 3PQ. **Foreign-2003**

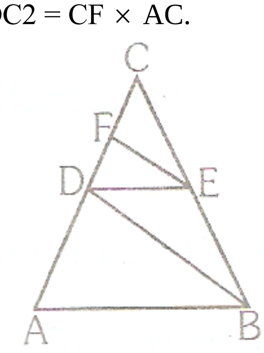
19. D is a point on the side BC of ΔABC such that $\angle ADC = \angle BAC$. Prove that $\frac{CA}{CD} = \frac{CB}{CA}$. **Delhi-2002:[NCERT]**

20. ABCD is a trapezium in which AB ∥ DC. The diagonals AC and BD intersect at O. Prove that $\frac{AO}{OC} = \frac{BO}{DO}$
Al-2004:[NCERT]

21. In a ΔABC, AD ⊥ BC and $\frac{BD}{AD} = \frac{AD}{DC}$. Prove that ABC is a right triangle, right angled at A. **Foreign-2004**

22. In a right angled triangle ABC, $\angle A = 90^0$ and AD ⊥ BC. Prove that $AD^2 = BD \times CD$. **Delhi-2004C, 2006**

23. In fig., AB ∥ DE and BD ∥ EF. Prove that DC2 = CF × AC. **Al-2004C : Delhi-2007**

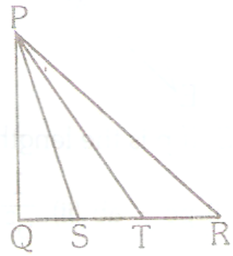


24. If one diagonal of a trapezium divides the other diagonal in the ratio of 1 : 2. prove that one of the parallel sides is double the other. **Foreign-2005**

25. In ΔABC, $AD \perp BC$, prove that $AB^2 + CD^2 = AC^2 + DB^2$. **Delhi-2005C, Al-2006 [NCERT]**

26. Prove that the sum of the squares of the sides of a rhombus is equal to sum of the squares of its diagonals. **Al-2005C [NCERT]**

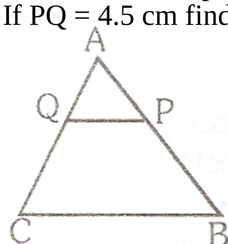
27. In figure, S and T trisect the side QR of a right triangle PQR. Prove that $8PT^2 = 3PR^2 + 5PS^2$.



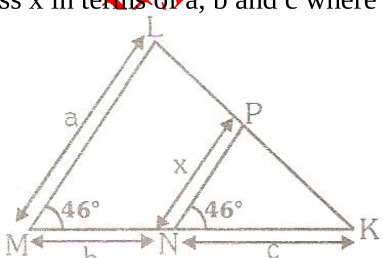
OR

If BL and CM are medians of a triangle ABC right-angled at A, then prove that $4(BL^2 + CM^2) = 5BC^2$. **Al-2006 C; Foreign-2009**

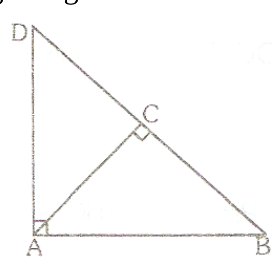
28. In the fig, P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm. If PQ = 4.5 cm find BC. **Delhi-2008**



29. In fig. $\angle M = \angle N = 46^0$ Express x in terms of a, b and c where a, b and c are lengths of LM, MN and NK respectively. **Delhi-2009**



30. In figure, ΔABC is a right triangle, right-angled at A and $AC \perp BD$. Prove that $AB^2 = BC. BD$. **Al-2009**



31. In a ΔABC, $DE \parallel BC$. If $DE = \frac{2}{3} BC$ and area of $\Delta ABC = 81$ cm^2, find the area of ΔADE. **Foregin-2009**

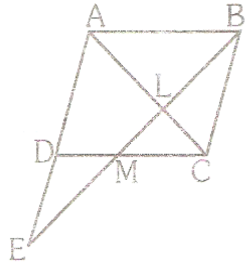
SHORT ANSWER TYPE QUESTIONS

1. P and Q are points on the sides CA and CB respectively of a ΔABC right-angled at C. prove that $AQ^2 + BP^2 = AB^2 + PQ^2$. **Delhi-1996, 2007**

2. ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE. **Al -1997**

3. In ΔABC, if AD is the median, show that $AB^2 + AC^2 = 2\,[AD^2 + BD^2]$. **Delhi-1997, 98**

4. In the given figure, M is the mid-point of the side CD of parallelogram ABCD. BM. When joined meets AC is L and AD produced in E. Prove that EL = 2BL. **Al-1998; Delhi-1999, Al-2009**



5. ABC is a right triangle, right-angled at C. if p is the length of the perpendicular from C to AB and a, b, c have the usual meaning, then prove that (i) pc = ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} = \frac{1}{b^2}$ **Delhi-1998, 98 C**

6. In an equilateral triangle PQR, the side QR is trisected at S. Prove that $9PS^2 = 7PQ^2$. **Al-1998, 98C [NCERT]**

7. If the diagonals of a quadrilateral divide each other proportionally, prove that it is trapezium. **Foreign-1999**

8. In an isosceles triangle ABC with AB = AC, BD is a perpendicular from B to the side AC. Prove that $BD^2 - CD^2 = 2CD.\,AD$. **Foreign-1999**

9. ABC and DBC are two triangles on the same base BC. If AD intersect BC at O. Prove that $\frac{ar.\Delta ABC}{ar.\Delta DBC} = \frac{AO}{DO}$ **Al-1999C; Delhi-2005**

10. In ΔABC, $\angle$A is acute. BD and CE are perpendiculars on AC and AB respectively. Prove that AB $\times$ AE = AC $\times$ AD. **Al-2003**

11. Points P and Q are on sides AB and AC of a triangle ABC in such a way that PQ is parallel to side BC. Prove that the median AD drawn from vertex A to side BC bisects the segment PQ. **Foreign -2003**

12. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

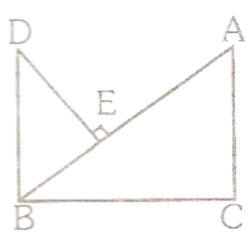
OR

Two Δs ABC and DBC are on the same base BC and on the same side of BC in which $\angle$A = $\angle$D = 90^0. If CA and BD meet each other at E, show that AE.ED. **Delhi-2008**

13. D and E are points on the sides CA and CB respectively of ΔABC right-angled at C. prove that $AE^2 + BD^2 = AB^2 + DE^2$.

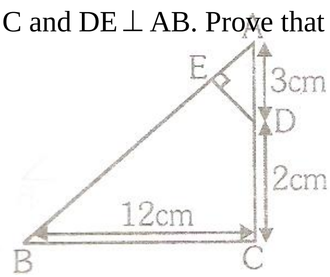
OR

In fig. DB $\perp$ BC, DE $\perp$ AB and AC $\perp$ BC. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$. **Al-2008**

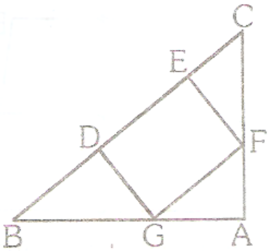


14. E is a point on the side AD produced of a $\|^{gm}$ ABCD and BE intersects CD at F. Show that ΔABC ~ ΔCFB. **Foreign-2008**

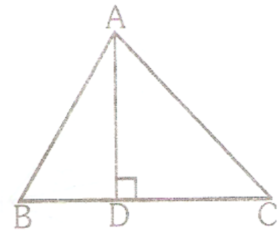
15. In fig, ΔABC is right angled at C and DE $\perp$ AB. Prove that ΔABC ~ ΔADE and hence find the lengths of AE and DE.



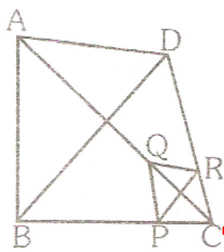
In fig, DEFG is a square and $\angle BAC = 90^0$. Show that $DE^2 = BD \times EC$ **Delhi-2009**



16. In fig, $AD \perp BC$ and $BD = \frac{1}{3}$ CD. Prove that $2CA^2 = 2AB^2 + BC^2$. **Al-2009**



17. In fig, two triangles ABC and DBC lie on the same side of base BC. P is a point on BC such that $PQ \parallel BA$ and $PR \parallel BD$. Prove that $QR \parallel AD$. **Foreign-2009**



Long Answer Type Questions

1. In a right triangle ABC, right-angled at C, P and Q are points on the sides CA and CB respectively which divide these sides in the ratio 1 : 2. Prove that **Al-1996C**

(i) $9AQ^2 = 9AC^2 + 4BC^2$ (ii) $0BP^2 = 9BC^2 + 4AC^2$ (iii) $9(AQ^2 + BP^2) = 13AB^2$.

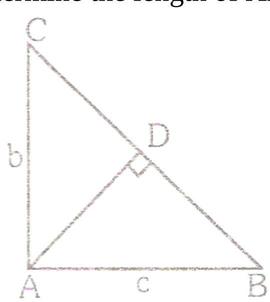
2. The ratio of the areas of similar triangles is equal to the ratio of the square on the corresponding sides, prove. Using the above theorem, prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal. **Delhi-1997C; 2005C; Foreign-2003**

3. Perpendiculars OD, OE and OF are drawn to sides BC, CA and AB respectively from a point O in the interior of a ΔABC. Prove that :

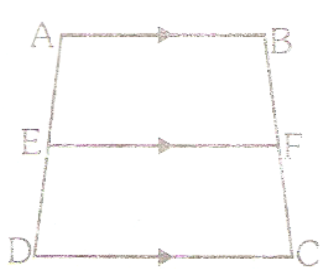
(i) $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$.

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$. **Delhi-1997C, [NCERT]**

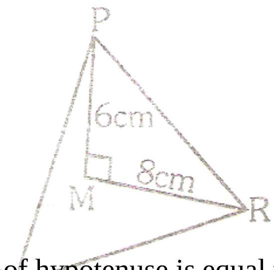
4. In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides. Prove. Using the above theorem, determine the length of AD in terms of b and C. **Al-1997 C**



5. If a line is drawn parallel to one side of a triangle, other two sides are divided in the same ratio, Prove. Using this result to prove the following : In the given figure, if ABCD is a trapezium in which $AB \parallel DC \parallel EF$, then $\frac{AE}{ED} = \frac{BF}{FC}$. **Foreign-1998**



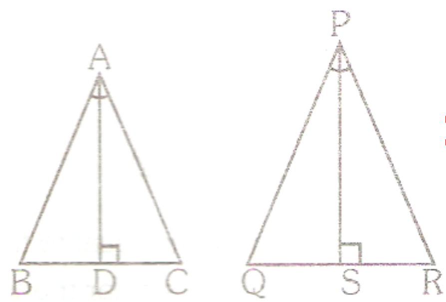
6. State and prove Pythagoras. Use the theorem and calculate are (Δ PMR) from the given figure. **Delhi-1998C, 2006**



7. In a right-angled triangle, the square of hypotenuse is equal to the sum of the squares of the two sides. Given that $\angle$B of ΔABC is an acute angle and AD $\perp$ BC. Prove that $AC^2 = AB^2 + BC^2 - 2BC.\ BD$. **Delhi-1999**

8. In a right triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using above, solve the following : In quadrilateral ABCD, find the length of CA, if CD $\perp$ DB, CD = 6 m, DB = 12 m and AB = 11 m. **Delhi-2000**

9. Prove that the ratio of the areas of two similar triangles is equal to the squares of their corresponding sides. Using the above, do the following



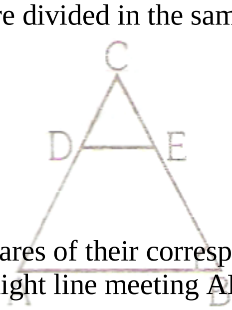
In fig, ΔABC and ΔPQR are isosceles triangles in which $\angle A = \angle P$. If $\frac{area(\Delta ABC)}{area(\Delta PQR)} = \frac{9}{16}$, find $\frac{AD}{PS}$. **Al-2000**

10. In a right-angled triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using the above result, find the length of the second diagonal of a rhombus whose side is 5 cm and one of the diagonals is 6 cm. **Al-2001**

11. In a triangle, if the square on one side is equal to the sum of the squares on the other two sides prove that the angle opposite the first side is a right angel.
Using the above theorem and prove that following : In triangle ABC, AD $\perp$ BC and BD = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$. **Al-2003**

12. In a right triangle, prove that the square on hypotenuse is equal to sum of the squares on the other two sides.Using the above result, prove that following : PQR is a right triangle right angled at Q. If S bisects QR, show that $PR^2 = 4PS^2 - 3PQ^2$. **Delhi-2004C**

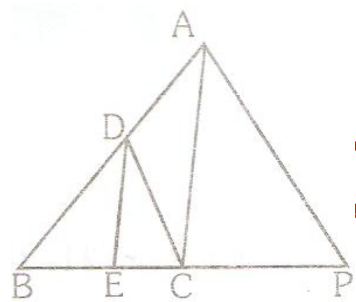
13. If a line is drawn parallel to one side of a trial prove that the other two sides are divided in the same ratio. Using the above result, prove from fig. that AD = BE if $\angle A = \angle B$ and DE $\parallel$ AB. **Al-2004C**



14. Prove that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides. Apply the above theorem on the following : ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 cm, prove that area of ΔAPQ is one-sixteenth of the area of ΔABC. **Delhi-2005**

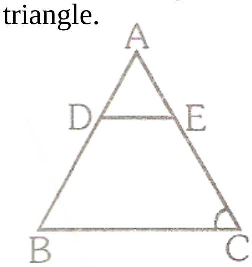
15. If a line is drawn parallel to one side of a triangle, prove that the other two sides are divided in the same ratio. Use the above to prove the following : In the given figure DE ∥ AC and DC ∥ AP.

Prove that $\frac{BE}{EC} = \frac{BC}{CP}$. **Al-2005**



16. In a triangle if the square on one side is equal to the sum of squares on the other two sides, prove that the angle opposite to the first side is a right angle. Using the above theorem to prove the following :
In a quadrilateral ABCD, $\angle B = 90^0$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90^0$. **Al-2205**

17. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following : In figure, DE ∥ AC and BD = CE. Prove that ABC is an isosceles triangle. **Delhi-2007, 2009**



18. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above for the following : If the areas of two similar triangles are equal, prove that they are congruent. **Al-2007**

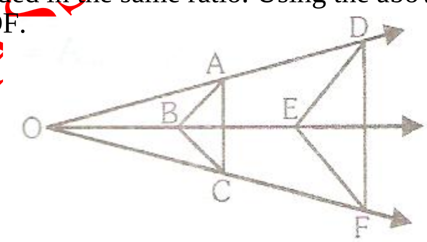
19. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above result, prove the following :

In a ΔABC, XY is parallel to BC and it divides ΔABC into two parts of equal area. Prove that $\frac{BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

Delhi-2008

20. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above, do the following :
The diagonals of a trapezium ABCD, with AB ∥ DC, intersect each other at the point O. If AB = 2 CD, find the ratio of the area of ΔAOB to the area of ΔCOD. **Al -2008**

21. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following : In the fig, AB ∥ DE and BC ∥ EF. Prove that AC ∥ DF. **Foreign-2008**



2. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Using the above, do the following : In a trapezium ABCD, AC and BD are intersecting at O, AB ∥ DC and AB = 2 CD. If area of ΔAOB = 84 cm^2, find the area of ΔCOD. **Delhi-2009**

VERY SHORT ANSWER TYPE QUESTIONS

2. 96 cm^2 **3.** $2\sqrt{(q-1)}$ **6.** 60^0 **8.** 13 m **9.** 4.25 **12.** 9 : 4 **14.** 4.9 cm **16.** 8.8 cm **18.** (iii) 16 : 81

28. 13.5 cm **19.** $\left(\frac{ac}{b+c}\right)$ **31.** 36 cm^2

SHORT ANSWER TYPE QUESTIONS

2. $2\sqrt{5}$ cm **15.** AE = $\frac{15}{13}$, DE = $\frac{36}{13}$

LONG ANSWER TYPE QUESTIONS

4. $\frac{bc}{\sqrt{b^2+c^2}}$ **6.** 24 cm^2 **8.** 13 cm **9.** 3 : 4 **10.** 8 cm **21.** 4 : 1 **23.** 21 cm^2

EXERCISE – 1 (FOR OLYMPIADS)

CHOOSE THE CORRECT ONE

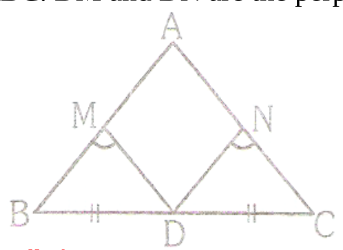
1. In a triangle ABC, if AB, BC and AC are the three sides of the triangle, then which of the statements is necessarily true?
(A) AB + BC < AC (B) AB + BC > AC (C) AB + BC = AC (D) $AB^2 + BC^2 = AC^2$

2. The sides of a triangle are 12 cm, 8 cm and 6 cm respectively, the triangle is :
(A) acute (B) obtuse (C) right (D) can't be determined

3. In an equilateral triangle, the incentre, circumcenter, orthocenter and centroid are:
(A) concylic (B) coincident (C) collinear (D) none of these

4. In the adjoining figure D is the midpoint of a ΔABC. DM and DN are the perpendiculars on AB and AC respectively and DM = DN, then the ΔABC is :

(A) right angled
(B) isosceles
(C) equilateral
(D) scalene

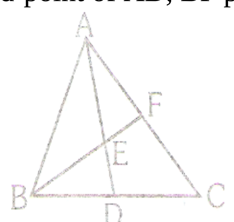


5. Triangle ABC is such that AB = 9 cm, BC = 6 cm, AC = 7.5 cm, Triangle DEF is similar to ΔABC, If EF = 12 cm then DE is :
(A) 6 cm (B) 16 cm (C) 18 cm (D) 15 cm

6. In ΔABC, AB = 5 cm, AC = 7 cm. If AD is the angle bisector of < A. Then BD : CD is :
(A) 25 : 49 (B) 49 : 25 (C) 6 : 1 (D) 5 : 7

7. In a ΔABC, D is the mid-point of BC and E is mid-point of AD, BF passes through E. What is the ratio of AF : FC

(A) 1 : 1
(B) 1 : 2
(C) 1 : 3
(D) 2 : 3

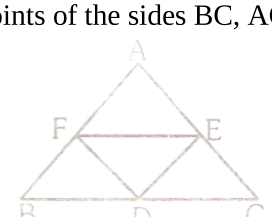


8. In a ΔABC, AB = AC and AD $\perp$ BC, then :
(A) AB < AD (B) AB > AD (C) AB = AD (D) AB $\leq$ AD

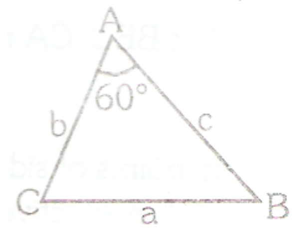
9. The difference between altitude and base of a right angled triangle is 17 cm and its hypotenuse is 25 cm. What is the sum of the base and altitude of the triangle is ?
(A) 24 cm (B) 31 cm (C) 36 cm (D) can't be determined

10. If AB, BC and AC be the three sides of a triangle ABC, which one of the following is true ?
(A) AB – BC = AC (B) (AB – BC) > AC (C) (AB – BA) < AC (D) $AB^2 - CB^2 = AC^2$

11. In the adjoining figure D, E and F are the mid-points of the sides BC, AC and AB respectively. ΔDEF is congruent to triangle :
(A) ABC
(B) AEF
(C) CDE , BFD
(D) AFE , BFD and CDE



12. In the adjoining figure $\angle BAC = 60^0$ and BC = a, AC = b and AB = c, then :



(A) $a^2 = b^2 + c^2$
(B) $a^2 = b^2 + c^2 - bc$
(C) $a^2 = b^2 + c^2 + bc$
(D) $a^2 = b^2 + 2bc$

13. If the medians of a triangle are equal, then the triangle is:
(A) right angled (B) isosceles (C) equilateral (D) scalene

14. The incentre of a triangle is determined by the:
(A) Medians (B) angle bisectors
(C) perpendicular bisectors (D) altitudes

15. The point of intersection of the angle bisectors of a triangle is :
(A) orthocenter (B) centroid (C) incentre (D) circumcentre

16. A triangle PQR is formed by joining the mid-points of the sides of a triangle ABC, 'O' is the circumcentre of ΔABC, then for ΔPQR, the point 'O' is :
(A) incentre (B) circumcentre (C) orthocenter (D) centroid

17. If AD, BE, CF are the altitudes of ΔABC whose orthocenter is H, then C is the orthocenter of :
(A) ΔABH (B) ΔBDH (C) ΔABD (D) ΔBEA

18. In an equilateral ΔABC, if a, b and c denote the lengths of perpendiculars from A, B and C respectively on the opposite sides, then:
(A) $a > b > c$ (B) $a > b < c$ (C) $a = b = c$ (D) $a = c \neq b$

19. Any two of the four triangles formed by joining the midpoints of the sides of a given triangle are:
(A) congruent (B) equal in area but not congruentAB > AD
(C) unequal in area and not congruent (D) none of these

20. The internal bisectors of $\angle$B and $\angle$C of ΔABC meet at O. If $\angle A = 80^0$ then $\angle$BOC is :
(A) 50^0 (B) 160^0 (C) 100^0 (D) 130^0

21. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is :
(A) centroid (B) incentre (C) circumcentre (D) orthocenter

22. Incentre of a triangle lies in the interior of :
(A) an isosceles triangle only (B) a right angled triangle only
(C) any equilateral triangle only (D) any triangle

23. In a triangle PQR, PQ = 20 cm and PR = 6 cm, the side QR is :
(A) equal to 14 cm (B) less than 14 cm (C) greater than 14 cm (D) none of these

24. If ABC is a right angled triangle at B and M, N are the mid-points of AB and BC, than $4(AN^2 + CM^2)$ is equal to-
(A) $4AC^2$ (B) $6\ AC^2$ (C) $5\ AC^2$ (D) $\frac{5}{4}\ AC^2$

25. ABC is a right angle triangle at A and AD is perpendicular to the hypotence. Then $\frac{BD}{CD}$ is equal to :
(A) $\left(\frac{AB}{AC}\right)^2$ (B) $\left(\frac{AB}{AD}\right)^2$ (C) $\frac{AB}{AC}$ (D) $\frac{AB}{AD}$

26. Let ABC be an equilateral triangle. Let BE $\perp$ CA meeting CA at E, then $(AB^2 + BC^2 + CA^2)$ is equal to :
(A) $2BE^2$ (B) $3\ BE^2$ (C) $4\ BE^2$ (D) $6\ BE^2$

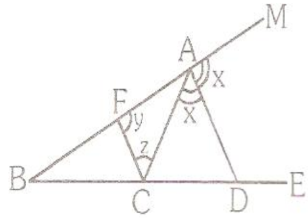
27. If D, E and F are respectively the mid-points of sides of BC, CA and AB of a ΔABC. If EF = 3 cm, FD = 4 cm, and AB = 10 cm, then DE, BC and CA respectively will be equal to :

(A) 6, 8 and 20 cm (B) 4, 6 and 8 cm (C) 5, 6 and 8 cm (D) $\frac{10}{3}$, 9 and 12 cm

28. In the right angle triangle $\angle C = 90^0$. AE and BD are two medians of a triangle ABC meeting at F. The ratio of the area of ΔABF and the quadrilateral FDCE is :

(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 2 : 3

29. The bisector of the exterior $\angle$A of ΔABC intersects the side BC produced to D. Here CF is parallel to AD.



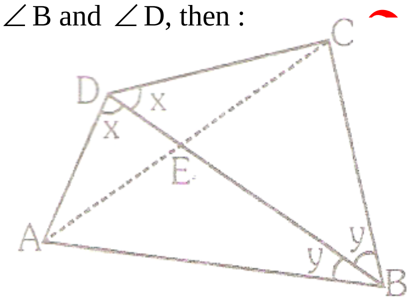
(A) $\frac{AB}{AC} = \frac{BD}{CD}$

(B) $\frac{AB}{AC} = \frac{CD}{BD}$

(C) $\frac{AB}{AC} = \frac{BC}{CD}$

(D) None of these

30. The diagonal BD of a quadrilateral ABCD bisects $\angle$B and $\angle$D, then :



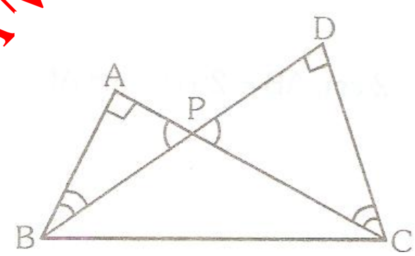
(A) $\frac{AB}{CD} = \frac{AD}{BC}$

(B) $\frac{AB}{BC} = \frac{AD}{CD}$

(C) $AB = AD \times BC$

(D) None of these

31. Two right triangles ABC and DBC are drawn on the same hypotenuse BC on the same side of BC. If AC and DB intersects at P, then



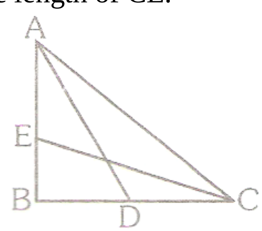
(A) $\frac{AP}{PC} = \frac{BP}{DP}$

(B) $AP \times DP = PC \times BP$

(C) $AP \times PC \times = BP \times DP$

(D) $AP \times BP \times = PC \times PD$

32. In figure, ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE:



(A) $2\sqrt{5}$ cm

(B) 2.5 cm

(C) 5 cm

(D) $4\sqrt{2}$ cm

33. In a ΔABC, AB = 10 cm, BC = 12 cm and AC = 14 cm. Find the length of median AD. If G is the centroid, find length of GA :

(A) $\frac{5}{3}\sqrt{7}, \frac{5}{9}\sqrt{7}$ (B) $5\sqrt{7}, 4\sqrt{7}$ (C) $\frac{10}{\sqrt{3}}, \frac{8}{3}\sqrt{7}$ (D) $4\sqrt{7}, \frac{8}{3}\sqrt{7}$

34. The three sides of a triangles are given. Which one of the following is not a right triangle ?

(A) 20, 21, 29 (B) 16, 63, 65

(C) 56, 90, 106 (D) 36, 35, 74

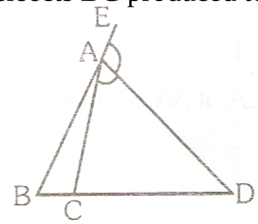
35. In the figure AD is the external bisector of $\angle$EAC, intersects BC produced to D. If AB = 12 cm, AC = 8 cm and BC = 4 cm, find CD.

(A) 10 cm

(B) 6 cm

(C) 8 cm

(D)9 cm



36. In ΔABC, AB2 + AC2 = 2500 cm2 and median AD = 25 cm, find BC.

(A) 25 cm (B) 40 cm (C) 50 cm (D) 48 cm

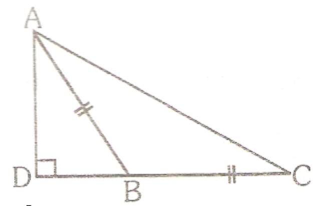
37. In the given figure, AB = BC and $\angle$BAC = 150. AB = 10 cm. Find the area of ΔABC.

(A) 50 cm^2

(B) 40 cm^2

(C) 25 cm^2

(D) 32 cm^2



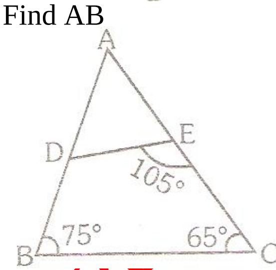
38. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and if AE = 10 cm. Find AB

(A) 16 cm

(B) 12 cm

(C) 15 cm

(D) 18 cm



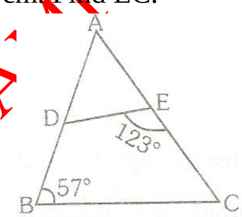
39. In the figure AD = 12 cm. AB = 20 cm and AE = 10 cm. Find EC.

(A) 14 cm

(B) 10 cm

(C) 8 cm

(D) 15 cm



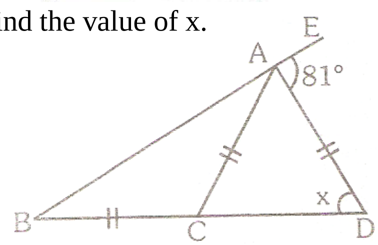
40. In the given fig, BC = AC = AD, $\angle$EAD = 81^0. Find the value of x.

(A) 45^0

(B) 54^0

(C) 63^0

(D) 36^0



41. What is the ratio of inradius to the circumradius of a right angled triangle?

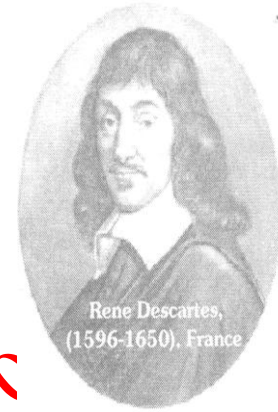
(A) 1 : 2 (B) 1 : $\sqrt{2}$ (C) 2 : 5 (D) Can't be determined

ANSWER KEY

Ans.	B	B	B	B	C	D	B	B	B	C	D	B	C	B	C
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	A	C	A	D	B	D	C	C	A	C	C	A	A	B
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**				
Ans.	C	A	D	D	C	C	C	C	A	B	D				

CO-ORDINATE GEOMETRY

★ INTRODUCTION



In the previous class, you have learnt to locate the position of a point in a coordinate plane of two dimensions, in terms of two coordinates. You have learnt that a linear equation in two variables, of the form ax + c = 0 (either $a \neq 0$ or $b \neq 0$) can be represented graphically as a straight line in the coordinate plane of x and y coordinates. In chapter 4, you have learnt that graph of a equation $ax^2 + bx + c = 0$, $a \neq 0$ is an upward parabola if a > 0 and a downward parabola if a < 0.

In this chapter, you will learn to find the distance between two given points in terms of their coordinates and also, the coordinates of the point which divides the line segment joining the two given points internally in the given ratio.

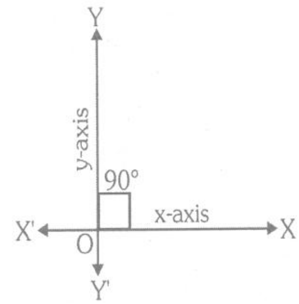
★ HISTORICAL FACTS

Rene Descartes (1596 - 1650), The 17th century French-Mathematician, was a thinker and a philosopher. He is called the father of Co-ordinate Geometry because he unified Algebra and Geometry which were earlier two distinct branches of Mathematics. Descartes explained that two numbers called co-ordinates are used to locate the position of a point in a plane.

He was the first Mathematician who unified Algebra and Geometry, so Analytical Geometry is also called Algebraic Geometry. Cartesian plane and Cartesian Product of sets have been named after the great Mathematician.

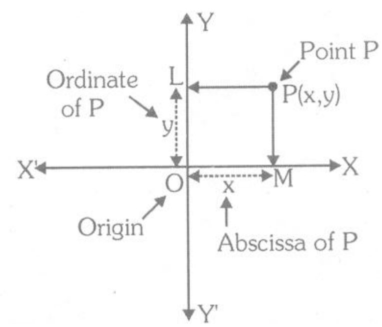
★ RECALL

Cartesian Co-ordinate system:



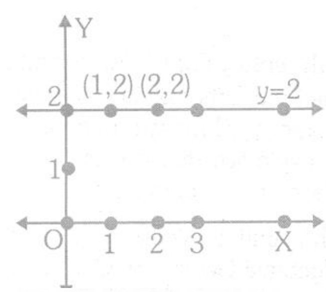
Let X'OX and Y'OY be two perpendicular straight lines intersecting each other at the point O. Then :

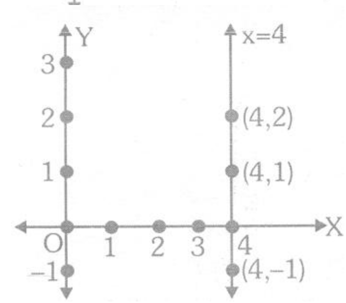
1. X'OX is called the x-axis or the axis of x.
2. Y'OY is called the y-axis or the axis of y.
3. The x-co-ordinate along OX is positive and along OX' negative, y-co-ordinate along OY (upward) is positive and along OY' (downward) is negative.
4. Both X'OX and Y'OY taken together in this order are called the rectangular axes because the angle between them is a right angle.
5. O is called the origin i.e., it is point of intersection of the axes of co-ordinates.



Co-ordinates of a point :

6. Abscissa of a point in the plane is its perpendicular distance with proper sign from y-axis.
7. Ordinate of a point in the plane is its perpendicular distance with proper sign from y-axis.
8. The y-co-ordinate any point on x-axis is zero.
9. The x-co-ordinate any point on x-axis is zero.
10. Any point in the xy-plane, whose y-co-ordinate is zero, lies on x-axis.
11. Any point in the xy-plane, whose x-co-ordinate is zero, lies on x-axis.
12. The origin has coordinates (0, 0).
13. The ordinates of all points on a horizontal line which is parallel to x-axis are equal i.e. y = constant = 2.
14. The abscissa of all points on a vertical line which is a line parallel to y-axis are equal i.e. x-constant = 4





Four Quadrants of a Coordinate plane :

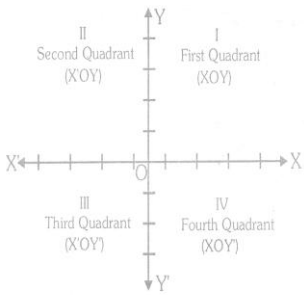
The rectangular axes X'OX and Y'OY divide the plane into four quadrants as below :

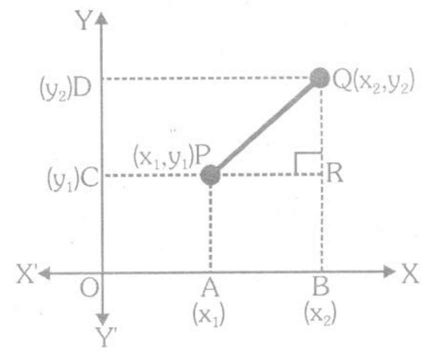
15. Any point in the I quadrant has (+ ve abscissa, + ve ordinate).
16. Any point in the II quadrant has (- ve abscissa, + ve ordinate).
17. Any point in the III quadrant has (- ve abscissa, - ve ordinate).
18. Any point in the IV quadrant has (+ ve abscissa, - ve ordinate).

★ DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) in a rectangular coordinate system is equal to
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Proof : X,OX and Y'OY are the rectangular coordinate axes. P(x_1, y_1) and Q (x_2, y_2) are the given points. We draw PA and QB perpendiculars on the x-axis : PC and QD perpendicular on the y-axis,





Now, CP (produced) meets BQ in R and PR $\perp$ BQ.
We find PR = AB = OB – OA = (x_2 – x_1)
and QR = BQ – BR = BQ – AP = CD – OC = (y_2 – y_1)
In right ΔPRQ, using Pythagoras theorem, we have

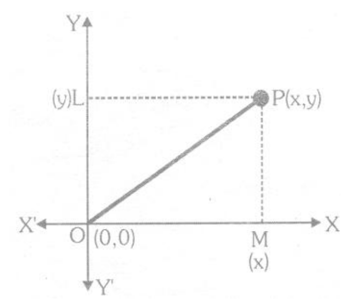
$$PQ^2 = PR^2 + QR^2 = (x_2-x_1)^2 + (y_1-y_2)^2 \quad \Rightarrow \quad PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

We have taken the positive square roof value because distance between two points is a non-negative quantity.
Distance of a Point from Origin:

The distance of a point (x, y) form origin is $\sqrt{x^2+y^2}$.

Proof : Let us take a point P (x,y) in the given plane of axes X'OX and Y'OY as shown in the fig. Here, the point P (x, y) is in the first quadrant but it can be taken anywhere in all the four quadrants.
We have to find the distance OP, i.e, the distance of the point P from the origin O.

Form the point P, draw PM $\perp$ OX and PL $\perp$ OY. Then we have
OM = x
MP = OL = y
OP2 = OM2 + MO2 = x2 + y2
Therefore, OP = $\sqrt{x^2+y^2}$

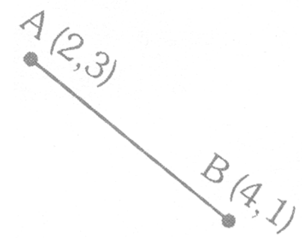


Test For Geometrical Figures :

(a)	For an isosclels	:	Prove that two sides are equal.
(b)	For an equilateral triangle	:	Prove that three sides are equal.
(c)	For a ritht-angled triangle	:	Prove that the sum of the squares of two sides is equal to the square of the third side.
(d)	For a square	:	Prove that all sides are equal and diagonals are equal.
(e)	For a rhombus	:	Prove that all sides are equal and diagonals are not equal.
(f)	For a rectangle	:	Prove that the opposites sides are equal and diagonals are also equal.
(g)	For a parallelogram	:	Prove that the opposite sides are equal in length and diagonals are not equal

Ex.1 Find the distance between the following pairs of points:
(a) (2,3), (4,1) (b) (-5, 7), (-1,3) (c) (a,b), (-1, - b) **[NCERT]**

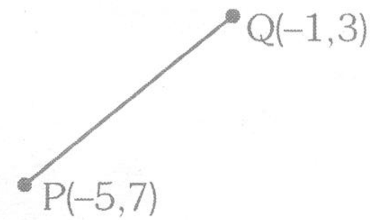
Sol. (a) The given points are : A (2,3), B (4,1).



Required distance $= AB = BA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(b) Distance between P(- 5, 7) and Q(-1, 3) is given by



PQ = QP $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1+5)^2 + (3-7)^2} = \sqrt{16+6} = \sqrt{32}$

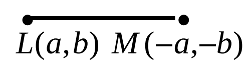
Required distance = PQ = QP = $4\sqrt{2}$ units

(c) Distance LM between L (a,b) and M (-a, -b) is given by

LM $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2}$

$= \sqrt{4(a^2 + b^2)} = 2\sqrt{a^2 + b^2}$ units



Ex.2 Find the points on x-axis which are at a distance of 5 units from the point A(-1, 4).

Sol. Let the point on x-axis be :P(x, 0).

Distance = PA = 5 units $\Rightarrow$ $PA^2 = 25$

$\Rightarrow$ $(x+1)^2 + (0-4)^2 = 25$ $\Rightarrow$ $x^2 + 2x + 1 + 16 = 25$

$\Rightarrow$ $x^2 + 2x + 17 = 25$ $\Rightarrow$ $x^2 + 2x - 8 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 + 4(8)}}{2} = \frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2}$$

$$= \frac{-2+6}{2}, \frac{-2-6}{2} = \frac{4}{2}, -\frac{8}{2} = 2, -4$$

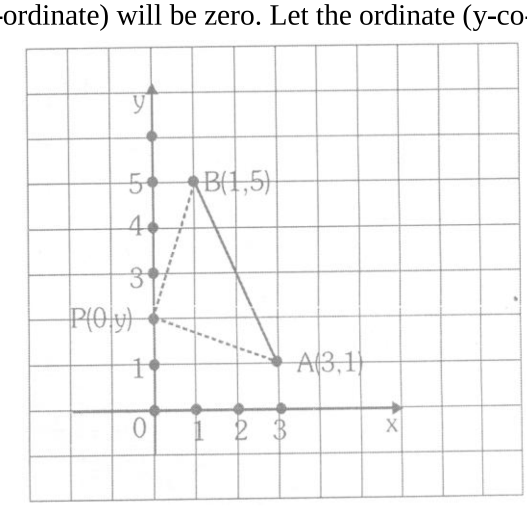
Required point on x-axis are (2, 0) and (-4, 0)

Verification: PA = $\sqrt{(2+1)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

PA = $\sqrt{(-4+1)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Ex.3 What point on y-axis is equidistant from the points (3, 1) and (1, 5)

Sol. Since the required point P(say) is on the y-axis, its abscissa (x-co-ordinate) will be zero. Let the ordinate (y-co-ordinate) of the point be y.



Therefore co-ordinates of the point P are : (0, y)

i.e., P (0, y)

Let A and B denote the points (3, 1) and (1, 5) respectively.
PA = PB …(given)

Squaring we get :
$PA^2 = PB^2$

$\Rightarrow$ $(0-3)^2 + (y-1)^2 = (0-1)^2 + (y-5)^2$

$\Rightarrow \quad 9+y^2 1-2y=1+y^2+25-10y$

$\Rightarrow \quad y^2-2y+10-y^2-10y+26 \Rightarrow -2y+10y=26-10 \Rightarrow 8y=16 \Rightarrow y=2$

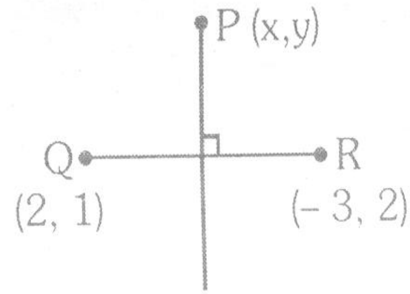
The required point on y-axis equidistant from A(3, 1) and B(1, 5) is P(0, 2).

Ex.4 If Q(2, 1) and R(-3, 2) and P(x, y) lies on the right bisector of QR then show that 5x – y +

Sol Let P(x, y) be a point on the right bisector of QR :

Q(2, 1) and R(-3, 2) are equidistant from P(x, y), then we must have :

PQ = PR



$\Rightarrow \quad PQ^2 = PR^2$

$\Rightarrow \quad (x-2)^2+(y-1)^2=(x+3)^2+(y-2)^2$

$\Rightarrow \quad (x^2-4x+4)\ +(y^2-2y+1)\ =(x^2+6x+9)\ +(y^2-4x+4)$

$\Rightarrow \quad -4x-2y\ +5=6x-4y+13$

$\Rightarrow \quad 10x-2y-8=0$

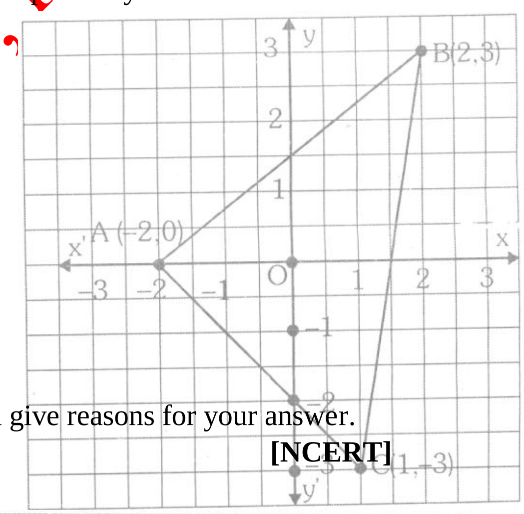
$\Rightarrow \quad 2(5x-y+4)\ =0$

$\Rightarrow \quad 5x-y+4=0$

Ex.5 The vertices of a triangle are (-2, 0), (2, 3) and (1, -3). Is the triangle equilateral : isosceles or scalene ?

Sol. We denote the given point (-2, 0), (2, 3) and (1, -3) by A, B and C respectively then :

$A(-2,0), B(2,3), C(1,-3)$



$AB=\sqrt{(2+2)^2+(3-0)^2}=\sqrt{(4)^2+(3)^2}=5$

$BA=\sqrt{(1-2)^2+(-3-3)^2}=\sqrt{(-1)^2+(-6)^2}=\sqrt{37}$

$BC=\sqrt{(-2-1)^2+(0+3)^2}=\sqrt{(-3)^2+(3)^2}=3\sqrt{2}$

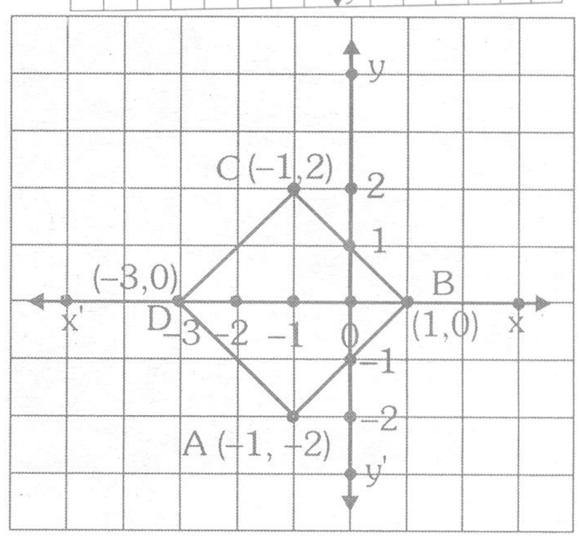
Thus we have $AB \neq BC \neq CA$

$\Rightarrow$ ABC is a scalene triangle

Ex.6 Name the quadrilateral formed, if any, by the following points, and give reasons for your answer.

$(-1,-2),(1,0),(-1,\ 2),(-3,0)$ **[NCERT]**

Sol. $A(-1,-2), B(1,0) C(-1,2), D(-3,\ 0)$



Determine distances : AB, BC, CD, DA, AC and BD.

AB = $\sqrt{(1+1)^2+(0+2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

BC = $\sqrt{(-1-1)^2+(2-0)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

CD = $\sqrt{(-3+1)^2+(0-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

DA = $\sqrt{(-1+3)^2+(-2-0)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

AB = BC = CD = DA

The sides of the quadrilateral are equal …………(1)

$$\left.\begin{aligned} AC &= \sqrt{(-1+1)^2+(2+2)^2} = \sqrt{0+16} = 4 \\ BD &= \sqrt{(-3+1)^2+(0-0)^2} = \sqrt{16+0} = 4 \end{aligned}\right\} \quad \text{Diagonal AC = Diagonal BD.........(2)}$$

From (1) and (2) we conclude that ABCD is a square.

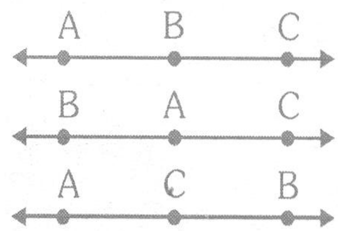
★ **COLLINEARITY OF THREE POINTS**

Let A, B and C three given points. Point A, B and C will be collinear, If the sum of lengths of any two line-
segments is equal to the length of the third line-segment.
In the adjoining fig. there are three point A, B and C.
Three point A, B and C are collinear if and only if

(i) AB + BC = AC

or (ii) AB + AC = BC

or (iii) AC + BC = AB



Ex.7 Determine whether the points (1, 5) (2, 3) and (-2, -11) are collinear. **[NCERT]**

Sol. The given points are : A(1, 5), B(2, 3) and C(-2, -11).
Let us calculate the distance : AB, BC and CA by using distance formula.

$AB = \sqrt{(2-1)^2+(3-5)^2} = \sqrt{(1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

$BC = \sqrt{(-2-2)^2+(-11-3)^2} = \sqrt{(-4)^2+(-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

$CA = \sqrt{(-2-1)^2+(-11-5)^2} = \sqrt{(-3)^2+(-16)^2} = \sqrt{9+296} = \sqrt{265}$

From the above we see that : AB + BC $\neq$ CA
Hence the above stated points A(1, 5) B(2, 3) and C(-2, -11) are not collinear.

★ **SECTION FORMULA**

Coordinates of the point, dividing the line-segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are given by $\left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

Proof. Let P (x, y) be the point dividing the line-segment joining A(x_1, y_1) internally in the ratio $m_1 : m_2$. We draw the perpendiculars AL, BM and PQ on the x-axis from the points A, B and P respectively. L, M and Q are the points on the x-axis where three perpendiculars meet the x-axis.

We draw AC $\perp$ PQ and PD $\perp$ BM. Here AC $\|$ x-axis and PD $\|$ x-axis.

$\Rightarrow$ AC $\|$ PD ($\because$ AC and PD both $\|$ x-axis)

$\Rightarrow$ $\angle$PAC = $\angle$BPD

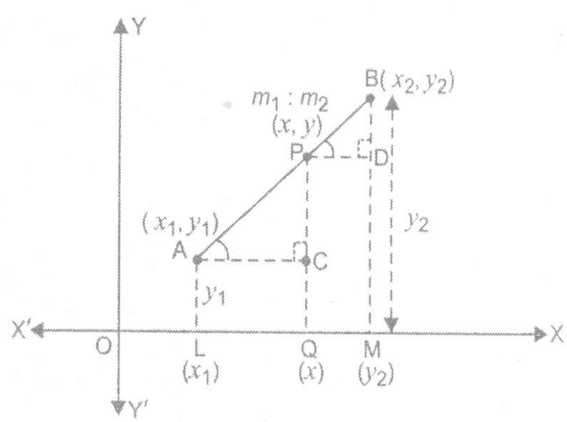
Thus, in ΔACP and ΔPDB, we have

$\angle$PAC = $\angle$BPD

and $\angle$ACP = $\angle$PDB = 90^0

Then by AA similarity criterion,

ΔACP ~ ΔPDB



$$\Rightarrow \quad \frac{AC}{PD} = \frac{PC}{BD} = \frac{AP}{PB} = \frac{m_1}{m_2}$$

$$\Rightarrow \quad \frac{AC}{PD} = \frac{m_1}{m_2} \text{........(1)} \quad \text{and} \quad \frac{PC}{BD} = \frac{m_1}{m_2} \text{.........(2)}$$

AC = LQ = OQ – OL = $(x - x_1)$
PD = QM = OM – OQ = $(x_2 - x)$
Putting in (1), we get

$$\frac{(x - x_1)}{(x_2 - x)} = \frac{m_1}{m_2} \quad \Rightarrow \quad m_2x - m_2x_1 = m_1x_2 - m_1x$$

$$\Rightarrow \quad m_1x + m_2x = m_1x_2 - m_2x_1 \quad \Rightarrow \quad (m_1 + m_2)x = m_1x_2 - m_1x_1$$

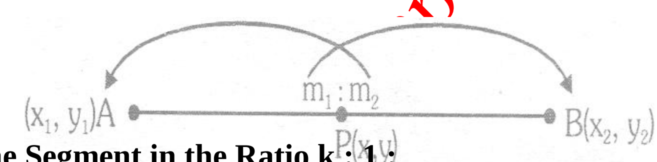
$$\Rightarrow \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

Now, PC = PQ – CQ = PQ – AL = $(y - y_1)$
BD = BM – DM = BM – PQ = $(y_2 - y)$
Putting in (2), we get

$$\frac{y - y_1}{y_2 - y} = \frac{m_1}{m_2} \quad \Rightarrow \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

Therefore, the coordinates of the point P are $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

Remark : To remember the section formula, the diagram given below is helpful:



Point Dividing a Line Segment in the Ratio k : 1 :

If P$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$ divides the line-segment, joining A(x_1, y_1) and B(x_2, y_2) internally in the ratio $m_1 : m_2$ we can express it as below :

P$\left(\frac{\frac{m_1}{m_2}x_2}{\frac{m_1}{m_2}+1}, \frac{\frac{m_1}{m_2}y_2 + y_1}{\frac{m_1}{m_2}+1}\right)$ (By dividing the numerator and the denominator by m_2)

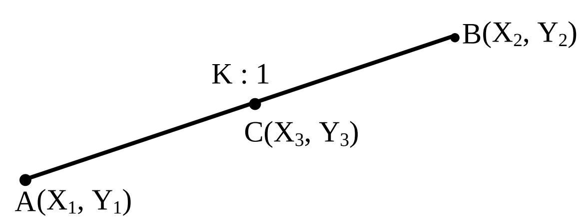
Putting $\frac{m_1}{m_2}$ = k, the ratio becomes k : 1 and the coordinates of P are expressed in the form

P$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2y_1}{k+1}\right)$

Therefore, the coordinates of the point P, which divides the line-segment joining A (x_1, y_1) and B(x_2, y_2) internally in the ratio k : 1, are given by $\left(\frac{kx_2 + x1}{k+1}, \frac{ky_2 + y1}{k+1}\right)$

Collinearity of three points :

Three given points A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) are said to be collinear if one of them must divide the line segment joining the other two points in the same ratio.



Remark : Three points are called non-collinear if one of them divides the line segment joining the other two points in different ratios

Mid-point Formula :

Coordinates of the mid-point of the line-segment joining (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.

The mid-point M (x,y) of the line-segment joining A (x_1, y_1) and B(x_2, y_2) divides the line-segment AB in the ratio 1 : 1. Putting $m_1 - m_2 = 1$in the section formula, we get the coordinates of the mid-point as $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

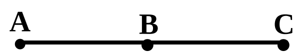
.

COMPETITION WINDOW

SECTION FORMULA FOR EXTERNAL DIVISION

The co-ordinates of line point which divides the line segment joining the point (x_1, y_1) and (x_2, y_2) externally in the ratio m : n are $\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n}\right)$

e.g., In the following case, C divides AB externally in ratio AC : BC

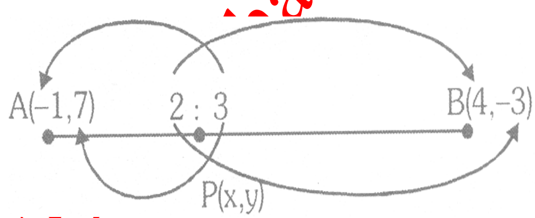


Let A = (1, 0), B = (4, 0) and 5 : 2 be the ratio in which C divides AB externally. Then co-ordinates of C are :

$$\left(\frac{5\times4-2\times1}{5-2},\frac{5\times0-2\times0}{5-2}\right)=[6,0]$$

Ex.8 Find the co-ordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2 : 3, **[NCERT]**

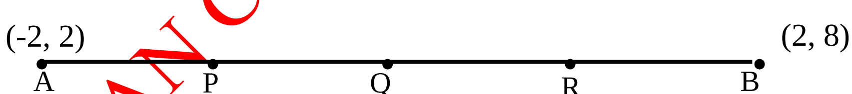
Sol. Let P(x, y) divides the line segment AB joining A(-1, 7) and B(4, -3) in the ratio 2 : 3. Ten by using section formula line the co-ordinates of P are given by :



$$\left(\frac{2\times4\times3\times(-1)}{2+3},\frac{2\times(-3)+3\times7}{2+3}\right)=P\left(\frac{8-3}{5},\frac{-6+21}{5}\right)=P\left(\frac{5}{5},\frac{15}{5}\right) = P(1, 3)$$

Hence the required point of division which divides the line segment joining A(-1, 7) and (4, -3) in the ratio 2 : 3 is P(1, 3).

Sol.



It is given that AB is divided into four equal parts : AP = PQ = QR = RB

Q is the mid-point of AB, then co-ordinates of Q are : $\left(\frac{-2+2}{2},\frac{2+8}{2}\right)=\left(\frac{0}{2},\frac{10}{2}\right)=(0,5)$

P is the mid-point of AQ, then co-ordinates of P are : $\left(\frac{-2+0}{2},\frac{2+5}{2}\right)=\left(\frac{-2}{2},\frac{7}{2}\right)=\left(-1,\frac{7}{2}\right)$

Also, R is the mid-point of QB, then co-ordinates of R are : $\left(\frac{0+2}{2},\frac{5+8}{2}\right)=\left(\frac{2}{2},\frac{13}{2}\right)=\left(-1,\frac{13}{2}\right)$

Hence, required co-ordinates of the points are :

$$P(-1\frac{7}{2}),Q(0,\ 5),R(1,\frac{13}{2})$$

Ex.10 If the point C(-1, 2) divides the lines segment AB in the ratio 3 : 4, where the co-ordinates of A are (2, 5), find the coordinates of B.

Sol. Let C (-1, 2) divides the line joining A (2, 5) and B (x, y) in the ratio 3 : 4. Then.

$$C\left(\frac{3x+8}{7}, \frac{3y+20}{7}\right) = C(-1, 2)$$

$$\Rightarrow \quad \frac{3x+8}{7} = -1 \quad \& \quad \frac{3y+20}{7} = 2$$

$$\Rightarrow \quad 3x+8 = -7 \quad \& \quad 3y+20 = 14$$

$$\Rightarrow \quad x = -5 \quad \& \quad y = -2$$

The coordinates of B are : B (-5, -2)

Ex.11 Find the ratio in which the line segment joining the points (1, -7) and (6, 4) is divided by x-axis

Sol. Let C (x, 0) divides AB in the ratio k : 1.

By section formula, the coordinates of C are given by :

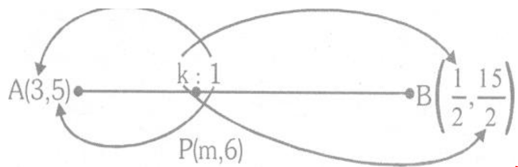
$$C\left(\frac{6k+1}{k+1}, \frac{4k-7}{k+1}\right) \quad \Rightarrow \quad \frac{4k-7}{k+1} = 0$$

$$\Rightarrow \quad 4k-7 = 0 \Rightarrow k = \frac{7}{4}$$

i,e., the x-axis divides AB in the ratio 7 : 4.

Ex.12 Find the value of m for which coordinates (3,5), (m,6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear.

Sol. Let P (m, 6) divides the line segment AB joining A (3,5) B $\left(\frac{1}{2}, \frac{15}{2}\right)$ in the ratio k : 1.



Applying section formula, we get the co-ordinates of P : $\left(\frac{\frac{1}{2}k+3\times1}{k+1}, \frac{\frac{15}{2}k+5\times1}{k+1}\right) = \left(\frac{k+6}{2(k+1)}, \frac{15k+10}{2(k+1)}\right)$

But P (m, 6) = P$\left(\frac{k+6}{2(k+1)}, \frac{15k+10}{2(k+1)}\right) \quad \Rightarrow \quad m = \frac{k+6}{2(k+)}$ and also $\frac{15k+10}{2(k+1)} = 6$

$$\Rightarrow \quad \frac{15k+10}{2(k+1)} = 6 \quad \Rightarrow \quad 15k+10 = 12(k+1)$$

$$\Rightarrow \quad 15k+10 = 12k+12 \quad \Rightarrow \quad 15k-12k = 12-10$$

$$\Rightarrow \quad 3k = 2 \quad \Rightarrow \quad k = \frac{2}{3}$$

Putting $k = \frac{2}{3}$ in the equation $m = \frac{k+6}{2(k+)}$ we get :

$$m = \frac{\left(\frac{2}{3}+6\right)}{2\left(\frac{2}{3}+1\right)} = \frac{\left(\frac{2+18}{3}\right)}{2\left(\frac{2+3}{3}\right)} = \frac{20}{3} \times \frac{3}{10} = \frac{20}{10} \qquad \left(\because k = \frac{2}{3}\right)$$

$$m = \frac{10\times2}{10} = 2$$

Required value of m is 2 $\Rightarrow$ m = 2

Ex.13 The two opposite vertices of a square are (-1, 2) and (3, 2). Find the co-ordinates of the other two vertices.

Sol. Let ABCD be a square and two opposite vertices of it are A(-1, 2) and C(3, 2) ABCD is a square.

$\Rightarrow \quad AB = BC$

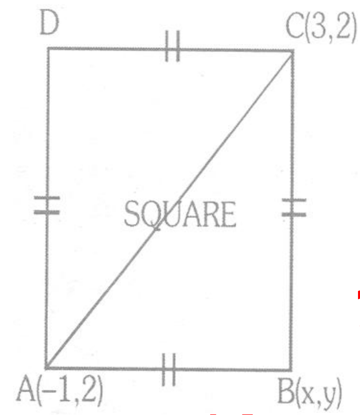
$\Rightarrow \quad AB^2 = BC^2$

$\Rightarrow \quad (x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$\Rightarrow \quad x^2 + 2x + 1 = x^2 - 6x + 9$

$\Rightarrow \quad 2x \;\; + 6x = 9 - 1 = 8$

$\Rightarrow \quad 8x = 8 \Rightarrow x = 1$



ABCD is right Δ at B, then

$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow \quad (3+1)^2 + (2-2)^2 = (x+)^2 + (y-2)^2 + (x-3)^2 = (y-2)^2$

$\Rightarrow \quad 16 = 2(y-2)^2 + (1+1)^2 + (1-3)^2$

$\Rightarrow \quad 16 = 2(y-2)^2 + 4 + 4 \Rightarrow 2(y-2)^2 = 16 - 8 = 8$

$\Rightarrow \quad (y-2)^2 = 4 \quad \Rightarrow \quad y - 2 = \pm 2 \Rightarrow \quad$ y = 4 and 0

i.e., when x = 1 then y = 4 and 0

Co-ordinates of the opposite vertices are : B(1, 0) or D(1, 4)

★ AREA OF A TRIANGLE

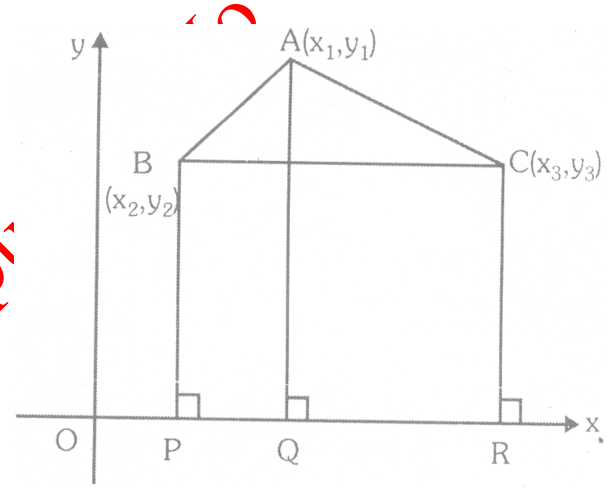
In your pervious classes, you have learnt to find the area of a triangle in terms of its base and corresponding altitude as below:

Area of triangle = $\frac{1}{2} \times$ base $\times$ altitude.

In case, we know the lengths of the three sides of a triangle, then the area of the triangle can be obtained by using the Heron's formula.

In this section, we will find the area of a triangle when the coordinates of its three vertices are given. The lengths of the three sides can be obtained by using distance formula but we will not prefer the use of Heron's formula.

Some times, the lengths of the sides are obtained as irrational numbers and the application of Heron's formula becomes tedious. Let us develop some easier way to find the area of a triangle when the coordinates of its vertices are given.



Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the given three points. Through A draw AQ $\perp$ OX, through B draw BP $\perp$ OX and through C draw CR $\perp$ OX.

Form the fig. AQ = y_1, BP = y_2 and CR = y_3, OP = x_2, OQ = x_1 and OR = x_3

$\Rightarrow \quad PQ = x_1 - x_2$; $QR = x_3 - x_1$ and $PR = x_3 - x_2$

Area of trapezium = $\frac{1}{2}$ (sum of parallel side) $\times$ distance between parallel lines

ar. (Δ ABC) = ar. (Trap.ABCD) + ar. (Trap.AQRC) – ar. (Trap. BPRC)

$= \frac{1}{2}(BP + AQ) \times PQ + \frac{1}{2}(AQ + CR) \times QR - \frac{1}{2}(PB + CR) \times PR$

$= \frac{1}{2}(y_2 + y_1)(x_1 - x_2) + \frac{1}{2}(y_2 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$

$= \frac{1}{2}\left| x_1(y_2 + y_1 - y_1 - y_3) + x_2(y_2 + y_3 - y_2 - y_1) + x_3(y_1 + y_3 - y_2 - y_3) \right|$

$= \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

Area of Δ ABC = $\frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

Condition of collinearity of three points :

The given points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ will be collinear if the area of the triangle formed by them must be zero because triangle can not be formed.

$\Rightarrow$ area of ΔABC = 0

$\Rightarrow$ $\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|=0$

$\Rightarrow$ $\boxed{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0}$

is the required condition for three points to be collinear.

Ex.14 The co-ordinates of the ΔABC are A(4, 1), D(3, 2) and C(0, K). Given that the area of ΔABC is 12 unit^2. Find the value of k.

Sol. Area of ΔABC formed by the given-points A(4, 1), B(-3, 2) and C(0, k) is

$= \frac{1}{2}\left|4(2-k)+(-3)(k-1)+0(1-2)\right|$

$= \frac{1}{2}\left|18-4k-3k+3\right| \quad = \frac{1}{2}(11-7k)$

But area of ΔABC = 12 unit^2 ……..(given)

$\frac{1}{2}\left| = \frac{1}{2}(11-7k)\right| = 24 \quad \Rightarrow \quad \left|(11-7k)\right| = 24$

$\pm(11-7k) = 24 \quad \Rightarrow \quad 11-7k = 24 \text{ or } -(11-7k) = 24$

$-7k = 24-11 = 13 \quad \Rightarrow \quad k = -\frac{13}{7}$

or $-(11-7k) = 24 \quad \Rightarrow \quad -11+7k = 24$

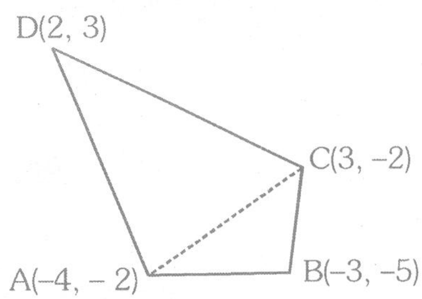
$\Rightarrow \quad 7k = 24+11 = 35 \quad \Rightarrow \quad k = \frac{35}{7} = 5$

Ex.15 Find the area of the quadrilateral whose vertices taken in order are (-4, -2), (-3, -5), (3, -2), and (2, 3). **[NCERT]**

Sol. Join A and C

The given points are A(-4, -2), B(-4, -2), C(-4, -2) and D(2, 3)

Area of ΔABC



$= \frac{1}{2}\left|(-4)(-5+2)-3(-2+2)+3(-2+5)\right|$

$= \frac{1}{2}\left|20-8-6+15\right| \qquad = \frac{21}{2} - 10.5$ sq. units

Area of ΔACD

$= \frac{1}{2}\left|(-4)(-2-3)+3(-2+2)+3(-2+5)\right|$

$= \frac{1}{2}\left|20+15\right| \quad = \frac{35}{2} = 17.5$ sq. units

Area of quadrilateral ABCD = ar. (ΔABC) + ar. (ΔACD) = (10.5 + 17.5) sq. units = 28 sq. units

Ex.16 Find the value of p for which the points (-1, 3), (2, p), (5, -1) are collinear.

Sol. The given points A (1, 3), B(2, p), C(5, -1) are collinear.

$\Rightarrow$ Area of ΔABC formed by these points should be zero.

$\Rightarrow$ The area of ΔABC = 0

$$\Rightarrow \quad \frac{1}{2}\left| x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right| = 0$$
$$\Rightarrow \quad -1(p+1)+2(-1-3)+5(3-p)=0$$
$$\Rightarrow \quad -p-1-8+15-5p=0$$
$$\Rightarrow \quad -6p+15-9=0 \Rightarrow 6p=-6 \Rightarrow p=1$$

Hence the value of p is 1.

COMPETITION WINDOW

AREA OF A QUADRILATERAL

If $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ and (x_4,y_4) are vertices of a quadrilateral, its area

$$\frac{1}{2}\left| (x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_4-x_4y_3)+(x_4y_1-x_1y_4)\right|$$

AREA OF A POLYEON

If $(x_1,y_1),(x_2,y_2),(x_3,y_3)\ \ldots\ldots\ (x_n,y_n)$ are vertices of a polygon of n sides, its area

$$\frac{1}{2}\left| (x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_4-x_4y_3)+\ldots.+(x_ny_1-x_1y_n)\right|$$

Remark : (i) If the area of a quadrilateral joining the four points is zero, the four points are collinear.
(ii) If two opposite vertex of a square are A(x_1, y_1) and C(x_2, y_2) then it's area is

$$\frac{1}{2}[(x_2-x_1)^2+(y_2-y_1)^2]$$

TRY OUT THE FOLLOWING

(i) Find the area of the quadrilateral formed by joining the four points (1, 1), (3, 4), (5, -2) & (4, -7).
(ii) Find the area of the pentagon whose vertices are A(1, 1), B(7, 21), C(7, -3) D(4, -7) and E(0, -3).
(iii) If the Co-ordinates of two opposite vertex of a square are (a, b) and (b, a), find the area of the square.

ANSWERS

(i) $\frac{41}{2}$ sq. nuits (ii) $\frac{137}{2}$ sq. nuits (i) $(a-b)^2$ sq. nuits

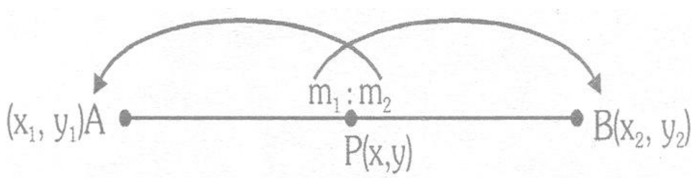
★ **SYNOPSIS**

⏩ ***Distance Formula : The distance between two points (x_1, y_2) and (x_2, y_2) in a rectangular coordinate system is equal to $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. The distance of a point (x, y) from origin is $\sqrt{x^2+y^2}$***

Test For Geometrical Figures :

(a)	For an isosceles triangle	:	Prove that at least two sides are equal
(b)	For an equilateral triangle	:	Prove that three sides are equal
(c)	For a right-angled triangle	:	Prove that the sum of the squares of two sides is equal to the square of the third side.
(d)	For a square	:	Prove that all sides are equal and diagonals are equal.
(e)	For a rhombus	:	Prove that all sides are equal and diagonals are not equal.
(f)	For a rectangle	:	Prove that the opposite sides are equal and diagonals are also equal.
(g)	For a parallelogram	:	Prove that the opposite sides are equal in length and diagonals are not equal.

- **Collinearity of three points :** Let A, B and C there given points. Point A, B and C will be collinear if the sum, of lengths of any two line-segment is equal to the length of the third line-segment.
  In the adjoining fig. there are three point A, B and C.
  Three points A, B and C are collinear if and only if
  (i) AB + BC = AC or (ii) AB + AC = BC or (iii) AC + BC = AB
- **Section Formula : Coordinates of the point, dividing the line-segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio : m_1 m_2 are given by** $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$



Mid-point Formula : Coordinates of the mid-point of the line-segment joining (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

- Area of triangle : $\boxed{\text{Area of } \Delta ABC = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+y_3(y_1-y_2)\right|}$

Condition of collinearity of three points : The given points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ will be collinear if the area of the triangle formed by them must be zero because triangle can not be formed.

$\Rightarrow$ area of $\Delta ABC = 0$

$\Rightarrow$ $\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+y_3(y_1-y_2)\right| = 0 \Rightarrow \boxed{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0}$

is the required condition for three points to be collinear.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

1. The distance between the points (a, b) and (-a, - b) is :
(A) $a^2 + b^2$ (B) $\sqrt{a^2 + b^2}$ (C) 0 (D) $2\sqrt{a^2 + b^2}$

2. The distance between points (a + b, b + c) and (a – b, c - b) is :
(A) $2\sqrt{a^2 + b^2}$ (B) $2\sqrt{a^2 + c^2}$ (C) $2.\sqrt{2b}$ (D) $\sqrt{a^2 - c^2}$

3. The distance between points A(1, 3) and B(x, 7) is 5. The value of x > 0 is :
(A) 4 (B) 2 (C) 1 (D) 3.

4. The distance between the points (a cos 20^0 + b sin 20^0,0) and (a sin 20^0 - b cos 20^0) is :
(A) (a + b) (B) (a – b) (C) $\sqrt{a^2 - b^2}$ (D) $\sqrt{a^2 + b^2}$

5. Mid-point of the line-segment joining the points (-5, 4) and (9, - 8) is :
(A) (–7, 6) (B) (2, – 2) (C) (7, – 6) (D) (–2, 2).

6. The co-ordinates of the points which divides the join of (–2, 2) and (–5, 7) in the ratio 2 : 1 is :
(A) (4, – 4) (B) (–3, 1) (C) (–4, 4) (D) (1, –3).

7. The co-ordinates of the points on x-axis which is equidistant from the points (5, 4) and (–2, 3) are :
(A) (2, 0) (B) (3, 0) (C) (0, 2) (D) (0, 3).

8. The co-ordinates of the points on y-axis which is equidistant from the points (3, 1) and (1, 5) are :
(A) (0, 4) (B) (0, 2) (C) (4, 0) (D) (2, 0).

9. The coordinates of the centre of a circle are (– 6, 1.5). If the ends of a diameter are (– 3, y) and (x, - 2) then:
(A) x = - 9, y = 5 (B) x = 5, y = - 9 (C) x = 9, y = 5 (D) None of these

10. The points (- 2, 2), (8, - 2) and (-4, - 3) are the vertices of a :
(A) equilateral Δ (B) isosceles Δ (C) right Δ (D) None of these

11. The points (1, 7), (4, 2) (- 1, 1) (- 4, 4) are the vertices of a :
(A) parallelogram (B) rhombus (C) rectangle (D) square.

12. The line segment joining (2, - 3) and (5, 6) is divided by x-axis in the ratio:
(A) 2 : 1 (B) 3 : 1 (C) 1 : 2 (D) 1 : 3.

13. The line segment joining the points (3, 5) and (- 4, 2) is divided by y-axis in the ratio:
(A) 5 : 3 (B) 3 : 5 (C) 4 : 3 (D) 3 : 4.

14. If (3, 2), (4, k) and (5, 3) are collinear then k is equal to :
(A) $\frac{2}{3}$ (B) $\frac{2}{5}$ (C) $\frac{5}{2}$ (D) $\frac{3}{5}$

15. If the points (p, 0), (0, q) and (1, 1) are collinear then $\frac{1}{p} + \frac{1}{q}$ is equal to :

(A) – 1 (B) 1 (C) 2 (D) 0

16. Two vertices of a triangle are (-2, -3) and (4, -1) and centroid is at the origin. The coordinates of the third vertex of the triangle are :

(A) (– 2, 3) (B) (–3, – 2) (C) (–2, 4) (D) (4, –2)

17. A (5, 1), B(1, 5) and C(–3, –1) are the vertices of Δ ABC. The length of its median AD is :

(A) $\sqrt{34}$ (B) $\sqrt{35}$ (C) $\sqrt{37}$ (D) 6

18. Three consecutive vertices of a parallelogram are (1, –2), (3, 6) and (5, 10). The coordinates of the fourth vertex are :

(A) (–3, 2) (B) (2,– 3) (C) (3, 2) (D) (– 2, –3)

19. The vertices of a parallelogram are (3, –2), (4, 0), (6,– 3) and (5,– 5). The diagonals intersect at the point M. The coordinates of the point M are :

(A) $\left(\frac{9}{2},\frac{5}{2}\right)$ (B) $\left(\frac{7}{2},\frac{5}{2}\right)$ (C) $\left(\frac{7}{2},\frac{3}{2}\right)$ (D) None of these

20. If two vertices of a parallelogram are (3, 2) and (–1, 0) and the diagonals intersect at (2, –5), then the other two vertex are :

(A) (1,– 10), (5, –12) (B) (1,– 12), (5, –10) (C) (2, –10) (D) (1, – 10), (2, – 12)

OBJECTIVE — **ANSWER KEY** — EXERCISE-4

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	D	C	A	C	B	C	A	B	A	C
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	D	A	D	C	C	C	C	C	A	B

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

SHORT ANSWER TYPE QUESTIONS

1. Find the distance between the points A and B in the following :

(i) $A(a+b, b-a), B(a-b, a+b)$ (ii) $A(1,-1), B(-\frac{1}{2},\frac{1}{2})$

2. Find the distance between the points A and B in the following :

(i) $A(8-2), B(3-6)$ (ii) $A(a+b, a-b), B(a-b, -a-b)$

3. A point P lies on the x-axis and has abscissa 5 and a point Q lies on y-axis and has ordinate – 12. Find the distance PQ.
4. Find a relation between x and y such that the point (x, y) is equidistant from (7, 1) and (3, 5).

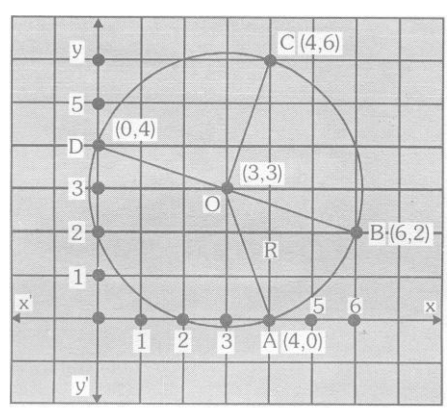
5. Using distance formula, show that the points A, B and C are collinear.

(i) $A(-1,-1), B(2,3), C(8,11)$ (ii) $A(-4,-2), B(-1,1), C(1, 3)$

6. Find a point on the x-axis which is equidistant from the points (5, 4) and (-2, 3).
7. Find a point on the x-axis which is equidistant from the points (-3, 4) and (2, 3).
8. Find the value of k, if the point (2, 3) is equidistant from the points A(k, 1) and B(7, k).
9. Find the value of k for which the distance between the point A(3k, 4) and B(2, k) is $5\sqrt{2}$ units.
10. Find the co-ordinates of the point which divides the line segment joining the points (1, -3) and (-3, 9) in the ratio 1 : 3 internally.
11. Find the mid-point of AB where A and B are the points (-5, 11) and (7, 3) respectively.
12. The mid-point of a line segment is (5, 8). If one end points is (3, 5), find the second end point.
13. The vertices of a triangle are A(3, 4), (7, 2) and C(-2, -5). Find the length of the median through the vertex A.
14. The co-ordinates of A and B are (1, 2) and (2, 3) respectively. Find the co-ordinates of R on line segment AB so that $\frac{AR}{RB} = \frac{4}{3}$.
15. Find the co-ordinates of the centre of a circle, the co-ordinates of the end points of a diameter being (-3, 8) and (5, 6)
16. Find the co-ordinates of the point where the diagonals of the parallelogram formed by joining the points (-2, -1), (1, 0), (4, 3) and (1, 2) meet.
17. Find the ratio in which the line segment joining the points (3, 5) and (-4, 2) is divided by y-axis.
18. In what ratio in does the point $\left(\frac{1}{2}, \frac{-3}{2}\right)$ divide the line segment joining the points (3, 5) and (-7, 9) ?
19. By using section formula, show that the points (-1, 2), (2, 5) and (5, 8) are collinear.
20. Find the distance of the point (1, 2) from the mid-point of the line segment joining the points (6, 8) and (2, 4).
21. Show that the mid-point of the line segment joining the points (5, 7) and (3, 9) is also the mid-point of the line segment joining the points (8, 6) and (0, 10).
22. Find the area of the triangle whose vertices are (3, 2) (-2, -3) and (2, 3).
23. For what value of m, the points (3, 5), (m, 6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear ?

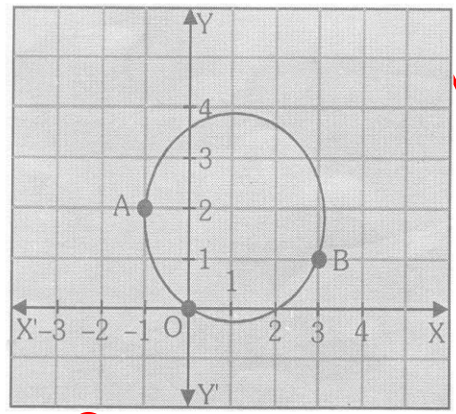
LONG ANSWER TYPE QUESTION

1. Prove that the points (1, 4), (3, 6) and (9, -2) are the vertices of an isosceles triangle.
2. Find the co-ordinates of the point equidistant from three given points A(5, 1), B(-3, -7) and C(7, -1).
3. Show that the points (7, 10), (-2, 5) and (3, - 4) are the vertices of an isosceles right triangle.
4. Prove that the points (0, 1), (1, 4), (4, 3) and (3, 0) are the vertices of a square.
5. Prove that the points (-4,- 1), (-2,- 4), (4, 0) and (2, 3) are the vertices of a rectangle.
6. If two vertices of an equilateral triangle are (0, 0) and $(3, \sqrt{3})$, find the third vertex of the triangle.
7. A (3, 4) and C (1, -1) are the two opposite angular points of a square ABCD. Find the co-ordinates two vertices
8. Find the co-ordinates of the point equidistant from the point A(-2, -3), B(-1, 0) and C(7, -6).
9. Show that (3, 3) is the centre of the circle passing through the points (4, 6), (0, 4), (6, 2) and (4, 0). What radius of the circle.
10. If A (2, -1), B(3, 4), C(-2, 3) and D (-3, -2) be four points in a co-ordinates plane, show that ABCD is a rhombus but not a square. Find the area of the rhombus.
11. In figure, find the co-ordinates of the centre of the circle which is drawn through the points A, B and O.



12. The line segment joining the points (3, -1) and (1, 2) is trisected at the points P and Q. If the co-ordinates of P and Q are (p, -2) and $\left(\frac{5}{2}, q\right)$ respectively, find the values of P and Q.

13. What will be the value of y if the point $\left(\frac{23}{5}, y\right)$, divides the line segment joining the points (5, 7) and (4, 5) in the ratio 2 : 3 internally.



14. Find the co-ordinates of the points which divide the line segment joining the points (-4, 0) and (0, 6) in 4 equal parts.

15. If the points (10, 5) (8, 4) and (6, 6) are the mid-points of the sides of a triangle, find its vertices.

16. Find the area of the quadrilateral ABCD formed by the points A (–2, –2), B (5, 1), C (2, 4) and (–1, 5)..

17. Find the point on the x-axis which is equidistant from the points (–2, 5) and (2, – 3). Hence, find the area of the triangle formed by these points.

18. A (4, 3), B (6, 5) and C (5, – 2) are the vertices of ΔABC.

(i) Find the co-ordinates of the centroid G of ΔABC. Find the area of ΔABC and compare it with area of ΔABC.

(ii) If D is the mid-point of BC, find the co-ordinates of D. Find the co-ordinates of a point P on AD such that AP : PD = 2 : 3. Find the area of ΔABC and compare it with area of ΔABC.

19. ABCDE is a polygon whose vertices are A(–1, 0), B(4, 0), C(4, 4), D(0, 7) and E(–6, 2). Find the area of the polygon.

20. Name the quadrilateral formed by joining the points (1, 2), (5, 4), (3, 8) and (–1, 6) in order. Find also the area of the region formed by joining the mid-points of the sides of this quadrilateral.

CO-ORDINATE GEOMETRY | **ANSWER KEY** | EXERCISE-2 (X)-

SHORT ANSWER TYPE QUESTION :

1. (i) $2\sqrt{a^2+b^2}$ units, **(ii)** $\frac{3\sqrt{2}}{2}$ units **2. (i)** $\sqrt{41}$ units, **(ii)** $2\sqrt{a^2+b^2}$ units **3.** 13 units

4. x – y = 2 **6.** (2, 0) **7.** (0, 6) **8.** k = 13 **9.** k = –1 or k = 3 **10.** (0, 0) **11.** (1, 7) **12.** (7, 11)

13. $\frac{\sqrt{122}}{2}$ units, **14.** $\left(\frac{11}{7}, \frac{18}{7}\right)$ **15.** (1, 7) **16.** (1, 1) **17.** 3 : 4 **18.** 1 : 3 **20.** 5 units **22.** 5 sq. unit

23. m = 2

LONG ANSWER TYPE QUESTIONS :

1. (2, – 4) **2.** $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$ **3.** $\left(\frac{9}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{5}{2}\right)$ **4.** (3, – 3) **5.** $\sqrt{10}$ units **6.** 24 sq. units

7. $\left(\frac{15}{14}, \frac{25}{14}\right)$ **8.** $p = \frac{7}{3}, q = 0$ **9.** $\frac{31}{5}$ **10.** $\left(-3, \frac{3}{2}\right)$, (–2, 3), $\left(-1, \frac{9}{2}\right)$ **11.** (8, 7), (12, 3), (4, 5) **16.** 26 sq. units

17. (–2, 0), 10 sq. units

18. **(i)** G (5, 2) ; ar (ΔGBC) = 2 sq. units ; ar (ΔGBC) : ar (ΔABC) = 1 : 3

(ii) $D\left(\frac{11}{2}, \frac{3}{2}\right); P\left(\frac{23}{5}, \frac{12}{5}\right)$; ar (ΔPBC) = $\frac{18}{5}$ sq. units ; ar (ΔPBC) : ar (ΔABC) = 3 : 5

19. 44 sq. units **20.** Square ; 10 sq. units.

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

1. Show that the point A(5, 6), B(1, 5), C(2, 1) and D(6, 2) are the vertices of a square. **[Delhi-2004]**

2. Determine the ratio in which the point P(m, 6) divide the join of A(–4, 3) and B(2, 8). Also find the value of m.

OR

A(3, 2) and B(–2, 1) are two vertices of a triangle ABC, whose centroid G has coordinates $\left(\frac{5}{3}, -\frac{1}{3}\right)$. Find the coordinates of the third vertex C of the triangle. **[Delhi-2004]**

3. Show that the points A(2, –2), B(14, 10), C(11, 13) and D(–1, 1) are the vertices of a rectangle. **[Al-2004]**

4. Prove that the coordinates of the centroid of a ΔABC, with vertices. A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are given by $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + _3}{2}\right)$ **[Al-2004]**

5. Determine the ratio in which the point (–6, a) divide the join of A(–3, –1) and B(–8, 9). Also find the value of a. **[Al-2004]**

6. Find the point on the x-axis which is equidistant from the points (–2, 5) and (2, –3) **[Al-2004]**

7. Prove that the points A(0, 1), B(1, 4), C(4, 3) and (3, 0) are the vertices of a square. **[Foreign-2004]**

8. Determine the ratio in which the point (a, –2) divide the join of A(–4, 3) and B(2, –4). Also find the value of a. **[Foreign-2004]**

9. Determine the ratio in which the point P(k, 2) divide the join of A(–3, 5) and B(5, 1). Also find the value of k. **[Foreign-2004]**

10. Determine the ratio in which the point P(b, 1) divide the join of A(7, –2) and B(–5, 6). Also find the value of b. **[Foreign-2004]**

11. The coordinates of the mid-point of the line joining the point (3p, 4) and (–2, 2q) are (5, p). Find the coordinates of p and q. **[Delhi-2004C]**

12. Two vertices of a triangle are (1, 2) and (3, 5). If the controid of the triangle is at the origin, find the coordinates of the third vertex.

OR

If ‘a’ is the length of one of the sides of an equilateral triangle ABC, base BC lies on x-axis and vertex B is at the origin, find the coordinates of the vertices of the triangle ABC. **[Delhi-2004C]**

13. Find the ratio in which the one-segment joining the points (6, 4) and (1, –7) is divided by x-axis . **[Al-2004C]**

OR

The coordinates of two vertices A and B of a tnangle An are (1, 4) and (5, 3) respectively. If the coordinates of the centroid of ΔABC are (3, 3), find the coordinates of the third vertex C. **[Al-2004C]**

14. Find the value of m for which the points with coordinates (3, 5), (m, 6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear. **[Al-2004C]**

15. Find the value of x such that PQ = QR where the coordinates of P,Q and R are (6,–1);(1,3) and (x,8)respectively.

OR

Find a point on x-axis which is equidistant from the points (7, 6) and (–3, 4). **[Delhi-2004]**

16. The line-segment joining the points (3, –4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, 2) and $\left(\frac{5}{3}, q\right)$ respectively, find the values of p and q. **[Delhi-2005]**

17. Prove that the points (0, 0), (5, 5) are vertices of a right isosceles triangle.

OR

If the point P(x, y) is equidistant from the point A(5, 1) and B(–1, 5), prove that 3x = 2y. **[Al-2005]**

18. The line joining the points (2, 1) and (5, –8) is trisected at the points p and Q. If point P lies on the line 2x – y + k = 0, find the value of k. **[Al-2005]**

19. Show that the points (0, –1); (2, 1); (0, 3) and (–2, 1) are the vertices of a square.

OR

Find the value of K such that the point (0, 2) is equidistant from the points (3, K) and (K, 5). **[Foreign-2005]**

20. The base BC of an equilateral ΔABC lies on y-axis. The coordinates of point C are (0, –3). If the origin is the mid-point of the base BC, find the coordinates of the points A and B. **[Foreign-2005]**

21. Find the coordinates of the point equidistant from the points A(1, 2), B(3, – 4) and C(5, –6).

OR

Prove that the points A(–4, –1), B(–2, –4), C(4, 0) and (2, 3) are the vertices of a rectangle. **[Delhi-2005C]**

22. Find the coordinates of the points which divide the line-segment joining the points (–4, 0) and (0, 6) in three equal parts. **[Delhi-2005C]**

23. Two vertices of ΔABC are given by A(2, 3) and B(–2, 1) and its centroid is G $\left(1, \frac{2}{3}\right)$. Find the coordinates of the third vertex C of the ΔABC. **[Al-2005]**

24. Show that the points A(1, 2), B(5, 4), C(3, 8) and D(–1, 6) are the vertices of a square.

OR

Find the co-ordinates of the point equidistant from three given points A(5, 1), B(–3, –7) and C(7,–1) **[Delhi-2006]**

25. Find the value of p for which the points (–1, 3), (2, p) and (5, –1) are collinear. **[Delhi-2006]**

26. If the points (10, 5), (8,4) and (6,6) are the mid. Points of the sides of a triangle, find its vertices. **[Foreign-2006]**

27. In what ratio is the line segment joining the points (–2, –3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

OR

If A (5, –1), B (–3, –2) and C (–1, 8) are the vertices of triangle ABC, find the length of median through A and the coordinates of the centroid. **[Delhi-2006C]**

28. If (–2, –1); (a, 0); (4, b) and C (1, 2) are the vertices of a parallelogram, find the values of a and b. **[Al-2006C]**

29. Show that the points (7, 10), (–2, 5) and (3, –4) are the vertices of an isosceles right triangle. **[Delhi-2007]**

30. In what ratio does the lines x – y – 2 = 0 divides the line segment joining (3, –1) and (8, 9) ? **[Delhi-2007]**

31. Three consecutive vertices of a parallelogram are (–2, 1); (1, 0) and (4, 3). Find the coordinates of the fourth vertex. **[Al-2007]**

32. If the point C(–1, 2) divides the line segment AB in the ratio 3 : 4 where the coordinates of A are (2, 5), find the coordinates of B. **[Al-2007]**

33. For what value of p, are the points (2, 1), (p, –1) and (–1, 3) collinear? **[Delhi-2008]**

34. Determine the ratio in which the line 3x + 4y – 9 = 0 divides joining the points (1, 3) and (2, 7). **[Delhi-2008]**

35. If the distances of P(x, y) from the points A(3, 6) and B(–3, 4) are equal, prove that 3x + y = 5. **[Delhi-2008]**

36. For what value of p, the points (–5, 1), (1, p) and (4, –2) are collinear? **[Delhi-2008]**

37. For what value of k, are the points (1, 1), (3, k) and (–1, 4) are collinear? **[Delhi-2008]**

OR

Find the area of the ΔABC with vertices A(–5, 7), B(–4, –5) and C(4, 5) **[Al-2008]**

38. If the point P(x, y) is equidistant from the points A(3, 6) and B(–3, 4) prove that 3x + y – 5 = 0. **[Al-2008]**

39. The point R divides the line segment AB, where A(–4, 0) and B(0, 6) such that AR = $\frac{2}{3}$ AB. Find the co-ordinates of R. **[Al-2008]**

40. The co-ordinates of A and B are (1, 2) and (2, 3) respectively. If P lies on AB find co-ordinates of P such that $\frac{AP}{PB} = \frac{3}{4}$. **[Al-2008]**

41. If A(4, – 8), B(3, 6) and C(5, – 4) are the vertices of a Δ ABC, D is the mid point of BC and P is a point on AD joining such that $\frac{AP}{PD} = 2$, find the co-ordinates of P. **[Al-2008]**

42. Find the value of k if the points (k, 3), (6, – 2) and (– 3, 4) are collinear. **[Foreign-2008]**

43. If P divides the join of A(–2, –2) and B(2, –4) such that $\frac{AP}{AB} = \frac{3}{7}$, find the co-ordinates of P. **[Foreign-2008]**

44. The mid points of the sides of a triangle are (3, 4), (4, 6) and (5, 7) . Find the co-ordinates of the vertices the triangle. **[Foreign-2008]**

45. Show that A(–3, 2), B(–5, –5), C(2, –3) and D(4, 4) are the vertices of a rhombus. **[Foreign-2008]**

46. Find the ratio in which the line 3x + y – 9 = 0 divides the line-segment joining the points (1, 3) and (2, 7). **[Foreign-2008]**

47. Find the distance between the points $\left(\frac{-8}{5}, 2\right)$ *and* $\left(\frac{2}{5}, 2\right)$. **[Delhi-2009]**

48. Find the point on y-axis which is equidistant from the points (5, –2) and (–3, 2).

OR

The line segment joining the points A(2, 1) and B(5, –8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by 2x – y + k = 0, find the value of k. **[Delhi-2009]**

49. If P (x, y) is any point on the line joining the points A(a, 0) and B(0, b), then show that $\frac{x}{a} + \frac{y}{b} = 1$. **[Delhi-2009]**

50. Find the point on x-axis which is equidistant from the points (2, – 5) and (– 2, 9) **[Delhi-2009]**

OR

The line segment joining the points P(3, 3), Q(6, – 6) is trisected at the points A and B such that A is nearer to P. If A also lies on the line given by 2x + y + k = 0, find the value of k.

51. If the points A P(4, 3) and B(x, 5) are on the circle with the centre O (2, 3), find the value of x. **[Al-2009]**

52. Find the ratio in which the point (2, y) divides the line segment joining the points A(–2, 2) and B(3, 7). Also find the value of y. **[Al-2009]**

53. Find the area of the quadrilateral ABCD whose vertices are A(–4, –2), B(–3, –5), C(3, –2) and D(2,3). **[Al-2009]**

54. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). **[Al-2009]**

55. If the mid-point of the line segment joining the points P(6, b – 2) and Q(2, – 3), find the value of b. **[Foreign-2009]**

56. Show that the points (–2, 5), (3, –4) and (7, 10) are the vertices of a right angled isosceles triangle.

OR

The centre of a circle is $(2\alpha - 1, 7)$ and it passes through the point (–3, – 1). If the diameter of the circle is 20 units, then find the value(s) of α. **[Foreign-2009]**

57. If C is a point lying on the line segment AB joining A(1, 1) and B(2, – 3) such that 3AC = CB, then find the co-ordinates of C. **[Foreign-2009]**

58. Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear. **[Foreign-2009]**

59. If the points (–2, 1), (a, b) and (4, – 1) are collinear and a – b = 1, then find the values of a and b. **[Foreign-2009]**

60. Find the value of K, if the points A(7, – 2), B(5, 1) and C(3, 2K) are collinear. **[Al-2010]**

61. Find the value of K, if the points A(8, 1), B(3, – 4) and C(2, K) are collinear. **[Al-2010]**

62. Point P divides the line segment joining the points A(–1, 3) and B(9, 8) such that $\frac{AP}{PB} = \frac{K}{1}$. If P lies on the line x – y + 2 = 0, find the value of K. **[Al-2010]**

63. If the points (p, q), (m, n) and (p – m, q – n) are collinear, show that pn = qm. **[Al-2010]**

CO-ORDINATE GEOMETRY CBSE | **ANSWER KEY** | EXERCISE-2 (X)-

SHORT ANSWER TYPE

2. 3 : 2, – 2/5 or (4, –4) **5.** 3 : 2, 5 **6.** (–2, 0) **8.** 2/7 **9.** 3 **11.** p = 4, q = 2

12. (–4, –7) or A$(a/2, \sqrt{3a}/2)$, B(0, 0), C(a, 0) **13.** 4 : 7 or (3, 2) **14.** 2 **15.** 5 or – 3 or (3, 0)

16. $p = \frac{7}{3}$, q = 0 **18.** k = – 8 **19.** k = 1 **20.** $(\pm 3\sqrt{3}, 0)$ and (0, 3) **21.** (11, 2) **22.** $\left(\frac{-8}{3}, 2\right), \left(\frac{-4}{3}, 4\right)$

23. (3, –2) **24.** (2, –4) **25.** p = 1 **26.** (4, 5), (8, 7), (12, 3) **27.** 2 : 3, (0, 1) or $\sqrt{65}, \left(\frac{1}{3}, \frac{5}{3}\right)$

28. a = 1 , b = 3 **30.** 2 : 3 **31.** (1 , 2) **32.** (–5, –2) **33.** p = 5 **34.** 6 : 25 **36.** – 1

37. – 2 or 53 sq. units **39.** $\left(-1, \frac{9}{2}\right)$ **40.** $\left(\frac{11}{7}, \frac{18}{7}\right)$ **41.** (4, – 2) **42.** $k = -\frac{3}{2}$ **43.** $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

44. (4, 5), (2, 3), (6, 9) **46.** 3 : 4 **47.** 2 **48.** (0, – 2) or – 8 **50.** (–7, 0) or – 8 **51.** 2 **52.** 4 : 1, 6

53. 28 sq. units **54.** 1 sq. units **55.** – 8 **56.** – 4 or 2 **57.** $\left(\frac{5}{4}, 0\right)$ **58.** x + 3y = 7 **59.** a = 1, b = 0

60. 2 **61.** – 5 **62.** $\frac{2}{3}$

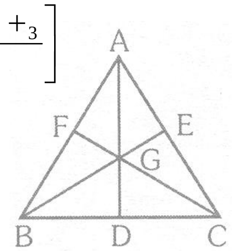
COMPETITION WINDOW

PROPERTIES OF TRIANGLES

Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ be the vertices of any ΔABC, then

1. **Centroid :** It is the point of intersection of the cedians. It divides the median in the ratio of 2 : 1.

Co-ordinates of controid : G $\left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 +_3}{3}\right]$



2. **Incentre :** It is the point of intersection of internal bisectors of the angle. Also it is the centre of the circle touching all the sides of a triangle.

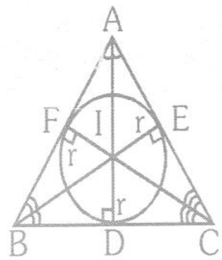
Co-ordinates of incentre $\left[\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right]$.Where a, b, c are the lengths of the sides of triangle.

The radius of incircle.

$$r=\frac{\Delta}{s}$$

Where Δ is the area of Triangle and

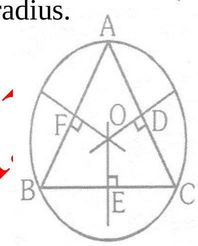
$$S=\frac{a+b+c}{2}$$



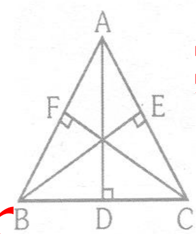
Remark : An angle bisector of a triangle divides the opposite side in the ratio of remaining sides.

E.g. AD divides BC in the ratio $\frac{BD}{DC}=\frac{AB}{AC}$.

3. **Circumcentre :** It is the point of intersection of perpendicular bisectors of the sides of a triangle. It is also the centre of the circle passing through the vertices of a triangle.
If O is the circumcentre of a triangle ABC, then OA = OB = OC = circumradius.



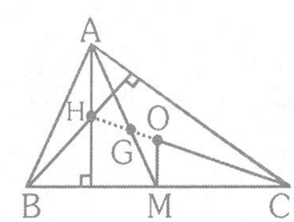
4. **Orthocentre :** It is the point of intersection of altitudes of a triangle.



Remark :

(i) In an equilateral triangle, the centroid, incentre, orthocenter and circumcentre coincide.
(ii) In an isosceles triangle, the centroid, incentre, orthocenter and circumcentre are collinear.
(iii) In a right angled triangle, the circumcentre is the mid point of hypotenuse and the orthocenter is the point where right angle is formed.
(iv) **Euler line :** The circumcentre O, the centroid G and the orthocenter H of a triangle are collinear, the line on which they lie is called Euler line. Also G divides HO in the ratio 2 : 1.

$$\frac{OG}{GH}=\frac{1}{2}$$



TRY OUT THE FOLLOWING

1. Two vertices of a triangle are (– 1, 6) are (5, 2). If its centroid is (0, – 3), find the third vertex.

2. If $\left(\frac{3}{2},0\right),\left(\frac{3}{2},6\right)$ and (–1, 6) are mid-points of the sides of a triangle, then find
(i) Centroid of the triangle (ii) In centre of the triangle

3. If a triangle has it's orthocenter at (1, 1) and circumcentre at $\left(\frac{3}{2},\frac{3}{4}\right)$ then, find the centroid.

ANSWERS

1. (–4, –15) **2.** (i) (2/3, 4) (ii) (1, 2) **3.** (4/3, 5/6)

COMPETITION WINDOW

CONDITIONS FOR A TRIANGLE TO BE ACUTE, OBTUSE OR RIGHT ANGLED

For an acute angled triangle, $a^2 + b^2 > c^2, b^2 + c^2 > a^2$ and $a^2 + b^2 > c^2$.

For an obtuse angled triangle, $a^2 + b^2 > c^2$ (if $\angle$C is obtuse)
$a^2 + c^2 > b^2$ (if $\angle$B is obtuse)
$b^2 + c^2 > a^2$ (if $\angle$A is obtuse)

For a right angled triangle, $a^2 + b^2 = c^2$ (if $\angle C = 90^0$)
$b^2 + c^2 = a^2$ (if $\angle A = 90^0$)
$a^2 + c^2 = b^2$ (if $\angle B = 90^0$)

Where a, b and c have their usual meanings.

COMPETITION WINDOW

CONCURRENCY OF LINES

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are said to be concurrent (lines passing through the same point) if $a_1(b_2c_3 - b_3c_2) + a_2(b_3c_1 - b_1c_3) + a_3(b_1c_2 - b_2c_1) = 0$

TRY OUT THE FOLLOWING

1. Prove that the lines 3x + y – 14 = 0, x – 2y = 0 and 3x – 8y + 4 = 0 are concurrent.
2. Show that the lines x – y – 6 = 0, 4x – 3y – 20 = 0 and 6x + 5y + 8 = 0 are concurrent. Also find their common point of intersection.
3. Find the value of λ, if the lines 3x – 4y – 13 = 0, 8x – 11y – 33 = 0 and 2x – 3y + λ = 0 are concurrent.

ANSWER

2. (2, – 4) **3.** $\lambda = -7$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

CHOOSE THE CORRECT ONE

1. The circumcentre of the triangle formed by the lines xy + 2x + 2y + 4 = 0 and x + y + 2 = 0 is :
(A) (– 1, – 2) (B) (– 1, – 1) (C) (– 2, – 2) (D) (0, 0)

2. The vertices of a triangle (a, b – c), (b, c – a) and (c, a – b), then it's centroid lies on :
(A) y-axis (B) x-axis (C) x = 0 (D) None of these

3. The points (1, 2), (3, 8) and (x, 20) are collinear if x =
(A) 4 (B) 5 (C) 6 (D) 7

4. For the triangle whose sides are along the lines x = 0 , y = 0 and $\frac{x}{6} + \frac{y}{8} = 1$, the incentre is :
(A) (3, 4) (B) (2, 2) (C) (2, 3) (D) (3, 2)

5. For the triangle whose sides are along the lines y = 15 , 3x – 4y = 0, 5x + 12y = 0, the incentre is :
(A) (1, 8) (B) (– 1, 8) (C) (8, 1) (D) None of these

6. The points D(2, 1), E(–1, –2) and F(3, 3) are the mid points of sides BC, CA and AB respectively of a ΔABC. The vertices A, B and C are :
(A) (0, 0), (6, 6), (– 2, – 4) (B) (0, 1), (6, 6), (2, 4)
(C) (1, 0), (3, 3), (– 2, – 4) (D) None of these

7. The number of integral values of m, for which x-coordinate of the point of intersection of the lines 3x + 4y = 9 and y = mx + 1 is also an integer, is :

(A) 2 (B) 0 (C) 4 (D) 1

8. The radius of the circle inscribed in the triangle formed by lines x = 0, y = 0, 4x + 3y – 24 = 0 is :

(A) 12 (B) 2 (C) $2\sqrt{2}$ (D) 6

9. In a ΔABC, if A is the point (1, 2) and equations of the median through B and C are respectively x + y = 5 and x = 4, then B is :

(A) (1, 4) (B) (7, – 2) (C) (4, 1) (D) (– 2, 7)

10. The straight line 3x + y = 9 divides the segment joining the points (1, 3) and (2, 7) in the ratio :

(A) 4 : 3 (B) 3 : 4 (C) 4 : 5 (D) 5 : 6

11. Two opposite vertices of a rectangle are (1, 3) and (5, 1). If the equation of a diagonal this rectangle is y = 2x + c, then the value of c is :

(A) – 4 (B) 1 (C) – 9 (D) None of these

12. The radius of the circle passing through the point (6, 2) two of whose diameters are x + y = 6 and x + 2y = 4 is :

(A) 10 (B) $2\sqrt{5}$ (C) 6 (D) 4

13. The straight lines x + y = 0, 3x + y – 4 = 0, x + 3y – 4 = 0 form a triangle which is :

(A) Isosceles (B) Equilateral (C) Right angled (D) None of these

14. The lines segment joining the points (1, 2) and (–2, 1) is divided by the line 3x + 4y = 7 in the ratio :

(A) 3 : 4 (B) 4 : 3 (C) 9 : 4 (D) 4 : 9

15. If a, b, c are in A. P. then the straight line ax + by + c = 0 will always pass through a fixed point whose co-ordinates are :

(A) (1, – 2) (B) (– 1, 2) (C) (1, 2) (D) (– 1, – 2)

16. The lines 8x + 4y = 1, 8x + 4y = 5, 4x + 8y = 3, 4x + 8y = 7 from a :

(A) Rhombus (B) Rectangle (C) Square (D) None of these

17. The incentre of the triangle formed by the lines y = 15, 12y = 5x and 3x + 4y = 0 is :

(A) (8, 1) (B) (– 1, 8) (C) (1, 8) (D) None of these

18. The area of triangle formed by the lines y = x, y = 2x and y = 3x + 4 is :

(A) 4 (B) 7 (C) 9 (D) 8

19. The triangle formed by the lines x + y = 1, 2x + 3y – 6 = 0 and 4x – y + 4 = 0 lies in the :

(A) First quadrant (B) Second quadrant (C) Third quadrant (D) Fourth quadrant

20. A line is drawn through the points (3, 4) and (5, 6). If the line is extended to a point whose ordinate is – 1, then the abscissa of that point is ;

(A) 0 (B) – 2 (C) 1 (D) 2

21. The area of the triangle whose sides are along the lines x = 0, y = 0 and 4x + 5y = 20 is :

(A) 20 (B) 10 (C) $\frac{1}{10}$ (D) $\frac{1}{20}$

22. If a, b, c are all distend, then the equations (b – c) x + (c – a) y + a – b = 0 and $(b^3 - c^3)$ x + $(c^3 - a^3)$ y + $a^3 - b^3 = 0$ represent the same line if :

(A) $a + b + c \neq 0$ (B) a + b c = 0

(C) a + b = 0 or b + c = 0 (D) None of these

23. The area of the quadrilateral with vertices at (4, 3), (2, – 1), (– 1, 2), (– 3, – 2) is :

(A) 18 (B) 36 (C) 54 (D) None of these

24. If α, β, γ are the real roots of the equation $x^3 - 3px^2 - 1 = 0$, then the centroid of the triangle with vertices $\left(\alpha \frac{1}{\alpha}\right), \left(\beta \frac{1}{\beta}\right)$ and $\left(\gamma \frac{1}{\gamma}\right)$ is at the point :

(A) (p, q) (B) (p/3, q/3) (C) (p + q, p – q) (D) (3p, 3q)

25. The co-ordinates of A, B C are (6, 3), (– 3, 5), (4, – 2) respectively and P is any point (x , y). the ratio of the areas of ΔABC and ΔABC is :

(A) $\left|\frac{x-y-2}{7}\right|$ (B) $\left|\frac{x+y-2}{7}\right|$ (C) $\left|\frac{x+y+2}{7}\right|$ (D) None of these

26. The area of a triangle is 5 square units. Two of its vertices are (2, 1) and (3, – 2). The third vertex lie on y = x + 3, the third vertex is :

(A) $\left(\frac{7}{2}, \frac{13}{2}\right) or \left(\frac{-3}{2}, \frac{3}{2}\right)$ (B) $\left(\frac{7}{2}, \frac{-13}{2}\right) or \left(\frac{-3}{2}, \frac{3}{2}\right)$

(C) $\left(\frac{7}{2},\frac{13}{2}\right) or \left(\frac{3}{2},\frac{3}{2}\right)$ (D) None of these

27. The point of intersection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$, lies on the line :

(A) x – y = 0 (B) $x+y=\frac{2ab}{a+b}$ (C) $x-y\frac{2ab}{a+b}$ (D) Both (A) and (B)

28. The point A divides the join of the points (–5, 1) and (3, 5) in the ratio k : 1 and co-ordinates of points B and C are (1, 5) and (7, – 2) respectively. If the area of ΔABC be 2 units, then k equals :

(A) 7, 9 (B) 6, 7 (C) $7, \frac{31}{9}$ (D) $9, \frac{31}{9}$

29. Q, R and S are the points on the line joining the points P(a, x) and T(b, y) such that PQ = QR = RS = ST, then $\left(\frac{5a+3b}{8},\frac{5x+3y}{8}\right)$ is the mid point of the segment :

(A) PQ (B) QR (C) RS (D) ST

30. The triangle with vertices A(2, 7), B(4, y) and C(–2, 6) is right angled at A if :

(A) y = – 1 (B) y = 0 (C) y = 1 (D) None of these

31. The co-ordinates with of the point which divides the line segment joining (– 3, – 4) and (– 8, 7) externally in the ratio 7 : 5 are :

(A) $\left(\frac{41}{2},\frac{69}{2}\right)$ (B) $\left(\frac{-41}{2},\frac{-69}{2}\right)$ (C) $\left(\frac{-41}{2},\frac{69}{2}\right)$ (D) None of these

32. The distance of the centroid from the orign of the friangle formed by the points (1, 1), (0, – 7) and (– 4, 0) is :

(A) $\sqrt{2}$ (B) $\sqrt{4}$ (C) $\sqrt{3}$ (D) $\sqrt{5}$

33. If A(4, – 3), B(3, – 2) and C(2, 8) are vertices of a triangle, then the distance of it's centroid from the y-axis is :

(A) $\frac{1}{2}$ (B) 1 (C) 3 (D) $\frac{1}{2}$

34. If (5, – 4) and (– 3, 2) are two opposite vertices of a square, then it's area is :

(A) 50 (B) 75 (C) 25 (D) 100

35. A(6, 3), B(–3, 5), C(4, – 2) and (x, 3x) are four points. If the areas of ΔDBC and ΔABC are in the ratio 1 : 2, then x is equal to :

(A) $\frac{11}{8}$ (B) 3 (C) $\frac{8}{11}$ (D) None of these

36. An equilateral triangle whose circumcentre is (–2, 5), one side is on y-axis, then length of side of the triangle is :

(A) 6 (B) $2\sqrt{3}$ (C) $4\sqrt{3}$ (D) 4

37. A(3, 4), and B(5, – 2) are two given points. If PA = PB and area of ΔPAB = 10. then P is :

(A) (7, 1) (B) (7, 2) (C) (– 7, 2) (D) (– 7, – 1)

38. The distance between foot of perpendiculars drawn from a point (– 3, 4) on both axes is :

39. Point P divides the line segment joining A(– 5, 1) and B(3, 5) internally in the ratio λ : 1. If Q = (1, 5), R = (7, – 2) and area of ΔPQR = 2, then λ equals :

(A) 23 (B) $\frac{29}{5}$ (C) $\frac{31}{9}$ (D) None of these

40. The area of an equilateral triangle whose two vertices are (1, 0) and (3, 0) and third vertex lying in the first quadrant is :

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) None of these

41. ABC is an isosceles triangle. If the co-ordinates of the base are B(1, 3) and C(–2, 7), the co-ordinates of vertex A is

(A) $\left(\frac{-1}{2},5\right)$ (B) (1, 6) (C) $\left(\frac{5}{6},6\right)$ (D) None of these

42. The area of the quadrilateral formed by the points $(a^2 + 2ab, b^2)$, $(a^2 + b^2, 2ab)$, $(a^2, b^2 + 2ab)$ and (a2 + b2 – 2ab, 4ab) is :
(A) Zero (B) $(a + b)^2$ (C) $a^2 + b^2$ (D) $(a – b)^2$

OBJECTIVE — **ANSWER KEY** — EXERCISE-4

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	B	D	C	C	A	A	B	B	B	A	B	A	D	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	A	C	A	B	B	B	B	A	A	B	A	D	C	B	A
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**			
Ans.	C	D	C	A	A	C	B	A	C	C	C	A			

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

CHOOSE THE COREECT ONE

1. The lines ax + 2y + 1 = 0, bx + 3y + 1 = 0 and cx + 4y + 1 = 0 are concurrent, then :
(A) a, b, c are in A.P. (B) a, b, c are in G.p. (C) a, b, c are in H.P. (D) None of these

2. If the lines x + 2ay + a = 0, x + 3by + b = 0 and x 4cy + c = 0 are concurrent, then a, b, c are in ($abc \neq 0$) :
(A) A.P. (B) G.P. (C) H.P. (D) None of these

3. If (0, β) lies on or inside the triangle formed by the lines 3x + y + 2 = 0, 3y – 2x – 5 = 0 and 4y + x – 14 = 0 then :
(A) $\frac{5}{2} \le \beta \le \frac{7}{3}$ (B) $\frac{5}{3} \le \beta \le \frac{7}{2}$ (C) $\frac{7}{3} \le \beta \le \frac{5}{2}$ (D) None of these

4. If a, x_1, x_2 are in G.P. with common ratio r1 and b, y_1, y_2 are in G.P. with common ratio s where s – r = 2, then the area of the triangle with vertices (a, b), (x_1, y_1) and (x_2, y_2) is :
(A) $\left| ab(r^2 - 1) \right|$ (B) $ab(r^2 – s^2)$ (C) $ab(s^2 – 1)$ (D) abrs

5. If the circumcentre of a triangle lies at the origin and the centroid is the middle point of the line joining the points $(a^2 + 1, a^2 + {}^1)$ and (2a, – 2a), then the co-ordinates of the orthocenter are :
(A) $\left[\frac{(a+1)^2}{4},\frac{(a-1)^2}{4}\right]$ (B) $\left[\frac{3}{4}(a+1)^2,\frac{3}{4}(a-1)^2\right]$
(C) $(3(a + 1)^2, 3 (a – 1)^2)$ (D) None of these

6. If every point on the line $(a_1 – a_2)$ x + $(b_1 – b_2)$ y = c is equidistant from the points (a_1, b_1) and (a_2, b_2) then 2c =
(A) $a_1^2 - b_1^2 + a_2^2 + b_2^2$ (B) $a_1^2 + b_1^2 + a_2^2 + b_2^2$ (C) $a_1^2 - b_1^2 - a_2^2 - b_2^2$ (D) None of these

7. A rectangle has two opposite vertices at the points (1, 2) and (5, 5). If the other vertices lie on the line x = 3, the co-ordinates of the vertex nearer the axis of x are :
(A) 3, 1 (B) (3, 2) (C) (3, 4) (D) (3, 6)

8. If the area of the triangle formed by the pair of lines $8x^2 – 6y^2 + y^2 = 0$ and the line 2x + 3y = a is 7, then a is equal
(A) 14 (B) $14\sqrt{2}$ (C) 28 (D) None of these

9. If the centroid of the triangle formed by the pair of lines $2y^2 + 5xy – 3x^2 = 0$ and x + y = k is $\left(\frac{1}{18},\frac{11}{18}\right)$, then the value of k is :
(A) – 1 (B) 0 (C) 1 (D) None of these

10. If x_1, x_2, x_3 are the abscissa of the points A_1, A_2, A_3 respectively where the lines $y = m_1, x$, $y = m_2x$, $y = m_3x$ meet the line $2x - y + 3 = 0$ such that m_1, m_2, m_3, are in A.P., then x_1, x_2, x_3 are in :
(A) A.P. (B) G.P. (C) H.P. (D) None of these

11. The area of the triangle with vertices $\left(1, \frac{\pi}{8}\right), \left(1, \frac{5\pi}{8}\right)$ and $\left(\sqrt{2}\,\frac{3\pi}{8}\right)$ is :
(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

12. An equilateral triangle whose orthocenter is (3, – 2), one side is on x-axis then vertex of triangle which is not on x-axis is :
(A) (3, – 6) (B) (1, – 2) (C) (9, – 2) (D) (3, – 3)

13. If O is the origin and the co-ordinates of A and B are (x_1, y_1) and (x_2, y_2) respectively then $OA \times OB \cos \angle AOB$ is equal to :
(A) $x_1y_1 + x_2y_2$ (B) $x_1x_2 + y_1y_2$ (C) $x_1y_2 + x_2y_1$ (D) $x_1x_2 - y_1y_2$

14. If the vertices of a triangle have integral co-ordinates, then the triangle is :
(A) Isosceles (B) Never equilateral (C) Equilateral (D) None of these

15. The circumcentre of the triangle formed by the points $(a \cos \alpha, a \sin \alpha)$, $(a \cos \beta, \sin \beta)$, $(a \cos \gamma, \sin \gamma)$ is
(A) (0, 0) (B) $\left[\left(\frac{a}{3}\right)(\cos\alpha + \cos\beta + \cos\gamma), \left(\frac{a}{3}\right)(\sin\alpha + \sin\beta + \sin\gamma)\right]$
(C) (a, 0) (D) None of these

16. The x co-ordinates of the incentre of the triangle where the mid point of the sides are (0, 1), (1, 1) and (1, 0) is
(A) $2+\sqrt{2}$ (B) $1+\sqrt{2}$ (C) $2-\sqrt{2}$ (D) $1+\sqrt{2}$

17. OPQR is a square and M and N are the mid points of the sides PQ and QR respectively, then ratio of area of square and the triangle OMN is :
(A) 4 : 1 (B) 2 : 1 (C) 8 : 3 (D) 4 : 3

18. The point with co-ordinates (2a, 3a), (3b, 2b) and (c, c) are collinear :
(A) For no value of a, b, c (B) For all value of a, b, c
(C) If a, $\frac{c}{5}$, b are in H.P. (D) If a, $\frac{2c}{5}$, b are in H.P.

19. If co-ordinates of orthocenter and centroid of a triangle are (4, – 1) and (2, 1), then co-ordinates of a point which is equidistant from the vertices of the triangle is :
(A) (2, 2) (B) (3, 2) (C) (2, 3) (D) None of these

20. If the line y = mx meets the lines x + 2y – 1 = 0 and 2x – y + 3 = 0 at the number of points having integral to :
(A) – 2 (B) 2 (C) – 1 (D) 1

21. A triangle is formed by the point O(0, 0), A(0, 21) and B(21, 0). The number of points having integral co-ordinates (both x and y) and strictly inside the triangle is :
(A) 190 (B) 305 (C) 181 (D) 206

22. The straight lines 5x + 4y = 0, x + 2y – 10 = 0 and 2x + y + 5 = 0 are :
(A) Concurrent (B) The sides of an equilateral triangle
(C) The sides of a right angled triangle (D) None of these

23. $A(a, b)$, $B(x_1, y_1)$ and $C(x_2, y_2)$ are the vertices of a triangle. If a, x_1, x_2 are in G.P. with common ratio r and b, y_1, y_2 are in G.P. with common ratio s, then area of ΔABC is :

(A) ab (r – 1) (s – 1) (s – r)
(B) $\frac{1}{2}$ ab (r + 1) (s + 1) (s – r)
(C) $\frac{1}{2}$ (r – 1) (s – 1) (s – r)
(D) ab (r + 1) (s + 1) (s – r)

24. If a, b, c are in G.P., then the line $a^2x + b^2y + ac = 0$, will always pass through the fixed point.
(A) (0, 1) (B) (1, 0) (C) (0, – 1) (D) (1, – 1)

25. The lies $\ell x + my + n = 0$, $mx + ny + \ell = 0$ are concurrent if :
(A) $\ell + m n = 0$
(B) $\ell + m - n = 0$
(C) $\ell - m + n = 0$
(D) $\ell^2 + m^2 + n^2 \neq \ell m + mn + n\ell$

26. The sides of a triangle are 3x + 4y, 4x + 3y and 5x + 5y units where x > 0, y > 0. The triangle
(A) Right angled (B) Acute angled (C) Obtuse angled (D) Isosceles

27. The lines x + 2y – 3 = 0, 2x + y – 3 = 0 and the line ℓ are concurrent. If the line ℓ passes through the origin, then its equation is :
(A) x – y = 0 (B) x + y = 0 (C) x + 2y = 0 (D) 2x + y = 0

28. Angles of the triangle formed by the lines $x^2 - y^2 = 0$, x = 7 are :
(A) $45^0, 90^0, 45^0$ (B) $30^0, 60^0, 90^0$ (C) $60^0, 60^0, 60^0$ (D) None of these

29. If the orthocenter and centroid of a triangle are (– 3, 5) and (3, 3) then it's circumcentre is :
(A) (6, 2) (B) (3, – 1) (C) (– 3, 5) (D) (– 3, 1)

30. A triangle with vertices (4, 0), (– 1, – 1), (3, 5) is : **[AIEEE-2002]**
(A) Isosceles and right angled
(B) Isosceles but not right angled
(C) right angled but not isosceles
(D) Neither right angled nor isosceles

31. The centroid of a triangle is (2, 3) and two of it's vertices are (5, 6) and (– 3, 4). The third vertex of the triangle is : **[AIEEE-2002]**
(A) (2, 1) (B) (2, – 1) (C) (1, 2) (D) (1, – 2)

32. If a vertex of a triangle is (1, 1) and the mid-points of two sides through this vertex are (– 1, 2) and (3, 2), then the centroid of the triangle is : **[AIEEE-2005]**
(A) $\left(-1\frac{7}{3}\right)$ (B) $\left(\frac{-1}{3},\frac{7}{3}\right)$ (C) $\left(1,\frac{7}{3}\right)$ (D) $\left(\frac{1}{3},\frac{7}{3}\right)$

33. If non zero numbers a, b, c are in H.P. then the straight the $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is : **[AIEEE-2005]**
(A) (1, – 2) (B) (1, – 1/2) (C) (– 1, 2) (D) (– 1, – 2)

34. The line parallel to x-axis passing through the intersection of the lines ax + 2by + 3b = 0 and bx – 2ay – 3a = 0 where (a, b) $\neq$ (0, 0) is : **[AIEEE-2005]**
(A) Above x-axis at a distance 3/2 from it
(B) Above x-axis at a distance 2/3 from it
(C) Below x-axis at a distance 3/2 from it
(D) Below x-axis at a distance 2/3 from it

35. Let A(h, k), B(1, 1) and C(2, 1) be the vertex of a right angled triangle with AC it's hypotenuse. If the area of the triangle is 1, then the set of value which 'k' can take is given by : **[AIEEE-2007]**

(A) {1, 3} (B) {0, 2} (C) {– 1, 3} (D) {– 3, – 2}

36. The orthocenter of the right triangle with vertices $\left[2,\frac{(\sqrt{3}-1)}{2}\right], \left(\frac{1}{2},-\frac{1}{2}\right)$ and $\left(2,-\frac{1}{2}\right)$ is : **[II T-1993]**

(A) $\left[\frac{3}{2},\frac{\sqrt{3}-3}{6}\right]$ (B) $\left[2,-\frac{1}{2}\right]$ (C) $\left[\frac{5}{4},-\frac{\sqrt{3}-2}{4}\right]$ (D) $\left[\frac{1}{2},\frac{1}{2}\right]$

37. The orthocenter of the triangle formed by the lines xy = 0 and x + y = 1 is : **[II T-1995]**

(A) $\left(\frac{1}{2},\frac{1}{2}\right)$ (B) $\left(\frac{1}{3},\frac{1}{3}\right)$ (C) (0, 0) (D) $\left(\frac{1}{4},\frac{1}{4}\right)$

38. If the vertices P,Q,R of a triangle PQR are rational points, which of the following points of the triangle PQR is (are) not always rational points(s) ? **[II T-1998]**

(A) Centroid (B) Incentre (C) Circumcentre (D) None of these

39. If P(1, 2), Q(4, 6), R(5, 7) and S(a, b) are the vertex of a parallelogram PQRS, then : **[II T-1998]**

(A) a = 2, b = 4 (B) a = 3, b = 4 (C) a = 2, b = 3 (D) a = 3, b = 5

40. The incentre of the triangle with vertex $(1,\sqrt{3})$, (0, 0) and (2, 0) is : **[II T-2000, AIEEE-2002]**

(A) $\left(1,\frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{2}{3},\frac{1}{\sqrt{3}}\right)$ (C) $\left(\frac{2}{3},\frac{\sqrt{3}}{2}\right)$ (D) $\left(1,\frac{1}{\sqrt{3}}\right)$

41. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are : **[II T-2007]**

(A) $\left(\frac{4}{3},3\right)$ (B) $\left(3,\frac{2}{3}\right)$ (C) $\left(3,\frac{4}{3}\right)$ (D) $\left(\frac{4}{3},\frac{2}{3}\right)$

OBJECTIVE — ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	B	A	D	C	A	C	C	C	B	A	B	B	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	C	D	D	C	A	A	C	C	A	C	A	A	A	A
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**				
Ans.	B	C	A	C	C	B	C	B	C	D	C				

PROBABILITY

★ INTRODUCTION

In our day-to-day conveersation, we generally use the phrases like :

(i) **Probably,** Satya will visit my house today

(ii) **Most probably,** Megha is preparing for CAT.

(iii) Khusboo is **quite sure** to be on the top.

(iv) **Chances** are high that Regi will head the organization,

The words' probably', 'most probably', 'quite sure', 'chances' etc involve an element of uncertainty.

Probability – Probability is the mathematical measurement of uncertainty.

Probability Theory – it is that branch of mathematics in which the degree of uncertainty (or certainty of occurrence of event) is measured numerically.

★ SOME BASIC CONCEPTS/TERMS

1. **Experiment :** An action or operation which can produce some well defined result is known as **experiment.**

2. **Deterministic experiment :** If we perform an experiment and repeat it under identical conditions, we get almost the same result every time, such an experiment is called a **deterministic experiment.**

3. **Random experiment :** An experiment is said to be a random experiment if it satisfies the following two conditions :

(i) It has more than one possible outcomes.

(ii) It is not possible to predict the outcome (result) in advance.

Ex. (i) Tossing a pair of fair coins. (ii) Rolling an unbiased die.

4. **Outcomes :** The possible results of a random experiment are called **outcomes.**

5. **Trial :** When an experiment is repeated under similar conditions and it does not give the same result each time but may result in any one of the several possible outcomes, the result is called a **trial.**

Ex. If a coin is tossed 100 times, then one toss of the coin is called a trial.

6. **Event :** The collection of all or some outcomes of a random experiment is called an **event.**

Ex. Suppose we toss a pair of coins simultaneously and let E be the event of getting exactly one head. Then, the event E contains HT and TH.

Ex. Suppose we roll a die and let E be the event of getting an even number. Then the event E contains 2, 4 and 6.

7. **Elementary or Simple Event :** An outcome of a trial is called an **elementary event.**

NOTE : An elementary event has only one element.

Ex. Let a pair of coins is tossed simultaneously. Then, possible outcomes of this experiment are.

HH : Getting H on first H on second (= E_1) [H = Head, T = Tail and E = event]

HT : Getting H on first T on second (= E_2)

TH : Getting T on first H on second (= E_3)

and TT : Getting T on first T on second (= E_4)

Here, E_1, E_2, E_3 and E_4 are the element events associated with the random experiment of tossing of two coins.

8. **Compound event or composite event or mixed event :** An event associated to a random experiment and obtained by combining two or more simple events associated to the same random experiment, is called a **compound event .**

OR

A compound event is an aggregate of some simple (elementary) event and is decomposable into simple events.

Ex. If we throw a die, then the event E of getting an odd number is a compound event because the event E contains three elements 1, 3 and 5, which is a compound of three simple events E_1, E_2, and E_3 containing 1, 3 and 5 respectively.

9. **Equally likely events :** The out comes of an experiment are said to be equally likely events if the chances of their happenings are neither less nor greater then other.

In other words, a given number o events are said to be equally likely if none of them is experiment to occur in preference to the others.

Ex. In tossing a coin, getting head (H) and tail (T) are equally likely events.

★ EXPERIMENTAL (OR EMPIRICAL) PROBABILITY

The experiment or empirical probability P(E) of an event is defined as

$$P(E) = \frac{Number\,of\,trials\,in\,which\,the\,event\,happened}{Total\,number\,of\,trials}$$

i.e., P (E) = $\frac{m}{n}$

NOTE : (i) These probabilities are based on results of an actual experiment.

(ii) These probabilities are only 'estimates', i.e., we may get different probabilities for the same event in various experiments.

★ THEORETICAL (OR CLASSICAL) PROBABILITY

The theoretical or classical probability of an event E, written as P(E), is defined as

$$P(E) = \frac{Number\,of\,outcomes\,favourable\,to\,E}{Number\,of\,all\,possible\,outcomes\,of\,the\,experiment}$$

Where the outcomes of the experiment are equally likely.

Ex. A die is thrown once (i) What is the probability of getting a number greater than 4? (ii) What is the probability of getting a number less than or equal to 4?

Sol. The possible outcomes are 1, 2, 3, 4, 5 and 6.

Let E = the event of getting a number greater than 4

and F = The event of getting a number less than or equal to 4

(i) The outcomes favorable to E are 5 and 6,

∴ the number of outcomes favorable to E is 2.

The therefore, P(E) = P(number greater than 4) = $\frac{4}{6} = \frac{2}{3}$

NOTE : Events E and F are not elementary events because event E has 2 outcomes and the event F has 4 outcomes,

<u>ANIMPORTANT REMARK</u>

In the experiment or empirical approach to probability, the probability of events are based on the results of actual experiment and adequate recordings of the happening of the events, while in theoretical approach to probability, we try to find (predict) the probabilities of the events without actually performing the experiment.

★ SOME SPECIAL EVENTS

⏩ **IMPOSSIBLE EVENT (OR NULL EVENT) :** An event is said to be an impossible event when none of the outcomes is favorable to the event.

The probability of an impossible event = 0 .

Ex. What is the probability, of getting

Sol. The possible outcomes are 1, 2, 3, 4, 5, 6.

Let E = the event of getting a number 8 in a single throw of a die.

Clearly, the number of outcomes favorable to E is 0 and the total number of possible outcomes is 6.

Therefore, P(E) = $\frac{0}{6} = 0$.

Here, E is an impossible event.

- **SURE (OR CERTAIN) EVENT :** An event is said to be a sure (or certain) event when all possible outcomes are favorable to the event.
  The probability of a sure event is 1.

Ex. What is the probability of getting a number less than 7 in a single of a die?

Sol. The possible outcomes are : 1, 2, 3, 4, 5, 6.
Let F = the event of getting a number 7 in a single throw of a dice. Clearly, the number of outcomes favorable to F are 1, 2, 3, 4, 5, 6. i.e., the number of outcomes favorable to F is 6.

Therefore, P(E) = $\frac{6}{6} = 1$.

Here, F is an impossible event.

- **COMPLEMENT OF AN EVENT :** Corresponding to every event E associated with random experiment, there is an event 'not E', which occurs only when E does not occur.
  The event $\overline{E}$, representing 'not E', is called the complement of the event E.
  E and $\overline{E}$, are also called complementary events.
  In general, P(E) + P($\overline{E}$) = 1
  i.e., P($\overline{E}$) = 1 – P(E) or P(not E) = 1 – P(E)

- **AN IMPORTANTR RESULT :** The probability of an event always lies between 0 and 1.
  i.e., $0 \le P(E) \le 1$
  PROOF Let m be the number of favorable outcomes of an event E and n be the total number of outcomes.
  Then, $0 \le m \le n$ [m cannot be negative integer and m cannot be greater than n]

$$\Rightarrow \quad 0 \le \frac{m}{n} \le \frac{n}{n} \Rightarrow 0 \le \frac{m}{n} \le 1 \Rightarrow a \le P(E) \le 1$$

Thus, the probability of an event always lies between 0 and 1.

NOTES : (i) $0 \le P(E) \le 1$
(ii) Let $E_1, E_2, E_3, \ldots, E_n$ be the n elementary events associated with a random experiment having exactly n outcomes. Then,
$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_n) = 1$

Ex. A bag contains 3 red balls, 4 white balls and 5 green balls. A ball is drawn at random.
Let R = the event of getting a red ball,
W = the event of getting a with ball,
and G = the event of getting a green ball,
Here, total number of balls (outcomes) = 3 + 4 + 5 = 12.

Then, P(R) $= \frac{3}{12}$ [Number of favorable outcomes = 3]

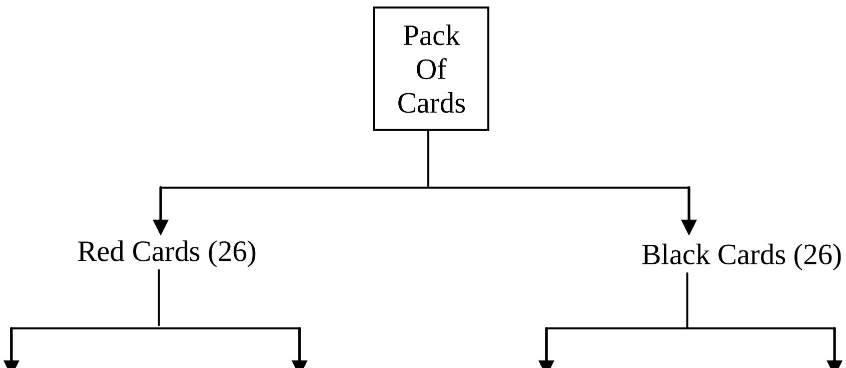
P(W) $= \frac{4}{12}$ [Number of favorable outcomes = 4]

And, P(G) $= \frac{5}{12}$ [Number of favorable outcomes = 5]

Clearly, $\frac{3}{12} + \frac{4}{12} + \frac{5}{12} + \frac{12}{12} = 1.$

i.e., P(R) + P(W) + P(G) = 1.

DESIGNATION OF PLAYING CARDS



(i) A beck (pack) of cards contains 52 cards, out of which there are 26 red cards and 26 black cards.
(ii) There are four suits each containing 13 cards.
(iii) The cards in each suit are ace (A). king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.
(iv) Kings, queens and jacks are called **face cards** (4 + 4 + 4 = 12).
(v) Kings, queens jacks and are called **honour cards** (4 + 4 + 4 + 4= 16).

JUST FOR YOU

1. $P(E) = \dfrac{\text{number of outcomes favourable to } E}{\text{number of all possible outcomes of the experiment}}$, where outcomes of the experiment are equally likely.
2. Probability of an impossible event = 0.
3. Probability of a sure event = 1.
4. P(E) + P(E) = 1, where E and $\overline{E}$ are complementary events.
5. $0 \le P(E) \le 1$
6. The sum of the probabilities of all the elementary events of an experiment is 1.

Ex.1 Complete the following statements :
(i) Probability of an event E + probability of the event 'not E' = ...
(ii) The Probability of an event that is certain to happen is......... Such an event is called..........
(iii) The Probability of an event is greater than or equal to......... and less than or equal to..........
(iv) $P(E) = \dfrac{\text{..............}}{\text{Total number of trials}}$

Sol. (i) 1 (ii) 1, sure or certain event (iii) 0, 1 (iv) number of trials in which event happened.

Ex.2 Which of the following experiments have equally likely outcomes ? Explain.
(i) A driver attempts to starts a car. The car starts or does not start.
(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.
(iii) A coin is tossed. It turns to be a head or a tail.
(iv) A monitor is nominated by the class teacher of a class. It is a boy or a girl.

Sol. (iii) because when a coin is tossed either a head or a tail turns.

Ex.3 Mach the following.

(i)	**P($\overline{E}$) =**	**(a) 0**
(ii)	**Probability of an impossible event**	**(b) 0.5**
(iii)	**Probability of an event cannot be more than**	**(c) 1 – P(E)**
(iv)	**A cards is drawn form a pack of 52 cards, then the probability of getting a red card is equal to**	**(d) 1**

Sol. (i) (c), (ii) (a), (iii) (d), (iv) (b)

Ex.4 If P(E) = 0.05, what is the probability of 'not E'?

Sol. We have P(E) = 0.05
$\therefore$ P(not E) = 1 – P(E) = 1 – 0.05 = 0.95
Therefore, P(not E) = 0.95.

Ex.5 Find the probability of getting a head when a coin is tossed once.

Sol. When a coin is tossed, then all possible outcomes are H and T
Total number of possible outcomes = 2
Let E be the event of getting a head
$\therefore$ number of favorable outcome = 1

Hence, required probability = P(E) = $\frac{Number of\ favourable outcomes}{Total number of\ possible outcomes} = \frac{1}{2}$

Ex.6 Two unbiased coins are tossed simultaneously. Find the probability of getting

(i) one head **(ii) one tail** **(iii) two heads**

(iv) at least one head **(ii) at most one tail** **(iii) no head.**

Sol. If two unbiased coins are tossed simultaneously, then all possible outcomes are :

HH, HT, TH, TT.

Total number of possible outcomes = 4.

(i) Let A_1 = the event of getting one head.

Then, favorable outcomes are HT, TH.

Number of favorable outcomes = 2.

Hence, required probability = P (getting one head) = $P(A_1) = \frac{2}{4} = \frac{1}{2}$

(ii) Let A_2 = the event of getting one tail.

Then, favorable outcomes are TH, HT.

Number of favorable outcome = 2.

Hence, required probability = P (getting one tail) = $P(A_2) = \frac{2}{4} = \frac{1}{2}$

(iii) Let A_3 = the event of getting two tail.

Then, favorable outcomes is HH

Number of favorable outcome = 1

Hence, required probability = P (getting two heads) = $P(A_3) = \frac{1}{4}$

(iv) Let A_4 = the event of getting at least one head.

Then, favorable outcomes are HT, TH, HH

Number of favorable outcome = 3

Hence, required probability = P (getting at least one head) = $P(A_4) = \frac{3}{4}$

(v) Let A_5 = the event of getting atmost one head.

Then, favorable outcomes are TT, HT, TH.

Number of favorable outcome = 3

Hence, required probability = P (getting atmost one head) = $P(A_5) = \frac{3}{4}$

(vi) Let A_6 = the event of getting no head.

Then, favorable outcomes are TT

Number of favorable outcome = 1

Hence, required probability = P (getting one head) = $P(A_6) = \frac{1}{4}$

Ex.7 Three unbiased coins are tossed together. Find the probability of getting

(i) one head **(ii) two heads** **(iii) all heads** **(iv) at least two heads**

Sol. If three unbiased coins are tossed together, then all possible outcomes are :

HHH, HHT, THH, HTT, THT, TTH, TTT

Total number of possible outcomes = 8

(i) Let A_1 = the event of getting one head.

Then, favorable outcomes are HHT, THT, THH.

Number of favorable outcomes = 3.

Hence, required probability = P (getting one head) = $P(A_1) = \frac{3}{8}$

(ii) Let A_2 = the event of getting two head.
Then, favorable outcomes are HHT, HTH, THH.
Number of favorable outcomes = 3.

Hence, required probability = P (getting two heads) = $P(A_2) = \frac{3}{8}$

(iii) Let A_3 = event of getting all heads.
Then, favorable outcomes are HHH
Number of favorable outcomes = 1

Hence, required probability = P (getting all head) = $P(A_3) = \frac{1}{8}$

(iv) Let A_4 = event of getting at least two heads.
Then, favorable outcomes are HHT, HTH, THH, HHH
Number of favorable outcomes = 4

Hence, required probability = P (getting at least two heads) = $P(A_4) = \frac{4}{8} = \frac{1}{2}$

Ex.8 A die is thrown once. Find the probability of getting
(i) a prime number (ii) a number lying between 2 and 6 (iii) an odd number.

Sol. If a die is thrown, then all possible outcomes are 1, 2, 3, 4, 5, 6.
Total number of possible outcomes = 6.

(i) Let A_1 = event of getting a prime number.
Then, the favorable outcomes are 2, 3, 5.
Number of favorable outcomes = 3.

Hence, required probability = P (getting a prime number) = $P(A_1) = \frac{3}{6} = \frac{1}{2}$

(ii) Let A_2 = event of getting a number lying between 2 and 6.
Then, the favorable outcomes are 3, 4, 5.
Number of favorable outcomes = 3.

Hence, required probability = P (getting a number lying between 2 and 6) = $P(A_2) = \frac{3}{6} = \frac{1}{2}$

(iii) Let A_3 = event of getting an odd number.
Then, the favorable outcomes are 1, 3, 5.
Number of favorable outcomes = 3.

Hence, required probability = P (getting an odd number) = $P(A_3) = \frac{3}{6} = \frac{1}{2}$

Ex.9 A die is thrown twice. What is the probability that
(i) 5 will not come up either time? (ii) 5 will come up at least once?

Sol. If a die is thrown twice, then all the possible outcomes are :
(1, 1), (1, 2), (1,3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2,3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3,3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4,3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5,3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6,3), (6, 4), (6, 5), (6, 6).
Total number of getting 5 not either time

(i) Let A_1 = event of getting 5 not either time.
Then, the favorable outcomes are: (1, 1), (1, 2), (1,3), (1, 4), (1, 6),
(2, 1), (2, 2), (2,3), (2, 4), (2, 6),
(3, 1), (3, 2), (3,3), (3, 4), (3, 6),

(4, 1), (4, 2), (4,3), (4, 4), (4, 6),
(6, 1), (6, 2), (6,3), (6, 4), (6, 6).

Number of favorable outcomes = 25

Hence, required probability = P (5 will not come up either time) = P(A_1) = $\frac{25}{36}$

(ii) Let A_2 = event of getting 5 at least once.
Then, the favorable outcomes are:
(1, 5), (2, 5), (3,5), (4, 5), (5, 1), (5, 2), (5, 3), (5,4), (5, 5), (5, 6), (6, 5).
Number of favorable outcomes = 11

Hence, required probability = P (5 will not come up at least once) = P(A_2) = $\frac{11}{36}$

Ex.10 A pair of dice is thrown simultaneously. Find the probability of getting
(i) a doublet
(ii) sum of the numbers on two dice is always 7
(iii) an even number on the first die and a multiple of 3 on the other.

Sol. If a pair of dice is thrown simultaneously, then all the possible outcomes are :
(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),(6,1), (6,2), (6,3), (6,4), (6,5), (6,6).
Total number of possible outcome = 36

(i) Let A_1 = event of getting a doublet.
Then, the favorable outcomes are (1, 1), (2, 2), (3,3), (4, 4), (5, 5), (6, 6).
Number of favorable outcomes = 6

Hence, required probability = P (getting a doublet) = P(A_1) = $\frac{6}{36} = \frac{1}{6}$

(ii) Let A_2 = event of getting a sum of numbers on two dice is always 7
Then, the favorable outcomes are (1, 6), (2, 5), (3, 4), (5, 2), (6, 1).
Number of favorable outcomes = 6

Hence, required probability = P (getting a sum of the numbers on two dice is always 7) = P(A_2) = $\frac{3}{36} = \frac{1}{6}$

(iii) Let A_3 = event of getting an even umber on the first die and a multiple of 3 on the other.
Then, the favorable outcomes are (2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6).
Number of favorable outcomes = 6
Hence, required probability = P (getting an even umber on the first die and a multiple of 3 on the other)
= P(A_3) = $\frac{6}{36} = \frac{1}{6}$

Ex.11 Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is
(a) 8 (b) 13 (iii) less than or equal to 12? (NCERT)

Sol. If two dice, one blue and one grey, are thrown at the same time, then all possible outcomes are :
(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),(6,1), (6,2), (6,3), (6,4), (6,5), (6,6).

Total number of possible outcome = 36

(i) Let A_1 = event of getting a sum two numbers appearing on the top o the dice is 8.
Then, the favorable outcomes are (2, 6), (2, 5), (4, 4), (5, 3), (6, 2).
Number of favorable outcomes = 5.

Hence, required probability = P(A_1) = $\frac{5}{36}$

(ii) Let A_2 = event of getting a sum two numbers appearing on the top o the dice is 13.
Then, the favorable outcomes = 0.

Hence, required probability = P(A_2) = $\frac{0}{36}$ = 0

(iii) Let A_3 = event of getting a sum two numbers appearing on the top o the dice is less than or equal to 12.

Then, the favorable outcomes = all the possible outcomes = 36.

Hence, required probability = $P(A_3) = \frac{36}{36} = 1$.

Ex.12 A cards is drawn at random from a well shuffled deck of 52 cards. Find the probability that the card drawn is –

(i) a red card (ii) a non-ace (iii) a king or a jack (iv) neither a king nor a queen.

Sol. If a card is drawn at random from a well shuffled deck of 52 cards, then total number of possible outcomes = 52

(i) Let A_1 = event of getting a red card.
Then, the favorable outcomes = 26.

Hence, required probability = P (getting a red card) = $P(A_1) = \frac{26}{52} = \frac{1}{2}$

(ii) Let A_2 = event of getting a non-ace
Then, the favorable outcomes = 48. [$\because$ there are 4 aces in a pack of playing cards]

Hence, required probability = P (getting a non-ace) = $P(A_2) = \frac{48}{52} = \frac{12}{13}$

(iii) Let A_3 = event of getting a king or a jack.
There are 4 king cards and 4 jack cards.
Hence, required probability = $P(A_3)$ = P (getting a king or a jack) = P (getting a king) + P (getting a jack)

$$= \frac{4}{52} + \frac{4}{52} + \frac{8}{52} + \frac{2}{13}$$

(iv) Let A_4 = event of getting neither a king nor a queen.
There are 4 king cards and 4 queen cards.
Hence, required probability = $P(A_4)$ = P (getting neither a king nor a queen)
= 1 – P (getting a king or a queen)
= 1 – P (getting a king) + P (getting a queen)]

$$= 1 - \left(\frac{4}{52} + \frac{4}{52}\right) = 1 - \frac{8}{52} = \frac{44}{52} = \frac{11}{13}$$

ALITER: Let A_4 : event of getting neither king nor queen.
$\therefore$ no, of favorable outcomes.
i.e., neither king nor queen cards = 52 – 8 = 44

Hence, $P(A_4) = \frac{44}{52} = \frac{11}{13}$

Ex.13 All the three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting

(i) a black face card (ii) a queen (iii) a black card

Sol. If all the three face cards of spades are removed from a well-shuffled pack of 52 cards, then there are 49 cards left in the pack.

(i) Let A_1 = event of getting a black face card.
There are 3 black face cards left. (face cards of club)

Hence, required probability = $P(A_1)$ = P (getting a black face card) = $\frac{3}{49}$

(ii) Let A_2 = event of getting a queen.
There are three queens left.

Hence, required probability = $P(A_2)$ = P (getting a queen) = $\frac{3}{49}$

(iii) Let A_3 = event of getting a black card.
There are 23 black cards left.

Hence, required probability = $P(A_3)$ = P (getting a black card) = $\frac{23}{49}$

Ex.14 Five cards, the-ten, jack, queen, king and ace of diamonds, are well-shuffled with their faces downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is

(a) an ace ? (b) a queen ?

Sol. There are five cards as the ten, jack, queen, king and ace of diamond.

(i) Let A = event of getting a queen

There is only one queen out of the five cards.

Hence, required probability = P(A) = P (getting a queen) = $\frac{1}{5}$

(ii) When a queen is drawn and put aside four cards, the ten, jack, king and ace are left. Therefore.

(a) required probability = P (getting an ace) = $\frac{1}{4}$

(b) required probability = P (getting a queen) = $\frac{0}{4}$ = 0.

Ex.15 A box contains 5 red. 4 green and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is

(i) white (ii) neither red nor white

Sol. Total number of balls in the box = 5 + 4 + 7 = 16 .

Let A_1 = event of getting a red ball

A_2 = event of getting a white ball.

(i) There are 7 white balls in the box.

Hence, required probability = $P(A_2)$ = P (getting a white ball) = $\frac{7}{16}$

(ii) There are 7 white and 5 red balls in the box.

Hence, required probability = P (getting neither red nor white ball)

= 1 – P (getting either red or white ball)

= 1 – P (getting a red) + P (getting a white ball)]

$= 1 - \left(\frac{5}{16} + \frac{7}{16}\right) = 1 - \frac{12}{16} = \frac{4}{16} = \frac{1}{4}$

ALITER P(getting neither red nor white ball) = P (getting a green ball) = $\frac{4}{16} = \frac{1}{4}$

Ex.16 A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of red ball, determine the number of blue balls in the bag.

Sol. There are 5 red balls in a bag.

Let number of blue balls be x.

Let A_1 = event of getting a red ball

and A_2 = event of getting a blue ball.

$P(A_1)$ = P (getting a red ball) = $\frac{5}{x+5}$

$P(A_2)$ = P (getting a blue ball) = $\frac{x}{x+5}$

$\because 2P(A_1) = P(A_2) \Rightarrow \frac{2\times 5}{x+5} = \frac{x}{x+5} \Rightarrow 10 = x \Rightarrow x = 10$

Hence, required number of blue balls = 10.

Ex.17 A box contains 5 red marbles, 8 white marbles and 4 green marbles one marble is taken out of the box at random. What is the probability that the marble taken out will be

(i) red (ii) white (iii) not green?

Sol. Total number of marbles in the box = 5 + 8 + 4 = 17.

Let A_1 = event of getting a red marble

A_2 = event of getting a white marble

and A_3 = event of getting a green marble.

(i) There are 5 red marbles in the box.

Hence, required probability = $P(A_1)$ = P (getting a red marble) $= \frac{5}{17}$

(ii) There are 8 white marbles in the box.

Hence, required probability = $P(A_2)$ = P (getting a white marble) $= \frac{8}{17}$

(iii) There are 4 green marbles in the box.

$\therefore$ $P(A_3)$ = P (getting a green marble) = $\frac{4}{17}$

Hence, required probability = P (not getting a green marble)

= 1 – P (getting a green marble) = 1 – $P(A_3)$ $= 1 - \frac{4}{17} = \frac{13}{17}$.

Ex.18 A box contains 19 balls bearing numbers 1, 2, 3,……, 19 respectively. A ball is drawn at random from the box. Find the probability that the number on the ball is –

(i) a prime number (ii) even number

(iii) divisible by 3 or 5 (iv) neither divisible by 5 nor by 10.

Sol. Total number of balls in the box = 19

$\therefore$ number of all possible outcomes = 19

(i) Let A_1 = event of getting a prime number.
Then, the favorable outcomes are 2, 3, 5, 7, 11, 13, 17, 19.
Number of favorable outcomes = 8.

Hence, required probability = P (getting a prime number) = $P(A_1)$ $= \frac{8}{19}$.

(ii) Let A_2 = event of getting an even number.
Then, the favorable outcomes are 2, 4, 6, 8, 10, 12, 14, 16, 18.
Number of favorable outcomes = 9.

Hence, required probability = P (getting an even number) = $P(A_2)$ $= \frac{9}{19}$.

(iii) Let A_3 = event of getting a number divisible by 3 or 5.
Then, the favorable outcomes are 3, 5, 6, 9, 10, 12, 15, 18.
Number of favorable outcomes = 8.

Hence, required probability = P (getting a number divisible by 3 or 5) = $P(A_3)$ $= \frac{8}{19}$.

(iv) Let A_4 = event of getting a number divisible by 5 or 10.

Then, the favorable outcomes are 5, 10, 15. Number of favorable outcomes = 3.

$\therefore$ P (getting a number divisible by 5 or 10) = $P(A_4)$ = $\frac{3}{19}$

Hence, required probability = P (getting a number neither divisible by 5 nor by 10)

= 1 – P (getting a number neither divisible by 5 nor by 10) $= 1 - \frac{3}{19} = \frac{16}{19}$

Ex.19 Seventeen cards numbered 1, 2, 3, 4,……, 16, 17 are put in a box and mixed thoroughly. One person drawn a card from the box. Find the probability that the number on the card is

(i) odd (ii) a prime (iii) divisible by 3 (iv) divisible by 2 and 3 both

Sol. There are seventeen cards in the box.
$\therefore$ number of all possible outcomes = 17.

(i) Let A_1 = event of getting an odd number.
Then, the favorable outcomes are 1, 3, 5, 7, 9, 11, 13, 15, 17.
Number of favorable outcomes = 9.

Hence, required probability = P (getting an odd number) = $P(A_1) = \frac{9}{17}$.

(ii) Let A_2 = event of getting a prime number.
Then, the favorable outcomes are 2, 3, 5, 7, 11, 13, 17.
Number of favorable outcomes = 7.

Hence, required probability = P (getting a prime number) = $P(A_2) = \frac{7}{17}$.

(iii) Let A_3 = event of getting a number divisible by 3.
Then, the favorable outcomes are 3, 6, 9, 12, 15.
Number of favorable outcomes = 5.

Hence, required probability = $P(A_3) = \frac{5}{17}$.

(iv) Let A_4 = event of getting a number divisible by 2 and 3 both.
Then, the favorable outcomes are 6, 12.
Number of favorable outcomes = 2.

Hence, required probability = $P(A_4) = \frac{2}{17}$.

Ex.20 Find the probability that a number selected at random from the numbers 1 to 15 not a prime number when each of the given numbers is equally likely to be selected.

Sol. The total given numbers = 25
Then, the favorable outcomes (prime numbers) are 2, 3, 5, 7, 11, 13, 17, 19, 23.
Number of favorable outcomes = 9.
Let A = event of getting a non-prime number.
∴ number of non-prime number = 25 – 9 = 16

∴ required probability = P (getting a non-prime numbers) = $P(A) = \frac{16}{25}$.

Ex.21 A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears
(i) a two digit number (ii) a perfect square number (iii) a number divisible by 5.

Sol. The total of discs = 90. Number of possible outcomes = 90.

(i) Let A_1 = event of getting a two digit number.
There are 9 single-digit numbers and 81 two-digit numbers.
Then, the number of favorable outcomes = 81.

Hence, required probability = $P(A_1)$ = P (getting a two-digit number) $= \frac{81}{90} = \frac{9}{10}$.

(ii) Let A_2 = event of getting a perfect square number.
Then, the number of favorable outcomes are 1, 4, 9, 16, 25, 36, 49, 64, 81.
Number of favorable outcomes = 9.

Hence, required probability = $P(A_2)$ = P (getting a perfect square number) $= \frac{9}{90} = \frac{1}{10}$.

(iii) Let A_3 = event of getting a number divisible by 5.
Then, the number of favorable outcomes are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90.
Number of favorable outcomes = 18.

Hence, required probability = $P(A_3)$ = P (getting a number divisible by 5) $= \frac{18}{90} = \frac{1}{5}$.

Ex.22 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not is defective. One pen is taken out at random from this from this lot. Determine the probability that the pen taken out is a good one.

Sol. There are 12 defective pens and 132 good pens.
∴ Total number of possible outcomes = 12 + 132 = 144.
Let A = event of getting a good pen
Then, the number of favorable outcomes = 132

Hence, required probability = P(A) = P (getting a good pen) $=\frac{132}{144}=\frac{11}{12}$.

Ex.23 A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it t her. What is the probability that (i) she will buy it (ii) she will not buy it?

Sol. There are 144 ball pens.

$\therefore$ total number of possible outcomes = 144.

(i) Let A_1 = event of buying a good pen.

There are 20 ball pens which are defective out of 144 ball pens.

$\therefore$ number of good ball pens = 144 – 20 = 124

Hence, required probability = $P(A_1)$ = P (buying a good pen) $=\frac{124}{144}=\frac{31}{36}$.

(ii) Let A_2 = event of not buying a good pen i.e., buying a defective pen.

Then, the number of favorable outcomes = 20.

Hence, required probability = $P(A_2)$ = P (not buying a good pen) $=\frac{20}{144}=\frac{5}{36}$.

Ex.24 Savita and Hamida are friend. What is the probability that both will have
(i) different birthdays (ii) the same birthday (ignoring leap year). (NCERT)

Sol. There are 365 days in a year.

$\therefore$ the total number of possible outcomes = 365.

(i) Let A_1 = event that Hamida's birthday is different from Savita's birthday Then, the number of favorable outcomes for her birthday = 365 – 1 = 364.

Hence, required probability = $P(A_1)$ = P (Hamida's birthday is different from Savita's birthday) $=\frac{364}{365}$.

(ii) Let A_2 = the event the Savita and Hamida have the same birthday

Hence, required probability = $P(A_2)$ = P (Savita and Hamida have the same birthday)

$= 1 - \text{P (both have different birthday)} = \frac{364}{365} = \frac{1}{365}$.

Ex.25 A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

Sol. There are 24 marbles in a jar.

$\therefore$ the total number of possible outcomes = 24.

Let number of green marbles be x.

(ii) Let A_1 = event of getting a green marble.

$\therefore$ required probability = $P(A_1)$ = P (getting a green marble) = $\frac{x}{24}$

But it is given that the probability of green marble is $\frac{2}{3}$

$\therefore \quad \frac{2}{3}=\frac{x}{24} \Rightarrow x=\frac{2\times 24}{3} \Rightarrow x=16$

So, number of green marbles = 16.

Hence, number of blue marble in jar = 24 – 16 = 8.

Ex.36 What is the probability that an ordinary year has 53 Sundays?

Sol. There are 365 day i.e., 52 weeks and 1 day in an ordinary year.

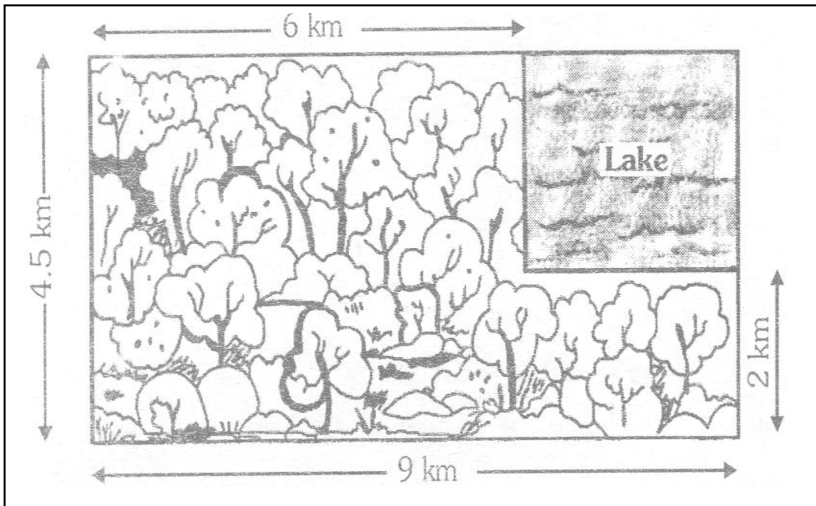
This 1 day can be anyone of the 7 days of the week.

$\therefore$ P (this day is sunday) = $\frac{1}{7}$

Also, 52 weeks have 52 Sundays.

Hence, required probability = P (an ordinary year has 53 sundays) $= \frac{1}{7}$.

Ex.27 A missing hellcopter is reported to have crashed somewhere in the rectangular region in figure. What is the probability that it crashed inside the lake shown in the figure? (NCERT)



Sol. The helicopter is equally likely to crash anywhere in the region.

Area of entire rectangular region, where the helicopter con crash = (4.5×9) km^2 = 40.5 km^2.

Area of the lake = (2.5×3) km^2 = 7.5 km^2.

Let A = the event that the helicopter crashed inside the lake.

Then, number of favorable outcomes = 7.5 km^2

Hence, required probability = P(A) = P (helicopter crashed in the lake) $= \frac{7.5}{40.5} = \frac{75}{405} = \frac{5}{27}$.

Ex.28 A piggy bank contains hundred 50p coins, fifty Re. 1 coins, twenty Rs. 2 coins and ten Rs. 5 coins. If it is likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50p coin (ii) will not be a Rs. 5 coin?

Sol. Number of 50p coins = 100

Number of Rs. 1 coins = 50

Number of Rs. 2 coins = 20

Number of Rs. 5 coins = 10

$\therefore$ total number of coins = 180.

$\therefore$ the total number of possible outcomes = 180.

(i) Let A_1 = event of getting 50p coin.

Then, the number of favorable outcomes = 100.

Hence, required probability = $P(A_1)$ = P (getting a 50p coin) $= \frac{100}{180} = \frac{5}{9}$.

(ii) Let A_2 = event of getting not a Rs. 5 coin.

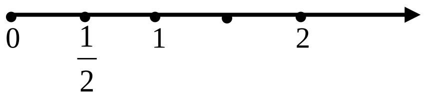
Hence, required probability = $P(A_2)$ = P (not getting Rs. 5 coin)

= 1 – P (getting a Rs. 5 coin)

$= 1 - = \frac{10}{180} = \frac{170}{180} = \frac{17}{18}$.

Ex.29 In a musical chair game, the person playing the music has been advised to stop playing the music at any time within two minutes after she/he starts playing. What is the probability that the music will stop within the first half minute after starting?

Sol. The possible outcomes are all the numbers between 0 and 2.
This is the portion of the number line from 0 to 2.



Let A = the event that the music is stopped within the first half-minute.

Then, the favorable outcomes are points on the number line from 0 to $\frac{1}{2}$

The distance from 0 to 2 is 2, while the distance from 0 to $\frac{1}{2}$ is $\frac{1}{2}$

Since all the outcomes are equally likely, therefore, the total distance = 2 and favorable to A = $\frac{1}{2}$.

Hence, required probability = P(A) = P (the music is stopped within the first half minute) $= \frac{\frac{1}{2}}{2} = \frac{1}{4}$

Ex.30 There are 40 students in class X of a school of whom 25 are girls and 15 are days. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical. Then she puts cards in a bag and stir them thoroughly. She, then draws one card from the bag. What is the probability that the name written on the card is the name of :
(i) a girl (ii) a boy? (NCERT)

Sol. There are 40 students out of which 25 are girls and 15 are days.
$\therefore$ number of all possible outcomes = 40.

(i) Let A_1 = event that the name written on the card is the name of a gils.
Then, the number of favorable outcomes = 25.

Hence, required probability = $P(A_1) = \frac{25}{40} = \frac{3}{8}$.

(ii) Let A_2 = event that the name written on the card is the name of a bot.
Then, the number of favorable outcomes = 15.

Hence, required probability = $P(A_2) = \frac{15}{40} = \frac{3}{8}$.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

1. If A be the event such that $P(A) = \frac{2}{5}$, then P(not A) is equal to

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{5}$ (d) None of these

An unbiased die is thrown (Q. NO. 2 to 6)

2. The probability of getting a prime number is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

3. The probability of getting a multiple of 3is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{3}{6}$ (d) $\frac{4}{6}$

4. The probability of getting a number greater than 1is

(a) $\frac{1}{6}$ (b) $\frac{2}{6}$ (c) $\frac{4}{6}$ (d) $\frac{5}{6}$

5. The probability of getting a number between 1 and 6 is

(a) $\frac{1}{6}$ (b) $\frac{2}{6}$ (c) $\frac{4}{3}$ (d) $\frac{2}{3}$

6. The probability of getting an odd number is

(a) $\frac{1}{6}$ (b) $\frac{2}{6}$ (c) $\frac{4}{6}$ (d) None of these

Two unbiased coins are tossed simultaneously (Q. No. 7 to 10)

7. The probability of getting one head is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) None of these

8. The probability of getting two heads

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) None of these

9. The probability of getting no head is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) None of these

10. The probability of getting at least one head is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) None of these

One card is drawn from a pack of 52 cards (Q. No. 11 to 14)

11. The probability of getting a jack card is

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{4}{13}$

12. The probability of getting a face card is

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{4}{13}$

13. The probability of getting a '10' of black suit is

(a) $\frac{1}{26}$ (b) $\frac{1}{13}$ (c) $\frac{2}{26}$ (d) None of these

14. The probability of getting a red and a king card is

(a) $\frac{5}{26}$ (b) $\frac{1}{13}$ (c) $\frac{7}{26}$ (d) None of these

15. A bag contains 4 red balls and 3 green balls. A ball is drawn at random. The probability a green ball is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

16. P(E) + P($\overline{E}$) is equal to
The probability of getting a jack card is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) None of these

Which one of the following cannot be the probability of an event (Q. No. 17 to 18)

17. (a) $\frac{1}{3}$ (b) $\frac{11}{36}$ (c) $\frac{-2}{3}$ (d) 1

18. (a) $\frac{2}{7}$ (b) 0 (c) $\frac{13}{29}$ (d) $\frac{5}{2}$

Choose the correct alternative for each of the following and justify your answer (Q. No. 19 to 22)

19. Probability of an impossible event is equal to

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) None of these

20. If $P(E_1)=\frac{1}{6}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{1}{6}$, where E_1, E_2, E_3 and E_4 are elementary events of a random experiment, then $P(E_4)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) None of these

21. Cards each marked with one of the numbers 4, 5, 6,…., 20 are placed in box and mixed thoroughly. One card is drawn at random from the box. Then, the probability of getting an even prime number is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) None of these

22. A bag contains 5 red and 4 black balls. A ball is drawn at random from the bag. Then, the probability of getting a black ball is

(a) $\frac{1}{5}$ (b) $\frac{4}{9}$ (c) $\frac{1}{5}$ (d) $\frac{1}{4}$

OBJECTIVE **ANSWER KEY** EXERCISE-4

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	C	B	D	D	D	A	B	A	C	A	C	A	D	C
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**								
Ans.	C	C	D	B	C	A	B								

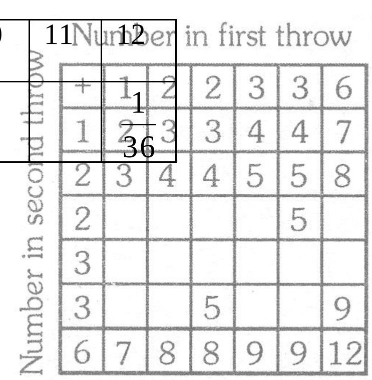
EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

SUBJCTIVE TYPE QUESTIONS

1. (a) Two dice are thrown at the same time. Complete the following table

Event : Sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(b) A die is numbered in such a way that its faces show the numbers 1, 2, 3, 4, 5, 6. It is thrown two times and the total score in two throws is noted.



Complete the following table which gives a few values of the total score on the two throws.

(c) Justify the statement : “Tossing a coin a fair of deciding which team should get the batting first at the beginning of a cricket game”

2. Which of the following experiment have not equally likely outcomes? Explain.
(i) A trial is made to answer a true-false question. The answer is right or wrong.
(ii) A baby is born. It is boy or a girl.
(iii) Kushagra appears in an interview. He is selected or not selected.
(iv) A die is thrown. It turns to be an even or an odd number.

3. Match the following :
A black die and a white die are thrown at the same time.

(i)	The probability of getting a total of 9.	(a) $\frac{1}{6}$
(ii)	The probability of getting a total of 10.	(b) $\frac{5}{36}$
(iii)	The probability of getting a total of more then 9.	(c) $\frac{1}{12}$
(iv)	The probability of getting the sum of the two numbers is 8.	(d) $\frac{1}{9}$

4. (a) The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow?
(b) If The probability of winning a game is 0.6, what is the probability of losing it?

5. Find the probability of getting a tail when a coin is tossed once.

6. (a) Two unbiased coins are tossed simultaneously. Find the probability of getting
(i) exactly one head (ii) exactly one tail (iii) two tails
(iv) at least one tail (v) atmost one tail (vi) no tail.
(b) Harpreet tosses two different coins simultaneously. What is the probability that she gets at least one head?

7. Three unbiased coins are tossed together. Find the probability of getting
(a) (i) one tail (ii) two tails (iii) all tails (iv) at least two tails

(b) (i) at most two tails (ii) at most two heads.

8. (a) A die is thrown once. Find the probability of getting
(i) a multiple of 2
(ii) a number lying between 1 and 5
(iii) an odd number.
(b) A child has a die whose six faces show the letters as given below

A	B	C	D	E	A

The die is thrown once. What is the probability of getting (i) A (ii) D?

9. A die is thrown twice. What is the probability that
(i) 3 will not come up either time?
(ii) 6 will come up at least once?

10. A pair of dice is thrown simultaneously. Find the probability of getting
(i) a multiple of 3 on both dice
(ii) sum of the numbers on two dice is always less than 7.
(iii) an odd number on the first die and a prime number on the other.

11. Two dice, one blue and green are thrown at the same time. What is the probability that sum of the two umbers appearing on the top of the dice is

(i) 9 (ii) greater than 10 (iii) less than or equal to 11

12. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(a) (i) a king of red colour (ii) a face card (iii) a red face card (iv) a jack of hearts

(b) (i) a spade (ii) the queen of diamonds (iii) neither a red card nor a queen.

(c) (i) a non-face card (ii) a black king or a red queen

13. (a) From a pack of 52 playing cards jacks, queens, kings and aces of red colour are removed. From a remaining, a card is drawn at random. Find the probability that the card drawn is

(i) a black queen (ii) a red card (iii) a ten (iv) a picture card [jacks, queens and kings are picture cards]

(b) All cards of ace, jack and queen are removed from a deck of playing cards. One card is drawn at random from the remaining cards. Find the probability that the card drawn is

(i) a face card

(ii) not a face card

14. Five cards – the ten, jack, queen, king and ace of diamonds are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is jack?

(ii) If the king is drawn and put aside, what is the probability that the second card picked up is

(a) a queen (b) a ten?

15. (a) A bag contains 7 red, 5 white and 3 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is

(b) (i) A bag contains 5 white balls, 7 red balls, 4 black and 2 blue balls. One ball is drawn at random from the bag. What is the probability that the ball drawn is

(1) white or blue (2) red or black (3) not white (4) neither white nor black

(ii) A bag contains 6 red balls, 8 white balls, 5 green balls and 3 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is

(1) white (2) red or black (3) not green (4) neither white nor black

(iii) A bag contains a red balls, a blue balls, a yellow balls, all the balls being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability that the she takes out the

(1) yellow ball (2) red ball (3) blue ball?

(iv) A box contains 7 red balls, 8 green balls, 5 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is

(1) white (2) neither red nor white

(c) Poonam buys a fish from a shop for her aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish 8 female fish. What is the probability that the fish taken out is a male fish?

16. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?

If 6 more black balls are put in the box, the probability of drawing a black ball is now doubled of what it was before, find x.

17. (a) A box contain 3 blue, 2 white and 4 red marbles. If a marble is drawn at random from the box. What is the probability of that it will be

(i) white (ii) blue (iii) red?

(b) A bag contain 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the

probability of getting
(i) a white ball or a green ball (ii) neither a green ball nor a red ball

18. (a) A box contains 20 balls bearing numbers 1, 2, 3,….., 20 respectively. A ball is drawn at random from the box what is the probability that the number on the ball is

(b) Find the probability that a number selected at random from the numbers 1, 2, 3, 4, 5,….., 34, 35 is a
(i) prime number (ii) multiple of 7 (iii) multiple of 3 or 5

(c) Cards bearing numbers 3 to 19 are put in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the card drawn is
(i) even (ii) a prime (iii) divisible by 2 and 3 both.

19. Fifteen cards numbered 1, 2, 3, 4,…..,14, 15 are put in a box and mixed thoroughly. A man drawn a card at random from the box. Find the probability that the number on the card is

(i) an odd number (ii) a multiple of 4 (iii) divisible by 5 (iv) divisible by 2 and 3 both.
(v) less than or equal to 10.

20. (a) There are 30 cards numbered from 1 to 30. One card is drawn at random. Find the probability that the number of the selected card is not divisible by 3.

(b) A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, and these are equally likely outcomes. What is the probability that it will point at
(i) 8 (ii) an odd number (iii) a number greater than 2 (iv) a number less than 9.

21. A box contains 50 discs which are numbered from 1 to 50. If one disc is drawn at random from the box, find the probability that it bears
(i) a two digit number less than (ii) a prime number (iii) a number divisible by 3

22. (i) A lot of 20 bulbs contains 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

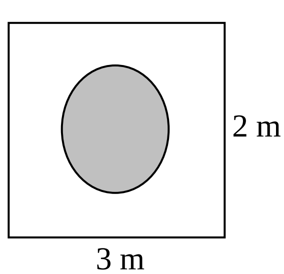
23. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujata, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that
(i) it is acceptable to Jimmy? (ii) it is acceptable to Sujata.

24. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

25. The probability of selecting a green marble at random from a jar that contains only green, white and yellow marbles is $\frac{1}{4}$. The probability of selecting a white marble at random from the same jar is $\frac{1}{3}$. If this jar contains 10 yellow marbles, what is the total number of marbles in the jar.

26. What is the probability that a leap year has 53 Sundays?

27. Suppose you drop a die at random on the rectangular region shown in figure. What is the probability that it will lend inside the circle with diameter 1 m?



28. A purse contains 10 five hundred rupee note, 20 hundred rupee notes, 30 fifty rupee note and 40 ten rupee note. If it is likely that one of the notes will fall out when the purse turns upside. What is the probability that the note

(i) will be a fifty rupee note (ii) will not be a five hundred rupee note.

29. In a musical chair game, the person playing the music has been advised to stop playing the music at any time within three minutes after she starts playing. What is the probability that the music will stop within the first half minute after starting?

30. There are 44 students in class X of a school of whom 32 are boys and 12 are girls. The class teacher has to selected one student as a class representative. He writes the name of each student on a separate card, the cards being identical. Then he puts cards in a bag and stir them thoroughly. He then drawn one card from the bag. What is the probability that the name written on the card is the name of

(i) a girl? (ii) a boy?

- **Subjective type Question**

1. (a)

Sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(b)

+	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2	3	4	4	5	5	8
3	4	5	5	6	6	9
3	4	5	5	6	6	9
6	7	8	8	9	9	12

2. (iii), because selection depends on number of factors, (constraints) **3.** (i)- (d), (ii)- (c), (iii)- (a), (iv)- (b)

4. (a) 0.15, (b) 0.4 **5.** $\frac{1}{2}$ **6.** (a) (i) $\frac{1}{2}$, (ii) $\frac{1}{2}$, (iii) $\frac{1}{4}$, (iv) $\frac{3}{4}$, (v) $\frac{3}{4}$, (vi) $\frac{1}{4}$, (b) $\frac{3}{4}$

7. (a) (i) $\frac{3}{8}$, (ii) $\frac{3}{8}$, (iii) $\frac{1}{8}$, (iv) $\frac{1}{2}$: (b) (i) $\frac{7}{8}$, (ii) $\frac{7}{8}$ **8.** (a) (i) $\frac{1}{2}$, (ii) $\frac{1}{2}$, (iii) $\frac{1}{2}$; (b) (i) $\frac{1}{3}$, (i) $\frac{1}{6}$

9. (i) $\frac{25}{36}$, (ii) $\frac{11}{36}$, **10.** (i) $\frac{1}{9}$, (ii) $\frac{5}{12}$, (iii) $\frac{1}{4}$, **11.** (i) $\frac{1}{9}$, (ii) $\frac{1}{12}$, (iii) $\frac{25}{36}$,

13. (a) (i) $\frac{1}{22}$, (ii) $\frac{9}{22}$, (iii) $\frac{1}{11}$, (iv) $\frac{3}{22}$; (b) (i) $\frac{1}{10}$, (ii) $\frac{9}{10}$, **14.** (i) $\frac{1}{5}$, (ii) $\frac{1}{4}$, (b) $\frac{1}{4}$,

15. (a) (i) $\frac{4}{5}$, (ii) $\frac{4}{5}$, (iii) $\frac{7}{15}$;

(b) (i) (1) $\frac{7}{18}$, (2) $\frac{11}{18}$, (3) $\frac{13}{18}$, (4) $\frac{1}{2}$; (ii) (1) $\frac{4}{11}$, (2) $\frac{9}{22}$, (3) $\frac{17}{22}$, (4) $\frac{1}{2}$; (iii) (1) $\frac{1}{3}$, (2) $\frac{1}{3}$, (3) $\frac{1}{3}$;

(iv) (1) $\frac{1}{4}$, (2) $\frac{2}{5}$, (c) $\frac{5}{13}$

16. $\frac{x}{12}, x = 3$ **17.** (a) (i) $\frac{2}{9}$, (ii) $\frac{1}{3}$, (iii) $\frac{4}{9}$; (b) (i) $\frac{3}{4}$, (ii) $\frac{7}{20}$

18. (a) (i) $\frac{1}{2}$, (ii) $\frac{13}{20}$, (iii) $\frac{2}{5}$, (iv) $\frac{9}{10}$; (b) (i) $\frac{11}{35}$, (ii) $\frac{1}{7}$ (iii) $\frac{16}{35}$; (c) (i) $\frac{8}{17}$, (ii) $\frac{7}{17}$, (iii) $\frac{3}{17}$,

19. (i) $\frac{8}{15}$, (ii) $\frac{1}{5}$, (iii) $\frac{1}{5}$, (iv) $\frac{2}{15}$, (v) $\frac{2}{3}$, **20.** (a) $\frac{2}{3}$; (b) (i) $\frac{1}{8}$, (ii) $\frac{1}{2}$, (iii) $\frac{3}{4}$, (iv) 1

21. (i) $\frac{2}{5}$, (ii) $\frac{3}{10}$, (iii) $\frac{8}{25}$ **22.** (i) $\frac{1}{5}$, (ii) $\frac{15}{19}$ **23.** (i) 0.88 (ii) 0.96 **24.** (i) 0.008 **25.** 24 **26.** $\frac{2}{7}$

27. $\frac{\pi}{24}$ **28.** (i) $\frac{3}{10}$, (ii) $\frac{9}{10}$ **29.** $\frac{1}{6}$ **30.** (i) $\frac{3}{11}$, (ii) $\frac{8}{11}$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

1. Mark Question :

1. From a well shuffled pack of cards, a card is drawn at random. Find the probability of getting a black queen. **[Delhi- 2008]**

2. A bag contains 4 red and 6 black balls. A ball is taken out of the bag at random. From the probability of getting a black ball. **[AI-2008]**

3. A die is thrown once. Find the probability of getting a number less than 3. **[Foreign-2008]**

4. Cards bearing numbers 3 to 20 are placed in a bag and mixed thoroughly. A card is taken out from the bag at random. What is the probability that the number on the card taken out is an even number? **[Delhi-2008 C]**

5. Two friends were born in the year 2000. What is the probability that they have the same birthday? **[AI-2008 C]**

OR

Two coins are tossed simultaneously. Find the probability of getting exactly one head. **[AI-2008]**

2 Mark Question :

1. A die is thrown once. Find the probability of getting **[Delhi-2008]**

(i) A prime number

(ii) A number divisible by 2.

2. Cards, carked with numbers 5 to 50, are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken card is :

(i) A prime number less than 10.

(ii) A number which is a perfect square

3. A pair of dice is thrown once. Find the probability of getting the same number on each dice. **[Foreign-2008]**

4. A bag contains 5 red, 4 blue and 3 green balls. A ball is taken out of the bag at random. Find the probability that the selected ball is (i) of red colour (ii) not of green colour.

OR

A card is drawn at random from a well-shuffled deck of playing cards. Find the probability of drawing a

(i) face card (ii) card which is neither a king nor a red card. **[Delhi-2008 C]**

5. Two dice are thrown simultaneously. Find the probability that the sum of the two numbers appearing on the top is less than or equal to 10.

OR

The king, queen and jack of diamonds are removed from a pack of 52 cards and than the pack is well shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (i) diamonds (ii) a jack. **[Al-2008 C]**

3 Marks Question :

1. A bag contains 5 black, 7 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) red (ii) black or white (iii) not black. **[Delhi-2004]**

2. A bag contains 7 black, 8 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) red (ii) black or white (iii) not black. **[Delhi-2004]**

3. A bag contains 6 black, 7 red and 2 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) red (ii) black or white (iii) not black. **[Delhi-2004]**

4. A bag contains 4 red, 5 black and 6 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) white (ii) red (iii) not black (iv) red or white. **[Al-2004]**

5. A bag contains 4 red, 5 black and 6 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) white (ii) red (iii) not black (iv) red or white. **[Al-2004]**

6. A bag contains 3 red, 5 black and 7 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) white (ii) red (iii) not black (iv) red or white. **[Al-2004]**

7. A bag contains 6 red, 5 black and 4 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) white (ii) red (iii) not black (iv) red or white. **[Al-2004]**

8. 15 cards, numbered 1, 2, 3,…., 15 are put in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the card drawn bears (i) an even number (ii) a number divisible by 2 or 3.**[Al-2004]**

9. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither an ace nor a king. **[Foreign-2004]**

10. Out of 400 bulbs in a box, 15 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective.

OR

Find the probability of getting 53 Fridays in a leap year. **[Al-2004 C]**

11. A bag contains 8 red, 6 white and 4 black balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i)red or white (ii) not black (iii) neither white nor black **[Al-2005]**

12. A bag contains 5 white balls, 7 red balls, 4 black balls, and 2 blue balls. Out ball is drawn at random from the bag. What is the probability that the ball drawn is :
(i) white or blue (ii) red or black (iii) not white (iv) neither white nor black **[Delhi-2006]**

13. A card is drawn at random from a well shuffled deck of playing cards. Find the probability that the card drawn is :
(i) a king or a jack (ii) a non ace (iii) a red card (iv) neither a king nor a queen **[Delhi-2006]**

14. A card is drawn at random from a well shuffled deck of playing cards. Find the probability that the card drawn is :
(i) a card of spade or an ace (ii) a red king (iii) neither a king nor a queen (iv) either a king or queen **[Delhi-2006]**

15. A box contains 19 balls bearing numbers 1, 2, 3, ….., 19. A ball is drawn at random from the box. What is the probability that the number of the ball is (i) a prime number (ii) divisible by 3 or 5 (iii) neither divisible by 5 nor by 10 (iv) an even number. **[Delhi-2006 C]**

16. Find the probability that a number selected at random from the numbers, …., 35 is a (i) prime number (ii) multiple of 7 (iii) multiple of 3 or 5. **[Delhi-2006 C]**

17. From a pack of 52 playing cards, jacks, queens, kings and aces of red colour are removed. From the remaining cards, a card is drawn at random. Find the probability that the card drawn is : (i) a black queen (ii) a red card (iii) a black jack (iv) a picture card (jacks, queens and kings are picture cards.) **[AI-2006 C]**

18. Cards marked with numbers 3, 4, 5, …, 50 are placed in a box and mixed thoroughly. One cards is drawn at random from the box. Find the probability that number on the drawn card is (i) divisible by 7 (ii) a number which is a perfect square. **[Delhi-2007]**

19. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue from the bag is thrice that of a red ball, find the number of blue balls in the bag. **[Delhi-2007]**

20. A box contains 5 red balls, 4 green balls and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is : (i) white (ii) neither red nor white. **[AI-2006]**

21. All the three face cards of spades are removed from a deck of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting a card of (i) a black face card (ii) a queen (iii) a black card. **[AI-2009]**

22. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (i) heart (ii) queen (iii) clubs. **[Delhi-2009]**

23. Two dice are thrown simultaneously. What is the probability that
(i) 5 will not come up on either of them?
(ii) 5 will come up on at least one?
(iii) 5 will come up at both dice? **[AI-2009]**

OR

A box has cards numbered 14 to 99. Cards are mixed thoroughly and a card is drawn from the bag at random. Find the probability that the number on the card, drawn from the box is :
(i) an odd number
(ii) a perfect square number
(iii) a number divisible by 7. **[Foreign-2009]**

SURFACE AREAS AND VOLUMES	ANSWER KEY	EXERCISE-2 (X)-CBSE

1 Mark :

1. $\frac{1}{26}$ **2.** $\frac{3}{5}$ **3.** $\frac{1}{3}$ 4. $\frac{1}{2}$ **5.** $\frac{1}{366}$ OR $\frac{1}{2}$

2 Marks :

1. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ **2.** (i) $\frac{1}{23}$ (ii) $\frac{5}{46}$ 3. $\frac{1}{6}$ **4.** (i) $\frac{5}{12}$ (ii) $\frac{3}{4}$ OR (i) $\frac{3}{13}$ (ii) $\frac{6}{13}$ **5.** $\frac{11}{12}$ OR (i) $\frac{10}{49}$, (ii) $\frac{3}{49}$

3 Marks :

1. (i) $\frac{7}{15}$ (ii) $\frac{8}{15}$ (iii) $\frac{2}{3}$ 2. (i) $\frac{1}{3}$ (ii) $\frac{2}{3}$ (iii) $\frac{8}{15}$ 3. (i) $\frac{7}{15}$ (ii) $\frac{8}{15}$ (iii) $\frac{3}{5}$ 4. (i) $\frac{2}{5}$ (ii) $\frac{4}{15}$ (iii) $\frac{2}{3}$ (iv) $\frac{2}{3}$

5. (i) $\frac{2}{5}$ (ii) $\frac{4}{15}$ (iii) $\frac{2}{3}$ (iv) $\frac{2}{3}$ 6. (i) $\frac{7}{15}$ (ii) $\frac{1}{5}$ (iii) $\frac{2}{3}$ (iv) $\frac{2}{3}$ 7. (i) $\frac{4}{15}$ (ii) $\frac{2}{5}$ (iii) $\frac{2}{3}$ (iv) $\frac{2}{3}$ 8. (i) $\frac{7}{15}$ (ii) $\frac{2}{3}$

9. $\frac{11}{13}$ 10. $\frac{77}{80}$ OR $\frac{2}{7}$ 11. (i) $\frac{7}{9}$ (ii) $\frac{7}{9}$ (iii) $\frac{4}{9}$ 12. (i) $\frac{7}{18}$ (ii) $\frac{11}{18}$ (iii) $\frac{13}{18}$ (iv) $\frac{1}{2}$ 13. (i) $\frac{2}{13}$ (ii) $\frac{12}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{11}{13}$

14. (i) $\frac{4}{13}$ (ii) $\frac{1}{26}$ (iii) $\frac{11}{13}$ (iv) $\frac{2}{13}$ 15. (i) $\frac{8}{19}$ (ii) $\frac{8}{19}$ (iii) $\frac{16}{19}$ (iv) $\frac{9}{19}$ 16. (i) $\frac{11}{35}$ (ii) $\frac{1}{7}$ (iii) $\frac{16}{35}$ 17. (i) $\frac{1}{22}$ (ii)

$\frac{9}{22}$ (iii) $\frac{1}{22}$ (iv) $\frac{3}{22}$ 18. (i) $\frac{1}{8}$ (ii) $\frac{5}{48}$ 19. 15 20. (i) $\frac{7}{16}$ (ii) $\frac{1}{4}$ 21. (i) $\frac{3}{49}$ (ii) $\frac{3}{49}$ (iii) $\frac{23}{49}$ 22. (i) $\frac{13}{49}$ (ii) $\frac{3}{49}$ (iii) $\frac{23}{49}$ **23.** (i) $\frac{25}{36}$ (ii) $\frac{11}{36}$ (iii) $\frac{1}{36}$ OR (i) $\frac{1}{2}$ (ii) $\frac{4}{43}$ (iii) $\frac{13}{86}$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

1. Find the probability of getting a head in a throw of a coin.

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) None of these

Directions (for Q. No. 2-5) : Two fair coins are tossed simultaneously. Find the probability of

2. Getting only one head

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

3. Getting two heads

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{8}$

4. Getting at least two heads

(A) $\frac{7}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{4}{5}$

5. Getting at least two heads

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 1

Directions (for Q. No. 6-12) : Three fair coins are tossed simultaneously. Find the probability of

6. Getting one head

(A) 0 (B) $\frac{3}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{8}$

7. Getting one tail

(A) 1 (B) $\frac{1}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{8}$

8. Getting at least one heads

(A) $\frac{7}{8}$ (B) $\frac{1}{8}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$

9. Getting two heads

(A) $\frac{3}{5}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{2}{5}$

10. Getting two heads

(A) $\frac{3}{8}$ (B) $\frac{7}{8}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

11. Getting at least one head and one tail

(A) $\frac{2}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{10}$ (D) $\frac{4}{3}$

12. Getting more heads than the number of tails

(A) 2 (B) $\frac{7}{8}$ (C) $\frac{5}{8}$ (D) $\frac{1}{2}$

13. Getting a number less than 7 but greater than 0

(A) 0 (B) $\frac{3}{4}$ (C) 1 (D) $\frac{7}{8}$

14. Getting a multiple of 3.

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{5}{6}$ (D) None of these

15. Getting a prime number

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{5}{7}$ (D) $\frac{5}{8}$

16. Getting an even number

(A) $\frac{1}{2}$ (B) $\frac{4}{5}$ (C) $\frac{5}{7}$ (D) $\frac{5}{8}$

Directions (for Q. No. 17 and 18) : A coin is tossed successively three times. Find the probability of

17. Getting exactly one head or two heads.

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{8}$

18. Getting no heads.

(A) 0 (B) 1 (C) $\frac{1}{8}$ (D) $\frac{7}{8}$

Directions (for Q. No. 19-27) : Two dice are rolled simultaneously. Find the probability of

19. Getting a total of 9

(A) $\frac{1}{3}$ (B) $\frac{1}{9}$ (C) $\frac{8}{9}$ (D) $\frac{9}{10}$

20. Getting a sum greater than 9

(A) $\frac{10}{11}$ (B) $\frac{5}{6}$ (C) $\frac{1}{6}$ (D) $\frac{8}{9}$

21. Getting a total of 9 or 11

(A) $\frac{2}{99}$ (B) $\frac{20}{99}$ (C) $\frac{1}{6}$ (D) $\frac{1}{10}$

22. Getting a doublet

(A) $\frac{1}{12}$ (B) 0 (C) $\frac{5}{8}$ (D) $\frac{1}{6}$

23. Getting a doublet of even number

(A) $\frac{5}{8}$ (B) $\frac{1}{12}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$

24. Getting a multiple of two on one die and a multiple of three on the other.

(A) $\frac{15}{36}$ (B) $\frac{25}{36}$ (C) $\frac{11}{36}$ (D) $\frac{5}{6}$

25. Getting the sum of numbers on the two faces divisible by 3 or 4.n even number

(A) $\frac{4}{9}$ (B) $\frac{1}{7}$ (C) $\frac{5}{9}$ (D) $\frac{7}{12}$

26. Getting the sum as a prime number

(A) $\frac{3}{5}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

27. Getting atleast one 5.

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{5}{36}$ (D) $\frac{11}{36}$

Directions (for Q. No. 28-35) : One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to drawn. Find the probability that

28. The card drawn is black

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{8}{13}$ (D) can't be determined

29. The card drawn is a queen

(A) $\frac{1}{12}$ (B) $\frac{1}{13}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

30. The card drawn is black and a queen

(A) $\frac{1}{13}$ (B) $\frac{1}{52}$ (C) $\frac{1}{26}$ (D) $\frac{5}{26}$

31. The card drawn is either black or a queen

(A) $\frac{15}{26}$ (B) $\frac{13}{17}$ (C) $\frac{7}{13}$ (D) $\frac{5}{26}$

32. The card drawn is either king or a queen

(A) $\frac{5}{26}$ (B) $\frac{1}{13}$ (C) $\frac{2}{13}$ (D) $\frac{12}{13}$

33. The card drawn is either a heart, a king or a queen

(A) $\frac{17}{52}$ (B) $\frac{21}{52}$ (C) $\frac{19}{52}$ (D) $\frac{9}{26}$

34. The card drawn is neither a spade nor a king

(A) 0 (B) $\frac{9}{13}$ (C) $\frac{1}{2}$ (D) $\frac{4}{13}$

35. The card drawn is neither a ace nor a king

(A) $\frac{11}{13}$ (B) $\frac{1}{2}$ (C) $\frac{2}{13}$ (D) $\frac{11}{26}$

36. From a well shuffled pack of 52 cards, three cards are drawn at random. Find the probability of drawing an ace, a king and a jack.

(A) $\frac{16}{5525}$ (B) $\frac{16}{625}$ (C) $\frac{16}{3125}$ (D) None of these

37. Four cards are drawn at random from a pack of 52 cards. Find the probability of getting all the four cards of same number.

(A) $\frac{17}{1625}$ (B) $\frac{1}{20825}$ (C) $\frac{7}{25850}$ (D) None of these

38. From a well shuffled pack of 52 cards, four cards are accidently dropped. Find the probability that one card is missing from each suit.

(A) $\frac{17}{20825}$ (B) $\frac{2197}{20825}$ (C) $\frac{197}{1665}$ (D) None of these

39. Four cards are drawn at random from a pack of 52 cards. Find the probability of getting all the four cards different number.

(A) $\frac{141}{4165}$ (B) $\frac{117}{833}$ (C) $\frac{264}{4165}$ (D) None of these

Directions (for Q. No. 40-13) : Four dice are thrown simultaneously. Find the probability that

40. All of them show the same face.

(A) $\frac{1}{216}$ (B) $\frac{15}{16}$ (C) $\frac{15}{36}$ (D) $\frac{1}{2}$

41. All of them show the different face.

(A) $\frac{3}{28}$ (B) $\frac{5}{18}$ (C) $\frac{15}{36}$ (D) $\frac{11}{36}$

42. Two of them show the same face and remaining two show the different faces.

(A) $\frac{4}{9}$ (B) $\frac{5}{9}$ (C) $\frac{11}{18}$ (D) $\frac{7}{9}$

43. At least two of them show the same face.

(A) $\frac{37}{72}$ (B) $\frac{11}{66}$ (C) $\frac{47}{72}$ (D) $\frac{25}{36}$

44. What is the probability that the number selected from the numbers 1, 2, 3, ….., 20, is a prime number when each of the given numbers is equally likely to be selected ?

(A) $\frac{7}{10}$ (B) $\frac{2}{15}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

45. Tickets numbered from 1 to 18 are mixed up together and then a ticket is drawn at random. Find the probability that the ticket has a number which is a multiple of 2 or 3.

(A) $\frac{1}{3}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$

46. In a lottery of 100 tickets numbered 1 to 100, two tickets are drawn simultaneously. Find the probability that both the tickets drawn have prime numbers.

47. In the previous question, find the probability that none of the tickets drawn has a prime number.

(A) $\frac{29}{66}$ (B) $\frac{17}{33}$ (C) $\frac{37}{66}$ (D) $\frac{17}{50}$

48. Find the probability that a leap year selected at random will contain 53 Sundays.

(A) $\frac{5}{7}$ (B) $\frac{3}{4}$ (C) $\frac{4}{7}$ (D) $\frac{2}{7}$

Directions (for Q. No. 49-53) : A bag contains 8 red and 4 green balls. Find the probability that

49. The ball drawn is red when one ball is selected at random.

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{5}{6}$

50. All the 4 balls drawn are red when 4 balls drawn at random.

(A) $\frac{17}{32}$ (B) $\frac{14}{99}$ (C) $\frac{7}{12}$ (D) None of these

51. All the 4 balls drawn are green when 4 balls drawn at random.

(A) $\frac{1}{495}$ (B) $\frac{7}{99}$ (C) $\frac{5}{12}$ (D) None of these

52. Two balls are red and one ball is green when three balls are drawn at random.

(A) $\frac{56}{99}$ (B) $\frac{112}{495}$ (C) $\frac{78}{495}$ (D) None of these

53. Three balls are drawn and none of them is red.

(A) $\frac{68}{99}$ (B) $\frac{7}{99}$ (C) $\frac{4}{495}$ (D) None of these

54. The odds in favor of an event are 2:7. find the probability of occurrence of this event.

(A) $\frac{2}{9}$ (B) $\frac{5}{12}$ (C) $\frac{7}{12}$ (D) None of these

55. The odds against of an event are 5:7. find the probability of occurrence of this event.

(A) $\frac{3}{8}$ (B) $\frac{7}{12}$ (C) $\frac{2}{7}$ (D) None of these

56. If there are two children in a family, find the probability that there is atleast one girl in the family.

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) None of these

57. From a group of 3 men and 2 women, two persons are selected at random. Find the probability that at least one woman is selected.

(A) $\frac{1}{5}$ (B) $\frac{7}{10}$ (C) $\frac{2}{5}$ (D) None of these

58. A box contains 5 defective and 15 non-defective bulbs. Two bulbs are chosen at random. Find the probability that both the bulbs are non-defective

(A) $\frac{5}{19}$ (B) $\frac{3}{20}$ (C) $\frac{21}{38}$ (D) None of these

59. In the previous question, find the probability that at least 3 bulbs are defective when 4 bulbs are selected at random.

(A) $\frac{31}{969}$ (B) $\frac{7}{20}$ (C) $\frac{1}{20}$ (D) None of these

OBJECTIVE — ANSWER KEY

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	A	B	B	C	D	D	A	B	C	D	D	C	B	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	A	B	C	B	C	C	D	B	C	C	B	D	A	B	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	C	C	C	B	A	A	B	B	C	A	B	B	C	C	C
Que.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	
Ans.	A	C	D	A	B	A	B	C	A	B	C	B	C	A	

QUADRATIC EQUATIONS

★ INTRODUCTION

When a polynomial f(x) is equated to zero, we get an equation which is known as a polynomial equation. If f(x) is a linear polynomial than f(x) = 0 is called a linear equation. For example, 3x – 2 = 0, 4t + $\frac{3}{5}$ = 0 etc. are linear equations. If f(x) is quadratic polynomial i.e., f(x) = ax^2 + bc + c, a $\neq$ 0, then f(x) = 0 i.e., ax_2 + bx + c = 0, a $\neq$ 0 is called a quadratic equation. Such equations arise in many real life situations. In this chapter, we will learn about quadratic and various ways of finding their zeros or roots. In the end of the chapter, we will also discuss some applications of quadratic equations in daily life situations.

★ HISTORICAL FACTS

On clay tables dated between 1800 BC and 1600 BC, the ancient Babylonians left the earliest evidence of the discovery of quadratic equations, and also gave early methods for solving them.
Indian mathematician Baudhayana who wrote a Sulba Sutra in ancient India circa 8th century BC first used quadratic equations of the form : $ax^2 = c$ and $ax^2 + bx = c$ and also gave methods for solving them.
Babylonian mathematicians from circa 400 BC and Chinese mathematicians from circa 200 BC used the method of completing the square to solve quadratic equations with positive roots, but did not have a general formula.
Euclid, a Greek mathematician, produced a more abstract geometrical method around 300 BC.
The first mathematician to have found negative solutions with the general algebraic formula was Brahmagupta (India, 7th century). He gave the first explicit (although still not completely general) solutions of the quadratic equations $ax^2 + bx = c$ as follows :
"To the absolute number multiplied by four times the [coefficient of the] square, add the square of the [coefficient of the] middle term; the square root of the same, less the [coefficient of the] middle term, being divided by twice the [coefficient of the] square is the value."

This is equivalent to : $$x = \frac{\sqrt{4ac + b^2} - b}{2a}$$

Muhammad ibn Musa al-Kwarizmi (Persia, 9th century) developed a set of formulae that worked for positive solutions.
Bhaskara II (1114-1185), an Indian mathematician-astronomer, solved quadratic equations with more than one unknown and is considered the originator of the equation.
Shridhara (India, 9th century) was one of the first mathematicians to give a general rule for solving a quadratic equation.

★ QUADRATIC EQUATIONS

A polynomial equations of degree two is called a quadratic equation.
Ex. $2x^2 - 3x + 1 = 0$, $4x - 3x^2 = 0$ and $1 - x^2 = 0$
General form of quadratic equations : $ax^2 + bx + c = 0$, where a,b,c, are real numbers and a $\neq$ 0.
Moreover, it is general form of a quadratic equation in standard form.
Types of Quadratic Equations : A quadratic equation can be of the following types :
(i) b = 0, c $\neq$ 0 i.e., of the type $ax^2 + c = 0$ **(Pure quadratic equation)**
(ii) b $\neq$ 0,c = 0 i.e., of the type $ax^2 + bx = 0$
(iii) b = 0, c = 0 i.e., of the type $ax^2 = 0$
(iv) b $\neq$ 0,c $\neq$ 0 i.e., of the type $ax^2 + bx + c = 0$ **(Mixed or complete quadratic equation)**
Roots of quadratic equation : x = α is said to be root of the quadratic equation ax2 + bx + c = 0, a $\neq$ 0 iff x = α satisfies the quadratic equation i.e. in other words the value of $a\alpha^2 + b\alpha + c$ is zero.
Solving a quadratic equation : The determination of all the roots of a quadratic equation is called solving the quadratic equation.

Ex.1 Check whether the following are quadratic equations :

(i) $(x + 1)^2 = (x - 3)$ (ii) $(x - 2)(x + 1) = (x - 1)(x + 3)$ (iii) $(x - 3)(2x + 1) = x(x + 5)$

Sol. (i) Here, the given equation is $(x + 1)^2 = 2(x - 3)$

$\Rightarrow$ $x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 2x - 2x + 1 + 6 = 0$

$\Rightarrow$ $x^2 + 7 = 0 \Rightarrow x^2 + 0.x + 7 = 0$, which is of the form $ax^2 + bx + c = 0$

Hence, $(x + 1)^2 = 2(x - 3)$ is a quadratic equation.

(ii) Here, the given equation is $(x - 2)(x + 3) = (x - 1)(x + 3)$

$\Rightarrow$ $x^2 + x - 2x - 2 = x^2 + 3x - x - 3 \Rightarrow x^2 - x^2 - x - 2x - 2 + 3 = 0 \Rightarrow -3 + 1 = 0$,

which is not of the form $ax^2 + bx + c = 0$

Hence, $(x - 2)(x + 1) = (x - 1)$ is not a quadratic equation.

(iii) Here, the given equation is $(x - 3)(2x + 1)\ x\ (x + 5)$

$\Rightarrow$ $2x^2 + x - 6x - 3 = x^2 + 5x \Rightarrow 2x^2 - x^2 - 5x - 5x - 3 = 0 \Rightarrow x^2 - 10x - 3 = 0$,

which is of the form $ax^2 + bx + c = 0$

Hence, $(x - 3)(2x + 1) = x(x + 5)$ is a quadratic equation.

Ex.2 In each of the following, determine whether the given values are the solution of the given equation or not :

(i) $\frac{2}{x^2} - \frac{5}{x} + 2 = 0; x = 5, x = \frac{1}{2}$ (ii) $a^2x^2 - 3abx + 2b^2 = 0; x = \frac{a}{b}, x = \frac{b}{a}$

Sol. (i) Putting x = 5 and $x = \frac{1}{2}$ in the given equation.

$$\frac{2}{(5)^2} - \frac{5}{5} + 2 \text{ and } \frac{2}{\left(\frac{1}{2}\right)^2} - \frac{5}{\left(\frac{1}{2}\right)} + 2$$

$\Rightarrow$ $\frac{2}{25} - 1 + 2 \text{ and } \frac{2}{\frac{1}{4}} - \frac{5}{\frac{1}{2}} + 2$

$\Rightarrow$ $\frac{2}{25} + 1 \text{ and } 8 - 10 + 2 \Rightarrow \frac{27}{25}$ and 0

i.e., x = 5 does not satisfy but $x = \frac{1}{2}$ satisfies the given equation.

Hence, x = 5 is not a solution but $x = \frac{1}{2}$ is a solution of $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$.

(ii) Putting $x = \frac{a}{b}$ and $x = \frac{b}{a}$ in the given equation.

$$a^2\left(\frac{a}{b}\right)^2 - 3ab\left(\frac{a}{b}\right) + 2b^2 \text{ and } a^2\left(\frac{b}{a}\right)^2 - 3ab\left(\frac{b}{a}\right) + 2b^2$$

$\Rightarrow$ $\frac{a^2}{b^2} + 2b^2 - 3a^2$ and 0

i.e., $x = \frac{a}{b}$ does not satisfy but $x = \frac{b}{a}$ satisfies the given equation.

Hence, $x = \frac{b}{a}$ is a solution but $x = \frac{a}{b}$ is not a solution of $a^2x^2 - 3abx + 2b^2 = 0$.

Ex.3 Find the values of p and q for which $x = \frac{3}{4}$ and $x = -2$ are the roots of the equation $px^2 + qx - 6 = 0$.

Sol. Since $x = \frac{3}{4}$ and $x = -2$ are the roots of the equation $px^2 + qx - 6 = 0$.

$\therefore$ $p\left(\frac{3}{4}\right)^2 + q\left(\frac{3}{4}\right) - 6 = 0$ and $p(-2)^2 + q(-2) - 6 = 0$

$\Rightarrow \quad p \times \frac{9}{16} + q \times \frac{3}{4} - 6 = 0$ and 4p – 2q – 6 = 0

$\Rightarrow \quad \frac{9p + 12q - 96}{16} = 0$ and 4p – 2q – 6 = 0

$\Rightarrow$ 9p + 12q – 96 = 0 and 4p – 2q – 6 = 0

$\Rightarrow$ 3p + 4q – 32 = 0 …(i)

and 2p – q – 3 = 0 …(ii)

Multiplying (2) by 4, we get 8p – 4q – 12 = 0 …(iii)

Adding (1) and (3), we get p = 4

Putting the value of p in equation (2), we get

$$23 \times 4 - q - 3 = 0 \Rightarrow q = 5$$

Hence, p = 4, q = 5.

★ METHODS OF SOLVING QUADRATIC EQUATIONS

Solution by factorization method

Algorithm :

Step-I : Factorize the constant term of the given quadratic equation.

Step-II : Express the coefficient of middle term as the sum or difference of the factors obtained in step-I. Clearly, the product of these two factors will be equal to the product of the coefficient of x^2 and constant term.

Step-III : Split the middle term in two parts obtained in stop-II

Step-IV : Factorize the quadratic equation obtained in step-III by grouping method.

Ex.4 Solve the following quadratic equation by factorization method $x^2 - 2ax + a^2 - b^2 = 0$

Sol. Factors of the constant term $a^2 - b^2$ are(a – b)&(a + b) also coefficient of the middle term = – 2a = –[(a – b)+(a + b)]

$\Rightarrow \quad x^2 - 2ax + a^2 - b^2 = 0$

$\Rightarrow \quad x^2 - \{(a - b) + (a + b)\} x + (a + b)(a - b) = 0$

$\Rightarrow \quad x^2 - (a - b)x - (a + b)x + (a - b)(a + b) = 0$

$\Rightarrow \quad x[x - (a - b)] - (a + b)[x - (a - b)] = 0$

$\Rightarrow \quad [x - (a - b)][x - (a + b)] = 0$

x – (a – b) = 0 or x – (a + b) = 0

x = a – b, x = a + b

Ex.5 Solve the quadratic equation $5x^2 = -16x - 12$ by factorization method.

Sol. $5x^2 = -16x - 12$

$5x^2 + 16x + 12 = 0$

$5x^2 + 10x + 6x + 12 = 0$

5x (x + 2) + 6 (x + 2) = 0

(x + 2) (5x + 6) = 0

x + 2 = 0 $\Rightarrow x = -2$

5x + 6 = 0 $\Rightarrow x = \frac{-6}{5}$

Solution by factorization method

Algorithm :

Stop-I : Obtain the quadratic equation. Let the quadratic equation be $ax^2 + bx + c = 0$, $a \neq 0$.

Stop-II : Make the coefficient of x^2 unite by dividing throughout by it, if it is not unity that is obtain

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Stop-III : Shift the coefficient term $\frac{c}{a}$ on R.H.S. to get $x^2 + \frac{b}{a} = -\frac{c}{a}$

Stop-IV : Add square of half of the coefficient of x. i.e., $\left(\frac{b}{2a}\right)^2$ on both sides to obtain.

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Stop-V : Write L.H.S. as the perfect square and simplify R.H.S. to get $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Stop-VI : Take square root of both sides to get $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Stop-VII : Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on R.H.S. i.e., $x = -\frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Ex.6 Solve : $9x^2 - 15x + 6 = 0$

Sol. Here, $9x^2 - 15x + 6 = 0$

$\Rightarrow$ $x^2 - \frac{15}{9}x + \frac{6}{9} = 0$ [Dividing throughout by 9]

$\Rightarrow$ $x^2 - \frac{5}{3}x + \frac{2}{3} = 0$

$\Rightarrow$ $x^2 - \frac{5}{3}x - \frac{2}{3} = 0$ [Shifting the constant term on RHS]

$\Rightarrow$ $x^2 - 2\left(\frac{5}{6}\right)x + \left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^2 - \frac{2}{3}$ [Adding square of half of coefficient x on both sides]

$\Rightarrow$ $\left(x - \frac{5}{6}\right)^2 = \frac{25}{36} - \frac{2}{3} \Rightarrow \left(x - \frac{5}{6}\right)^2 = \frac{25-24}{36} \Rightarrow \left(x - \frac{5}{6}\right)^2 = \frac{1}{36}$

$\Rightarrow$ $x - \frac{5}{6} = \pm\frac{1}{6}$ [Taking square root of both sides]

$\Rightarrow$ $x = \frac{5}{6} = \pm\frac{1}{6}$ $\Rightarrow$ $x = \frac{5}{6} + \frac{1}{6} = 1$ or, $x = \frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

$\Rightarrow$ $x = 1$ or, $x = \frac{2}{3}$

Ex.7 Solve the equation $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$ by the method of completing the square.

Sol. We have,

$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

$\Rightarrow$ $x^2 - (\sqrt{3}+1)x = -\sqrt{3}$

$\Rightarrow$ $x^2 - 2\left(\frac{\sqrt{3}+1}{2}\right)x + \left(\frac{\sqrt{3}+1}{2}\right)^2 = -\sqrt{3} + \left(\frac{\sqrt{3}+1}{2}\right)^2$

$\Rightarrow$ $\left(x - \frac{\sqrt{3}+1}{2}\right)^2 = \frac{-4\sqrt{3} + (\sqrt{3}+1)^2}{4}$

$\Rightarrow \quad \left(x - \frac{\sqrt{3}+1}{2}\right)^2 = \left(\frac{\sqrt{3}-1}{2}\right)^2 \qquad \Rightarrow x - \frac{\sqrt{3}+1}{2} = \pm\frac{\sqrt{3}-1}{2}$

$\Rightarrow \quad x - \frac{\sqrt{3}+1}{2} \pm \frac{\sqrt{3}-1}{2} \qquad \Rightarrow x = \sqrt{3}, 1$

Hence, the roots are $\sqrt{3}$ and 1.

Solution by Quadratic Formula "Sreedharacharya's Rule"

Consider quadratic equation $ax^2 + bx + c = 0, a \neq 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ The roots of x are

$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \frac{-b + \sqrt{D}}{2a}$ or, $x = \frac{-b - \sqrt{D}}{2a}$, where $D = b^2 - 4ac$

Thus, if $D = b^2 - 4ac \geq 0$, then the quadratic equation ax2 + bx + c = 0 has real roots α and β given by

$\alpha = \frac{-b + \sqrt{D}}{2a}$ and $\beta = \frac{-b - \sqrt{D}}{2a}$

Discriminant : If $ax^2 + bx + c = 0, a \neq 0 (a, b, c \in R)$ is a quadratic equation, then the expression $b^2 - 4ac$ is known as its discriminant and is generally denoted by D or Δ.

Ex.8 Solve the quadratic equation $x^2 - 6x + 4 = 0$ by using quadratic formula (Sreedharacharya's Rule).

Sol. On comparing the given equation $x^2 - 6x + 4 = 0$ with the standard quadratic equation $ax^2 + bx + c = 0$, we get a = 1, b = – 6, c = 4

Hence the required roots are

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)} = \frac{6 \pm \sqrt{36-16}}{2} = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm \sqrt{4 \times 5}}{2}$$

$$= \frac{2(3 \pm \sqrt{5})}{2} = 3 \pm \sqrt{5}$$

COMPETITION WINDOW

SOLUTIONS OF EQUATIONS REDUCIBLE TO QUADRATIC FORM

Equations which are not quadratic at a glance but can be reduced to quadratic equations by suitable transformations Some of the common types are :

Type-I : $ax^4 + bx^2 + c = 0$

This can be reduced to a quadratic equation by substituting $x^2 = y$ i.e., $ay^2 + by + c = 0$

e.g. Solve $2x^4 - 5x^2 + 3 = 0$

Putting $x^2 = y$, we get $2y^2 - 5y + 3 = 0$

$\Rightarrow \quad (2y-3)(y-1) = 0 \Rightarrow y = \frac{3}{2}$ or 1

$\Rightarrow \quad (2y-3)(y-1) = 0 \Rightarrow y = \frac{3}{2}$ or 1

Type-II : aip(x)$\}^2$ + b.p (x) + c = 0 where p(x) is an expression in 'x'

Put p(x) = y, $\{p(x)\}^2 = y^2$ to get the quadratic equation $ay^2 + by + c = 0$.

e.g. Solve $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0, x \in R$

Putting $x^2 + 3x = y$, we get y2 – y – 6 = 0

Solving, we get y = 3 or – 2

$\Rightarrow \quad x^2+3x=3$ or $x^2+3x=-2$

$\Rightarrow \quad x\frac{-3\pm\sqrt{21}}{2}$ or $x=-2$ or -1.

Type-III : ap$(x)+\frac{b}{p(x)}=c$, where p(x) is an expression in x.

Put p(x) = y to obtain the quadratic equation $ay^2 - cy + b = 0$.

e.g. Solve $\frac{x}{x+1}+\frac{x+1}{x}=\frac{34}{15}$

Putting $\frac{x}{x+1}=y$, we get, $y=\frac{1}{y}=\frac{34}{15}$

$\Rightarrow \quad 15y^2-34y+15=0 \Rightarrow y=\frac{5}{3}$ or $\frac{3}{5}$

$\Rightarrow \quad \frac{x}{x+1}=\frac{5}{3}$ or $\frac{x}{x+1}=\frac{3}{5} \Rightarrow x=\frac{-5}{2}$ or $\frac{3}{2}$

Type-IV : (i) $a\left[x^2+\frac{1}{x^2}\right]+b\left[x+\frac{1}{x}\right]+c=0$ **(ii)** $a\left[x^2+\frac{1}{x^2}\right]+b\left[x-\frac{1}{x}\right]+c=0$

If the coefficient of b in the given equation contains $x+\frac{1}{x}$, then replace $x^2+\frac{1}{x^2}$ by $\left(x^2+\frac{1}{x^2}\right)^2-2$ and put $x+\frac{1}{x}=y$. In case the coefficient of b is $x-\frac{1}{x}$, then replace $x^2+\frac{1}{x^2}$ by $\left(x-\frac{1}{x}\right)^2+2$ and put $x-\frac{1}{x}=y$.

e.g. Solve $9\left[x^2+\frac{1}{x^2}\right]-9\left[x+\frac{1}{x}\right]-52=0$

Putting $x+\frac{1}{x}=y$, we get : $9(y^2-2)-9y-52=0$

$\Rightarrow \quad y=\frac{10}{3}$ or $y=-\frac{7}{3} \quad \Rightarrow x+\frac{1}{x}=\frac{10}{3}$ or $x+\frac{1}{x}=-\frac{7}{3}$

$\Rightarrow \quad x=\frac{1}{3}$ or 3 or $x=\frac{-7\pm\sqrt{13}}{6}$

Type-V : (x + a) (x + b) (x + c) (x + d) + k = 0, such that a + b = c + d.

Rewrite the equation in the form

{(x + a) (x + b)} . {(x + c) (x + d)} + k = 0

Put $x^2 + x(a + b) = x^2 + x(c + d) = y$ to obtain a quadratic equation in y i.e. (y + ab) (y + cd) = k.

e.g. Solve (x + 1) (x + 2) (x + 3) (x + 4) = 120

$\because$ 1 + 4 = 2 + 3, we write the equation in the following form :

{(x + 1) (x + 4)} . {(x + 2) (x + 3)} = 120

$\Rightarrow$ $(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$

Putting $x^2 + 5x = y$, we get (y + 4) (y + 6) = 120

$\Rightarrow$ y = – 16 or 6

$\Rightarrow$ $x^2 + 5x = -16$ or $x^2 + 5x = 6$

$\Rightarrow$ x = – 6 or 1 ($x^2 + 5x + 16$ has no real solution)

Type-VI : $\sqrt{ax+b}=(cx+d)$

Square both sides to obtain $(ax + b) = (cx + d)^2$

or $c^2x^2 + (2cd - a)x + d^2 - b = 0$

Reject those values of x, which do not satisfy both $ax+b\geq 0$ and $cx+d\geq 0$

e.g. Solve : $\sqrt{2x+9}+x=13$

$\Rightarrow$ $(2x + 9) = (13 - x)^2$ (on squaring both sides)

$\Rightarrow$ $x^2 - 28x + 160 = 0$

$\Rightarrow$ x = 20 or 8

x = 20 does not satisfy $2x+9 \geq 0$. So, x = 8 is the only root.

Type-VII : $\sqrt{ax^2+bx+c} = dx+e$

Square both sides to obtain the quadratic equation $x^2 (a - d^2) + x (b - 2de) + (c - e^2) = 0$. solve it and reject those value of x which do not satisfy ax2 + bx + c $\geq$ 0 and dx + e $\geq$ 0.

e.g. Solve : $\sqrt{3x^2+x+5} = x-3$

$\Rightarrow$ $3x^2 + x + 5 = (x - 3)^2$ (On squaring both sides)

$\Rightarrow$ $2x^2+7x-4=0 \Rightarrow x=\frac{1}{2}$ or -4

No value of x satisfy $3x^2+x+5 \geq 0$ and $x-3 \geq 0$

Type-VIII : $\sqrt{ax+b} \pm \sqrt{cx+d} = e$

Square both sides and simplify in such a manner that the expression involving radical sing on one side and all other terms are on the other side. square both sides of the equation thus obtained and simplify it to obtain a quadratic in x. Reject these values which do not satisfy ax + b $\geq$ 0 and cx + d $\geq$ 0.

e.g. Solve : $\sqrt{4-x} + \sqrt{x+9} = 5$

$\Rightarrow$ $\sqrt{4-x} = 5 - \sqrt{x+9}$

$\Rightarrow$ $x+15 = 5\ \sqrt{x+9}$ (on squaring both sides)

$\Rightarrow$ $(x+15)^2 = 25\ \sqrt{x+9}$ (on squaring both sides)

$\Rightarrow$ x = 0 or – 5

Clearly, x = 0 and x = – 5 satisfy 4 – x $\geq$ 0 and x + 9 $\geq$ 0.

Hence, the roots are 0 and – 5

★ **NATURE OF THE ROOTS OF THE QUADRATIC EQUATION**

Let the quadratic equation be ax2 + bx + c = 0. …(i)

Where a $\neq$ b and a, b, c $\in$ R.

The roots of the given equation are given by $x = \frac{-b \pm \sqrt{D}}{2a}$.

i.e., of α and β are two roots of the quadratic equation (i). Them.

$$\alpha = \frac{-b+\sqrt{D}}{2a} \text{ and } \beta = \frac{-b-\sqrt{D}}{2a}$$

Now, the following cases are possible.

Case-I : When D > 0.

Roots are real and unequal (distinct).

The roots are given by $\alpha = \frac{-b+\sqrt{D}}{2a}$ and $\beta = \frac{-b-\sqrt{D}}{2a}$

Remark : Consider a quadratic equation ax2 + bx+ c = 0. where a,b,c $\in$ Q, a $\neq$ b and D > 0 them :

(i) If D is a perfect square, then roots are rational and unequal.

(ii) If D is not a perfect square, then roots are irrational and unequal. If one root is of the form $p+\sqrt{q}$ (where p is rational and $\sqrt{q}$ is a surd) then the other root will be $p-\sqrt{q}$.

Case-II : When D = 0.

Roots are real and equal and each root $\alpha = \dfrac{-b}{2a} = \beta$

Case-III : When D < 0.

No real roots exist. Both the roots are imaginary.

Remark : If D < 0, the roots are of the form a $\pm$ ib (a,b$\in$R&I = $\sqrt{-1}$). If one root is a + ib, then other root will be a – ib.

e.b. $x^2 - 3x + 12 = 0$ has D = – 39 < 0

$\therefore$ It's roots are, $\alpha = \dfrac{-b+\sqrt{D}}{2a}$ and $\beta = \dfrac{-b-\sqrt{D}}{2a}$

or $\alpha = \dfrac{3+\sqrt{-39}}{2}$ and $\beta = \dfrac{3-\sqrt{-39}}{2}$

or $\alpha = \dfrac{3}{2}+\dfrac{i\sqrt{39}}{2}$ and $\beta = \dfrac{3}{2}-\dfrac{i\sqrt{39}}{2}$

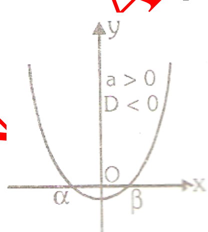
COMPETITION WINDOW

GEOMETRICAL REPRESENTATION OF QUADRATIC EXPRESSION

Consider the quadratic expression, y = ax2 + bx + c, a $\neq$ 0 & a,b,c $\in$ R then :

(i) The graph between x,y is always a parabola. If a > 0, then the shape of the parabola is concave upwards & if a < 0 then the shape of the parabola is concave downwards.

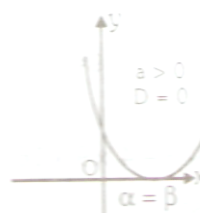
(ii) The graph of y = $ax^2 + bx + c$ can be divided into 6 categories which are as follows :
(Let the roots of the equation $ax^2 + bx + c = 0$ be α and β)



Fig(i)
Roots are real and distinct

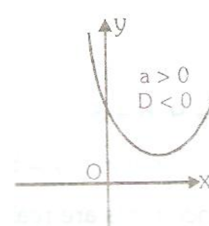
$ax^2+bx+c>0 \forall x \in (-\infty,\alpha)\cup(\beta,\infty)$

$ax^2+bx+c<0 \forall x \in (\alpha,\beta)$



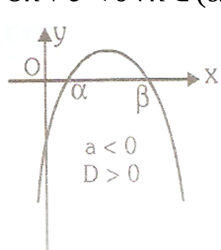
Fig(i)
Roots are coincident

$ax^2+bx+c>0 \forall x \in R-(\alpha)$

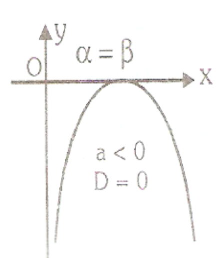


Fig(iii)
Roots are complex conjugates

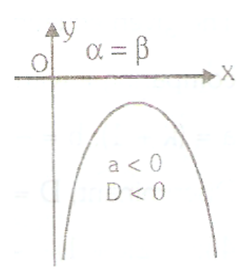
$ax^2+bx+c>0 \forall x \in R$



Fig(iv)



Fig(v)



Fig(vi)

$ax^2 + bx + c < 0 \forall x \in (\alpha, \beta)$ $\quad ax^2 + bx + c > 0 \forall x \in R - (\alpha)$ $\quad ax^2 + bx + c > 0 \forall x \in R$

$ax^2 + bx + c > 0 \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$

Remark : (i) The quadratic expression $ax^2 + bx + c > 0 \ \forall \ x \in R \Rightarrow a > 0, D < 0$ (fig (iii))

(ii) The quadratic expression $ax^2 + bx + c > 0 \ \forall \ x \in R \Rightarrow a > 0, D < 0$ (fig (vi))

Ex.9 Find the nature of the roots of the following equations. If the real roots exist, find them.

(i) $2x^2 - 6x + 3 = 0$ (ii) $2x^2 - 3x + 5 = 0$

Sol. (i) The given equation $2x^2 - 6x + 3 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 2, b = -6$ and $c = 3$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (-6)^2 - 4.2.3 = 36 - 24 = 12 > 0$

$\because$ $D > 0$, roots are real and unequal.

Now, by quadratic formula, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Hence the roots are $x = \frac{3 + \sqrt{3}}{2}, \frac{3 - \sqrt{3}}{2}$

(ii) Here, the given equation is $2x^2 - 3x + 5 = 0$;

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 2, b = -3$ and $c = 5$.

$\therefore$ Discriminant, $D = b^2 - 4ac = 9 - 4 \times 2 \times 5 = 9 - 40 = -31$

$\because$ $D < 0$, the equation has no real roots.

Ex.10 Find the value of k for each of the following quadratic equations, so that they have real and equal roots :

(i) $9x^2 + 18kx + 16 = 0$ (ii) $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

Sol. (i) The given equation $9x^2 + 8kx + 16 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 9, b = 8k$ and $c = 16$.

$\therefore$ Discriminant, $D = b^2 - 4ac = (8k)^2 - 4 \times 9 \times 16 = 64k^2 - 576$

Since roots are real and equal, so

$D = 0 \Rightarrow 64k^2 - 576 = 0 \Rightarrow 64k^2 = 576$

$\Rightarrow$ $k^2 = \frac{576}{64} = - \Rightarrow k = \pm 3$

Hence, $k = 3, -3$

(ii) The given equation $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = (k + 1), b - 2(k - 1)$ and $c = 1$

$\therefore$ Discriminant, $D = b^2 - 4ac = 4(k - 1)^2 - 4(k + 1) \times 1$

$=$ $4(k^2 - 2k + 1) - 4k - 4$

$\Rightarrow$ $4k^2 - 8k + 4 - 4k - 4 = 4k^2 - 12k$

Since roots are real and equal, so

$D = 0 \Rightarrow 4k^2 - 12k = 0 \Rightarrow 4k(k - 3) = 0$

$\Rightarrow$ either $k = 0$ or $k - 3 = 0 \Rightarrow k = 0$ or $k = 3$

Hence, k = 0, 3.

Ex.11 Find the set of value of k for which the equations $kx^2 + 2x + 1$ has distinct real roots.

Sol. The given equation is $kx^2 + 2x + 1 = 0$

$\therefore$ $D = (4 - 4 \times k \times 1) = 4c - 4k$

For distinct and real roots, we must have, D > 0.

Now, $D = (4 - 4k) > 0 \Leftrightarrow 4 > 4k \Leftrightarrow 4k < 4 \Leftrightarrow k < 1$.

$\therefore$ Required set = $\{k \in R : k < 1\}$

Ex.12 Find the of k for which the equations $5x^2 - kx + 4 = 0$ has real roots.

Sol. The given equation is $5x^2 - kx + 4 = 0$

$\therefore$ $D = k^2 - 4 \times 5 \times 4 = k^2 - 80$

For real roots, we must have, $D \geq 0$.

Now, $D > 0 \Leftrightarrow k^2 - 80 \geq 0 \Leftrightarrow k^2 \geq 80 \Leftrightarrow k \geq \sqrt{80}$ or $k \leq -\sqrt{80} \Leftrightarrow k \geq 4\sqrt{5}$ or $k \leq -4\sqrt{5}$.

COMPETITION WINDOW

ROOTS UNDER PARTICUALR CASES

(A) Let the quadratic equation $ax^2 + bx + c = 0$ has real roots and

(i) If b = 0 $\Leftrightarrow$ roots are of equal magnitude but of opposite sign.

(ii) If c = 0 $\Leftrightarrow$ one roots is zero and the other is $-\frac{b}{a}$

(iii) If a = c $\Leftrightarrow$ roots are of opposite sign.

(iv) If a > 0 c < 0, a < 0 c > 0 $\Leftrightarrow$ roots are of opposite sign.

(v) If a > 0, b > 0, c < 0, a < 0, b < 0, c < 0 $\Leftrightarrow$ both roots are negative $(\alpha + \beta < 0 \,\&\, \alpha\beta > 0)$

(vi) If a > 0, b < 0, c > 0, a < 0, b > 0, c < 0 $\Leftrightarrow$ both roots are positive $(\alpha + \beta < 0 \,\&\, \alpha\beta > 0)$

(vii) If a + b + c = 0 $\Leftrightarrow$ One of the roots is 1 and the other roots is $\frac{c}{a}$.

(viii) If a = 1 b, c $\in$ Z and the roots are rational numbers, then these roots must be integers.

(ix) If a, b, c $\in$ Q and D is a perfect square $\Leftrightarrow$ roots are rational.

(x) (A) If a, b, c $\in$ Q and D is positive but not a perfect square $\Leftrightarrow$ roots are irrational.

(B) If $ax^2 + bx + c = 0$ is satisfied by more then two values, it is an identity and a = b = c = 0 and vice versa

(C) The quadratic equation whose roots are reciprocal of the roots of $ax^2 + bx + c = 0$ is $cx^2 + bx + a = 0$ (i.e. the coefficients are writer in reverse order).

★ **SUM & PRODUCT OF THE ROOTS**

Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.

Then $\alpha = \frac{-b + \sqrt{+b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{+b^2 - 4ac}}{2a}$

$\therefore$ The sum of roots $\alpha + \beta = -\frac{b}{a} = -\frac{Ceoff.of\ x}{Ceoff.of\ x^2}$

and product of roots = $\alpha.\beta = \frac{c}{a} = -\frac{\text{costan}\,t\,term}{coefficient\,of\,x^2}$

★ FORMATION OF QUADRATIC EQUATION

Consider the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.
Let α and β be the roots of the quadratic equation

$\therefore \quad \alpha + \beta = -\frac{b}{a}$ and $\alpha.\beta = \frac{c}{a}$

Hence the quadratic equation whose roots are α and β is given by

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e. x^2 – (sum of the roots) x + product of the roots = 0

Ex.13 Form the quadratic equation in each of the following cases when the roots are :

(i) $2+\sqrt{5}2-\sqrt{5}$ (ii) a and $\frac{1}{a}$

Sol. (i) Here roots are $\alpha = 2+\sqrt{5}$ and $\beta = 2-\sqrt{5}$

$\therefore$ Sum of roots $= \alpha + \beta(2+\sqrt{5})+(2-\sqrt{5})$

$\therefore$ $\alpha + \beta = 4$

and product of the roots $= \alpha.\beta = (2+\sqrt{5}).(2-\sqrt{5}) = 4-5 = -1$

$\therefore$ $\alpha\,\beta = -1$

$\therefore$ Required equation is
x^2 – (sum of the roots) x + product of the roots = 0

or $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

or $x^2 - (4)x + (-1) = 0$

$\therefore$ $x^2 - 4x - 1 = 0$

(ii) Here roots are a and $\frac{1}{a}$

$\therefore$ $\alpha + \beta = a + \frac{1}{a}$ and $\alpha.\beta = a \times \frac{1}{a} = 1$

Here the required equation is $x^2 - \left(a + \frac{1}{a}\right)x + 1 = 0$

COMPETITION WINDOW

CONDITION FOR TWO QUADRATIC EQUATION TO HAVE A COMMON ROOT

Suppose that the quadratic equation $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$ (where a, a' $\neq$ 0 and ab' – a'b $\neq$ 0) have a common root. Let this common root be α Than $a\alpha^2 b\alpha + c = 0$ and $a'\alpha^2 b'\alpha + c' = 0$
Solving the above equations, we get,

$$\frac{\alpha^2}{bc' - b'c} = \frac{\alpha}{a'c - ac'} = \frac{1}{ab' - a'b}$$

$\Rightarrow \quad \alpha^2 = \frac{bc' - b'c}{ab' - a'b}$ and $\alpha = \frac{a'c - ac'}{ab' - a'b}$

Eliminating α, we get : $\frac{(a'c - ac')^2}{(ab - a'b)^2} = \frac{bc' - b'c}{ab' - a'b}$

$\Rightarrow \quad (a'c - ac')^2 = (bc' - b'c)(ab' - a'b)$

This is the required condition for two quadratic equations to have a common root. To obtain the common root, make coefficient of x^2 in both the equation same and subtract one equation from the other to obtain a linear equation in x. Solve it for x to obtain the common root.

Ex. For which value of k will the equations $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ have one common root.

Sol. Let the common root be α than, $\alpha^2 - k\alpha - 21 = 0$ and $\alpha^2 - 3k\alpha - 35 = 0$.

Solving by Cramer rule, we have : $\frac{\alpha^2}{-35k-63k} = \frac{\alpha}{-21-35k} = \frac{1}{-3k+k}$

$\therefore \quad \alpha = \frac{-98k}{-56} = \frac{-7k}{4}$ and $\frac{7k}{4} = \frac{28}{k} \Rightarrow 7k^2 = 28 \times 4 \Rightarrow k = \pm 4$

CONDITION FOR TWO QUADRATIC EQUATION TO HAVE THE SAME ROOT

Two quadratic equations $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$ have the same roots if and only if

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

★ APPLICATIONS OF QUADRATIC EQUATIONS

Algorithm :

The method of problem solving consists of the following three steps :-

Stop-I : Translating the word problem in to symbolic language (mathematical statement) which means identifying relationships existing in the problem & then forming the quadratic equation.

Stop-II : Solving the quadratic equation thus formed

Stop-III: Interpreting the solution of the equation which means translating the result of mathematical statement into verbal language.

Type-I : Problems Based On Numbers.

Ex.14 The difference of two numbers is 3 and their product is 504. Find the numbers.

Sol. Let the required numbers be x and (x – 3). Then,

$x(x-3) = 504$

$\Rightarrow x^2 - 3x - 504 = 0 \Rightarrow x^2 - 24x + 21x - 504 = 0$

$\Rightarrow x(x-24) + 21(x-24) = 0 \Rightarrow (x-24)(x+21) = 0$

$\Rightarrow x - 24 = 0$ or $x + 21 = 0 \Rightarrow x = 24$ or $x = -21$

If x = – 21, then the numbers are – 21 and – 24.

Again, if x = 24, then the numbers are 24 and 21.

Hence, the numbers are – 21, – 24 or 24, 21

Ex.15 The sum of the square of two consecutive odd positive integers is 290. find the integers.

Sol. Let the two consecutive odd positive integers be x and (x + 2). Then.

$x^2 + (x+2)^2 = 290$

$\Rightarrow x^2 + x^2 + 4x = 290 \Rightarrow x^2 + 2x - 143 = 0$

$\Rightarrow x^2 + 13x - 11x - 143 = 0 \Rightarrow x(x+13) - 11(x+13) = 0$

$\Rightarrow (x+13)(x-11) = 0 \Rightarrow x = -13$ or $x = 11$

If x = – 21, then the numbers are – 21 and – 24.

But – 13, is not an odd positive integer.

Hence, the required integers are 11 and 13.

Type-II : Problems Based On Ages :

Ex.16 Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two fifth of Varun's age. Find their present ages.

Sol. Let the present ages of Varun and Swati be x years and y years respectively.

Seven years ago,

Varun's age = (x – 7) years and Swati's age = (y – 7) years.

$\therefore \quad (x-7) = 5(y-7)^2 \Rightarrow x - 7 = 5(y^2 - 14y + 49)$

$\Rightarrow x = 5y^2 - 70y + 245 + 7 \Rightarrow x = 5y^2 - 70y + 252$ …(i)

Three years hence,

Varun's age = (x + 3) years and Swati's age = (y + 3) years.

$\therefore \quad (y+3) = \frac{2}{5}(x+3) \Rightarrow 5y + 15 = 2x + 6 \Rightarrow x = \frac{5y+9}{2}$ …(ii)

From (i) and (ii) we get $5y^2 - 70y + 252 = \frac{5y+9}{2}$

$\Rightarrow 10y^2 - 140y + 504 = 5y + 9 \Rightarrow 10y^2 - 145y + 495 = 0 \Rightarrow 2y^2 - 29y + 99 = 0$

$\Rightarrow 2y^2 - 18y - 11y + 99 = 0 \Rightarrow 2y(y-9) - 11(y-9) = 0$

$\Rightarrow$ (y – 9) (2y – 11) = 0 $\Rightarrow$ y = 9 or $y = \frac{11}{2}$

$\because$ $y = \frac{11}{2}$ is not possible $\left[\because \frac{11}{2} < 7\right]$

So, y = 9.

$\therefore$ $\frac{5\times9+9}{2} = 27$ [From (ii)]

Hence, the Varun's present age is 27 years and Swati's present age is 9 years..

Type-III : Problems Based On Geometrical Concepts :

Ex.17 The length of the hypotenuse of a right triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

Sol. Let ΔABC be a right triangle, right angled at B.

Let AB = x. Then

AC = (2x + 1) and BC = (2x + 1) – 2 = 2x – 1

$\Rightarrow$ ΔABC, $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow$ $(2x + 1)^2 = x^2 + (2x – 1) \Rightarrow 4x^2 + 4x + 1 = x^2 + 4x^2 – 4x + 1$

$\Rightarrow$ $x^2 = 8x \Rightarrow x = 8$ cm,

$\therefore$ BC = 2x – 1 = 2$\times$8 + 1 = 15 cm

AC = 2x + 1 = 2$\times$8 + 1 = 17 cm

Hence, the sides of the given triangle are 8cm, 15 cm and 17 cm.

Type-IV : Problems Based On Perimeter/Age :

Ex.18 Is it possible to design a rectangular park of perimeter 80 cm and area 400 m^2? If so, find its length and breadth.

Sol. Let the length and breadth of the rectangular park be ℓ and b respectively. Then,

2 (ℓ + b) = 80

ℓ + b = 40 $\Rightarrow$ ℓ = (40 – b)

And area of the park = 400 m^2

$\therefore$ ℓb = 400

$\Rightarrow$ (40 – b) b = 400 $\Rightarrow$ $40b – b^2 = 400$

$\Rightarrow$ $b^2 – 40b + 400 = 0 \Rightarrow b^2 – 20b + 400 = 0$

$\Rightarrow$ b (b – 20) – 20 (b – 20) = 0 $\Rightarrow$ (b – 20) (b – 20) = 0

$\Rightarrow$ $(b – 20)^2 = 0 \Rightarrow b – 20 \Rightarrow b = 20$ m

$\therefore$ ℓ = 40 – b = 40 – 20 = 20 m

Hence, length and breadth of the park are 20 m and 20 m respectively.

Thus, it is possible to design a rectangular park of perimeter 80 m and area 400 m^2

Type-V : Problems Based On Time and Distance :

Ex.19 A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less tor the same journey. Find the speed of the train.

Sol. Let the speed of the train be x km/h. Then,

Time taken to cover the distance of 360 km = $\frac{360}{x}$ hours.

If the speed of the train increased by 5 km/h. Them,

Time taken to cover the same distance = $\left(\frac{360}{x+5}\right)$h

According to the question, $\frac{360}{x} - \frac{360}{x+5} = 1$

$\Rightarrow$ $\frac{360(x+5)-360x}{x(x+5)} = 0 \Rightarrow 360x + 1800 - 360x = x^2 + 5x$

$\Rightarrow \quad x^2+5x-1800=0 \Rightarrow x^2+45x-40x-1800=0$

$\Rightarrow \quad x(x+45)-40(x+45)=0 \Rightarrow (x+45)(x-40)=0$

$\Rightarrow \quad x=-45$ or $x=40$

But the speed can not be negative.

Hence, the speed of the train is 40 km/h.

Type-VI : Problems Based On Time and Work :

Ex.20 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank respectively. Find the time in which each tap can separately fill the tank.

Sol. Let the tap of larger diameter takes x hours to fill the tank. Then, the tap of smaller diameter takes (x + 10) hours to fill the tank.

$\therefore$ The portion of tank filled by the larger tap in one hour = $\frac{1}{x}$, the portion of tank filled by the smaller tap in one hour = $\frac{1}{x+10}$

And the portion of tank filled by both the smaller and the larger tap in one hour = $\frac{1}{9\frac{3}{8}}=\frac{8}{75}$

$\therefore \quad \frac{1}{x}+\frac{1}{x+10}=\frac{8}{75}$

$\Rightarrow \quad \frac{x+10+x}{x(x+10)}=\frac{8}{75} \Rightarrow \frac{2x+10}{x^2+10x}=\frac{8}{75}$

$\Rightarrow \quad 15x + 750 = 8x^2 + 80x \Rightarrow 8x^2 - 70x - 750 = 0$

$\Rightarrow \quad 4x^2 - 35x - 375 = 0 \Rightarrow 4x^2 - 60x + 25x - 375 = 0$

$\Rightarrow \quad 4x (x - 15) + 25 (x - 15) = 0 \Rightarrow (x - 15) (4x + 25) = 0$

$\Rightarrow \quad x = 15$ or $x=\frac{-25}{4}$

But the speed can not be negative.

Hence, the larger tap takes 15 hours and the smaller tap takes 25 hours.

Type-VI : Miscellaneous Problems :

Ex.21 300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students.

Sol. Let the number of students be x. Then,

The number of apples received by each student = $\frac{300}{x}$

if there is 10 more students, i.e., (x + 10) students. Then,

The number of apples received by each student = $\frac{300}{x+10}$

According to the question, $\frac{300}{x}-\frac{300}{x+10}=1$

$\Rightarrow \quad \frac{300x+3000-300x}{x(x+10)}=1 \Rightarrow 3000=x^2+10x$

$\Rightarrow \quad x^2+10x-3000=0 \Rightarrow x^2+60x-50x-3000=0$

$\Rightarrow \quad x(x+60)-50(x+60)=0 \Rightarrow (x+60)(x-50)=0$

$\Rightarrow \quad x=-60$ or $x=50$

But the number of students can not be negative.
Hence, the number of students is 50.

SYNOPSIS

Quadratic Equation : A quadratic equation in one variable x is of the form $ax^2 + bx + c = 0$, $a \neq 0$ where a, b and c are real numbers.

Roots of the quadratic equation : A real number α is said to be a root of the quadratic equation or a zero of the quadratic polynomial if and only if α satisfies the equation i.e., which make LHS = RHS.

Sreedharacharya formula : $ax^2 + bx + c = 0$, $a \neq 0$, $b^2 - 4ac \geq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \alpha \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Nature of roots : A quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has :

(i) No real roots if D < 0. (ii) Two distinct real roots if D > 0. (iii) Two equal real roots if D = 0.

Relation between roots of equation : $ax^2 + bx + c = 0$, $a \neq 0$

Sum of roots = $\alpha + \beta = \frac{-b}{a}$, Product of roots = $\alpha\beta = \frac{c}{a}$

Formation of quadratic equation when roots are given : $ax^2 + bx + c = 0$ $[x^2 - (\alpha + \beta)x + \alpha\beta]$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOSSE THE CORRECT ONE

1. Which of the following quadratic expression can be expressed as a product of real linear factors?

(A) $x^2 - 2x + 3$ (B) $3x^2 - \sqrt{2x} - \sqrt{3}$ (C) $\sqrt{2x^2} - \sqrt{5x} + 3$ (D) None of these

2. Two candidates attempt to solve a quadratic equation of the form $x^2 + px + q = 0$. One starts with a wrong value of p and finds the roots to be 2 and 6. The other starts with a wrong value of q and finds the roots to be 2 and – 9. Find the correct roots of the equation :

(A) 3, 4 (B) – 3, – 4 (C) 3, – 4 (D) – 3, 4

3. Solve for x : $15x^2 - 7x - 36 = 0$

(A) $\frac{5}{9}, -\frac{4}{3}$ (B) $\frac{9}{5}, -\frac{4}{3}$ (C) $\frac{5}{9}, -\frac{3}{4}$ (D) None of these

4. Solve for y : $7y^2 - 6y - 13\sqrt{7} = 0$

(A) $\sqrt{7}, 2\sqrt{7}$ (B) $3, \frac{2}{\sqrt{7}}$ (C) $\frac{13}{\sqrt{7}}, -\sqrt{7}$ (D) None of these

5. Solve for x : $6x^2 + 40x = 31$

(A) $\frac{3}{8}, \frac{2}{5}$ (B) $\frac{3}{8}, \frac{3}{2}$ (C) $0, \frac{8}{3}$ (D) $\frac{8}{3}, \frac{5}{2}$

6. Determine k such that the quadratic equation $x^2 + 7(3 + 2k) - 2x(1 + 3k) = 0$ has equal roots :

(A) 2, 7 (B) 7, 5 (C) $2, -\frac{10}{9}$ (D) None of these

7. Discriminant of the roots of the equation $-3x^2 + 2x - 8 = 0$ is

(A) – 92 (B) – 29 (C) 39 (D) 49

8. The nature of the roots of the equation $x^2 - 5x + 7 = 0$ is

(A) No real roots (B) 1 real root (C) Can't be determined (D) None of these

9. The roots of $a^2x^2 + abx = b^2$, $a \neq 0$ are :

(A) Equal (B) Non-real (C) Unequal (D) None of these

10. The equation $x^2 - px + q = 0$ p, q $\in$ R has no real roots if :

(A) $p^2 > 4q$ (B) $p^2 < 4q$ (C) $p^2 = 4q$ (D) None of these

11. Determine the value of k for which the quadratic equation $4x^2 - 3kx + 1 = 0$ has equal roots :

(A) $\pm\left[\frac{2}{3}\right]$ (B) $\pm\left[\frac{4}{3}\right]$ (C) ± 4 (D) ± 6

12. Find the value of k such that the sum of the square of the roots of the quadratic equation $x^2 - 8x + k = 0$ is 40 :

(A) 12 (B) 2 (C) 5 (D) 8

13. Find the value of p for which the quadratic equation $x^2 + p(4x + p - 1) + 2 = 0$ has equal roots :

(A) $-1, \frac{2}{3}$ (B) 3, 5 (C) $1, -\frac{4}{3}$ (D) $\frac{4}{3}, 2$

14. The length of a hypotenuse of a right triangle exceeds the length of its base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle (in cm) :

(A) 6, 8 10 (B) 7, 24, 25 (C) 8, 15, 17 (D) 7, 40, 41

15. A two digit number is such that the product of it's digits is 12. When 9 is added to the number, the digits interchange their places, find the number :

(A) 62 (B) 34 (C) 26 (D) 43

16. A plane left 40 minutes late due to bad weather and in order to reach it's destination, 1600 km away in time, it had to increase it's speed by 400 km/h from it's usual speed. Find the usual speed of the plane :

(A) 600 km/h (B) 750 km/h (C) 800 km/h (D) None of these

17. The sum of the squares of two consecutive positive odd numbers is 290. Find the sum of the numbers :

18. A shopkeeper buys a number of books for Rs. 80. If he had bought 4 more for the same amount, each book would have cost Re. 1 less. How many books did he buy?

(A) 8 (B) 36 (C) 24 (D) 28

19. The squares have sides x cm and (x + 4) cm. The sum of their areas is 656 cm^2. find the sides of the square.

(A) 8 cm, 12 cm (B) 12 cm, 15 cm (C) 6 cm, 10 cm (D) 16 cm, 20 cm

20. The real values of a for which the quadratic equation $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite signs are given by :

(A) a > 6 (B) a > 9 (C) 0 < a < 4 (D) a < 0

OBJECTIVE — **ANSPER KEY** — **EXERCISE -I**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	B	B	C	D	C	A	A	C	B	B	A	A	C	B
Que.	16	17	18	19	20										
Ans.	C	B	B	D	C										

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. State which of the following equations are quadratic equation :

(i) $3x+\frac{1}{x}-8=0$ (ii) $18x^2-6x=0$ (iii) $x^2-5x=7-6x^3$ (iv) $x^2=25$ (v) $6x^5+3x^2-7=0$

(vi) $x+\frac{1}{x^2}=3$ (vii) $5x^2+6x=7$ (viii) $5x^3-2x-3=0$ (ix) $\frac{3x}{4}-\frac{5x^2}{8}=\frac{7}{8}$

(x) $\sqrt{x}+\frac{1}{\sqrt{x}}=4$ (xi) $(x+1)(x+3)=0$ (xii) $(2x+1)(3x+2)=6(x-1)(x-2)$

(xiii) $16x^2-3=(2x+5)(5x-3)$ (xiv) $(x-2)^2+1=2x-3$ (xv) $x(x+1)+8=(x+2)(x-2)$

(xvi) $x(2x-3)=x^2+1$ (xvii) $(x+2)^3=x^3-4$ (xviii) $x^2+\frac{2}{x^2}=3$

2. Represent each of the following situations in the form of a quadratic equation :

(i) The sum of the squares of two consecutive positive integers is 545. We need to find the integers.

(ii) The hypotenuse of a right triangle is 25 cm. The difference between the length of the other two sides of the triangle is 5 cm. We need to find the lengths of these sides.

(iii) One year ago, the father was 8 times as old as his son. Now his age is square of the son's age. We need to find their present ages.

(iv) Ravi and Raj together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out the number of toys produced on that day.

(v) A cottage industry product a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys product in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

3. In each of the following determine whether the given values are the solutions of the given equation or not :

(a) (i) $x^2 - 7x + 12 = 0$; x = 3, x = 4

(ii) $x^2-\sqrt{2}x-4=0$; $x=-\sqrt{2}, x=-2\sqrt{2}$

(iii) $10x-\frac{1}{x}=3$; $x\neq 0, x=\frac{1}{2}, x=\frac{-1}{2}$

(iv) $\frac{a}{(x-b)}+\frac{b}{(x-a)}=2$; $(x\neq a,b)$; $x=(a+b), x=\frac{a+b}{2}$

(b) (i) $x^2-(\sqrt{2}+\sqrt{3})x+\sqrt{6}=0$; $x=\sqrt{2}, x=\sqrt{3}$

(ii) $\frac{x}{a}+\frac{b}{x}=\frac{a+b}{a}$; $(x\neq 0), x=a, x=b$

4. In each of the following find the value of k for which the given value is a solution of the given equation :

(i) $(x+3)(2x-3k)=0$: $x=6$ (ii) $3\sqrt{7}x^2-4x+k=0$: $x=\frac{\sqrt{7}}{3}$

5. Find the value of p and q for which $x=\frac{2}{3}$ and x = – 3 are the roots of the equation $px^2 + 7x + q = 0$.

SHORT ANSWER TYPE QUESTIONS

Find the solutions of the following quadratic equations by factorization method and check the solutions (1-24) :

1. $27x^2 - 12 = 0$

2. $3\left(\frac{x}{2}+1\right)^2=27$

3. $16(x-4)^2 = 9(x+3)^2$

4. $x^2 - 300 = 0$

5. $x^2 + (a-b)x = ab$

6. $(3x + a)(3x + b) = ab$

7. $x^2-(1+\sqrt{2})x+\sqrt{2}=0$

8. $3\sqrt{7}x^2+4x-\sqrt{7}=0$

9. $\sqrt{3}y^2+11y+6\sqrt{3}=0$

10. $abx^2 - (a^2 + b^2)x + ab = 0$

11. $x^2-\frac{x}{12}-\frac{1}{12}=0$

12. $x^2+\left(a+\frac{1}{a}\right)x+1=0$

13. $x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$

14. $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

15. $\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0$

16. $\frac{5}{x-5}+\frac{4}{x}=\frac{3}{x-3}$

17. $\frac{5}{x-5}+\frac{2}{x-2}=\frac{3}{x-3}+\frac{4}{x-4}$

18. $\frac{x}{x+1}+\frac{x+1}{x}=\frac{34}{15}$

19. $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$

20. $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

21. $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5$

22. $\frac{4x-3}{2x+1}-10\left(\frac{2x+1}{4x-3}\right)=3$

23. $2\left(\frac{x+2}{2x-3}\right)-9\left(\frac{2x-3}{x+2}\right)=3$

24. $\frac{x}{2x+1}+\frac{2x+1}{x}=\frac{58}{21}$

Find the roots of each of the following quadratic equations by the method of completing the squares (25 - 29)

25. $x^2 - 6x + 4 = 0$

26. $2x^2 - 5x + 3 = 0$

27. $\sqrt{5}x^2+9x+4\sqrt{5}=0$

28. $(5z + 2a)(3z + 4b) = 8ab$

29. $2\sqrt{2}x^2+\sqrt{15}x+\sqrt{2}=0$

30. Find the solutions of $3x^2-2\sqrt{6}x+2=0$ by the method of completing the squares when
(i) x is a rational number (ii) x is a real number

31. Find the solutions of $15x^2 + 3 = 17x$, when (i) x is a rational number (ii) x is a real number.

32. Find the solutions of $5x^2 - 6x - 2 = 17x$, when (i) x is a rational number (ii) x is a real number.

Find the roots of each of the following quadratic equations by using the quadratic formula (33 - 50) :

33. $4x^2 + 3x + 5 = 0$

34. $x^2 - 16x + 64 = 0$

35. $3x^2 - 5x + 2 = 0$

36. $2x^2 2\sqrt{2}x+1=0$

37. $3x^2 2\sqrt{5}x-5=0$

38. $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

39. $x+\frac{1}{x}=3, x\neq 0$

40. $\frac{x-2}{4}=\frac{x+2}{x}, x\neq 0$

41. $y-\frac{15}{4y}+1=0, y\neq 0$

42. $\frac{x-3}{x+3}-\frac{x+3}{x-3}=6\frac{6}{7}, x\neq -3,3$

43. $\frac{x+2}{x-2}+\frac{x-2}{x+2}=\frac{5}{2}, x\neq 2,-2$

44. $\frac{x}{x-1}+\frac{x-1}{x}=4, x\neq 0,1$

45. $\frac{1}{x}-\frac{1}{x-2}=3, x\neq 0,2$

46. $\left(\frac{2x-3}{x-1}\right)-4\left(\frac{x-1}{2x-3}\right)=3, x\neq 1,\frac{3}{2}$

47. $(x^2 - 2x)^2 - 4(x^2 - 2x) + 3 = 0$

48. $(y^2 - 4y)^2 + 11(y^2 + 4y) + 28 = 0$

49. $2\left(\frac{x}{x+1}\right)^2-5\left(\frac{x}{x+1}\right)+2=0, x\neq -1$

50. $(x^2 + 3x + 2)^2 - 8(x^2 + 3x) - 4 = 0$

51. Find the nature of the roots of the following equations. If the real roots exist, find them :

(a) (i) $6x^2 + x - 2 = 0$ (ii) $2x^2+5\sqrt{3}x+6=0$ (iii) $2x^2 - 6x + 3 = 0$ (iv) $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

(b) Find the discriminant of roots of equation $3x^2-2x+\frac{1}{3}=0$ and hence find the nature of its roots. Find them, if they are real.

(c) What is the nature of roots of the quadratic equation $4x^2 - 12x - 9 = 0$?

52. Find the value of k for each of the following quadratic equations, so that they have two real and equal roots :

(a) (i) $2x^2 + kx + 3 = 0$ (ii) $kx^2 - 2\sqrt{5}x + 4 = 0$ (iii) $4x^2 - 2(k + 1)x + (k + 4) = 0$
(iv) $(k - 3)x^2 + 4(k - 3)x + 4 = 0$

(b) (i) $x^2 - 2(k + 1)x + k^2 = 0$ (ii) $(k + 4)x^2 + (x + 1)x + 1 = 0$ (iii) $kx^2 - 2\sqrt{5}x + 4 = 0$
(iv) $2kx^2 - 40x + 25 = 0$

(c) (i) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$ (ii) $x^2 - kx + 4 = 0$ (iii) $2x^2 - (k - 2)x + 1 = 0$

53. Determine the value(s) of p for which the quadratic equation $2x^2 + px + 8 = 0$ has (i) real roots.

54. Show that the equation $x^2 + px - 1 = 0$ has (i) real and distinct roots for all real values of p.

55. (a) If – 2 is a root of the quadratic equation $x^2 + px + 2 = 0$ and the quadratic equation $2x^2 + px + k = 0$ has equal roots, find the value of k.

(b) If – 2 is a root of the quadratic equation $x^2 + px + 2 = 0$ and the quation $2x^2 + px + q = 0$ has equal roots, find the value of p and q.

56. If the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$ or ad = ac.

57. Prove that both the roots of the equation $(x + a)(x + b) + (x + b)(x + c) + (x + c)(x + a) = 0$ are always real and can not be equal unless a = b = c.

58. (a) If the root of the equation $x^2 + 2cx + ab = 0$ are real and unequal, prove that the equation $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ has no real roots.

(b) Prove that the equation $x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$ has no real roots, if $ad \neq bc$.

(c) If p,q,r and s are real number such that pr = 2(q + s) than show that atleast one of the equation $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ has real roots.

SBJECTIVE	ANSPER KEY	EXERCISE -2 (x)-CBSE

- **VERY SHORT ANSWER TYPE QUESTIONS**

1. Equations in questions No. (i), (ii), (iv), (vii), (ix), (xiii), (xiv), (xvi) and (xvii) are quadratic equations.

2. (i) $x^2 + x - 272 = 0$, where x is the smaller integer. (ii) $x^2 + 5x - 300 = 0$, where x is the length of one side.
(iii) $x^2 - 8x + 7 = 0$, where x (in years) is the present age of son.
(iv) $x^2 - 45x + 324 = 0$, where x is the number of marbles with Ravi.
(v) $x^2 - 55x + 750 = 0$, where x (in km/h) is the speed of the train.

3. (a) (i) Both are solution (ii) $x = -\sqrt{2}$ is a solution but $x = -2\sqrt{2}$ is not a solution.
(iii) $x = \frac{1}{2}$ is a solution but $x = \frac{-1}{2}$ is not a solution. (iv) Both are solution
(b) (i) Both are solution (ii) Both are solution

4. (i) k = 4, (ii) $k = -\sqrt{7}$ **5.** p = 3, q = – 6

- **SHORT ANSWER TYPE QUESTIONS**

1. $\frac{2}{3}, -\frac{2}{3}$ **2.** 4, -8 **3.** 1, 25 **4.** $10\sqrt{3}, -10\sqrt{3}$ **5.** b, - a **6.** $0, -\frac{(a+b)}{3}$ **7.** $\sqrt{2}$ **8.** $-\frac{\sqrt{7}}{3}, \frac{\sqrt{7}}{7}$ **9.** $-\frac{2}{\sqrt{3}}, -3\sqrt{3}$

10. $\frac{a}{b}, \frac{b}{a}$ **11.** $\frac{1}{3}, -\frac{1}{4}$ **12.** $a, \frac{1}{a}$ **13.** $\frac{-a}{a+b}, -\frac{(a+b)}{a}$ **14.** –a, –b **15.** – 1 **16.** 12, – 2 **17.** $0, \frac{7}{2}$ **18.** $-\frac{5}{2}, \frac{3}{2}$

19. 5, – 1 **20.** $6, \frac{40}{13}$ **21.** $-10, -\frac{1}{5}$ **22.** $-\frac{4}{3}, \frac{1}{8}$ **23.** $\frac{11}{5}, \frac{5}{8}$ **24.** $3, -\frac{7}{11}$ **25.** $3 \pm \sqrt{5}$ **26.** $1, \frac{3}{2}$ **27.** $-\sqrt{5}, \frac{-4}{\sqrt{5}}$

28. $0, -\frac{6a+20b}{15}$ **29.** No solution **30.** (i) No solution (ii) $\sqrt{\frac{2}{3}}$ **31.** (i) No solution (ii) $\frac{17+\sqrt{109}}{30}, \frac{17-\sqrt{109}}{30}$

32. (i) No solution (ii) $\frac{3 \pm \sqrt{19}}{5}$ **33.** No solution **34.** 8 **35.** $1, \frac{2}{3}$ **36.** $\frac{1}{\sqrt{2}}$ **37.** $\frac{\sqrt{5}}{3}, -\sqrt{5}$ **38.** $\frac{-2b}{a}, \frac{-2b}{3a}$

39. $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$ **40.** $3+\sqrt{17}, 3-\sqrt{17}$ **41.** $\frac{3}{2}, -\frac{5}{2}$ **42.** $-4\frac{9}{4}$ **43.** 6, – 6 **44.** $\frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}$ **45.** $\frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}$

46. $\frac{1}{2}, \frac{4}{3}$ **47.** –1, 3, $1+\sqrt{2}, 1-\sqrt{2}$ **48.** – 2, – 1, $\frac{-3 \pm \sqrt{21}}{2}, \frac{-3 \pm \sqrt{5}}{2}$ **49.** – 2, 1 **50.** 1, 0, – 3 – 4

51. (a) (i) $\frac{1}{2}, \frac{-2}{3}$ (ii) $\frac{-\sqrt{3}}{2}, -2\sqrt{3}$ (iii) $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$ (iv) $\frac{-2b}{a}, \frac{-2b}{3a}$ (b) $\frac{1}{3}, \frac{1}{3}$ (c) Roots are real and unequal

52. (a) (i) $k = \pm 2\sqrt{6}$ (ii) $k = \frac{5}{4}$ (iii) = – 3, 5 (iv) k = 4 ; (b) (i) $k = \frac{-1}{2}$ (ii) k = 5, – 3 (iii) $k = \frac{5}{4}$ (iv) k = 8
(c) (i) k = 14 (ii) k = ± 4 (iii) $k = 2 \pm \sqrt{2}$ **53.** (i) p = ± 8 (ii) p ≤ – 8 or p ≥ 8, p ∈ R **55.** $k = \frac{9}{4}$, (b) p = 3, $q = \frac{9}{8}$

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

APPLICATIONS TO WORD PROBLEMS

1. Find the numbers whose sum is 40 and product 375.
2. The difference between two integers is 4. Their product is 221. Find the numbers.
3. The sum of a natural number and its reciprocal is $\frac{65}{8}$. Find the natural numbers.
4. Divide 27 into two parts such that the sum of their reciprocals is $\frac{3}{20}$.
5. The sum of two numbers is 12 and the sum of their squares is 74. Find the natural numbers.
6. Find two consecutive natural numbers, the sum of whose squares is 145.
7. Find two consecutive positive even integers, whose product is 224.
8. The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.
9. Find two consecutive multiples of 3 whose product is 270.
10. A number consists of two digits whose product is 18. If 27 is added to the number, the digits interchange their places. Find the number
11. A two-digit number contains the smaller of the two digits in the unit place. The product of the digits is 40 and the difference between the digits is 3. Find the number.
12. The sum of numerator and denominator of a certain fraction is 10. If 1 is subtracted from both the numerator and denominator, the fraction is decreased by $\frac{2}{21}$. Find the fraction.
13. Two years ago, a man's age was three times the square of his son's age. In three years time, his age will be four times his son's age. Find their present ages.
14. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. Find the time taken by the first pipe alone to fill the tank.
15. A booster pump can be used for filling as well as for emptying a tank. The capacity of the tank is 2400 m^3. The emptying capacity of the tank is 10 m^3 per minute higher than its filling capacity and the pump needs 8 minutes lesser to empty the tank than it needs to fill it. What is the filling capacity of the pump?
16. Albert goes to his friend's house which is 15 km away from his house. He covers half of the distance at a speed of x km/hr and the remaining at (x + 2) km/hr. If he takes 2 hrs 30 min. to cover the whole distance, find x.
17. (i) A train covers a distance of 780 km at x km/hr. Had the speed been (x – 5) km/hr, the time taken to cover the same distance would have been increased by 1 hour. Write down an equation in x and solve it to evaluate x.
(ii) A train covers a distance of 600 km at x km/hr. Had the speed been (x + 20) km/hr, the time taken to cover the same distance would have been reduced by 5 hour. Write down an equation in x and solve it to evaluate x.
18. By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car.
19. The distance by road between two towns A and B, is 216 km, and by rail it is 208 km. A car travels at a speed of x km/hr and the train travels at a speed which is 16 km/hr faster than the car.
(i) Write down the time taken by the car to reach town B from A, in terms of x.
(ii) Write down the time taken by the train to reach town B from A, in terms of x.
(iii) If the train takes 2 hours less than the car to reach town B, obtain an equation in x and solve it .

(iv) Hence, find the speed of the train .

20. Car A travels x km for every litre of petrol, while car B travels (x + 5) km for every litre of petrol.

(i) Write down the number of litres used by car A and B in covering a distance of 400 km.

(ii) If car A used 4 litres of petrol more than car B in covering 400 km, write an equation in x and solve it to determine the number of litres of petrol used by car B for the journey.

21. The speed of a boat in still water is x km/hr and the speed of the stream is 3 km/hr.

(i) Write the speed of the boat upstream, in terms of x.

(ii) Write the speed of the boat downstream, in terms of x.

(iii) If the boat goes 15 km upstream and 22 km downstream in 5 hours, write an equation in x to represent the statement.

(iv) Solve the equation to evaluate x.

22. The hypotenuse of right triangle is 20 m. If the difference between the lengths of other sides be 4 m. find the other sides.

23. The length of the sides of a right triangle are (2x – 1) m, and (4x + 1) m, where x > 0. Find :

(i) The value of x. (ii) The area of the triangle.

24. Two squares have sides x cm and (x + 5) cm. The sum of their areas is 697 sq. cm.

(i) Express this as an algebraic equation in x.

(ii) Solve this equation to find the sides of the squares .

25. The length of a rectangle is 8 metres more than its breadth and its area is 425 m^2.

(i) Taking x metres as the breadth of the rectangle, write an equation in x that represents the above statement.

(ii) Solve the above equation and find the dimensions of the rectangle.

26. The ratio between the length and the breadth of a rectangular field is 3 : 2. If only the length is increased by 5 metres, the new area of the field will be 2600 sq. metres. What is the breadth of the rectangular field?

27. The perimeter of a rectangular plot of land is 114 metres and its area is 810 square metres.

(i) Take the length of plot as x metres. Use the perimeter 114 m to write the value of the breadth in terms of x.

(ii) Use the values of length, breadth and area to write an equation in x.

(iii) Solve the equation to find the length and breadth of the plot.

28. Write a rectangular garden 10 m wide and 20 m long, we wish to pave a walk around the borders of uniform width so as to leave an area of 96 m^2 for flowers. Haw wide should the walk be ?

29. The area of right-angle triangle is 96 m^2. If the base is three times its altitude, find the base.

30. The length of the parallel sides of trapezium are (x + 8) cm and (2x +3) cm, and the distance between them is (x + 4) cm. If its area is 590 cm^2, find the value of x.

31. A man buys an article for Rs. x and sells it for Rs. 56 at a gain of x%. Find the value of x.

32. Rohit is on tour and he has Rs. 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by Rs. 3. For how many days Rohit is on tour?

33. Rs. 6400 were divided equally among x persons. Had this money been divided equally among (x + 14) persons, each would have got Rs. 28 less. Find the value of x.

34. Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member increased Rs. 10. Find how many students went for the picnic.

35. A shopkeeper buys x books for Rs. 720.

(i) Write the cost of 1 book in terms of x.

(ii) If the cost of per book were Rs. 5 less, the number of books that could be bought for Rs. 720 would be 2 more. Write down the equation in x for the above situation and solve it to find x.

36. A piece of cloth costs Rs. 35. If the length of the piece would have been 4 m longer and each metre costs Rs. 1 less, the cost would have remained unchanged. How long is the piece?

37. A fruit seller-bought x apples for Rs. 1200.

(i) Write the cost price of each apple in terms of x.

(ii) If 10 of the apple were rotten and he sold each of the rest at Rs. 3 more than the cost price of each, write the selling price of (x – 10) apples.

(iii) If he made a profit of Rs. 60 in this transaction, from an equation in x and solve it to evaluate x.

38. Vibha and Sanya distribute Rs. 100 each in charity. Vibha distributes money to 5 more people than Sanya and Sanya gives each Re 1 more than Vibha. How many people are recipients of the charity?

SBJECTIVE | **ANSWER** | **EXERCISE -3 (x)-CBSE**

Applications To Word Problems

1. 15, 25 **2.** 13, 17 or 13, – 17 **3.** 8 **4.** 15, 12 **5.** 5, 7 **6.** 8, 9 **7.** 14, 16 **8.** 27, 29, 31 **9.** 15, 18

10. 36 **11.** 85 **12.** $\frac{3}{7}$ **13.** 29 years, 5 years **14.** 15 hours **15.** 50 m^3/ min **16.** x = 4

17. (i) $x^2 - 5x - 3900 = 0$, x = 65 (ii) $x^2 + 20x - 2400 = 0$, x = 40 **18.** 30 km/hr

19. (i) $\frac{216}{x}$ hrs (ii) $\frac{208}{(x+16)}$ hrs (iii) $x^2 + 12x - 1728 = 0$, x = 36 (iv) 52 km/hr

20. (i) $\left(\frac{400}{x}\right)$ litres and $\left(\frac{400}{x+5}\right)$ litres (ii) $\frac{400}{x+5} - \frac{400}{(x+5)} = 4, x = 20$. Car B consumed 16 litres.

21. (i) (x – 3) km/hr (ii) x + 3 km/hr (iii) $\frac{15}{(x+3)} + \frac{22}{(x+3)} = 5$ (iv) x = 8 **22.** 16 m, 12,

23. (i) x = 3 (ii) 30 m^2 **24.** (i) $x^2 + 5x - 336 = 0$ (ii) 16 cm, 21 cm **25.** (i) $x^2 + 8x - 425 = 0$ (ii) 17 m, 25 m

26. 40 m **27.** (i) Breadth = (57 – x) m (ii) $x^2 - 57x + 810 = 0$ (iii) ℓ = 30 m, b = 27 m **28.** 2 m **29.** 24 m

30. x = 16 **31.** x = 40 **32.** 20 days **33.** x = 50 **34.** 16 **35.** (i) Rs. $\left(\frac{720}{x}\right)$ (ii) $x^2 + 2x - 228 = 0$ x = 16 **36.** 10 m

37. (i) Rs. $\left(\frac{1200}{x}\right)$ (ii) Rs. (x – 10) $\left(\frac{1200}{x} + 3\right)$ (iii) $x^2 - 30x - 4000 = 0$, x = 80 **38.** 45

EXERCISE – 4 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD QUESTIONS

SHORT ANSWER TYPE QUESTIONS

1. Find the values of k so that (x – 1) is a factor of $k^2x^2 - 2kx\ 3$. **[CBSE-Delhi-2003]**
2. Solve using the quadratic formula : $x^2 - 4x + 1 = 0$ **[ICSE-2003]**
3. Solve for x : $4x^2 - 2(a^2 + b^2)\,x + a^2b^2 = 0$ **[CBSE-Delhi-2004]**
4. Solve for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$ **[CBSE-Delhi-2004]**
5. Solve for x : $9x^2 - 9\,(a + b)\,x = [2a^2 + 5ab + 2b^2] = 0$ **[CBSE-Delhi-2004]**
6. Using quadratic formula, solve the following quadratic equation for x : $p^2x^2 + (p^2 - q^2)\,x - q^2 = 0$ **[CBSE-AI-2004]**
7. Using quadratic formula, solve the following quadratic equation for x : $x^2 - 2x + (a^2 - b^2) = 0$ **[CBSE-AI-2004]**
8. Using quadratic formula, solve the following quadratic equation for x : $x^2 - 4x + 4a^2 - b^2 = 0$ **[CBSE-AI-2004]**
9. Solve for x : $9x^2 - 6a^2x + (a^4 - b^4) = 0$ **[CBSE-Foreign-2004]**
10. Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$ **[CBSE-Foreign-2004]**
11. Solve for x : $16x^2 - 8a^2x + (a^4 - b^4) = 0$ **[CBSE-Foreign-2004]**
12. Solve for x : $36x^2 - 12ax + (a^2 - b^2) = 0$ **[CBSE-Delhi-2004C]**
13. Solve the equation $3x^2 - x - 7 = 0$ and give your answer correct to two decimal places. **[ICSE-2004]**
14. Solve for x : $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ **[CBSE-Foreign-2005]**

    OR

    Solve for x : $x^2 - 2\,(a^2 + b^2)\,x + (a^2 - b^2)^2 = 0$ **[CBSE-Delhi-2006C]**
15. Solve $x^2 - 5x - 10 = 0$ and give your answer correct to two decimal places **[ICSE-2005]**
16. Using quadratic formula, solve for x : $9x^2 - 3\,(a + b)\,x + ab = 0$

    OR

    The sum of the square of two consecutive natural numbers is 421. Find the numbers. **[CBSE-Dehli-2005C]**
17. Using quadratic formula, solve the following for x : $9x^2 - 3\,(a^2 + b^2)\,x + a^2\,b^2 = 0$

    OR

    The sum of the square of three consecutive positive integers is 50. Find the integers. **[CBSE-AI-2005C]**
18. Rewrite the following as a quadratic equation in x and then solve for x : $\frac{4}{x} - 3 = \frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$

    [CBSE-AI-2006C]
19. Solve $2x - \frac{1}{x} = 7$ and give your answer correct to 2 decimal places. **[ICSE-2006]**
20. Solve $x^2 - 3x - 9 = 0$ and give your answer correct to 2 decimal places. **[ICSE-2007]**
21. Find the roots of the following equation : $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$; $x \neq -4, 7$ **[CSBE-Delhi-2008]**
22. Is x = – 2 a solution of the equation $x^2 - 2x + 8 = 0$? **[CSBE-AI-2008]**
23. Is x = – 3 a solution of the equation $2x^2 + 5x + 3 = 0$? **[CSBE-AI-2008]**
24. Is x = – 4 a solution of the equation $2x^2 + 5x - 12 = 0$? **[CSBE-AI-2008]**
25. Show that x = – 3 is a solution of $x^2 + 6x + 9 = 0$. **[CSBE-Foreign-2008]**
26. Show that x = – 3 is a solution of $2x^2 + 6x - 3 = 0$. **[CSBE-Foreign-2008]**
27. Show that x = – 2 is a solution of $3x^2 + 13x + 14 = 0$. **[CSBE-Foreign-2008]**
28. Find the discriminant of the equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$. **[CSBE-AI-2009]**
29. The sum of two numbers is 8. Determine the numbers if the sum of their reciprocals is $\frac{8}{15}$. **[CSBE-AI-2009]**
30. Write the nature of roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$. **[CSBE-Foreign-2009]**

LONG ANSWER TYPE QUESTIONS

1. An aeroplane traveled a distance of 400 km at an average speed of x km/hr. On the return journey, the speed was increased by 40 km/hr. Write down an expression for the time taken for (i) the onward journey, (ii) the return journey. If the return journey took 30 minutes less than the onward journey, write an equation in x and find the value of x. **[ICSE-2002]**

2. In an auditorium, seats were arranged in rows and columns. The number of rows was equal to number of seats in each row. When the number of rows was doubled and the number of seats in each row was reduced by 10, the total number of seats increased by 300. Find (i) the number of rows in the original arrangement, (ii) the number of seats in the auditorium after rearrangement. **[ICSE-2003]**

3. Solve for x : $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5$; given that $x \neq -3,\ x \neq \frac{1}{2}$ **[CSBE-Delhi-2004]**

4. Solve for x : $2\left(\frac{x-1}{x+3}\right)-7\left(\frac{x+3}{x-1}\right)=5$; given that $x \neq -3,\ x \neq 1$ **[CSBE-Delhi-2004]**

5. Solve for x : $2\left(\frac{2x+3}{2x+1}\right)-10\left(\frac{2x+1}{2x-3}\right)=3$; given that $x \neq 3,\ x \neq \frac{-3}{2}$ **[CSBE-Delhi-2004]**

6. Solve for x : $2\left(\frac{4x-3}{2x+1}\right)-9\left(\frac{2x-3}{x+2}\right)=3$; given that $x \neq \frac{-1}{2};\ x \neq \frac{3}{4}$

OR

300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students. **[CSBE-Al-2004]**

7. Solve for x : $2\left(\frac{x+2}{2x-3}\right)-9\left(\frac{2x-3}{x+2}\right)=3$; given that $x \neq \frac{3}{2};\ x \neq -2$

OR

An aeroplane takes one hour less for a journey of 1200 km if its speed is increased by 100 km/hour from its usual speed. Find the its usual speed. **[CSBE-Foreign-2004]**

8. A two digit number is four times the sum of its digits and is also equal to twice the product of its digits. Find the number **[CSBE-Delhi-2004C]**

9. A two digit number is seven times the sum of its digits and is also equal to 12 less than three times the product of its digits. Find the number **[CSBE-Delhi-2004C]**

10. A two digit number is 5times the sum of its digits and is also equal to 5 more than twice the product of its digits. Find the number **[CSBE-Delhi-2004C]**

11. The sum of two number a and b is 15, and the sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$. Find the number **[CSBE-Delhi-2005]**

OR

12. The sum of two number is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the number **[CSBE-Delhi-2005]**

13. The sum of two number is 18. The sum of their reciprocals is $\frac{1}{4}$. Find the number **[CSBE-Delhi-2005]**

14. A two digit number is such that the product of its digits is 15. If 18 is added to the number, the digits interchange their places. Find the number **[CSBE-Al-2005]**

15. A two digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number **[CSBE-Al-2005]**

16. A two digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number **[CSBE-Al-2005]**

17. The sum of the square of two natural number is 34. If the first number is one less than twice the second number, find the number **[CBSE-Foreign-2005]**

18. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hour from its usual speed. Find the usual speed of the train. **[CSBE-Delhi-2005C, 2006]**

19. Solve for $x\frac{x+1}{x-1}+\frac{x-2}{x+2}=3:(x\neq 1,-2)$ **[CSBE-Al-2005C]**

OR

Aeroplane left 30 minutes later than its scheduled time and in order to reach destination 1500 km away in time, it has to increase its speed by 250 km/h from its usual speed. Determine its usual speed.

20. Solve for x : $\frac{1}{a+b+x}+\frac{1}{a}+\frac{1}{x}:a\neq 0, b\neq 0, x\neq 0$

OR

Solve for x : $abx^2 + (b^2 - ac)x - bc = 0$ **[CSBE-Delhi-2005]**

21. Solve for x : $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

OR

Solve for x : $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}\ (x\neq 2,4)$ **[CSBE-Al-2005]**

22. By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car. **[ICSE-2005]**

23. Solve for x : $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

OR

A two digit number is such that the product of its digits is 35. When 18 is added to number, the digits interchange their places. Find the number. **[CBSE-Dehli-2006]**

24. Using quadratic formula, solve the equation : $a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$

OR

The sum of two natural numbers is 8. Determine the numbers if the sum of their reciprocals is $\frac{8}{15}$.

[CBSE-Al-2006]

25. Solve for x : $(a + b)^2x^2 + 8(a^2 - b^2)x + 16(a - b)^2 = 0$

OR

Two number differ by 3 and their product is 504. Find the number. **[CBSE-Foreign-2006]**

26. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speeds of the two trains. **[CBSE-Foreign-2006]**

27. Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two-fifth of Varun's age. Find their present ages. **[CBSE-Delhi-2006C]**

28. A 2-digit number is such that product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

OR

A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for the journey. Find the original speed of the train **[CBSE-Al-2006C]**

29. A shopkeeper buys x books for Rs. 720. (i) Write the cost of 1 book in terms of x, (ii) If the cost per book were Rs. 5 less, the number of books that could be bought for Rs. 720 would be 2 more.
Write down the equation in x for the above situation and solve it to find x. **[ICSE-2006]**

30. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

OR

By increasing the list price of a book by Rs. 10 a person can buy 10 less books for Rs. 1200. Find the original list price of the book. **[CBSE-Delhi-2007]**

31. The numerator of a fraction is one less than its denominator. If three is added to each of the numerator and denominator, the fraction is increased by $\frac{3}{28}$. Find the fraction.

OR

The difference of squares of two natural numbers is 45. The square of the smaller umber is four times the larger number. Find the numbers. **[CBSE-AI-2007]**

32. Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member increased by Rs. 10. Find how many students went for the picnic. **[ICSE-2008]**

33. In a class test, the sum of the marks obtained by P in mathematics and science is 28. Had he got 3 more marks in mathematics and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

OR

The sum of the areas of two squares is 640 m^2. If the difference in their perimeters be 64 m, find the sides of the two squares. **[CBSE-Delhi-2008]**

34. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **[CBSE-AI-2008]**

35. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to it's hole at the base of pillar. Seeing the snake the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?

OR

Two difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the two numbers. **[CBSE-Foreing-2008]**

36. The sum of the squares of two consecutive odd numbers is 394. Find the numbers. **[CBSE-Delhi-2009]**

37. Solve the following equation for x : $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

OR

If (– 5) is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k. **[CBSE-AI-2009]**

38. A trader bought a number of articles for Rs. 900. Five articles were found damaged. He sold each of the remaining articles at Rs. 2 more than what he paid for it. He got a profit of Rs. 80 on the whole transaction. Find the number of articles he bought.

OR

Two years ago a man's age was three times the square of his son's age. Three years hence his age will be four times his son's age. Find their present ages. **[CBSE-Foreing-2009]**

39. A girl is twice as old as her sister. Four years hence. The product of their ages (in yeras) will be 160. Find their present ages. **[CBSE-AI-2010]**

SBJECTIVE	ANSPER KEY	EXERCISE -4 (x)-CBSE

- **SHORT ANSWER TYPE QUESTION**

1. (–1, 3) **2.** $[2+\sqrt{3}, 2-\sqrt{3}]$ **3.** $x = \frac{a^2}{2}, \frac{b^2}{2}$ **4.** $x = \frac{(a^2+b^2)}{2}. \frac{(a^2-b^2)}{2}$ **5.** $x = \frac{(2a+b)}{3}. \frac{(a+2b)}{3}$ **6.** $\frac{q^2}{p^2}, 1$

7. a + b, a – b **8.** 2a + b, 2a – b **9.** $\frac{(a^2+b^2)}{3}, \frac{(a^2-b^2)}{3}$ **10.** $\frac{(a+b)}{3}\frac{(a-b)}{3}$ **11.** $\frac{(a^2+b^2)}{4}, \frac{(a^2-b^2)}{4}$

12. $\frac{(a+b)}{6}\frac{(a-b)}{6}$ **13.** 1.70, – 1.37 **14.** $\frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$ or $(a+b)^2, (a-b)^2$ **15.** 6.53, – 1.53 **16.** $\frac{a}{3}.\frac{b}{3}$ or 14, 15

17. $\frac{a^2}{3}.\frac{b^2}{3}$ or 3, 4, 5 **18.** x = – 2, 1 **19.** 3.64 – 0.14 **20.** 4.85, – 1.85 **21.** 2, 1 **22.** No **23.** No

24. Yes 28.64 29. 3and 5

- **LONG ANSWER TYPE QUESTION**

1. (i) $\left(\frac{400}{x}\right)$ hrs. (ii) $\left(\frac{400}{x+40}\right)$ hrs; x = 160 km/hr 2. (i) 30 (ii) 1200 3. x = – 10 1/5 4. $x = -\frac{23}{5}, -1$

6. $x = -\frac{4}{3}, \frac{1}{8}$ or 50 7. $x = \frac{5}{8}, \frac{11}{5}$ or 300 km/hr 8. 36 9. 84 10. 45 11. 5,10 12. 4,12 13. 6,12 14. 35 15. 45

16. 27 17. 5 and 3 18. 25 km/hr 19. x = – 5, 2 or 750 km/hr 20. x = – a, – b or $x = \frac{c}{b}, \frac{-b}{a}$

21. $x = \frac{1}{b^2}, -\frac{1}{a^2}$ or $x = \frac{5}{2}, 5$ 22. 30 km/hr 23. $x = \frac{-2b}{3a}, \frac{3a}{4b}$ or 57 24. $x = \frac{-3a^2}{b^2}, \frac{4b^2}{a^2}$ or 3 and 5

25. $x = \frac{-4(a-b)}{a+b}$ or 21, 24 or – 21, – 24 26. 40 km/hr, 50 km/hr 27. 9 years, 27 years 28. 92 or 45 km/hr

29. (i) Rs. $\left(\frac{720}{x}\right)$ (ii) x2 + 2x – 288 = 0, x = 16 **30.** 10 and 5 or Rs. 30 **31.** 3/4 or 9 and 6 **32.** 16

33. Marks in maths : 12(9), Marks in science : 16(19) **34.** 6 km/hr or 25 hrs and 15 hrs

35. 12 m or (7 and 3) or (– 3 and – 7) **36.** 13 and 15 **37.** $\frac{2a+b}{3}, \frac{a+2b}{3}$ or p = 7 and $k = \frac{7}{4}$

38. 75 or son's age = 5 years and man's age = 29 years. **29.** 6 years and 12 years

EXERCISE – 5 (FOR OLYMPIADS)

CHOOSE THE CORRECT ONE

1. The roots of the equation (x – a) (x – b) (x – c) + (x – b) (x – c) +(x – c) (x – a) = 0 are :

(A) Real (B) Not real (C) Imaginary (D) Rational

2. The integral values of k for which the equation (k – 2) x^2 + 8x + k + 4 = 0 has both the roots real, distinct and negative is :

(A) 0 (B) 2 (C) 3 (D) – 4

3. If the roots of the equation $\frac{x^2-bx}{ac-c} = \frac{m-1}{m+1}$ are equal and of oppositve sign, then the value of m will be :

(A) $\frac{a-b}{a+b}$ (B) $\frac{b-a}{a+b}$ (C) $\frac{a+b}{a-b}$ (D) $\frac{b+a}{b-a}$

4. If α, β are the roots of the equation x^2 + 2x + 4 = 0, then $\frac{1}{\alpha^3}\frac{1}{\beta^3}$ is equal to :

(A) $-\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 32 (D) $\frac{1}{32}$

5. If $x = \sqrt{6+\sqrt{6+\sqrt{6+...to\infty}}}$, then :

(A) x is an irrational number (B) 2 < x < 3

(C) x = 3 (D) None of these

6. If α, β are the roots of the equation $x^2 + 7x + 12 = 0$, then the equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ is :

(A) $x^2 + 50x + 49 = 0$ (B) $x^2 - 50x + 49 = 0$ (C) $x^2 - 50x - 49 = 0$ (D) $x^2 + 12x + 7 = 0$

7. The values of k (k > 0) for which the equation $x^2 + kx + 64 =$ and $x^2 - 8x + k = 0$ both will have real roots is :

(A) 8 (B) 16 (C) – 64 (D) None of these

8. If α, β are the roots of the equation $x^2 + bx - c = 0$, then the equation whose roots are b and c is :

(A) $x^2 + \alpha x - \beta = 0$ (B) $x^2 - [(\alpha+\beta) + \alpha\beta]x - \alpha(\alpha+\beta) = 0$

(C) $x^2 + (\alpha\beta + \alpha + \beta)x + \alpha\beta(\alpha+\beta) = 0$ (D) $x^2 + (\alpha\beta + \alpha + \beta)x - \alpha\beta(\alpha+\beta) = 0$

9. Solve for y : $9y^4 - 29y^2 + 20 = 0$

(A) $\pm 2, \pm\frac{2}{3}$ (B) $\pm 3, \pm\frac{3}{\sqrt{5}}$ (C) $\pm 1, \pm\frac{2\sqrt{5}}{3}$ (D) None of these

10. Solve for x : $x^6 - 26x^3 - 27 = 0$

(A) – 1, 3 (B) 1, 3 (C) 1, – 3 (D) – 1, – 3

11. Solve : $\sqrt{2x+9} + x = 3$:

(A) 4, 16 (B) 8, 20 (C) 2, 8 (D) None of these

12. Solve : $\sqrt{2x+9} - \sqrt{x-4} = 3$

(A) 4, 16 (B) z8, 20 (C) 2, 8 (D) None of these

13. Solve for x : $2\left[x^2 + \frac{1}{x^2}\right] - 9\left[x + \frac{1}{x}\right] + 14 = 0$:

(A) $\frac{1}{2}$, 1, 2 (B) 2, 4, $\frac{1}{3}$ (C) $\frac{1}{3}$, 4, 1 (D) None of these

14. Solve x : $6\left[x^2 + \frac{1}{x^2}\right] - 25\left(x + \frac{1}{x}\right) + 12 = 0$:

(A) $-\frac{1}{3}, -\frac{1}{2}$, 2, 3 (B) $\frac{1}{3}, \frac{1}{2}$, 2, 3 (C) $\frac{1}{3}, \frac{1}{2}, -2, -3$ (D) None of these

15. Solve for x : $\sqrt{x^2 + x - 6} - x + 2 = \sqrt{x^2 - 7x + 10},\ x \in R$:

(A) 2, 6, $-\frac{10}{3}$ (B) 2, 6 (C) – 2 , – 6 (D) None of these

16. Solve for x : $3^{x+2} + 3^{-x} = 10$

(A) – 3, – 2 (B) – 2, 0 (C)2 , 3 (D) None of these

17. Solve for x : (x + 1) (x + 2) (x + 3) (x + 4) = 24 (x $\in$ R) :

(A) 0, – 5 (B) 0, 5 (C) 0, – 2 (D) 0, 2

18. The sum of all the real roots of the equation $|x-2|^2 + |x-2|^2 - 2 = 0$ is :

(A) 2 (B) 3 (C) 4 (D) None of these

19. If a, b $\in$ {1, 2, 3, 4}, then the number of quadratic equation of the form ax2 + bx + 1 = 0, having real roots is :

(A) 6 (B) 7 (C) 8 (D) None of these

20. The number of real solutions of $x - \frac{1}{x^2-4} = 2 - \frac{1}{x^2-4}$ is :

(A) 0 (B) 1 (C) 2 (D) Infinite

21. If $(2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1} = \frac{2}{2-\sqrt{3}}$, then x is equal to :

(A) 0 (B) 1 (C) 2 (D) Both (A) and (C)

22. The quadratic equation $3x^2 + 2(a^2 + 1)x + a^2 - 3a + 2 = 0$ possesses roots of opposite sign then a lies in :
(A) $(-\infty, 0)$ (B) $(-\infty, 1)$ (C) (1, 2) (D) (4, 9)

23. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has :
(A) No solution (B) One solution (C) Two solution (D) More than two solution

24. The number of real solutions of the equation $2|x|^2 - 5|x| + 2 = 0$ is :
(A) 0 (B) 4 (C) 2 (D) None of these

25. The number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$:
(A) 0 (B) 2 (C) 3 (D) 6

26. The number of real solutions of the equation $2^{3x^2-7x+4} = 1$ is :
(A) 0 (B) 4 (C) 2 (D) Infinitely many

27. If the equation $(3x)^2 + (27 \times 3^{1/k} - 15)x + 4 = 0$ has equal roots, then k =
(A) – 2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 0

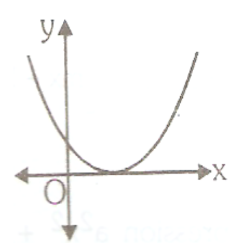
28. If $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots + \infty}}}$, then x is :
(A) 1 (B) 2 (C) 3 (D) None of these

29. Equation $ax^2 + 2x + 1$ has one double root if :
(A) a = 0 (B) a = – 1 (C) a = 1 (D) a = 2

30. Solve for x : (x + 2) (x – 5) (x – 6) (x + 1) = 144 :
(A) –1, –2, –3 (B) 7, – 3, 2 (C) 2, – 3, 5 (D) None of these

31. If f(x) = $\frac{2x+5}{x^2+x+5}$, then find f(f(– 1))
(A) $\frac{149}{155}$ (B) $\frac{155}{147}$ (C) $\frac{155}{149}$ (D) $\frac{147}{155}$

32. What does the following graph represent?
(A) Quadratic polynomial has just one root.
(B) Quadratic polynomial has equal one roots.
(C) Quadratic polynomial has no root.
(D) Quadratic polynomial has equal roots and constant term is non-zero.



33. Consider a polynomial $ax^2 + bx + c$ such that zero is one of it's roots then :
(A) $c = 0, x = \frac{-b}{a}$ satisfies the polynomial equation
(B) $c \neq 0, x = \frac{-b}{a}$ satisfies the polynomial equation
(C) $, x = \frac{-b}{a}$ satisfies the polynomial equation
(D) Polynomial has equal roots.

34. For a parabola opening upwards and above x-axis, quadratic will have :
(A) Equal roots and a = 0 (B) Unequal roots and $a \neq 0$
(C) No roots, a > 0 (D) No roots, a < 0

35. The equation $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$ has :

(A) An extraneous root between – 5 and – 1.

(B) An extraneous root between – 10 and – 6.

(C) Two extraneous roots.

(D) A real root between 20 and 25.

36. Consider a quadratic polynomial $f(x) = ax^2 - x + c$ such that $ac > 1$ and it's graph lies below x-axis then :

(A) $a < 0, c > 0$ (B) $a < 0, c < 0$ (C) $a > 0, c > 0$ (D) $a > 0, c < 0$

37. If α, β are the roots of a quadratic equation $x^2 - 3x + 5 = 0$, then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is :

(A) $x^2 + 4x + 1 = 0$ (B) $x^2 - 4x + 4 = 0$ (C) $x^2 - 4x - 1 = 0$ (D) $x^2 + 2x + 3 = 0$

OBJECTIVE — ANSPER KEY — EXERCISE -5

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	A	B	C	B	B	C	C	A	B	B	A	A	B
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	B	A	C	B	A	D	C	A	B	A	C	B	B	C	B
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**								
Ans.	C	D	A	C	B	B	B								

EXERCISE – 6 (FOR IIT-JEE/AIEEE)

CHOOSE THE CORRECT ONE

Based on Graph of Quadratic Expression

1. If the expression $\left[mx - 1 + \frac{1}{x}\right]$ is non negative for all positive real x, then the minimum value of m must be :

(A) $\frac{-1}{2}$ (B) 0 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

2. The expression $a^2x^2 + bx + 1$ will be positive for all $x \in R$ if :

(A) $b^2 > 4a^2$ (B) $b^2 < 4a^2$ (C) $4b^2 > a^2$ (D) $4b^2 < 4a^2$

3. If x be real, then 3x2 + 14x + 11 > 0 when :

(A) $x < -\frac{3}{2}$ (B) $x > -\frac{3}{4}$ (C) $x > -2$ (D) Never

4. For what value of a the curve y = x2 + ax + 25 touches the x-axis :

(A) 0 (B) ± 5 (C) ± 10 (D) None of these

5. The integer k for which the inequality $x^2 - 2(4k - 1)x$ + 15k2 – 2k – 7 > 0 is valid for any x is :

(A) 2 (B) 3 (C) 4 (D) 6

6. The value fo the expression $x^2 - 2bx + c$ will be positive for all real x if :

(A) $b^2 - 4c > 0$ (B) $b^2 - 4c < 0$ (C) $c^2 < b$ (D) $b^2 < c$

7. If the roots fo the quadratic equation $ax^2 + bx + c = 0$ are imaginary then for all values of a, b, c and x R the expression $a^2x^2 + abx + ac$ is :

(A) Positive (B) Non-negative (C) Negative (D) May be positive, zero or negative

Based on Maximum & Minimum Value of the Expression :

8. The range of $y = \frac{x+2}{2x^2 + 3x + 6}$, if x is real, is :

(A) $-\frac{1}{13} \le y \le \frac{1}{3}$ (B) $\frac{1}{13} \le y \le \frac{1}{3}$ (C) $-\frac{1}{13} \le y \le \frac{1}{13}$ (D) None of these

9. If $x \in R$ and $k = \frac{(x^2 - x + 1)}{(x^2 + x + 1)}$, then :

(A) $x \le 0$ (B) $\frac{1}{3} \le k \le 3$ (C) $k \ge 5$ (D) None of these

10. For all real values of x, the maximum value of the expression $\frac{x}{x^2-5x+9}$ is :

(A) 1 (B) 45 (C) 90 (D) None of these

11. If x be real then the maximum and minimum value of the expression $\frac{x^2-3x+4}{x^2+3x+4}$ are

(A) 2, 1 (B) $7, \frac{1}{7}$ (C) $5, \frac{1}{5}$ (D) None of these

12. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is : **[AIEEE-2006]**

(A) $\frac{17}{7}$ (B) $\frac{1}{4}$ (C) 41 (D) None of these

Based on the Concept of Common Roots :

13. The value of k, so that the equation $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have one root in common is :

(A) – 2, – 3 (B) $-3, -\frac{27}{7}$ (C) – 5, – 6 (D) None of these

14. If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \neq 0$, then the common factor is :

(A) (x – 3) (B) (x – 6) (C) (x – 8) (D) None of these

15. The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is :

(A) 0, 2 (B) 0, – 2 (C) 2, – 2 (D) None of these

16. If the equation $x^2 + bx + c = 0$ and $x^2 + cx + b = 0$, $(b \neq c)$ have a common root then :

(A) b + c = 0 (B) b + c = 1 (C) b + c + 1 = 0 (D) None of these

17. If both the roots of the equation $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then 2r – p is equal to :

(A) 1 (B) – 1 (C) 2 (D) 0

18. If every pair from among the equation $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$ and x^2 rx + pq = 0 has a common root, then the sum of three common roots is :

(A) 2 (p + q + r) (B) p + q + r (C) – (p + q + r) (D) pqr

19. If $x^2 - ax - 21 = 0$ and $x^2 - 3ax + 35 = 0$; a > 0 have a common root, then a is equal to :

(A) 1 (B) 2 (C) 4 (D) 5

20. The values of a for which the quadratic equation $(1 - 2a)x^2 - 6ax - 1 = 0$ and $ax^2 - x + 1 = 0$ have at least one root in common are :

(A) $\frac{1}{2}, \frac{2}{9}$ (B) $0, \frac{1}{2}$ (C) $\frac{2}{9}$ (D) $0, \frac{1}{2}, \frac{2}{9}$

21. If the quadratic equation $2x^2 + ax + b = 0$ and $2x^2 + bx + a = 0$ $(a \neq 0)$ and $ax^2 - x + 1 = 0$ have a common root, the value of a + b is :

(A) – 3 (B) – 2 (C) – 1 (D) 0

22. If the equation $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have a common root and $b \neq c$, then their other roots will satisfy the equation :

(A) $x^2 - (b + c)x + bc = 0$ (B) $x^2 - ax + bc = 0$

(C) $x^2 + ax + bc = 0$ (D) None of these

23. If both the roots of the equation $x^2 + mx + 1 = 0$ and $(b - c)x^2 + (c - a)x + (a - b) = 0$ are common then :

(A) m = – 2 (B) m = – 1 (C) m = 0 (D) m = 1

24. The quadratic equation $x^2 - 6x + a = 0$ and $x^2 - cx + ab = 0$ have one common root. The other roots of first and second equation are integers in the ratio 4 : 3. Then common root is : **[AIEEE-2008]**

(A) 1 (B) 4 (C) 3 (D) 2

Miscellaneous :

25. Solve $x^2 - 5x + 4 = 0$:

(A) $1 < x < 4$ (B) $-4 < x < 1$ (C) $x < 1$ and $x > 4$ (D) None of these

26. Solve $-x^2 + 6x - 8 = 0$:

(A) $-2 < x < 4$ (B) $-4 < x < -2$ (C) $2 < x < 4$ (D) None of these

27. For all $x \in R$, $x^2 + 2ax + 10 - 3a > 0$ then the interval in which 'a' lies is : **[IIT Screening-2004]**

(A) $(-\infty, -5)$ (B) $(-5, 2)$ (C) $(5, \infty)$ (D) $(2, 5)$

28. The solution set contained in R of the inequation : $3^x + 3^{1-x} - 4 < 0$ is : **[EAMCET-2003]**

(A) (1, 3) (B) (0, 1) (C) (1, 2) (D) (0, 2)

29. The number of real solution of the equation $x^2 - 3|x| + 2 = 0$ is : **[AIEEE-2003]**

(A) 3 (B) 2 (C) 4 (D) 1

30. Product of real roots the equation $t^2x^2 + |x| + 9 = 0$: **[AIEEE-2002]**

(A) Is always positive (B) Is always negative (C) Does not exist (D) None of these

31. For the equation 3x2 + px + 3 = 0, p > 0. If one of the roots is square of the other, then p =

(A) $\frac{1}{2}3$ (B) 1 (C) 3 (D) $\frac{2}{3}$ **[IIT Screening-2000]**

32. The roots of the equation $|x^2 - x - 6| = x + 2$ are :

(A) – 2, 1, 4 (B) 0, 2, 4 (C) 0, 1, 4 (D) – 2, 2, 4

33. If α, β are the roots of $x^2 + x + 1 = 0$, the equation whose roots are $(\alpha^{19}, \beta^7$ is : **[IIT 1994]**

(A) $x^2 - x - 1 = 0$ (B) $x^2 - x + 1 = 0$ (C) $x^2 + x - 1 = 0$ (D) $x^2 + x + 1 = 0$

34. The equation of the smallest degree with real coefficients having 1 + 1 as one of the roots is :

(A) $x^2 + x + 1 = 0$ (B) $x^2 - 2x + 2 = 0$ (C) $x^2 + 2x + 2 = 0$ (D) $x^2 + 2x - 2 = 0$

[Kerala Engineering -2002]

35. If a, b, c, d are positive reals such that a + b + c + d = 2 and M = (a + b) (c + d), then :

(A) $0 < M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

[IIT Screening-2000]

36. Let a, b, c be real numbers such that 4a + 2b + c = 0 and ab > 0; then the quadratic equation $ax^2 + bx + c = 0$ has :

(A) Real roots (B) Non-real roots **[IIT 1990]**

(C) Purely imaginary roots (D) Only one real roots

37. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then the biquadratic P(x) Q(x) = 0 has :

(A) All the four roots real (B) No real roots **[IIT 1989]**

(C) Atleast imaginary roots (D) Two equal roots

38. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has : **[IIT 1989]**

(A) Two roots (B) Infinitely many roots (C) Only one roots (D) No root

39. Number of values of x satisfying the equation $(15+4\sqrt{14})^t + (15-4\sqrt{14})^t = 30$, where $t = x^2 - 2|x|$:

(A) 0 (B) 2 (C) 4 (D) 6

40. The of values of x which satisfy the expression : $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{16})^{x^2-3} = 10$

(A) $\pm 2, \pm\sqrt{3}$ (B) $\pm\sqrt{2}, \pm 4$ (C) $\pm 2, \pm\sqrt{2}$ (D) $\pm\sqrt{2}, \pm\sqrt{3}$

41. If α and β are the roots of the equation $ax^2 + bx + c$, where (a,b,c) > 0, then α and β are :

(A) Rational numbers (B) Real and negative (C) Negative real parts (D) None of these

42. The number of quadratic equation which remain unchanged by squaring their roots, is :

(A) 0 (B) 2 (C) 4 (D) Infinitely many

43. If the equation $(\lambda^2 + 5\lambda + 6)x^2 + (\lambda^2 - 3\lambda + 2)x + (\lambda^2 - 4) = 0$ has more than two roots, then the value of λ is

(A) 2 (B) 3 (C) 1 (D) – 2

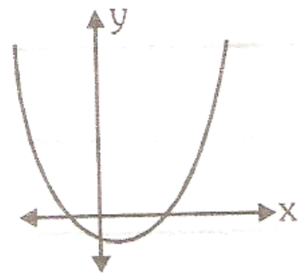
44. Find all the integral values of a for which the quadratic equation (x – a) (x – 10) + 1 = 0 has integral roots ?

(A) 12, 8 (B) 4, 6 (C) 2, 0 (D) None of these

45. If one root of the quadratic equation $px^2 + qx + r = 0$ ($p \neq 0$) is a surd $\dfrac{\sqrt{a}}{\sqrt{a} + \sqrt{a-b}}$ where p,q,r,a,b are all rationals then the other root is :

(A) $\dfrac{\sqrt{a}}{\sqrt{a} + \sqrt{a-b}}$ (B) $a + \dfrac{\sqrt{a(a-b)}}{b}$ (C) $\dfrac{a + \sqrt{a(a-b)}}{b}$ (D) $\dfrac{\sqrt{a} - \sqrt{a-b}}{\sqrt{b}}$

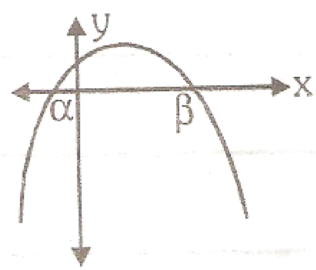
46. Graph of $y = ax^2 + bx + c$ is given adjacently. What conclusions can be drawn from the graph :



(i) a > 0 (ii) b < 0 (iii) c < 0 (iv) $b^2 - 4ac > 0$

(A) (i) and (iv) (B) (ii) and (iii) (C) (i), (ii) & (iv) (D) (i), (ii), (iii) & (iv)

47. The adjacently figure shows the graph of $y = ax^2 + bc + c$. Then which of the following is correct :



(i) a > 0 (ii) b > 0 (iii) c > 0 (iv) $b^2 < 4ac$

(A) (i) and (iv) (B) (ii) and (iii) (C) (iii) & (iv) (D) None of these

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OBJECTIVE					ANSWER KEY									EXERCISE -6
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	C	B	B	C	B	D	A	A	B	A	B	C	B	C	B
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	D	B	C	C	B	A	A	D	C	C	B	B	C	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	C	D	D	B	A	A	C	D	C	C	C	C	A	A	C
Que.	**46**	**47**													
Ans.	D	B													

AREAS RELATED TO CIRCLES

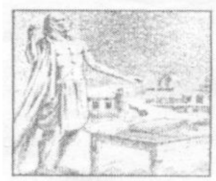
★ **INTRODUCTION**

In earlier classes, we have studied methods of finding perimeters and area of simple plane figures such as rectangles, squares, parallelograms, triangle and circles. In our daily life, we come across many objects which are related to circular shape in some form or the other. For example, cycle wheels, wheel arrow, drain cover, bangles, flower beds, circular paths etc. That is why the problem of finding perimeters and areas related to circular figures is of great practical importance. In this chapter, we shall discuss problems on finding the area of some combinations of plane figures involving circles or parts of circles. Let us first recall the concepts related to their perimeter and area of a circle.

★ **HISTORICAL FACTS**

Mensuration is that branch of mathematics which studies the method of measurements. Measurement is a very important human activity. We measure the length of a cloth for stitching. The area of a wall for painting, the perimeter of a plot for fencing. We do many other measurements of similar nature in our daily life. All these measurements, we shall study in this chapter called Mensuration.

π (pi) occupies the most significant place in measurement of surface area as well as volume of various solid and plane figures. The value of π is not exactly known. The story of the accuracy by which the value of π was estimated is an interesting one.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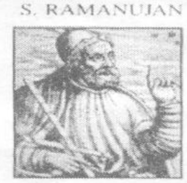
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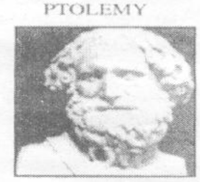

ARCHIMEDES

Mathematically $\pi = \dfrac{Cirumferene\ of\ a\ circle}{Diameter\ of\ the\ circle}$

$$\pi = \frac{(10+4)\ x\ 8+62000}{20{,}000} = \frac{62832}{20{,}000} = 3.1416$$

According to **S. Ramanujan,** the value of π

$$\pi = \frac{355}{113}$$

According to **Archimedes ;** the value of π is given below :

$$\pi = \frac{Cirumference\ of\ a\ circle}{Diameter\ of\ the\ circle} = 3\frac{10}{71} < \pi < 3\frac{10}{70}$$

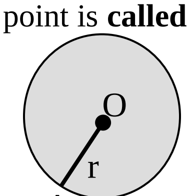
Ptolemy, $\pi = 3\dfrac{17}{120}$

The **Egyptians,** π = $\left(\dfrac{16}{9}\right)^2 = 3.160$

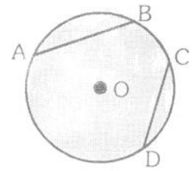
Note: π (pi) is an irrational number. It cannot be expressed as the ratio of whole numbers. However, the ratio 22 : 7 is often used as approximation for it.

★ **RECALL**

(A) **Circle:** Circle is the locus of a point which moves in such a manner that its distance from a fixed point O remains constant (the same). The fixed point is **called the centre** O and the constant distance OA is **called its radius.**

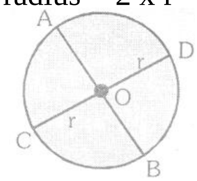


(B) **Chord :** A line segment joining two points on a circle is called a chord of the circle. If fig. AB and CD are two chords of the circle.

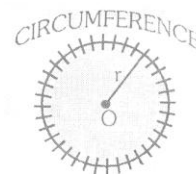


(C) **Diameter :** A chord passing through the centre of the circle is called the diameter. In fig, AOB and COD are diameter of the circle i.e., the diameter is the largest chord of the circle.

Length of diameter = Twice the radius = 2 x r = AOB = COD



(D) **Circumference :** The perimeter of the circle or the length of boundary of the circle is called its circumference i.e. the distance covered by traveling once around a circle is called the perimeter or circumference. The circumference of a circle is given by $2\pi r$. It is well-known fact that the ratio of the circumference of a circle to its diameter bears a constant ratio.

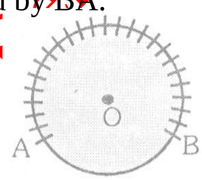


$$\pi = \frac{Circumference\ of\ a\ circle}{Diameter\ of\ the\ circle}$$

$\Rightarrow$ Circumference = π x diameter = π x 2r = $2\pi r$ where r is the radius of the circle.

(E) **Arc.** Any part of a circle is called an arc of the circle. Two points A and B on a circle divides it into two arcs. In general one arc is greater than other. The smaller arc is called minor arc and greater arc is called major arc.

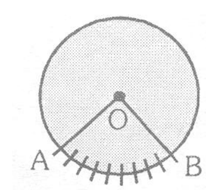
In the given fig, AB is an arc of a circle with centre O, denoted by $\overset{\frown}{AB}$. The remaining part of the circle shown by the dotted lines is represented by BA.



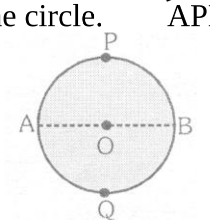
(F) **Central Angle :** Angle subtended by an arc at the centre of a circle is called its central angle. In fig. the centre of the circle is O.

Central angle made by $\overset{\frown}{AB}$ at the centre O = $\angle AOB$ - θ

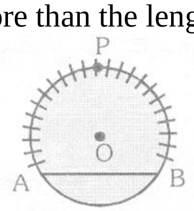
If $\theta^{o} < 180^{o}$ then the arc $\overset{\frown}{AB}$ is called the minor arc and the arc $\overset{\frown}{AB}$ is called major arc.



(G) **Semi-circle :** A diameter divides a circle into two congruent arcs. Each of these two arc is called a semicircle. In the given fig. of circle with centre O, $\overset{\frown}{APB}$ and $\overset{\frown}{BQA}$ are semicircles. Is the half of the circle.

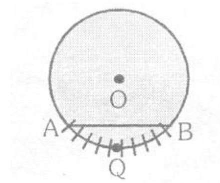


(H) **Major arc :** An arc whose length is more than the length of the semi-circle is called a major arc:



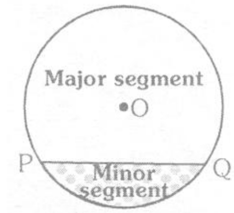
Major arc = $\overset{\frown}{APB}$

(I) **Minor arc :** An arc whose length is less than the length of semi-circle is called a minor arc.



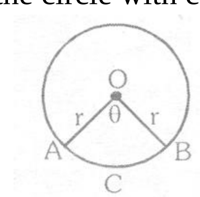
Minor arc = $\widehat{BQA}$

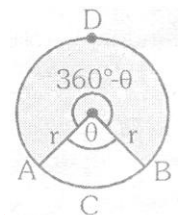
(J) **Segment :** A segment of a circle is the region bounded by an arc and its chord, including the arc and the chord.



The shaded segment containing the minor arc is called a minor segment, while the unshaded segment containing the major arc is called the major segment.

(K) **Sector of a circle :** A sector of a circle is a region enclosed by an arc and its two bounding radii. In the fig OACBO is a sector of the circle with centre O.





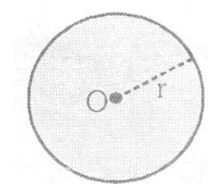
If arc AB is a minor arc then OACBO is a called the minor segment of the circle. The remaining part OADBO of the circle is called the major sector of the circle.

★ **FORMULA**

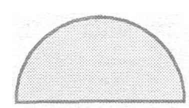
1. **For a circle of radius = r units, we have**

(a) Circumference of the circle = $(2\pi r)$ units = (πd) units,
Where d is the diameter.

(b) Area of the circle = (πr^2) sq. units.



II. **For a semi-circle of radius = r units, we have**

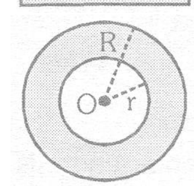


(a) Area of the semi-circle = $(\frac{1}{2}\pi r^2)$ sq. units

(b) Perimeter of the semi-circle = $(\pi r + 2r)$ units.

III. **Area of a Circular Ring :**

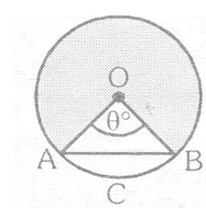
If R and r be the outer and inner radii of a ring, then
Area of the ring = $\pi (R^2 - r^2)$ sq. units



IV **Results on Sectors and Segments :**

Suppose an arc ACB makes an angle θ at the centre O of a circle of radius = r units. Then :

(a) Length of arc ACB = $\left(\frac{2\pi r\theta}{360}\right)$ units



(b) Area of sector OACBO = $\left(\frac{\pi r^2\theta}{360}\right)$ sq. units

$= \frac{1}{2} x\, r\, x \left(\frac{2\pi r\theta}{360}\right)$ sq. units = $\left(\frac{1}{2} x \text{ radius } x \text{ arc length}\right)$ sq. units

(c) Perimeter of sector OACBO = length of arc ACB + OA + OB = $\left(\frac{2\pi r\theta}{360} + 2r\right)$ units.

(d) Area of segment ACBA = (Area of sector OACBO) – (Area of ΔOAB) = $\left(\frac{\pi r^2\theta}{360} - \frac{1}{2}r^2 \sin\theta\right)$ sq. units.

(e) Perimeter of segment ACBA = arc ACB + chord AB) units.

(f) Area of Major segment BDAB = (Area of circle) – (Area of segment ACBA).

V. Rotations Made By a Wheel:

(a) Distance moved by a wheel in 1 revolution = Circumference of the wheel

(b) Number of rotations made by a wheel in unit time = $\frac{Distance\ moved\ by\ it\ in\ unit\ time}{Circuference\ of\ the\ wheel}$

VI. Facts About Clocks:

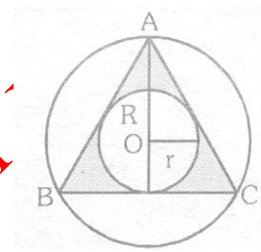
(a) Angle described by minute hand in 60 minutes = 360^o

(b) Angle described by minute hand in 5 minutes = $\left(\frac{360}{60} x\ 5\right)^o$ = 30^o

(c) Angle described by hour hand in 12 hours = 360^o.

Angle described by hour hand in 1 hour = 30^o.

VII. In an equilateral triangle of side a units, we have:



(a) Height of the triangle, h = $\frac{\sqrt{3}}{2}$ a units.

(b) Area of the triangle = $\left(\frac{\sqrt{3}}{4}a^2\right)$ sq. units.

(c) Radius of incircle, r = $\frac{1}{3}$h = $\left(\frac{1}{3}.\frac{\sqrt{3}}{2}a\right) = \left(\frac{a}{2\sqrt{3}}\right)$ units.

(d) Radius of circumcircle, R = $\frac{2}{3}h = \left(\frac{2}{3}.\frac{\sqrt{3}}{2}a\right) = \left(\frac{a}{\sqrt{3}}\right)$ units.

Thus, r = $\frac{a}{2\sqrt{3}}$ and R = $\frac{a}{\sqrt{3}}$

Ex.1 Calculate the circumference and area of a circle of radius 5.6 cm.

Sol. We have :

$Circum\ ference\ of\ the\ circle = 2\pi r = \left(2\ x\frac{22}{7}x5.6\right)$ cm = 35.2 cm.

Area of the circle = $\pi r^2 = \left(\frac{22}{7}x5.6x5.6\right)$ cm^2 = 98.56 cm^2.

Ex.2 The circumference of a circle is 123.2 cm. Calculate :

(i) the radius of the circle in cm,

(ii) the area of the circle, correct to nearest cm^2.

Sol. (i) Let the radius of the circle be r cm.

Then, its circumference = (2πr) cm.

$\therefore$ 2πr = 123.2 $\Rightarrow$ 2 x $\frac{22}{7}$ x r = 123.2 $\Rightarrow$ r = (123.2 x $\frac{7}{44}$) = 19.6 cm.

$\therefore$ Radius of the circle = 19.6 cm.

(ii) Area of the circle = πr^2 = ($\frac{22}{7}$ x 19.6 x 19.6) cm^2= 1207.36 cm^2.

$\therefore$ Area of the circle, correct to nearest cm^2 =m 1207 cm^2.

Ex.3 The area of a circle is 301.84 cm^2. Calculate:

(i) the radius of the circle in cm.

(ii) the circumference of the circle, correct to nearest cm

Sol. (i) Let the radius of the circle be r cm.

Then, its area = πr^2 cm^2 = 301.84

$\Rightarrow \frac{22}{7} \times r^2 = 301.84$

$\Rightarrow r^2 = \left(301.84 \times \frac{7}{22}\right) = 96.04 \Rightarrow r = \sqrt{96.04} = 9.8$ cm.

$\therefore$ Radius of the circle = 9.8 cm.

(ii) Circumference of the circle = $2\pi r = \left(2 \times \frac{22}{7} \times 9.8\right)$ cm = 61.6 cm.

$\therefore$ Circumference of the circle, correct to nearest cm = 62 cm.

Ex.4 The perimeter of a semi-circular protractor is 32.4 cm. Calculate :

(i) the radius of the protractor in cm,

(ii) the area of the protractor in cm^2.

Sol. (i) Let the radius of the protractor be r cm.

Then, its perimeter = $(\pi r + 2r)$ cm.

$\therefore \pi r + 2r = 32.4 \Rightarrow (\pi + 2)r = 32.4$

$\Rightarrow \left(\frac{22}{7} + 2\right) r = 32.4 \Rightarrow \frac{36}{7} r = 32.4 \Rightarrow r = \left(32.4 \times \frac{7}{36}\right)$ cm = 6.3 cm.

Radius of the protractor = 6.3 cm.

(ii) Area of the protractor = $\frac{1}{2}\pi r^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 6.3 \times 6.3\right)$ cm^2 = 62.37 cm^2.

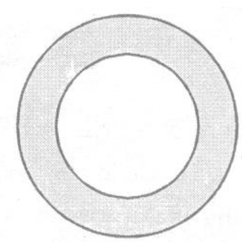
$\therefore$ Area of the protractor = 62.37 cm^2.

Ex.5 The area enclosed by the circumferences of two concentric circles is 346.5 cm^2. If the circumference of the inner circle is 88 cm, calculate the radius of the outer circle.

Sol. Let the radius of inner circle be r cm.

The, its circumference = $(2\pi r)$ cm.

$\therefore 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = \left(88 \times \frac{7}{44}\right)$ = 14 cm.



$\therefore$ Radius of the inner circle is, r = 14 cm.

Let the radius of the outer circle be R cm.

Than, area of the ring = $(\pi R^2 - \pi r^2)$ cm^2

$= \pi (R^2 - r^2)$ cm^2 = $\frac{22}{7} \times [R^2 - (14)^2]$ cm^2

$= \left(\frac{22}{7} R^2 - 616\right)$ cm^2

$\therefore \frac{22}{7} R^2 - 616 = 346.5 \Rightarrow \frac{22}{7} R^2 = 962.5$

$\Rightarrow R^2 = \left(962.5 \times \frac{7}{22}\right) = 306.25 \Rightarrow R = \sqrt{306.25} = 17.5$ cm.

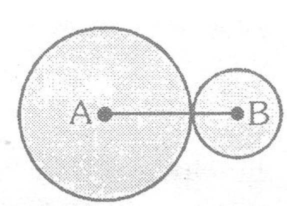
Hence, the radius of the outer circle is 17.5 cm.

Ex.6 Two circles touch externally. The sum of their areas is 130π sq. cm and distance between their centres is 14 cm. Determine the radii of the circles.

Sol. Let the radii of the given circles be R cm and r cm respectively. As the circles touch externally, distance between their centres = (R + r) cm.

$\therefore$ R + r = 14 ...(i)

Sum of their areas = $(\pi R^2 + \pi r^2)$ cm^2 = $\pi (R^2 + r^2)$ cm^2.



$\therefore \quad \pi(R^2 + r^2) = 130\pi$

$\Rightarrow \quad R^2 + r^2 = 130 \qquad \ldots\text{(ii)}$

We have the identity, $(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

$(14)^2 + (R - r)^2 = 2 \times 130 \qquad$ [From (i) and (ii)]

$(R - r)^2 = 64$

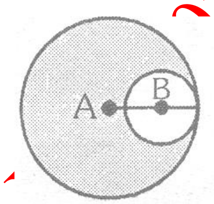
$R - r = 8 \qquad \ldots\text{(iii)}$

On solving (i) and (iii), we get R = 11 and r = 3.

Hence, the radii of the given circles are 11 cm and 3 cm.

Ex.7 Two circles touch internally. The sum of their areas is 116π sq. cm and the distance between their centres is 6 cm. Find the radii of the given circles.

Sol. Let the radii of the given circles be R cm and r cm respectively. As the circles touch internally, distance between their centres = (R – r) cm.



$\therefore \quad R - r = 6 \qquad \ldots\text{(i)}$

Sum of their areas = $(\pi R^2 + \pi r^2)$ cm^2 = $\pi(R^2 + r^2)$ cm^2

$\therefore \; \pi(R^2 + r^2) = 116\pi \Rightarrow R^2 + r^2 = 116 \qquad \ldots\text{(iii)}$

We have the identity, $(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

$\Rightarrow \quad (R + r)^2 + 6^2 = 2 \times 116 \qquad$ [Using (i) and (ii)]

$\Rightarrow \quad (R + r)^2 = 196$

$\Rightarrow \quad R + r = \sqrt{196} = 14 \qquad \ldots\text{(iii)}$

On solving (i) and (iii), we get R = 10 and r = 4.

Hence, the radii of the given circles are 10 cm and 4 cm.

Ex.8 The wheel of a cart is making 5 revolutions per second. If the diameter of the wheel is 84 cm, find its speed in km/hr. Give your answer, correct to nearest km.

Sol. Radius of the wheel = 42 cm.

Circumference of the wheel = $2\pi r = \left(2 \times \frac{22}{7} \times 42\right)$ cm = 264 cm.

Distance moved by the wheel in 1 revolution = 264 cm.

Distance moved by the wheel in 5 revolutions = (264 x 5) cm = 1320 cm.

$\therefore$ Distance moved by the wheel in 1 second = 1320 cm.

Distance moved by the wheel in 1 hour = (1320 x 60 x 60) cm.

$= \left(\frac{1320 \times 60 \times 60}{100 \times 1000}\right)$ km

$\therefore$ Speed of the cart = $\left(\frac{1320 \times 60 \times 60}{100 \times 1000}\right)$ km/hr = 47.52 km/hr.

Hence, the speed of the cart, correct to nearest km/hr is 48 km/hr.

Ex.9 The diameter of the driving wheel of a bus is 140 cm. How many revolutions must the wheel make in order to keep a speed of 66 km/hr?

Sol. Distance to be covered in 1 min. = $\left(\frac{66 \times 1000}{60}\right)$ m = 1100 m.

Radius of the wheel = $\left(\frac{140}{2}\right)$ cm = 70 cm = 0.70 m.

Circumference of the wheel = $2\pi r = \left(2 \times \frac{22}{7} \times 0.70\right)$ m = 4.4 m.

$\therefore$ Number of revolutions per minute = $\left(\frac{1100}{4.4}\right)$ = 250.

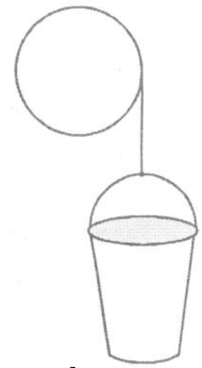
Hence, the wheel must make 250 revolutions per minute.

Ex.10 A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the bucket ascends in 1 min. 28 seconds with a uniform speed of 1.1 m / sec, calculate the number of complete revolutions the wheel makes in raising the bucket.

Sol. Time taken by bucket to ascend = 1 min. 28 sec. = 88 sec. Speed = 1.1 m/sec.

Length of the rope = Distance covered by bucket to ascend

= (1.1m x 88) m = (1.1 x 88 x 100) cm = 9680 cm.



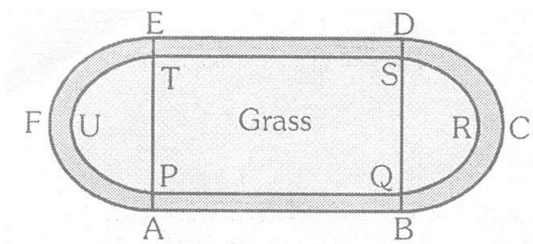
Radius of the wheel = $\frac{77}{2}$ cm.

Circumference of the wheel = $2\pi r = 2x\left(\frac{22}{7}x\frac{77}{2}\right)$ cm = 242 cm.

$\therefore$ Number of revolutions = $\frac{Length\ of\ the\ rope}{Circumference\ of\ the\ wheel} = \left(\frac{9680}{242}\right)$ = 40.

Hence, the wheel makes 40 revolutions to raise the bucket.

Ex.11 The figure shows a running track surrounding a grass enclosure PQRSTU. The enclosure consists of a rectangle PQST with a semi-circular region at each end. Given, PQ = 200 m and PT = 70 m.



(i) Calculate the area of the grassed enclosure in m^2.

(ii) Given that the track is of constant width 7 m, calculate the outer perimeter ABCDEF to the track.

Sol. (i) Diameter of each semi-circular region of grassed enclosure = PT = 70 m,

$\therefore$ Radius of each one of them = 35 m.

Area of grassed enclosure

= (Area of rect. PQST) + 2 x $\frac{1}{2}\pi r^2$ = $\left[(200x70)+\frac{22}{7}x35x35\right]$ m^2 = 17850 m^2.

(ii) Diameter of each outer semi-circle of the track = AE = (PT + 7 + 7) m = 84 m.

$\therefore$ Radius of each one of them = 42 m.

Outer perimeter ABCDEF = (AB + DE + semi-circle BCD + semi-circle EFA)

= (2PQ + 2 x circumference of semi-circle with radius 42 m)

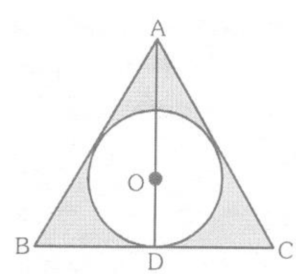
= (2 x 200 + 2 x π x 42) m = $\left[2\ x\ 200+2\ x\frac{22}{7}x\ 42\right]$ m = 664 m.

Ex.12 In an equilateral triangle of side 24 cm, a circle is inscribed, touching its sides. Find the area of the remaining portion of the triangle. Take $\sqrt{3}$ = 1.73 and π = 3.14.

Sol. Let ΔABC be the given equilateral triangle in which a circle is inscribed.

Side of the triangle, a = 24 cm.

Height of the triangle, h = $\left(\frac{\sqrt{3}}{2}x\,a\right)cm = \left(\frac{\sqrt{3}}{2}x\ 24\right)cm = 12\sqrt{3}\,cm.$



Radius of the incircle, r = $\frac{1}{3}h = \left(\frac{1}{3}x12\sqrt{3}\right)cm = 4\sqrt{3}\,cm.$

$\therefore$ Required Area = Area of the shaded region

= (Area of ΔABC) – (Area of incircle)

$$= \left(\frac{\sqrt{3}}{4} x\ 24\ x\ 24 - \pi\, x\, 4\sqrt{3}\, x\, 4\sqrt{3}\right) \text{cm}^2$$

$= (144\sqrt{3} - 3.14 \text{ x } 48)\ \text{cm}^2 = (144 \text{ x } 1.73 - 3.14 \text{ x } 48)\ \text{cm}^2$

$= [48 \text{ x } (3 \text{ x } 1.73 - 3.14)]\ \text{cm}^2 = (48 \text{ x } 2.05)\ \text{cm}^2 = 98.4\ \text{cm}^2$

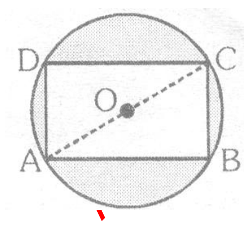
Ex.13 In the given figure, a circle circumscribes a rectangle with sides 12 cm and 9 cm. Calculate :

(i) the circumference of the circle to nearest cm,

(ii) the area of the shaded region, correct to 2 places of decimal, in cm^2.

Take $\pi = 3.14$.

Sol. Let ABCD be the rectangle with AB = 12 cm and BC = 9 cm.



$\therefore$ AC = $\sqrt{AB^2 + BC^2} = \sqrt{(12)^2 + 9^2} = \sqrt{225} = 15\,\text{cm}$.

Let O be the mid-point of AC.

Then, O is the centre and OA, the radius of the circum-circle.

$\therefore$ Radius, OA = $\frac{1}{2}$ AC = $\left(\frac{1}{2} x 15\right)$ cm = 7.5 cm.

$\therefore$ (i) Circumference of the circle = $2\pi r$ = (2 x 3.14 x 7.5) cm = 47.1 cm.

Hence, the circumference of the circle, correct to nearest cm is 47 cm.

(ii) Area of shaded region = (Area of the circle) – (Area of the rectangle)

$$= \left[\left(3.14 x \frac{15}{2} x \frac{15}{2}\right) - (12 x 9)\right] cm^2$$

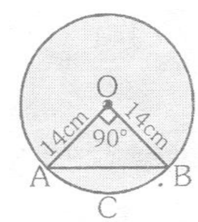
$= (176.625 - 108)\ \text{cm}^2 = 68.625\ \text{cm}^2 = 68.63\ \text{cm}^2$.

Ex.14 A chord of a circle of radius 14 cm makes a right angle at the centre. Calculate :

(i) the area of the minor segment of the circle, (ii) the area of the major segment of the circle.

Sol. Let AB be the chord of a circle with centre O and radius 14 cm such that $\angle$AOB = 90^o.

Thus, r = 14 cm and $\theta = 90^o$.



(i) Area of sector OACB = $\frac{\pi r^2 \theta}{360} = \left(\frac{22}{7} x 14 x \frac{90}{360}\right)$ cm^2 = 154 cm^2.

Area of ΔOAB = $\frac{1}{2} r^2 \sin\theta = \left(\frac{1}{2} x 14 x 14 x \sin 90^o\right)$ cm^2 = 98 cm^2.

$\therefore$ Area of minor segment ACBA = (Area of sector OACB) – (Area of ΔOAB) = (154 – 98) cm^2 = 56 cm^2.

(ii) Area of major segment BDAB

= (Area of the circle) – (Area of minor segment ACBA)

$= \left[\left(\frac{22}{7} x 14 x 14\right) - 56\right]$ cm^2 = (616 – 56) cm^2 = 560 cm^2.

Ex.15 The minute hand of a clock is 10.5 cm long. Find the area swept by it in 15 minutes.

Sol. Angle described by minute hand in 60 minutes = 360^o.

Angle described by minute hand in 15 minutes = $\left(\frac{360}{60} x 15\right)^o$ = 90^o.

Thus, required area is the area of a sector of a circle with central angle, $\theta = 90^o$. and radius, r = 10.5 cm.

Required area = $\left(\frac{\pi r^2}{360}\right) = \left(\frac{22}{7} x 10.5 x 10.5 x \frac{90}{360}\right)$ cm^2 = 86.63 cm^2.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. If the radii of two circles are 7 cm and 24 cm, then the radius of circle having area equal to the sum of the areas of the two circles, is

(A) 31 cm (B) 25 cm
(C) 17 cm (D) 28 cm

2. The cost of fencing a circular field at the rate of Rs. 24 per metre is RS. 5280. Then the cost of ploughing the field, at the rate of 50 paise/m^2, is

(A) Rs. 2875 (B) Rs. 3850
(C) Rs. 1925 (D) Rs. 1825

3. The inner circumference of a circular track is 220 m, and the track is 14 m wide. The cost of leveling the track, at 50 paise/m^2, is

(A) Rs. 1848 (B) Rs. 1663.2
(C) Rs. 1478.4 (D) None of these

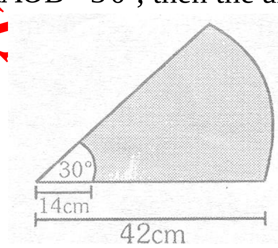
4. The area of a sector, of a circle with radius 7 cm and angle of the sector is 60^o. is

(A) $\frac{144}{3}$ cm^2 (B) $\frac{154}{21}$ cm^2
(C) $\frac{150}{7}$ cm^2 (D) $\frac{77}{3}$ cm^2

5. In a circle of radius 21 cm, an are subtends an angle of the centre. The area of the segment formed by the corresponding chord of the arc is

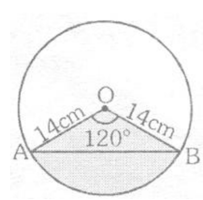
(A) 40.63 cm^2 (B) 421.73 cm^2
(C) 429.43 cm^2 (D) 40.27 cm^2

6. AB and CD are respectively arcs of two concentric circles of radii 42 cm and 14 cm and centre O as shown in the adjoining figure. If $\angle AOB = 30^o$, then the area of the shaded region is



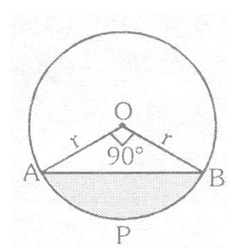
(A) $\frac{1232}{3} cm^2$ (B) $\frac{1220}{3} cm^2$
(C) 411cm^2 (D) None of these

7. In the given figure, the shaded area is



(A) 205.03 cm^2
(B) 205.04 cm^2
(C) 205.33 cm^2
(D) 205.35 cm^2

8. In the given figure, the area of the segment APB is



(A) $\frac{1}{4}\pi r^2$

(B) $\frac{1}{4}(\pi-2)r^2$

(C) $\frac{1}{4}(\pi-1)r^2$

(D) None of these

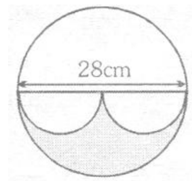
9. In the given figure, the area of shaded region is

(A) 462 cm^2

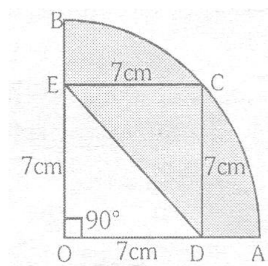
(B) 308 cm^2

(C) 616 cm^2

(D) 154 cm^2



10. In the given figure, ODCE is a square then the area of shaded region is



(A) 52.5 cm^2 (B) 24.5 cm^2 (C) 49 cm^2 (D) None of these

OBJECTIVE				ANSWER KEY					EXERCISE	
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	A	D	D	A	C	B	D	A

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

Remark : Take $\pi = \frac{22}{7}$, unless mentioned otherwise.

1. A sheet is 11 cm long and 2 cm wide. Circular pieces 0.5 cm in diameter are cut from it to prepare discs. Calculate the number of discs that can be prepared.

2. Find the circumference and area of a circle of radius 17.5 cm.

3. Find the circumference and area of a circle of diameter 91 cm.

4. Find the circumference and area of a circle of radius 15 cm. (Take $\pi = 3.14$)

5. The circumference of a circle is 123.2 cm. Taking $\pi = \frac{22}{7}$, calculate:

(i) the radius of the circle in cm;

(ii) the area of the circle in cm^2, correct to the nearest cm^2;

(iii) the effect on the area of the circle if the radius is doubled

6. Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 m^2.

7. The area of a circle is 394.24 cm^2. Calculate : (i) the radius of the circle, (ii) the circumference of the circle.

8. Find the perimeter and area of a semi-circular of a plate of radius 25 cm (Take $\pi = 3.14$).

9. The perimeter of a semi-circular metallic plate is 86.4 cm. Calculate the radius and area of the plate.

10. The circumference of a circle exceeds its diameter by 180 cm. Calculate
(i) the radius (ii) the circumference and (iii) the area of the circle.

11. A copper wire when bent in the form of a square encloses an area of 272.25 cm^2. If the same wire is bent into the form of a circle, what will be the area enclosed by the wire?

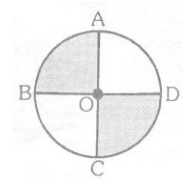
12. A copper wire when bent in the form of a equilateral triangle has an area of $121\sqrt{3}\ cm^2$. If the same wire is bent into the form of a circle, find the area enclosed by the wire.

13. The circumference of a circle field is 528 m.

14. The cost of leveling a circular field at Rs2 per sq. metre is Rs 33957. Calculate:
(i) the area of the field; (ii) the radius of then field; (iii) the circumference of the field;
(iv) the cost of fencing it at Rs 2.75 per metre.

15. The cost of fencing a circular field at Rs 9.50 per metre is Rs 2926. Find the cost of ploughing the field at Rs 1.50 per sq. metre.

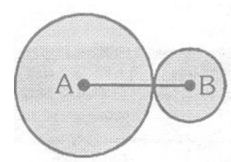
16. AC and BD are two perpendicular diameters of a circle ABCD. Given that the area of the shaded portion is 308 cm^2, calculate : (i) the length of AC; and (ii) the circumference of the circle.



17. The sum of the radii of two circles is 140 cm and the difference of their circumference is 88 cm. Find the radii of the two circles.

18. The sum of the radii of two circles is 84 cm and the difference of their areas is 5544 cm^2. Calculate the radii of the two circles .

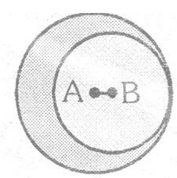
19. Two circles touch externally. The sum of their areas is $117\pi\ cm^2$ and the distance between their centres is 15 cm. Find the radii of the two circles.



20. Two circles touch internally. The sum of their areas is and the distance between then centres is 4 cm. Find the radii of the circles.

21. Find the area of a ring whose outer and inner radii are 19 cm and 16 cm respectively.

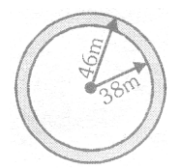
22. A path of width 8 m runs around a circular park whose radius is 38 m. Find the area of the path.

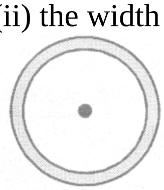


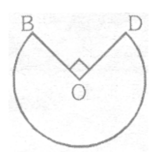
23. The areas of two concentric circles are 962.5 cm^2 and 1386 cm^2 respectively. Find the width of the ring.

24. The area enclosed between two concentric circles is 770 cm^2. If the radius of the outer circle is 21 cm, calculate the radius of the inner circle.

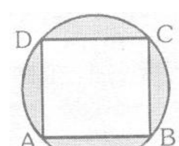
25. In the given figure, the area enclosed between two concentric circles is 808.5 cm^2. The circumference of the outer circle is 242 cm. Calculate : (i) the radius of the inner circle, (ii) the width of the ring.





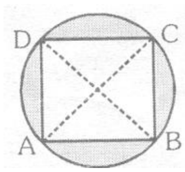


26. Find the area of a circle circumscribing an equilateral triangle of side 15 cm. [Take π = 3.14].



27. Find the area of a circle inscribed in an equilateral triangle of side 18 cm. [Take π = 3.14].

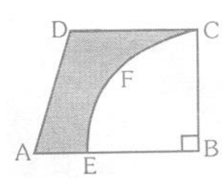
28. The shape of the top of a table in a restaurant is that of a segment of a circle with centre O and $\angle BOD = 90^o$. BO = OD = 60 cm. Find: (i) the area of the top of the table; (ii) the perimeter of the table. [Take π = 3.14].



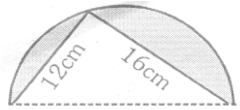
29. In the given figure, ABCD is a square of side 5 cm inscribed in a circle. Find:
(i) the radius of the circle, (ii) the area of the shaded region. [Take π = 3.14]

30. In the given figure, ABCD is a rectangle inscribed in a circle. If two adjacent sides of the rectangle be 8 cm and 6 cm, calculate : (i) the radius of the circle; and (ii) the area of the shaded region. [Take π = 3.14].

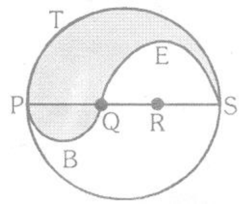
31. In the given figure, ABCD is a piece of cardboard in the shape of a trapezium in which AB || DC, $\angle ABC = 90^o$. From this piece, quarter circle BEFC is removed. Given DC = BC = 4.2 cm and AE = 2 cm.
Calculate the area of the remaining piece of the cardboard.



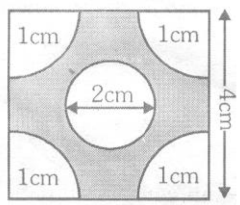
32. Find the perimeter and area of the shaded region in the given figure. (Take π = 3.142).



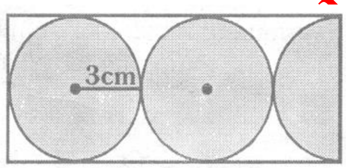
33. In the given figure, PQRS is a diameter of circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters. If PS = 12 cm, find the perimeter and the area of the shaded region. [Take π = 3.14].



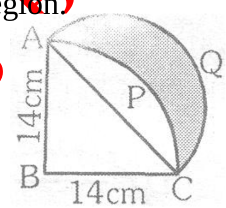
34. Find the perimeter and area of the shaded region shown in the figure. The four corners are circle quadrants and at the centre, there is a circle. [Take π = 3.14].



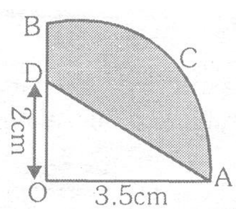
35. In the given figure, find the area of the unshaded portion within portion within the rectangle. [Take π = 3.14].



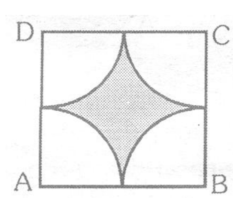
36. In the given figure, ABCP is a quadrant of circle of radius 14 cm. With AC as diameter, a semi-circle is drawn. Find the area of the shaded region.



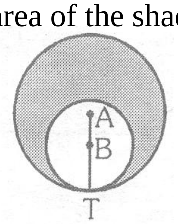
37. In the given, OACB is a quadrant of a circle. The radius OA = 3.5 cm, OD = 2 cm. Calculate the area of the shaded portion.



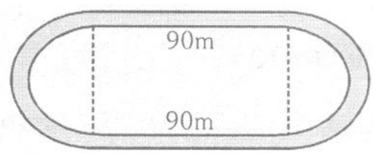
38. In the given figure, ABCD is a square of side 14 cm and A, B, C, D are centres of circular arcs, each of radius 7 cm. Find the area of the shaded region.



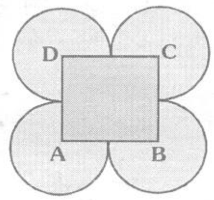
39. In the given figure, two circles with centres A and B touch each other at the point T. If AT = 14 cm and AB = 3.5 cm, find the area of the shaded region.



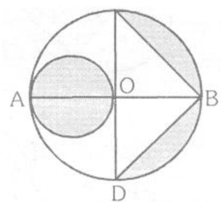
40. In the adjoining figure, the inside perimeter of a running track with semi-circular ends and straight parallel sides is 312 m. The lengths of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.



41. In the given figure, ABCD is a square of side 7 cm and A, B, C, D are centres of equal circles which touch externally in pairs. Find the area of the shaded region.



42. In the given figure, AB is the diameter of a circle with centre O and OA = 7 cm. Find the area of the shaded region.



43. The diameter of a wheel is 1.26 m. How far will it travel in 500 revolutions?

44. The wheel of the engine of a train $4\frac{2}{7}$ m in circumference makes 7 revolutions in 3 seconds. Find the speed of the train in km per hour.

45. A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 30 revolutions?

46. A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.

AREAS RELATED TO CIRCLES | **ANSWER KEY** | **EXERCISE – 2 (X)-CBSE**

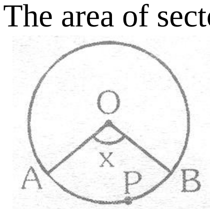
1. 88 **2.** 110 cm, 962.5 cm^2 **3.** 286 cm, 6506.5 cm^2 **4.** 94.2 cm, 706.5 cm^2
5. (i) 19.6 cm (ii)mm 1207 cm^2 (iii) area becomes four times. **6.** 56 m **7.** (i) 1.2 cm (ii) 70.4cm
8. 128.5 cm, 981.25 cm^2 **9.** 168. cm, 443.52 cm^2 **10.** (i) 42 cm (ii) 264 cm (iii) 5544 cm^2 **11.** 346.5 cm^2
12. 346.5 cm^2 **13.** (i) 84 m (ii) 22176 m^2 (iii) Rs 33264 **14.** (i) 16978.5 m^2 (ii) 73.5 m (iii) 462 m (iv) Rs 1270.50 **15.** Rs 11319 **16.** (i) 28 cm (ii) 88 cm **17.** 77 cm, 63cm **18.** 52.5 cm; 31.5 cm
19. 9 cm, 6 cm **20.** 11 cm, 7 cm **21.** 330 cm^2 **22.** 2112 m^2 **23.** 3.5 m^2 **24.** 14 cm
25. 35 cm, 3.5 cm **26.** 235.5 cm^2 **27.** 84.78 cm^2 **28.** (i) 8478 cm^2 (ii) 402.60 cm
29. (i) $\frac{5}{2}\sqrt{2}$ cm (ii) 14.25 cm^2 **30.** (i) 5 cm (ii) 30.5 cm^2 **31.** 7.28 cm^2 **32.** 59.4 cm, 61.1 cm^2
33. 37.68 cm, 37.68 cm^2 **34.** 20.56 cm, 9.72 cm^2 **35.** 19.35 cm^2 **36.** 98 cm^2 **37.** 6.125 cm^2
38. 42. cm^2 **39.** 269.5 cm^2 **40.** 636.57 m^2 **41.** 164.5 cm^2 **42.** 66.5 cm^2 **43.** 1980 m
44. 36 km/hr **45.** 50 **46.** 14 m

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

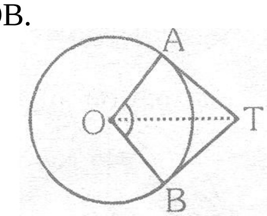
VERY SHORT ANSWER TYPE QUESTIONS

1. In the fig. O is the centre of a circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle. Find x.

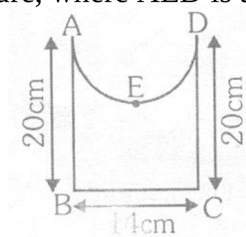


[Delhi-2008]

2. In fig., if $\angle ATO = 40^{o}$, find $\angle$ AOB. **[AI-2008]**



3. Find the perimeter of the given figure, where AED is a semi circle and ABCD is a rectangle. **[AI-2008]**

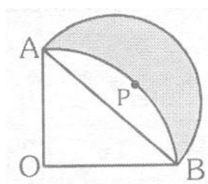


4. If the diameter of a semicircular protractor is 14 cm, then find it's perimeter. **[AI-2009]**

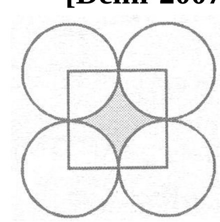
5. The length of the minute hand of a wall clock is 7 cm. How much area does it sweep in 20 minutes? **[Foreign-2009]**

SHORT ANSWER TYPE QUESTIONS

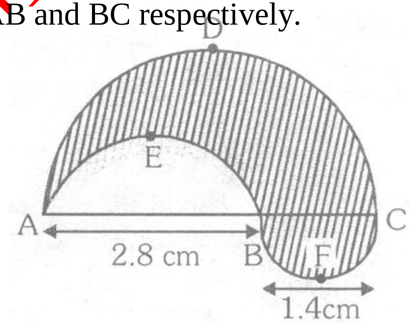
1. In fig. AOBPA is quadrant of a circle of radius 14 cm. A semicircle with AB as diameter is draw. Find the area of the shaded region.



2. Four circles are described about the four corners of a square of a square so that each touches two of the others as shown in fig. Find the area of the shaded region. Each side of the square is 14 cm. (Take π = 22/7) **[Delhi-2007]**

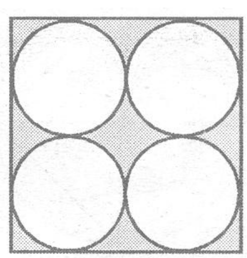


3. In the fig., find the perimeter of shaded region where ADC, AEB and BFC are semicircles on diameters AC, AB and BC respectively.

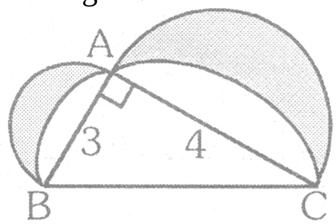


OR

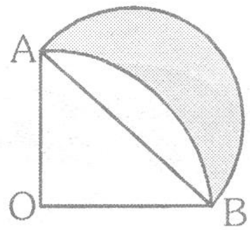
Find the area of the shaded region in the fig., where ABCD is a square of side 14 cm. **[Delhi-2008]**



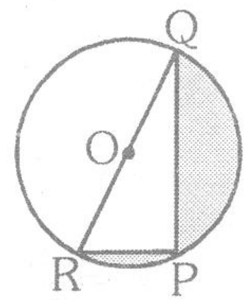
4. In fig., ABC is a right-angled triangle, right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region. **[AI-2008]**



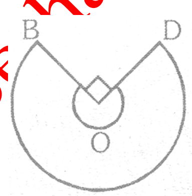
5. In the fig., ABC is a quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of the shaded region. **[Foreign-2008]**



6. In fig., PQ = 24 cm, PR = 7 cm and O is the centre of the circle. Find the area of shaded region. (Take π = 3.14) **[Delhi-2009]**

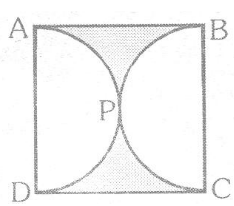


7. In figure, the shape of the top of a table in a restaurant is that of a sector of a circle with centre O and $\angle$BOD = 90°. If BO = OD = 60 cm, find.
(i) the area of the top of the table (ii) The perimeter of the table top. (Take π = 3.14)



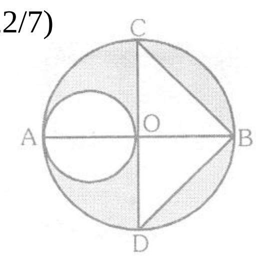
OR

In fig., ABCD is a square of side 14 cm and APD and BPC are semicircles. Find the area of shaded region.

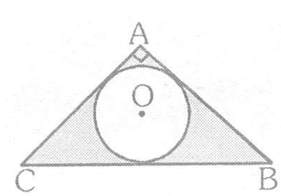


(Take π = 22/7) **[Foreign-2009]**

8. In fig., AB and CD are two perpendicular diameters of a circle with centre O. If OA = m7 cm. find the area of the shaded region. (Take π = 22/7) **[AI-2010]**



LONG ANSWER TYPE QUESTIONS



1. In fig., ABC is a right triangle right angled at A. Find the area of shaded region if AB = 6 cm, BC = 10 cm and O is the centre of the incircle of ΔABC. (Take π = 3.14) **[Delhi-2009]**

2. The area of an equilateral triangle is $49\sqrt{3}\,cm^2$. Taking each angular point as centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles. (Take $\sqrt{3}$ = 1.73)

AREAS RELATED TO CIRCLES | **ANSWER KEY** | **EXERCISE – 3 (X)-CBSE**

- **VERY SHORT ANSWER TYPE QUESTIONS**

  1. 100^o **2.** 100^o **3.** $(7\pi + 54)$ cm **4.** 36 cm **5.** $\frac{154}{3}\,cm^2$

- **SHORT ANSWER TYPE QUESTIONS**

  1. 98 cm^2 **2.** 42 cm^2 **3.** 13.2 cm or 42 cm^2 **4.** 6 sq. units **5.** 98 cm^2 **6.** 161.3 cm^2

- **LONG ANSWER TYPE QUESTIONS**

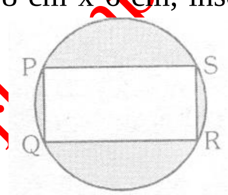
  1. 11.44 cm^2 **2.** 7.77 cm^2

EXERCISE – 4 (FOR OLYMPIADS)

CHOOSE THE CORRECT ONE

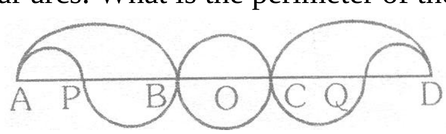
1. In the adjoining figure PQRS is a rectangle 8 cm x 6 cm, inscribed in the circle. The area of the shaded portion will be :

(A) 48 cm^2
(B) 42.50 cm^2
(C) 32.50 cm^2
(D) 30.5 cm^2



2. In the adjoining figure AB = CD = 2BC = 2BP = 2 CQ. In the middle, a circle with radius 1 cm is drawn. In the rest figure all are the semicircular arcs. What is the perimeter of the whole figure?

(A) 4 π
(B) 8π
(C) 10 π
(D) None of these



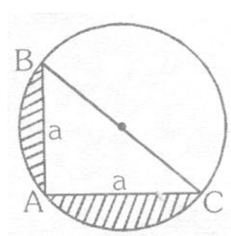
3. If BC passes through centre of the circle, then the area of the shaded region in the given figure is :

(A) $\frac{a^2}{2}(3-\pi)$

(B) $a^2\left(\frac{\pi}{2}-1\right)$

(C) $2a^2(\pi - 1)$

(D) $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$



4. Two circles of unit radii, are so drawn that the centre of each lies on the circumference of the other. The area of the region common to both the circles, is :

(A) $\frac{(4\pi-3\sqrt{3})}{12}$ (B) $\frac{(4\pi-6\sqrt{3})}{12}$

(C) $\frac{(4\pi - 3\sqrt{3})}{6}$ (D) $\frac{(4\pi - 6\sqrt{3})}{6}$

5. The area of the largest possible square inscribed in a circle of unit radius (in square unit) is :

(A) 3 (B) 4 (C) $2\sqrt{3\pi}$ (D) 2

6. The area of the largest triangle that can be inscribed in a semicircle of radius r is:

(A) r^2 cm^2 (B) $\left(\frac{r}{3}\right)^2 cm^2$ (C) $r\sqrt{2}$ cm^2 (D) $3\sqrt{3r}$ cm^2

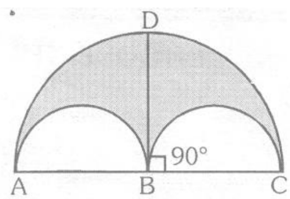
7. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is :

(A) $6\sqrt{3r}$ (B) 6r (C) 3r (D) 12r

8. If a regular circumscribes a circle of radius r, then its perimeter is :

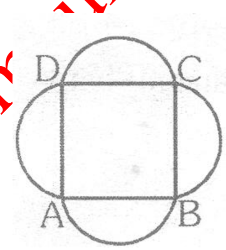
(A) $4\sqrt{3r}$ (B) $6\sqrt{3r}$ (C) 6r (D) $12\sqrt{3r}$

9. In the adjoining figure there are three semicircles in which BC = 6 cm and BD = $6\sqrt{3}$ cm. What is the area of the shaded region (in cm):



(A) 12π
(B) 9π
(C) 27π
(D) 28π

10. ABCD is a square of side a cm. AB, BC, CD and AD all are the chords of circles with equal radii each. It the chords subtends an angle of 120^o at their respective centres, find the total area of the given figure, where arcs are part of the circles:



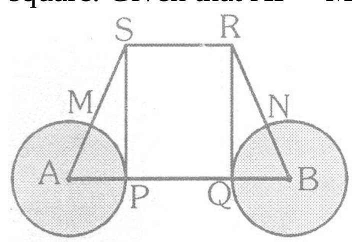
(A) $\left[a^2 + 4\left(\frac{\pi a^2}{9} - \frac{a^2}{3\sqrt{2}}\right)\right]$

(B) $\left[a^2 + 4\left(\frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}}\right)\right]$

(C) [9a^2 - 4π + $3\sqrt{3a^2}$]

(D) None of these

11. In the adjoining figure PQRS is a square and MS = RN and A, P, Q and B lie on the same line. Find the ratio of the area of two circles to the area of the square. Given that AP = Ms.



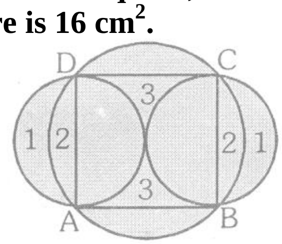
(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{3\pi}{2}$

(D) $\frac{6}{\pi}$

Direction for questions number (12 to 14) : In the adjoining figure ABCD is a square. A circle ABCD is passing through all the four vertices of the square. There are two more circles on the sides AD and BC touching each other inside the square, AD and BC are the respective diameters of the two smaller circles. Area of the square is 16 cm^2.



12. What is the area of region 1?

(A) 2.4 cm^2 (B) $\left(2-\frac{\pi}{4}\right)cm^2$

(C) 8 cm^2 (D) $(4\pi - 2)$ cm^2

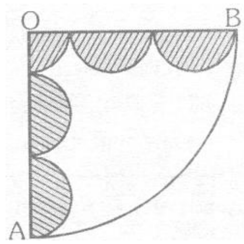
13. What is the area of region 2?

(A) $3(\pi - 2)$ cm^2 (B) $(\pi - 3)$ cm^2 (C) $(2\pi - 3)$ cm^2 (D) $4(\pi - 2)$ cm^2

14. What is the area of region 3?

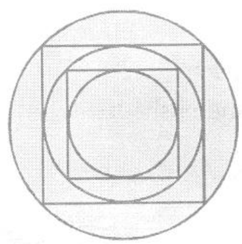
(A) $(4 - 4\pi)$ cm^2 (B) $4(4 - \pi)$ cm^2 (C) $(4\pi - 2)$ cm^2 (D) $(3\pi + 2)$ cm^2

15. A circular paper is folded along its diameter, then again it is folded to form a quadrant. Then it is cut as shown in the figure, after it the paper was reopened in the original circular shape. Find the ratio of the original paper to that of the remaining paper? (The shaded portion is cut off from the quadrant. The radius of quadrant OAB is 5 cm and radius of each semicircle is 1 cm) :



(A) 25 : 16 (B) 25 : 9 (C) 20 : 9 (D) None of these

Directions for questions number 16-18 : A square is inscribed in a circle then another circle is inscribed in the square. Another square is then inscribed in the circle. Finally a circle is inscribed in the innermost square. Thus there are 3 circles and 2 squares as shown in the fig. The radius of the outer-most circle is R.



16. What is the radius of the inner-most circle?

(A) $\frac{R}{2}$ (B) $\frac{R}{\sqrt{2}}$ (C) $\sqrt{2}R$ (D) None of these

17. What is the sum of areas of all the squares shown in the figure?

(A) $3R^2$ (B) $3\sqrt{2}R^2$ (C) $\frac{3}{\sqrt{2}}R^2$ (D) None of these

18. What is the ratio of sum of circumferences of all the circles to the sum of perimeters of all the squares?

(A) $(2+\sqrt{3})\pi R$ (B) $(3+\sqrt{2})\pi R$ (C) $3\sqrt{3}\pi R$ (D) None of these

Directions for questions number 19-21 : A regular hexagon is inscribed in a circle of radius R. Another circle is inscribed in the hexagon. Now another hexagon is inscribed in the second (smaller) circle.

19. What is the sum of perimeters of both the hexagons?

(A) $(2+\sqrt{3})R$ (B) $3(2+\sqrt{3})R$ (C) $3(3+\sqrt{2})R$ (D) None of these

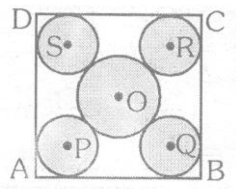
20. What is the ratio of area of inner circle to the outer circle?

(A) 3 : 4 (B) 9 : 16 (C) 3 : 8 (D) None of these

21. If there are some more circles and hexagons inscribed in the similar way as given above, then the ratio of each side of outermost hexagon (largest one) to that of the fourth (smaller one) hexagon is (fourth hexagon means the hexagon which is inside the third hexagon from the outerside.):

(A) $9 : 3\sqrt{2}$ (B) 16 : 9 (C) $8 : 3\sqrt{3}$ (D) None of these

22. In the adjoining diagram ABCD is a square with side 'a' cm. In the diagram the area of the larger circle with centre 'O' is equal to the sum of the areas of all the rest four circles with equal radii, whose centres are P, Q, R, and S. What is the ratio between the side of square and radius of a smaller circle?



(A) $(2\sqrt{2}+3)$ (B) $(2+3\sqrt{2})$ (C) $(4+3\sqrt{2})$ (D) Can't be determined.

23. There are two concentric circles whose areas are in the ratio of 9 : 16 and the difference between their diameters is 4 cm. What is the area of the outer circle?

(A) 32 cm^2 (B) 64π cm^2 (C) 36 cm^2 (D) 48 cm^2

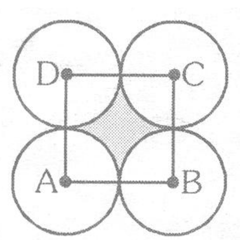
24. ABCD is a square, 4 equal circles are just touching each other whose centres are the vertices A, B, C, D of the square. What is the ratio of shaded to the unshaded area within square?

(A) $\frac{8}{11}$

(B) $\frac{3}{11}$

(C) $\frac{5}{11}$

(D) $\frac{6}{11}$



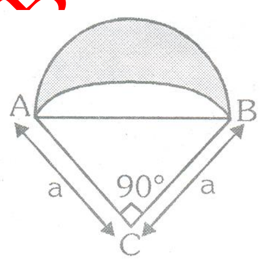
25. In the adjoining figure ACB is a quadrant with radius 'a'. A semicircle is drawn outside the quadrant taking AB as a diameter. Find the area of shaded region :

(A) $\frac{1}{4}(\pi - 2a^2)$

(B) $\left(\frac{1}{4}\right)(\pi a^2 - a^2)$

(C) $\frac{a^2}{2}$

(D) Can't be determined



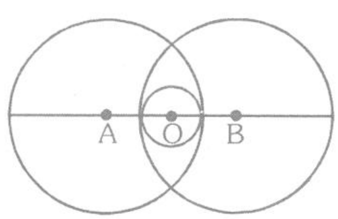
26. There are two circles intersecting each other. Another smaller circle with centre O, is lying between the common region of two larger circles. Centre of the circle (i.e., A, O and B) are lying on a straight line. AB = 16 cm and the radii of the larger circles are 10 cm each. What is the area of the smaller circle?

(A) 4π cm^2

(B) 2π cm^2

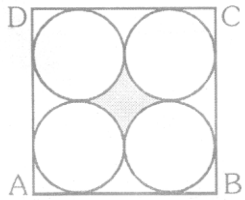
(C) $\frac{4}{\pi}$ cm^2

(D) $\frac{\pi}{4}$ cm^2



27. ABCD is a square, inside which 4 circles with radius 1 cm, each are touching each other. What is the area of the shaded region?

(A) $(2\pi - 3)\ \text{cm}^2$
(B) $(4 - \pi)\ \text{cm}^2$
(C) $(16 - 4\pi)\ \text{cm}^2$
(D) None of these



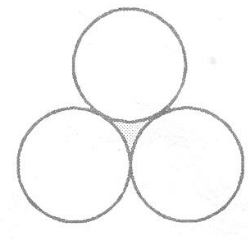
28. Three circles of equal radii touch each other as shown in figure. The radius of each circle is 1 cm. What is the area of shaded region?

(A) $\left(\frac{2\sqrt{3}-\pi}{2}\right)\ \text{cm}^2$

(B) $\left(\frac{3\sqrt{2}-\pi}{3}\right)\ \text{cm}^2$

(C) $\frac{2\sqrt{3}}{\pi}\ \text{cm}^2$

(D) None of these



OBJECTIVE **ANSWER KEY** **EXERCISE – 4**

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	D	C	D	C	D	A	B	A	C	B	B	C	D	B	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**		
Ans.	A	A	D	B	A	C	B	B	B	C	A	B	A		

ARITHMETIC PROGRESSIONS

★ INTRODUCTION

Consider the following arrangement of numbers :

(i) 1, 3, 5, 7, …….. (ii) 3, 6, 12, 24, …….. (iii) 1, 4, 9, 16, ……..

In each of the above arrangements, we observe some patterns. In (i) we find that the succeeding terms are obtained by adding a fixed number [i.e. 2], in (ii) by multiplying with a fixed number [i.e. 2], in (iii) we find that they are squares of natural numbers.

In this chapter, we shall discuss one of these patterns in which succeeding terms are obtained by adding a fixed number to the preceding terms. We shall also see how to find their n^{th} terms and the sum of n consecutive terms, and use this knowledge in solving some daily life problems.

★ HISTORICAL FACTS

Gauss was a very talented and gifted mathematician of 19^{th} century who developed the formula :

$1 + 2 + 3 + 4 + \ldots + (n - 1) + n = \frac{n(n+1)}{2}$ for the sum of first n natural numbers at the age of 10. He did this in the following way :

$$S = 1 + 2 + 3 \ldots\ldots + (n - 2) + (n - 1) + n$$
$$S = \underline{n + (n - 1) + (n - 2) + \ldots\ldots + 3 + 2 + 1}$$
$$2S = (n + 1) + (n + 1) + (n + 1) + \ldots + (n + 1) + (n + 1) + (n + 1)$$
$$= (n + 1)\,(1 + 1 + 1 + \ldots \text{ up to n times})$$
$$2S = (n + 1)\,n \Rightarrow S = \frac{n(n+1)}{2}$$

Even when he was a little child of three he could read and make mathematical calculation himself. Gauss proved the fundamental theorem of Algebra when he was 20 years old. His contribution to mathematics has been immense because his formulae were used in applied field of Astronomy, Differential Geometry and Electricity widely all over the world by scientists.

★ SEQUENCE

In our daily life, we come across the arrangement of numbers or objects in an order such as arrangement of students in a row as per their roll numbers, arrangement of books in the library, etc.

An arrangement of numbers depends on the given rule :

Given Rule	Arrangement of numbers
Write 3 and then add 4 successively	3. 7, 11, 15, 19,….
Write 3 and then multiply 4 successively	3, 12, 48, 192,…..
Write 4 and then subtract 3 successively	4, 1, - 2, -5,…..
Write alternately 5 and – 5	5, - 5, 5, -5,…

Thus, a sequence is an ordered arrangement of numbers according to a given rule.

Terms of a Sequence : The individual numbers that form a sequence are the terms of a sequence.

For example : 2, 4, 6, 8, 10,…… forming a sequence are called the first, second third, fourth and fifth,…. terms of the sequence.

The terms of a sequence in successive order is denoted by ‘T’$_n$ or ‘a’$_n$ The nth term ‘T’n is called the general terms of the sequence.

★ SERIES

The sum of terms of a sequence is called the series of the corresponding sequence. $T_1 + T_2 + T_3 + \ldots$ is an infinite series, where as $T_1 + T_2 + T_3 + \ldots + T_{n-1} + T_n$ is a finite series of n terms.

$$S_n = T_1 + T_2 + T_3 + \ldots + T_{n-2} + T_{n-1} + T_n$$
$$S_{n-1} = T_1 + T_2 + T_3 + \ldots + T_{n-2} + T_{n-1}$$
$$S_n - S_{n-1} = T_n$$

OR $\boxed{T_n = S_n - S_{n-1}}$

Ex.1 Write the first five terms of the sequence, whose nth term is $a_n = \{1 + (-1)^n\}n$.

Sol. $a_n = \{1 + (-1)^n\}n$

Substituting n = 1, 2, 3, 4 and 5, we get

$a_1 = \{1 + (-1)^1\}\ 1 = 0$; $a_2 = \{1 + (-1)^2\}\ 2 = 4$;

$a_3 = \{1 + (-1)^3\}\ 3 = 0$; $a_4 = \{1 + (-1)^4\}\ 4 = 8$;

$a_5 = \{1 + (-1)^5\}\ 5 = 0$

Thus, the required terms are : 0, 4, 0, 8 and 0.

Ex.2 Find the 20th term of the sequence whose nth term is, $a_n = \dfrac{n(n-2)}{n+3}$

Sol. $a_n = \dfrac{n(n-2)}{n+3}$. Putting n = 20, we obtain $a_{20} = \dfrac{20(20-2)}{20+3}$

Thus, $a_{20} = \dfrac{360}{23}$

Ex3. The Fibonacci sequence is defined by $a_1 = 1 = a_2$; $a_n = a_{n-1} + a_{n-2}$ for n > 2. Find $\dfrac{a_{n+1}}{a_n}$, for n = 1,2,3,4,5,

Sol. We have $a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$

Substituting n = 3, 4, 5 and 6, we get,

$a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

and $a_6 = a_5 + a_4 = 5 + 3 = 8$

Now, we have to find $\dfrac{a_{n+1}}{a_n}$ for n = 1, 2, 3, 4 and 5

For, n = 1, $\dfrac{a_2}{a_1} = \dfrac{1}{1} = 1$

n = 2, $\dfrac{a_3}{a_2} = \dfrac{2}{1} = 2$

n = 3, $\dfrac{a_4}{a_3} = \dfrac{3}{2}$

n = 4, $\dfrac{a_5}{a_4} = \dfrac{5}{3}$

n = 5, $\dfrac{a_6}{a_5} = \dfrac{8}{5}$

Hence, the required values are 1, 2, $\dfrac{3}{2}, \dfrac{5}{3}$ and $\dfrac{8}{5}$

COMPETITION WINDOW

SERIES OF NATURAL NUMBERS

1. The sum of first n natural numbers i.e. 1 + 2 + 3 +……+ n is usually written as $\sum n$.

$$\boxed{\sum n = \frac{n(n+1)}{2}}$$

2. The sum of squares of first n natural numbers i.e. $1^2 + 2^2 + 3^2 + \ldots + n^2$ is usually written as $\sum n^2$.

$$\boxed{\sum n^2 = \frac{n(n+1)(2n+1)}{6}}$$

3. The sum of cubes of first n natural numbers i.e. $1^3 + 2^3 + 3^3 + \ldots + n^3$ is usually written as $\sum n^3$.

$$\boxed{\sum n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\sum n\right)^2}$$

★ **PROGRESSON**

It is not always possible to write each and every sequence of some rule.
For example of prime numbers 2, 3, 5, 7, 11,… cannot be expressed explicitly by stating a rule and we do not have any expression for writing the general term of this sequence.
The sequence that follows a certain pattern is called a progression. Thus, the sequence 2, 3, 5, 7, 11,… is not a progression. In a progression, we can always write the nth term.

Consider the following collection of numbers : (i) 1, 3, 5, 7… (ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

From the above collection of numbers, we observe that
(i) Each term is greater than the previous by 2.
(ii) In each term the numerator is 1 and the denominator is obtained by adding 1 to the preceding denominator.
Thus, we observe that the collection of numbers given in (i) and (ii) follow a certain pattern and as such are all progressions.

★ **ARITHMETIC PROGRESSIONS**

An arithmetic progression is that list of numbers in which the first term is given and each term, other than the first term is obtained by adding a fixed number 'd' to the preceding term.
The fixed term **'d'** is known as the **common difference** of the arithmetic progression. It's value can be positive, negative or zero. The **first term** is denoted by **'a'** or **'a_1'** and the **last term** by 'ℓ'.
Ex. Consider a sequence 6, 10, 14, 18, 22, …

Hence, $a_1 = 6, a_2 = 10, a_3 = 14, a_4 = 18, a_5 = 22$
$a_2 - a_1 = 10 - 6 = 4$
$a_3 - a_2 = 14 - 10 = 4$
$a_4 - a_3 = 18 - 14 = 4$

Therefore, the sequence is an arithmetic progression in which the first term a = 6 and the common difference d = 4.

Symbolical form : Let us denote the first term of an AP by a_1, second term by a_2,….nth term by a_n and the common difference by d. Then the AP becomes $a_1, a_2, a_3, \ldots, a_n$.
So, $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$.

General form : In general form, an arithmetic progression with first term 'a' and common difference 'd' can be represented as follows :

a, a + d, a + 2d, a + 3d, a + 4d,….

Finite AP : An AP in which there are only a finite number of terms is called a finite AP. It may be noted that each such AP has a last term.

Ex. (a) The heights (in cm) of some students of school standing in a queue in the morning assembly are 147, 148, 149,…., 157.

(b) The minimum temperatures (in degree Celsius) recorded for a week in the month of January in a city arranged in ascending order are – 3.1, - 3.0, - 2.9, - 2.8, - 2.7, - 2.6, - 2.5

Infinite AP : An AP in which the number of terms is not finite is called infinite AP. It is note worthy that such APs do not have a last term.

Ex. (a) 1, 2, 3, 4,……

(b) 100, 70, 40, 10,….

Least Information Required : To know about an AP, the minimum information we need to know is to know both – the first term a and the common difference d.

For instance if the first term a is 6 and the common difference d is 3, then AP is 6, 9, 12, 15,…

Similarly, when

a = - 7, d = - 2, the AP is – 7, - 11, -13,….

a = 1.0, d = 0.1, the AP is 1.0, 1.1, 1.2, 1.3,…

So if we know what a and d are we can list the AP.

Ex.4 In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs.150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs. 10000 is deposited at compound interest 8% per annum. **[NCERT]**

Sol. (i) Taxi fare for 1 km = Rs. 15 = a_1

Taxi fare for 2 kms = Rs. 15 + 8 = Rs. 23 = a_2

Taxi fare for 3 kms = Rs. 23 + 8 = Rs. 31 = a_3

Taxi fare for 4 kms = Rs. 31 + 8 = Rs. 39 = a_4 and so on.

$a_2 - a_1$ = Rs. 23 – 15 = Rs. 8

$a_3 - a_2$ = Rs. 31 – 23 = Rs. 8

$a_4 - a_3$ = Rs. 39 – 31 = Rs. 8

i.e., $a_{k+1} - a_k$ is the same every time.

So, this list of numbers form an arithmetic progression with the first term a = Rs 15 and the common difference d = Rs. 8

(ii) Amount of air present in the cylinder = x units (say) = a_1

Amount of air present in the cylinder after one time removal of air by the vacuum pump $= x - \frac{x}{4} = \frac{3x}{4}$ units a_2

Amount of air present in the cylinder after two time removal of air by the vacuum pump $= -\frac{3x}{4} - \frac{1}{4}\left(\frac{3x}{4}\right) = \frac{3x}{4} - \frac{3x}{16} = \frac{9x}{16}$ units $= \left(\frac{3}{4}\right)^2$ x units = a_3

Amount of air present in the cylinder after three times removal of air by the vacuum pump $= \left(\frac{3}{4}\right)^2 x - \frac{1}{4}\left(\frac{3}{4}\right)^2 x \Rightarrow \left(1-\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 x \Rightarrow \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)^2 x = \left(\frac{3}{4}\right)^3$ x units = a_4 and so on.

$a_2 - a_1 = \frac{3x}{4} - x = \frac{x}{4}$ units

$a_3 - a_2 = \left(\frac{3}{4}\right)^2 x - \frac{3}{4}x = -\frac{3}{16}$ x units

As $a_2 - a_1 \neq a_3 - a_2$, this list of numbers does not form an AP.

(iii) Cost of digging the well after 1 metre of digging = Rs. 150 = a_1
Cost of digging the well after 2 metres of digging = Rs. 150 + 50 = Rs 200 = a_2
Cost of digging the well after 3 metres of digging = Rs. 200 + 50 = Rs 250 = a_3
Cost of digging the well after 4 metres of digging = Rs. 250 + 50 = Rs 300 = a_4
and so on.

$a_2 - a_1$ = Rs 200 – 150 = 50
$a_3 - a_2$ = Rs 250 – 200 = 50
$a_4 - a_3$ = Rs 300 – 250 = 50

i.e., $a_{k-1} - a_k$ is the same every time. So this list of numbers forms an AP with the first term a = Rs. 150 and the common difference d = Rs.50

(iv) Amount of money after 1 year = Rs. 10000 $\left(1+\frac{8}{100}\right) = a_1$

Amount of money after 2 year = Rs. 10000 $\left(1+\frac{8}{100}\right)^2 = a_2$

Amount of money after 3 year = Rs. 10000 $\left(1+\frac{8}{100}\right)^3 = a_3$

Amount of money after 4 years = Rs. 10000 $\left(1+\frac{8}{100}\right)^4 = a_4$

$a_2 - a_1$ = Rs. 10000 $\left(1+\frac{8}{100}\right)^2$ - Rs. 10000 $\left(1+\frac{8}{100}\right)$

= Rs. 10000 $\left(1+\frac{8}{100}\right)\left(1+\frac{8}{100}-1\right) \Rightarrow$ Rs. 10000 $\left(1+\frac{8}{100}\right)\left(\frac{8}{100}\right)$

$a_3 - a_2$ = Rs. 10000 $\left(1+\frac{8}{100}\right)^3$ - Rs. 10000 $\left(1+\frac{8}{100}\right)^2 \Rightarrow$ Rs. 10000 $\left(1+\frac{8}{100}\right)^2\left(1+\frac{8}{100}-1\right)$ = Rs. 10000 $\left(1+\frac{8}{100}\right)^2\left(\frac{8}{100}\right)$

As $a_2 - a_1 \neq a_3 - a_2$, this list of numbers does not form an AP.

Ex.5 Write first four terms of the AP, when the first term a and the common difference d are given as follows
(i) a = 4, d = 5 (ii) a = - 1.25, d = - 0.25

Sol. (i) a = 4, d = 5

First term,	a = 4
Second term	= 4 + d = 4 + 5 = 9
Third term	= 9 + d = 9 + 5 = 14
Fourth term	= 14 + d = 14 + 15 = 19

Hence, first four terms of the given AP are 4, 9, 14, 19.

(ii) a = – 1.25, d = – 0.25

First term = a = – 1.25
Second term = - 1.25 + d = - 1.25 + (- 0.25) = - 1.50
Third term = - 1.50 + d = - 1.50 + (-0.25) = - 1.75
Fourth term = - 1.75 + d = - 1.75 + (-0.25) = - 2.00
Hence, first four terms of the given AP are – 1.25, - 1.50, - 1.75, - 2.00

Ex.6 For the AP $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$,….. write the first term a and the common difference d. Also write the next two terms after the given last term - $\frac{3}{2}$.

Sol. We have $a_1 = \frac{3}{2}, a_2 = \frac{1}{2}, a_3 = -\frac{1}{2}, a_4 = -\frac{3}{2}$ and so on.

Thus, $a = \frac{3}{2}$

$a_2 = a_1 = \left(\frac{1}{2}\right) - \left(\frac{3}{2}\right) = -1,$

$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right) = -1,$

$a_4 - a_3 = \left(-\frac{3}{2}\right) - \left(-\frac{1}{2}\right) = -1$, and so on.

⇒ $d = -1$

Now, we find the successor of - $\frac{3}{2}$.

$a_5 = \left(-\frac{3}{2}\right) + d = \left(-\frac{3}{2}\right) + (-1) = -\frac{5}{2}$

Then $a_6 = a_5 + d = \left(-\frac{5}{2}\right) + (-1) = -\frac{7}{2}$

Hence, the next two terms after the given term $-\frac{3}{2}$ *are* $-\frac{5}{2}, -\frac{7}{2}$.

COMPETITION WINDOW

GEOMETRIC PROGRESSION

1. A sequence of non-zero numbers $a_1, a_2, a_3, \ldots, a_n$ is said to be a geometric sequence or G.P.

iff $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots\ldots$

i.e. iff $\frac{a_{n+1}}{a_n}$ = a constant for all n.

This constant is called the common ratio of the G.P. and is usually denoted by 'r'. e.g., 3, 9, 27, 81,….
A general G.P. is a, ar, ar^2,…
When the terms of a geometric sequence are added, we get a geometric series.

HARMONIC PROGRESSION

A sequence of non-zero numbers $a_1, a_2, \ldots a_n$ is said to be a harmonic sequence or H.P.

iff $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots \frac{1}{a_n}$ are in A.P.

e.g., (i) 12, 6, 4, 3….. (ii) 10, 30, - 30, - 10, - 6,…..

A general H.P. is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} + \ldots$

Where is the first term and d is the common difference of the A.P.

★ **GENERAL TERM OF AN ARITHEMETIC PROGRESSION**

The formula for writing general term or the nth term of an arithmetic progression is

$$a_n = a + (n - 1)d$$

Where, a is the first term of arithmetic progression,

and d is the common difference of arithmetic progression.

★ **r^{th} TERM OF FINITE ARITHMETIC PROGRESSION FROM THE END**

Let there be an arithmetic progression with first term a and common difference d. If there are n terms in the arithmetic progression, then

$$r^{th} \text{ term from the end} = a + (n - r)d$$

Also, if ℓ is the last term of the arithmetic progression then r^{th} term from the end is the r^{th} term of an arithmetic progression whose first term is ℓ and common difference is – d.

$$\text{rth term form the end} = \ell + (r - 1)(- d)$$

Ex.7 Find the 30th term of the AP : 10, 7, 4…. **[NCERT]**

Sol. The given A.P. is 10, 7, 4,…..

Here, a = 10, d = 7 – 10 = - 3 and n = 30

we have $a_n = a + (n - 1)d$

So, $a_{30} = 10 + (30 - 1)(-3)$

$\Rightarrow$ $a_{30} = 10 - 87$ $\Rightarrow$ $a_{30} = - 77$

$\therefore$ The 30^{th} term of the given AP is – 77.

Ex.8 The 6^{th} term of an arithmetic progression is – 10 and the 10^{th} term is – 26. Determine the 15^{th} term of the AP.

Sol. Let first term and the common difference of the AP be a and d respectively.

6^{th} term = - 10 (Given)

$\Rightarrow$ a + 5 (6 – 1)d = - 10 [$\because$ $a_n = a + (n - 1)d$]

$\Rightarrow$ a + 5d = - 10 ….(i)

10^{th} term = - 26 (Given)

$\Rightarrow$ a + (10 – 1)d = - 26

$\Rightarrow$ a + 9d = - 26 ….(ii)

Solving (i) and (ii) we get

a = 10, d = - 4

Therefore, 15^{th} term of the AP

= a + (15 – 1)d [$\because$ $a_n = a + (n - 1)d$]

= $a + 14d$

= $10 + 14(-4)$

= $10 - 56 = -46$

Hence, the 15^{th} term of AP is – 46.

Ex.9 Find the 6th term from the end of the AP 17, 14, 11,…., - 40.

Sol. The given AP 17, 14, 11,…., - 40

Here, $a = 17$, $d = 14 - 17 = -3$, $\ell = -40$

Let there be n terms in the given AP.

Then, nth term = - 40

$\Rightarrow$ $a + (n-1)d = -40$ $[\because a_n = a + (n-1)d]$

$\Rightarrow$ $17 + (n-1)(-3) = -40$

$\Rightarrow$ $(n-1)(-3) = -40 - 17$

$\Rightarrow$ $(n-1)(-3) = -57$

$\Rightarrow$ $n - 1 = \frac{-57}{-3}$

$\Rightarrow$ $n - 1 = 19$

$\Rightarrow$ $n = 19 + 1$

$\Rightarrow$ $n = 20$

Hence, there are 20 terms in the given AP, Now, 6th term from the end

= $a + (20-6)d$ $[\because$ rth term from the end $= a + (n-r)d]$

= $a + 14d$

= $17 + 14(-3)$

= $17 - 42 = -25$

Hence, the 6th term from the end of the given AP is – 25.

Ex.10 is 200 any term of the sequence 3, 7, 11, 15,…?

Sol. The given sequence is 3, 7, 11, 15,…

$a_2 - a_1 = 7 - 3 = 4$

$a_3 - a_2 = 11 - 7 = 4$

$a_4 - a_3 = 15 - 11 = 4$

As $a_{k+1} - a_k$ is the same for k = 1, 2, 3, etc., the given sequence form an AP.

Here, $a = 3$, $d = 4$

Let 200 be the nth term of the given sequence. Then,

$a_n = 200$

$\Rightarrow a + (n-1)d = 200$ $\Rightarrow 3 + (n-1)4 = 200$

$\Rightarrow (n-1) = \frac{197}{4}$ $\Rightarrow n = \frac{197}{4} + 1 \Rightarrow n = \frac{201}{4}$.

But n should be a positive integer. So, 200 is not term of the given sequence.

COMPETITION WINDOW

GENERAL TERM OF A.G.P.

The nth terms of a G.P. is a, ar^2,….ar^{n-1} is $\boxed{T_n = ar^{n-1}}$

rth TERM FROM THE END OF FINITE G.P.

Let a be the first term and r be the common ratio of a finite G.P. consisting of n terms, then

$$\boxed{\text{rth term from the end} = ar^{n-r}}$$

Also, if ℓ is the last term of the G.P. then

$$\boxed{\text{rth term from the end} = \ell\left[\frac{1}{r}\right]^{n-1}}$$

GENERAL TERM OF A H.P.

To find the nth term of an H.P., find the nth term of the corresponding A.P. obtained by the reciprocals of the terms of the given H.P. Now the reciprocal of the nth term of an A.P., will be the nth term of the H.P.

Try out the Following:

1. Find the 9th term and the general term of the progression $\frac{1}{4}, \frac{-1}{2}, 1, -2....$

2. Which term of the G.P. 5, 10, 20, 40, ….is 5120?

3. The fourth, seventh and the last term of a G.P. are 10, 80 and 2560 respectively Find the first term and the number of terms in the G.P.

4. Find the 9th term of progression $\frac{1}{7}+\frac{1}{14}+\frac{1}{21}+\frac{1}{28}+....$

5. If the pth term of a H.P. is qr and it's qth term is pr, then find it's rth term.

6. Find the 6th term of the series $2 + 1\frac{3}{4} + 1\frac{5}{9}, ...$

ANSWER KEY

1. 64; $(-1)^{n-1}\,2^{n-3}$ **2.** 11th term **3.** $\frac{10}{8}$, 12 terms **4.** $\frac{1}{63}$ **5**. pq **6.** $\frac{7}{6}$

★ **SELECTION OF TERMS IN AN AP**

Sometimes we require certain number of terms in AP. The following ways of selecting terms are generally very convenient.

Number of terms	Terms	Common difference
3	a-d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + 3d, a + 5d	2d

It should be noted that in case of an odd number of terms, the middle term is a and the common difference is d while in case of an even number of terms the middle terms are a – d, a + d and the common difference is 2d.

Remark–1 : If the sum of terms is not given, then select terms as a, a + d, a + 2d,….

Remark–2 : If three numbers a, b, c in order are in AP. Then

$$b - a = \text{Common difference} = c - b$$
$$\Rightarrow \quad b - a = c - b$$
$$\Rightarrow \quad 2b = a + c$$

Thus, a, b, c are in AP if and only if $\boxed{2b = a + c}$

Remark–3 : If a, b, c are in AP, then b is known as the arithmetic mean (AM) between a and c.

Remark–4 : If a, x, b are in AP Then,

$$2x = a + b \Rightarrow x = \frac{a+b}{2}$$

Thus, AM between a and b is $\frac{a+b}{2}$.

Ex.11 The sum of three numbers in AP is -3, and their product is 8. Find the numbers.

Sol. Let the numbers be (a – d), a, (a + d). Then,

Sum = - 3 $\Rightarrow$ (a – d) + a + (a + d) = - 3 $\Rightarrow$ 3a = - 3 $\Rightarrow$ a = - 1

Now, product = 8

$\Rightarrow$ (a – d) (a) (a +d) = 8

$\Rightarrow$ $a(a^2 - d^2) = 8$

$\Rightarrow$ $(-1)(1 - d^2) = 8$ [$\because$ a = - 1]

$\Rightarrow$ $d^2 = 9 \Rightarrow d = \pm 3$

If d = 3, the numbers are – 4, -1, 2. If d = - 3, the numbers are 2, - 1, - 4

Thus, the numbers are – 4, - 1, 2 or 2, - 1, - 4

Ex.12 Find four numbers in AP, whose sum is 20 and the sum of whose squares is 120.

Sol. Let the numbers be (a – 3d), (a – d), (a + d), (a + 3d). Then,

Sum = 20

$\Rightarrow$ (a – 3d) + (a –d) + (a + d) + (a + 3d) = 20 $\Rightarrow$ 4a = 20 $\Rightarrow$ a = 5

Now sum of the squares = 120

$\Rightarrow$ $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$

$\Rightarrow$ $4a^2 + 20d^2 = 120$

$\Rightarrow$ $a^2 + 5d^2 = 30$ $\Rightarrow$ $25 + 5d^2 = 30$

$\Rightarrow$ $25 + 5d^2 = 30$ $\Rightarrow$ $5d^2 = 5 \Rightarrow d = \pm 1$

If d = 1, then the numbers are 2, 4, 6, 8. If d = - 1, then the numbers are 8, 6, 4, 2.

Thus, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2.

Ex.13 If 2x, x + 10, 3x + 2 are in AP. Find the value of x.

Sol. Since, 2x, x + 10, 3x + 2 are in AP.

$\therefore$ 2 (x + 10) = 2x + (3x + 2)

$\Rightarrow$ 2x + 20 = 5x + 2

$\Rightarrow$ 3x = 18

$\Rightarrow$ x = 6.

COMPETITION WINDOW

SELECTION OF TERMS IN G.P.

Sometimes it is required to select a finite number of terms in G.P. It is always convenient if we select the terms in the following manner :

No. of Terms	Terms	Common Ratio
3	$\frac{a}{r}$, a, ar	R
4	$\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar^3	r^2
5	$\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2	r

If the product of the numbers is not given, then the numbers are taken as a, ar, ar^2, ar^3,…

TRY OUT THE FOLLOWING

1. If the sum of three numbers in G.P. is 38 and their product is 1728, find them.

2. Find the three numbers in G.P. is whose sum is 13 and the sum of whose squares is 91.

3. Find four numbers in G.P. whose sum is 85 and product is 4096.

4. Three numbers are in G.P. whose sum is 70. If the extremes be each multiplied by 4 and the means by 5, they will be in A.P. Find the numbers.

5. Find four numbers in G.P. in which the third term is greater than the first by 9 and the second term is greater than the fourth by 18.

6. The product of first three terms of a G.P. is 1000. If 6 is added to it's second term and 7 added to its third term, the terms become in A.P. Find the G.P.

ANSWERS

1. 8, 12, 18, or 18, 12, 8 **2.** 1, 3, 9 or 9, 3, 1 **3.** 1, 4, 16, 64 or 64, 16, 4, 1

4. 10, 20, 40, or 40, 20, 10 **5.** 3, - 6, 12, -24 **6.** 5, 10, 20,… or 20, 10, 5…

★ SUM TO N TERMS OF AN ARITHMETIC PROGRESSION

The sum S_n of n terms of an arithmetic progression with first term 'a' and common difference 'd' is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

OR

$$S_n = \frac{n}{2}[a + \ell]$$

Where ℓ = last term.

Remark–1 : In the formula $S_n = \frac{n}{2}[2a + (n - 1) d]$, there are four quantities viz. S_n, a, n and d. If any three of these are known, the fourth can be determined. Sometimes, two of these quantities are given.
In such a case, remaining two quantities are provided by some other relation.

Remark–2 : If the sum S_n of n terms of a sequence is given, then n^{th} term a^n of the sequence can be determined by using the following formula :

$\mathbf{a_n = S_n - S_{n-1}}$

i.e., the nth term of an AP is the difference of the sum to first n terms and the sum to first (n – 1) terms of it.

Ex.14 Find the sum of the AP: $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,...., to 11 terms. **[NCERT]**

Sol. Here, $a = \frac{1}{15}$

$d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$

n = 11

We know that

$\Rightarrow \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow \quad S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right] \Rightarrow S_{11} = \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$

$\Rightarrow \quad S_{11} = \frac{11}{2}\left[\frac{3}{10}\right] \Rightarrow S_{11} = \frac{33}{20}$

So, the sum of the first 11 terms of the given AP is $\frac{33}{20}$.

Ex.15 Find the sum : 34 + 32 + 30 + …. + 10

Sol. 34 + 32 + 30 + … + 10

This is an AP

Here, $a = 34$

$d = 32 - 34 = -2$

$\ell = 10$

Let the number of terms of the AP be n.

We know that

$a_n = a + (n - 1)d$

$\Rightarrow \quad 10 = 34 + (n - 1)(-2) \Rightarrow (n - 1)(-2) = -24$

$\Rightarrow \quad n - 1 = \frac{-24}{-2} = 12 \Rightarrow n = 13$

Again, we know that

$S_n = \frac{n}{2}(a + \ell) \Rightarrow S_{13} = \frac{13}{2}(34 + 10)$

$\Rightarrow \quad S_{13} = 286$

Hence, the required sum is 286.

Ex.16 Find the sum of all natural numbers between 100 and 200 which are divisible by 4.

Sol. All natural numbers between 100 and 200 which are divisible by 4 are

104, 108, 112, 116,….196

Here, $a_1 = 104$

$a_2 = 108$

$a_3 = 112$

$a_4 = 116$

$\vdots \quad \vdots$

$\therefore \quad a_2 - a_1 = 108 - 104 = 4$

$a_3 - a_2 = 112 - 108 = 4$

$a_4 - a_3 = 116 - 112 = 4$

$\vdots \quad \vdots \quad \vdots$

$\therefore \quad a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots(= 4 \text{ each})$

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, $a = 104$

$d = 4$

$\ell = 196$

Let the number of terms be n. Then

$\ell = a + (n - 1)d$

$\Rightarrow$ 196 = 104 + (n – 1)4

$\Rightarrow$ 196 – 104 = (n – 1)4

$\Rightarrow$ 92 = (n – 1) 4

$\Rightarrow$ (n – 1) 4 = 92

$\Rightarrow$ $n - 1 = \frac{92}{4}$

$\Rightarrow$ n – 1 = 23

$\Rightarrow$ n = 23 + 1 $\Rightarrow$ n = 24

Again, we know that

$$S_n = \frac{n}{2}(a + \ell)$$

$\Rightarrow$ $S_{24} = \left(\frac{24}{2}\right)(104 + 196)$

= (12) (300) = 3600

Hence, the required sum is 3600.

Ex.17 Find the number of terms of the AP 54, 51, 48,…so that their sum is 513.

Sol. The given AP is 54, 51, 48,….

Here, a = 54, d = 51 – 54 = - 3

Let the sum of n terms of this AP be 513.

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow$ $513 = \frac{n}{2}[2(54) + (n-1)(-3)]$ $\Rightarrow$ $513 = \frac{n}{2}[108 - 3n + 3]$

$\Rightarrow$ $513 = \frac{n}{2}[111 - 3n]$ $\Rightarrow$ 1026 = n [111 -3n]

$\Rightarrow$ $1026 = 111n - 3n^2$ $\Rightarrow$ $3n^2 - 111n + 1026 = 0$

$\Rightarrow$ $n^2 - 37n + 342 = 0$ [Dividing throughout by 3]

$\Rightarrow$ $n^2 - 18n - 19n + 342 = 0$

$\Rightarrow$ n(n – 18) – 19(n – 18) = 0

$\Rightarrow$ (n – 18) (n – 19) = 0

$\Rightarrow$ n – 18 = 0 or n – 19 – 0

$\Rightarrow$ n = 18, 19

Hence, the sum of 18 terms or 19 terms of the given AP is 513.

Note : Actually 19^{th} term

= a_{19}

= a + (19 – 1) d [$\because a_n = a + (n-1)d$]

= a + 18d

= 54 + 18 (-3)

= 54 – 54 = 0

Ex.18 Find the AP whose sum to n terms is $2n^2 + n$.

Sol. Here, $S_n = 2n^2 + n$ (Given)

Put n = 1, 2, 3, 4,…., in succession, we get

$S_1 = 2(1)^2 + 1 = 2 + 1 = 3$

$S_2 = 2(2)^2 + 2 = 8 + 2 = 10$

$S_3 = 2(3)^2 + 3 = 18 + 3 = 21$

$S_4 = 2(4)^2 + 4 = 32 + 4 = 36$

and so on.

$\therefore$ $a_1 = S_1 = 3$

$a_2 = S_2 - S_1 = 10 - 3 = 7$

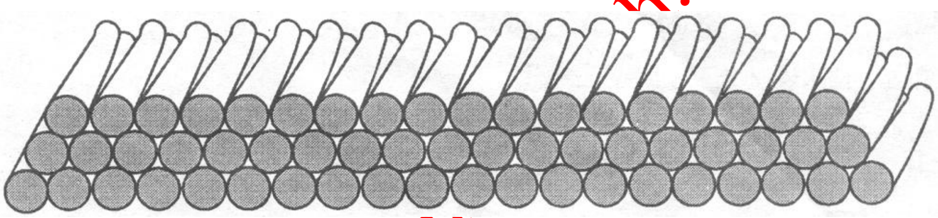
$a_3 = S_3 - S_2 = 21 - 10 = 11$

$a_4 = S_4 - S_3 = 36 - 21 = 15$

and so on.

Hence, the required AP is 3, 7, 11, 15,…

Ex.19 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?



[NCERT]

Sol. The number of logs in the bottom row, next row, row next to it and so on form the sequence 20, 19, 18, 17, …….

$a_2 - a_1 = 19 - 20 = -1$

$a_3 - a_2 = 18 - 19 = -1$

$a_4 - a_3 = 17 - 18 = -1$

i.e., $a_{k+1} - a_k$ is the same every time.

So, the above sequence forms an AP.

Here, a = 20

d = - 1

$S_n = 200$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow$ $200 = \frac{n}{2}[2(20) + (n-1)(-1)]$ $\Rightarrow$ $200 = \frac{n}{2}[40 - n + 1]$

$\Rightarrow$ $200 = \frac{n}{2}[41 - n]$ $\Rightarrow$ $400 = n[41 - n]$

$\Rightarrow$ $n[41 - n] = 400$ $\Rightarrow$ $41n - n^2 = 400$

$\Rightarrow$ $n^2 - 41n + 400 = 0$ $\Rightarrow$ $n^2 - 25n - 16n + 400 = 0$

$\Rightarrow$ $n(n - 25n) - 16(n - 25) = 0$ $\Rightarrow$ $(n - 25)(n - 16) = 0$

$\Rightarrow$ $n - 25 = 0$ or $n - 16 = 0$ $\Rightarrow$ $n = 25$ or $n = 16$

$\Rightarrow$ $n = 25, 16$

Hence, the number of rows is either 25 or 16.

Now, number of logs in row

= Number of logs in 25th row

= a_{25}

= $a + (25 - 1)d$ $\quad [\because a_n = a + (n-1)d]$

= $a + 24d$

= $20 + 24(-1) \Rightarrow 20 - 24 = -4$

Which is not possible.

Therefore, n = 16 and

Number of log in top row

= Number of logs in 16th row

= a_{16}

= $a + (16 - 1)d$ $\quad [\because a_n = a + (n-1)d]$

= $a + 15d$

= $20 + 15(-1)$

= $20 - 15 = 5$

Hence, the 200 logs are placed in 16 rows and there are 5 logs in the top row.

COMPETITION WINDOW

SUM OF n TERMS OF A.G.P.

If S_n is the sum of first n terms of the G.P. a, ar, ar^2,….,

i.e., $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$, then $S^n = \frac{a(1-r^n)}{1-r}$ or $\frac{a(r^n-1)}{r-1}$, $r \neq 1$

Also, $S^n = \frac{a - r\ell}{1-r}$ or $\frac{r\ell - a}{r-1}$, $r \neq 1$, where ℓ is the last term i.e. the nth term .

For r = 1, Sn = na

SUM OF AN INFINITE G.P.

The sum of an infinite G.P. with first term a and common ratio r, where $-1 < r < 1$, is

$$S_\infty = \frac{a}{1-r}$$

If $r \geq 1$, then the sum of an infinite G.P. tends to infinity.

SUM OF n TERMS OF A H. P.

There is no specific formula to find the sum of n terms of H.P. To solve the questions of this progression, first of all convert it in A.P. then use the properties of A.P.

TRY OUT THE FOLLOWING

1. Find the sum of seven terms of the G.P. 3, 6, 12, ….

2. Find the sum to 7 terms of the sequence $\left[\frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}\right], \left[\frac{1}{5^4}+\frac{2}{5^5}+\frac{3}{5^6}\right], \left[\frac{1}{5^7}+\frac{2}{5^8}+\frac{3}{5^9}\right]$.

3. Find the sum of the series 2 + 6 +18 +… + 4374.

4. How many terms of the sequence 1, 4, 16, 64…. will make the sum 5461?

5. Find the sum to infinite of the G.P. $\frac{-5}{4}, \frac{5}{16}, \frac{-5}{64}$, ….

6. The first term of a G.P. is 2 and the sum of infinity is 6. Find the common ratio

7. If each term of an infinite G.P. is twice the sum of the terms following it, then find the common ratio of the G.P.

ANSWERS

1. 381 **2.** $\frac{19}{62}\left[1-\frac{1}{5^{21}}\right]$ **3.** 6560 **4.** – 1 **5.** $\frac{2}{3}$ **6.** $\frac{1}{3}$

★ **PROPERTIES OF ARITHMETICAL PROGRESSIONS**

1. If a constant is added to or subtracted from each term of an A.P., then the resulting sequence is also an A.P. with the same common difference.

2. If each term of a given A.P. is multiplied or divided by a non-zero constant K, then the resulting sequence is also an A.P. with common difference Kd or d/K, where d is the common difference of the given A.P.

3. In a finite A.P., the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of first and last term.

4. Three numbers a, b, c, are in A.P. iff 2b = a + c.

5. A sequence is an A.P. iff it's nth term is a linear expression in n i.e., $a_n = A_n + B$ are constants. In such a case, the coefficient of n is the common difference of the A.P.

6. A sequence is an A.P. iff the sum of it's first n terms is of the form $An^2 + Bn$, where A, B are constants. independent of n. In such a case, the common difference is 2 A.

7. If the terms of an A.P. are chosen at regular intervals, then they form an A.P.

COMPETITION WINDOW

ARITHMETIC MEANS

1. If there numbers a, b, c are in A.P. then b is called the arithmetic mean (A.M.) between a and c.

2. The arithmetic mean between two numbers a and b is $\frac{a+b}{2}$

3. $A_1, A_2 \ldots, A_n$ are said to be n A.M.s between two numbers a and b. iff $a, A_1, A_2, \ldots, A_n, b$ are in A.p.
Let d be the common difference of the A.P.
Clearly, b = (n + 2)th term of the A.P.

$\Rightarrow$ b = a + (n + 1) d

$$\Rightarrow d = \frac{b-a}{n+1}$$

Hence, $A_1 = a + d = a + \frac{b-a}{n+1}$, $A_2 = a + 2d = a + \frac{2(b-a)}{n+1}$

……………………………………………………

……………………………………………………

$A_n = a + nd = a + \frac{n(b-a)}{n+1}$

4. The sum of n A.M.'s between two numbers a and b is n times the single A.M. between then i.e., $n\left[\frac{a+b}{2}\right]$

GEOMETRIC MEANS

1. If three non-zero numbers a, b, c are in G.P. then b is called the geometric mean (G.M.) between a and b.

2. The geometric mean between two positive numbers a and b is $\sqrt{ab}$

HARMONIC MEAN

1. If three non-zero numbers a, b, c are in H.P., then is called the harmonic mean (H.M.) between a and b,

2. The harmonic mean between numbers a and b is $\frac{2ab}{a+b}$

Remark : If A, G, H denote respectively, the A.M., the G.M. and the H.M. between two distinct positive numbers, then

(i) A, G, H are in G.P. (ii) A > G > H

★ **SYNOPSIS**

1. **Sequence :** A sequence is an ordered arrangement of numbers according to a given rule.
2. **Terms :** The numbers in a sequence are called its terms.
3. **Series :** The sum of terms of a sequence is called the series of the corresponding sequence.
4. **Progression :** A progression is a sequence whose terms obey a certain pattern.
5. **Arithmetic Progression :** Arithmetic progression is a sequence if the difference of a term a and its predecessor is always constant.
6. **Common Difference :** The difference between two successive terms of an A.P. is called common difference.
7. **General Term :** General term or nth term or last term of an A.P. is $T_n = \ell = a_n = a + (n - 1) d$, where 'a' is the first term and 'd' the common difference.
8. **Sum of n terms of an A.P. :** $S_n = \frac{n}{2}\{2a + (n-1)d\} = \frac{n}{2}\{a + \ell\}$

Where ℓ = last term = a + (n – 1) d

nth term, $a_n = S_n - S_{n-1}$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. If the sum of first n terms of an AP be $3n^2 - n$ and it's common difference is 6, then its first term is :

(A) 2 (B) 3 (C) 1 (D) 4

2. If 7^{th} and 13^{th} terms of an A.P. be 34 and 64, respectively, then it's 18^{th} term is :

(A) 87 (B) 88 (C) 89 (D) 90

3. The sum of all 2-digit odd numbers is :

(A) 2475 (B) 2530 (C) 4905 (D) 5049

4. The fourth term of an A.P. is 4. Then the sum of the first 7 terms is :

(A) 4 (B) 28 (C) 16 (D) 40

5. In an A.P. $s_1 = 6$, $s_7 = 105$, then $s_n : s_{n-3}$ is same as :

(A) (n + 3) : (n – 3) (B) (n + 3) : n (C) n : (n – 3) (D) None of these

6. In an A.P. $s_3 = 6$, $s_6 = 3$, then it's common difference is equal to :

(A) 3 (B) – 1 (C) 1 (D) None of these

7. The number of terms common to the two A.P. s

2 + 5 + 8 + 11 + ….+ 98 and 3 + 8 + 13 + 18 + … + 198

(A) 33 (B) 40 (C) 7 (D) None of these

8. (p + q)th and (p – q)th terms of an A.P. are respectively m and n, The P^{th} term is :

(A) $\frac{1}{2}$(m + n) (B) $\sqrt{mn}$ (C) m + n (D) mn

9. The first, second and last terms of an A.P. are a, b and 2a. The number of terms in the A.P. is:

(a) $\frac{b}{b-a}$ (B) $\frac{b}{b+a}$ (C) $\frac{a}{b-a}$ (D) $\frac{a}{a+b}$

10. Let s_1, s_2, s_3 be the sums of n terms of three series in A.P., the first term of each being 1 and the common differences 1, 2, 3 respectively. If $s_1 + s_3 = \lambda s_2$, then the value of λ is :

(A) 1 (B) 2 (C) 3 (D) None of these

11. Sum of first 5 terms of an A.P. is one fourth of the sum of next five terms. If the first term = 2, then the common difference of the A.P. is :

(A) 6 (B) – 6 (C) 3 (D) None of these

12. If x, y, z are in A.P., then the value of (x + y – z) (y + z –x) is equal to :

(A) $8yz - 3y^2 - 4z^2$ (B) $8yz - 3z^2 - 4y^2$ (C) $8yz + 3y^2 - 4z^2$ (D) $8yz - 3y^2 + 4z^2$

13. The number of numbers between 105 and 1000 which are divisible by 7 is :

(A) 142 (B) 128 (C) 127 (D) None of these

14. If the numbers a, b, c, d, e form an A.P. then the value of a – 4b + 6c – 4d + e is equal to :

(A) 1 (B) 2 (C) 0 (D) None of these

15. If s_n denotes the sum of first n terms of an A.P., whose common difference is d, then $s_n - 2s_{n-1} + s_{n-2}$ (n > 2) is equal to :

(A) 2d (B) – d (C) d (D) None of these

16. The sum of all 2-digited numbers which leave remainder 1 when divided by 3 is:

(A) 1616 (B) 1602 (C) 1605 (D) None of these

17. The first term of an A.P. of consecutive integers is $p^2 + 1$. The sum of 2p + 1 terms of this series can be expressed as :

(A) $(p + 1)^2$ (B) $(2p + 1) (p + 1)^2$ (C) $(p + 1)^3$ (D) $p3 + (p + 1)^3$

18. If the sum of n terms of an AP is $2n^2$ 5n, then its nth term is –

(A) 4n – 3 (B) 3n – 4 (C) 4n + 3 (D) 3n + 4

19. If the last term of an AP is 119 and the 8th term from the end is 91 then the common difference of the AP is –

(A) 2 (B) 4 (C) 3 (D) – 3

20. If (a_n) = {2.5, 2.51, 2.52,….} and $\{b_n\}$ = {3.72, 3.73, 3.74,…} be two AP's then $a_{100005} - b_{100005}$ =

(A) – 1.22 (B) 1.22 (C) 1.2 (D) – 1.02

OBJECTIVE					ANSWER KEY									EXERCISE – 1
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	C	A	B	A	B	C	A	A	B	B	A	C	C	C
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	C	D	C	B	A										

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS

SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE

1. Write the first five terms of each of the sequences, whose nth terms are:

(i) $a_n = \frac{(-1)^n(2n+1)}{6}$ (ii) $a_n = \frac{1}{n} + (-1)^n$ (iii) $a_n = n\left[\frac{n^2+5}{4}\right]$ (iv) $a_n = (-1)^{n-1}\, 5^{n+1}$

2. Find the indicated terms in each of the following sequences whose nth terms are:

(i) $a_n = 2^n + n^3$; a^3 (ii) $a_n = \frac{n^2 - n + 1}{n}$, a_{10} (iii) $a_n = (-1)^{n-1}n^3$; a_9 (iv) $a_n = (n - 1)(2 - n)(3 + n)$; a_{20}

3. Write the first five terms of each of the following sequences and obtain the corresponding series.
(i) $a_1 = 1$, $a_n = a_{n-1} + 2$, $n \geq 2$ (ii) $a_1 = 4$, $a_{n+1} = 2na_n$

4. Write the first term a and the common difference d of the AP : -5, -1, 3, 7….

5. Write the first term a and the common difference d of the AP : -1.1, -3.1, - 5.1, - 7.1……

6. Write the arithmetic progression when first term a = - 1 and common difference d = $\frac{1}{2}$.

7. Write the arithmetic progression when first term a = - 1.5 and common difference d = - 0.5.

8. Find the common difference and write the next four terms of the AP : $1, \frac{1}{4}, \frac{3}{2}, \ldots..$

9. Write the sequence with nth term, $a_n = 3 + 4n$.

10. Find out whether the sequence 3, 3, 3, 3, … is an AP. If it is, find out the common difference.

11. Find the common difference and write the next two terms of the AP : 1.8, 2.0, 2.2, 2.4…

12. Find out whether the sequence $1^2, 3^2, 5^2\ 7^2, \ldots$ is an AP. If it is, find out the common difference.

13. Find the common difference and write the next two terms of the AP $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \ldots.$

14. Find the 18th term of the AP. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2} \ldots.$

15. Which term of the AP : 84, 80, 76,… is 0?

16. Is 302 a term of the AP : 3, 8, 13,….?

17. How many terms are there in the AP : 7, 13, 19,…..205 ?

18. Find the sum of the arithmetic progression : -26, -24, -22,…. to 36 terms.

19. Show that the sequence defined by $a_n = 2n^2 + 3$ is not an A.P.

20. Find the 14th term of the A.P. 9, 5, 1, - 3…

21. Find the nth term of the sequence m – 1, m – 3, m – 5,…

22. Is – 150 a term of the A.P. 11, 8, 5, 2….

SHORT ANSWER TYPE QUESTIONS

1. Show that the sequence defined by $a_n = 5n - 7$ is an AP. Find its common difference.

2. Prove that no matter what the real numbers a and b are, the sequence with nth term a + nb is always an AP. What is the common difference?

3. The first term of an AP is 5, the common difference is 3 and the last term is 80, find the number of terms.

4. If the nth term of the AP. 9, 7, 5… is same as the nth term of the AP. 15, 12, 9,…., find n.

5. Find the 12th term from the end of the arithmetic progression : 3, 5, 7, 9,…..201?

6. Find n if the given value of x is the nth term of the AP : $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots.;\ x = 550.$

7. Which term of the AP : 3, 10, 17,…. will be 84 more than its 13th term?

8. Find the 8th term from the end of the AP : 7, 10, 13,….184

9. Find the value of x for which (8x + 4), (6x – 2) and (2x + 7) are in AP.
10. If the sum of a certain number of terms starting from first of an AP 25, 22, 19,…, is 116. Find the last term.
11. How many terms of he sequence 18, 16, 14,…. Should be taken so that their sum is zero?
12. Find the sum of first n odd natural numbers.
13. Find the sum of all even integers between 101 and 999.
14. Find the sum : $7 + 10\frac{1}{2} + 14 + \ldots + 84$.
15. Find the sum of the first 15 terms of the sequence having nth term as : $a_n = 3 + 4n$.
16. The 6th and 17th terms of an AP. are 19 and 41 respectively, find the 40th term.
17. In a certain AP, the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.
18. Find the second term and nth term of an AP whose 6th term is 12 and the 8th term is 22.
19. An AP consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32nd term.
20. Find $a_{30} - a_{20}$ for the AP : a, a + d, a + 2d, a + 3d,….
21. Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?
22. Find the term of the arithmetic progression 9, 12, 15, 18,…. Which is 39 more than its 36th term.
23. The sum of three terms of AP. Is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.
24. The sum of three numbers in AP. is 12 and the sum of their cubes is 288. Find the numbers.
25. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.
26. How many terms are there in the AP whose first and fifth terms are – 14 and 2 respectively and the sum of the terms is 40?
27. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
28. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.
29. In an AP if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms?
30. Find the sum of the first 25 terms of an AP whose nth terms is given by $a_n = 2 - 3n$.
31. If x, x + 10 and 3x + 2 are in A.P., find the value of x.
32. If x + 1, 3x and 4x + 2 are in A.P., find the fifth term of A.P.
33. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P., then prove that $2b^2 = a^2 + c^2$.
34. If a, $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ and b are in A.P. and $a \neq b$, then find the value of n.
35. Find the nth term and 100th term of the sequence 7 + 3 – 1 – 5….
36. Which term of the A.P., 3, 8, 13, 18,…., is 78?
37. Which term of the A.P., 21, 18, 15,… is – 81?
38. Which term of the A.P., 121, 117, 113,…., is it's first negative term?
39. Which term of the sequence 114, 109, 104,… is the first negative term?
40. How many terms are there in the A.P. $-1, \frac{-5}{6}, -\frac{2}{3}, \frac{-1}{2}, \ldots, \frac{10}{3}$?
41. How many two digit numbers are divisible by 3?
42. How many three digit numbers are divisible by 5?
43. If the 5th and 21st terms of an A.P. are 14 and – 14 respectively, then which term of the A.P. is zero?
44. In an A.P., the fourth term exceeds four times the 12th term by one and the third term exceeds twice the tenth term by five, find the A.P.
45. Determine the A.P. whose fourth term is 15 and the difference of 6th term from 10th term is 16.
46. The 4th term of an A.P. is zero, prove that its 25th term is triple its 11th term.
47. If the pth term of an A.P. is $\frac{1}{q}$, and qth term of an A.P. is $\frac{1}{q}$, then show that its (pq)th term is 1.
48 . The sum of three numbers in A.P. is 21 and their product is 231. Find the numbers.
49. The sum of three numbers in A.P. is 3 and their product is – 35. Find the numbers.

50. Divided 20 into four parts which are in A.P. such that the product of the first and fourth and the product of the second and third is in the ratio 2 : 3.

51. If the sum of first n terms of an A.P. is given by $s_n = 5n^2 + 3n$, find the nth term of the A.P.

52 The sum of the first 9 terms of an A.P. is 81 and the sum of it's first 20 terms is 400. Find the first term, the common difference and the sum up to 15th term

53. The sum of n terms of two arithmetic progressions are in the ratio (3n + 8) : (7n + 15).Find the ratio of their (i) 12th terms (ii) 15th terms.

54. If $s_n = n^2p$ and $s_m = m^2p$, $m \neq n$ is an A.P. prove that $s_p = p^3$.

55. The first, second and the last terms of an A.P. are m, n and 2m respectively. Show that it's sum is $\frac{3mn}{2(n-m)}$

56. If the mth term of an A.P. is 20 and nth term is 10, then show that sum of it's first (m + n) terms is $\frac{m+n}{2}\left[30+\frac{10}{m-n}\right]$.

57. If s_1, s_2, s_3 are the sum of n terms of three arithmetic progressions, the first term of each being unity and the respective common difference being 1, 2, 3; prove that $s_1 + s_3 = 2s_2$.

58. Two A.P.'s have the same common difference. If the first term of the two A.P's are 3 and 8 respectively, find the difference between their sum to first 30 terms.

59. If in an A.P., the sum of 12 terms is equal to 18 and the sum of 18 terms. is equal to 12, then prove that the sum of 30 terms is – 30.

60. Find the sum of all two digit natural numbers which are divisible by 4.

61. Find the sum of all 3-digit natural numbers which are divisible by 13.

62. Find the sum of all 2-digit numbers which when divided by 5 leave remainder 1.

63. Find the sum of all multiples of 9 lying between 300 and 700.

LONG ANSWER TYPE QUESTION

1. If pth, qth and rth terms of an AP are a, b, c respectively, then show that :
(i) a(q – r) + b (r – p) + c (p – q) = 0, (ii) (a – b)r + (b – c)p + (c – a)q = 0

2. In a garden bed, there are 23 rose plants in the first row, twenty one in the second row, nineteen in the third row and so on. There are five plants in the last row. How many rows are there of rose plants?

3. If the mth term of an AP is $\frac{1}{n}$ and the nth term is $\frac{1}{m}$, show that the sum of mn terms is $\frac{1}{2}$(mn + 1).

4. If the sum of m terms of an AP is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.

5. The sum of the first p,q,r terms of an AP. are a,b,c respectively. Show that:

$$\frac{a}{q}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

6. A manufacturer of radio sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the product increases uniformly by a fixed number every year, find (i) the production in the first year
(ii) the total product in 7 years and (ii) the product in the 10th year.

7. A man is employed to count Rs. 10710. He counts at the rate of Rs. 180 per minute for half an hour. After this he counts at the rate of Rs 3 less every minute thatn the preceding minute. Find the time taken by him to count the entire amount.

8. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid, find the value of the first installment.

9. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are 2.5 metre apart, what is the length of the wood required for the rungs?

10. The digits of a positive integer, having three digits, are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

11. Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by $\frac{1}{2}$ km each succeeding hour. After how many hours will the second car overtake the first car if both cars go non – stop?

12. A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will it take him to clear the loan?

13. 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more workers dropped the third day and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.

14. Along a road lie an odd number of stones place at intervals of 10 metres. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried the job with one of the end stones by carrying them in succession. In carrying all the stones he covered a distance of 3 km. Find the number of stones.

15. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120^o, and the common difference is 5^o. Find the number of sides of the polygon.

16. A man saved Rs. 16500 in ten years after the first he saved Rs. 100 more than he did in the preceding year. How much did he save in the first year?

17. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid, find the value of the first installment.

18. A piece of equipment cost a certain factory Rs. 600.000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year and so on. What will be it's value at the end of 10 years, all percentages applying to the original cost?

ARITHMETIC PROGRESSIONS	ANSWER KEY	EXERCISE – 2 (X)–CBSE

- **VERY SHORT ANSWER TYPE QUESTIONS**

1. (i) $\frac{-1}{2}, \frac{5}{6}, \frac{-7}{6}, \frac{3}{2}$ and $\frac{-11}{6}$ (ii) $0, \frac{3}{2}, \frac{-2}{3}, \frac{5}{4}$ and $\frac{-4}{5}$ (iii) $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21$ and $\frac{75}{2}$ (iv) 25, - 125, 625, - 3125 and 15625

2. (i) 35 (ii) $\frac{91}{10}$ (iii) 729 (iv) – 7866 **3.** (i) 1, 3, 5, 7, 9 ; 1 + 3 + 5 + 7 +9 (ii) 4, 8, 32, 192, 1536 : 4 + 8 32 + 192 + 1536

4. a = - 5, d = 4 **5.** a = -1.1, d = -2 **6.** $-1, -\frac{1}{2}, 0, \frac{1}{2}, 1$, ……. **7.** –1.5, -2, -2.5, -3,….

8. $\frac{5}{4}$; $a_4 = \frac{11}{4}, a_5 = \frac{16}{4}, a_6 = \frac{21}{4}, a_7 = \frac{26}{4}$ **9.** 7, 11, 15, 19,…. **10.** Yes, d = 0 **11.** d = 0.2, a_5 = 2.6, a_6 = 2.8

12. No **13.** $d = \frac{1}{4}; a_5 = 1, a_6 = \frac{5}{4}, a_7 = \frac{26}{4}$ **14.** $35\sqrt{2}$ **15.** 22nd term **16.** No **17.** 34

18. 324 **20.** – 43 **21.** m – 2n + 1 **22.** No

- **SHORT ANSWER TYPE QUESTIONS**

1. 5 **2.** b **3.** 26 **4.** 7 **5.** 179 **6.** 100 **7.** 25th **8.** 163 **9.** $\frac{15}{2}$ **10.** 4 **11.** 19 **12.** n^2 **13.** 246950

14. $\frac{2093}{2}$ **15.** 525 **16.** 87 **18.** a_2 = - 8, a_n = 5n – 18 **19.** 69 **20.** 10d **21.** 100 **22.** 49th **23.** 1, 7, 13

24. 2, 4, 6 or 6, 4, 2 **26.** 10. **27.** 38,6973 **29.** 1150 **30.** – 925 **31.** x = 9 **32.** 24 **34.** 1

35. 11 – 4n, - 389 **36.** 16th **37.** 15th, 8th **38.** 32nd **39.** 24th **40.** 27 **41.** 30 **42.** 180

43. 3rd **44.** 27, 25, 23, 21 **45.** 3, 7, 11, 15, 19,…. **48.** 3, 7, 11 or 11, 7, 3 **49.** – 5, 1, 7 or 7, 1, - 5

50. 2, 4, 6 and 8 **51.** 10 n – 2 **52.** a = 1, d = 2, s_{15} = 225 **53.** (i) 7 : 16 (ii) 95 : 218 **58.** 150 **60.** 1188

61. 37674 **62.** 963 **63.** 21978

- **LONG ANSWER TYPE QUESTIONS**

2. 10 rows **6.** **(i)** 550 **(ii)** 4375 **(iii)** 775 **7.** 89 minutes **8.** Rs. 51 **9.** 3.5 minters **10.** 852

11. 9 hours **12.** 20 months **13.** 25 days **14.** 25 stones **15.** 9 sides **16.** Rs. 1200 **17.** Rs. 51 **18.**10500

EXERCISE – 3 (FOR SCHOOL/BOARDS EXAMS)

PERVIOUS YEARS BOARD (CBSE) QUESTIONS

1. The nth term (t_n) of an Arithmetic progression is given by $t_n = 7n + 1$. Find the sum of the first 30 terms of Arithmetic progression. **[Foreign – 2004]**

2. The 10th term of an Arithmetic progression (A.P.) is 57 and its 15th term is 87. Find the Arithmetic Progression. **[Foreign – 2004]**

3. If the sum of first n terms of an A.P. is given by $S_n = 3n^2 + 2n$, find the nth term of the A.P.

OR

If m times the mth terms of an A.P. is equal to n times its nth term, find its (m + n)th term.

[Delhi-2004C]

4. How many terms of the A.P.3,5,7,… must be taken so that the sum is 120? **[Delhi-2004C]**

5. If the sum of first n terms of an A.P. is given by $S_n = 4n^2 – 3n$, find the nth term of the A.P.

[Delhi-2004C]

6. If the sum of first of an A.P. is given by $S_n = 2n^2 + 5n$, find the nth term of the A.P. **[Delhi-2004C]**

7. Find the sum of first 15 terms of an A.P. whose nth term is $9 – 5n$.

OR

If the sum to first n terms of an A.P. is given by $S_n = 5n^2 + 3n$, find the nth term of the A.P.

[AI-2004C]

8. Find 10[th] term from end of an A.P. 4, 9, 14,…, 254 **[Delhi-2005]**

9. Find the number of terms of the A.P. 54, 51, 48,….so that their sum is 513.

OR

If the nth term of an A.P. is (2n + 1), find the sum of first n terms of the A.P. **[Delhi-2005]**

10. Find the sum of all two digits odd positive numbers. **[AI-2005]**

11. The 8th term of an Arithmetic Progression is zero. Prove that its 38th term is triple of it 18th term.

[AI-2005]

12. Find the sum of all two digit positive numbers divisible by 3. **[Foreign-2005]**

13 If fifth term of the A.P. is zero, show that its 33rd term is four times its 12th term **[Foreign-2005]**

14. Which term of the A.P. 5, 9, 13,... is 81? Also find the sum 5 + 9 + 13 +… + 81 **[Delhi-2005C]**

15. The sum of first n terms of an A.P. is given by $(n^2 + 3n)$. Find the 20th term of the progression.

[Delhi-2005C]

16. Find the sum of the first 51 terms of the A.P. whose 2nd term is 2 and 4th term is 8. **[AI-2005C]**

17. The sum of the first n terms of an A.P. is given by $S_n = 3n^2 – n$. Determine the A.P. and its 25[th] term.

OR

The sum of three numbers in A.P. is 27 and their product is 405. Find the numbers. **[AI-2005C]**

18. The 6[th] term of an Arithmetic progression (A.P.) is -10 and the 10th term is – 26 Determine the 15th term of the A.P. **[Delhi-2006]**

19. Find the sum of all the natural numbers less than 100 which are divisible by 6. **[AI-2006]**

20. How many terms are there in A.P. whose first term and 6th term are – 12 and 8 respectively and sum of all its terms is 120?

21 Using A.P., find the sum of all 3-digit natural numbers which are multiples of 7. **[Delhi-2006C]**

22. In an A.P. the sum of first n terms is $\frac{5n^2}{2}+\frac{3n}{2}$ Find its 20th term. **[AI-2006C]**

23. Find the sum of first 25 terms of an A.P. whose nth term is 1 – 4n. **[Delhi-2007]**

24. Which term of the A.P. 3, 15, 27, 39, …will be 132 more than its 54th term? **[Delhi-2007]**

25. In an A.P., the sum of its first n terms is $n^2 + 2n$. Find its 18th term. **[AI-2007]**

26. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively. Find the sum of all terms of this A.P. **[AI-2007]**

27. The nth term of an A.P. is 7 – 4n. Find its common difference. **[Delhi-2008]**

28. The sum of n terms of an A.P. is $5n^2 – 3n$. Find the A.P. Hence, find its 10th term. **[Delhi-2008]**

29. The nth term of an A.P. is 6n + 2. Find it's common difference. **[Delhi-2008]**

30. Find the 10th term from the end of the A.P. 8, 10, 12,…., 126 **[Delhi-2008]**

31. Write the next term of the A.P. $\sqrt{8}, \sqrt{18}, \sqrt{32}$,…. **[AI-2008]**

32. The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P. **[AI-2008]**

33. The first term of an A.P. is p and it's common difference is q. Find it's 10th terms is. **[AI-2008]**

34. For what value of n are the terms of two A.P.'s 63, 65, 67,….., and 3, 10, 17 ,…. equal?

OR

If m times the mth tem of an A.P. is equal to n times it's nth term, find the (m + n)th term of the A.P. **[Foreign-2008]**

35. In an A.P. the first term is 8, nth term is 33 and sum to first n terms is 123. Find n and d, the common difference. **[Foregin-2008]**

36. In an A.P., the term is 22, nth term is – 11, and sum to first n terms is 66. Find n and d, the common difference. **[Foreign-2008]**

37. In an A.P., the first term is 22, nth term is – 11, and sum to first n terms is 66. Find n and d, the common difference. **[Foreign-2008]**

38. For what value of p, are 2p – 1, 7 and 3p three consecutive terms of an A.P.? **[Delhi-2009]**

39. If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 – 4n$, then find its nth term. **[Delhi-2009]**

40. The sum of 4th and 8th terms of an A.P. is 24 and sum of 6th and 10th terms is 44. Find A.P. **[Delhi-2009]**

41. If S_n the sum of first n terms an A.P. is given by $S_n = 5n^2 + 3n$, then find its nth term. **[Delhi-2009]**

42. The sum of 5th 9th terms of an A.P., is 72 and the sum of 7th and 12th terms is 97. Find the A.P. **[Delhi-2009]**

43. If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a. **[AI-2009]**

44. Which term of the A.P. 3, 15, 27, 39,… will be 120 more than it's 21str term? **[AI-2009]**

45. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P. **[AI-2009]**

46. Which term of the A.P. 4, 12, 20, 28,…. will be 120 more than it's 21st term? **[AI-2009]**

47. For what value of k, are the numbers x, 2x + k and 3x + 6 three consecutive terms of an A.P.? **[Foreign-2009]**

48. The 17th term of an A.P. exceeds its 10th term by 7. Find the common difference. **[Foreign-2009]**

49. If 9th term of an A.P. is zero, prove that its 29th term is double of it's 19th term. **[Foreign-2009]**

50. If 5th term of an A.P. is zero, prove that it's 23rd term is three times it's 11th term. **[Foreign-2009]**

51. If the 7th term of an A.P. is zero, prove that it's 27th term is five times it's 11th term. **[Foreign-2009]**

SUBJECTIVE | **ANSWER KEY** | **EXERCISE-4 (X)-CBSE**

1. 3285 **2.** 3, 9, 15, 21… **3.** 6n – 1 or 0 **4.** 10 terms **5.** 8n – 7 **6.** 4n + 3
7. – 465 or 10n – 2 **8.** 209 **9.** 18 or 19 OR n (n + 2) **10.** 2,475 **12.** 1,665 **14.** 860
15. 42 **16.** 3,774 **17.** 146 OR (3, 9,15) or (15, 9,3) **18.** – 46 **19.** 816 **20.** 12
21. 70, 336 **22.** 99 **23.** – 1275 **24.** 65th term **25.** 38 **26.** 5412 **27.** – 4
28. 2, 12, 22..; $a_{10} = 92$ **29.** 6 **30.** 108 **31.** $5\sqrt{2}$ **32.** – 13, - 8, - 3
33. p + 9q **34.** n = 13 or $a_{m+n} = 0$ **35.** n = 6, d = 5 **36.** n = 15, d = - 3
37. n = 12, d = - 3 **38.** p = 3 **39.** 6n – 7 **40.** – 13, – 8, – 3 **41.** 10n – 2
42. 6, 11, 16, 21,… **43.** a = $\frac{7}{5}$ **44.** 31st term **45.** a = 2, $a_{13} = 26$ **46.** 36th term
47. k = 3 **48.** d = 1

EXERCISE – 4 (FOR OLYMPIADS)

CHOSSE THE CORRECT ONE

1. For a series whose nth term is $\frac{n}{x} + y$, the sum of r terms is :

(A) $\frac{r(r+1)}{2x} + ry$ (B) $\frac{r(r-1)}{2x}$ (C) $\frac{r(r-1)}{2x} - ry$ (D) $\frac{r(r+1)}{2y} - rx$

2. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., then $\left[\frac{1}{a} + \frac{1}{b} - \frac{1}{c}\right]\left[\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right]$ is equal to :

(A) $\frac{4}{ac} - \frac{3}{b^2}$ (B) $\frac{b^2 - ac}{a^2b^2c^2}$ (C) $\frac{4}{ac} - \frac{1}{b^2}$ (D) None of these

3. The sum of first 24 terms of an A.P. $a_1, a_2, a_3, \ldots$; if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, is equal to :

(A) 90 (B) 180 (C) 900 (D) 1800

4. A student read common difference of an AP is – 2 instead of 2 and got the sum of first five terms as – 5. The actual sum of first five terms is :

(A) 25 (B) – 25 (C) – 35 (D) 35

5. The sum of n terms of two A.P's are in the ratio of (7n + 1) : (4n + 27). The ratio of their 11th terms is –

(A) 2 : 3 (B) 4 : 3 (C) 5 : 4 (D) 5 : 6

6. If $1^2 + 2^2 + 3^2 + \ldots n^2 = 1015$, then the value of n is :

(A) 13 (B) 14 (C) 15 (D) None of these

7. The sum of the series $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \ldots$ upto 9 terms is:

(A) $-\frac{5}{6}$ (B) $-\frac{1}{2}$ (C) 1 (D) $-\frac{3}{2}$

8. The sum of first n odd natural numbers is:

(A) n^2 (B) 2n (C) $\frac{n(n-1)}{2}$ (D) $\frac{n(n+1)}{2}$

9. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference will be:

(A) $\pm$ 1 (B) $\pm$ 2 (C) ± 3 (D) ± 4

10. If A.M between two numbers is 5 and their G.M. is 4, then their H.M. is:

(A) $\frac{16}{5}$ (B) $\frac{14}{5}$ (C) $\frac{11}{5}$ (D) None of these

11. If A is the single A.M. between two numbers a and b and S is the sum of n A.M.'s between them, then $\frac{S}{A}$ depends upon:

(A) n, a, b (B) n, a (C) n, b (D) n

12. If the A.M. between the roots of a quadratic equation is 8 and the G.M. is 5, then the equation is:

(A) $x^2 + 10x - 25 = 0$ (B) $x^2 - 8x + 5 = 0$
(C) $x^2 - 16x + 25 = 0$ (D) $x^2 - 16x - 25 = 0$

13. If c is the harmonic mean between a and b, then $\frac{c}{a} + \frac{c}{b}$ is equal to:

(A) 2 (B) $\frac{a+b}{ab}$ (C) $\frac{ab}{a+b}$ (D) None of these

14. If a,b,c,d,e,f are in A.P. then e-c is equal to :

(A) 2(c – a) (B) 2(f – d) (C) 2(d – c) (D) d – c

15. 20th term of the series : $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \ldots$ is :

(A) $\frac{441}{4}$ (B) $\frac{443}{2}$ (C) $\frac{445}{2}$ (D) $\frac{439}{2}$

16. If the value of $1^3 + 2^3 + 3^3 + \ldots + n^3 = 2025$, then the value of $1 + 2 + 3 + \ldots + n$ is:

(A) 675 (B) 81 (C) 45 (D) 285

17. If the value of $1 + 2 + 3 + \ldots + n$ is 55, then the value of $1^3 + 2^3 + 3^3 + \ldots + n^3$ is:

(A) 165 (B) 385 (C) 3025 (D) 555

18. The nth term of the series $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \ldots$ is :

(A) $\frac{n-1}{2}$ (B) $\frac{n^2+1}{2}$ (C) $\frac{n+1}{2}$ (D) $\frac{n^2-1}{2}$

19. $1^2 + 1 + 2^2 + 2 + 3^2 + 3 + \ldots + n^2 + n$ is equal to :

(A) $\frac{n(n+1)}{2}$ (B) $\left[\frac{n(n+1)}{2}\right]^2$ (C) $\frac{n(n+1)(n+2)}{3}$ (D) $\frac{n(n+)(n+2)(n+3)}{4}$

20. The next term of the sequence $\frac{1}{4}, \frac{1}{36}, \frac{1}{144} \ldots$ is :

(A) $\frac{1}{576}$ (B) $\frac{1}{400}$ (C) $\frac{1}{1296}$ (D) None of these

21. If the sum of first n natural numbers is one-fifth of the sum of their squares, then n equals:

(A) 5 (B) 6 (C) 7 (D) 8

22. The nth term of the series 1 + 3 + 6 +10 + 15 + … is :

(A) $\frac{n(n+1)}{2}$ (B) $n^2 - n + 1$ (C) n (n + 1) (D) None of these

23. The sum of the series $1^2 + 1 + 2^2 + 2 + 3^2 + 3 + \ldots$ + up to n terms is :

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n+1)(n+2)}{3}$ (C) $\left[\frac{n(n+1)}{2}\right]^2$ (D) None of these

24. The nth term of the sequence $1, \sqrt{2}, 3^{\frac{1}{3}}, 2^{\frac{1}{2}}, \ldots$ is :

(A) $n^{\frac{1}{n}}$ (B) n^n (C) $\left(\frac{1}{n}\right)^n$ (D) None of these

25. The sum of n terms of the series $(1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + ,,,$ is :

(A) $\frac{n(n+1)}{2}$ (B) $\frac{-n(n+1)}{2}$ (C) $-n(2n + 1)$ (D) None of these

26. If A_1 and A_2 be the two A.M.s between two numbers p and q, then $(2A_1 - A_2)(2A_2 - A_1)$ is equal to :

(A) p + q (B) p – q (C) pq (D) None of these

27. If $\frac{1}{a}, \frac{a^n + b^n}{a^{n+1} + b^{n+1}}, \frac{1}{b}$ are in A.P., then n is equal to :

(A) 0 (B) – 1 (C) $\frac{1}{2}$ (D) None of these

28. If $S_n = nP + \frac{1}{2}n(n-1)Q$ where S_n denotes the sum of the first n terms of an A.P., then the common difference of the A.P. is

(A) P + Q (B) 2P + 3Q (C) 2Q (D) Q

29. If a,b,c are positive reals, then least value of $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ is :

(A) 1 (B) 6 (C) 9 (D) None of these

30. The sum of first four terms of an A.P. is 56 and sum of last four terms is 112. If the first term is 11, then the number of terms is :

(A) 10 (B) 12 (C) 11 (D) None of these

31. For an A.P., $S_{2n} = 3S_n$. The value of $\frac{S_{3n}}{S_n}$ is equal to :

(A) 4 (B) 6 (C) 8 (D) 10

32. The ratio of the 7th to the (n – 1)th mean between 1 and 31, when n arithmetic means are inserted between then, is 5 : 9. The value of n is :

(A) 12 (B) 13 (C) 14 (D) 15

33. The first, second and last terms of an A.P. are a,b, and 2a respectively, the sum of the series is :

(A) $\frac{3ab}{2(b+a)}$ (B) $\frac{3ab}{2(b-a)}$ (C) $\frac{3ab}{2(a-b)}$ (D) None of these

34. Sum of first m terms of an A.P. is 0. If a be the first term of the A.P., then the sum of next n terms is :

(A) $\frac{-a(m+n)m}{m-1}$ (B) $\frac{-a(m+n)n}{m-1}$ (C) $\frac{-a(m+n)n}{n-1}$ (D) $\frac{-a(m+n)m}{n-1}$

35. If A_1 and A_2 be the two A.M.s between two numbers a and b, then $A_2 - A_1$ is equal to

(A) a + b (B) b – a (C) $\frac{b-a}{3}$ (D) None of these

36. The sum of terms equidistant from the beginning and end in an A.P. is equal to :

(A) Last term (B) First term

(C) Sum of the first and the last term (D) None of these

37. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then bc^2, ca^2, ab^2 are in:

(A) A.P. (B) G.P. (C) H.P. (D) None of these

38. If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. and $a_1 > 0$ for all I, then : $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} =$

(A) $\frac{n}{\sqrt{a_1}+\sqrt{a_n}}$ (B) $\frac{n}{\sqrt{a_n}-\sqrt{a_1}}$ (C) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ (D) None of these

39. If a,b,c are in A.P. and also $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., then:

(A) $a = b \neq c$ (B) $a \neq b = c$ (C) $a = b = c$ (D) $a \neq b \neq c$

40. If a,b,c are in H.P., then $\frac{a-b}{b-c}$ equals :

(A) $\frac{b}{a}$ (B) $\frac{a}{b}$ (C) $\frac{a}{c}$ (D) None of these

41. An, A.P. consists pf n (odd) terms and it's middle term is m. Then the sum of the A.P. is :

(A) 2mn (B) $\frac{1}{2}mn$ (C) mn (D) mn^2

42. If A,G and H denote respectively the A.M., G.M. and H.M. between two positive numbers a and b, then A-G is equal to :

(A) a – b (B) $\frac{2ab}{a+b}$ (C) $\frac{1}{2}(\sqrt{a}-\sqrt{b})^2$ (D) None of these

43. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference is :

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

OBJECTIVE							ANSWER KEY						EXERCISE – 4	
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	A	C	D	B	B	D	A	C	A	D	C	A	C	A
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	C	C	C	B	C	A	D	A	C	C	B	D	C	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**		
Ans.	B	C	B	B	C	C	A	C	C	C	C	C	C		

EXERCISE – 5 (FOR IIT – JEE/AIEEE)

CHOOSE THE CORRECT ONE

1. The sum of the n terms of the series $\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+\ldots$ is : **[Kerala Engineering-2003]**

(A) $\frac{3^n(2n+1)+1}{2(3^n)}$ (B) $\frac{3^n(2n+1)-1}{2(3^n)}$ (C) $\frac{n3^n-1}{2(3^n)}$ (D) $\frac{3^n-1}{2}$

2. If the third term of a G.P. is p, then the product of it's first 5 terms is : **[Kerala Engineering-2003]**

(A) p^3 (B) p^2 (C) p^{10} (D) p^5

3. If $a_1, a_2, \ldots\ldots a_n$ are n A.M's between a and b, then $2\sum_{i=1}^{n} a_i =$ **[Kerala Engineering-2003]**

(A) ab (B) n (a + b) (C) $\frac{n(a+b)}{ab}$ (D) $\frac{a+b}{n}$

4. $4^{\frac{1}{2}} x 4^{\frac{1}{4}} x 4^{\frac{1}{8}}$ x ….. to ∞ is a root of the equation : **[Kerala Engineering-2003]**

(A) $x^2 - 4 = 0$ (B) $x^2 - 4x + 6 = 0$ (C) $x^2 - 5x + 4 = 0$ (D) $x^2 - 3x + 2 = 0$

5. If a,b,c are in A.P., then which one of the following is not true? **[Kerala Engineering-2003]**

(A) a + k, b + k, c + k are in A.P. (B) ka, kb, kc are in A.P.

(C) a^2, b^2, c^2 are in A.P. (D) a + b, c + a, b + c are in A.P.

6. The sum of the series: $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots.+\frac{1}{\sqrt{n^2-1}+\sqrt{n^2}}$ is equal to **[AMU–2002]**

(A) $\frac{2n+1}{\sqrt{n}}$ (B) $\frac{\sqrt{n}+1}{\sqrt{n}+\sqrt{n-1}}$ (C) $\frac{n+\sqrt{n^2-1}}{2\sqrt{n}}$ (D) n – 1

7. Sum of infinite number of terms of a G.P. is 20 and sum of their squares is 100. The common ratio of the G.P. is : **[AIEEE–2002]**

(A) 5 (B) $\frac{3}{5}$ (C) $\frac{8}{5}$ (D) $\frac{1}{5}$

8. $1^3 - 2^3 + 3^3 - 4^3 + \dots + 9^3 =$ **[AIEEE–2002]**

(A) 425 (B) – 425 (C) 475 (D) – 475

9. If $y = x - x^2 + x^3 - x^4 + \dots$ to ∞, then the value of x will be $(-1 < x < 1)$:

(A) $y + \frac{1}{y}$ (B) $\frac{y}{1+y}$ (C) $y - \frac{1}{y}$ (D) $\frac{y}{1-y}$

10. The two geometric means between 1 and 64 are: **[Kerala Engineering-2002]**

(A) 1 and 64 (B) 8 and 16 (C) 4 and 16 (D) 3 and 16

11. The sum of infinite terms of the geometric progression $\frac{\sqrt{2}+1}{\sqrt{2}-1}, \frac{1}{2-\sqrt{2}}, \frac{1}{2}, \dots$ is :

[Kerala Engineering-2002]

(A) $\sqrt{2}\left(\sqrt{2}+1\right)^2$ (B) $\left(\sqrt{2}+1\right)^2$ (C) $5\sqrt{2}$ (D) $3\sqrt{2}+\sqrt{5}$

12. If the nth term of the geometric progression, $5, -\frac{5}{2}, \frac{5}{4}, -\frac{5}{8}, \dots is \frac{5}{1024}$, then the value of n is

[Kerala Engineering-2002]

(A) 11 (B) 10 (C) 9 (D) 4

13. In a harmonic progression, pth term is q and qth term is p, then the (pq)th term is:

(A) $\frac{p+q}{pq}$ (B) 0 (C) $\frac{pq}{p+q}$ (D) 1

14. Suppose a,b,c are A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$; then the value of a is:

[IIT Screening-2002]

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

15. Let the positive numbers a,b,c,d be in A.P., then abc, abd, acd, bcd are: **[IITScreening-2001]**

(A) Not in A.P./G.P./H.P. (B) In A.P.

(C) In G.P. (D) In H.P.

16. If the sum of the first 2n terms of the A.P. 2, 5, 8,… is equal to the sum of first n term of the A.P.57, 59, 61,… then n equals : **[IIT Scereening-2001]**

(A) 10 (B)12 (C)11 (D) 13

17. If $\frac{3+5+7+\dots upto\, n\, terms}{5+8+11+\dots upto\, 10\, terms} = 7$, then the value of n is:

(A) 35 (B) 36 (C) 37 (D) 40

18. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in: **[DCE-2000]**

(A) G.P. (B) A.P. (C) H.P. (D) None of these

19. Consider an infinite geometric series with first term a and common ratio r. If it's sum is 4 and the second term is $\frac{3}{4}$, then : **[IIT-Screening-2000]**

(A) $a = \frac{7}{4}, r = \frac{3}{7}$ (B) $a = 2, r = \frac{3}{8}$ (C) $a = \frac{3}{2}, r = \frac{1}{2}$ (D) $a = 3, r = \frac{1}{4}$

20. If 4th term of an H.P. is 5 and 5th term is 4, then it's 20th term is:

(A) Zero (B) $\frac{4}{5}$ (C) 1 (D) $\frac{5}{4}$

21. H.M. between two numbers is 4. The A.M. 'A' and the G.M. 'G' between them satisfy the relation $2A + G^2 = 27$. The numbers are:

(A) 6, 3 (B) 4, 2 (C) 6, 9 (D) 3, 5

22. The sum of an infinite G.P. is 3. The sum of the series formed by squaring its terms is also 3. The series is:

(A) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...$
(B) $\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+...$
(C) $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+...$
(D) $1-\frac{1}{3}+\frac{1}{3^2}+-\frac{1}{3^3}+...$

23. If first three terms of a sequence $\frac{1}{16}, a, b, \frac{1}{6}$ are in G.P. and last three are in H.P., then values of a and b are respectively :

(A) $-\frac{1}{4}, 1$
(B) $\frac{1}{12}, \frac{1}{9}$
(C) Both (A) and (B) are true
(D) $\frac{1}{9}, \frac{1}{12}$

24. The sum of few terms of a ratio series is 728, if common ratio is 3 and last term is 486, then first term of the series is :

(A) 1 (B) 2 (C) 3 (D) 4

25. The 6th term of a G.P. is 32 and it's 8th term is 128; the common ratio of the G.P. is :

(A) – 1 (B) 2 (C) 4 (D) – 4

26. Let $a_1, a_2, \ldots, a_{10}$ be A.P., $h_1, h_2, \ldots, h_{10}$ be in H.P. If $a_1 = h_1 = 2$, $a_{10} = h_{10} = 3$, then $a_4 h_7$ = **[IIT]**

27. In a G.P., the first term is a, second term is b and the last term is c, then sum of the series is: **[AMU]**

28. If H is the harmonic mean between a and b, then $\frac{H+a}{H-a}+\frac{H+b}{H-b}$ is equal to: **[AMU]**

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) None of these

29. Let A_1, A_2 be two AMs and G_1, G_2 be two GMs between a and b, then $\frac{A_1+A_2}{G_1 G_2}$ =

(A) $\frac{a+b}{2ab}$ (B) $\frac{2ab}{a+b}$ (C) $\frac{a+b}{ab}$ (D) $\frac{a+b}{\sqrt{ab}}$

30. a,b,c are three unequal numbers such that a,b,c are in A.P. ; b – a, c – b, a are in G.P. then a : b :c : : **[Karnataka-CET]**

(A) 1 : 2 : 4 (B) 2 : 3: 5 (C) 1 : 2: 3 (D) 1 :3 : 5

31. The first two terms of an infinite G.P. are together equal to 5 and every term is 3 times the sum of all the terms that follow it ; the common ratio of the G.P. is : **[AMU]**

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) 3 (D) 4

32. The eighth term of a G.P. is 128 and common ratio is 2. The product of it's first five terms is:

(A) 4^6 (B) 4^3 (C) 4^5 (D) 4^4

33. The sum of infinity of $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+\ldots$ is : **[AMU]**

(A) $\frac{3}{16}$ (B) $\frac{1}{5}$ (C) $\frac{1}{24}$ (D) $\frac{1}{16}$

34. p,q,r are in A.P. and each is numerically less than 1. Let : **[Karnataka-CET]**

$x = 1 + p + p^2 + \ldots\ldots$ to ∞
$y = 1 + q + q^2 + \ldots\ldots$ to ∞
$z = 1 + r + r^2 + \ldots\ldots$ to ∞, then x,y,z are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

35. If the numbers p,q,r are in A.P., then m^{7p}, m^{7q}, m^{7r} (m > 0) are in :

(A) A.P. (B) G.P. (C) H.P. (D) None of these

36. If the pth, qth and rth terms of a G.P. are ℓ, m and n respectively, then $\ell^{q-r} m^{r-p} n^{p-q}$ is :

(A) 1 (B) 0 (C) pqr (D) ℓmn

37. $\frac{1}{1x2}+\frac{1}{2x3}+\frac{1}{3x4}+....+\frac{1}{n(n+1)}$ equals : **[AMU]**

(A) $\frac{n+1}{n}$ (B) $\frac{n(n+1)}{6}$ (C) $\frac{n}{n+1}$ (D) $\frac{n^2}{n+1}$

38. Sum of n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+....$ is equal to :

(A) $2^n - n - 1$ (B) $1 - 2^{-n}$ (C) $2^n - 1$ (D) $n + 2^{-n} - 1$

39. If $a^x = b^y = c^z$ and a,b,c are in G.P. then x, y,z are in :

(A) A.P. (B) G.P. (C) H.P. (D) None of these

40. If a_n be the nth term of a G.P. of positive numbers and $\sum_{n=1}^{100} a_{2n} = \alpha, \sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the common ratio of the G.P. is : **[IIT]**

(A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$ (C) $\sqrt{\frac{\alpha}{\beta}}$ (D) $\sqrt{\frac{\beta}{\alpha}}$

41. If p,q,r are in A.P. and x,y,z are in G.P, then $x^{q-r} y^{r-p} z^{p-q}$ is equal to :

(A) p + q + r (B) xyz (C) 1 (D) px + qy +rz

42. If a,b,c are in A.P., then $10^{ax + 10}$, $10^{bx + 10}$, $10^{cx + 10}$, $x \neq 0$ are in :

(A) A.P. (B) G.P. only when x > 0

(C) G.P. for all x (D) G.P. only when x < 0

43. If the sum of roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in :

(A) G.P. (B) H.P. (C) A.P. (D) None of these

44. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta\ \gamma, \delta$ are in G.P., then integral values of p and q are respectively.

(A) – 2, – 32 (B) – 2, 3 (C) – 6, 3 (D) – 6,– 32

45. If a,b,c,d and x are all real and $(a^2 + b^2 + c^2)\ x^2 - 2\,(ab + bc + cd)\,x + (b^2 + c^2 + d^2) \leq 0$ then :

(A) a,b,c,d are in G.P. (B) a,b,c,d are in A.P. (C) a,b,c,d are in H.P. (D) None of these

OBJECTIVE — ANSWER KEY — EXERCISE – 4

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	B	C	C	D	B	A	D	C	A	A	D	D	D
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	C	A	B	D	C	A	B	C	B	B	D	B	C	C	C
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	B	C	A	C	B	A	C	D	C	A	C	C	B	A	A

Important Notes

CIRCLES

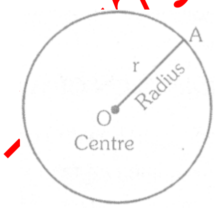
★ INTRODUCTION

In class IX, we have studied that a circle is a collection of all points in a plane which are at a constant distance from a fixed point. The fixed point is called the centre and the constant distance is known as the radius. We have also studied various terms related to a circle like chord, segment, sector, are etc. Now we shall study properties of a line touching a circle at one point.

★ RECALL

Circle

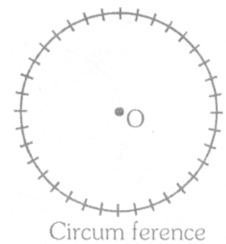
A circle is the locus of a point which moves in such a way that it is always at the constant distance from a fixed point in the plane. **The fixed point** 'O' is called the centre of the circle. **The constant distance** 'OA' between the centre (O) and the moving point (A) is called the **Radius** of the circle.



Circumference

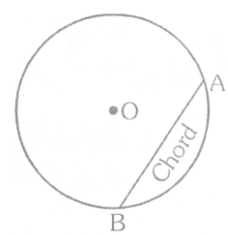
The distance round the circle is called the circumference of the circle.

$2\pi r$ = circumference of the circle
= Perimeter of the circle.
= boundary of the circle
r is the radius of the circle.



Chord

The chord of a circle is a line segment joining any two points on the circumference. AB is the chord of the circle with centre O. In fig. AB is the chord of the circle.

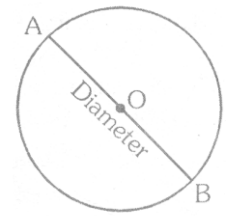


Diameter

A line segment passing through the centre of the circle and having its end points on the circle is called diameter. If r is the radius of the circle then the diameter of the circle is twice the radius i.e., d = 2r

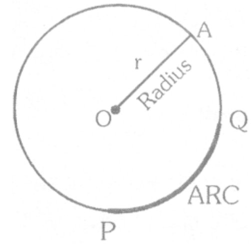
AOB is a diameter of the circle whose centre is O

AOB = OA + OB = r + r + 2r.



Arc of a circle

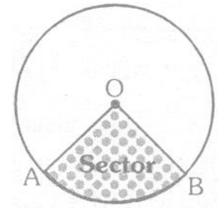
If P and Q be any two points on the circle then the circle is divided into two pieces each of which is an arc. Now we denote the arc from P to Q in counter clock-wise direction by $\overset{\frown}{PQ}$ and the are from Q to P in clock-wise direction by $\overset{\frown}{QP}$.



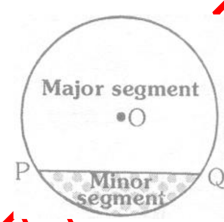
Sector of a circle

The part of a circle bounded by two radii and arc is called sector. In fig, the part of the plane region enclosed0 by $\overset{\frown}{AB}$ and its bounding radii OA and OB is a sector of the circle with centre O.

Segment of a circle

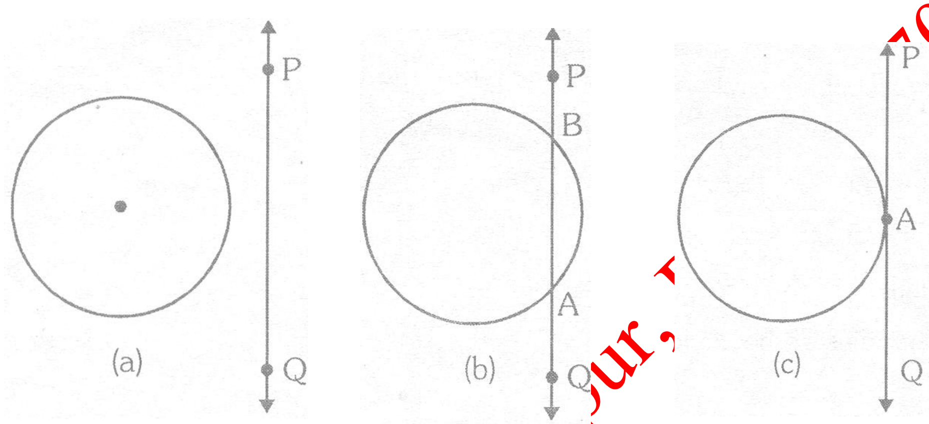


Let PQ be a chord of a circle with centre O and radius r, then PQ divides the region enclosed by the circle into two parts. Each part is called a segment of the circle. The part containing the minor arc is called the **minor segment** and the part containing the major arc is called the **major segment.**



★ INTERSECTION OF A CIRCLE AND A LINE

Consider a circle with centre O and radius r and a line PQ in a plane. We find that there are three different positions a line can take with respect to the circle as given below in fig.



(a) The line PQ does not intersect the circle.
In fig. (a) the line PQ and the circle have no common point. In this case PQ is called a non-intersecting line with respect to the circle.

(b) The line PQ intersect the circle in more than one point. In fig. (b), there are two common points A and B between the line PQ and the circle and we call line PQ as a secant of the circle.

(c) The line intersect the circle in a single point i.e. the line intersect the circle in only one points In fig. (c) you can verify that there is only one point ‘A’ which is common to the line PQ in the given circle. In this case the line is called a tangent to the circle.

Secant

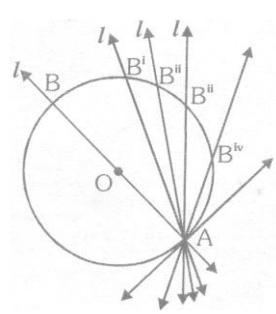
A secant is a straight line that cuts the circumference of the circle at two distinct (different) points i.e., if a circle and a line have two common points then the line is said to be secant to the circle.

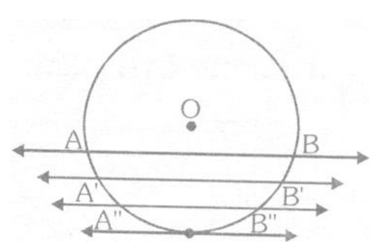
Tangent

A tangent is a straight line that meets the circle at one and only one point. This point ‘A’ is called point of contact or point of tangency in fig. (c).

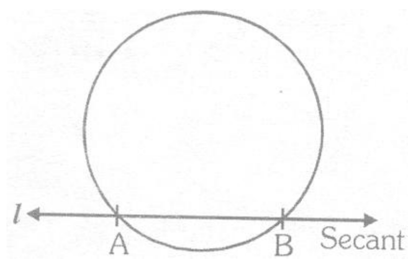
Tangent as a limiting case of a secant

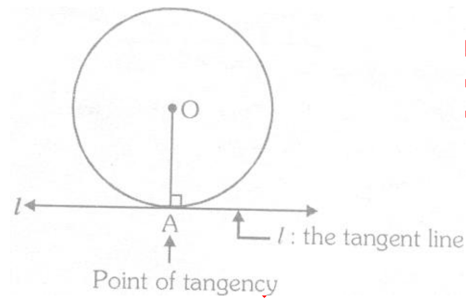
In the fig. the secant ℓ cuts the circle at A and B. If this secant ℓ is turned around the point A, keeping A fixed then B moves on the circumference closer to A. In the limiting position, B coincides with A. The secant ℓ becomes the tangent at A. Tangent to a circle is a secant when the two end points of its corresponding chord coincide.





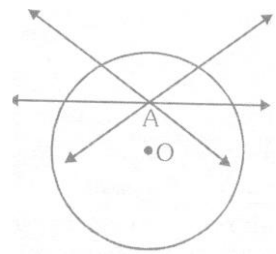
In the fig. ℓ is a secant which cuts the' circle at A and B. If the secant is moved parallel to itself away from the centre, then the points A and B come closer and closer to each other. In the limiting position, they coincide into a single point at A, the secant ℓ becomes the tangent at A. Thus a tangent line is the limiting case of a secant when the two points of intersection of the secant and a circle coincide with the point A. i.e., the common point of the tangent and the circle is called the point of contact and the tangent is said to touch the circle at the common point.





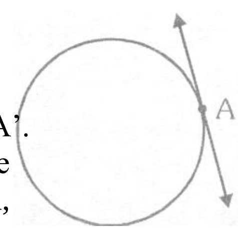
Note: The line containing the radius through the point of contact is called normal to the circle at the point.

★ NUMBER OF TANGENTS TO ACIRCLE FROM A POINT

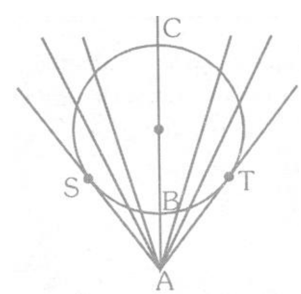


1. If a point A lies inside a circle, no line passing through 'A' can be a tangent to the circle. i.e., No tangent can be drawn from the point A.

2. If A lies on the circle, then one and only one tangent can be drawn to pass through 'A'.
i.e. Exactly one tangent can be drawn through A.

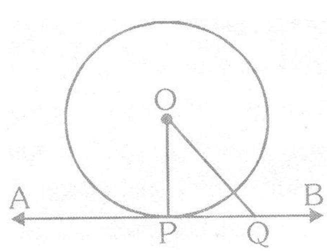


3. If A lies outside the circle then exactly two tangents can be drawn through 'A'. In the fig., a secant ABC is drawn from a point 'A' outside the circle, if the secant is turned around A in the clockwise direction, in the limiting position, it becomes a tangent at T. Similarly if the secant is turned in the anti-clockwise direction, in the limiting position, it becomes a tangent at S. Thus from a point A outside a circle only two tangents can be drawn. The points S and T where the lines touch the circle are called the points of contact.



★ PROPERTIES OF TANGENT TO A CIRCLE

Theorem-1 : The tangent at any point of a circle and the radius through the point are perpendicular to each other.
Given : A circle with centre O. AB is a tangent to the circle at a point P and OP is the radius through P.
To prove : OP $\perp$ AB.
Construct : Take a point Q, other than P, on tangent AB. Join OQ.



Proof :

	STATEMENT	REASON
1.	Since Q is a point on tangent AB, other than the point P, so Q will lie outside the circle $\therefore$ OQ will intersect the circle at some point R.	Tangent at P intersects the circle at points P only.
2.	$\therefore$ OR < OQ $\Rightarrow$ OP < OQ	Part is less than the whole. OR = OP = radius.
3.	Thus, OP is shorter than any other line segment joining O to any point of AB.	
4.	OP $\perp$ AB	Of all line segments drawn from O to line AB, the perpendicular is the shortest

Hence, proved.

Remark 1 : A pair of tangents drawn at two points of a circle are either parallel or they intersect each other at a point outside the circle.

Remark 2 : If two tangents drawn to a circle are parallel to each other, then the line-segment joining their points of contact is a diameter of the circle.

Remark 3 : The distance between two parallel tangents to a circle is equal to the diameter of the circle, i.e., twice the radius.

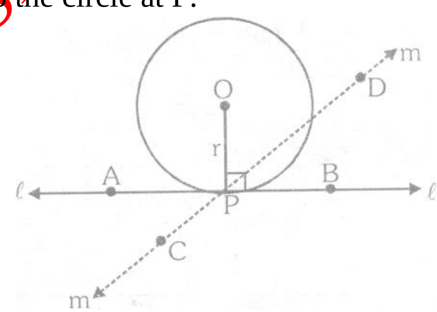
Remark 4 : A pair of tangents drawn to a circle at the end point of a diameter of a circle are parallel to each other.

Remark 5 : A pair of tangents drawn to a circle at the end points of a chord of the circle, other than a diameter, intersect each other at a point outside the circle.

Corollary 1: A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle

Given: O is the centre and r be the radius of the circle. OP is a radius of the circle. Line ℓ is drawn through P so that OP $\perp \ell$

To prove: Line ℓ is tangent to the circle at P.



Construction: Suppose that the line ℓ is not the tangent to the circle at P. Let us draw another straight line m which is tangent to the circle at P. Take two points A and B (other that P) on the line ℓ and two points C and D on m.

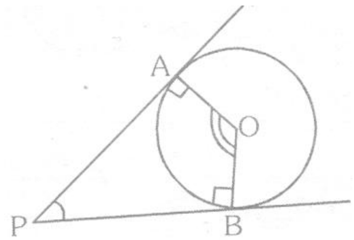
Proof:

	STATEMENT	REASON
1.	OP $\perp \ell$ $\Rightarrow \angle$OPB=90^o	Given
2.	OP $\perp$ m	By theorem

	$\Rightarrow \angle OPD = 90^o$ $\Rightarrow \angle OPD = \angle OPB$	Each = 90^o

But a part cannot, be equal to whole. This gives contradiction. Hence, our supposition is wrong. Therefore, the line ℓ is tangent to the circle at P

Corollary 2: If O be the centre of a circle and tangents drawn to the circle at the points A and B of the circle intersect each other at P, then $\angle AOB + \angle APB = 180^o$.



Proof:

	STATEMENT	REASON
1.	$OP \perp PA$ & $OB \perp PB$ $\Rightarrow \angle OPB = \angle OBP = 90^0$	By theorem
2.	$\angle AOB + \angle OBP + \angle OAP + \angle APB = 360^o$ $\Rightarrow \angle AOP + 90^o + 90^o + \angle APB = 360^o$ $\Rightarrow \angle AOB + \angle APB + 180^o = 360^o$ $\Rightarrow \angle AOB + \angle APB = 360^o - 180^o$ $\Rightarrow \angle AOB + \angle APB = 180^o$	

Hence, Proved

Theorem-2: If two tangents are drawn to a circle from an exterior point, then

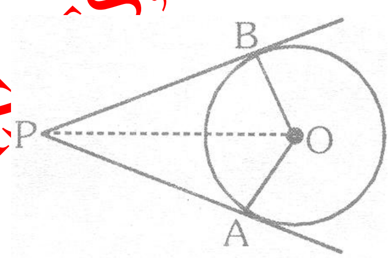
(i) the tangents are equal in length

(ii) the tangents subtend equal angles at the centre

(iii) the tangents are equally inclined to the line joining the point and the centre of the circle.

Given : PA and PB are two tangents drawn to a circle with centre O, from an exterior point P.

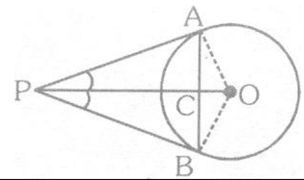
To prove: (i) PA = PB (ii) $\angle AOP = \angle BOP$, (iii) $\angle APO = \angle BPO$.



Proof:

	STATEMENT	REASON
1.	In ΔAOP and ΔBOP: OA = OB $\angle OAP = \angle OBP = 90^o$ OP = OP $\therefore \Delta AOP \cong \Delta BOP$	Radii of the same circle. Radius through point of contact is perpendicular to the tangent Common.
2.	Hence, we have (i) PA = PB (ii) $\angle AOP = \angle BOP$ (iii) $\angle APO = \angle BPO$.	 c.p.c.t c.p.c.t c.p.c.t

Corollary 3: If PA and PB are two tangents from a point to a circle with centre 0 touching it at A and B prove that OP is perpendicular bisector of AB.



Proof:

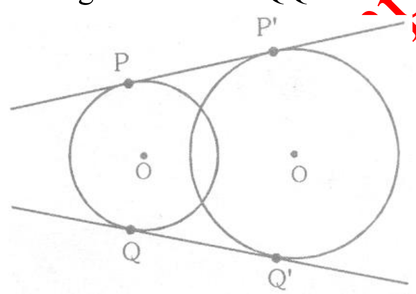
	STATEMENT	REASON
1.	For Δ ACP and Δ BCP	
	(i) PA= PB	Lengths of two tangents from P are equal
	(ii) PC = PC	Common
	(iii) $\angle$ACP = $\angle$BPC	PO bisector $\angle$APB
2.	ΔACP $\cong$ ΔBCP	SAS congruency
3.	AC = BC	c.p.c.t
4.	$\angle ACP = \angle BCP = \frac{1}{2} \times 180^o = 90^o$	

Therefore, OP is perpendicular bisector of AB.
Hence proved.

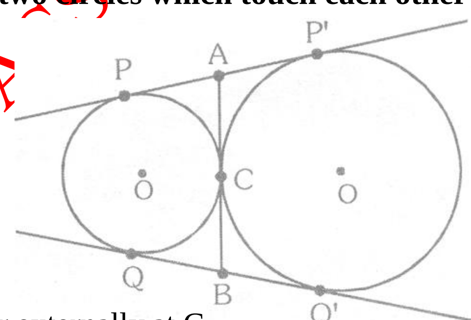
★ **COMMON TANGENTS OF TWO CIRCLES**

Two circles in a plane, either intersect each other in two points or touch each other at a point or they neither intersect nor touch each other.

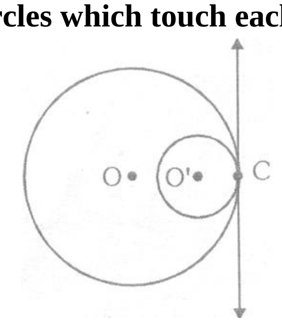
Common Tangent of two intersecting circles : Two circles intersect each other in two points A and B. Here, PP’ and QQ’ are the only two common tangents. The case where the two circles are of unequal radii, we find the common tangents PP’ and QQ’ are not parallel.



Common tangents of two circles which touch each other externally at a point:



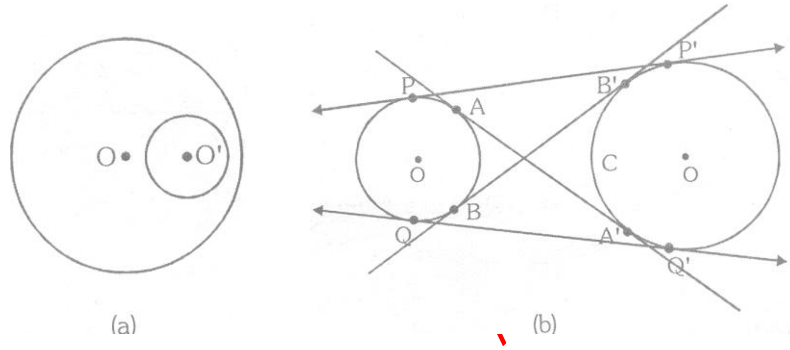
Two circles touch other externally at C.
Here, PP’, QQ’ and AB are the three common tangents drawn to the circles.
Common tangents of two circles which touch each other internally at a point:



Two circles touch other internally at C. Here, we have only one common tangent of the two circles.

Common tangents of two non-intersecting and non-touching circles:

Here, we observe that in figure (a), there is no common tangent but in figure (b) there are four common tangents PP' QQ', AA and BB'.

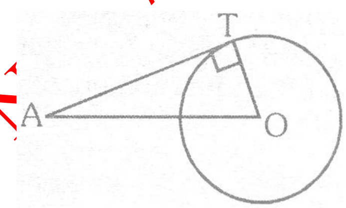


Ex.1. A point A is 26 cm away from the centre of a circle and the length of tangent drawn from A to the circle is 24 cm. Find the radius of the circle.

Sol. Let O be the centre of the circle and let A be a point outside the circle such that OA = 26 cm.

Let AT be the tangent to the circle.

Then, AT = 24 cm. Join OT.



Since the radius through the point of contact is perpendicular to the tangent, we have $\angle OTA = 90^o$. In right ΔOTA, we have

$$OT^2 = OA^2 - AT^2$$

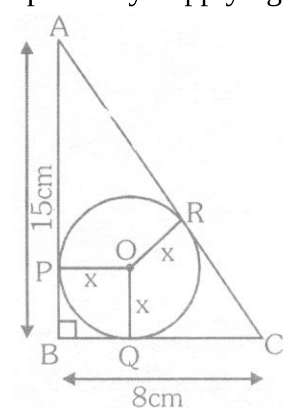
$$= [(26)^2 - (24)^2] = (26 + 24)(26 - 24) = 100.$$

$\Rightarrow$ $OT = \sqrt{100} = 10$ cm.

Hence, the radius of the circle is 10 cm.

Ex.2. In the given figure, ΔABC is right-angled at B, in which AB = 15 cm and BC = 8 cm. A circle with centre O has been inscribed in ΔABC. Calculate the value of x, the radius of the inscribed circle.

Sol. Let the inscribed circle touch the sides AB, BC and CA at P, Q and R respectively. Applying Pythagoras theorem on right ΔABC, we have



$$AC^2 = AB^2 + BC^2 = (15)^2 + (8)^2 = (225 + 64) = 289$$

$\Rightarrow$ $AC = \sqrt{289} = 17$ cm.

Clearly, OPBQ is a square.

$[\because \angle OPB = 90^o, \angle PBQ = 90^o, \angle OQB = 90^o$ and $OP = OQ = x$ cm$]$

$\therefore BP = BQ = x$ cm.

Since the tangents to a circle from an exterior point are equal in length, we have AR = AP and CR = CQ.

Now, AR = AP = (AB – BP) = (15 – x) cm

CR = CQ = (BC – BQ) = (8 – x) cm.

$AC = AR + CR \Rightarrow 17 = (15 - x) + (8 - x) \Rightarrow 2x = 6 \Rightarrow x = 3.$

Hence, the radius of the inscribed circle is 3 cm.

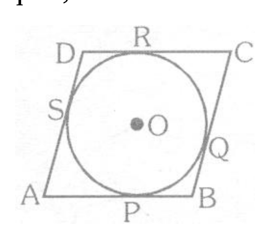
Ex.3 If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

Sol. Let ABCD be a parallelogram whose sides AB, BC, CD and DA touch a circle at the points P, Q, R and S respectively.

Since the lengths of tangents drawn from an external point to a circle are equal, we have

AP = AS, BP = BQ, CR = CQ and DR = DS.

$\therefore$ AB + CD = AP + BP + CR + DR



= AS + BQ + CQ + DS
= (AS + DS) + (BQ + CQ)
= AD + BC

Now, AB + CD = AD + BC

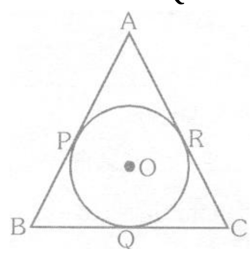
$\Rightarrow$ 2AB = 2BC [$\therefore$ Opposite sides of a || gm are equal]

$\Rightarrow$ AB = BC

$\therefore$ AB = BC = CD = AD.

Hence, ABCD is a rhombus.

Ex.4 In the given figure, the in circle of ΔABC touches the sides AB, BC and CA at the points P, Q, R respectively. Show that AP + BQ + CR = BP + CQ + AR = $\frac{1}{2}$(Perimeter of ΔABC



Sol. Since the lengths of two tangents drawn from an external point to a circle are equal, we have

AP = AR, BQ = BP and CR = CQ

$\therefore$ AP + BQ + CR = AR + BP + CQ ….(i)

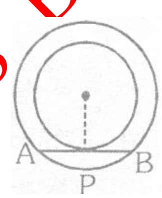
Perimeter of ΔABC = AB + BC + CA = AP + BP + BQ + CQ + AR + CR
= (AP + BQ + CR) + (BP + CQ + AR)

= 2(AP + BQ + CR) [Using (i)]

$\therefore$ AP + BQ + CR = BP + CQ + AR = $\frac{1}{2}$ (Perimeter of ΔABC).

Ex.5 In two concentric circles, prove that a chord of larger circle which is tangent to smaller circle is bisected at the point of contact.

Sol. Let there be two concentric circles, each with centre O.



Let AB be a chord of larger circle touching the smaller circle at P. Join OP.

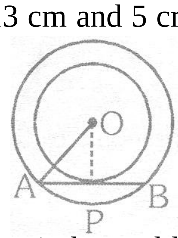
Since OP is a radius of smaller circle and APB is a tangent to it at the point P, so OP $\perp$ AB.

But the perpendicular from the centre to a chord, bisects the chord.

$\therefore$ AP = PB

Hence, AB is bisected at the point P.

Ex.6 Two concentric circles are of radii 13 cm and 5 cm. Find the length of the chord of the outer circle which touches the inner circle.



Sol. Let O be the centre of the concentric circles and let AB be a chord of the outer circle, touching the inner circle at P. Join OA and OP.

Now, the radius through the point of contact is perpendicular to the tangent.

$\therefore$ OP $\perp$ AB.

Since, the perpendicular from the centre to a chord, bisects the chord, AP = PB. Now, in right ΔOPA, we have OA = 13 cm and OP = 5 cm.

$\therefore$ $OP^2 + AP^2 = OA^2 \Rightarrow AP^2 = OA^2 - OP^2 = (13^2 - 5^2) = (169 - 25) = 144$.

$\Rightarrow$ AP = $\sqrt{144}$ = 12 cm.

$\therefore$ AB = 2AP = (2 x 12) cm = 24 cm.

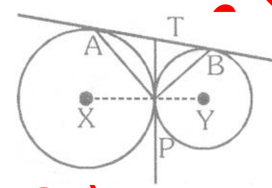
Hence, the length of chord AB = 24 cm.

Ex.7 In the given figure, PT is a common tangent to the circles touching externally at P and AB is another common tangent touching the circles at A and B. Prove that:

(i) T is the mid-point of AB

(ii) $\angle APB = 90^o$

(iii) If X and Y are centres of the two circles,
show that the circle on AB as diameter touches the line XY.



Sol. (i) Since the two tangents to a circle from an external point are equal, we have

TA = TP and TB = TP.

$\therefore$ TA = TB [Each equal to TP]

Hence, T bisects AB, i.e., T is the mid-point of AB.

(ii) TA = TP $\Rightarrow$ $\angle TAP = \angle TPA$

TB = TP $\Rightarrow$ $\angle TBP = \angle TPB$

$\therefore$ $\angle TAP + \angle TBP = \angle TPA + \angle TPB = \angle APB$

$\Rightarrow$ $\angle TAP + \angle TBP = \angle APB = 2\angle APB$

$\Rightarrow$ $2\angle APB = 180^o$ [$\therefore$ The sum of the $\angle$s of a Δ is 180^o]

$\Rightarrow$ $\angle APB = 90^o$

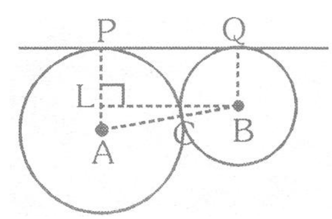
(iii) Thus, P lies on the semi-circle with AB as diameter.

Hence, the circle on AD as diameter touches the line XY.

Ex.8 Two circles of radii 25 cm and 9cm touch each other externally. Find the length of the direct common tangent.

Sol. Let the two circles with centres A and B and radii 25 cm and 9 cm respectively touch each other externally at a point C.

Then, AB = AC + CB = (25 + 9) cm = 34 cm.



[$\therefore$ Radius through point of contact is perpendicular to the tangent]

Draw, BL $\perp$ AP.

Then, PLBQ is a rectangle.

Now, LP = BQ = 9 cm and PQ = BL

$\therefore$ AL = (AP – LP) = (25 – 9) cm = 16 cm.

From right f ΔALB, we have

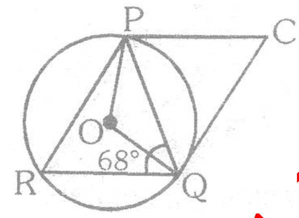
$AB^2 = AL^2 + BL^2 \Rightarrow BL^2 = AB^2 - AL^2 = (34)^2 - (16)^2 = (34 + 16)(34 - 16) = 900$

$\Rightarrow$ BL = $\sqrt{900}$ = 30 cm.

$\therefore$ PQ = BL = 30 cm.

Hence, the length of direct common tangent is 30 cm.

Ex.9 In the given figure, PQ = QR, $\angle RQP = 68^o$, PC and CQ are tangents to the circle with centre O. Calculate the values of : (i) $\angle QOP$ (ii) $\angle QCP$



Sol. (i) In ΔPQR,

$PQ = QR \Rightarrow \angle PRQ + \angle QPR$ [$\angle$s opp. to equal sides of a Δ are equal]

Also, $\angle QPR + \angle RQP + \angle PRQ = 180^o$ [Sum of the $\angle$s of a Δ is 180^o]

$\Rightarrow \quad 68^o + 2\angle PRQ = 180^o$

$\Rightarrow \quad 2\angle PRQ = (180^o - 68^o) = 112^o$

$\Rightarrow \quad \angle PRQ = 56^o$.

$\therefore \quad \angle QOP = 2\angle PRQ = (2 \text{ x } 56^o) = 112^o$. [Angle at the centre is double the angle on the circle]

(ii) Since the radius through the point of contact is perpendicular to the tangent, we have

$\angle OQC = 90^o$ and $\angle OPC = 90^o$.

Now, $\angle OQC + \angle QOP + \angle OPC + \angle QCP = 360^o$ [Sum of the $\angle$s of a quad. Is 360^o)

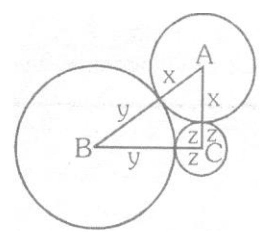
$\Rightarrow 90^o + 112^o + 90^o + \Rightarrow \angle QCP = 360^o$.

$\Rightarrow \angle QCP = (360^o - 292^o) = 68^o$.

Ex.10 With the vertices of ΔABC as centres, three circles are described, each touching the other two externally. If the sides of the triangle are 9 cm, 7 cm and 6 cm, find the radii of the circles.

Sol. Let AB = 9 cm, BC = 7 cm and CA = 6 cm.

Let x, y, z, be the radii of circles with centres A, B, C respectively.



Then, x + y = 9, y + z = 7 and z + x = 6.

Adding, we get $2(x + y + z) = 22 \Rightarrow x + y + z = 11$.

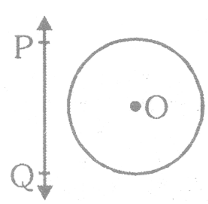
$\therefore x = [(x + y + z) - (y + z)] = (11 - 7)$ cm = 4cm.

Similarly, y = (11 – 6) cm = 5 cm and z = (11 – 9) cm = 2 cm.

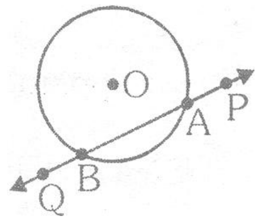
Hence, the radii of circles with centres A, B, C are 4 cm, 5cm and 2 cm respectively.

★ SYNOPSIS

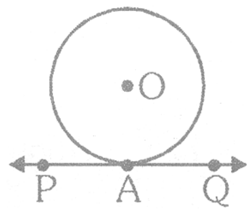
➤ If a circle and a line have no common point, then line is called a non-intersecting line with respect to the circle.



- If a circle and a line have two common points or a line intersect a circle in two distinct points, then line is called secant to the circle.



- If a line and a circle have only one point common, or a line intersect the circle in only one point, then it is called tangent to the circle.



- There is only one tangent at a point of the circle.
- The common point of the tangent and the circle is called the point of contact.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- The line containing the radius through the point of contact of tangent is called the normal to the circle at the point.
- There is no tangent to the circle passing through a point lying inside the circle.
- There are exactly two tangents to a circle through a point lying outside the circle.
- The length of the segment of the tangent from the external point and the point of contact with the circle is called the length of the tangent.
- The lengths of tangents drawn from an external point to a circle are equal.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. A point P is 10 cm from the centre of a circle. The length of the tangent drawn from P to the circle is 8 cm. The radius of the circle is equal to

(A) 4 cm (B) 5 cm (C) 6 cm (D) None of these.

2. A point P is 25 cm from the centre of a circle. The radius of the circle is 7 cm and length of the tangent drawn from P to the circle is x cm. The value of x =

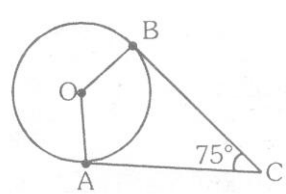
(A) 20 cm (B) 24 cm (C) 18 cm (D) 12 cm.

3. In fig, O is the centre of the circle, CA is tangent at A and CB is tangent at B drawn to the circle. if <ACB = 75^o, then <AOB =

(A) 75^o

(B) 85^o

(C) 95^o



(D) 105^{o}

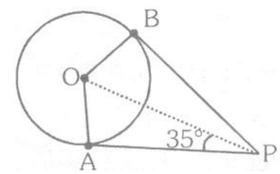
4. In figure 10.75, PA and PB are the two tangents drawn to the circle. 0 is the centre of the circle. A and B are the points of contact of the tangents PA and PB with the circle. If $\angle OPA = 35^{o}$, then $\angle POB =$

(A) 55^{o}

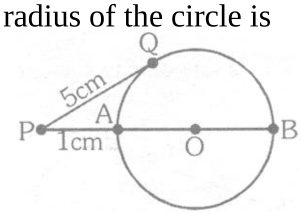
(B) 65^{o}

(C) 75^{o}

(D) 85^{o}



5. In fig, O is the centre of the circle. PQ is tangent to the circle and secant PAB passes through the centre O. If PQ = 5 cm and PA = 1 cm, then the radius of the circle is



(A) 8 cm (B) 12cm (C) 10cm (D) 6cm

6. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q such that OQ = 12cm. Length PQ is

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}\ cm$

7. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25cm. The radius of the circle is

(A) 7 cm (B) 12 cm (C) 15cm (D) 24.5 cm

8. The length of the tangent from a point A at a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is

(A) $\sqrt{7}\ cm$ (B) 7 cm (C) 5 cm (D) 25cm

9. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80^{o} then $\angle POA$ is equal to

(A) 50^{o} (B) 60^{o} (C) 70^{o} (D) 80^{o}

10. If TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110^{o}$, then, $\angle PTQ$ is equal to

(A) 60^{o} (B) 70^{o} (C) 80^{o} (D) 90^{o}

11. PQ is a tangent to a circle with centre O at the point P. If ΔOPQ is an isosceles triangle, then $\angle OQP$ is equal to

(A) 30^{o}` (B) 45^{o} (C) 60^{o} (D) 90^{o}

12. Two circle touch each other externally at C and AB is a common tangent to the circles. Then, $\angle ACB =$

(A) 60^{o} (B) 45^{o} (C) 30^{o} (D) 90^{o}

13. ABC is a right angled triangle, right angled at B such that BC = 6 cm and AB = 8 cm. A circle with centre O is inscribed in ΔABC. The radius of the circle is

(A) 1 cm (B) 2 cm (C) 3 cm (D) 4 cm

14. PQ is a tangent drawn from a point P to a circle with centre O and QOP is a diameter of the circle such that $\angle POR = 120^{o}$, then $\angle OPQ$ is

(A) 60^{o} (B) 45^{0} (C) 30^{o} (D) 90^{o}

15. If four sides of a quadrilateral ABCD are tangential to a circle, then

(A) AC + AD = BD + CD (B) AB + CD = BC + AD

(C) AB + CD = AC + BC (D) AC + AD = BC + DB

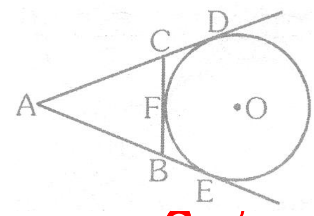
16. The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is

(A) $\sqrt{7}$ cm (B) $2\sqrt{7}$ cm (C) 10 cm (D) 5 cm

17. AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that CD = 4 cm, then AB is equal to

(A) 4 cm (B) 6 cm (C) 8 cm (D) 12 cm

18. In the adjoining figure, if AD, AE and BC are tangents to the circle at D, E and F respectively. Then,



(A) AD = AB + BC + CA (B) 2AD = AB + BC + CA

(C) 3AD = AB + BC + CA (D) 4AD = AB + BC + CA

OBJECTIVE					**ANSWER KEY**					**EXERCSE**
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	C	B	D	A	B	D	A	C	A	B
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans.	B	D	B	C	B	B	C	B		

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

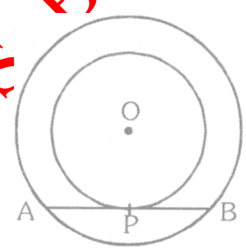
SUBJECTIVE TYPE QUESTIONS

SHORT ANSWER TYPE QUESTIONS

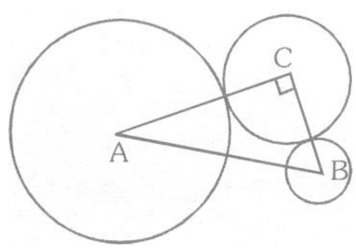
1. Find the length of the tangent drawn to a circle of radius 8 cm, from a point which is at a distance of 10 cm from the centre of the circle.

2. A point P is 7 cm away from the centre of the circle and the length of tangent drawn from P to the circle is 15 cm. Find the radius of the circle.

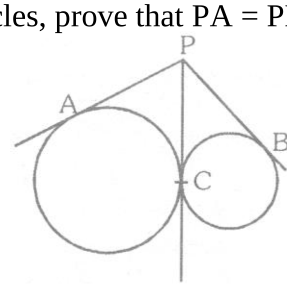
3. There are two concentric circles, each with centre O and of radii 10 cm and 26 cm respectively. Find the length of the chord AB of the outer circle which touches the inner circle at P.



4. A and B are centres of circles of radii 9 cm and 2 cm such that AB = 17 cm and C is the centre of the circle of radius r cm which touches the above circles externally. If $\angle ACB = 90^{\circ}$, write an equation in r and solve it.



5. Two circles touch each other externally at a point C and P is a point on the common tangent at C. If PA and PB are tangents to the two circles, prove that PA = PB.



6. Two circles touch each other internally. Prove that the tangents drawn to the two circles from any point on the common tangent are equal in length.

7. Two circles of radii 18 cm and 8 cm touch externally. Find the length of a direct common tangent to the two circles.

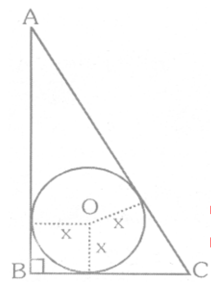
8. Two circles of radii 8 cm and 3 cm have their centres 13 cm apart. Find the length of a direct common tangent to the two circles.

9. Two circles of radii 8 cm and 3 cm have a direct common tangent of length 10 cm. Find the distance between their centres, up to two places of decimal.

10. With the vertices of ΔPQR as centres, three circles are described, each touching the other two externally. If the sides of the triangle are 7 cm, 8 cm and 11 cm, find the radii of the three circles.

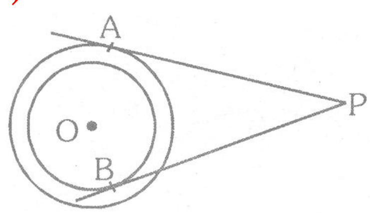
LONG ANSWER TYPE QUESTIONS

1. ΔABC is right-angled triangle with AB = 12 cm and AC = 13 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of x, the radius of the inscribed circle.

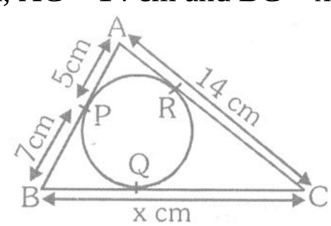


2. PQR is a right-angled triangle with PQ = 3 cm and QR = 4 cm. A circle which touches all the sides of the triangle is inscribed in the triangle. Calculate the radius of the circle.

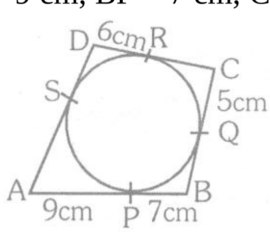
3. In the given figure, O is the centre of each one of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to outer and inner circle respectively. If PA = 10 cm, find the length of PB, up to two places of decimal.



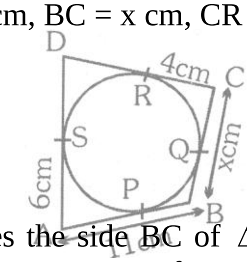
4. In the given figure, ΔABC is circumscribed. The circle touches the sides AB, BC and CA at P, Q, R respectively. If AP = 5 cm, BP = 7 cm, AC = 14 cm and BC = x cm, find the value of x.



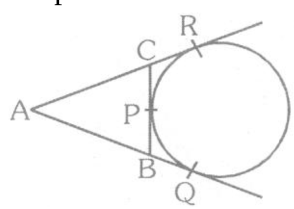
5. In the given figure, quadrilateral ABCD is circumscribed. The circle touches the sides AB, BC, CD and DA at P, Q, R, S respectively. If AP = 9 cm, BP = 7 cm, CQ = 5 cm and DR = 6 cm, find the perimeter of quad. ABCD.



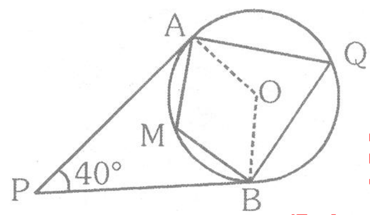
6. In the given figure, the circle touches the sides AB, BC, CD and DA of a quadrilateral ABCD at the points P, Q, R,S respectively. If AB = 11 cm, BC = x cm, CR = 4 cm and AS = 6 cm, find the value of x.



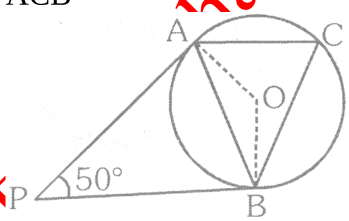
7. In the given figure, a circle touches the side BC of ΔABC at P and AB and AC produced at Q and R respectively. If AQ = 15 cm, find the perimeter of ΔABC.



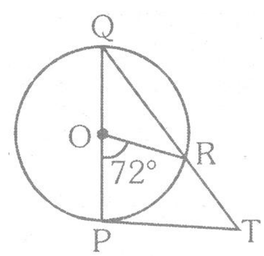
8. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 40^o$, find $\angle AQB$ and $\angle AMB$.



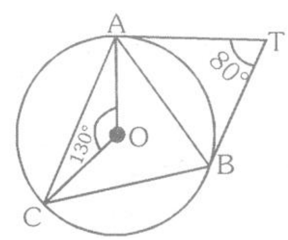
9. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 50^o$, find:
(i) $\angle AOB$ (ii) $\angle OAB$ (iii) $\angle ACB$



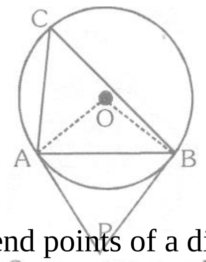
10. In the given figure PQ is a diameter of a circle with centre O and PT is a tangent at. QT meets the circle at R. If $\angle POR = 72^o$, find $\angle PTR$.



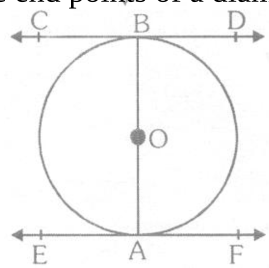
11. In the given figure, O is the centre of the circumcircle of ΔABC. Tangents at A and B intersect at T. If $\angle ATB = 80^o$ and $\angle AOC = 130^o$, Calculate $\angle CAB$.



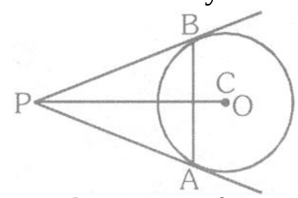
12. In the given figure, PA and PB are tangents to a circle with centre O and ΔABC has been inscribed in the circle such that AB = AC. If $\angle BAC = 72^o$, calculate (a) $\angle AOB$ (B0 $\angle APB$.



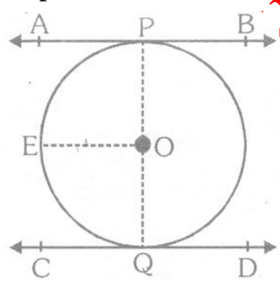
13. Show that the tangent lines at the end points of a diameter of a circle are parallel.



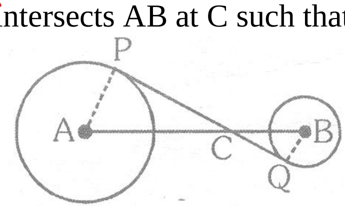
14. Prove that the tangents at the extremities of any chord make equal angles with the chord.



15. Show that the line segment joining the points of contact of two parallel tangents passes through the centre.

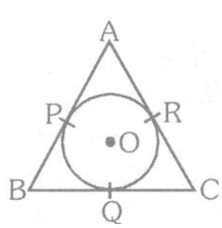


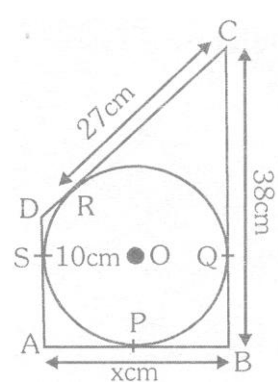
16. In the given figure, PQ is a transverse common tangent to two circles with centres A and B and of radil 5 cm and 3 cm respectively. If PQ intersects AB at C such that CP = 12 cm, calculate AB.

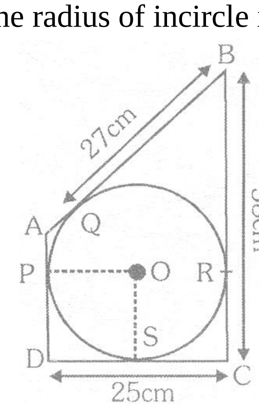


17. ΔABC is an isosceles triangle in which AB = AC, circumscribed about a circle. Prove that the base is bisected by the point of contact.

18. In the given figure quadrilateral ABCD is circumscribed and AD $\perp$ AB. If the radius of incircle is 10 cm, find the value of x.







19. In the given figure, a circle is inscribed in quad. ABCD. If BC = 38 cm, BQ = 27 cm, DC = 25 cm and AD $\perp$ DC, find the radius of the circle.

CIRCLE	ANSWER KEY	EXERCISE – 2 (X) – CBSE

SHORT ANSWER TYPE QUESTIONS:

1. 6 cm **2.** 8 cm **3.** 48 cm **4.** $r^2 + 11r - 102 = 0$, r = 6 **7.** 24 cm **8.** 12 cm
9. 11.8 cm **10.** 5 cm, 2 cm, 6 cm

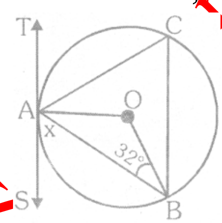
LONG ANSER TYPE QUESTIONS:

1. 2 cm **2.** 1 cm **3.** 10.95 cm **4.** 16 cm **5.** 54 cm **6.** x = 9 **7.** 30 cm
8. $\angle AQB = 70^o$, $\angle AMB = 110^o$ **9.** (i) 130^o, (ii) 25^o, (iii) 65^o **10.** 54^o **11.** 65^o
12. (a) 108^o, (b) 72^o **16.** 20.8 cm **18.** 21 cm **19.** 14 cm

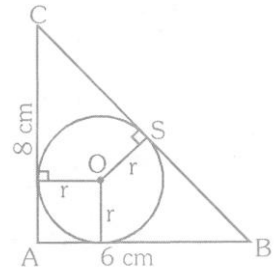
EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD QUESTIONS
VERY SHORT ANSWER QUESTIONS

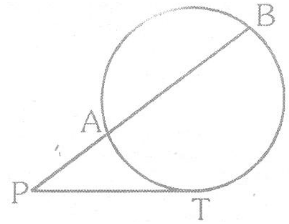
1. In the given figure, TAS is a tangent to the circle, with centre O, at the point A. If $\angle OBA = 32^o$, find the value of x. **[Delhi-1996C]**



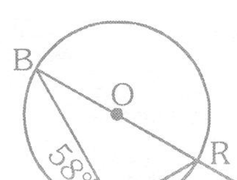
2. In the given figure, ABC is a right angled triangle right angled at A, with AB = 6cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of r, the radius of the inscribed circle. **[AI-1998]**



3. In the given figure, PT is tangent to the circle at T. If PA = 4 cm and AB = 5 cm, find PT. **[Delhi-19980]**

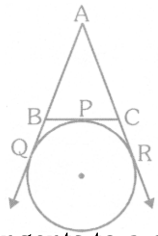


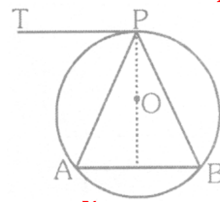
4. In the figure, O is the centre of the circle, PQ is tangent to the circle at A. If $\angle PAB = 58^o$, find $\angle ABQ$ and $\angle AQB$.



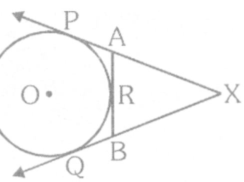
5. In figure, a circle touches the side BC of ΔABC at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of ΔABC.

6. A tangent PT is drawn parallel to a chord AB as shown in figure. Prove that APB is an isosceles triangle. **[Foreign – 2000]**

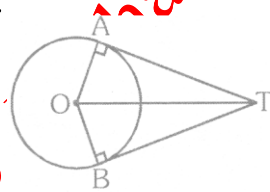




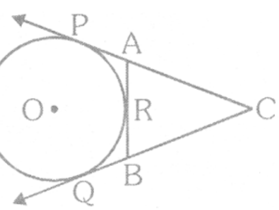
7. In figure, XP and XQ are two tangents to a circle with centre O from a point X outside the circle. ARB is tangent to circle at R. Prove that XA + AR = XB + BR. **[Delhi-2003]**



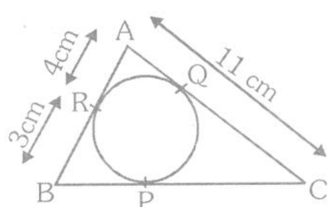
8. In fig, if $\angle ATO = 40^o$, find $\angle AOB$. **[AI-2008]**



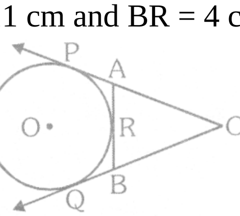
9. In fig., CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then find the length of BR. **[Delhi-2009]**



10. In fig., ΔABC is circumscribing a circle. Find the length of BC. **[AI-2009]**



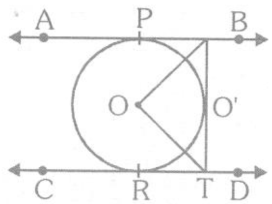
11. In fig., CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC. **[AI-2010]**



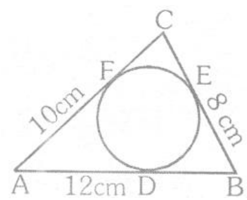
SHORT ANSWER TYPE QUESTIONS

1. If ΔABC is isosceles with AB = AC, prove that the tangent at A to the circumcircle of ΔABC is parallel to BC. **[AI-1998C]**

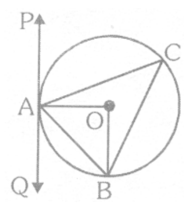
2. In figure, AB and CD are two parallel tangents to a circle with centre O. ST is tangent segment between the two parallel tangents touching the circle at Q. Show that $\angle SOT = 90^o$. **[AI-2000]**



3. A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm as shown in figure. Find AD, BE and CF. **[Delhi-2001]**



4. PAQ is a tangent to the circle with centre O at a point A as shown in figure. If $\angle OBA = 35^o$, find the value of $\angle BAQ$ and $\angle ACB$.

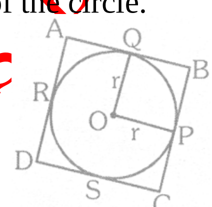


5. AB is diameter and AC is a chord of a circle such that $\angle BAC = 30^o$. If then tangent at C intersects AB produced in D, prove that BC = BD. **[Delhi-2003]**

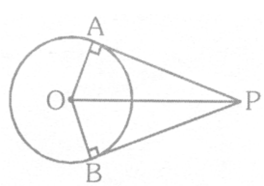
6. ABC is an isosceles triangle in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact.

OR

In the fig., a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^o$. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius (r) of the circle.

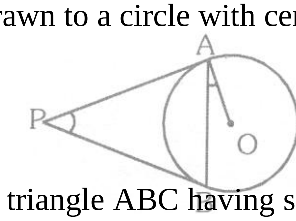


7. In fig., OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle. **[AI-2008]**



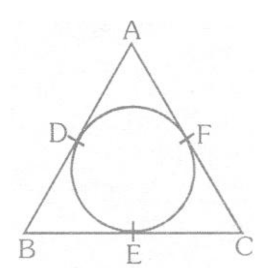
8. Prove that a parallelogram circumscribing a circle is a rhombus. **[Foreign-2008]**

9. Two tangents PA and PB are drawn to a circle with centre O from and external point P. Prove that $\angle APB = 2\angle OAB$.

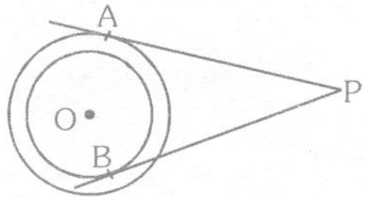


10. In fig., a circle is inscribed in a triangle ABC having side BC = 8 cm, AC = 10 cm and AB = 12 cm. Find AD, BE and CF

[Foreign–2009]

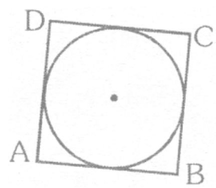


11. In fig., there are two concentric circles with centre O and of radii 5 cm and 3 cm. From an external Point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP. **[AI – 2010]**

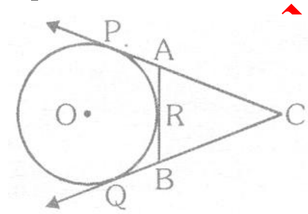


LONG ANSWER TYPE QUESTIONS

1. Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above, prove the following :
A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.
[Delhi-2008, AI-2009]

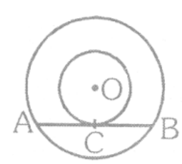


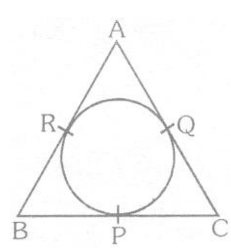
2. Prove that the lengths of the tangents drawn from an external point to a circle are equal. Using the above, do the following:
In the fig., TP and TQ are tangents from T to the circle with centre O and R is any point on the circle. If AB is a tangent to the circle at R, prove that TA + AR = TB + BR. **[AI-2008]**



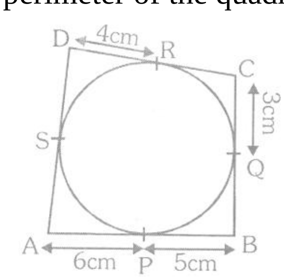
3. Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above do the following :
ABC is an isosceles triangle in which AB = AC, circumscribe about a circle as shown in the fig. Prove that the base is bisected by the point of contact. **[Foreign-2008]**

4. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following:
In fig., O is the centre of the two concentric circles. AB is a chord of the larger circle touching the small circle at C. Prove that AC = BC.
[AI-2009]





5. Prove that the length of the tangents drawn from an external point to a circle are equal. Using the above, do the following :
In fig, quadrilateral ABCD is circumscribing a circle. Find the perimeter of the quadrilateral ABCD.
[Foreign-2009]



CIRCLE	ANSWER KEY	EXERCISE-3 (X)-CBSE

VERY SHORT ANSWER TYPE QUESTIONS:

1. $x = 58^o$ **2.** r = 2 cm **3.** PT = 6 cm **4.** 32^o, 26^o **5.** 10 cm **8.** 100^o
9. 4 cm **10.** 10cm **11.** 7 cm

SHORT ANSWER TYPE QUESTIONS:

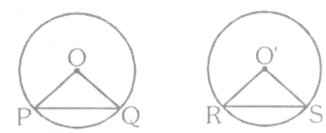
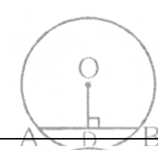
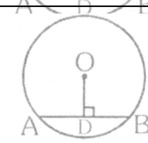
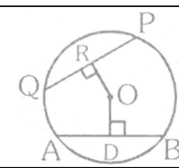
3. 7 cm, 5cm, 3 cm **4.** 55^o and 55^o **6.** 11 cm **10.** AD = 7 cm, BE = 5 cm and CF = 3 cm
11. $4\sqrt{10}$ cm

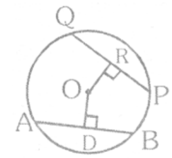
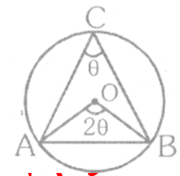
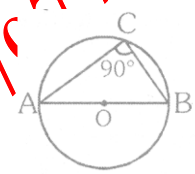
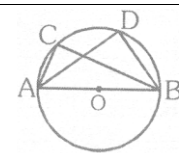
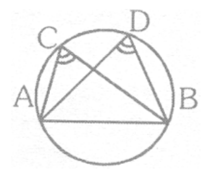
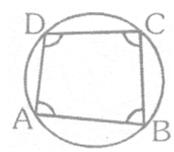
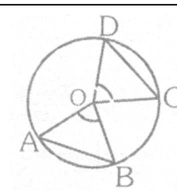
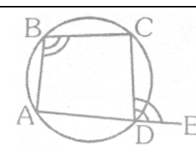
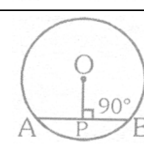
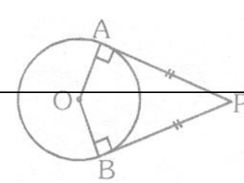
LONG ANSWER TYPE QUESTIONS:

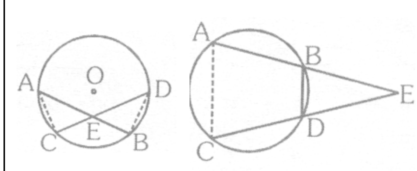
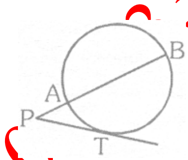
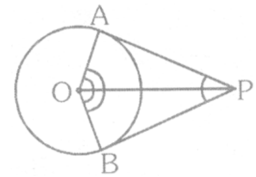
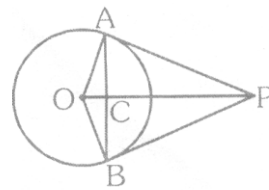
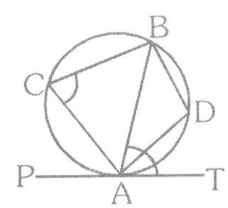
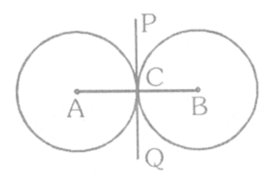
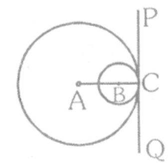
5. 36 cm

COMPETITION WINDOW

SOME IMPORTANT THEOREMS:

S. No.	Theorem	Diagram
1.	In a circle (or in congruent circles) equal chords are made by equal arcs. {OP = OQ} = {O'R = O'S} and $\overset{\frown}{PQ} = \overset{\frown}{RS}$ $\therefore$ PQ = RS	
2.	Equal arcs (or chords) subtend equal angles at the centre i.e., if $\overset{\frown}{PQ} = \overset{\frown}{AB}$ (or PQ = AB) $\therefore$ $\angle POQ = \angle AOB$	
3.	The perpendicular from the centre of a circle to a chord bisects the chord i.e., if OD $\perp$ AB $\therefore$ AB = 2AD = 2BD	
4.	The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord. $\therefore$ AD = DB $\therefore$ OD $\perp$ AB	
5.	Perpendicular bisector of a chord passes through the centre. i.e., if OD $\perp$ AB and AD = DB $\therefore$ O is the centre of the circle.	
6.	Equal chords of a circle (or of congruent circles) are equidistant from the centre. $\therefore$ AB = PQ	

	∴ OD = OR	
7.	Chords which are equidistant from the centre in a circle (or in congruent, circles) are equal. ∴ OD = OR ∴ AB = PQ	
8.	The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. m ∠AOB = 2m∠ACB.	
9.	Angle in a semicircle is a right angle.	
10.	Angle in the same segment of a circle are equal i.e., ∠ACB = ∠ADB	
11.	If line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle. ∠ACB = ∠ADB ∴ Points A, C, D, B are co cyclic i.e., lie on the circle	
12.	The sum of pair of opposite angles of a cyclic quadrilateral is 180^o ∠DAB + ∠BCD = 180^o and ∠ABC + ∠CDA = 180^o (converse of this theorem is also true)	
13.	Equal chords (or equal arcs) of a circle (or congruent circles subtend equal angles at the centre. AB = CD (or $\overset{\frown}{AB} = \overset{\frown}{CD}$) ∴ ∠AOB = ∠COD	
14.	If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. m∠CDE = m∠ABC	
15.	A tangent at any point of a circle is perpendicular to the radius through the point of contact. (converse of this theorem is also true)	
16.	The lengths of two tangents drawn from an external point to a circle are equal. i.e., AP = BP	

17.	If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E) then AE x BE = CE x DE	
18.	If PB be a secant which intersects the circle at A and B and PT be a tangent at T then PA . PB = $(PT)^2$	
19.	From an external point from which the tangents are drawn to the circle with centre O, then (a) They subtend equal angles at the centre. (b) They are equally inclined to the line segment joining the centre of that point. $\angle AOP = \angle BOP$ and $\angle APO = \angle BPO$	
20.	If P is an external point from which the tangents to the circle with centre O touch it at A and B then OP is the perpendicular bisector of AB. $OP \perp AB$ and AC = BC	
21.	**Alternate Segment Theorem :** If from the point of contact of a tangent, a chord is drawn then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram. $\angle BAT = \angle BCA$ and $\angle BAP = \angle BDA$	
22.	The point of contact of two tangents lies on the straight line joining the two centres. (a) When two circles touch externally then the distance between their centres is equal to sum of their radii i.e. AB = AC + BC (a) When two circles touch internally then the distance between their centres is equal to the difference between their radii i.e. AB = AC – BC	

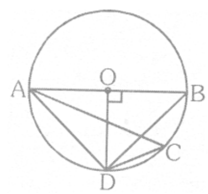
EXERCISE – 4 (FOR OLMPLADS]

CHOOSE THE CORRECT ONE

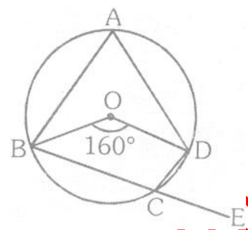
1. If the diagonals of cyclic quadrilateral are equal, then the quadrilateral is
(A) rhombus (B) square (C) rectangle (D) none of these

2. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is a
(A) rectangle (B) square (C) parallelogram (D) cyclic quadrilateral

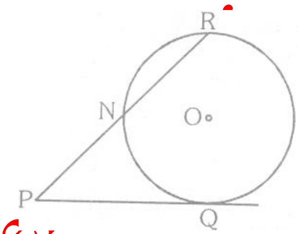
3. In the given figure, AB is the diameter of the circle. Find the value of $\angle ACD$:
(A) 30^o
(B) 60^o
(C) 45^o
(D) 25^o



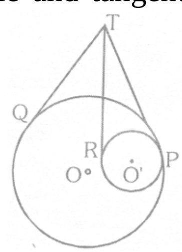
4. Find the value of $\angle DCE$:
(A) 100^o
(B) 80^o
(C) 90^o
(D) 75^o



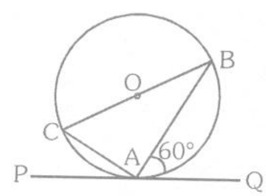
5. In the given figure, PQ is the tangent of the circle. Line segment PR intersects the circle at Nand R.PQ = 15 cm, PR = 25 cm, find PN:
(A) 15 cm
(B) 10 cm
(C) 9 cm
(D) 6 cm



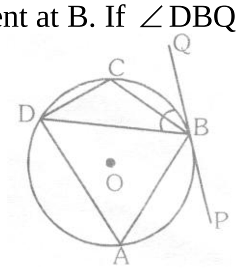
6. In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find TQ : TR
(A) 8 : 7
(B) 7 : 8
(C) 5 : 4
(D) 1 : 1



7. In the given figure, PAQ is the tangent. BC is the diameter of the circle. m $\angle BAQ = 60^o$, find m $\angle ABC$:
(A) 25^o
(B) 30^o
(C) 45^o
(D) 60^o



8. ABCD is a cyclic quadrilateral PQ is a tangent at B. If $\angle DBQ = 65^o$, then $\angle BCD$ is :
(A) 35^o
(B) 85^o
(C) 115^o
(D) 90^o

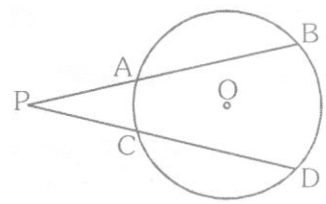


9. In the given figure, AP = 2 cm, BP = 6 cm and CP = 3 cm. Find DP:

(A) 6 cm
(B) 4 cm
(C) 2 cm
(D) 3 cm

10. In the given figure, AP = 3 cm, BA = 5 cm and CP = 2 cm. Find CD :

(A) 12 cm
(B) 10 cm
(C) 9 cm
(D) 6 cm



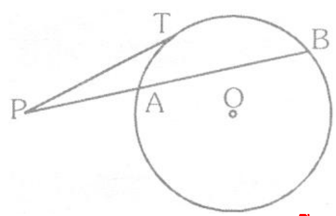
11. In the figure, tangent PT = 5 cm, PA = 4 cm, find AB :

(A) $\frac{7}{4}$ cm

(B) $\frac{11}{4}$ cm

(C) $\frac{9}{4}$ cm

(D) can't be determined



12. Two circles of radii 13 cm and 5 cm touch internally each other. Find the distance between their centres :

(A) 18 cm (B) 12 cm (C) 9 cm (D) 8 cm

13. Three circles touch each other externally. The distance between their centre is 5 cm. 6 cm and 7 cm. Find the radii of the circles :

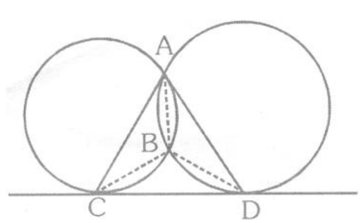
(A) 2 cm, 3 cm, 4 cm (B) 3 cm, 4 cm, 1 cm
(C) 1 cm, 2.5 cm, 3.5 cm (D) 1 cm, 2 cm, 4 cm

14. If AB is a chord of a circle, P and Q are two points on the circle different from A and B, then:

(A) the angle subtended by AB at P and Q are either equal or supplementary .
(B) the sum of the angles subtended by AB at P and Q is always equal two right angles.
(C) the angles subtended at and Q by AB are always equal.
(D) the sum of the angles subtended at P and Q is equal to four right angles.

15. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B, then: $\angle$ CAD + $\angle$CBD = ?

(A) 120°
(B) 90°
(C) 360°
(D) 180°



16. In a circle of radius 5 cm, AB and AC are the two chords such that AB = AC = 6 cm. Find the length of the chord BC.

(A) 4.8 cm (B) 10.8 cm (C) 9.6 cm (D) none of these

17. In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is :
(A) 23 cm (B) 30 cm (C) 15m cm (D) none of these

18. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :
(A) $\sqrt{3}:2$ (B) $\sqrt{3}:1$ (C) $\sqrt{5}:1$ (D) none of these

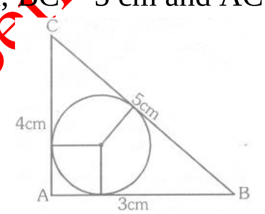
19. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the other circle which is outside the inner circle, is of length :
(A) $2\sqrt{2}\,cm$ (B $3\sqrt{2}\,cm$ (C) $2\sqrt{3}\,cm$ (D) $4\sqrt{2}\,cm$

20. Through any given set of four points P, Q, R, S it is possible to draw:
(A) atmost one circle (B) exactly one circle (C) exactly two circles (D) exactly three circles

21. The distance between the centers of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is:
(A) 4 cm (B) 6 cm (C) 8 cm (D) 10 cm

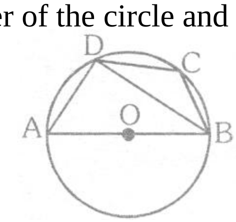
22. The number of common tangents that can be drawn to two given circles is at the most :
(A) 1 (B) 2 (C) 3 (D) 4

23. ABC is a right angled triangle AB = 3 cm, BC = 5 cm and AC = 4 cm, then the inradius of the circle is :
(A) 1 cm
(B) 1.25 cm
(C) 1.5 cm
(D) none of these

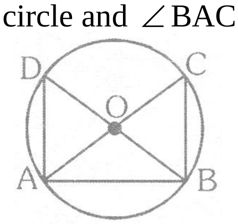


24. A circle has two parallel chords of lengths 6 cm and 8 cm. If the chords are 1 cm apart and the centre is on the same side of the chords, then a diameter of the circle is of length:
(A) 5 cm (B) 6 cm (C) 8 cm (D) 10 cm

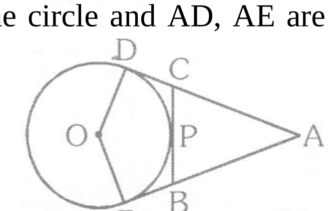
25. In the adjoining figure AB is a diameter of the circle and $\angle BCD = 130^o$. What is the value of $\angle ABD$?
(A) 30^o
(B) 50^o
(C0 40^o
(D) None of these



26. In the given figure O is the centre of the circle and $\angle BAC = 25^o$. then the value of $\angle ADB$ is :
(A) 40^o
(B) 55^o
(C) 50^o
(D) 65^o

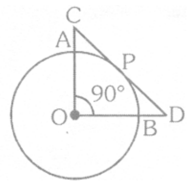


27. In the given circle O is the centre of the circle and AD, AE are the two tangents. BC is also a tangent, then:

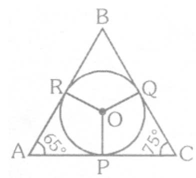


(A) AC + AB = BC
(B) 3AE = AB + BC + AC
(C) AB + BC + AC = 4AE
(D) 2AE = AB + BC + AC

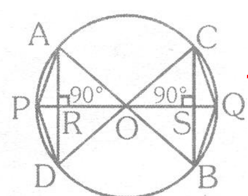
28. In a circle O is the centre and $\angle COD$ is right angle. AC = BD and CD is the tangent at P. What is the value of AC + CP, if the radius of the circle is 1 metre?
(A) 105 cm
(B) 141.4 cm
(C) 138.6 cm
(D) Can't be determined



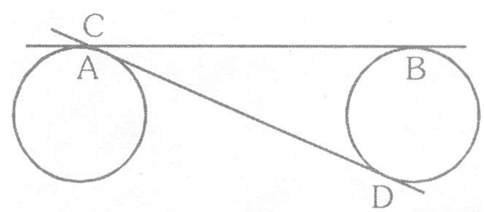
29. In a triangle ABC, O is the centre of incircle PQR, $\angle BAC = 65^o$, $\angle BCA = 75^o$, find $\angle ROQ$:
(A) 80^o
(B) 120^o
(C) 140^o
(D) Can't be determined



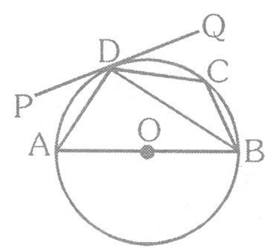
30. In the adjoining figure O is the centre of the circle. $\angle AOD = 120^o$. If the radius of the circle be 'r', then find the sum of the areas of quadrilaterals AODP and OBQC:
(A) $\frac{\sqrt{3}}{2}r^2$
(B) $3\sqrt{3}r^2$
(C) $\sqrt{3}r^2$
(D) None of these



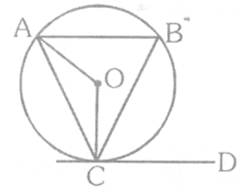
31. There are two circles each with radius 5 cm. Tangent AB is 26 cm. The length of tangent CD is :
(A) 15 cm
(B) 21 cm
(C) 24 cm
(D) Can't be determined



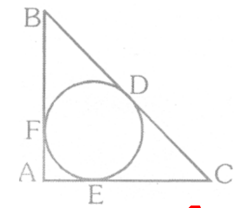
32. In the adjoining figure O is the centre of the circle and AB is the diameter. Tangent PQ touches the circle at D. $\angle BDQ$ 48^o. Find the value of $\angle DBA : \angle DCB$:
(A) $\frac{22}{7}$
(B) $\frac{7}{22}$
(C) $\frac{7}{12}$
(D) Can't be determined



33. In the given diagram O is the centre of the circle and CD is a tangent, $\angle CAB$ and $\angle ACD$ are supplementary to each other $\angle OAC$ 30^o. Find the value of $\angle OCB$:
(A) 30^o
(B) 20^o
(C) 60^o
(D) None of these

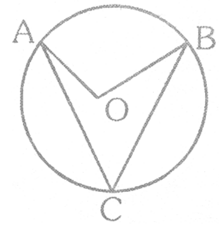


34. In the given diagram an incircle DEF is circumscribed by the right angled triangle in which AF = 6 cm and EC = 15 cm. Find the difference between CD and BD:



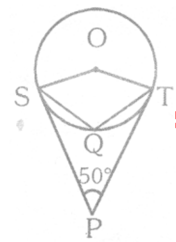
(A) 1 cm
(B) 3 cm
(C) 4 cm
(D) Can't be determined

35. In the adjoining figure 'O' is the centre of circle, $\angle CAO = 25^o$ and $\angle CBO = 35^o$. What is the value of $\angle AOB$?



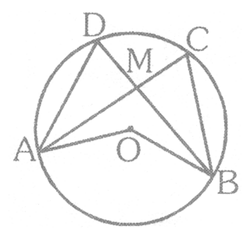
(A) 55^o
(B) 110^o
(C) 120^o
(D) Data insufficient

36. In the given figure 'O' is the centre of the circle SP and TP are the two tangents at S and T respectively. $\angle SPT$ is 50^o, the value of $\angle SQT$ is :



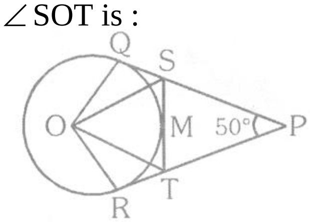
(A) 125^o
(B) 65^o
(C) 115^o
(D) None of these

37. In the given figure of circle, 'O' is the centre of the circle $\angle AOB = 130^o$. What is the value of $\angle DMC$?



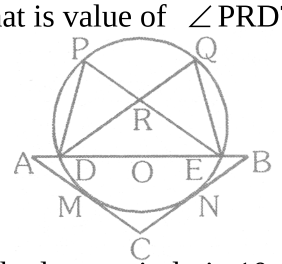
(A) 65^o
(B) 125^o
(C) 85^o
(D) Can't be determined

38. In the adjoining figure 'O' is the centre of the circle of the circle and PQ, PR and ST are the three tangents. $\angle QPR = 50^o$, then the value of $\angle SOT$ is :



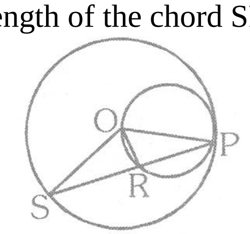
(A) 30^o
(B) 75^o
(C) 65^o
(D) Can't be determined

39. ABC is an isosceles triangle and AC, BC are the tangents at M and N respectively. DE is the diameter of the circle. $\angle ADP = \angle BEQ = 100^o$. What is value of $\angle PRD$?



(A) 60^o
(B) 50^o
(C) 20^o
(D) Can't be determined

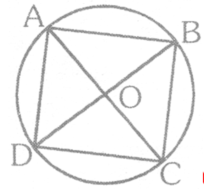
40. In the adjoining figure the diameter of the larger circle is 10 cm and the smaller circle touches internally the lager circle at P and passes through O, the centre of the larger circle. Chord SP cuts the smaller circle at R and OR is equal to 4 cm. What is the length of the chord SP?



(A) 9 cm
(B) 12 cm
(C) 6 cm

(D) $8\sqrt{2}$ cm

41. In the given figure ABCD is a cyclic quadrilateral DO = 8 cm and CO = 4 cm. AC is the angle bisector of $\angle$BAD. The length of AD is equal to the length of AB. DB intersects diagonal AC at O, then what is the length of the diagonal AC?’



(A) 20 cm
(B) 24 cm
(C) 16 cm
(D) None of these

OBJECTIVE						ANSEWER KEY									EXERCISE – 4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	C	D	C	B	C	D	B	C	B	B	C	D	A	A	D
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	B	B	D	A	C	B	A	D	C	D	D	B	C	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**				
Ans.	C	B	A	A	C	C	D	C	C	C	A				

Important Notes

CONSTRUCTIONS

★ **INTRODUCTION**

In class IX, we have discussed a number of constructions with the help of ruler and compass e.g. bisecting a line segment, bisecting an angle, perpendicular bisector of line segment, some more constructions of triangles etc. with their justifications. In this chapter we will discuss more constructions by using the knowledge of the earlier construction.

★ **DIVISION OF A LINE SEGMENT**

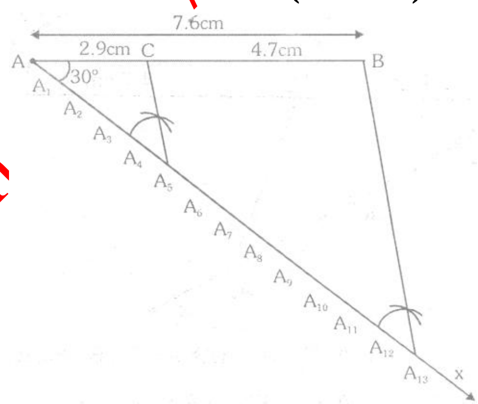
Let us divide the given line segment in the given ratio say 5 : 8. This can be done in the following two ways:

(i) Use of Basic Proportionality Theorem.

(ii) Constructing a triangle similar to a given triangle.

Construction – 1: Draw a segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts . (NCERT)

Steps of Constructions:



Step 1 : Draw any ray AX making an angle of 30^o with AB.

Step 2 : Locate 13 points : A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12} and A_{13} So that:

$$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = ... = A_{11}\ A_{12} = A_{12}\ A_{13}$$

Step 3 : Join B with A_{13}.

Step 4 : Through the point A_5, draw a line $A_5C \| A_{13}B$ such that $\angle AA_5C$ = corr. $\angle AA_{13}B$ intersecting AB at a point C.

Then AC : CB = 5 : 8.

Let us see how this method gives us the required division.

Since A_5C is parallel to A_{13} B.

Therefore $\frac{AA_5}{A_5A_{13}} = \frac{AC}{CB}$ (Basic Proportionality Theorem)

By construction, $\frac{AA_5}{A_5A_{13}} = \frac{5}{8}$

Therefore $\frac{AC}{CB} = \frac{5}{8}$

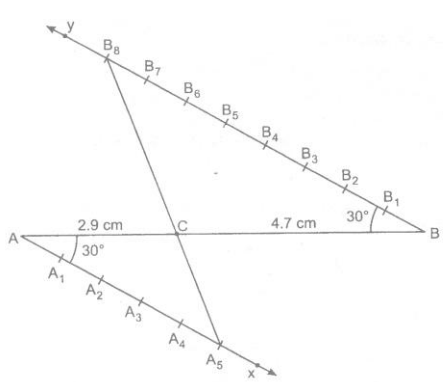
This given that C divides AB in the ratio 5 : 8.

By measurement, we find, AC = 2.9 cm, CB = 4.7 cm.

By Calculation: AC = $\frac{7.6x5}{13} = \frac{38}{13} = 2.9$

BC = $\frac{7.6x8}{13} = \frac{60.8}{13} = 4.67 = 4.7$ cm.

Alternative Solutions



Step 1 : Draw a line segment AB = 7.6 cm and to be divided in the ratio 5 : 8.

Step 2 : Draw any ray AX making an angle of 30^o with AB.

Step 3 : Draw a ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$. i.e. $\angle ABY$ = corr. $\angle BAX$.

Step 4 : Locate the points A_1, A_2, A_3, A_4, A_5, on AX and B_1, B_2, B_3, B_4, B_5, B_6, B_7, and B_8 on BY such that :

$AA_1 = A_1A_2 = \ldots\ldots = A_4A_5 = BB_1 = B_1B_2 = \ldots\ldots = B_6B_7 = B_7B_8$.

Step 5 : Join A_5B_8. Let it intersect AB at a point C. then AC : CB = 5 : 8.

Here ΔAA_5C is similar to ΔBB_8C

Then $$\frac{AA_5}{BB_8} = \frac{AC}{BC}$$

Since by construction, $$\frac{AA_5}{BB_8} = \frac{5}{8} \text{ Therefore } \frac{AC}{CB} = \frac{5}{8}$$

By measurement : AC = 2.9 cm, BC = 4.7 cm.

Constructions – 2 : Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle. (NCERT)

Sol. First all we are to construct a triangle ABC with given sides, AB = 6 cm, BC = 7 cm, CA = 5 cm. Given a triangle ABC, we are required to construct a triangle whose sides are $\frac{7}{5}$ of the corresponding sides of ΔABC.

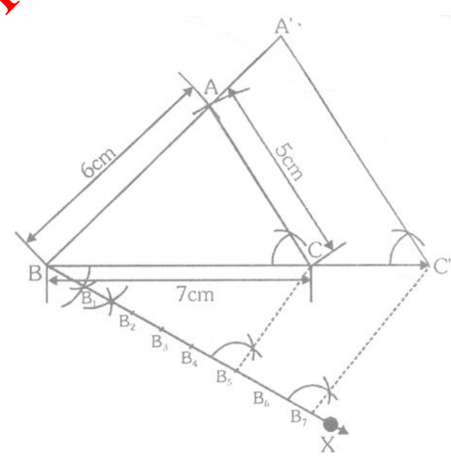
Steps of Construction :

Step 1 : Draw any ray BX making an angle of 30^{o} with the base BC of ΔABC on the opposite side of the vertex A.

Step 2 : Locate seven points B_1, B_2, B_3, B_4, B_5, B_6 and B_7 on BX so that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

[Note that the number of points should be greater of m and n in the scale factor $\frac{m}{n}$.]



Step 3 : Join B_5 (the fifth point) to C and draw a line through B_7 parallel to B_5C, intersecting the extended line segment BC at C'.

Step 4 : Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.

Then, A'B'C is the required triangle.

For justification of the construction.

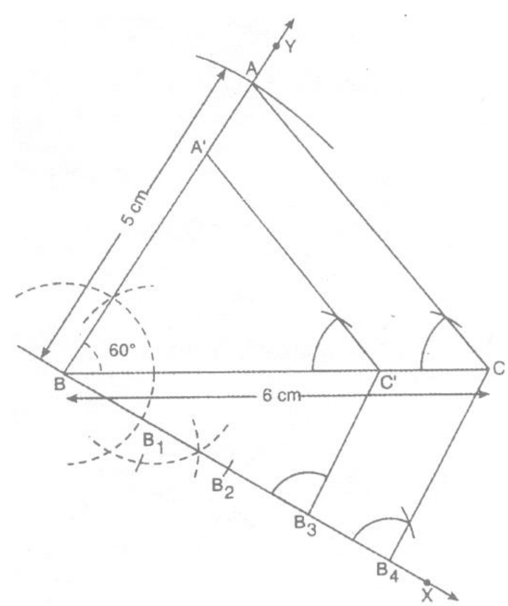
$$\Delta ABC \approx \Delta A'BC'$$

Therefore, $$\frac{AB}{A'B} = \frac{AC}{A'C} = \frac{BC}{BC'}$$

But $$\frac{BC}{BC'} = \frac{BB_5}{BB_7} = \frac{5}{7}$$

Therefore $$\frac{A'B}{AB} = \frac{A'C}{AC} = \frac{BC'}{BC} = \frac{7}{5}$$

Construction – 3 : Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^{o}$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC. (NCERT)



Sol. Given a triangle ABC, we are required to construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

Step of Constructions :

Step 1 : Draw a line segment BC = 6 cm.

Step 2 : At B construct $\angle CBY = 60^o$ and cut off AB = 5 cm, join AB and AC. ABC is the required Δ.

Step 3 : Draw any ray BX making an acute angle say 30^o with BC on the opposite side of the vertex A, $\angle CBX = 30^o$ downwards.

Step 4 : Locate four (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX, so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

Step 5 : Join B_4C and draw a line through B_3 (the 3rd point) parallel to B_4C to intersect BC at C'.

Step 6 : Draw a line through C' parallel to the line CA to intersect BA at A'.

Then A'BC' is the required triangle whose each side is $\frac{3}{4}$ times the corresponding sides of them ΔABC,

Let us now see how this construction gives the required triangle.

For justification of the construction.

$$\frac{BC'}{C'C} = \frac{3}{1}$$

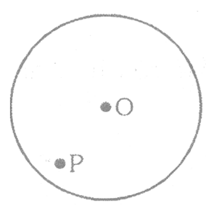
Therefore $$\frac{BC}{BC'} = \frac{BC' + C'C}{BC'} = \frac{BC'}{BC'} + \frac{C'C}{BC'} = 1 + \frac{1}{3} = \frac{4}{3}$$

$\Rightarrow$ $BC' = \frac{3}{4}$ BC, Also C'A' is parallel to CA.

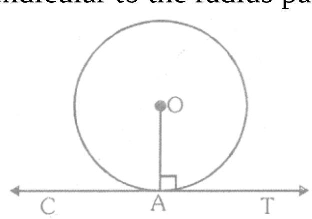
Therefore $\Delta A'BC' \approx \Delta ABC \Rightarrow \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$

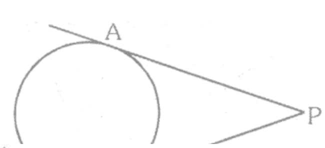
★ **CONTRUCTION OF TANGENTS TO A CIRCLE**

(a) If a point lies inside a circle, we can not draw any tangent to the circle i.e., No tangent is possible in this case



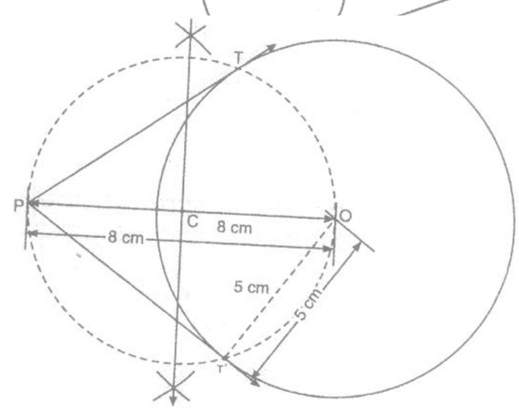
(b) If a point lies on the circle, then there is only one tangent to the circle at this point. The tangent to a circle at any point is perpendicular to the radius passing through the point of contact.





(c) Two tangents are drawn from an external point to circle, they are equal in length.

Construction 4 : Draw a circle of radius 5 cm. From a point 8 cm away from its centre, construct pair of tangents to the circle measure their lengths.



Sol.

Steps of Construction :

Step-1 : Draw a circle with radius 5 cm whose centre is O.

Step-2 : Take a point P at a distance 8 cm from the centre O such that OP = 8cm.

Step-3 : Bisect the line segment OP at the point C such that OC = CP = 4 cm.

Step-4 : Taking C as centre and OC as arc, draw a dotted circle to intersect the given circle at the points T and T'.

Step-5 : Join PT and PT'

PT and PT' are the required pair of tangents to the circle.

By measurement we obtain PT = PT' = 6.2 cm (Answer)

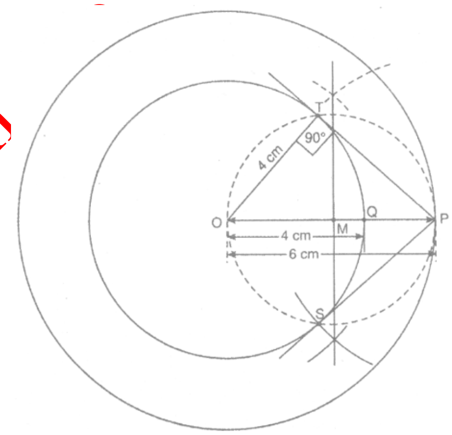
Verification: PT = PT' = $\sqrt{8^2 - 5^2} = \sqrt{64-25} = \sqrt{39} = 6.2$ cm (Answer)

Construction 5. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

(NCERT)

Sol.

Steps of Construction:



Step 1 : Draw two concentric circles with centre O and radii 4 cm and 6 cm such that OP = 6 cm, OQ = 4 cm.

Step 2 : Join OP and bisect it at M. i.e. M is the mid-point of OP i.e. OM = PM = 3 cm.

Step 3 : Taking M as centre with OM as radius draw a circle intersecting the smaller circle in two points namely T and S.

Step 4 : Join PT and PS.

PT and PS are the required tangents from a point P to the smaller circle, whose radius is 4 cm. By measurement: PT = 4.5 cm.

Verification. OTP is right Δ at T

$OP^2 = OT^2 + PT^2$

$6^2 = 4^2 + PT^2 \Rightarrow PT^2 = 36 - 16 = 20$

PT = $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5} = 2 \times 2.24 = 4.48$ cm

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

1. Draw a line segment of length 7.5 cm and divide it internally in the ratio 3 : 2. Measure the two parts.
2. Divide a line segment 8.8 cm long internally in the ratio 4 : 7 and measure the two parts.
3. Draw a line segment of length 13.5 cm and divide it internally in the ratio 2 : 3 : 4. Measure each part.
4. Construct a triangle with sides AB = 4 cm. BC = 5 cm and AC = 6 cm and then another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

5. Construct a triangle ABC whose sides are 4 cm, 5 cm, 7 cm. Construct another triangle similar to ΔABC and with sides $\frac{2}{3}$ rd of the corresponding sides of triangle ABC.

6. Draw a right triangle in which the sides (other than hypotenuse) are of length 5 cm and 12 cm. Then construct another triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the given triangle.

7. Construct an isosceles triangle whose base is 6 cm and altitude 3 cm and then another triangle whose sides are $\frac{4}{5}$ times the corresponding sides of the isosceles triangle.

8. Draw a triangle ABC with sides BC = 8 cm, $\angle B = 30^o$, $\angle A = 45^o$. Then construct a triangle whose sides are $\frac{5}{4}$ times the corresponding sides of ΔABC.

9. Construct a ΔABC, whose perimeter is 10.5 cm and base angles are 60^o and 45^o. Construct another Δ whose sides are $\frac{4}{3}$ of the corresponding sides of the ΔABC.

10. Draw two tangents to a circle of radius 4 cm from a point P at a distance 7 cm from its centre. Also measure the length of the two tangents. Are they equal? Give reasons for your answer.

11. Construct a circle with radius equal to 3 cm. Draw two tangents to it inclined at an angle of 60^o at their point of intersection. Measure their lengths and verify the results by calculation.

12. Draw two tangents to a circle of radius 4 cm inclined at an angle of 45^o to each other.

13. Construct a tangent to a circle of radius 3 cm from a point on the concentric circle of radius 5 cm and measure its length. Also verify the measurement by actual calculation.

14. Draw a circle of radius 2.5 cm. Take two points P and Q on one of its extended diameter each at a distance of 7.5 cm from its centre. Draw tangents to the circle from these two points P and Q.

15. Draw a line segment AB of length 10 cm. Taking A as centre, draw a circle of radius 5 cm and taking B as centre, draw a circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

1. Draw a line segment AB = 7 cm. Divide it internally in the ratio of (i) 3 : 5, (ii) 5 : 3. **[2000 C]**
2. From a point P on the circle of radius 4 cm, draw a tangent to the circle with using the centre. Also write the steps of construction. **[2000]**
3. Draw circle of radius 4.5 cm. Take a point P on it. Construct a tangent at the point P without using the centre of the circle. Write the steps of construction. **[2001]**
4. Divide a line segment of length 5.6 cm internally in the ratio (i) 3 : 2 (ii) 2 : 3. **[2001]**
5. Construct a ΔABC in which base AB = 6 cm, $\angle C = 60^o$ and the median CD = 5 cm. Construct a $\Delta AB'C'$ similar to ΔABC with base AB' = 8 cm. **[2002]**
6. Draw a circle of radius 3.5 cm. From a point P on the circle draw a tangent to the circle without using its centre.. **[2003]**
7. Draw a circle of radius 5 cm. Take a point P on it, without using the centre of the circle, construct a tangent at the point P. Write the steps of construction also. **[2003]|**

8. Draw a circle of diameter 12 cm. From a point P, 10 cm away from its centre, construct a pair of tangent to the circle. Measure the lengths of the tangent segments. **[2004 C]**

9. Draw a circle of radius 3.5 cm. Form a point P, outside the circle at a distance of 6 cm from the centre of circle, draw two tangent to the circle. **[2005]**

10. Construct a ΔABC in which AB = 6.5 cm, $\angle B = 60^o$ and BC = 5.5 cm. Also construct a triangle AB'C' similar to ΔABC, whose each side is $\frac{3}{2}$ of the corresponding side of the ΔABC. **[Delhi-2008]**

11. Draw a ΔABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^o$. Construct a $\Delta AB'C'$ similar to ΔABC such that sides of $\Delta AB'C'$ are $\frac{3}{4}$ of the corresponding sides of ΔABC. **[AI-2008]**

12. Draw a right triangle in which the sides containing the right angle are 5 cm and 4 cm. Construct a similar triangle whose sides are $\frac{5}{3}$ times the sides of the above triangle. **[Foreign-2008]**

13. Construct a ΔABC in which BC = 6.5 cm, AB = 4.5 cm and $\angle ABC = 60^o$. Construct a triangle similar to this triangle whose sides are $\frac{3}{4}$ of the corresponding sides of triangle ABC. **[Delhi-2008]**

14. Draw a right triangle in which sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle. **[AI-2009]**

15. Draw a circle of radius 3 cm. From a point P, 6 cm away from it's centre, construct a pair of tangents to the circle. Measure the lengths of the tangents. **[Foreign-2009]**

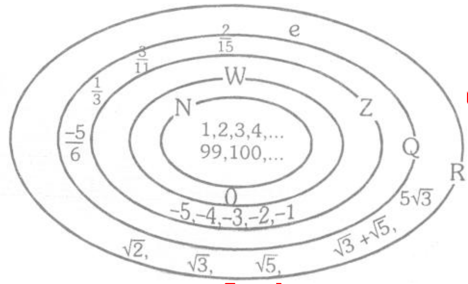
16. Construct a triangle ABC in which AB = 8 cm, BC = 10 cm and AC = 6 cm. Then construct another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of ΔABC. **[AI-2010]**

17. Construct a triangle ABC in which BC = 9 cm, $\angle B = 60^o$ and AB = 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of ΔABC. **[AI-2010]**

18. Construct a triangle ABC in which BC = 8 cm, $\angle B = 60^o$ and $\angle C = 45^o$. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC. **[AI-2010]**

REAL NUMBERS

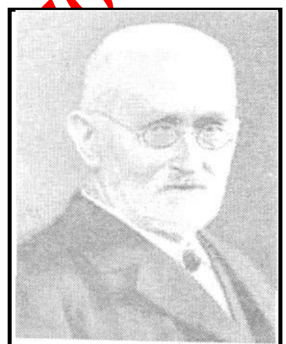
★ INTRODUCTION



"God gave us the natural number, all else is the work of man". It was exclaimed by Leopold Kronecker (1823-1891). The reputed German Mathematician. This statement reveals in a nut shell the significant role of the universe of numbers played in the evolution of human though.

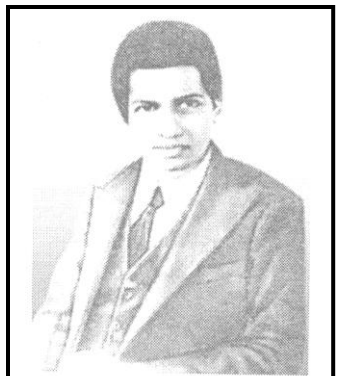
N	:	The set of natural number,
W	:	The set of whole numbers,
Z	:	The ser of Integers ,
Q	:	The set of rationales,
R	:	The set of Real Numbers.

★ HISTORICAL FACTS



Richard Dedekind
(1831-1916)

Dedekind was the first modern mathematician to publish in 1872 the mathematically rigorous definition of irrational numbers. He gave explanation of their place in the real Numbers System. He was able to demonstrate the completeness of the real number line. He filled in the "holes' in the system of Rational numbers with irrational Numbers. This innovation the made Richard Dedekind an immortal figure in the history of Mathematics.



Srinivasa Ramanujan
(1887-1920)

Srinivasa Ramanujan (1887-1920) was one of the most outstanding mathematician that India produced. He worked on history of Numbers and discovered wonderful properties of numbers. He stated intuitively many complicated result in mathematics. Once a great mathematician Prof. Hardy come of India to see Ramanujan. Prof. Hardy remarked that the he has traveled in a taxi with a rather dull number viz. 1729. Ramanujan jumped up and said, Oh! No. 1729 is very interesting number. It is the smallest number which can be expressed as the sum of two cubes in two different ways.

viz $1729 = 1^3 + 12^3$,

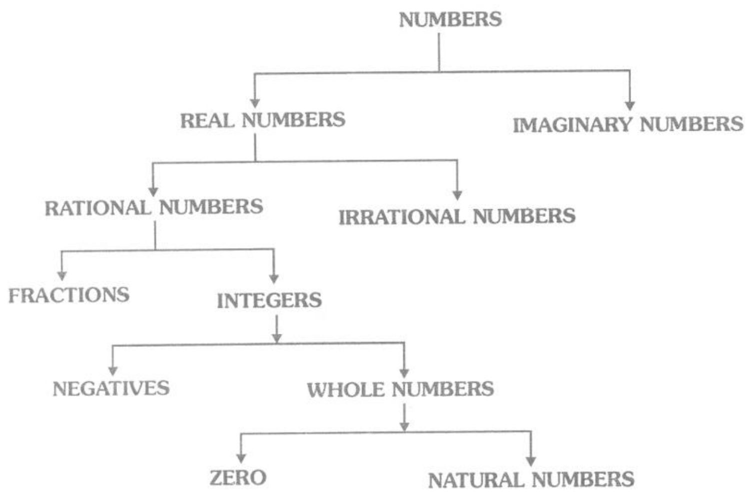
$1729 = 9^3 + 10^3$,

$\Rightarrow 1729 = 1^3 + 12^3 = 9^3 + 10^3$

★ RECALL

In our day to life, we deal with different types of numbers which can be broadly classified as follows.

CLASSIFICATION OF NUMBERS



(i) **Natural numbers (N) :** N + {1, 2, 3, 4… ∞ }

Remark :
- (i) The set N is infinite i.e. it has unlimited members.
- (ii) N has the smallest element namely '1'.
- (iii) N has no largest element. i.e., give me any natural number, we can find the bigger number from the given number.
- (iv) N does not contain '0' as a member. i.e., '0' is not a member of the set N.

(ii) **Whole numbers (W)** W = {0, 1, 2, 4… ∞ }

Remark :
- (i) The set of whole number is infinite (unlimited elements)
- (ii) This set has the smallest members as '0'. i.e. '0' the smallest whole number. i.e., set W contain '0' as a member.
- (iii) The set of whole numbers has no largest member.
- (iv) Emery natural number is a whole number.
- (v) Non-zero smallest whole number is '1'.

(iii) **Integers (I or Z) :** I or Z = { $-\infty$ …-3, -2, -1, 0, +2, +3…+ ∞ }
Positive integers : {1, 2, 3…}, **Negative integers :** {…. -4, -3, -2, -1}

Remark :
- (i) This set Z is infinite .
- (ii) It has neither the greatest nor the lest element.
- (iii) Every natural number is an integer.
- (iv) Every whole number is an integer.
- (iv) The set of non-negative integer = {0, 1, 2, 3, 4,…..}
- (v) The set of non-positive integer = {…….-4, -3, -2, -1, 0}

(iv) **Rational numbers :-** These are real numbers which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Ex. $\frac{2}{3}, \frac{37}{15}, \frac{-17}{19}, -3, 0, 10, 4.33, 7.123123123.........\,......$

Remark :

(i) Every integer is a rational number.
(ii) Every terminating decimal is a rational number .
(iii) Every recurring decimal is a rational number.
(iv) A non-terminating repeating decimal is called a recurring decimal.
(v) Between any two rational numbers there are an infinite number of rational numbers. This property is known as the density rational numbers.
(vi) If a and b are two rational numbers then $\frac{1}{2}(a+b)$ lies between a and b.

$$a < \frac{1}{2}(a+b) < b$$

n rational number between two different rational numbers a and b are :

$$a+\frac{(b-a)}{n+1};a+\frac{2(b-a)}{n+1};a+\frac{3(b-a)}{n+1};a+\frac{4(b-a)}{n+1};.........a+\frac{n(b-a)}{n+1};$$

(vii) Every rational number can be represented either as a terminating decimal or a non-termination repeating (recurring) decimals.
(viii) Types of rational numbers :- (a) Terminating decimal numbers and
(b) Non-termination repeating (recurring) decimal numbers

(v) Irrational numbers :- A number is called irrational number , if it can not be written in the form $\frac{p}{q}$, where p & q are integers and q$\neq$ 0. All Non-terminating & Non-repeating decimal numbers are Irrational numbers .

Ex. $\sqrt{2},\sqrt{3},\ 3\sqrt{2},\ 2+\sqrt{3},\ \sqrt{2+\sqrt{3}},\ \ \pi,\ e,\ etc$

(vi) Real numbers :- The totality of rational numbers and irrational numbers is called the set of real numbers i.e. rational numbers and irrational numbers taken together are called real numbers .
Every real number is either a rational number or an irrational number.

★ NATURE OF THE DECIMAL EXPANSION OF RATIONAL NUMBERS

Theorem -1 : Let x be a rational number whose decimal expansion terminates. Then we can express x in the form $\frac{p}{q}$, where p and q are co-primes, and the prime factorisation of q is of the form $2^m \times 5^n$, where m, n are non-negative integers.

Theorem-2 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is the $2^m \times 5^n$, where m, n are non-negative integers. Then , x has a decimal expansion which terminates.

Theorem-3 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^m \times 5^m$, where m, n are non-negative integers. Then , x has a decimal expansion which is non-terminating repeating

Ex. (i) $\frac{189}{125}=\frac{189}{5^3}=\frac{189}{2^0\times 5^3}$

we observe that prime factorization of the denominators of these rational numbers are of the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, $\frac{189}{125}$ has terminating decimal expansion.

(ii) $\frac{17}{6}=\frac{17}{2\times 3}$

we observe that the prime factorization of the denominator of these rational numbers are not of the form $2^m \times 5^n$, where m, n are non-negative integers. Hence $\frac{17}{6}$ has non-terminating and repeating decimal expansion

(iii) $\frac{17}{8} = \frac{17}{2^3 \times 5^0}$

So, the denominator 8 of $\frac{17}{8}$ is of the form $2^m \times 5^n$, where m, n are non-negative integers. Hence $\frac{17}{8}$ has terminating decimal expansion.

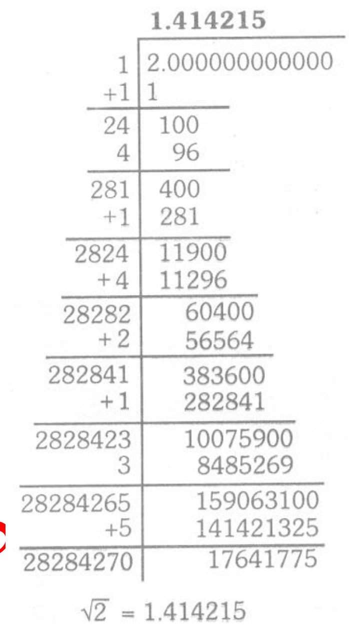
(iv) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Clearly, 455 is not of the form $2^m \times 5^n$, So, the decimal expansion of $\frac{64}{455}$ is non-terminating repeating.

★ **PROOF OF IRRATIONALITY OF $\sqrt{2}, \sqrt{3}, \sqrt{5}$,...............**

Ex.1 **Prove that $\sqrt{2}$ is not a rational number or there is no rational whose square is 2. (CBSE (outside Delhi) 2008).**

Sol. Let us find the square root of 2 by long division method as shown below.



Clearly, the decimal representation of $\sqrt{2}$ is neither terminating nor repeating.
We shall prove this by the method of contradiction .
If possible, let us assume that $\sqrt{2}$ is a rational number.

Then $\sqrt{2} = \frac{a}{b}$ where a, b are integers having no common factor other than 1.

$\Rightarrow \left(\sqrt{2}\right)^2 = \left(\frac{a}{b}\right)^2$ (squaring both sides)

$2 = \frac{a^2}{b^2}$

$a^2 = 2b^2$

$\Rightarrow$ 2 divides a^2

$\Rightarrow$ 2 divides a

Therefore let $a = 2c$ for some integer c.

$\Rightarrow a^2 = 4c^2$.

$\Rightarrow 2b^2 = 4c^2$

$\Rightarrow \quad b^2 = 2c^2$

$\Rightarrow \quad$ 2 divides b^2

$\Rightarrow \quad$ 2 divides b

Thus, 2 is a common factor of a and b.

But, it contradicts our assumption that a and b have no common factor other than 1.

So, our assumption that $\sqrt{2}$ is a rational, is wrong.

Hence, $\sqrt{2}$ is irrational.

Ex.2 Prove that $\sqrt[3]{3}$ is irrational .

Sol. Let $\sqrt[3]{3}$ be rational $= \frac{p}{q}$, where p and q $\in$ Z and p, q have no common factor except 1 also q > 1.

$\therefore \frac{p}{q} = \sqrt[3]{3}$

Cubing both sides

$\frac{p^3}{q^3} = 3$

Multiply both sides by q^2

$\frac{p^3}{q} = 3q^2$, Clearly L.H.S is rational since p, q have no common factor.

$\therefore$ p^3, q also have no common factor while R.H.S. is an integer .

$\therefore$ L.H.S. $\neq$ R.H.S. which contradicts our assumption that $\sqrt[3]{3}$ is Irrational .

Ex. 3 Prove that $2 + \sqrt{3}$ is irrational . **[Sample paper (CBSE) 2008]**

Sol. Let $2 + \sqrt{3}$ be a rational number equals to r

$\therefore \quad 2 + \sqrt{3} = r$

$\sqrt{3} = r - 2$

Here L.H.S. is an irrational number while R.H.S. r – 2 is rational. $\therefore$ S.H.S. $\neq$ R.H.S

Hence it contradicts our assumption that $2 + \sqrt{3}$ is rational .

$\therefore 2 + \sqrt{3}$ is irrational.

Ex.4 Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Sol. Let $\sqrt{2} + \sqrt{3}$ be rational number say 'x' $\Rightarrow x = \sqrt{2} + \sqrt{3}$

$x^2 = 2 + 3 + 2\sqrt{3}.\sqrt{2} = 5 + 2\sqrt{6}$

$\Rightarrow \quad x^2 = 5 + 2\sqrt{6} \Rightarrow \sqrt{6} = \frac{x^2 - 5}{2}$

As x, 5 and 2 are rationales $\Rightarrow \frac{x^2 - 5}{2}$ is a rational number .

$\Rightarrow \sqrt{6} = \frac{x^2 - 5}{2}$ is a rational number

Which is contradiction of the fact that $\sqrt{6}$ is a irrational number.

Hence our supposition is wrong $\Rightarrow \sqrt{2} + \sqrt{3}$ is an irrational number .

★ **EUCLID'S DIVISION LEMMA OR EUCLID'S DIVISION ALGORITHM**

For any two positive integers **a** and **b** there exist unique integers q and r such that

A = bq + r, where 0 $\leq$ r < b.

Let us consider a = 217, b = 5 and make the division of 217 by 5 as under :

Divined
$\downarrow$
Divisor $\rightarrow$ 5) 217(43 $\leftarrow$ **Quotient**
20
17
15
2 $\leftarrow$ **Remainder**

i.e.

Dividends	**=**	**Divisor**	**×**	**Quotient**	**+**	**Remainder**
(a)	**=**	**(b)**	**×**	**(q)**	**+**	**(r)**

e.g. **(i)** Consider number 23 and 5, then :

23 = 5 × 4 + 3

Comparing with a = bq + r

we get, a = 23, b = 5, q = 4, r = 3 and 0 $\leq$ r < b (as 0 < 3 < 5)

(ii) Consider positive integers 18 and 4

18 = 4 × 4 + 2

For 18 (= a) and 4 (= b) we have q = 4, r = 2 and 0 $\leq$ r < b

In the relations a = bq + r, where 0 $\leq$ r < b is nothing but a statement of the long division of number a by b in which q is the quotient obtained and r is the remainder.

Ex.5 Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m +1 for some integer m.

Sol. Let **a** and **b** are two positive integers such that **a** is greater than **b**, then :

a = bq + r ; where q and r are also positive integers and 0 $\leq$ r < b

Taking b = 3, we get :

a = 3q7 + r ; where 0 $\leq$ r < 3

$\Rightarrow$ The value of positive integer **a** will be

3q + 0, 3q + 1 or 3q + 2

i.e., 3q, 3q + 1or 3q + 2.

Now we have to show that square of positive integers 3q, 3q + 1 and 3q + 2 can be expressed as 3m or 3m +1 for some integer m.

$\therefore$ **Square of 3q = $(3q)^2$**

= $9q^2 = 3(3q^2) = 3m$; where m is some integer and $m = 3q^2$

Square of 3q + 1 = (3q + 1)2

= $9q^2 = 6q + 1$

= $3(3q^2 + 2q) + 1 = 3m + 1$ for some integer and m $3q^2 + 2q$.

Square of 3q + 2 = $(3q + 2)^2$

= $9q^2 = 12q + 4$

= $9q^2 = 12q + 3 + 1$

= $3(3q^2 + 4q + 1) + 1 = 3m + 1$ for some integer and $m = 3q^2 + 4q + 1$.

$\therefore$ The square of any positive integer is either of the form 3m or 3m +1 for some integer m.

Ex.6 Show that one and only out of n; + 2 or n + 4 is divisible by 3, where is any positive integer.

Sol. Consider any two positive integers **a** and **b** such that **a** is grater than **b**, then according to Euclid's division algorithm –

$a = bq + r$; where q and r positive integers and $0 \le r < b$

Let $a = n$ and $b = 3$, then

$a = bq + r \Rightarrow n = 3q\ r$; where $0 \le r < 3$.

$r = 0 \Rightarrow n = 3q + 0 = 3q$

$r = 1 \Rightarrow n = 3q + 1$

and $r = 2 \Rightarrow n = 3q + 2$

if $n = 3q$; **n is divisible by 3**

If $n = 3q + 1$; then $n + 2 = 3q + 1 + 2$

$= 3q + 3$; which is divisible by 3

$\Rightarrow$ **n + 2 is divisible by 3**

If $n = 3q + 2$; then $n + 4 = 3q + 2 + 4$

$= 3q + 6$; which is divisible by 3

$\Rightarrow$ **n + 4 is divisible by 3**

Hence , if n is any positive integer, then one and only one out n, n + 2 or n + 4 is divisible by 3.

★ APPLICATION OF EUCLID'S DIVISION LEMMA FOR FINDING H.C.F. OF POSITIVE INTEGERS

Algorithm :

Consider positive integers 418 and 33

Step. (a) Taking bitter number (418) as a and smaller number (33) as b.

Express the numbers are $a = bq + r$

$418 = 33 \times 12 + 22$

Step. (b) Now taking the divisor 33 and remainder 22, apply the Euclid's division method to get.

$33 = 22 \times 1 + 11$ [Expressing as $a = bq + r$]

Step. (c) Again with new divisor 22 and new remainder 11, apply the Euclid's division algorithm to get

$22 = 11 \times 2 + 0$

Step. (d) Since, the remainder = 0 so we can not proceed further.

Step. (e) The last divisor is 11 and we say H.C.F. of 418 and 33 = 11

Ex.7 Use Euclid's algorithm to find the HCF of 4052 and 12576.

Sol. Using $a = bq + r$, where $0 \le r < b$.

Clearly, 12576 > 4052 [a = 12576, b = 4051]

$\Rightarrow$ $12576 = 4051 \times 3 + 420$

$\Rightarrow$ $4052 = 420 \times 9 + 272$

$\Rightarrow$ $402 = 272 \times 1 + 148$

$\Rightarrow$ $272 = 148 \times 1 + 124$

$\Rightarrow$ $148 = 124 \times 1 + 24$

$\Rightarrow$ $124 = 24 \times 5 + 4$

$\Rightarrow$ $24 = 4 \times 6 + 0$

The remainder at this stage is 0. So, the divisor at this stage, i.e., 4 is the HCF of 12576 and 4052.

Ex.8 Find the HCF of 1848, 3058 and 1331.

Sol. Two numbers 1848 and 3058 , where 3058 > 1848

3058 = 1848 × 1 + 1210

1848 = 1210 × 1 638 [Using Euclid's division algorithm to the given number 1848 and 3058]

1210 = 638 × 1 572

638 = 572 × 1 + 66

527 = 66 × 8 + 44

66 = 44 × 1 + 22

44 = 22 × 2 + 0

Therefore HCF of 1848 and 3058 is 22.

HCF (1848 and 3058) = 22

Let us find the HCF of the numbers 1331 and 22.

1331 = 22 × 60 + 11

22 = 11 × 2 + 10

∴ HCF of 1331and 22 is 11

⇒ HCF (22, 1331) = 11

Hence the HCF of the given numbers 1848, 3058 and 1331 is 11.

HCF (1848, 3058, 1331) = 11

Ex.9 Using Euclid's division, find the HCF of 56, 96 and 404 **[Sample paper (CBSE) - 2008]**

Sol. Using Euclid's division algorithm, to 56 and 96.

96 = 56 × 1 + 40

56 = 40 × 1 + 16

40 = 16 × 2 + 8

16 = 8 × 2 + 0

Now to find HCF of 8 and 404

We apply Euclid's division algorithm to 404 and 8

404 = 8 × 50 + 4

8 = 4 × 2 + 0

Hence 4 is the HCF of the given numbers 56, 96 and 404.

★ **THE FUNDAMENTAL THEOREM OF ARITHMETIC**

Statement – "Every composite number can be factorized as a product of prime numbers in a unique way, except for the order in which the prime numbers occur.

e.g. **(i)** $90 = 2 \times 3 \times 3 \times 5 = 2\times3^2 \times 5$

(ii) $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$

(iii) $12600 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$

In general, a composite number is expressed as the product of its prime factors written in ascending order of their values.

COMPETITION WINDOW

NUMBER OF FACTORS OF A NUMBER

To get number of factors (or divisors) of a number N, express N as

$N = a^p . b^q . c^r . d^s$………(a, b, c, d are prime numbers and p, q, r, s are indices)

Then the number of total divisors or factors of N = (p + 1) (q + 1) (r + 1) (s + 1) ……

Eg. $540 = 2^2 \times 3^3 \times 5^1$

∴ total number of factors of 540 = (2 + 1) (3 + 1) (1 + 1) = 24

SUM OF FACTORS OF A NUMBER

The sum of all factors of $N = \dfrac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)(d^{s+1}-1)}{(a-1)(\text{b}-1)(\text{c}-1)()\text{d}-1}$

Eg. $270 = 2 \times 3^3 \times 5$

∴ Sum of factors of $270 = \dfrac{(2^{1+1}-1)(3^{3+1}-1)(5^{1+1}-1)}{(2-1)(3-1)()5-1} = \dfrac{3\times80\times24}{1\times2\times4} = 720$

PRODUCT OF FACTORS

The product of factors of composite number N = $N^{n/2}$, where n is the total number of factors of N.

Eg. $360 = 2^3 \times 3^2 \times 5^1$

∴ No. of factors of 360 = (3 + 1) (2 + 1) (1 + 1) = 24

Thus, the product of factors = $(360)^{24/2} = (360)^{12}$

NUMBER OF ODD FACTORS OF A NUMBER

To get the number of odd factors of a number N, express N as

$N = (p_1^a \times p_2^b \times p_3^c \times ....) \times (e^x)$

(where p^1, p^2, p^3………are the odd prime factors and e is the even prime factor)

Then the total number of odd factors = (a + 1) (b + 1) (c + 1)…….

Eg. $90 = 2^1 \times 3^2 \times 5^1$

∴ Total number of odd factors of 90 = (2 + 1)(1 + 1) = 6

NUMBER OF EVEN FACTORS OF A NUMBER

Number of even factors of a number = Total number of factors – Total number of odd factors.

NUMBER OF WAYS TO EXPRESS A NUMBER AS A PRODUCT OF TWO FACTORS

Let n be the number of total factors of a composite number .

Case – 1 : If the composite number is not a perfect square then number of ways of expressing the composite number as a product of two factors $= \dfrac{n}{2}$

Case – 2 : If the composite number is a perfect square then

(a) Number of ways of expressing the composite number as a product of two factors $= \dfrac{n+1}{2}$

(b) Number of ways of expressing the composite number as a product of two distinct factors $= \dfrac{(n-1)}{2}$

★ USING THE FUNDAMENTAL THEOREM OF ARITHMETIC TO FIND H.C.F. AND L.C.M.

For any two number a and b.

(a) L.C.M. (Least common multiple) = Product of each prime factor with highest powers

$$\text{L.C.M. (a,b)} = \frac{\text{Product of the numbers or } (a \times b)}{\text{H.C.F. (a, b)}}$$

(b) H.C.F. (Highest common factor) = Product of common prime factor with lowest power.

$$\text{H.C.F. (a, b)} = \frac{\text{Product of the numbers or } (a \times b)}{\text{L.C.M. (a, b)}}$$

Remark : The above relations hold only for two numbers.

COMPETITION WINDOW

For any three positive integers p, q, r –

HCF (P, q, r) × LCM (p, q, r) $\neq$ p × q × r

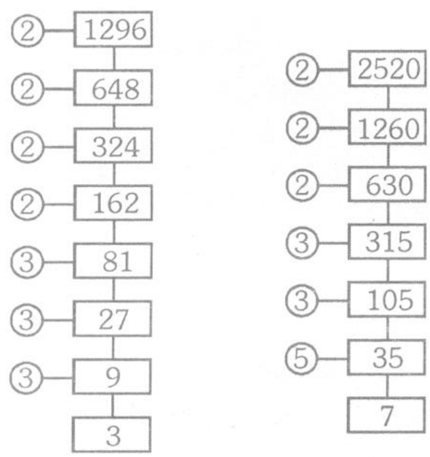
However , the following results hold good for the three positive integers p, q and r :

$$\text{LCM (p, q, r)} = \frac{\text{p.q.r.HCF (p, q, r)}}{\text{HCF (p, q) . HCF (q, r) . HCF (p, r)}} \qquad \text{HCF (p, q, r)} = \frac{\text{p.q.r.LCM (p, q, r)}}{\text{LCM (p, q) . LCM (q, r) . LCM (p, r)}}$$

Ex.10 Find the L.C.M. and H.C.F. of 1296 and 2520 by applying the fundamental theorem of arithmetic method i.e. using the prime factorisation method.

Sol. $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times = 2^4 \times 3^4$

$2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2^3 \times 3^2 \times 5 \times 7$



L.C.M. = $2^4 \times 3^4 \times 5 \times 7 = 45360$

H.C.F. = $2^3 \times 3^2 \times = 72$

Ex.11 Given that H.C.F. (306, 657) = 9. Find L.C.M. (306, 657)

Sol. H.C.F. (306, 657) = 9 means H.C.F. of 306 and 657 = 9

Required L.C.M. (306, 657) means required L.C.M. of 306 and 657.

For any two positive integers ;

$$\text{their L.C.M.} = \frac{\text{Product to the number}}{\text{Their H.C.F.}}$$

i.e., L.C.M. (306, 657) $= \dfrac{306 \times 657}{9} = 22,338$

Ex.12 Given that L.C.M. (150, 100) = 300, find H.C.F. (150, 100)

Sol. L.C.M. (150, 100) = 300

$\Rightarrow$ L.C.M. of 150 and 100 = 300

Since, the product of number 150 and 100 = 150 × 100

$$\frac{\text{Product of 150 and 100}}{\text{L.C.M.(150. 100)}} \qquad \frac{150 \times 100}{300}$$

And , we know : H.C.F. (150, 100) = = = 50

Ex.13 Explain why 7 × 13 +13 are composite numbers :

Sol. (i) Let 7 × 11 × 13 + 13 = (7 × 11 + 1) × 13

= (77 + 1) × 13 = 78 × 13 $\Rightarrow$ 7 × 11 × 13 + 13 = 2 × 3 × 13 × 13

= 2 × 3 × 13^2 is a composite number as powers of prime occur.

COMPETITION WINDOW

HCF AND LCM OF FRACTIONS

HCF of Fractions : The greatest common fraction is called the CHF of the given fractions.

HCf of fractions = $\dfrac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$

For example : The HCF of $\dfrac{4}{3},\dfrac{4}{9},\dfrac{2}{15},\dfrac{36}{21} = \dfrac{HCF\ \ of\ \ 4,4,2,36}{LCM\ \ of\ \ 3,9,15,21} = \dfrac{2}{315}$

Called the LCM of the fractions.

LCM of fractions = $\dfrac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$

For example : The LCM of $\dfrac{4}{3},\dfrac{4}{9},\dfrac{2}{15},\dfrac{36}{21} = \dfrac{LCM\ \ of\ \ 4,4,2,36}{HCF\ \ of\ \ 3,9,15,21} = \dfrac{36}{3} = 12$

HCF AND LCM OF DECIMALS

HCF

Step-1 : First of all equate the number of places in all the numbers by using zeros, wherever required .
Step-2 : Then considering these number as integers find the HCF of these numbers.
Step-3 : Put the decimal point in the resultant value as many places before the right most digit as that of in the every equated number.

Ex. Find the HCF of 0.0005, 0.005, 0.15, 0.175, 0.5 and 3.5

Sol. 0.0005 $\Rightarrow$ 5 0.0050 $\Rightarrow$ 50 0.1500 $\Rightarrow$ 15 0.1750 $\Rightarrow$ 1750

0.5000 $\Rightarrow$ 5000 3.5000 $\Rightarrow$ 35000

Then the HCF of 5, 50, 1500, 1750, 5000, and 35000 is 5. So, the HCF of the given numbers is 0.0005.

LCM

Step-1: First of all equate the number of places in all the given numbers by putting the minimum possible number of zeros at the end of the decimal numbers. wherever even required.
Step-2 : Now consider the equated numbers as integers and then find the LCM of these numbers.
Step-3 : Put the decimal point in the LCM of the number as many places as that of in the equated numbers.

Ex. Find LCM of 1.8, 0.54 and 7.2.

Sol. $\left.\begin{matrix}1.8\\0.54\\7.2\end{matrix}\right\} \rightarrow \left.\begin{matrix}1.80\\0.54\\7.20\end{matrix}\right\} \rightarrow \begin{matrix}180\\54\\720\end{matrix}$ **;** Now the LCM of 180, 54 and 720 is 2160. Therefore the required LCM is 21.60.

★ **SYNOPSIS**

1. **Euclid Division Algorithm :** Given any two positive integers a and b, b $\neq$ 1. a > b and a is not divisible by b, there exists two (unique) integers q and r such that

$$a = q + r\ ,\ \text{where}\ \ r < b$$

2. **Prime Factorization Theorem :** Every composite number can be expressed as a product of prime factors, and the decomposition is unique, apart from the order of factors.

(The fundamental Theorem of Arithmetic)

i.e. given any composite number x, we can find unique prime factors $p_1, p_2, p_3 \ldots\ldots\ldots p_n$ such that
$x = p_1 \times p_2 \times p_3 \times \ldots\ldots \times p_n$

3. **HCF and LCM of two numbers :** Let a, b be given numbers, Let each of these is expressed at a product of prime factors.

(i) The product of the smaller powers of the common prime numbers is the HCF.

(ii) The product of the prime numbers is either or both of these expression taken with greater power is the required LCM.

(iii) HCF (a, b) × LCM (a, b) = a × b

4. Rational Numbers $\frac{p}{q}, q \neq 0$ has a terminating decimal expansion if the prime factors of q are only 2's and 5's or both

5. Let $x = \frac{p}{q}$ be a rational number such the prime factorization of q is of the form $2^n . 5^m$ where n, m are non-negative integers, then x ahs a decimal expansion which terminates.

6. A rational number $\frac{p}{q}, q \neq 0$ has terminating repeating decimal expansion if the prime factors of q are other than 2 and 5 or both .

7. Let $x = \frac{p}{q}$ be a rational number such that the prime factorization of q is not of the form $2^n . 5^m$, where n and m are non negative integers, then x has a decimal expansion which is non-terminating repeating .

8. **Irrational Numbers :** $\sqrt{2}, \sqrt{3}, \sqrt{5}, 3\sqrt{3}, \sqrt{2}+\sqrt{3}, \pi, e$ are all irrational numbers. numbers which are expressed as non-terminating and non-repeated decimals are called the irrational numbers.

9. Real Numbers are a combination of the rational numbers and the irrational numbers .

SOLVED NCERT EXERCISE

EXERCISE : 1.1

1. **Use Euclid's division algorithm to find the HCF of :**

(i) **135 and 225** **(ii)** **196 and 38220** **(iii)** **867 and 255.**

Sol. **(i)** **135 and 225,** Start with the lager integer, that is, 225. Apply the division lemma to 225 and 135, to get.

$$225 = 135 \times 90$$

Since the remainder 90 $\neq$ 0, we apply the division lemma to 135 and 90 to get

$$138 = 90 \times 1 + 45$$

We consider the new divisor 90 and the new remainder 45, and apply the division lemma to get

$$90 = 45 \times 2 + 0$$

The remainder has now become zero, so our procedure stops.

Since the divisor at this stage is 45, the HCF of 225 and 135 is 45.

[Rest Try Yourself]

2. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Sol. Let us start with taking a, where a is any positive odd integer. We apply the division algorithm, with a and b = 6. Since $0 \leq r < 6$, the possible remainders are 0, 1, 2, 3, 4, 5. That is , a can be 6q or 6q + 1, or 6q +2 , or 6q + 3, or 6q + 4 or 6q + 5 where q is the quotient, However, since a is odd, we do not consider the cases 6q, 6q + 2 and 6q + 4 (since all the three are divisible by 2). Therefore, any positive odd integer is of the form 6q + 1, or 6q + 3, 6q + 5.

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march ?

Sol. Hint : Find HCF of 616 & 32

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Sol. Let a be any odd positive integer. We apply the division lemma with a and b = 3. Since $0 \leq r < 3$, the possible remainders are 0, 1 and 2. That is, a can be 3q, or 3q +1, or 6q + 2, where q is the quotient .

Now, $(3q)^2 = 6q^2$

which can be written in the form 3m, since 9 is divisible by 3.

Again , $(3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1$

Which can be written in the form 3m + 1 since $9q^2 + 6q$, i.e., $3(3q^2 + 2q)$ is divisible by 3.

Lastly, $(3q + 2)^2 = 9q^2 + 12q + 4 = (9q^2 + 12q + 3) + 1 = 3(3q^2 + 4q + 1) + 1$

which can be written in the form 3m + 1, since $9q^2 + 12q + 3$, i.e., $3(3q^2 + 4q + 1)$ is divisible by 3.

Therefore, the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, + 1 or 9m + 8

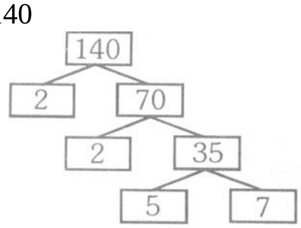
Sol. **Try Yourself**

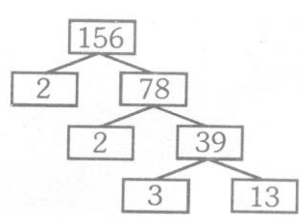
EXERCISE : 1 . 2

1. Express each number as product of its prime factors

(i) 140 **(ii) 156** **(iii) 3825** **(iv) 5005** **(v) 7429**

Sol. (i) 140 (ii) 156





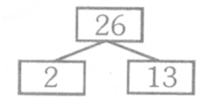
So, $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$ So, $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

[Rest Try Yourself]

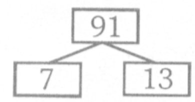
2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Sol.(i) 26 and 91



So, $26 = 2 \times 13$



So, $91 = 7 \times 13$

Therefore, LCM (26, 91) = $2 \times 7 \times 13 = 182$

HCF (26, 91) = 13

Verification LCM × HCF = $182 \times 13 = 2366$ and $26 \times 91 = 2366$

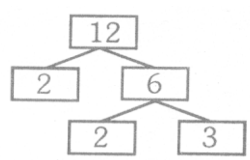
i.e., LCM × HCF product of two numbers.

[Rest Try Yourself]

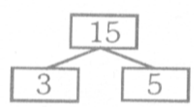
3. Find the LCM and HCF of the following integers by applying the prime factorization method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

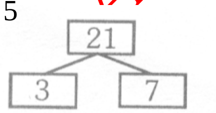
Sol.(i) 12, 15 and 21



So, $12 = 2 \times 2 \times 3 = 2^2 \times 3$



So, $15 = 3 \times 5$



So, $21 = 3 \times 7$

Therefore, HCF (12, 15, 21) = 3 ; LCM = (12, 15, 21) = $2^2 \times 3 \times 5 \times 7 = 420$ **[Rest Try Yourself]**

4. Given that HCF (306, 657) = 9. find LCM (306, 657)

Sol. $$\text{LCM}(306, 657) = \frac{306 \times 657}{HCF\ \ (306, 657)} = \frac{306 \times 657}{9} = 22338$$

5. Check whether 6” can end with the digit 0 for any natural number n.

Sol. If the number 6^n, for any neural number n, end with digit 0, then it would be divisible by 5. That is the prime factorization of 6^n, would contain the prime number 5. This is not possible because $6^n = (2 \times 3)^n = \times 3^n$; so the only prime in the factorization of 6^n are 2 and 3 and the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 6^n, So, there is no nature number n for which 6^n, ends with the digit zero.

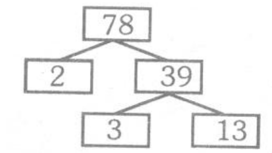
6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol. (i) $7 \times 11 \times 13 + 13 = (7 \times 11 + 1) \times 13$

$= (77 + 1) \times 13$

$= 78 \times 13 = (2 \times 3 \times 13) \times 13$

So, $78 = 2 \times 3 \times 13 = 2 \times 3 \times 13^2$



Since, $7 \times 11 \times 13 + 13$ can be expressed as a product of primes, therefore, it is a composite number.

(ii) **[Try Yourself]**

7. **There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point ?**

Sol. [Hint : Take LCM of 18 and 12]

EXERCISE : 1 . 3

1. **Prove $\sqrt{5}$ is irrational.**

Sol. Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find co prime integers a and b ($\neq 0$) such that

$$\sqrt{5} = \frac{a}{b}$$

$\Rightarrow$ $\sqrt{5}\, b = a$

Squaring on both sides, we get

$5b^2 = a^2$

Therefore, 5 divides a^2.

Therefore, 5, divides a

So, we can write a = 5c for same integer c.

Substituting for a, we get

$5b^2 = 25c^2$

$\Rightarrow$ $b^2 = 5c^2$

This means that 5 divides b^2, and so 5 divides b.

Therefore , a and b have at least 5 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

This contradiction arose because of our incorrect assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is irrational.

2. **prove that $3 + 2\sqrt{5}$ is irrational.**

Sol. Let us assume, to the contrary, $3 + 2\sqrt{5}$ is rational.

That is, we can find co prime integers a and b (b $\neq 0$) such that $3 + 2\sqrt{5} = \frac{a}{b}$

Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \dfrac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \dfrac{a-3b}{2b} = \sqrt{5} \Rightarrow \dfrac{a}{2b} - \dfrac{3}{2} = \sqrt{5}$

Since a and b are integers, we get $\dfrac{a}{2b} - \dfrac{3}{2}$ is rational , and so $\dfrac{a-3b}{2b} = \sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has risen because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

3. Prove that the following are irrationals :

(i) $\dfrac{1}{\sqrt{2}}$ **(ii)** $7\sqrt{5}$ **(iii)** $6+\sqrt{2}$

Sol. [Try yourself]

EXERCISE : 1 . 4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

(i) $\dfrac{13}{3125}$ **(ii)** $\dfrac{17}{8}$ **(iii)** $\dfrac{64}{455}$ **(iv)** $\dfrac{15}{1600}$ **(v)** $\dfrac{29}{343}$

(vi) $\dfrac{23}{2^3 5^2}$ **(vii)** $\dfrac{129}{2^2 5^7 7^5}$ **(viii)** $\dfrac{16}{15}$ **(ix)** $\dfrac{35}{50}$ **(x)** $\dfrac{77}{210}$

Sol. (i) $\dfrac{13}{3125} = \dfrac{13}{5^5}$

Hence , q = 5^5 , which is of the form $2^n 5^m$ (n = 0, m = 5). So, the rational number $\dfrac{13}{3125}$ has a terminating decimal expansion.

(ii) $\dfrac{17}{8} = \dfrac{17}{2^3}$

Hence, q = 2^3, which is of the form $2^n\ 5^m$ (n = 3, m = 0). So, the rational number $\dfrac{17}{8}$ has a terminating decimal expansion.

(iii) $\dfrac{64}{455} = \dfrac{64}{5\times7\times13}$

Hence, q=5×7×13, which is not of the form 2n5m. so, the rational number $\dfrac{64}{455}$ has a non-terminating repeating decimal expansion.

[Rest Try Yourself]

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Sol. (i) $\dfrac{13}{3125}$

$= \dfrac{13}{5^5} = \dfrac{13\times2^5}{5^5\times2^5} = \dfrac{416}{10^5} = 0.00416$

(ii) $\dfrac{17}{8} = \dfrac{17}{2^3}$

$$= \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 5^3}{10^3} = \frac{2125}{10^3} = 2.125$$

[Rest Try Yourself]

3. **The following real numbers have decimal expansions as given below. In each case, decide whether they are rational, or not. If they are rational, and of the form $\dfrac{p}{q}$, what can you say about the prime factors of q ?**

(i) 43.123456789

(ii) 0.120 1200 12000 120000……..

34.1$\overline{23456789}$

Sol. (i) 43.123456789

Since, the decimal expansion terminates, so the given real number is rational and therefore of the form $\dfrac{p}{q}$

43.123456789

$$= \frac{43123456789}{1000000000}$$

$$= \frac{43123456789}{10^9}$$

$$= \frac{43123456789}{(2 \times 5)^9}$$

$$= \frac{43123456789}{2^9 5^9}$$

Hence, q = $2^9\,5^9$

The prime factorization of q is of the form $2^n\,5^m$, where n = 9, m = 9

(ii) 0.120 1200 12000 120000……

Since, the decimal expansion is neither terminating nor non-ter4mnating repeating, therefore, the given real number is not rational .

(iii) **Try Yourself**

EXERCISE – 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

Choose The Correct One

1. $\sqrt{2}$ is –

(A) An integer (B) A rational number

(C) An irrational number (D) None of these

2. $\frac{1}{\sqrt{3}}$ is –

(A) A rational number (B) An irrational number

(C) a whole number (D) None of these

3. $7\sqrt{3}$ is –

(A) An irrational (B) A natural number

(C) A rational number (D) None of these

4. $5-\sqrt{3}$ is –

(A) An integer (B) A rational number

(C) An irrational number (D) None of these

5. $\pi = \frac{\text{Circumference of the circle}}{\text{Diameter of the circle}}$

(A) A rational number (B) A whole number

(C) A positive integer (D) None of these

6. HCF(p, q) × LCM (p, q) =

(A) p + q (B) $\frac{p}{q}$ (C) p × q (D) p^q

7. HCF(p, q, r) . LCM (p, q, r) =

(A) $\frac{pq}{r}$ (B) $\frac{qr}{p}$ (C) p, q, r (D) None of these

8. If $\sqrt[3]{32} = 2^x$ then x is equal to

(A) 5 (B) 3 (C) $\frac{3}{5}$ (D) $\frac{5}{3}$

9. 0.737373… =

(A) $(0.73)^3$ (B) $\frac{73}{100}$ (C) $\frac{73}{99}$ (D) None of these

10. If p is a positive prime integer, then $\sqrt{p}$ is –

(A) A rational number (B) An irrational number

(C) a positive integer (D) None of these

11. LCM of three numbers 28, 44, 132 is –

(A) 528 (B) 231 (C) 462 (D) 924

12. If a is a positive integer and p be a prime number and p divides a^2, then

(A) a divides p (B) p divides a (C) p^2 divides a (D) None of these

13. Evaluate $\sqrt[3]{\left(\frac{1}{6}\right)^{-2}}$

(A) 4 (B) 16 (C) 32 (D) 64

14. If $a = \frac{a+\sqrt{3}}{2-\sqrt{3}}$, $b = \frac{2-\sqrt{3}}{2+\sqrt{3}}$ then the value of a + b is –

(A) 14 (B) – 14 (C) $8\sqrt{3}$ (D) $-\sqrt{3}$

15. If $x = 0.1\overline{6}$, then 3x is –

(A) $0.\overline{48}$ (B) $0.\overline{49}$ (C) $0.\overline{5}$ (D) 0.5

16. Find the value of x then $\left(\frac{3}{5}\right)^{2x-3} = \left(\frac{5}{3}\right)^{x-3}$

(A) x = 2 (B) x = - 2 (C) x = 1 (D) x = - 1

17. $1.\overline{3}$ is equal to –

(A) 3/4 (B) 2/3 (C) 4/3 (D) 2/5

18. The product of $4\sqrt{6}$ and $3\sqrt{24}$ is –

(A) 124 (B) 134 (C) 144 (D) 154

19. If x = (7 + $4\sqrt{3}$), then the value of $x^2 + \frac{1}{x^2}$ is –

(A) 193 (B) 194 (C) 195 (D) 196

20. If $16 \times 8^{n+2} = 2^m$, then m is equal to –

(A) n + 8 (B) 2n +10 (C) 3n + 2 (D) 3n + 10

ANSWER KEY **EXERCISE - 1**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	B	A	C	D	C	D	D	C	B	D	B	B	B	A
Que.	16	17	18	19	20										
Ans.	A	C	C	B	D										

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions

1. Show that product of two numbers 60 and 84 is equal to the product of their HCF and LCM.

2. The product of two numbers is 396 × 576 and their LCM is 6336. Find their HCF.

3. Without actually performing the long division, state whether the following rational numbers have a terminating decimal expansion or a non-terminating repeating decimal expansion :

(i) $\frac{1}{7}$ (ii) $\frac{1}{11}$ (iii) $\frac{22}{7}$ (iv) $\frac{3}{5}$ (v) $\frac{7}{20}$ (vi) $\frac{2}{13}$ (vii) $\frac{27}{40}$ (viii) $\frac{13}{125}$ (ix) $\frac{23}{7}$ (x) $\frac{42}{100}$

4. Write down the decimal expansions of the following rational numbers :

(i) $\frac{241}{2^3 5^2}$ (ii) $\frac{19}{256}$ (iii) $\frac{25}{1600}$ (iv) $\frac{9}{30}$ (v) $\frac{133}{2^3 5^4}$

5. Show that 5309 and 3072 are prime to each other.

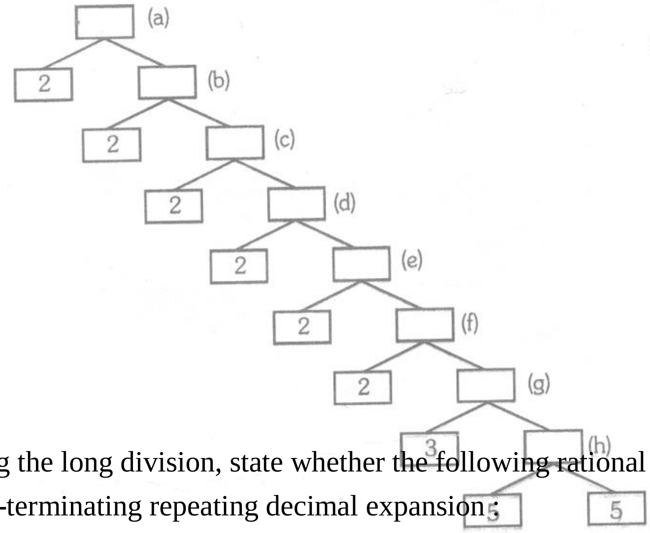
6. The HCF of two numbers is 119 and their LCM is 11781. If one of the numbers is 1071, find the other.

7. The LCM of two numbers is 2079 and their HCF is 27. If one of the numbers is 189, find the other.

8. Find the prime factorization of the following numbers:

(i) 10000 (ii) 2160 (iii) 396 (iv) 4725 (v) 1188

9. Find the missing numbers in the following factorization :



10. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion :

(i) $\frac{11}{125}$ (ii) $\frac{19}{128}$ (iii) $\frac{32}{405}$ (iv) $\frac{15}{3200}$ (v) $\frac{29}{2401}$

11. Write down the decimal expansions of the following rational numbers :

(i) $\frac{5}{8}$ (ii) $\frac{12}{125}$ (iii) $\frac{13}{625}$ (iv) $\frac{7}{64}$ (v) $\frac{7}{8}$

Short Answer Type Questions

12. Use Euclid's algorithm to find the HCF of 4052 and 12576.

13. Find the HCF 84 and 105. using Euclid's algorithm.

14. Find the HCF of 595 and 107, using Euclid's algorithm.

15. Find the HCF of 861 and 1353, using Euclid's algorithm.

16. Find the HCF of 616 and 1300, using Euclid's algorithm.

17. Show that every positive even integer is of the form 2q, and that very positive odd integer is of the form 2q + 1, where q is some integer .

18. Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some inerter.

19. Show that one and only out of n, n + 2 or n + 4 is divisible by 3, where n is any positive integer.

20. Find the greatest length which can be contained exactly in 10 m 5 dm 2cm 4mm and 12m 7dm 5cm 2mm.

21. Find the greatest measure which is exactly contained in 10 liters 857 millilitres and 15 litres 87 millilitres.

22. Consider the number 4^n, where n is a natural number. Check whether there is any value of $n \in N$ for which 4^n ends with the digit zero.

23. Find the LCM and HCF of 6 and 20 by the prime factorization method.

24. Find the HCF of 12576 and 4052 by using the prime factorization method.

25. Find the HCF and LCM of 6, 72 and 120 using the prime factorization method.

26. Find the prime factors of the following numbers:

(i) 1300 (ii) 13645 (iii) 3456

27. Find the LCM and HCF of 18, 24, 60, 150

28. Find the HCF and LCM of 60, 32, 45, 80, 36, 120

29. Split 4536 and 18511 into their prime factors and hence find their LCM and HCF.

30. Prove that $\sqrt{5}$ is irrational.

31. Prove that $\sqrt{7}$ is irrational.

32. Prove that $\frac{1}{\sqrt{3}}$ is irrational.

33. Prove that $3\sqrt{5}$ is irrational.

34. Prove that $3-\sqrt{3}$ is irrational.

35. Prove that $7+\sqrt{2}$ is irrational.

36. Prove that $5-\sqrt{5}$ is irrational.

37. Prove that $3\sqrt{2}$ is irrational.

38. Use Euclid's division lemma to find the HCF of

(i) 13281 and 15844 (ii) 1128 and 1464 (iii) 4059 and 2190

(iv) 10524 and 12752 (v) 10025 and 14035

39. What is the greatest number by which 1037 and 1159 can both be divided exactly ?

40. Find the greatest number which both 2458090 and 867090 will contain an exact number of times.

41. Find the greatest weight which can be contained exactly in 3 kg 7 hg 8 dag 1 g and 9 kg 5 dag 4 g.

42. Find the LCM of the following using prime factorization method. :

(i) 72, 90, 120

(ii) 24, 63, 70

(iii) 455, 117, 338

(iv) 225, 240, 208

(v) 2184, 2730, 3360

43. Prove that $\sqrt{3}$ is irrational.

44. Prove that $2\sqrt{2}$ is irrational.

45. Prove that $\frac{1}{\sqrt{5}}$ is irrational.

46. Prove that $7+\sqrt{3}$ is irrational.

47. Prove that $8-\sqrt{2}$ is irrational.

REAL NUMBERS **ANSWER KEY** **EXERCISE – 2 (X) - CBSE**

- **Very Short Answer Type Questions**

2. 36.

3. (i) Non-terminating repeating ; (ii) Non-terminating repeating ; (iii) Non-terminating repeating

(iv) Terminating ; (v) Terminating (vi) Non-terminating repeating (vii) Terminating ; (viii) Terminating

(ix) Non-terminating repeating ; (x) Terminating

4. (i) 1.205 ; (ii) 0.07421875 ; (iii) 0.015625 ; (iv) 0.0266 **6.** 1309 **7.** 297

8. (i) $2^4 \times 5^4$; (ii) $2^4 \times 3^3 \times 5$; (iii) $2^2 \times 3^2 \times 11$; (iv) $3^3 \times 5^2 \times 7$; (v) $2^2 \times 3^3 \times 11$

9. (a) 4800 ; (b) 2400 ; (c) 1200 ; (d) 600 ; (e) 300 ‘ (f) 150 ‘ (g) 75 ‘ (h) 25

10. (i) Terminating ; (ii) Terminating ; (iii) Non-terminating repeating ;(iv) Terminating ;(v)Non-terminating repeating

11. (i) 0.625 ; (ii) 0.96 ; (iii) 0.0208 ; (iv) 0.109375 ; (v) 0.875

- **Short Answer Type Questions**

12. 4 **13.** 21 **14.**119 **15.** 123 **16.** 4 **20.** 4mm **21.** 141 mmllilitres **22.** No **23.** 60, 2

24. 4 **25.** 6, 360 **26.** (i) $2^2 \times 5^2 \times 13$; (ii) 3×5×7×13 ; (iii) $2^7 \times 3^3$ **27.** 1800, 6 **28.** 1, 1440

29. 149688, 567 **38.** (i) 233 ; (ii) 24 ; (iii) 3 ; (iv) 4 ; (v) 2005. **39.** 61 **40.** 10 **41.** 1 hg 9 dag 9 g

42. (i) 360 ; (ii) 2520 ; (iii) 106470 ; (iv) 46800 ; (v) 43680

EXERCISE – 3 (FOR SCHOOL / BOARD EXAMS

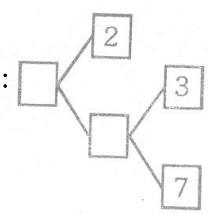
PREVIOUS YEARS BOARD (CBSE) QUESTIONS

Questions Carrying 1 Mark

1. If $\frac{p}{q}$ is a rational number (q ≠ 0), what is condition of q so that the decimal representation of $\frac{p}{q}$ is terminating ? **[Delhi-2008]**

2. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$. **[AI-2008]**

3. Complete the missing entries in the following factor tree : **Foreign - 2008]**



4. The decimal expansion of the rational number $\frac{43}{2^4 5^3}$, will terminate after how many places of decimals ? **[Delhi - 2009]**

5. Find the [HCF × LCM] for the numbers 100 and 190. **[AI - 2009]**

6. Find the [HCF × LCM] for the numbers 105 and 120. **[AI - 2009]**

7. Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion. **[Foreign - 2009]**

8. The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, write the other number. **[Foreign - 2009]**

Questions Carrying 3 Marks

9. Show that $5-2\sqrt{3}$ is an irrational number **[Delhi - 2008]**

10. Show that $2-\sqrt{3}$ is an irrational number **[Delhi - 2008]**

11. Show that $5+3\sqrt{2}$ is an irrational number **[Delhi - 2008]**

12. Prove that $\sqrt{3}$ is an irrational number . **[Delhi – 2009/AI-2008]**

13. Use Euclid’s Division Lemma to show the square of any positive integer is either of the form 3m or 3m + 1 for some integer m. **[Foreign – 2008 / AI-2008]**

14. Prove that $\sqrt{2}$ is an irrational number. **[Delhi – 2009/AI-2008]**

15. Prove that $\sqrt{5}$ is an irrational number. **[Delhi – 2009/AI-2008]**

16. Prove that $3+\sqrt{2}$ is an irrational number. **[AI-2008]**

17. Prove that $5-2\sqrt{3}$ is an irrational number. **[AI-2008]**

18. Prove that $3+5\sqrt{2}$ is an irrational number. **[AI-2009]**

19. Show that the square of any positive odd integers is of the form 8m + 1, for some integer m. **[Foreign-2009]**

20. Prove that $7+3\sqrt{2}$ is not a rational number. **[ForeignAI-2009]**

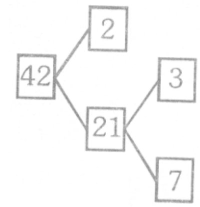
REAL NUMBERS — ANSWER KEY — EXERCISE -3 (X) – CBSE

1. $q = 2^n \times 5^m$, where n and m are whole numbers.

2. $\sqrt{2} = 1.41..........,\ \sqrt{3} = 1.73........$

$\therefore$ One rational no. between $\sqrt{2}$ and $\sqrt{3}$ is 1.5.

3.



4. After 4 decimal ; $\frac{43}{2^4 5^3} = \frac{43}{2000} = 0.0215$

5. HCF × LCM = 100× 190 = 19000

6. HCF × LCM = 105 × 120 = 12600

7. $\frac{51}{1500} = \frac{17}{500}; 500 = 2^2 \times 5^3 (2^m \cdot 5^n)$. So, it has terminating expansion.

8. Other number $= \frac{9 \times 360}{45} = 72$

EXERCISE – 4 (FOR OLYMPIADS)

Choose The Correct One

1. The greatest possible number with which when we divide 37 and 58, leaves the respective remainder of 2 and 3, is -
(A) 2 (B) 5 (C) 10 (D) None of these

2. The largest possible number with which when 60 and 98 are divided, leaves the remainder 3 in each case, is –
(A) 38 (B) 18 (C) 19 (D) None of these

3. The largest possible number with which when 38, 66 and 80 are divided the remainders remain the same is –
(A) 14 (B) 7 (C) 28 (D) None of these

4. What is the least possible number which when divided by 24, 32 or 42 in each case it leaves the remainder 5 ?
(A) 557 (B) 677 (C) 777 (D) None of these

5. In Q.N. 4, how many numbers are possible between 666 and 8888 ?
(A) 10 (B) 11 (C) 12 (D) 13

6. What is the least number which when divided by 8, 12 and 16 leaves 3 as the remainder in each case, but when divided by 7 leaves no remainder ?
(A) 147 (B) 145 (C) 197 (D) None of these

7. What is the least possible number which when divided by 18, 35 or 42 leaves 2, 19, 26 as the remainders respectively ?
(A) 514 (B) 614 (C) 314 (D) None of these

8. What is the least possible number which when divided by 2, 3, 4, 5, 6 leaves the remainders 1, 2, 3, 4, 5 respectively ?
(A) 39 (B) 48 (C) 59 (D) None of these

9. In Q.No. 8, what is the least possible 3 digit number which is divisible by 11 ?
(A) 293 (B) 539 (C) 613 (D) None of these

10. How many numbers lie between 11 and 1111 which when divided by 9 leave a remainder of 6 and when divided by 21 leave a remainder of 12 ?
(A) 18 (B) 28 (C) 8 (D) None of these

11. If x divides y (written as x | y) and y | z, (x, y, z $\in$ z) then –

(A) x | z (B) z | y (C) z | x (D) None of these

12. If x | y, where x > 0, y > 0 (x, y ∈ z) then –

(A) $x < y$ (B) $x = y$ (C) $x \leq y$ (D) $x \geq y$

13. If a | b, then gcd of a and b is –

(A) a (B) b (C) ab (D) Can't be determined

14. If gcd of b and c is g and d | b & d | c, then –

(A) d = g (B) g | d (C) d | g (D) None of these

15. If x, y ∈ R and | x | + | y | = 0, then –

(A) $x > 0,\ y < 0$ (B) $x < 0, y > 0$ (C) $x = 0,\ y = 0$ (D) None of these

16. If a, b, c ∈ R and $a^2 + b^2 + c^2 = ab + bc + ca$, then –

(A) a = b = c (B) a = b = c = 0 (C) a, b, c are distinct (D) None of these

17. If x, y ∈ R and $x < y \Rightarrow x^2 > y^2$ then –

(A) $x > 0$ (B) $y > 0$ (C) $x < 0$ (D) $y < 0$

18. If x, y ∈ R and $x > y \Rightarrow |x| > |y|$, then –(A)(B)(C)(D)

(A) $x > 0$ (B) $y > 0$ (C) $x < 0$ (D) $y < 0$

19. If x, y ∈ R and $x > y \Rightarrow |x| < |y|$, then –

(A) $x < 0$ (B) $x > 0$ (C) $y > 0$ (D) $y < 0$

20. π and e are –

(A) Natural numbers (B) Integers (C) Rational numbers (D) Irrational numbers.

21. If a, b ∈ R and a < b, then –

(A) $\frac{1}{a} < \frac{1}{b}$ (B) $\frac{1}{a} > \frac{1}{b}$ (C) $a^2 > b^2$ (D) Nothing can be said

22. If x is a non-zero rational number and xy is irrational , then y must by –

(A) a rational number (B) an irrational number (C) non-zero (D) an integer

23. The arithmetical fraction that exceeds it's square by the greatest quantity is –

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) None of these

24. If x and y are rational numbers such that $\sqrt{xy}$ is irrational , then $\sqrt{x}+\sqrt{y}$ is –

(A) Rational (B) Irrational (C) Non-real (D) None of these

25. If x and y are positive real numbers, then –

(A) $\sqrt{x}+\sqrt{y} > \sqrt{x+y}$ (B) $\sqrt{x}+\sqrt{y} < \sqrt{x+y}$

(C) $\sqrt{x}+\sqrt{y} = \sqrt{x+y}$ (D) None of these

26. If $(\sqrt{2}+\sqrt{3})^2 = a + b\sqrt{6}$, where a, b ∈ Q, then –

(A) a = 5, b = 6 (B) a = 5, b = 2 (C) a = 6, b = 5 (D) None of these

27. If x ∈ R, then | x | =

(A) x (B) –x (C) max {x, -x} (D) min {x, -x}

28. $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{125}}$ is equal to –

(A) $\sqrt{5}(5+\sqrt{2})$ (B) $\sqrt{5}(2+\sqrt{2})$ (C) $\sqrt{5}(\sqrt{2}+1)$ (D) $\sqrt{5}(3+\sqrt{2})$

29. $\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}$ is equal to –

(A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{\sqrt{2}}$ (C) $\frac{\sqrt{2}}{\sqrt{3}}$ (D) $\sqrt{6}$

30. The expression $\frac{\sqrt{3}-1}{2\sqrt{2}-\sqrt{3}-1}$ is equal to –

(A) $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ (B) $\sqrt{6}-\sqrt{4}+\sqrt{3}-\sqrt{2}$

(C) $\sqrt{6}-\sqrt{4}-\sqrt{3}+\sqrt{2}$ (D) None of these

31. If x, y, z are real numbers such that $\sqrt{x-1}+\sqrt{y-2}+\sqrt{z-3}=0$ then the values of x, y, z are respectively

(A) 1, 2, 3 (B) 0, 0, 0

(C) 2, 3, 1 (D) None of these

32. If a, b, c $\in$ R and a > b $\Rightarrow$ ac < bc, then –

(A) $c \geq 0$ (B) $c \leq 0$

(C) c > 0 (D) c < 0

33. If a, b, c $\in$ R and ac = bc $\Rightarrow$ a = b, then –

(A) $c \geq 0$ (B) $c \leq 0$

(C) $c = 0$ (D) $c \neq 0$

34. Between any two distinct rational numbers –

(A) There lie infinitely many rational numbers.

(B) There lies only one rational number.

(C) There lie only finitely many numbers.

(D) There lie only rational numbers.

35. The total number of divisors of 10500 except 1 and itself is –

(A) 48 (B) 50

(C) 46 (D) 56

36. The sum is the factors of 19600 is –

(A) 54777 (B) 33667

(C) 5428 (D) None of these

37. The product of divisors of 7056 is –

(A) $(84)^{48}$ (B) $(84)^{44}$

(C) $(84)^{45}$ (D) None of these

38. The number of odd factors (or divisors) of 24 is –

(A) 2 (B) 3 (C) 1 (D) None of these

39. The number of even factors (or divisors) of 24 is –

(A) 6 (B) 4 (C) 8 (D) None of these

40. In how many ways can 576 be expressed as a product of two distinct factors ?

(A) 10 (B) 11 (C) 21 (D) None of these

OBJECTIVE							ANSWER KEY							EXERCISE - 4
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	A	B	D	A	B	C	B	A	A	C	A	C	C
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A	D	B	A	D	D	B	B	B	A	B	C	D	D	A
Que.	31	32	33	34	35	36	37	38	39	40					
Ans.	A	D	D	A	C	A	C	A	A	A					

COMPETITION WINDOW

COMPLEX NUMBERS

The idea of complex numbers was introduced, so that all algebraic equations could have solutions. Over the real numbers, the square root on negative number is not defined.

Leonhard Euler for the first time introduced the symbol iota (i) in 1748, (i is the first letter of Latin word 'imaginaries'] for $\sqrt{-1}$ with the property $i^2 = -1$.

$i = \sqrt{-1}$ so $i^2 = -1$.

Imaginary Numbers : Square root of a negative number is called imaginary number, e.g. $\sqrt{-1}, \sqrt{-2}, \sqrt{-9/4}$ etc.

$\sqrt{-2}$ can be written as

$\sqrt{-2} = \sqrt{2} \times \sqrt{-1} = \sqrt{2}i$

Remark : 1. If a, b are positive real numbers, then $\sqrt{-a} \times \sqrt{-b} = -\sqrt{ab}$

2. For any two real number $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ is not valid if a and b both are negative.

3. For any positive real number a, we have $\sqrt{-a} = \sqrt{-1 \times a} = \sqrt{-1} \times = \sqrt{a} = i\sqrt{a}$

E.g 1. $\sqrt{-144} = \sqrt{-1 \times 144} = \sqrt{-1} \times \sqrt{144} = 12i$

2. $\sqrt{-4} \times \sqrt{-\frac{9}{4}} = 2i\left(\frac{3i}{2}\right) = 3i^2 = -3$

3. $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9} = 5i + 6i + 6i = 17i$

Integral powers of i : We have $i = \sqrt{-1}$ so $i^2 = -1,\ i^3 = -i,\ i^4 = 1$

For any n $\in$ N, we have

$i^{4n} = 1$,

$i^{4n+2} = 1$,

$i^{4n+2} = -1$,

$i^{4n+3} = -i$

E.g. 1. $i^{35} = i^3 = -i$

2. $i^{-999} = \frac{1}{i^{999}} = \frac{1}{i^3} = \frac{i}{i^4} = \frac{i}{1} = i$

3. $\left[i^{19} + \left(\frac{1}{i}\right)^{25}\right]^2 = \left[i^{19} + \frac{1}{i^{25}}\right]^2 = \left[i^3 + \frac{1}{i}\right]^2 = \left[-i + \frac{i^3}{i^4}\right]^2 = [-i + i^3]^2 = (-i - i)^2 = 4i^2 = -4$

Complex Numbers : If a, b are two real numbers, then a number of the form a + ib is called a complex number.
e.g. 7 + 2i, -1 + i, 3 -2i etc

If z = a + ib is a complex number, then 'a' is called the real part of z (Re (z)) and 'b' is called the imaginary part of z(Im(z)).

Equality of complex numbers : Two complex numbers $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are equal if $a_1 = a_2$ and $b_1 = b_2$.

Algebra of complex combers : Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ then

(i) $z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2) = (a_1+a_2) + i\,(b_1 + b_2)$

(ii) $z_1 - z_2 = (a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + i\,(b_1 - b_2)$

(iii) $z_1 \cdot z_2 = (a_1 + ib_1) \cdot (a_2 + ib_2) = (a_1a_2 - b_1b_2) + i\,(a_1b_2 + b_1a_2)$

(iv) $\dfrac{z_1}{z_2} = \dfrac{(a_1 + ib_1)}{(a_2 + ib_2)} = \dfrac{(a_1a_2 + b_1b_2)}{a_2^2 + b_2^2} + i\dfrac{(b_1a_2 - a_1b_2)}{a_2^2 + b_2^2}$

Multiplicative Inverse of a complex number : Corresponding to every non-zero complex number z = a + ib , there exists a complex number $z^{-1} = x + iy$ such that

$z \cdot z^{-1} = 1 \ (z \neq 0)$

$$z^{-1} = \frac{1}{z} = \frac{1}{a+ib} = \frac{a-ib}{(a+ib)(a-ib)} = \frac{a-ib}{a^2+b^2}$$

Conjugate of a complex number : Let z = a + ib be a complex number. Then the conjugate of z is dented by $\bar{z}$ and is equal to a – ib .

Thus z = a +ib $\Rightarrow$ $\bar{z}$ = a – ib

E.g. if z = 3 + 4i $\Rightarrow$ $\bar{z}$ = 3 – 4i

Modulus of a complex number : The modulus of a complex number z = a + ib is denoted by | z | and is defined as

$|z| = \sqrt{a^2 + b^2}$

| z | is also called the absolute value of z.

EXERCISE – 5 (FOR ITT-JEE/AIEEE)

Choose The Correct One

1. The value of i^{457} is -

(A) 1 (B) – 1 (C) i (D) – i

2. The value $i^{37} + \dfrac{1}{i^{67}}$ is –

(A) 1 (B) – 1 (C) 2i (D) –2

3. The value of $\left(i^{41} + \dfrac{1}{i^{257}}\right)^9$ is –

(A) 1 (B) 0 (C) –1 (D) 2

4. The value of $(i^{77} + i^{70} + i^{87} + i^{414})^3$

(A) – 8 (B) – 6 (C) 6 (D) 8

5. The value of the expression $\dfrac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$ is -

(A) –1 (B) 1 (C) 0 (D) i

6. The standard form of (1 + i) (1 + 2i) is –

(A) 3 + I (B)–3 + i (C) 1 – 3i (D) 1 – + 3i

8. The standard form of $\dfrac{(1+i)(1+\sqrt{3i})}{(1-i)}$ is –

(A) $-\sqrt{3}+i$ (B) $\sqrt{3}-i$ (C) $1-i\sqrt{3}$ (D) $1+i\sqrt{3}$

9. The standard form of $\dfrac{3-4i}{(4-2i)(1+i)}$ is –

(A) $\dfrac{1}{4}+\dfrac{3}{4}i$ (B) $\dfrac{1}{4}-\dfrac{3}{4}i$ (C) $\dfrac{3}{4}+\dfrac{1}{4}i$ (D) $\dfrac{3}{4}-\dfrac{1}{4}i$

10. If (x + iy) (2 – 3i) = 4 + i , then real values of x and y are –

(A) $x=5,\ y=14$ (B) $x=\dfrac{13}{5},\ y=\dfrac{14}{13}$

(C) $x=\dfrac{5}{13},\ y=\dfrac{14}{13}$ (D) None of these

11. If $\dfrac{(1+i)x-2i}{3+i}+\dfrac{(2-3i)y+i}{3ii}=i,$ then real values of x and y are –

(A) $x=3,\ y=-1$ (B) $x=-1,\ y=3$

(C) $=1,\ y=-2$ (D) $x=-1,\ y=-3$

12. The conjugate of 4 – 5i is –

(A) 4 + 5i (B) – 4 – 5i (C) – 4 + 5i (D) 4 – 5i

13. The conjugate of $\dfrac{1}{3+5i}$ is –

(A) $\dfrac{1}{34}(3+5i)$ (B) 3 + 5i (C) $\dfrac{1}{3-5i}$ (D) $\dfrac{34}{3-5i}$

14. The conjugate of $\dfrac{(1+i)(2+i)}{3+i}$ is –

(A) $\dfrac{3}{5}+\dfrac{4}{5}i$ (B) $\dfrac{3}{5}-\dfrac{4}{5}i$ (C) $-\dfrac{3}{5}-\dfrac{4}{5}i$ (D) $\dfrac{3}{5}+\dfrac{4}{5}i$

15. The multiplicative inverse of 1 – i is –

(A) 1 + i (B) $\dfrac{1}{1+i}$ (C) $\dfrac{1}{2}+\dfrac{1}{2}i$ (D) None of these

16. The multiplicative inverse of $(1+\sqrt{3})^2$ is –

(A) $-\dfrac{1}{8}-\dfrac{i\sqrt{3}}{8}$ (B) $(1-i\sqrt{3})^2$ (C) $\dfrac{1}{8}+\dfrac{i\sqrt{3}}{8}$ (D) None of these

17. The value of $2x^3 + 2x^2 - 7x + 72$, when $x=\dfrac{3-5i}{2}$ is –

(A) 4 (B) – 4 (C) 2 (D) 0

18. The value of $x^4 + 4x^3 + 6x^2 + 4x + 9$. when $x=-1+\sqrt{2}$ is –

(A) 12 (B) 10 (C) 14 (D) 8

19. If $a+ib=\dfrac{c+i}{c-i}$, where c is real, then $a^2 + b^2$ =

(A) i (B) 1 (C) – 1 (D) 0

20. If $(x + iy)^{1/3} = a + ib$, x, y, a b $\in$ R, then $\frac{x}{a} + \frac{y}{b} =$

(A) 4 (B) $4(a^2 + b^2)$ (C) $4(a^2 - b^2)$ (D) $(a^2 - b^2)$

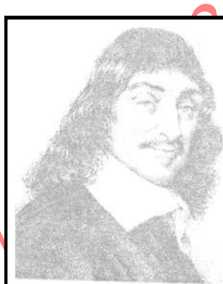
OBJECTIVE							ANSWER KEY							EXERCISE - 5
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	B	A	A	B	D	A	B	C	A	A	A	B	C
Que.	16	17	18	19	20										
Ans.	A	A	A	B	C										

POLYNOMIALS

★ INTRODUCTION

In class IX, have studied the polynomials in one variable and their degrees. We have also learnt about the values the zeros of a polynomial. In the this chapter, we wil discuss more about the zeros of a polynomial and the relationship between the zeros and the coefficients of a polynomial with particular reference to quadratic polynomials. In addition, statement and simple problems on division algorithm for polynomials with real coefficients will be discussed.

★ HISTORICAL FACTS


Rene Descartes

Determining the roots of polynomials, or ‘solving algebraic equations”, is among the oldest problems in mathematics. However, elegant and practical notation we use today only developed beginning in the 15th century. Before that, equations were written out in words. For example, an algebra problem from the Chinese Arithmetic in Nine Sections, begins “Three sheaf of good crop, two sheaf of mediocre crop, and one sheaf of bad crop are sold for 29 dou”. We would write 3x + 2y + z = 29.

The earliest known use of the equal sign is in Robert Recorder’s The Whetstone of Witte, 1557. The signs + for addition, - for subtraction, and the use of letter for and unknown appear in Michael Stifel’s Arithmetical Integra, 1544. **Rene Descartes**, in La geometric, 1637, introduced the concept of the graph of polynomial equation. He popularized the use of letters from the beginning of the alphabet to denote constants and letters from the end of the alphabet to denote variables, as can be seen in the general formula for a polynomial, where the a’s denote constants and x denotes a variable. Descartes introduced the use of superscripts to denote exponents as well.

★ RECALL

(i) **Polynomials :** An algebraic expression of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x^1 + a_0 x^0$ where $a_n \neq 0$ and $a_0, a_1, a_2, ...a_n$ are real numbers and each power of x is a positive integer, is called a polynomial.

Hence, a_n, a_{n-1}, a_{n-2}, are coefficients of $x^n, x^{n-1}................x^0$ and $a_n x^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, ...$ are terms of the polynomial . Here the term $a_n x^n$ is called the ***leading term*** and its coefficient a_n, the ***leading coefficient***, For example : $p(u) = \frac{1}{2}u^3 - 3u^2 + 2u - 4$ is a polynomial in variable u.

$\frac{1}{2}u^3, -3^2, 2u, -4$ are know as terms of polynomial and $\frac{1}{2}, -3, 2, -4$ are their respective coefficients.

$6x^{-2}$	This is NOT a polynomial term	Because the variable has a negative exponent
$\frac{1}{x^2}$	This is NOT a polynomial term	Because the variable is in the denominator
sqrt (x)	This is NOT a polynomial term	Because the variable is inside a radical
$4x^2$	This IS a polynomial term	Because it obeys all the rules

(ii) **Types of Polynomials :** Generally we divide the polynomials in three categories .

Types of Polynomials

Based on number of distinct variables | **Based on degree** | **Based on number of terms**

Polynomials classified by number of distinct variables

Number of distinct variables	Name	Example
1	Univariate	$x + 9$
2	Bivariate	$x + y + 9$
3	Trivariate	$x + y + z + 9$

Generally, a polynomial in more than one variable is called a **multivariate polynomial.** A second major way of classifying polynomials is by their degree. Recall that the degree of a term is the sum of the exponents on variables, and that the degree of a polynomial is the largest degree of any one term.

Polynomials classified by degree

Degree	Name	Example
$-\infty$	Zero	0
0	(non-zero) constant	1
1	Linear	$x + 1$
2	quadratic	$x^2 + 1$
3	cubic	$x^3 + 2$
4	quadratic (or biquadratic)	$x^4 + 3$
5	quintic	$x^5 + 4$
6	sextic (or hexic)	$x^6 + 5$
7	septic (or heptic)	$x^7 + 6$
8	octic	$x^8 + 7$
9	nonic	$x^9 + 8$
10	decic	$x^{10} + 9$

Usually, a polynomial of degree n, for n grater than 3, is called a polynomial of degree n, although the phrases quartic polynomial and quintic polynomial are sometimes used.

The polynomial 0, which may be considered to have no terms at all, is called the **zero polynomial.** Unlike other constant polynomials, its degree is not zero. Rather the degree of the zero polynomial is either left explicitly undefined , or defined to be negative (either -1 or $-\infty$)

Polynomials classified by number of non-zero terms

Number of non-zero terms	Name	Example
0	zero polynomial	0
1	monomial	x^2
2	binomial	$x^2 + 1$
3	trinomial	$x^2 + x + 1$

If a polynomial has only one variable, then the terms are usually written either from highest degree to lowest degree ("descending powers") or from lowest degree to highest degree ("ascending powers").

(iii) Value of a Polynomial : If p(x) is a polynomial in variable x and α is any real number, then the value obtained by replacing x by α in p(x) is called value of p(x) at x = α and is denoted by p(x).

For example : Find the value of $p(x) = x^3 - 6x^2 + 11x - 6$ at $= -2$

$\Rightarrow p(-2) = (-2)^3 - 6(-2)^2 + 11(-2) - 6 = -8 - 24 - 22 - 6 \Rightarrow p(-2) = -60$

(iv) Zero of a Polynomial : A real number α is zero of the polynomial p(x) if p(α) = 0.

For example : consider $p(x) = x^3 - 6x^2 + 11x - 6$

$$p(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$$
$$p(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$$
$$p(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$$

Thus, 1, 2 and 3 are called the zero of polynomial p(x).

★ GEOMETRICAL MEANING OF THE ZEROS OF A POLYNOMIAL

Geometrically the zeros of a polynomials f(x) are the x-co-ordinates of the points where the graph y = f(x) intersects x-axis. To understand it, we will see the geometrical representations of linear and quadratic polynomials.

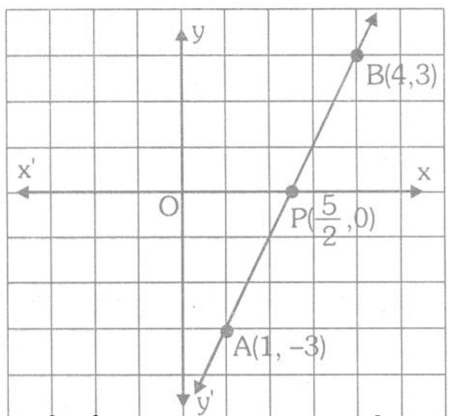
Geometrical Representation of the zero of a Linear Polynomial

Consider a linear polynomial, y = 2y – 5.

The following table lists the values of y corresponding to different values of x.

x	1	4
y	- 3	32

On plotting the points A(1, -3) and B(4, 3) and joining them, a straight line is obtained.



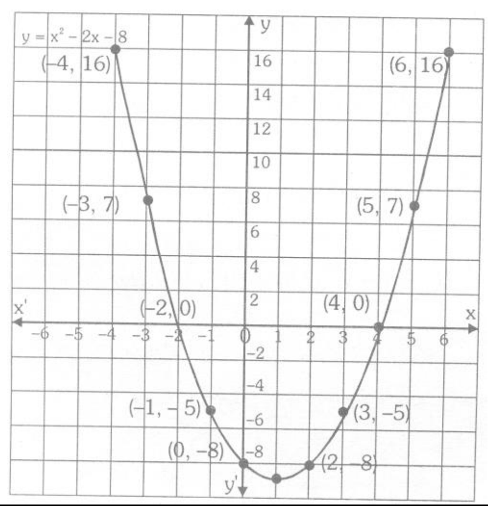
From, graph we observer that the graph of y = 2x – 5 intersects the x-axis at $\left(\frac{5}{2}, 0\right)$ whose x-coordinate is $\frac{5}{2}$, Also, zero of 2x – 5 is $\frac{5}{2}$.

Therefore, we conclude that the linear polynomial as + b has one and only one zero, which is the x-coordinate of the point where the graph of y = ax + b intersects the x-axis

Geometrical Representation of the zero of a quadratic Polynomial :

Consider quadratic polynomial, y = x2 – 2x – 8,

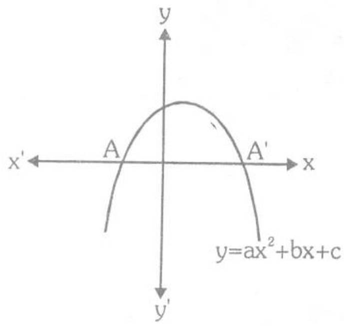
The following table gives the values of y or f(x) for various values of x.



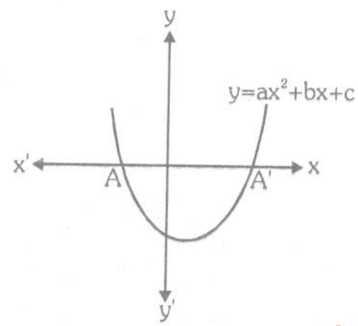
x	– 4	– 3	– 2	– 1	0	1	2	3	4	5	6
$y = x^2 - 2x - 8$	16	7	0	–5	–8	–9	–8	–5	0	7	16

On plotting the points (-4, 16), (-3, 7)(-2, 0), (-1, -5), (0, -8), (1, -9), (2, -8), (3, -5), (4, 0), (5, 7) and (6, 16) on a graph paper and drawing a smooth free hand curve passing through these points, the curve thus obtained represents the graph of the polynomial y = x2 – 2x – 8. This is called a parabola.

It is clear from the table that –2 and 4 are the zeros of the quadratic polynomial x2 – 2x – 8. Also, we observe that –2 and 4 are the x-coordinates of the points where the graph of y = x2 – 2x – 8 intersects the x-axis.
Consider the following cases –
Case-I : Here, the graph cuts x–axis at two distinct points A and A'.
The x-coordinates of A and A' are two zeroes of the quadratic polynomial ax2 + bx + c.

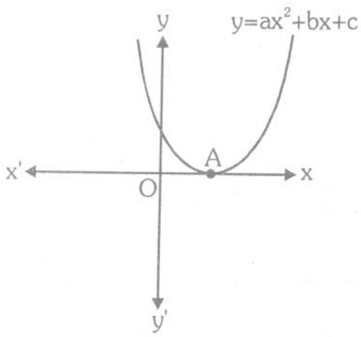


(i)

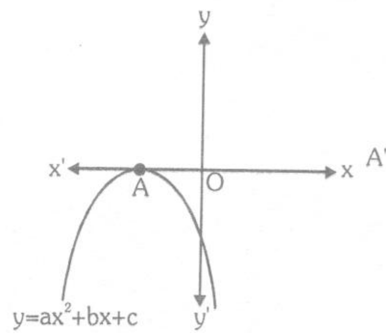


(ii)

Case-II : Here, the graph cuts the x-axis at exactly one point, i.e., at two coincident points. So, the two points A and A' of Case (i) coincide here to become one point A.

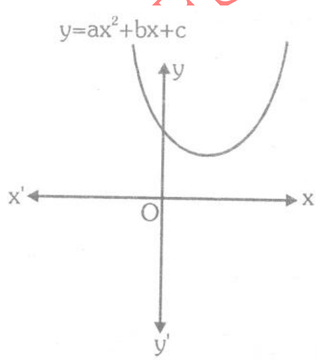


(iii)

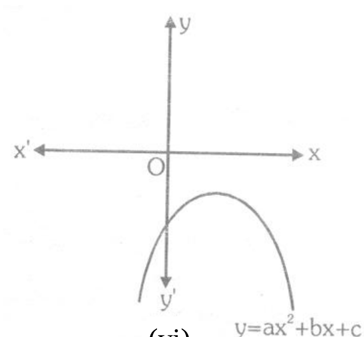


(iv)

The x-coordinate of A is the only zero for the quadratic polynomial ax2 + bx + c in this case.
Case-III : Here, the graph is either completely above the x-axis or completely below the x-axis, So, it does not cut the x-axis at any point.



(v)



(vi)

So, the quadratic polynomial ax2 + bx + c has no zero in this case.
So, you can see geometrically that a quadratic polynomial can have either two distinct zeroes or one zero, or no zero. This also means that a polynomial of degree 2 has atmost two zeroes.

Remark : In general given a polynomial p(x) of degree n, the graph of y = p(x) intersects the x-axis at atmost n points. Therefore, a polynomial p(x) of degree n has atmost n zeros.

★ Relationship Between The Zeros And Coefficients Of A Polynomial

For a linear polynomial ax + b, $(a \neq 0)$, we have,

$$\frac{(\text{constant term})}{(\text{coefficient of x})}$$

zero of a linear polynomial $= -\frac{b}{a} = -$

For a quadratic polynomial ax2 + b + c (a$\neq$0), with α and β as it's zeros, we have

$$\text{Sum of zeros } = \alpha + \beta = -\frac{b}{a} = -\frac{\text{(coefficient of x)}}{\text{(coefficient of } x^2)}$$

$$\text{Product of zeros } = \alpha\beta = \frac{c}{a} = \frac{\text{(constant term)}}{\text{(coefficient of } x^2)}$$

If α and β are the zeros of a quadratic polynomial f(x). Then polynomial f(x) is given by

f(x) = K{x^2 –($\alpha + \beta$)x+$\alpha\ \beta$ }

or f(x) = K{x^2 – (sum of the zeros) x + product of the zeros}

where K is a constant .

COMPETITION WINDOW

RELATIONSHIP BETWEEN THE ZEROS AND COEFFICIENTS OF CUBIC POLYNOMIAL

For a cubic polynomial $ax^3 + bx^2 + cx + d$ (a$\neq$0), with α , β and λ at its zeros, we have :

Sum of three zeros $= \alpha + \beta + \lambda = \frac{-b}{a}$

Sum of the product of its zeros taken two at a time $= \alpha\beta + \beta\lambda + \lambda\alpha = \frac{c}{a}$

Product of its zeros $= \alpha\beta\lambda = \frac{-d}{a}$

The cubic polynomial whose zeros are α , β and λ is given by

$$f(x) = \{x^3 - (\alpha + \beta + \lambda)x^2 + (\alpha\beta + \beta\lambda + \lambda\alpha)x - \alpha\beta\lambda\}$$

RELATIONSHIP BETWEEN THE ZEROS AND COEFFICIENTS OF A BI-QUADRATIC POLYNOMIAL

For a bi-quadratic polynomial $ax^4 + bx^3 + cx^2 + dx + e$ (a$\neq$0), with α, β, λ and δ as its zeros, we have :

Sum of four zeros $= \alpha + \beta + \lambda + \delta = \frac{-b}{a}$

Sum of the product of its zeros taken two at a time $= \alpha\beta + \alpha\lambda + \alpha\delta + \beta\lambda + \beta\delta + \lambda\delta = \frac{c}{a}$

Sum of the product of its zeros taken three at a time $= \alpha\beta\lambda + \alpha\beta\delta + \beta\lambda\delta + \lambda\delta\alpha = \frac{-b}{a}$

Product of all the four zeros $= \alpha\beta\lambda\delta = \frac{e}{a}$

The bi-quadratic polynomial whose zeros are α, β, λ and δ is given by

$$f(x) = \{x^4 - (\alpha + \beta + \lambda + \delta)x^3 + (\alpha\beta + \alpha\lambda + \alpha\delta + \beta\lambda + \beta\delta + \lambda\delta)x^2 - (\alpha\beta\lambda + \alpha\beta\delta + \beta\lambda\delta + \lambda\delta\alpha)x + \alpha\beta\lambda\delta\}$$

Ex. 1 Find the zeros of the quadratic polynomial $x^2 + 7x + 12$, and verify the relation between the zeros and its coefficients

Sol. We have,

f(x) = $x^2 + 7x + 12 = x^2 + 4x + 3x + 12$

$\Rightarrow$ f(x) = x (x + 4) + 3 (x + 4)

$\Rightarrow$ f(x) = (x + 4)(x + 3)

The zeros of f(x) are given by

f(x) = 0

$\Rightarrow$ $x^2 + 7x + 12 = 0$

$\Rightarrow$ (x + 4)(x + 3) = 0

$\Rightarrow$ x + 4 = 0 or, x + 3 = 0

$\Rightarrow$ x = - 4 or x = - 3

Thus, the zeros of f(x) = $x^2 + 7x + 12$ are $\alpha = -4$ and $\beta = -3$

Now, sum of the zeros $= \alpha + \beta = (-4) + (-3) = -7$

and $-\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^2} = -\dfrac{7}{1} = -7$

$\therefore$ Sum of the zeros = $-\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$

Product of the zeros $= \alpha\beta = (-4) \times (-3) = 12$

and, $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{12}{1} = 12$

$\therefore$ Product of the zeros $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Ex.2 Find the zeros of the quadratic polynomial f(x) = $abx^2 + (b^2 + ac)x + be$ and verify the relationship between the zeros and its coefficients.

Sol. f(x) = $abx^2 + (b^2 + ac)x + bc = abx^2 + b^2x + acx + bc$.

= bx (ax + b) + c (ax + b) = (ax + b) (bx + c)

So, the value of f(x) is zero when ax + b = 0 or bx + c = 0, i.e. $x = \dfrac{-b}{a}$ or $x = \dfrac{-c}{b}$

Therefore, $\dfrac{-b}{a}$ and $\dfrac{-c}{b}$ are the zeros (or roots) of f(x).

Now, sum of zeros $= \left(\dfrac{-b}{a}\right) + \left(\dfrac{-c}{b}\right) = \dfrac{-b^2 - ac}{ab} = \dfrac{-(b^2 + ac)}{ab} = -\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$

Product of zeros $= \left(\dfrac{-b}{a}\right)\left(\dfrac{-c}{b}\right) = \dfrac{bc}{ab} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

★ **SYMMETRIC FUNCTIONS OF THE ZEROS**

Let α, β be the zeros of a quadratic polynomial, then the expression of the form $\alpha + \beta; (\alpha^2 + \beta^2)$; $\alpha\beta$ are celled the functions of the zeros. By symmetric function we mean that the function remain invariant (unaltered) in values when the roots are changed cyclically. In other words, an expression involving α and β which remains unchanged by interchanging α and β is called symmetric function of α and β.

Some useful relations involving α and β are :-

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

(ii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

(iii) $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

(v) $\alpha^3 - \beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$

(vi) $\alpha^4 - \beta^4 = (\alpha^2+\beta^2)(\alpha+\beta)(\alpha-\beta) = [(\alpha+\beta)^2 - 2\alpha\beta](\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

(vii) $\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2 = [(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

(viii)

$\alpha^5 + \beta^5 = (\alpha^3+\beta^3)(\alpha^2+\beta^2) - \alpha^2\beta^2(\alpha+\beta) = [(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)][(\alpha+\beta)^2 - 2\alpha\beta] - (\alpha\beta)^2(\alpha+\beta)$

Ex.3 **If α and β are the zeros of the quadratic polynomial f(x) = ax^2 + bx + c then calculate :**

(i) $\alpha^2 + \beta^2$ **(ii)** $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Sol. Since α and β are the zeros of the quadratic polynomial
f(x) = ax^2 + bx + c

$\therefore \quad \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

(i) We have,

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 = 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$

(ii) We have, $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta} = \frac{\left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)}{\frac{c}{a}} \Rightarrow \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{3abc - b^3}{a^2c}$

Ex.4 **If α and β are the zeros of the quadratic polynomial p(s) = 3s^2 – 6s + 4, find the value of** $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$

Sol. Since α and β are the zeros of the polynomial p(s) = 3s^2 - 6s + 4.

$\therefore \quad \alpha + \beta = \frac{-(-6)}{3} = 2$ and $\alpha\beta = \frac{4}{3}$

We have $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{\alpha^2+\beta^2}{\alpha\beta} + 2\left(\frac{\beta+\alpha}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + \frac{2(\alpha+\beta)}{\alpha\beta} + 3\alpha\beta = \frac{(2)^2 - 2\times\frac{4}{3}}{\frac{4}{3}} + \frac{2\times 2}{\frac{4}{3}} + 3\times\frac{4}{3} = 8$

Ex.5 **If α and β are the roots (zeros) of the polynomial f(x) = x^2 - 3x + k such that $\alpha - \beta = 1$, find the value of k.**

Sol. Since α and β are the roots (zeros) of the polynomial f(x) = x^2 - 3x + k.

$\therefore \quad \alpha + \beta = \frac{-(-3)}{1} = 3$ and $\alpha\beta$ = k.

We have $\alpha - \beta = 1 \Rightarrow (\alpha-\beta)^2 = (1)^2 \Rightarrow \alpha^2 - 2\alpha\beta + \beta^2 = 1$

$\Rightarrow \quad (\alpha^2+\beta^2) - 2\alpha\beta = 1 \Rightarrow \{(\alpha+\beta)^2 - 2\alpha\beta\} - 2\alpha\beta = 1$

$\Rightarrow \quad (\alpha+\beta)^2 - 4\alpha\beta = 1 \Rightarrow (3)^2 - 4\times k = 1$

$\Rightarrow \quad 9 - 4k = 1 \Rightarrow 4k = 8 \Rightarrow k = 2$

Hence, the value of k is 2.

Ex.6 **If α, β are the zeros of the polynomial f(x) = $2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k for this to be possible .**

Sol. Since α and β are the zeros of the polynomial f(x) = $2x^2 + 5x + k$

$\therefore \quad \alpha + \beta = \frac{-5}{2}$ and $\alpha\beta = \frac{k}{2}$

Now, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

$\Rightarrow \quad (\alpha^2 + \beta^2 + 2\alpha\beta) - \alpha\beta = \frac{21}{4}$

$\Rightarrow \quad (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$

$\Rightarrow \quad \frac{25}{4} - \frac{k}{2} = \frac{21}{4} \qquad \left[\because \alpha + \beta = -\frac{5}{2} \text{ and } \alpha\beta = \frac{k}{2}\right]$

$\Rightarrow \quad -\frac{k}{2} = -1$

$\Rightarrow \quad k = 2$

Ex.7 **Find a quadratic polynomial each with the given numbers as the sum and product of its zeros prospectively.**

(i) $\frac{1}{4}, -1$ **(ii)** $\sqrt{2}, \frac{1}{3}$

Sol. We know that a quadratic polynomial which the sum and product of its zeros are given i given by –

f(x) = k{x^2 - (Sum of the zeros) x + Product of the zeros}, where k is a constant.

(i) Required quadratic polynomial f(s) is given by

$$f(x) = k\left(x^2 - \frac{1}{4}x - 1\right)$$

(ii) Required quadratic polynomial f(s) is given by

$$f(x) = k\left(x^2 - \sqrt{2}x + \frac{1}{3}\right)$$

Ex.8 **If α, β are the zeros of the polynomial $ax^2 + bx + c$, find a polynomial whose zeros are $\frac{1}{a\alpha + b}$ and $\frac{1}{a\beta + b}$**

Sol. since α and β are the zeros of the polynomial $ax^2 + bc + c$.

$\therefore \quad \alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Since $\frac{1}{a\alpha + b}$ and $\frac{1}{a\beta + b}$ are the zeros of the require polynomial

$\therefore \quad$ sum of the zeros $= \frac{1}{a\alpha + b} + \frac{1}{a\beta + b} = \frac{a\beta + b + a\alpha + b}{(a\alpha + b)(a\beta + b)}$

$$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2} = \frac{a\times\left(\frac{-b}{a}\right)+2b}{a^2\times\left(\frac{c}{a}\right)+ab\times\left(\frac{-b}{a}\right)+b^2} = \frac{b}{ac}$$

Product of the zeros $= \left(\frac{1}{a\alpha+b}\right)\left(\frac{1}{a\beta+b}\right) = \frac{1}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

$$= \frac{1}{a^2\times\frac{c}{a}+ab\times\left(\frac{-b}{a}\right)+b^2} = \frac{1}{ac}$$

Hence, the required polynomial = x^2 – (sum of zeros) x + product of zeros $= x^2 - \left(\frac{b}{ac}\right)x + \frac{1}{ac}$

★ DIVISION ALGORITHM FOR POLYNOMIALS

If f(x) is a polynomial and g(x) is a non-zero polynomial, then there exist two polynomials q(x) and r(x) such that f(x) = g(x) ×q(x) + r(x) , where r(x) = 0 or degree r(x) < degree g(x). In other words,

Dividend = Divisor × Quotient + Remainder

Remark : If r(x) = 0, then polynomial g(x) is a factor of polynomial f(x).

Ex.9 **divide the polynomial $2x^2 + 3x + 1$ by the polynomial x + 2 and verify the division algorithm .**

Sol. We have

$$\begin{array}{r|l} & 2x-1 \\ x+2 & 2x^2+3x+1 \\ & \underline{\overset{-}{2x^2}+\overset{-}{4x}} \\ & \quad -x+1 \\ & \quad \underline{\overset{+}{-x}\overset{+}{-2}} \\ & \qquad 3 \end{array}$$

Clearly, quotient = 2x -1 and remainder = 3

Also, (x + 2) (2x - 1) + 3 = $2x^2 + 4x – x – 2 + 3 = 2x^2 + 3x + 1$

i.e., $2x^2 + 3x + 1 = (x + 2)(2x - 1) + 3$. Thus, Dividend = Divisor × Quotient + Remainder.

Ex.10 **Check whether the polynomial $t^2 – 3$ is a factor of the polynomial $2t^4 + 3t^3 – 2t^2 = 9t – 12$, by dividing the second polynomial by the first polynomial.**

Sol. We have

$$\begin{array}{r|l} & 2t^2+3t+4 \\ t^2-3 & 2t^4+3t^3-2t^2-9t-12 \\ & \underline{\overset{-}{2t^4} \quad \overset{+}{-6t^2}} \\ & \quad 3t^3+4t^2-9t-12 \\ & \quad \underline{\overset{-}{3t^3} \quad \overset{+}{-9t}} \\ & \qquad 4t^2-12 \\ & \qquad \underline{\overset{-}{4t^2} \quad \overset{+}{-12}} \\ & \qquad\quad 0 \end{array}$$

Since the remainder is zero, therefore, the polynomial $t^2 – 3$ is a factor of the polynomial $2t^4 + 3t^3 – 2t^2 – 9t – 12$.

$$\begin{array}{r|l} & 2x^2-3x+1 \\ x^2-2 & 2x^4-3x^3-3x^2+6x-2 \\ & \underline{\overset{-}{2x^4} \quad \overset{+}{-4x^2}} \\ & \quad -3x^3+x^2+6x-2 \\ & \quad \underline{\overset{+}{-3x^3} \quad \overset{-}{+6x}} \\ & \qquad x^2-2 \\ & \qquad \underline{\overset{-}{x^2} \quad \overset{+}{-2}} \\ & \qquad\quad 0 \end{array}$$

Ex.11 Find all the zeros of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeros are $\sqrt{2}$ and $\sqrt{2}$.

Sol. Let p(x) $2x^4 - 3x^3 - 3x^2 + 6x - 2$ be the given polynomial. Since two zeros are $\sqrt{2}$ and $-\sqrt{2}$ so, $(x - \sqrt{2})$ and $(x + \sqrt{2})$ are both factors of the given polynomial p(x).

Also, $(x - \sqrt{2}) = (x^2 - 2)$ is a factor of the polynomial. Now , we divide the given polynomial by $x^2 - 2$.

By division algorithm, we have

$$2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$$

$$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})\,(x + \sqrt{2})\,(2x^2 - 2x - x + 1)$$

$$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})\,(x + \sqrt{2})\,\{2x(x - 1) - (x - 1)\}$$

$$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})\,(x + \sqrt{2})\,(x - 1)\,(2x - 1)$$

When p(x) = 0, $x = \sqrt{2}, \; -\sqrt{2}, \; 1$ and $\frac{1}{2}$

Hence, all the zeros of the polynomial $2x^4 - 3x^3 - 3x^2 + 6x - 2$ are $\sqrt{2}, \; -\sqrt{2}, \; 1$ and $\frac{1}{2}$

Ex.12 On dividing $f(x) = x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x)

Sol. Here, Divided $= x^3 - 3x^2 + x + 2$,

Quotient $= x - 2$,

Remainder $= -2x + 4$ and Divisor = g(x).

Since **Dividend = Divisor × Quotient + Remainder**

So , $x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$$\Rightarrow \quad g(x) \times (x - 2) = x^3 - 3x^2 + x + 2 + 2x - 4$$

$$\Rightarrow \quad g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2} = \frac{(x - 2)(x^2 - x + 1)}{x - 2} = x^2 - x + 1$$

Hence, $g(x) = x^2 - x + 1$.

★ **SYNOPSIS**

1. The highest power of the variable (x) in a polynomial p(x) is called a degree of polynomial p(x).
2. A polynomial of degree one is called linear polynomial :
   p(x) = ax + b, where $a \neq 0$ | a = coefficient of x ; b = constant term
3. A polynomial of a degree two is called quadratic polynomial :
   $p(x) = ax^2 + bx + c$, where $a \neq 0$ | a = coefficient of x^2; b = coefficient of x; c = constant term
4. A polynomial of degree three is called a cubic polynomial : $p(x) = ax^3 + bx2 + cx + d, a \neq 0$.
5. The zeros of a polynomial p(x) are precisely the x-coordinates of the point where the graph of y = p(x) intersects the x-axis.
6. The graph of the quadratic function $y = ax^2 + bx + c, a \neq 0$ is a parabola .
7. The parabola opens upwards if a > 0 and opens downwards if a < 0 .
8. A polynomial of degree n can have at most n zeros. So the quadratic polynomial can have at most two zeros and a cubic polynomial can have at most three zeros.
9. If α, β are the zeros of a quadratic polynomial $ax^2 + bx + c, a \neq 0$ then

Sum of its zeros $= \alpha + \beta = -\frac{b}{a}$ and Product of its zeros $= \alpha\beta = \frac{c}{a}$.

10. If α, β, γ are the zeros of a cubic polynomial $ax^3 + bx^2 + cx + d$, $a \neq 0$ then

Sum of its zeros $= \alpha + \beta + \gamma = -\frac{b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

Sum of the products of zeros taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

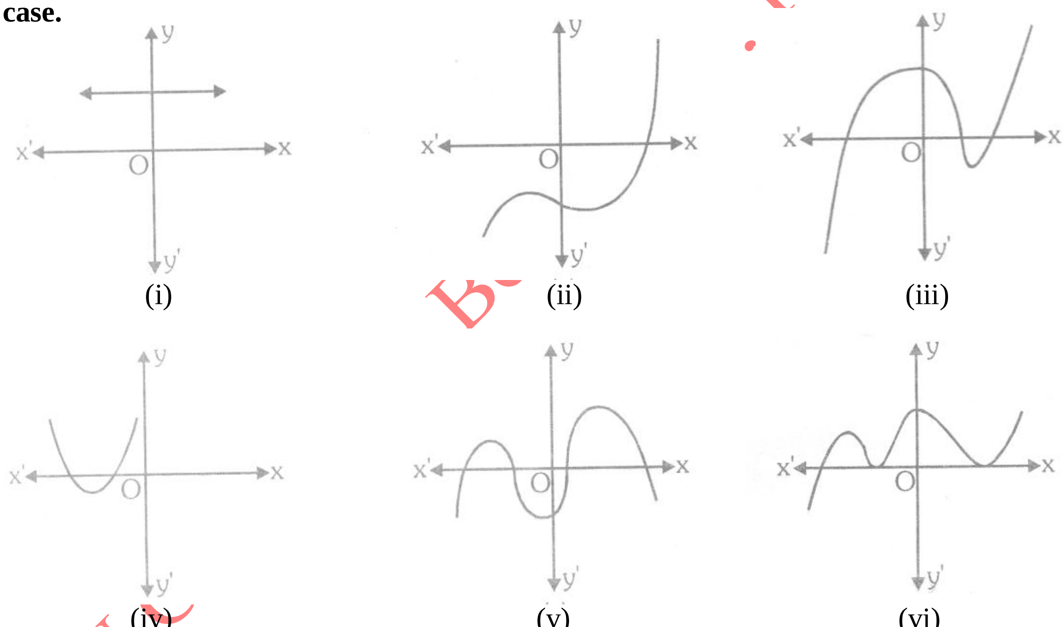
Product of its zeros $= \alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

11. The division algorithm states that given any polynomial p(x) and any non-zero polynomial g(x) then we can find quotient polynomial q(x) and remainder polynomial r(x) such that :
p(x) = g(x). q(x) + r(x) where deg. of r(x) < degree of g(x), deg of r(x) = 0.

SOLVED NCERT EXERCISE

EXERCISE : 2 . 1

1. The graph of y – p(x) are given in fig below, for some polynomials p(x). Find the number of zeros of p(x), in each case.



Sol. (i) Graph of y = p(x) does not intersect the x-axis. Hence, polynomial p(x) has no zero.

(ii) Graph of y = p(x) intersects the x-axis at one and only one point.
Hence, polynomial p(x) has **one end only one** real zero.
[Rest Try Yourself]

EXERCISE : 2.2

1. Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients.

(i) $x^2 - 2x - 8$ **(ii) $4s^2 - 4s + 1$** **(iii) $6x^2 - 3 - 7x$** **(iv) $4u^2 + 8u$**
(v) $t^2 - 15$ **(vi) $3x^2 - x - 4$**

Sol. (i) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x - 4) + 2(x - 4) = (x + 2)(x - 4)$
Zeros are –2 and 4.

$\frac{\text{-Coefficient of x}}{\text{Coefficient of } x^2}$

Constant term

Sum of the zeros = (–2) + (4) = 2 = $\frac{-(-2)}{1}$ =

Product of the zeros $= (-2)(4) = -(8) = \frac{(-8)}{1} =$

(ii) $4s^2 - 4s + 1 = (2s - 1)^2$

The two zeros are $\frac{1}{2}, \frac{1}{2}$

Sum of the two zeros $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{\text{-Coefficient of x}}{\text{Coefficient of } x^2}$

Product of two zeros $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

[Rest Try Yourself]

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $\frac{1}{4}, -1$ **(ii)** $\sqrt{2}, \frac{1}{3}$ **(iii)** $0, \sqrt{5}$ **(iv) 1, 1** **(v)** $-\frac{1}{4}, \frac{1}{4}$ **(iv) 4, 1**

Sol. (i) Let the quadratic polynomial be $ax^2 + bx + c$

Then $-\frac{b}{a} = \frac{1}{4}$ and $\frac{c}{a} = -1$

i.e., $\frac{b}{a} = \frac{-1}{4}$ and $\frac{c}{a} = \frac{-1}{1}$

We select a = LCM (4, 1) = 4

Then $\frac{b}{4} = \frac{-1}{4}$ and $\frac{c}{4} = -1 \Rightarrow b = -1$ and $c = -4$.

Substituting $a = 4, b = -1, c = -4$ in $ax^2 + bx + c$, we get the required polynomial $4x^2 - x - 4$

(ii) $-\frac{b}{a} = \sqrt{2}, \quad \frac{c}{a} = \frac{1}{3}$

$\Rightarrow \frac{b}{a} = \frac{-\sqrt{2}}{1}, \quad \frac{c}{a} = \frac{1}{3}$

Select a = LCM (1, 3) = 3.

Then $\frac{b}{3} = -\sqrt{2}$ and $\frac{c}{a} = \frac{1}{3} \Rightarrow b = -3\sqrt{2}$ and c = 1.

Substituting $a = 3,\ b = -3\sqrt{2}$ and c = 1 in $ax^2 + bx + c$, we get the required polynomial $3x^2 - 3\sqrt{2}\,x + 1$

[Rest Try Yourself]

EXERCISE : 2.3

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5,\ g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$.

$x^2 - 2\,)\ x^3 - 3x^2 + 5x - 3\ (\ q(x) = (x - 3)$

$x^3 \quad - 2x$

$- \quad +$

$-3x^2 + 7x - 3$

Sol. (i)

Hence, Quotient q(x) = x – 3 and Remainder r(x) = 7x – **9**

[Rest Try Yourself]

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

(i) $t^2 – 3, 2t^4 + 3t^3 – 9t –12$ **(ii) $x^2 + 3x + 1 , 3x^4 + 5x^3 – 7x^2 + 2x + 2$**

(iii) $x^3 – 3x + 1, x^5 – 4x^3 + x^2 + 3x + 1$

Sol. (i) $t^2 - 3 \,\overline{)\, 2t^4 + 3t^3 - 2t^2 - 9t - 12 \,}$ $q(t)\ 2t^2 + 3t + 4$

$2t^4 \quad - 6t^2$

$- \quad +$

$3t^3 + 4t^2 - 9t - 12$

$3t^3 \quad - 9t$

$- \quad +$

$4t^2 - 12$

$4t^2 - 12$

$- \quad +$

Remainder = 0

Hence, $t^2 – 3$ is a factor of $2t^4 + 3t^3 – 2t^2 – 9t – 12$

[Rest Try Yourself]

3. Obtain all other zeros of $3x^4 + 6x^3 – 2x^2 – 10x – 5$, If two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Two of the zeros of $3x^4 + 6x^3 – 2x^2 – 10x – 5$, are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$\Rightarrow \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)$ is a factor of the polynomial .

i.e., $x^2 - \frac{5}{3}$ is a factor.

i.e., $(3x^2 – 5)$ is a factor of the polynomial. Then we apply the division algorithm as below :

$3x^2 - 5 \,\overline{)\, 3x^4 + 6x^3 - 2x^2 - 10x - 5 \,}$ $q(x) = x^2 + 2x + 1$

$3x^4 \quad - 5x^2$

$- \quad +$

$6x^3 + 3x^2 - 10x - 5$

$6x^3 \quad - 10x$

$- \quad +$

$3x^2 - 5$

$3x^2 - 5$

$- \quad +$

×

The other two zeros will be obtained from the quadratic polynomial $q(x) = x^2 + 2x + 1$

Now $x^2 + 2x + 1 = (x + 1)^2$.

Its zeros are –1 , –1.

Hence, all other zeros are –1 , –1.

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x – 2 and –2x + 4, respectively. Find g(x).

[Try Yourself]

5. Give examples of polynomial p(x), g(x) and r(x), which satisfy the division algorithm and
(i) deg p(x) = deg q(x)   (ii) deg q(x) = deg r(x)   (iii) deg r(x) = 0,

Sol. (i) $p(x) = 2x^2 + 2x + 8$, $g(x) = 2x^0 = 2$; $q(x) = x^2 + x + 4$; $r(x) = 0$

(ii) $p(x) = 2x^2 + 2x + 8$, $g(x) = x^2 + x + 9$; $q(x) = 2$; $r(x) = -10$

(iii) $p(x) = x^3 + x + 5$; $g(x) = x^2 + 1$; $g(x) = x$; $r(x) = 5$.

EXERCISE – 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

Choose The Correct One

1. Quadratic polynomial having zeros 1 and –2 is -

(A) $x^2 - x + 2$ (B) $x^2 - x - 2$
(C) $x^2 + x - 2$ (D) None of these

2. If (x – 1) is a factor of $k^2x^3 - 4kx - 1$, then the value of k is –

(A) 1 (B) – 1
(C) 2 (D) – 2

3. For what value of a is the polynomial $2x^4 - ax^3 = 4x^2 + 2x + 1$ divisible by 1 – 2x ?

(A) a = 25 (B) a = 24 (C) a = 23 (D) a = 22

4. If one of the factors of $x^2 + x - 20$ is (x + 5), then other factor is -

(A) (x – 4) (B) (x – 5) (C) (x – 6) (D) (x – 7)

5. If α, β be the zeros of the quadratic polynomial $2x^2 + 5x + 1$, then value of $\alpha + \beta + \alpha\beta =$

(A) – 2 (B) – 1 (C) 1 (D) None of these

6. If α, β be the zeros of the quadratic polynomial $2 - 3x - x^2$, then $\alpha + \beta =$

(A) 2 (B) 3 (C) 1 (D) None of these

7. Quadratic polynomial having sum of it's zeros 5 and product of it's zeros – 14 is-

(A) $x^2 - 5x - 14$ (B) $x^2 - 10x - 14$
(C) $x^2 - 5x + 14$ (D) None of these

8. If x = 2 and x = 3 are zeros of the quadratic polynomial $x^2 + ax + b$, the values of a and b respectively are :

(A) 5, 6 (B) – 5 , – 6 (C) –5, 6 (D) 5, 6

9. If 3 is a zero of the polynomial $f(x) = x^4 - x^3 - 8x^2 + kx + 12$, then the value of k is -

(A) – 2 (B) 2 (C) – 3 (D) $\frac{3}{2}$

10. The sum and product of zeros of the quadratic polynomial are – 5 and 3 respectively the quadratic polynomial is equal to -

(A) $x^2 + 2x + 3$ (B) $x^2 – 5x + 3$ (C) $x^2 + 5x + 3$ (D) $x^2 + 3x – 5$

11. On dividing $x^3 – 3x^2 + x + 2$ by polynomial g(x), the quotient and remainder were x – 2 and 4 – 2x respectively then g(x) :

(A) $x^2 + x + 1$ (B) $x^2 + x – 1$

(C) $x^2 – x – 1$ (D) $x^2 – x + 1$

12. If the polynomial $3x^3 – x3 – 3x + 5$ is divided by another polynomial x – 1 – x2 , the remainder comes out to be 3, then quotient polynomial is –

(A) 2 – x (B) 2x – 1 (C) 3x + 4 (D) x – 2

13. If sum of zeros $= \sqrt{2}$, product of its zeros $= \frac{1}{3}$. The quadratic polynomial is –

(A) $3x^2 - 3\sqrt{2}x + 1$ (B) $\sqrt{2}x^2 + 3x + 1$

(C) $3x^2 - 2\sqrt{3}x + 1$ (D) $\sqrt{2}x^2 + x + 3$

14. If $-\frac{1}{3}$ is the zeros of the cubic polynomial $f(x) = 3x^3 – 5x^2 – 11x – 3$ the other zeros are :

(A) – 3 , –1 (B) 1, 3 (C) 3, –1 (D) –3, 1

15. If α and β are the zeros of the polynomial $f(x) = 6x^2 – 3 – 7x$ then $(\alpha + 1)(\beta + 1)$ is equal to –

(A) $\frac{5}{2}$ (B) $\frac{5}{3}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

16. Let $p(x) = ax^2 + bx + c$ be q quadratic polynomial. It can have at most –

(A) One zero (B) Two zeros

(C) Three zeros (D) None of these

17. The graph of the quadratic polynomial $ax^2 + bx + c, a \neq 0$ is always-

(A) Straight line (B) Curve

(C) Parabola (D) None of these

18. If 2 and $-\frac{1}{2}$ as the sum and product of its zeros respectively then the quadratic polynomial f(x) is –

(A) $x^2 – 2x – 4$ (B) $4x^2 – 2x + 1$

(C) $2x^2 + 4x – 1$ (D) $2x^2 – 4x – 1$

19. If α and β are the zeros of the polynomial $f(x) = 16x^2 + 4x – 5$ then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to –

(A) $\frac{2}{5}$ (B) $\frac{5}{2}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

20. If α and β are the zeros of the polynomial $f(x) = 15x^2 – 5x + 6$ then $\left(1 + \frac{1}{\alpha}\right)\left(1 + \frac{1}{\beta}\right)$ is equal to –

(A) $\frac{13}{3}$ (B) $\frac{13}{2}$ (C) $\frac{16}{3}$ (D) $\frac{15}{2}$

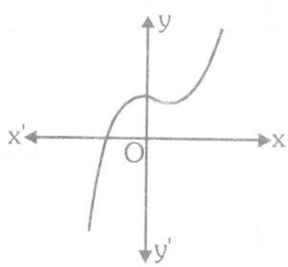
OBJECTIVE							ANSWER KEY							EXERCISE - 1
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**C**	**A**	**A**	**A**	**A**	**D**	**A**	**C**	**B**	**C**	**D**	**D**	**A**	**C**	**B**
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	**B**	**C**	**D**	**D**	**A**										

EXERCISE – 2 (FOR SCHOOL / BOARD EXAMS)

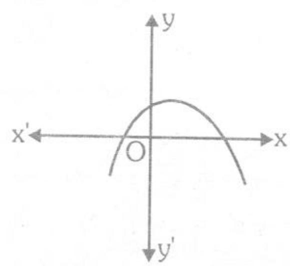
SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions

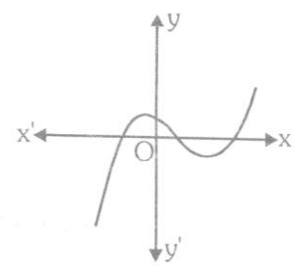
1. Look at the graph in fig given below. Each is the graph of y = p(x) , where p(x) is a polynomial. For each of the graph, find the number of zeros of p(x).



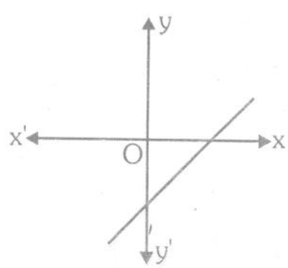
(i)



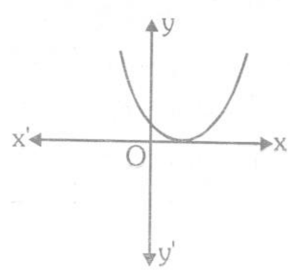
(ii)



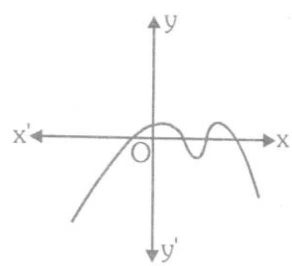
(iii)



(iv)

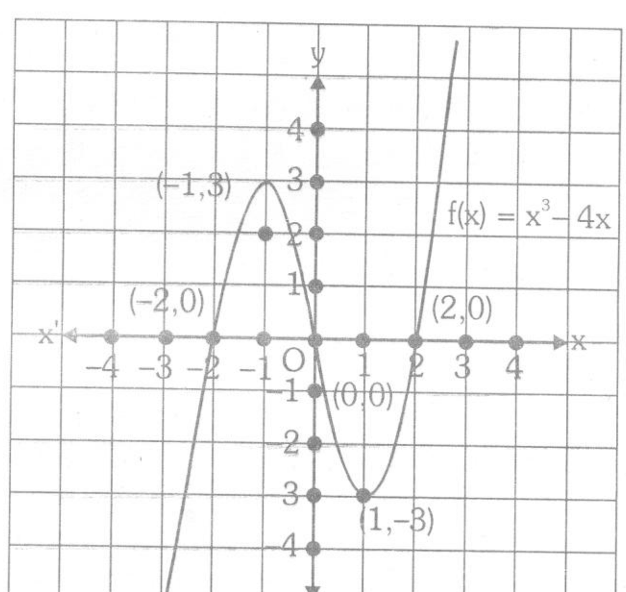


(v)



(vi)

2. Consider the cubic polynomial $f(x) = x^3 – 4x$. Find from the fig, the number of zeros of the above stated polynomial

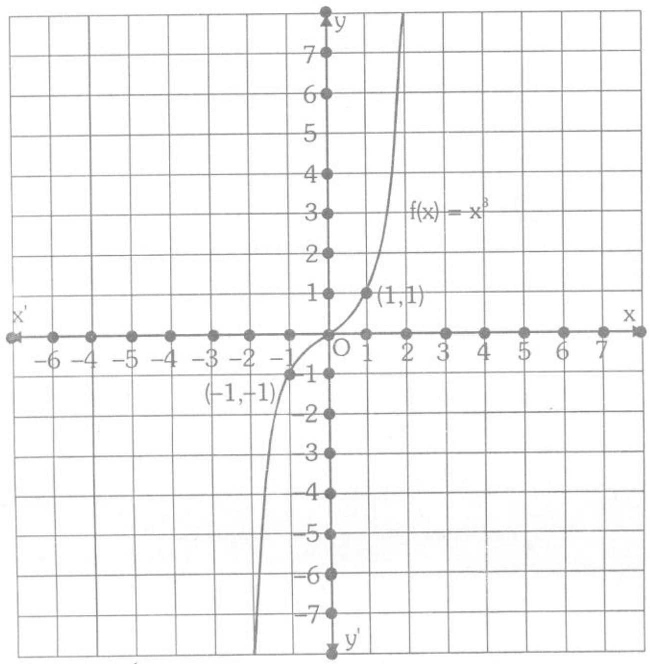


3. Let $f(x) = x^3$

The graph of the polynomial is shown in fig.

(i) Find the number of zeros of polynomial f(x).

(ii) Determine the co-ordinates of the points, at which the graph intersects the x-axis



Short answer Type Questions

1. Find the zeros of the following quadratic polynomial and verify the relationship between the zeros and their coefficients.

(i) $6x^2 - x - 1$ (ii) $25x(x + 1) + 4$ (iii) $4x^2 + 4x + 1$ (iv) $48y^2 - 13y - 1$ (v) $63 - 2x - x^2$
(vi) $2x^2 - 5x$ (vii) $49x^2 - 81$ (viii) $4x^2 - 4x - 3$

2. Find a quadratic polynomial each with the given numbers as the zeros of the polynomial .

(i) $3+\sqrt{7}, 3-\sqrt{7}$ (ii) $2\sqrt{3}, -2\sqrt{3}$ (iii) $-\frac{3}{7}, -\frac{2}{3}$ (iv) $\sqrt{3}, 3\sqrt{3}$ (v) $2+3\sqrt{2}, 2-3\sqrt{2}$ (vi) $\frac{8}{3}, \frac{5}{2}$

3. Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively .

(i) $4\sqrt{3}, 9$ (ii) $2\sqrt{3}-1, 3-\sqrt{3}$ (iii) $0, -\frac{1}{4}$ (iv) $\frac{-10}{\sqrt{3}}, 7$ (v) $\frac{5}{6}, \frac{25}{9}$ (vi) $\frac{-2\sqrt{5}}{3}, -\frac{5}{3}$ (vii) $-\sqrt{3}, \frac{1}{4}$ (viii) $-\frac{6}{5}, \frac{9}{25}$

(ix) $\sqrt{2}, -12$

4. If α and β are the zeros of the polynomial f(x) $5x^2 + 4x - 9$ then evaluate the following :

(i) $\alpha - \beta$ (ii) $\alpha^2 + \beta^2$ (iii) $\alpha^2 - \beta^2$ (iv) $\alpha^3 + \beta^3$ (v) $\alpha^3 - \beta^3$ (vi) $\alpha^4 - \beta^4$

5. If one of the zeros of the quadratic polynomial $2x^2 + px + 4$ is 2, find the other zero. Also find the value of p.

6. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is the reciprocal of the other, find the value of a .

7. If the product of zeros of the polynomial $ax^2 - 6x - 6$ is 4, find the value of a.

8. Find the zeros of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeros and the coefficients of the polynomial .

9. Determined if 3 is a zero of p(x) $= \sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} - \sqrt{4x^2 - 14x + 6}$

10. If α and β be two zeros of the quadratic polynomial $ax^2 + bx + c$, then valuate :

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$ (iii) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ (iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

11. Find the value of k :

(i) If α and β are the zeros of the polynomial $x^2 - 5x + k$ where $\alpha - \beta = 1$

(ii) If α and β are the zeros of the polynomial x2 – 8x + k such that $\alpha^2 + \beta^2 = 40$.

(iii) If α and β are the zeros of the polynomial $x^2 - 6x + k$ such that $3\alpha + 2\beta = 20$.

12. If 2 and 3 are zeros of polynomial $3x^2 - 2kx + 2m$, find the values of k and m.

13. If one zeros of polynomial $3x^2 = 8x + 2x + 1$ is seven times the other, then find the zeros and the value of k.

14. If α and β are the zeros of the polynomial $2x^2 - 4x + 5$. Form the polynomial where zeros are :

(i) $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ (ii) $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ (iii) $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$

15. If α and β are the zeros of the quadratic polynomial $x^2 - 3x + 2$, find a quadratic polynomial whose zeros are :

(i) $\frac{1}{2\alpha + \beta}$ and $\frac{1}{2\beta + \alpha}$ (ii) $\frac{\alpha - 1}{\alpha + 1}$ and $\frac{\beta - 1}{\beta + 1}$

16. If the sum of the squares of zeros of the polynomial $5x^2 + 3x + k$ is $-\frac{11}{25}$, find the value of k .

17. If one zero of the quadratic polynomial $2x^2 - (3k + 1)x - 9$ is negative of the other, find the value of k .

18. If α and β are the two zeros of the quadratic polynomial $x^2 - 2x + 5$, find a quadratic polynomial whose zeros are $\alpha + \beta$ and $\frac{1}{\alpha} + \frac{1}{\beta}$

19. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$, prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

20. Apply the division algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by g(x) as given below :

(i) $p(x) = 3x^3 + 2x^2 + x + 1$; $g(x) = x^3 + 3x + 2$

(ii) $p(x) = x^6 + x^4 - x^2 - 1$; $g(x) = x^3 - x^2 + x - 1$

(iii) $p(x) = 2x^5 + 3x^4 + 4x^2 + 3x + 2$; $g(x) = x^3 + x^2 + x + 1$

(iv) $p(x) = x^3 - 3x^2 - x + 3$; $g(x) = x^2 - 4x + 3$

21. Find the quotient q(x) and remainder r(x) of the following when f(x) is divided by g(x). Verify the division algorithm.

(i) $f(x) = x^6 + 5x^3 + 7x + 3$; $g(x) = x^2 + 2$

(ii) $f(x) = x^4 + 2x^2 + 1$; $g(x) = x^3 + 1$

(iii) $f(x) = 4x^4 - 7x^2 + 18x - 1$; $g(x) = 2x + 1$

(iv) $f(x) = 5x^3 - 70x^2 + 153x - 342$; $g(x) = x^2 - 10x + 6$

22. Check whether g(y) is a factor of f(y) by applying the division algorithm :

(i) $f(y) = 2y^4 + 3y^3 - 2y^2 - 9y - 12$, $g(y) = y^2 - 3$

(ii) $f(y) = 3y^4 + 5y^3 - 7y^2 + 2y + 2$, $g(y) = y^2 + 3y + 1$

(iii) $f(y) = y^5 - 4y^3 + y^2 + 3y + 1$, $g(y) = y^3 - 3y + 1$

23. (a) If 1 is the zero of $f(x) = k^2x^2 - 3kx - 1$ then find the value (s) of k.

(b) If 1 and – 2 are the zeros of $f(x) = x^3 + 10x^2 + ax + b$, then find the values of a and b.

(c) Find p and q such that 3 and – 1 are the zeros of $f(x) = x^4 + px^3 + qx^2 + 12x - 9$.

(d) If 3 is the zero of $f(x) = x^4 - x^3 - 8x^2 + kx + 12$, then find the value of k.

24. (a) Find all the zeros of $3x^3 + 16x^2 + 23x + 6$ if two its zeros are – 3 and – 2 .

(b) Determine all the zeros of $4x^3 + 12x^2 - x - 3$ if two of its zeros are $-\frac{1}{2}$ and $\frac{1}{2}$.

(c) Determine all the zeros of $x^3 + 5x^2 - 2x - 10$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$

(d) Determine all the zeros of $4x^3 + 12x^2 - x - 3$ if one of its zeros is $\frac{5}{2}$

(e) Determine all the zeros of $4x^3 + 5x^2 - 180x - 225$ if one of its zeros is $-\frac{5}{4}$.

25. (a) Find all the zeros of $3x^4 - 10x^3 + 5x^2 + 10x - 8$ if three of its zeros are 1, 2 and – 1 .
(b) Obtain all the zeros of $2x^4 + 5x^3 - 8x^2 - 17x - 6$ if three of its zeros are – 1, –3, 2.
(c) Determine all the zeros of $x^4 - x3 - 8x^2 + 2x + 12$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

26. (a) Obtain all other zeros of the polynomial $2x^3 - 4x - x^2 + 2$ if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.
(b) Find all the zeros of $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeros are $2+\sqrt{3}$ and $2-\sqrt{3}$.
(c) Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeros are 2 and – 2 .
(d) Find all the zeros of the polynomial $2x^4 + 7x3 - 19x^2 - 14x + 30$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

27. (a) On dividing $f(x) = 3x^3 + x^2 + 2x + 5$ by a polynomial $g(x) = x^2 + 2x + 1$, the remainder $r(x) = 9x + 10$. Find the quotient polynomial q(x).
(b) On dividing f(x) be a polynomial $x - 1 - x^2$, the quotient q(x) and remainder r(x) are (x – 2) and 3 respectively . Find f(x).
(c) On dividing $x^5 - 4x^3 + x^2 + 3x + 1$ by polynomial g(x), the quotient and remainder are $(x^2 - 1)$ and 2 respectively. Find g(x).
(d) On dividing $f(x) = 2x^5 + 3x^4 + 4x^3 + 4x^2 + 3x + 2$ by a polynomial g(x), where $g(x) = x^3 + x^2 + x + 1$, the quotient obtained as $2x^2 + x + 1$. Find the remainder r(x).

POLYNOMIALS **ANSWER KEY** **EXERCISE – 2 (X) CBSE**

- **Very Short Answer Type Questions**

1. (i) One zero, (ii) Two zero, (iii) One zero, (v) One zero, (vi) Four zeros
2. Three zeros **3.** (i) One zero, (ii) (0, 0)

- **Short Answer Type Questions**

1. (i) $-\frac{1}{3},\frac{1}{2}$ (ii) $-\frac{1}{5},\frac{-4}{5}$, (iii) $\frac{-1}{2},\frac{-1}{2}$, (iv) $\frac{1}{3},\frac{-1}{16}$, (v) 7, -9 , (vi) $0,\frac{5}{2}$ (vii) $\frac{9}{7},\frac{-9}{7}$, (viii) $\frac{3}{2},\frac{-1}{2}$

2. (i) $x^2 - 6x + 2$, (ii) $x^2 - 12$, (iii) $21x^2 + 33x + 6$, (iv) $x^2 - 4\sqrt{3}x + 9$, (v) $x^2 - 4x - 14$, (vi) $6x^2 - 31x + 40$

3. (i) $x^2 - 4\sqrt{3}x + 9$, (ii) $x^2 - (2\sqrt{3}-1)x + (3-\sqrt{3})$, (iii) $4x^2 - 1$, (iv) $3x^2 + 10\sqrt{3}x + 21$ (v) $18x^2 - 15x + 50$.
(vi) $3x^2 + 2\sqrt{5}x - 5$, (vii) $4x^2 + 4\sqrt{3}x + 1$ (viii) $25x^2 + 30x + 9$, (ix) $x^2 - \sqrt{2}x - 12$

4. (i) $\frac{14}{5}$, (ii) $\frac{106}{25}$, (iii) $\frac{-56}{25}$, (iv) $\frac{-604}{125}$ (v) $\frac{854}{125}$ (vi) $\frac{-5936}{125}$ **5.** p = - 6 , other zero = 1 **6.** a = 3 **7.** $a = \frac{-31}{2}$

8. 2 and $\frac{-2}{5}$ **9.** Yes **10.** (i) $\frac{b^2 - 2ac}{a^2}$ (ii) $\frac{3abc - b^3}{a^3}$ (iii) $\frac{3abc - b^3}{c^3}$ (iv) $\frac{3abc - b^3}{a^2c}$ **11.** (i) 6 (ii) 12 (iii) –16

12. $k = \frac{15}{2},\ m = 9$ **13.** $\frac{1}{3},\frac{7}{3}, k = \frac{-5}{3}$ **14.** (i) $\frac{1}{5}(5x^2 - 4x + 2)$ (ii) $\frac{1}{25}(25x^2 + 4x + 4)$ (iii) $\frac{1}{5}(5x^2 - 8x + 8)$

15. (i) $20x^2 - 9x + 1$ (ii) $3x^2 - x$ **16.** 2 **17.** $-\frac{1}{3}$ **18.** $5x^2 - 12x + 4$

20. (i) q(x) = 3, $r = 2x^2 - 8x - 5$, (ii) $q(x) = x^3 + x^2 + x + 1$, r(x) = 0, (iii) $q(x) = 2x^2 + x + 1$, (x) = x + 1,
(iv) q(x) = x + 1, r(x) = 0

21. (i) $q(x) = x^4 - 2x^2 + 5x + 4$, r(x) = - (3x + 5), (ii) q(x) = x, $r(x) = 2x^2 - x + 1$,
(iii) $q(x) = 2x^3 - x^2 - 3x + \frac{11}{2}$, $r(x) = -\frac{13}{2}$, (iv) q(x) = 5x – 20, r(x) = – 127x – 22

22. (i) g(y) is a factor of f(y), (ii) g(x) is a factor of f(x), (iii) g(t) is not a factor of f(t)

23. (a) k = $\pm$1, (b) a = 7, b = -18, (c) p = - 8, q = 12, (d) k = 2

24. (a) $-2, -3, -\frac{1}{3}$, (b) $\frac{1}{2}, -\frac{1}{2}, 3$, (c) $\sqrt{2}, -\sqrt{2}, -5$, (d) $\frac{5}{2},\frac{3}{2},\frac{1}{2}$, (e) $-\frac{5}{4}, 3\sqrt{5}, -3\sqrt{5}$

25. (a) 1, 2, $-1, \frac{4}{3}$ (b) $-1, -3, 2, -\frac{1}{2}$, (c) $\sqrt{2}, -\sqrt{2}, 3, -2$

26. (a) $\frac{1}{2}$, (b) $2 \pm \sqrt{3}, 1, -\frac{1}{2}$, (c) 2, – 2 , 5 and – 6, (d) $\pm\ \sqrt{2}, \frac{3}{2}$ and – 5

27. (a) q(x) = 3x – 5 , (b) f(x) = $-x^3 + 3x^2 - 3x + 5$, (c) g(x) = $x^3 - 3x + 1$, (d) r(x) = x + 1,

EXERCISE – 3 (FOR SCHOOL / BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

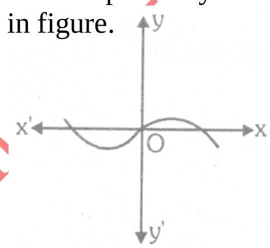
Questions Carrying 1 Mark

1. Write the zeros of the polynomial $x^2 + 2x + 1$. **[Delhi – 2008]**

2. Write the zeros of the polynomial $x^2 - 2x - 6$. **[Delhi – 2008]**

3. Write a quadratic polynomial , the sum and product of whose zeros are 3 are – 2 respectively. **[Delhi – 2008]**

4. Write the number of zeros of the polynomial y = f(x) whose graph is given in figure. **[AI– 2008]**



5. If (x + a) is a factor of $2x^2 + 2ax + 5x + 10$, find a **[Foreign – 2008]**

6. For what value of k, (- 4) is a zero of the polynomial $x^2 - x - (2x + 2)$? **[Delhi – 2009]**

7. For what value of p, (-4) is a zero of the polynomial $x^2 - 2x - (7p + 3)$? **[Delhi – 2009]**

8. If 1 is a zero of the polynomial p(x) = $ax^2 - 3(a - 1)x - 1$, then find the value of a. **[AI– 2009]**

9. Write the polynomial , the product and sum of whose zeros $-\frac{9}{2}$ and $-\frac{3}{2}$ respectively **[Foreign – 2009]**

10. Write the polynomial , the product and sum of whose zeros are $-\frac{13}{5}$ and $-\frac{3}{5}$ respectively **[Foreign – 2009]**

Questions Carrying 2 Marks

11. Find the zeros of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeros and the co-efficient of the polynomial . **[Delhi – 2008]**

12. Find the zeros of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeros and the coefficients of the quadratic polynomial **[Delhi – 2008]**

13. Find the quadratic polynomial sum of whose zeros is 8 and their product is 12. Hence, find the zeros of the polynomial . **[AI– 2008]**

14. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other. Find the value of 'a' **[AI– 2008]**

15. If the product of zeros of the polynomial $ax^2 - 6x - 6$ is 4, find the value of 'a' **[AI– 2008]**

16. Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of it's zeros are 2 and – 2 . **[Foreign – 2008]**

17. Find all the zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of it's zeros are $\sqrt{2}$ and $-\sqrt{2}$ **[Foreign – 2008]**

18. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out of be (ax +b) , find a and b. **[Delhi – 2009]**

19. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be px + q. Find the values of p and q. **[Delhi – 2009]**

20. Find all the zeros of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of it's zeros are $-\sqrt{2}$ and $\sqrt{2}$ **[AI – 2009]**

21. Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of it's zeros are $-\sqrt{3}$ and $\sqrt{3}$. **[AI – 2009]**

22. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by polynomial $2x^2 - 5$, then find the value of the a and b **[Foreign – 2009]**

POLYNOMIALS	ANSWER KEY	EXERCISE –3 (X)- CBSE

1. x = - 1 **2.** 3, -2 **3.** $x^2 - 3x - 2$ **4.** 3 **5.** 2 **6.** 9 **7.** 3 **8.** a = 1 **9.** $2x^2 + 3x - 9$ **10.** $5x^2 + 3x - 13$

11. $\left[\frac{-1}{3},\frac{3}{2}\right]$ **12.** $\left[\frac{-2}{2},2\right]$ **13.** $x^2 - 8x + 12$; (6, 2) **14.** 3 **15.** $\frac{-3}{2}$ **16.** 2, -2 , -6 and 5 **17.** $\sqrt{2}, -\sqrt{2} - 5$ and $\frac{3}{2}$

18. a = 1, b = 2 **19.** p = 2, q = 3 **20.** $-\sqrt{2}, \sqrt{2}$ and -3 **21.** $-\sqrt{3}, \sqrt{3}$ and $-\frac{1}{2}$ **22.** a = - 20, b = - 25

EXERCISE – 4 (FOR OLYMPIADS)

Choose The Correct One

1. If α, β and γ are the zeros of the polynomial $2x^3 - 6x^2 - 4x + 30$. then the value of $(\alpha\beta + \beta\gamma + \gamma\alpha)$ is
(A) – 2 (B) 2 (C) 5 (D) –30

2. If α, β and γ are the zeros of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} =$
(A) $-\frac{b}{a}$ (B) $\frac{c}{d}$ (C) $-\frac{c}{d}$ (D) $-\frac{c}{a}$

3. If α, β and γ are the zeros of the polynomial $f(x) = ax^3 - bx^2 + cx - d$, then $\alpha^2 + \beta^2 + \gamma^2 =$
(A) $\frac{b^2 - ac}{a^2}$ (B) $\frac{b^2 + 2ac}{b^2}$ (C) $\frac{b^2 - 2ac}{a}$ (D) $\frac{b^2 - 2ac}{a^2}$

4. If α, β and γ are the zeros of the polynomial $f(x) = x^3 + px^2 - pqrx + r$, then $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} =$
(A) $\frac{r}{p}$ (B) $\frac{p}{r}$ (C) $-\frac{p}{r}$ (D) $-\frac{r}{p}$

5. If the parabola $f(x) = ax^2 + bx + c$ passes through the points (-1, 12), (0, 5) and (2, -3) , the value of a + b + c is –
(A) - 4 (B) -2 (C) Zero (D) 1

6. If a, b are the zeros of $f(x) = x^2 + px + 1$ and c, d are the zeros of $f(x) = x^2 + qx + 1$ the value of
E = (a – c) (b – c) (a + b) (b + d) is –
(A) $p^2 - q^2$ (B) $q^2 - p^2$ (C) $q^2 + p^2$ (D) None of these

7. If α, β are zeros of $ax^2 + bx + c$ then zeros of $a^3x^2 + abcx + c^3$ are -
(A) $\alpha\beta, \alpha + \beta$ (B) $\alpha^2\beta, \alpha\beta^2$ (C) $\alpha\beta, \alpha^2\beta^2$ (D) α^3, β^3

8. Let α, β be the zeros of the polynomial $x^2 - px + r$ and $\frac{\alpha}{2}, 2\beta$ be the zeros of $x^2 - qx + r$, Then the value of r is –
(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$ (C) $\frac{2}{9}(q-2)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

9. When $x^{200} + 1$ is divided by $x^2 + 1$, the remainder is equal to –
(A) x + 2 (B) 2x – 1 (C) 2 (D) - 1

10. If $a(p+q)^2 + 2bpq + c = 0$ and also $a(q + r)^2 + 2bqr + c = 0$ then pr is equal to –
(A) $p^2 + \frac{a}{c}$ (B) $q^2 + \frac{c}{a}$ (C) $p^2 + \frac{a}{b}$ (D) $q^2 + \frac{a}{c}$

11. If a, b and c are not all equal and α and β be the zeros of the polynomial $ax^2 + bx + c$, then value of $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2)$ is :
(A) 0 (B) positive (C) negative (D) non-negative

12. Two complex number α and β are such that $\alpha + \beta = 2$ and $\alpha^4 + \beta^4 = 272$, then the polynomial whose zeros are α and β is –
(A) $x^2 - 2x - 16 = 0$ (B) $x^2 - 2x + 12 = 0$ (C) $x^2 - 2x - 8 = 0$ (D) None of theses

13. If 2 and 3 are the zeros of $f(x) = 2x^3 + mx^2 - 13x + n$, then the values of m and n are respectively –
(A) -5 , - 30 (B) -5, 30 (C) 5, 30 (D) 5, -30

14. If α, β are the zeros of the polynomial $6x^2 + 6px + p^2$, then the polynomial whose zeros are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ is –

(A) $3x^2 + 4p^2x + p^4$
(B) $3x^2 + 4p^2x - p^4$
(C) $3x^2 - 4p^2x + p4$
(D) None of theses

15. If c, d are zeros of $x^2 - 10ax - 11b$ and a, b are zeros of $x^2 - 10cx - 11d$, then value of a + b + c + d is –

(A) 1210
(B) -1
(C) 2530
(D) -11

16. If the ratio of the roots of polynomial $x^2 + bx + c$ is the same as that of the ratio of the roots of $x^2 + qx + r$, then –

(A) $br^2 = qc^2$
(B) $cq^2 = rb^2$
(C) $q^2c^2 = b^2r^2$
(D) $bq = rc$

17. The value of p for which the sum of the squares of the roots of the polynomial $x^2 - (p-2)x - p - 1$ assume the least value is -

(A) -1
(B) 1
(C) 0
(D) 2

18. If the roots of the polynomial $ax^2 + bx + c$ are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$ then the value of $(a + b + c)^2$ is-

(A) $b^2 - 2ac$
(B) $b^2 - 4ac$
(C) $2b^2 - ac$
(D) $4b^2 - 2ac$

19. If α, β and γ are the zeros of the polynomial $x^3 + a_0x^2 + a_1x + a_2$, then $(1-\alpha^2)(1-\beta^2)(1-\gamma)$ is

(A) $(1 - a_1)^2 + (a_0 - a_2)^2$
(B) $(1 + a_1)^2 - (a_0 + a_2)^2$
(C) $(1 + a_1)^2 + (a_0 + a_2)^2$
(D) None of these

20. If α, β, γ are the zeros of the polynomial $x^3 - 3x + 11$, then the polynomial whose zeros are $(\alpha+\beta)(\beta+\gamma)$ and $(\gamma+\beta)$ is –

(A) $x^3 + 3x + 11$
(B) $x^3 - 3x + 11$
(C) $x^3 + 3x - 11$
(D) $x^3 - 3x - 11$

21. If α, β, γ are such that $\alpha+\beta+\gamma = 2$, $\alpha^2+\gamma^2 = 6$, $\alpha^3+\beta^3+\gamma^3 = 8$, then $\alpha^4+\beta^4+\gamma^4$ is equal to –

(A) 10
(B) 12
(C) 18
(D) None of these

22. If α, β are the roots of $ax^2 + bx + c$ and $\alpha + k$, $\beta + k$ are the roots of $px^2 + qx + r$, then k =

(A) $-\frac{1}{2}\left[\frac{a}{b} - \frac{p}{q}\right]$
(B) $\left[\frac{a}{b} - \frac{p}{q}\right]$
(C) $\frac{1}{2}\left[\frac{b}{a} - \frac{q}{p}\right]$
(D) (ab – pq)

23. If α, β are the roots of the polynomial $x^2 - px + q$, then the quadratic polynomial, the roots of which are $(\alpha^2-\beta^2)(\alpha^3-\beta^3)$ and $\alpha^3\beta^2 + \alpha^2\beta^3$:

(A) $px^2 - (5p + 7q)x - (p^6q^6 + 4p^2q^6) = 0$
(B) $x^2 - (p^5 - 5p^3q + 5pq^2)x + (p^6q^2 - 5p^4q^3 + 4p^2q^4) = 0$
(C) $x^2 - (p^3q - 5p^5 + p^4q) - (p^6q^2 - 5p^2q^6) = 0$
(D) All of the above

24. The condition that $x^3 - ax^2 + bx - c = 0$ may have two of the roots equal to each other but of opposite signs is :

(A) ab = c
(B) $\frac{2}{3}a = bc$
(C) $a^2b = c$
(D) None of these

25. If the roots of polynomial $x^2 + bx + ac$ are α, β and roots of the polynomial $x^2 + ax + bc$ are α, γ then the values of α, β, γ respectively are –

(A) a,b,c
(B) b,c,a
(C) c,a,b
(D) None of these

26. If one zero of the polynomial $ax^2 + bx + c$ is positive and the other negative then (a, b, c $\in$ R, a $\neq$ 0)

(A) a and b are of opposite signs.
(B) a and c are of opposite signs.
(C) b and c are of opposite signs.
(D) a,b,c are all of the same sign.

27. If α, β are the zeros of the polynomial $x^2 - px + q$. then $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$ is equal to -

(A) $\frac{p^4}{q^2} + 2 - \frac{4p^2}{q}$
(B) $\frac{p^4}{q^2} - 2 + \frac{4p^2}{q}$
(C) $\frac{p^4}{q^2} + 2q - \frac{4p^2}{q}$
(D) None of these

28. If α, β are the zeros of the polynomial $x^2 - px + 36$ and $\alpha^2 + \beta^2 = 9$, then p =

(A) ± 6
(B) $\pm$ 3
(C) $\pm$ 8
(D) ± 9

29. If α, β are zeros of $ax^2 + bx + c$, $ac \neq 0$, then zeros of $cx^2 + bx + a$ are –

(A) $-\alpha, \ -\beta$
(B) $\alpha, \frac{1}{\beta}$
(C) $\beta, \frac{1}{\alpha}$
(D) $\frac{1}{\alpha}, \frac{1}{\beta}$

30. A real number is said to be algebraic if it satisfies a polynomial equation with integral coefficients. Which of the following numbers is not algebraic :

(A) $\frac{2}{3}$
(B) $\sqrt{2}$
(C) 0
(D) π

31. The bi-quadratic polynomial whose zeros are $1, \ 2, \ \frac{4}{3}, -1$ is :

(A) $3x^4 - 10x^3 + 5x^2 + 10x - 8$
(B) $3x^4 + 10x^3 - 5x^2 + 10x - 8$
(C) $3x^4 + 10x^3 + 5x^2 - 10x - 8$
(D) $3x^4 - 10x^3 - 5x^2 + 10x - 8$

32. The cubic polynomials whose zeros are $4, \frac{3}{2}$ and -2 is :

(A) $2x^3 + 7x^2 + 10x - 24$
(B) $2x^3 + 7x^2 - 10x - 24$
(C) $2x^3 - 7x^2 - 10x + 24$
(D) None of these

33. If the sum of zeros of the polynomial $p(x) = kx^3 - 5x^2 - 11x - 3$ is 2, then k is equal to :

(A) $k = -\frac{5}{2}$
(B) $k = \frac{2}{5}$
(C) k = 10
(D) $k = \frac{5}{2}$

34. If $f(x) = 4x^3 - 6x^2 + 5x - 1$ and α, β and γ are its zeros , then $\alpha\beta\gamma =$

(A) $\frac{3}{2}$
(B) $\frac{5}{4}$
(C) $-\frac{3}{2}$
(D) $\frac{1}{4}$

35. Consider $f(x) = 8x^4 - 2x^2 + 6x - 5$ and $\alpha, \beta, \gamma . \delta$ are it's zeros then $\alpha + \beta + \gamma + \delta =$

(A) $\frac{1}{4}$
(B) $-\frac{1}{4}$
(C) $-\frac{3}{2}$
(D) None of these

36. If $x^2 - ax + b = 0$ and $x^2 = px + q = 0$ have a root in common and the second equation has equal roots, then –

(A) b + q = 2ap
(B) $b + q = \frac{ap}{2}$
(C) b + q = ap
(D) None of these

OBJECTIVE — ANSWER KEY — EXERCISE - 4

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	D	B	C	B	B	D	C	B	D	C	B	C	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	B	B	B	D	C	C	B	A	C	B	A	D	D	D
Que.	31	32	33	34	35	36									
Ans.	A	C	D	D	D	B									

EXERCISE - 5 (FOR IIT-JEE/AIEEE)

Choose The Correct One

1. If the sum of the two zeros of $x^3 + px^2 + qx + r$ is zero, then pq = **[EAMCET - 2003]**

(A) – r (B) r (C) 2r (D) –2r

2. Let $a \neq 0$ and p(x) be a polynomial of degree grater than 2. If p(x) leaves remainders a and – a when divided respectively by x + a and x – a , the remainder when p(x) is divided by $x^2 - a^2$ is **[EAMCET - 2003]**

(A) 2x (B) – 2x (C) x (D) – x

3. If one root of the polynomial $x^2 + px + q$ is square of the other root, then **[IIT-Screening - 2003]**

(A) $p^3 - q(3p - 1) + q^2 = 0$ (B) $p^3 - q(3p + 1) + q^2 = 0$

(C) $p^3 + q(3p - 1) - q^2 = 0$ (D) $p^3 + q(3p + 1) - q^2 = 0$

4. If α, β are the zeros of $x^2 + px + 1$ and γ, δ be those of $x^2 + qx + 1$, then the value of $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) =$ **[DCE-2000]**

(A) $p^2 - q^2$ (B) $q^2 - p^2$ (C) p^2 (D) q^2

5. The quadratic polynomial whose zeros are twice the zeros of $2x^2 - 5x + 2 = 0$ is – **[Kerala Engineering -2003]**

(A) $8x^2 - 10x + 2$ (B) $x^2 - 5x + 4$ (C) $2x^2 - 5x + 2$ (D) $x^2 - 10x + 6$

6. The coefficient of x in $x^2 + px + q$ was taken as 17 in place of 13 and it's zeros were found to be – 2 and – 15. The zeros of the original polynomial are - **[Kerala Engineering -2003]**

(A) 3, 7 (B) – 3 , 7 (C) – 3 , – 7 (D) – 3, – 10

7. If $\alpha + \beta = 4$ and $\alpha^2 + \beta^2 = 44$, then α, β are the zeros of the polynomial . **[Kerala Engineering -2003]**

(A) $2x^2 - 7x + 6$ (B) $3x^2 + 9x + 11$ (C) $9x^2 - 27x + 20$ (D) $3x^2 - 12x + 5$

8. If α, β, γ are the zeros of the polynomial $x^3 + 4x + 1$, then $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1} =$

(A) 2 (B) 3 (C) 4 (D) 5 **[EAMCET-2003]**

9. If α, β are the zeros of the quadratic polynomial $4x^2 - 4x + 1$, then $\alpha^3 + \beta^3$ is –

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) 16 (D) 32

10. The value of 'a', for which one root of the quadratic polynomial $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2$ is twice as large as the other, is - **[AIEEE -2003]**

(A) $-\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) $\frac{1}{3}$

11. Let α, β be the zeros of $x^2 + (2 - \lambda)x - \lambda$. The values of λ for which $\alpha^2 + \beta^2$ is minimum is –

(A) 0 (B) 1 (C) 2 (D) 3 **[AMU-2002]**

12. If 1 + 2i is a zero of the polynomial $x^2 + bx + c$, b, c $\in$ R , then (b, c) is given by –

(A) (2. – 5) (B) (- 3, 1) (C) (-2, 5) (D) (3, 1)

13. If 2 + i is a zero of the polynomial $x^3 - 5x^2 + 9x - 5$, the other zeros are –

(A) 1 and 2 – i (B) – 1 and 3 + i (C) 0 and 1 (D) None of these

14. The value of λ for which one zero of $3x^2 - (1 + 4\lambda)x + \lambda^2 + 2$ may be one-third of the other is –

(A) 4 (B) $\frac{33}{8}$ (C) $\frac{17}{4}$ (D) $\frac{31}{8}$

15. If 1 – i is a zero of the polynomial $x^2 + ax + b$, then the values of a and b are respectively .

(A) 2, 1 (B) – 2 , 2 **[Tamil Nadu Engineering 2002]**

(C) 2, 2 (D) 2, - 2

16. If the sum of the zeros of the polynomial $x^2 + px + q$ is equal to the sum of their squares, then –

(A) $P^2 - q^2 = 0$ (B) $p^2 + q^2 = 0$ (C) $p^2 + p = 2q$ (D) None of these

17. Let α, β be the zeros of the polynomial (x – a) (x – b) – c with $c \neq 0$. then the zeros of the polynomial $(x-\alpha)(x-\beta)$ + c are : **[IIT-1992, AIEEE - 2002]**

(A) a, c (B) b, c (C) a, b (D) a + c, b + c

18. If p, q are zeros of $x^2 + px + q$. then **[AIEEE - 2002]**

(A) p = 1 (B) p = 1 or 0 (C) p = - 2 (D) p = - 2 or 0

19. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3, \beta^2 = 5\beta - 3,$ then the polynomial whose zeros are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is : **[AIEEE - 2002]**

(A) $3x^2 - 25x + 3$ (B) $x^2 - 5x + 3$

(C) x2+5x – 3 (D) $3x^2 - 19x + 3$

20. If $\alpha \neq \beta$ and the difference between the roots of the polynomials $x^2 + ax + b$ and $x^2 + bx + a$ is the same, then **[AIEEE - 2002]**

(A) a + b + 4 = 0 (B) a + b – 4 = 0 (C) a – b + 4 = 0 (D) a – b – 4 = 0

21. If the zeros of the polynomial $ax^2 + bx + c$ be in the ratio m : n, then

(A) $b^2 mn = (m^2 + n^2) ac$ (B) $(m + n)^2 ac = b^2 mn$

(C) $b^2 (m^2 + n^2) = mnac$ (D) None of these

COMPREHENSION BASED QUESTIONS

Maximum and Minimum value of a quadratic expression :

At $x = \frac{-b}{2a}$, we get the maximum or minimum value of the quadratic expression, $y = ax^2 + bx + c$

(i) When a > 0, the expression $ax^2 + bx + c$ gives minimum value $= \frac{4ac - b^2}{4a}$

(ii) When a < 0, the expression $ax^2 + bx + c$ gives maximum value $= \frac{4ac - b^2}{4a}$

Based on above information, do the following questions :

22. The minimum value of the expression $4x^2 + 2x + 1$ $(x \in R)$ is -

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

23. If x be real , the maximum value of $7 + 10x - 5x^2$ is –

(A) 12 (B) 15 (C) 16 (D) 18

24. If p and q $(\neq 0)$ are the zeros of the polynomial $x^2 + px + q$, then the least value of $x^2 + px + q$ $(x \in R)$ is –

(A) $-\frac{1}{4}$ (B) $\frac{1}{4}$ (C) $-\frac{9}{4}$ (D) $\frac{9}{4}$

25. If x is real, the minimum value of $x^2 - 8x + 17$ is –

(A) – 1 (B) 0 (C) 1 (D) 2

OBJECTIVE — ANSWER KEY — EXERCISE - 5

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	A	B	B	D	D	C	A	B	B	C	A	D	B
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	C	C	B	D	A	B	C	A	C	C					

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

★ INTRODUCTION

In class IX, we have read about linear equations in two variables. A linear equation is a rational and integral equating of the first degree.

For example, the equations : 3x + 2y = 7, $2x - \sqrt{3}y = \sqrt{5}$, $y - 4x = \sqrt{3}$ are linear equations in two variables, since in each case

(i) Neither x nor y is under a radical sign i.e., x and y rational.

(ii) Neither x nor y in the denominator.

(ii) The exponent f x and y in each term is one.

In general, ax + by + c = 0 : a, b, c $\in$ R ; $a \neq 0$ and $b \neq 0$ is a linear equation in two variables. A linear equation in two variables has an infinite number of solutions. The graph of a linear equation in two variables is always a straight line . In this chapter, we shall study about systems of linear equations in two variables, solution of system of linear equations in two variables and graphical and algebraic methods of solving a system of linear equations in two variables. In the end of the chapter, we shall be discussing some applications of linear equations in two variables in simple problems areas.

★ HISTORICAL FACTS

Diophantus, the last genius of Alexandria and the best algebraic mathematician of the Greek-Roman Era, has made a unique contribution in the development of Algebra and history of mathematics. He was born in the 3rd century and lived for 84 years. Regarding his age it has been told in VINODIKA of Greek collections.

"He spent one-sixth of his life in childhood, his beard grew after one twelfth more, after another one-seventh he married, five years later his son was born, the son lived to half the father's age, and Diophantus died four years after his son."

i.e. $\frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + \frac{x}{2} + 4 = x \Rightarrow 9x = 756 \Rightarrow x = 84$ Years.

He was known as the father of Algebra. Arithimetica is his famous book.

★ RECALL

(i) Equation : An statement of equality of two algebraic expressions which involve one or more unknown quantities is known as an equation.

(ii) Linear Equation : An equation in which the maximum power of variable is one is called a linear equation.

(iii) Linear Equation in One Variable : An equation of the form ax + b = 0 where x is a variable, a, b are real number and $a \neq 0$ is called a linear equation in one variable.

(iv) Linear Equation in Two Variables : An equation of the form ax + by + c = 0 , where a, b, c are real number , $a \neq 0$, $b \neq 0$ and x, y are variables is called linear equation in two variables.

Any pair values of x & y which satisfies the equation ax + by + c = 0 is called a root or solution it.

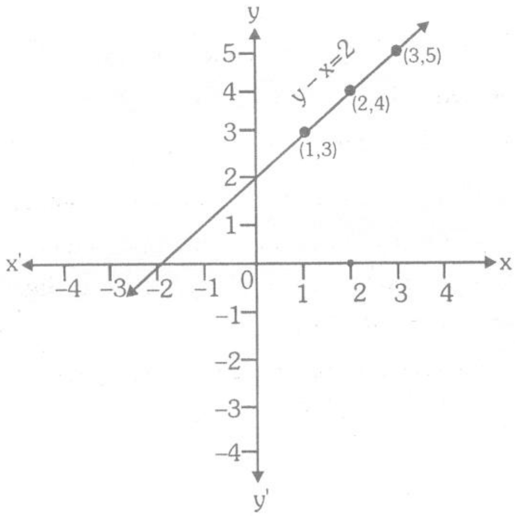
Ex. (x = 1, y = 1) is a solution of 4x – y – 3 = 0 .

Remark : A linear equation in two variables have infinite number of solutions.

(v) Graph of a Linear Equation in two Variables : Assume y – x = 2 be a linear equation in two variables. The following table exhibits the abscissa and ordinates of points on the line represented by the equation y – x = 2

x	1	2	3
y	3	4	5

Plotting the points (1, 3), (2, 4) and (3, 5) one the graph paper and drawing the line joining them we obtain the graph of line represented by the given equation as shown in fig.



★ **SIMULTANEOUS LINEAR EQUATIONS IN TWO RABBLES**

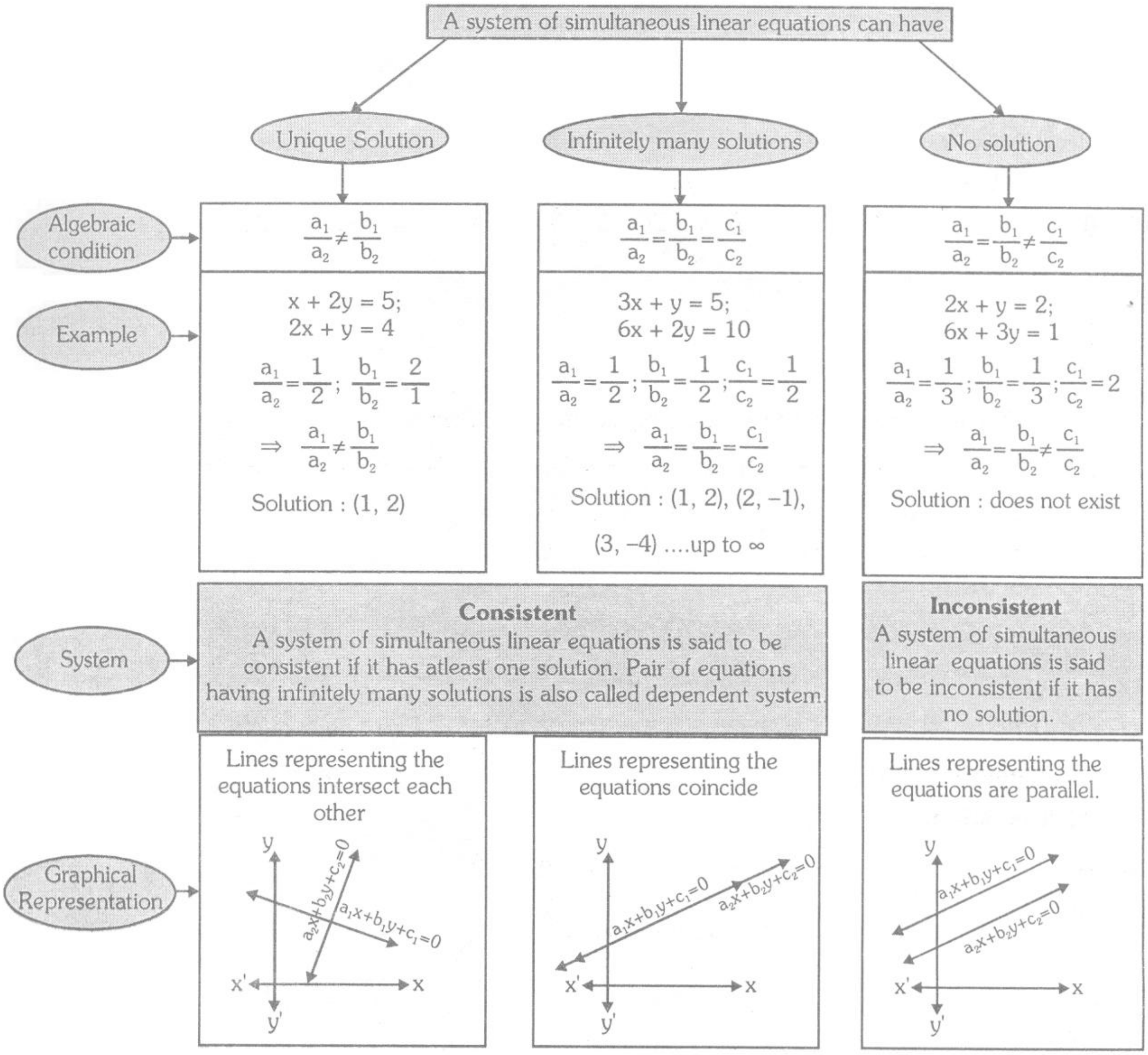
A pair of linear equations in two variables is said to form a system of simultaneous linear equations.
General Form : $a_1x + b_1y\ c_1 = 0$
and $a_2x + b_2y + c_2 = 0$, where a_1, a_2, b_1, b_2, c_1 and c_2 are era number ; $a_1^2 + b_1^2 \neq 0$ and $a_2^2 + b_2^2 \neq 0$ and x, y are variables.
Ex. Each of the following pairs of linear equations form a system of two simultaneous linear equations in two variables.
(i) $x – 2y = 3,\ 2x + 5y = 5$ (ii) $3x+5y+7=0,\ 5x + 2y + 9 = 0$

★ **SOLUTION OF THE SYSTEM OF EQUATIONS**

Consider the system of simultaneous linear equations : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

A pair of value of the variables x and y satisfying each one of the equations in a given system of two simultaneous linear equations in x and y is called a solution of the system .

Ex. x = 2, y = 3 is a solution of the system of simultaneous linear equations.
$2x + y = 7$, $3x + 2y = 12$
The given equations are $2x + y = 7$ ……(i)
$3x + 2y = 12$ ……(ii)
Put x = 2, y = 3 in LHS of equation (i), we get
LHS = $2 \times 2 + 3 = 7$ = RHS
Put x = 2, y = 3 in LHS of equation (ii), we get
LHS = $3 \times 2 + 2 \times 3 = 12$ = RHS
The value x = 2, y = 3 satisfy both equations (i) and (ii).



Hence x = 2, y = 3 is a solution of the given system. **Remark :** An equation involving two variables cannot give value of both the variables. For values of both the variables we required two equations. Similarly for three variables we require three equations and so on, i.e. to find n variables we need n equates.

★ **HOMOGENEOUS SYSTEM OF EQUATIONS**

A system of simultaneous equations is said to be homogenous, if all of the constant terms are zero.

General Form : $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$

Homogeneous equation of the form ax + by = 0 is a line passing through the origin.

Therefore , the system is always consistent.

(i) When $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ the system of equation has only one solution.

(ii) When $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ the system of equation has infinitely many solutions.

Ex.1 On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following points of linear equations are consistent of inconsistent.

(i) 3x + 2y = 5, 2x – 3y = 7 **(ii) 2x – 3y = 8, 4x – 6y = 9**

Sol. **(i)** We have, $3x + 2y = 5 \Rightarrow 3x + 2y - 5 = 0$ and $2x - 3y = 7 \Rightarrow 2x - 3y - 7 = 0$

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3} \text{ and } \frac{c_1}{c_2} = \frac{5}{7}$$

$$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, the given pair of linear equations is consistent.

(ii) We have, $2x - 3y = 8 \Rightarrow 2x - 8 = 0$ and $4x - 6y = 9 \Rightarrow 4x - 6y - 9 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given pair of linear equations is inconsistent .

Ex.2 **For what value of k, the system of equations x + 2y = 5, 3x + ky + 15 = 0 has**
(i) a unique solution
(ii) No solution ?

Sol. We have , $x + 2y = 5 \Rightarrow x + 2y - 5 = 0$ and $3x + kt + 15 = 0$.

(i) The required condition for unique solution is : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\therefore \quad \frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$$

Hence, for all real values of k except 6, the given system of equations will have a unique solution.

(ii) The required condition for no solution is : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\therefore \quad \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15} \Rightarrow \frac{1}{3} = \frac{2}{k} \text{ and } \frac{2}{k} \neq \frac{-5}{15}$$

$$\Rightarrow \quad k = 6 \text{ and } \frac{2}{k} \neq \frac{-1}{3} \Rightarrow k = 6 \text{ and } k \neq -6$$

Hence the given system of equations will have no solution when k = 6.

Ex.3 **Find the value of k for which the system of equations 4x + 5y = 0, kx + 10y = 0 has infinitely many solution .**

Sol. The given system is of the form $a_1x + b_1y = 0$, $a_2x + b_2y = 0$

$a_1 = 4$, $a_2 = k$, $b_1 = 5$, $b_2 = 10$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, the system has infinitely many solutions.

$$\frac{4}{k} = \frac{5}{10} \Rightarrow k = 8$$

Ex.4 **Find the value of a and b for which the given system of equations has an infinite number of solutions :**
2x + 3y = 7 ; (a + b + 1) x + (a + 2b + 2) y = 4 (a + b) + 1

Sol. We have $2x + 3y = 7 \Rightarrow 2x + 3y - 7 = 0$

and $(a + b + 1) x + (a + 2b + 2) y = 4 (a + b) + 1$

$\Rightarrow$ (a + b + 1) x + (a + 2b + 2) y – {4 (a + b) + 1} = 0

The required condition for an infinite number of solutions is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \quad \frac{2}{a+b+1} = \frac{3}{a+2b+2} = \frac{-7}{-\{4(a+b)+1\}}$

$\Rightarrow \quad \frac{2}{a+b+1} = \frac{3}{a+2b+2}$ and $\frac{3}{a+2b+2} = \frac{7}{4(a+b)+1}$

$\Rightarrow$ 2a + 4b + 4 = 3a + 3b + 3 and 12a + 12b + 3 = 7a + 14b + 14

$\Rightarrow$ a – b – 1 = 0 and 5a – 2b = 11

$\Rightarrow$ a – b = 1 …(i)

and 5a – 2b = 11 …(ii)

Multiplying (i) by 2 we get 2a – 2b = 2 …(iii)

Subtracting (iii) from (ii) we get 3a = $\Rightarrow$ a = $\frac{9}{3}$ = 3

Put a = 3 in (i), we get 3 – b = $\Rightarrow$ b = 2

Hence, the given system of equations will have infinite number of solutions when a = 3 and b = 2.

★ GRAPHICAL METHOD OF SOLVING A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

To solve a system of two linear equations graphically,

(i) Draw graph of he first equation.

(ii) On the same pair of axes, draw graph of the second equation.

(iii)(a) If the two lines intersect at a point, read the coordinates of the point of intersection of obtain the solution and verify your answer.

(b) If the two lines are parallel, there is no point of intersection, write the system as inconsistent. Hence , no solution

(c) If the two lines have the same graph, then write the system as consistent with infinite number of solutions

Ex.5 Which of the following pairs of linear equations are consistent / inconsistent ? If consistent, obtain the solution graphically.

(i) x + 2y – 3 = 0, 4x + 3y = 2 (ii) 3x + y = 1, 2y = 2 – 6x (iii) 2x – y = 2, 2y – 4x = 2

Sol. (i) x + 2y – 3 = 0 $\Rightarrow$ y = $\frac{3-x}{2}$

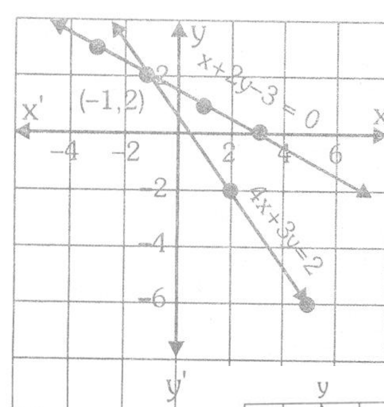
x	1	2	-3
y	1	0	3

Points are (1, 1), (3, 0), (-3, 3)

4x + 3y $\Rightarrow$ y = $\frac{2-4x}{3}$

x	2	-1	5
y	-2	2	-6

points are (2, -2), (-1, 2), (5, -6)



From the graph, we see that the two lines intersect at a point (-1, 2)

So the solution of the pair of linear equations is x = - 1, y = 2

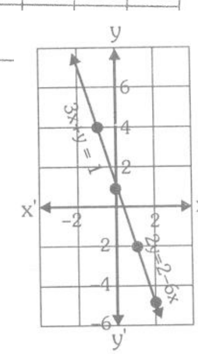
i.e., the given pair of equations is consistent.

(ii) 3x + y = 1$\Rightarrow$ y = 1 – 3x

x	0	1	2
y	1	-2	-5

2y = 2 – 6x $\Rightarrow$ y = $\frac{2-6x}{2}$

X	-1	1	-2
y	4	-2	7



Points are (0, 1), (1, -2), (2, -5)   Points are (-1, 4), (1, -2), (-2, 7)

The two equations have the same graph. Thus system is consistent with infinite number of solutions, i.e., the system is dependent.

(iii) $2x - y = 2 \Rightarrow y = 2x - 2$

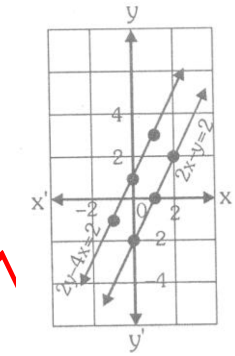
x	0	1	2
y	-2	0	2

Points are (0, -2), (1, 0), (2, 2)

$2y - 4x = 2 \Rightarrow y = \dfrac{4x+2}{2}$

x	0	1	-1
y	1	3	-1

Points are (0, 1), (1, 3), (-1, -1)



The graph of the system consists of two parallel lines. Thus, the system is inconsistent. It has no solution.

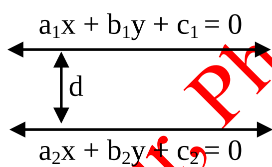
COMPETITION WINDOW

DISTANCE BETWEEN TWO PARALLEL LINES

Consider pair of parallel lines

$a_1x + b_1y + c_1 = 0$ …(i)

$a_2x + b_2y + c_2 = 0$ …(ii)



∵ The lines are parallel .

∴ $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = k\,(\text{say}) \Rightarrow a_1 = a_2 k \;\&\; b_1 = b_2 k$

Putting these values in (i), we get : $a_2kx + b_2ky + c_1 = 0$ or $a_2x + b_2y + \dfrac{c_1}{k} = 0$

or $a_2x + b_2y + c_3 = 0$ …(iii) $\left[c_3 = \dfrac{c_1}{k}\right]$

Clearly in equation (ii) and (iii), coefficients of x and y are same but the constant term is different in both the equations. The perpendicular distance (d) between the two lines can be calculated by using the following formula :

$$d = \frac{|c_2 - c_3|}{\sqrt{a_2^2 + b_2^2}}$$

e.g, The distance between the parallel liens 3x – 4y 9 = 0 and 6x – 8y – 15 = 0 can be calculated as follows :

$3x - 4y + 9 = 0$ …(i), $6x - 8y - 15 = 0$ or $3x - 4y - \dfrac{15}{2} = 0$ …(ii)

Required perpendicular distance, $d = \dfrac{\left|9 - \left(-\dfrac{15}{2}\right)\right|}{\sqrt{(3)^2 + (4)^2}} = \dfrac{\left|9 + \dfrac{15}{2}\right|}{\sqrt{25}} = \dfrac{33}{10}$

★ ALGEBRAIC METHOD OF SOLVING A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Some times, graphical number does not given an accurate answer. While reading the co-ordinate of a point on a graph paper we are likely to make an error. So we require some precise method to obtain accurate result. The algebraic methods are given below :

(i) Method of elimination by substitution.

(ii) Method of elimination by equating the coefficients.

(iii) Method of cross multiplication.

★ ALGEBRAIC SOLUTION BY SUBSTITUTION METHOD

To solve a pair linear equations in two variables x and y by substitution method, we follow the following steps :

Step – I : Write the given equations

$a_1x + b_1y + c_1 = 0$ ..(i)

and $a_2x + b_2y + c_2 = 0$ ...(ii)

Step –II : Choose one of the two equations and express y in terms of x (or x in terms of y), i.e. express, one variable in terms of the other.

Step –III : Substitute this value of y obtained in step-II, in the other equation to get a linear equation in x.

Step-IV : Solve the linear equation obtained in step-III and get the value of x.

Step-V : Substitute this value of x in the relation obtained in step-II and find the value of y.

Ex.6 Solve for x and y : 4x + 3y = 24, 3y – 2x = 6.

Sol. $4x + 3y = 24$ …(i)

$3y - 2x = 6$ …(ii)

From equation (i), we get

$$y = \frac{24-4x}{3} \quad \text{…(iii)}$$

Substituting in equation (ii), we get

$$3\left(\frac{24-4x}{3}\right) - 2x = 6$$

$\Rightarrow 24 - 4x - 2x = 6$

$\Rightarrow -6x = -24 + 6$

$\Rightarrow 6x = 18$

$\Rightarrow x = 3$

Substituting x = 3 in (iii), we get

$$\Rightarrow y = \frac{24-12}{3}$$

$$\Rightarrow \frac{12}{3} = 4$$

Hence , x + 3, y = 4

Ex.7 Solve the following pair of linear equations by the substitution method.

$\sqrt{2}x + \sqrt{3}y = 0$ **and** $\sqrt{3}x - \sqrt{8}y = 0$

Sol. We have,

$\sqrt{2}x + \sqrt{3}y = 0$ …(i)

and $\sqrt{3}x - \sqrt{8}y = 0$ …(ii)

From (i), we get $y = \frac{-\sqrt{2}x}{\sqrt{3}}$ ...(iii)

Substituting $y = \frac{-\sqrt{2}x}{\sqrt{3}}$ in (ii), we get $\sqrt{3}x - \sqrt{8}\left(\frac{-\sqrt{2}x}{\sqrt{3}}\right) = 0$

$\Rightarrow \quad \sqrt{3}x + \frac{4x}{\sqrt{3}} = 0 \Rightarrow 3x + 4y = 0 \Rightarrow 7x = 0 \Rightarrow x = 0$

Substituting x = 0 in (iii), we get $y = \frac{-\sqrt{2} \times 0}{\sqrt{3}} = 0$

Hence, the solution is x = 0 and y = 0.

★ **ALGEBRAIC SOLUTION BY ELIMINATION METHOD**

To solve a pair of linear equations x and y by elimination method, we follow the following steps :

Step-I : Write the given equation

$a_1x + b_1y + c_1 = 0$ ...(i)

and $a_2x + b_2y + c_2 = 0$ ...(ii)

Step-II : Multiply the given equations by suitable numbers so that the coefficient of one of the variables are numerically equal .

Stpe-III : If the numerically equal coefficients are opposite in sign , then add the new equations otherwise subtract

Step-IV : Solve the linear equations in one variable obtained in step–III and get the value of one variable .

Step-V : Substitute this value of the variable obtained in step-IV in any of the two equations and find the value of the other variable.

Ex.8 Solve the following pair of linear equations by elimination method : 3x + 4y = 10 and 2x – 2y = 2.

Sol. We have, 3x + 4y = 10 ...(i)

and 2x – 2y = 2 ...(ii)

Multiplying (ii) by 2, we get 4x – 4y = 4 ...(iii)

Adding (i) and (iii), we get 7x = 14 ⇒ x = 2

Putting x = 2 in equation (ii), we get 2 × 2 – 2y = 2 ⇒ y = 1

Hence, the solution is x = 2 and y = 1.

Ex.9 Solve : ax + by = c, bx + ay = 1 + c

Sol. ax + by = c ...(i)

bx + ay = 1 + c ...(ii)

Adding (i) and (ii), we get

(a + b) x + (a + b) y = 2c + 1

$\Rightarrow \quad x + y = \frac{2c+1}{a+b}$ ...(iii)

Subtracting (ii) and (i), we get

(a – b) x – (a – b) y = – 1

$$\Rightarrow \quad x - y = \frac{-1}{a-b} \qquad \ldots\text{(iv)}$$

Adding (iii) and (iv), we get

$$2x = \frac{2x+1}{a+b0} - \frac{-1}{a-b} = \frac{2ac-2bc+a-b-a-b}{a^2-b^2}$$

$$\Rightarrow \quad 2x = \frac{2ac-2bc-2b}{a^2-b^2}$$

$$\Rightarrow \quad x = \frac{ac-bc-b}{a^2-b^2}$$

Subtracting (iv) from (iii) we get

$$2y = \frac{2c+1}{a+b} + \frac{-1}{a-b} = \frac{2ac-2bc+a-b+a+b}{a^2-b^2}$$

$$\Rightarrow \quad 2y = \frac{2ac-2bc+2a}{a^2-b^2}$$

$$\Rightarrow \quad y = \frac{ac-bc+a}{a^2-b^2}$$

Hence , $x = \frac{ac-bc-b}{a^2-b^2}, y = \frac{ac-bc+a}{a^2-b^2}$

★ ALGEBRAIC SOLUTIONS BY CROSS-MULTIPLICATION METHOD

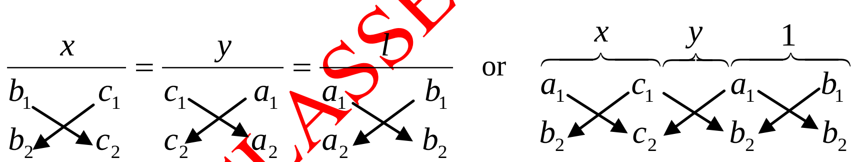
Consider the system of linear equations

$a_1x + b_1y + c_1 = 0$ …(i)

$a_2x + b_2y + c_2 = 0$ …(ii)

To solve it by cross multiplication method , we follow the following steps :

Step-I : Write the coefficients as follows :



The arrows between the two numbers indicate that they are to be multiplied. The products with upward arrows are to be subtracted from the products with downward arrows.

To apply above formula, all the terms must be in left to the equal sign in he system of equations –

Now, by above mentioned rule, equation (i) reduces to

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Case-I : If $a_1b_2 - a_2b_1 \neq 0 \Rightarrow$ x and y have some finite value, with unique solution for the system of equations.

Case-II : If $a_1b_2 - a_2b_1 = 0 \Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$

Here two cases arise :

(a) If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} = \lambda(\lambda \neq 0)$

Then $a_1 = a_2\lambda$, $b_1 = b_2\lambda$, $c_1 = c_2\lambda$

Put these values in equation $a_1x + b_1y + c_1 = 0$ …(i)

$\Rightarrow$ $a_2\lambda x + b_2\lambda + c_2\lambda = 0$

$\Rightarrow$ $\lambda(a_2x + b_2y + c_2) = 0$ but $\lambda \neq 0$

$\Rightarrow$ $a_2x + b_2y + c_2 = 0$ …(ii)

So (i) and (ii) are dependent, so there are infinite number of solutions.

(b) If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2} \Rightarrow a_1b_2 - b_1a_2 = 0$

But $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow$ $x = \dfrac{\text{Finite value}}{0}$ = does not exist

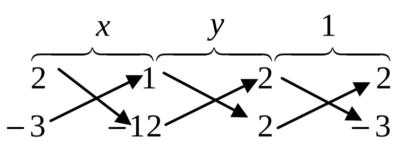
and $y = \dfrac{\text{Finite value}}{0}$ = does not exist

So system of equations is inconsistent.

Ex10 Solve by cross-multiplication method : x + 2y + 1 = 0 and 2x – 3y – 12 = 0

Sol. We have, x + 2y + 1 = 0 and 2x – 3y – 12 = 0

By cross-multiplication method , we have



$$\therefore \quad \frac{x}{2\times(-12)-(-3)\times 1} = \frac{y}{1\times 2-(-12)\times 1} = \frac{1}{1\times(-3)-2\times 2}$$

$$\Rightarrow \quad \frac{x}{-24+3} = \frac{y}{2+12} = \frac{1}{-3-4} \Rightarrow \frac{x}{-21} = \frac{y}{14} = \frac{1}{-7}$$

$$\Rightarrow \quad x = \frac{-21}{-7} = 3 \text{ and } y = \frac{14}{-7} = -2$$

Hence the solution is x = 3 and y = – 2 .

Ex.11 Solve by cross-multiplicaiton method : (a – b) x + (a + b) y = 2 $(a^2 – b^2)$, (a + b) x – (a – b) y = 4ab.

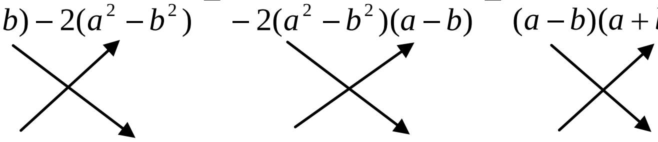
Sol. Writing the equations in the standard form, we get .

$(a – b) x + (a + b) y – (a^2 – b^2) = 0$

$(a + b) x – (a – b) y – 4ab = 0$

Applying the cross-multiplication method, we get

$$\frac{x}{(a+b)-2(a^2-b^2)} = \frac{y}{-2(a^2-b^2)(a-b)} = \frac{1}{(a-b)(a+b)}$$



$-(a-b) \quad -4ab \quad -4ab \quad (a+b) \quad (a+b)-(a-b)$

Simplification of the expression under x :

$-4ab(a+b)-2(a-b)(a^2-b^2)$

$= -2(a+b)[2ab+(a-b)^2]$

$= -2(a+b)(2ab+a^2+b^2-2ab)$

$= -2(a+b)(a^2+b^2)$

Simplification of the expression under y :

$-2(a^2-b^2)(a+b)+4ab(a-b)$

$= -2(a-b)[(a+b)(a+b)-2ab]$

$= -2(a-b)(a^2+b^2+2ab-2ab)$

$= -2(a-b)(a^2+b^2)$

Simplification of the expression under 1 :

$-(a-b)^2-(a+b)^2$

$= -(a^2+b^2-2ab)-(a^2+b^2+2ab)$

$= -2(a^2+b^2)$

Hence, $\dfrac{x}{-2(a+b)(a^2-b^2)} = \dfrac{y}{-2(a-b)(a^2-b^2)} = \dfrac{1}{-2(a^2+b^2)}$

$\Rightarrow \dfrac{x}{a+b} = \dfrac{y}{a-b} = \dfrac{1}{1}$

$\Rightarrow$ x = (a + b) and y = (a – b)

★ **EQUATIONS OF THE FORM ax + by = c AND bx + ay = d, WHERE a $\neq$ b.**

To solve the equations of the form. :

ax + by = c …(i)

and bx + ay = d …(ii)

where a $\neq$ b, we follow the following steps :

Step-I : Add (i) and (ii) and obtain $(a+b)x+(b+a)y = c+d$, i.e., $x+y = \dfrac{c+d}{a+b}$ …(iii)

Step-II : Subtract (ii) from (i) and obtain $(a-b)x-(a-b)y = c-d$, i.e., $x-y = \dfrac{c-d}{a-b}$ …(iv)

Step-III: Solve (iii) and (iv) to get x and y.

Ex.12 Solve for x and y : 47x + 31y = 63, 31x + 47y = 15.

Sol. We have,

47x + 31y = 63…(i) and 31x + 47y = 15…(ii)

Adding (i) and (ii), we get : 78x + 78y = 78 $\Rightarrow$ x + y = 1 …(iii)

Subtracting (ii) from (i), we get : 16x – 16y = 48 $\Rightarrow$ x – y = 3 …(iv)

Now , adding (iii) and (iv), we get : 2x = 4 $\Rightarrow$ x = 2

Putting x = 2 in (ii), we get : 2 + y = 1 $\Rightarrow$ y = - 1

Hence, the solution is x + 2 and y = - 1

★ **EQUATIONS REDUCIBLE TO LINER EQUATIONS IN TWO VARIABLES**

Equations which contain the variables, only in the denominators, are called reciprocal equations. These equations can be of the following types and can be solved by the under mentioned method :

Type-I : $\frac{a}{u}+\frac{b}{v}=c$ and $\frac{a'}{u}+\frac{b'}{v}=c'\forall a,b,c,a',b',c'\in R$

Put $\frac{1}{u}=x$ and $\frac{1}{y}=v$ and find the value of x and y by any method described earlier.

Then $u=\frac{1}{x}$ and $v=\frac{1}{y}$

Type-II : au +bv = cuv and a'u + b'v = c'uv ∀ a,b,c,a',b',c' ∈ R

Divide both equations by uv and equations can be converted in the form explained in (i).

Type-III : $\frac{a}{lx+my}+\frac{b}{cx+dy}=k,\frac{a'}{lk+my}+\frac{b'}{cx+dy}=k'\forall a,b,k,a',b',k'\in R$

Put $\frac{1}{lx+my}=u$ and $\frac{1}{cx+dy}=v$

Then equations are au + vc = k and a'u + b'v = k

Find the values of u and v and put in Ix + my = $\frac{1}{u}$ and cx + dy = $\frac{1}{v}$

Again solve for x and y, by any method explained earlier.

Ex.13 **Solve for x and y :** $\frac{3a}{x}-\frac{2b}{y}+5=0$ **and** $\frac{a}{x}+\frac{3b}{y}-2=0(x\neq 0,y\neq 0)$

Sol. We have, $\frac{3a}{x}-\frac{2b}{y}+5=0$ and $\frac{a}{x}+\frac{3b}{y}-2=0$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$. Then, the given equations can be written as

3au – 2bv = - 5…(i) and au +3bv = …(ii)

Multiplying (i) by 3 and (ii) by 2, we get

9au – 6bv = - 17 …(iii) and 2au + 6bv = 4…(iv)

Adding (iii) and (iv), we get 11au = - 11 $\Rightarrow u=\frac{-1}{a}$

Put $u=\frac{-1}{a}$ in equation (ii), we get a$\left(\frac{-1}{a}\right)+3bv=2\Rightarrow 3bv=3\Rightarrow v=\frac{1}{b}$

But $\frac{1}{x}=u$ and $\frac{1}{y}=v$

Therefore, $\frac{1}{a}=\frac{-1}{a}\Rightarrow x=-a$ and $\frac{1}{y}=\frac{1}{b}\Rightarrow y=b$ $\quad [\because u=\frac{-1}{a},v=\frac{1}{b}]$

Hence the solution is x = - a and y = b.

Ex.14 **Solve** $\frac{57}{x+y}+\frac{6}{x-y}=5$ **and** $\frac{38}{x+y}+\frac{21}{x-y}=9.$

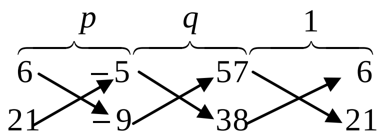
Sol. We have, $\frac{57}{x+y}+\frac{6}{x-y}=5\Rightarrow\frac{57}{x+y}+\frac{6}{x-y}-5=0$

and $\frac{38}{x+y}+\frac{21}{x-y}=9\Rightarrow\frac{38}{x+y}+\frac{21}{x-y}-9=0$

Let $\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$. Then. the given equations can be written as

57p + 6q – 5 = 0 and 38p + 21q – 9 = 0

By cross-multiplication method, we have



$\therefore \quad \dfrac{p}{6\times(-9)-21\times(-5)}=\dfrac{q}{(-5)\times 38-(-9)\times 57}=\dfrac{1}{57\times 21-38\times 6}$

$\Rightarrow \quad \dfrac{p}{-54+105}+\dfrac{q}{-190+513}=\dfrac{1}{1197-228}$

$\Rightarrow \quad \dfrac{p}{51}=\dfrac{q}{323}=\dfrac{1}{969}\Rightarrow p=\dfrac{51}{969}=\dfrac{1}{18}$ and $q=\dfrac{323}{969}=\dfrac{1}{3}$

But $\dfrac{1}{x+y}=p$ and $\dfrac{1}{x-y}=q$. therefore

$\dfrac{1}{x+y}=\dfrac{1}{19}\Rightarrow x+y=19$ …(i)

and $\dfrac{1}{x-y}=\dfrac{1}{3}\Rightarrow x-y=3$ …(ii)

adding (i) and (ii) , we get $2x = 22 \Rightarrow x = 11$

Put x = 11 in (i),we get $11 + y = 19 \Rightarrow y = 8$

Hence, the solution is x = 11 and y = 8.

Ex.15 Solve for x and y : $\dfrac{7x-2y}{xy}=5, \dfrac{8x+7y}{xy}=15.$

Sol. $\dfrac{7x-2y}{xy}=5, \Rightarrow \dfrac{7}{y}-\dfrac{2}{x}=5$ …(i)

$\dfrac{8x+7y}{xy}=15\Rightarrow \dfrac{8}{y}+\dfrac{7}{x}=15$ …(ii)

Putting $\dfrac{1}{y}=u$ and $\dfrac{1}{x}=v$, we get

7u – 2v = 5 …(iii)

8u 7v = 15 …(iv)

Multiplying (iii) by 7 and (iv) by 2 and adding we get

49u – 14v = 35

and 16u + 14v = 30

65u = 65 $\Rightarrow$ u = 1 $\Rightarrow \dfrac{1}{y}$ = 1 or y = 1

Substituting u = 1 in (iii) we get : 7 – 2v = 5 $\Rightarrow$ v = 1 $\Rightarrow \dfrac{1}{x}$ = 1 or x = 1

Hence, x = 1, y = 1.

★ APPLICATIONS OF LINEAR EQUATIONS IN TWO VARIABLES

In This section, we will study about some applications of simultaneous linear equations in solving variety of word problems related to our day-to-day life situations. The following examples are self-explanatory and will give some insight to the solution to such problems.

Type-I :Based on Articles And Their Costs / Quantities

Ex.16 7 audio cassettes and 3 video cassettes cost Rs. 1110, which 5 audio cassettes and 4 video cassettes cost Rs. 1350. Find the cost of an audio cassette and a video cassette.

Sol. Let the cost of an audio cassette and a video cassette be Rs. x and Rs. y respectively .

The cost of 7 audio cassettes and 3 video cassettes = Rs. 1110

⇒ $7x + 3y = 1110$ …(i)

The cost of 5 audio cassettes and 4 video cassettes = Rs. 1350

⇒ $5x + 4y = 1350$ …(ii)

Multiplying (i) by 4 and (ii) by 3, we get

$28x + 12y = 4440$ …(iii)

$15x + 12y = 4050$ ..(iv)

Subtracting (iv) from (iii), we get $13x = 390 \Rightarrow x = 30$

Putting x = 30 in (i), we get $7 \times 30 + 3y = 1110 \Rightarrow 210 + 3y = 1110$

⇒ $3y = 900 \Rightarrow y = 300$

Hence, the cost of an audio cassette is Rs. 30 and that of a video cassette is Rs. 300.

Type-II: Based on numbers

Ex.17 The sum of the digits of a two-digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. Find the number .

Sol. Let the digit at ten's place by x and that at unit's place be y. Then,

$x + y = 12$ …(i)

And , the two digits number = 10x + y

Now, according to the equation,

$(10y + x) = (10x + y) + 18 \Rightarrow 9y - 9x = 18 \Rightarrow y - x = 2$ …(ii)

Adding (i) and (ii), we get $2y = 14 \Rightarrow y = 7$

Put y = 7 in (i), we get $x + y = 12 \Rightarrow x = 5$

Hence, the enquired number is $(10 \times 5 + 7)$, i.e., 57.

Type-III: Based on Fractions

Ex.18 If we add 1 to ht numerator and subtract 1 from the denominator, a fraction reduces to 1, It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Sol. Let the required fraction be $\frac{x}{y}$. Then

$$\frac{x+1}{x-y} = 1 \Rightarrow x+1 = y-1 \Rightarrow x-y = -2 \quad \text{…(i)}$$

and $$\frac{x}{x+y} = \frac{1}{2} \Rightarrow 2x = y+1 \Rightarrow 2x - y = 1 \quad \text{…(ii)}$$

Subtracting (i) from (ii), we get x = 3

Put x = 3 in (i), we get $3 - y = -2 \Rightarrow y = 5$

Hence, the fraction is $\frac{3}{5}$

Type-IV : Based on Ages

Ex.19 Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of father and son.

Sol. Let the present ages of the father and the son be x years and y years respectively .

Two years ago, Father's age = (x – 2) years and son's age = (y – 2) years

∴ $(x - 2) = 5(y - 2) \Rightarrow x - 5y = -8$ …(i)

Two years later, father's age = (x + 2) years and son's age = (y + 2) years

∴ $(x + 2) = 3(y + 2) + 8 \Rightarrow x + 2 = 3y + 6 + 8 \Rightarrow x - 3y = 12$ …(ii)

Subtracting (i) from (ii), we get 2y = 20 $\Rightarrow$ y = 10

Putting y = 10 in (ii), we get x – 3 × 10 = 12 $\Rightarrow$ x = 42

Hence , the present ages of father and son are 42 years and 1 years respectively.

Type-III: Based on Geometrical Applications

Ex.20 The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Sol. Let the larger angle be x^0 and the smaller angle by y^0. Then,

$x + y = 180$ …(i)

and $x = y + 18 \Rightarrow x - y = 18$ …(ii)

Adding (i) and (ii) , we get 2x = 198 $\Rightarrow$ x = 99

Putting x = 99 in (i), we get 99 + y = 18 $\Rightarrow$ y = 81

Hence the required angles are 99^0 and 81^0.

Type-III: Based on Time, Distance and Speed.

Formulae to be used :

1. **(a) Speed =** $\mathbf{\frac{Distance}{Time}}$

(b) Distance = Speed × Time

(c) Time = $\mathbf{\frac{Distance}{Speed}}$

2. Let speed of a boat in still water = u km/h

and speed of the current = v km/h. Then ,

(a) Speed of a boat downstream = (u + v)km/h

(b) Speed of a boat upstream = (u – v)km/h.

Ex.21 A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car it takes him 4 hours. But, if he travels 130 km by train and rest by car, be takes 18 minutes, longer. Find the speed of the train and that of the car.

Sol. Let the speeds of the train and that of the car be x km/h and y km/h respectively .

$$\frac{250}{x} + \frac{120}{y} = 4 \qquad \left[\therefore \text{Time} = \frac{\text{Distance Speed}}{\text{Distance}}\right] \qquad \ldots(i)$$

And if he covers 130 km by train and 240 km by car it takes 4 hours and 18 minutes.
Therefore,

$$\frac{130}{x} + \frac{240}{y} = 4 + \frac{18}{60} \qquad \left[\therefore 18 \text{ minutes} = \frac{18}{60} \text{ hours}\right]$$

$$\frac{130}{x} + \frac{240}{y} = \frac{43}{10} \qquad \ldots(ii)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$ Then, the equations (i) and (ii), can be written as

$250u + 120v = 4$ …(iii)

and $130u + 140v = \frac{43}{10}$ …(iv)

Multiplying (iii) by 2, we get 500u + 240v = 8 …(v)

Subtracting (iv) from (v), we get 370u = $8-\frac{43}{10} \Rightarrow 370u = \frac{37}{10} \Rightarrow u = \frac{1}{100}$

Putting $u = \frac{1}{100}$ in (iii), we get $250\times\frac{1}{100}+120v = 4 \Rightarrow \frac{5}{2}+120v = 4$

$\Rightarrow \quad 120v = 4-\frac{5}{2} \Rightarrow v = \frac{3}{120\times 2} = \frac{1}{80}$

but $\quad u = \frac{1}{x}$ and $v = \frac{1}{y}.$

Therefore, $\frac{1}{x} = \frac{1}{100} \Rightarrow x = 100$ and $\frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$

Hence the speeds of the train and that of the car are 100 km/h and 80 km/h respectively .

Type-VII : Miscellaneous

Ex.22 8 man and 12 boys can finish is a piece of work in 10 days while 6 man and 8 boys can finish it in 14 days. Find the time taken by one men alone and that by one boy alone to finish the work .

Sol. Let one man alone can finish the work in x days and one alone can finish the work in y days. Then. the work done by one man in one day $= \frac{1}{x}$ and the work done by one boy in on e day $= \frac{1}{y}$

According to the question, $10\left(\frac{8}{x}+\frac{12}{y}\right) = 1 \Rightarrow \frac{1}{x}+\frac{3}{y} = \frac{1}{40}$ …(i)

Also, $14\left(\frac{6}{x}+\frac{8}{y}\right) = 1 \Rightarrow \frac{3}{x}+\frac{4}{y} = \frac{1}{28}$ …(ii)

Multiplying (i) by 4 and (ii) by 3, we get : $\frac{8}{x}+\frac{12}{y} = \frac{1}{10}$ …(iii) and $\frac{9}{x}+\frac{12}{y} = \frac{3}{28}$ …(iv)

Subtracting (iii) from (iv), we get $\frac{1}{x} = \frac{3}{28}-\frac{1}{10} \Rightarrow \frac{1}{x} = \frac{2}{280} \Rightarrow x = 140$

Putting x = 140 (ii), we get $\frac{3}{140}+\frac{4}{y} = \frac{1}{28} \Rightarrow \frac{4}{y} = \frac{1}{28}-\frac{3}{140}$

$\Rightarrow \quad \frac{4}{y} = \frac{5-3}{140} \Rightarrow \frac{4}{y} = \frac{2}{140} \Rightarrow y = 180$

Hence, one man alone can finish the work in 140 days and one by alone can finish the work in 280 days.

★ **SYNOPSIS**

(i) Two linear equations in the same two variables are called a pair of linear equations in two variables, or briefly, a linear pair. The most general form of a linear pair is :

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where a1, a_2, b_1, b_2, c_1, c_2 are real numbers , such that $a_1^2 + b_1^2 \neq a_2^2 + b_2^2 \neq 0.$

(ii) A pair of linear equations in two variables can be represented, and solved , by the

(a) Graphical method (b) Algebraic method

(iii) **Graphical Method :** The graph of the pair of linear equations in two variables is represented by a pair of lines.

(a) If the pair intersects at a point, then that point is the unique common solution of the two equations. In this case, the pair is consistent.

(b) If the pair coincide, then it has infinitely many solutions – each point on the line being a solution. In this case, the pair is consistent (dependent).

(c) If two lines are parallel, then the pair has no solution, and is called inconsistent.

(iv) **Algebraic Method :** We have discussed the following methods for finding the solutions (s) of a pair of linear equations

(a) Substitution method. (b) Elimination method. (c) Cross-multiplication method.

(v) If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise :

(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ **: In this case the pair of linear equations is consistent.**

(b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: In this case the **pair of linear equations is inconsistent.**

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$: In this case the **pair of linear equations is dependent and consistent.**

(iv) There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a linear pair.

SOLVED NCERT EXERCISE

EXERCISE : 3 . 1

1. After tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three sears from now, I shall be three times as old as you will be". (Isn't this interesting ?) Represent this situation algebraically and graphically .

Sol. Let the present age of Aftab's daughter = x years.

and the present age of Aftab = y years (y > b)

According to the given conditions

Seven years ago, $(y - s) = 7 \times (x - 7)$

i.e., $y - 7 = 7x - 49$

i.e., $7x - y - 42 = 0$ …(i)

Three years later, $(y + 3) = 3 \times (x + 3)$

i.e., $y + 3 = 3x + 9$

i.e., $3x - y + 6 = 0$ …(ii)

Thus, the algebraic relations are $7x - y - 42 = 0$, $3x - y + 6 = 0$.

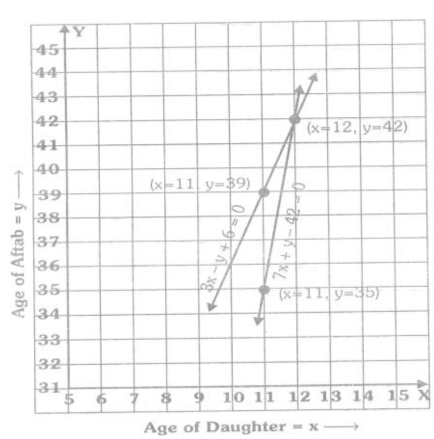
Now, we represent the problem graphically as below :

$7x - y - 42 = 0$ …(i)

Age of Aftab's daughter = x	11	12
Age of Aftab = y = 7x – 42	35	42

$3x - y + 6 = 0$ …(ii)

Age of Aftab's daughter = x	11	12
Age of Aftab = y = 3x + 6	39	42



From the graph, we find that x = 12

and $\quad y = 42$

Thus, the present age of Aftab's daughter = 12 years

and the present age of Aftab = 42 years

2. The coach of a cricket team buys 3 bats and 6 balls for Rs. 3900. Later, she buys another bat and 3 more balls of the same kind for Rs. 1300. Represent this situation algebraically and geometrically .

Sol. [Try Yourself]

3. The cost of 2 kg of apples and 1 kg of grapes on a day found to be Rs. 160. After a month, the cost of 4 kg of apples and 2 g of grapes is Rs. 300. Represent the situation algebraically and geometrically .

Sol. [Try Yourself]

EXERCISE : 3 . 2

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys , find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

Sol. (i) Let the number of boys be x and the number of girls be y.

According to the given conditions

$x + y = 10$ and $y = x + 4$

We get the required pair of linear equations as

$$x + y - 10 = 0, \; x - y + 4 = 0$$

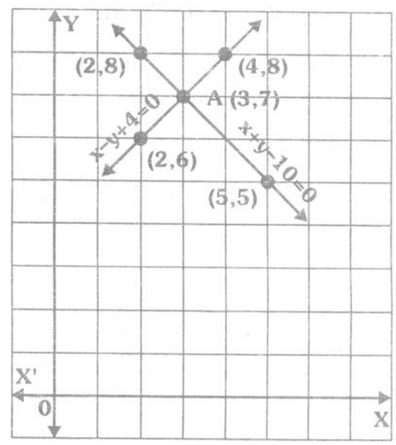
Graphical Solution

$x + y - 10 = 0 \quad \ldots(i)$

x	2	5
y = 10 – x	8	5

$x - y + 4 = 0 \quad \ldots(ii)$

x	2	5
y = x + 4	8	5



From the graph, we have : x = 3, y = 7 common solution of the two linear equations.

Hence, the number of boys = 3 and the number of girls = 7.

(ii) **[Try Yourself]**

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at point, are parallel or coincident .

(i) $5x - 4y + 8 = 0 \; ; \; 7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0 \; ; \; 18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0 \; ; \; 2x - y + 9 = 0$

Sol. (i) $5x - 4y + 8 = 0 \quad \ldots(i)$

$7x + 6y - 9 = 0 \quad \ldots(ii)$

$$\frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = -\frac{2}{3}$$

$\Rightarrow \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ Lines represented by (i) and (ii)

Intersect at a point

[Rest Try Yourself]

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5 ; 2x - 3y = 7$ **(ii) $2x - 3y = 8 ; 4x - 6y = 9$**

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7 ; 9x - 10y = 14$ **(iv) $5x - 3y = 11 ; -10x + 6y = -22$**

(v) $\frac{4}{3}x + 2y = 8 ; 2x + 3y = 12$

Sol. (i) $3x + 2y - 5 = 0$ …(i)

$2x - 3y - 7 = 0$ …(ii)

$\frac{a_1}{a_2} = \frac{3}{2} ; \frac{b_1}{b_2} = -\frac{2}{3}$ $\Rightarrow \frac{a1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ The equations have a unique solution.

Hence, consistent.

[Rest Try Yourself]

4. Which of the following pairs of linear equations are consistent / inconsistent ? If consistent, obtain the solution graphically :

(i) $x + y = 5, 2x + 2y = 10$ **(ii) $x - y = 8, 3x - 3y = 16$**

(iii) $2x + y - 6 = 0, 4x - 2y - 4 = 0$ **(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$**

Sol. (i) $x + y = 5$ …(i)

$2x + 2y = 10$ …(ii)

$\frac{a_1}{a_2} = \frac{1}{2}, = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$

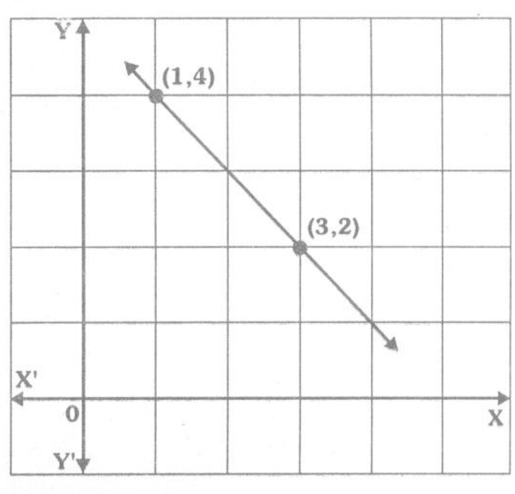
i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the pair of linear equations is consistent.

(i) and (ii) are same equations and hence the graph

Is coincident straight line.

x	1	3
y = 5 – x	4	2



[Rest Try Yourself]

5. Half the perimeter of a rectangular garden, whose length is 4 m more that its width, is 36 m. Find the dimensions of the garden

Sol. [Try Yourself]

6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(i) Intersecting **(ii) Parallel** **(iii) Coincident lines**

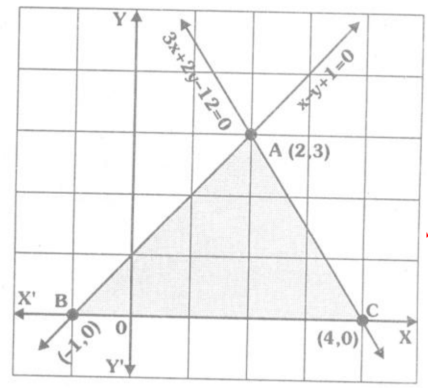
Sol. (i) $2x + 3y - 8 = 0$ (Given equation)

$3x + 2y + 4 = 0$ (New equation)

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the graph of the two equations will be two intersecting lines. **[Rest Try Yourself]**

7. **Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.**



Sol. x – y + 1 = 0 …(i)

x	1	3
y = x + 1	2	4

3x+2y – 12 = 0 …(ii)

x	0	4
$y = \frac{12-3x}{2}$	6	0

The vertices of the triangle are
A (2, 3), B (-1, 0) and C (4, 0)

EXERCISE : 3.3

1. **Solve the following pair of linear equations by the substitution method.**

(i) x + y = 14, x – y = 4

(ii) s – t = 3, $\frac{s}{3}+\frac{t}{2}=3$

(iii) 3x – y = 3, 9x – 3y = 9

(iv) 0.2x + 0.3y = 1.3, 0.4 + 0.5y = 2.3

(v) $\sqrt{2}x+\sqrt{3}y=0, \sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2, \frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

Sol. (i) x + y = 14 …(i)

x – y = 4 …(ii)

From (ii) y = x – 4 …(iii)

Substituting y from (iii) in (i), we get

x + x – 4 = 14 ⇒ 2x = 18 ⇒ x = 9

Substituting x = 9 in (iii), we get

y = 9 – 4 = 5,

i.e., y = 5

x = 9, y = 5

(ii) s – t = 3 …(i)

$\frac{s}{3}+\frac{t}{2}=6$ …(ii)

From (i) s = t + 3 …(iii)

Substituting s from (iii) in (ii), we get

$\frac{t+3}{3}+\frac{t}{2}=6 \quad \Rightarrow 2(t+3)+3t=36$

⇒ 5t + 6 = 36 ⇒ t = 6

From (iii), s = 6 + 3 = 9

Hence, s = 9 , t = 6

[Rest Try Yourself]

2. **Solve 2x + 3y = 11 and 2x – 4y = - 24 and hence find the value of 'm' for which y = mx = 3**

Sol. **[Try Yourself]**

3. **Form the pair of linear equations for the following problems and find their solution by substitution method.**

(i) **The difference between two number is 26 and one number is three times the other. Find them.**

(ii) **The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.**

(iii) **The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. Later, she buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball.**

(iv) **The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 105 and for a journey of 15 km, the charge paid is Rs. 155. what are the fixed charges and the charge per kilometer ? How much does a person have to pay for travelling a distance of 25 km ?**

(v) **A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator is becomes $\frac{5}{6}$. Find the fraction.**

(vi) **Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their the present ages ?**

Hints (i) Let the two numbers be x and y (x > y). Then, x – y = 26 and x = 3y.

(ii) Let the supplementary angles by x and y (x > y) Then, x + y = 180 and x – y = 18.

(iii) Try Yourself

(iv) Let fixed charge be Rs x and charge per km be Rs y. Then, x + 10y = 105 and x + 15y = 155.

(v) Let $\frac{x}{y}$ be the fraction where x and y are positive integers. $\frac{x+2}{y+2} = \frac{9}{11}$ and $\frac{x+3}{y+3} = \frac{5}{6}$

(vi) Let x (in years) be the present age of Jacob's son and y (in years) be the present age of Jacob. Then, (x + 5) = 3 (x + 5) and (y – 5) = 7 (x – 5)

EXERCISE : 3 . 4

1. **Solve the following pair of equations by the elimination method and the substitution method.**

(i) x + y = 5 and 2x – 3y = 4 **(ii) 3x + 4y = 10 and 2x – 2y = 2**

(iii) 3x – 5y – 4 = 0 and 9x = 2y + 7 **(iv)** $\frac{x}{2} + \frac{2y}{3} = -1$ **and** $x - \frac{y}{3} = 3$

Sol. (i) Solution By Elimination Method:

x + y = 5 …(i)

2x – 3y = 4 …(ii)

Multiplying (i) by 3 and (ii) by 1 and adding we get 3(x + y) + 1 (2x – 3y) = 3 × 5 + 1 × 4

$\Rightarrow$ 3x + 3y + 2x – 3y = 19

$\Rightarrow$ 5x = 19 $\Rightarrow$ x = $\frac{19}{5}$

From (i), substitution x = $\frac{19}{5}$, we get

$\frac{19}{5}$ + y = 5 $\Rightarrow$ y = 5 – $\frac{19}{5}$ $\Rightarrow$ y = $\frac{6}{5}$

Hence , x = $\frac{19}{5}$, y = $\frac{6}{5}$

[Rest Try Yourself]

(i) Solution By Substitution Method:

x + y = 5 …(i)

2x – 3y = 4 …(ii)

From (i), y = 5 – x …(iii)

Substituting y from (iii) in (ii),

2x – 3(5 – x) $\Rightarrow$ 2x – 15 + 3x = 4

$\Rightarrow$ 5x = 19 $\Rightarrow$ x = $\frac{19}{5}$

Then from (iii), y = 5 – $\frac{19}{5}$ $\Rightarrow$ y = $\frac{6}{5}$

Hence , x = $\frac{19}{5}$, y = $\frac{6}{5}$

2. **Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method.**

(i) **If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?**

(ii) **Five years ago Nuri was thrice as old as Sonu. Ten years late, Nuri will be twice as old as Sonu. How old are Nuri and Sonu ?**

(iii) **The sum of the digits of a tow-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.**

(iv) **Meena went to a bank to withdraw Rs. 2000. She asked the cashier to give her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received.**

(v) **A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs. 27 for a book kept for seven days, while Susy paid Rs. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.**

Hints (i) Let the fraction be $\frac{x}{y}$. Then $\frac{x+1}{y-1}=1$; $\frac{x}{y+1}=\frac{1}{2}$

(ii) Try yourself

(iii) Let x be the digit at unit place and y be the digit at tens place of the number. So, number = x + 10y. Then x + y = 9 and 9[x + 10y] = 2[y + 10x].

(iv) Let x and y be the number of Rs. 50 and Rs. 100 notes respectively.
Then, x + y = 25 and 50x + 100y = 2000

(v) Try yourself

EXERCISE : 3 . 5

1. **Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.**

(i) x – 3y – 3 = 0; 3x – 9y – 2 = 0 (ii) 2x + y = 0; 3x + 2y = 8 (iii) 3x – 5y = 20; 6x – 10y = 40 (iv) x – 3y – 7 = 0; 3x – 3y – 15 = 0

Sol. (i) x – 3y – 3 = 0, 3x – 9y – 2 = 0

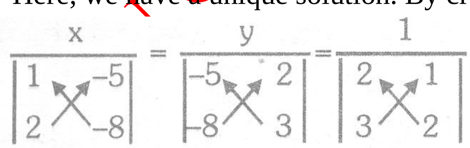
$$\frac{a_1}{a_2}=\frac{1}{3},\frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3},\frac{c_1}{c_2}=\frac{3}{2}\Rightarrow\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Hence, no solution .

(ii) 2x + y = 5 …(i) and 3x + 2y = 8 …(ii)

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}\qquad\left(\frac{a_1}{a_2}=\frac{2}{3},\frac{b_1}{b_2}=\frac{1}{2}\right)$$

Here, we have a unique solution. By cross multiplication, we have



$$\Rightarrow\quad\frac{x}{\{(1)(-8)-(2)(-5)\}}=\frac{y}{\{(-5)(3)-(-8)(2)\}}=\frac{1}{\{(2)(2)-(3)(1)\}}$$

$$\Rightarrow\quad\frac{x}{(-8+10)}=\frac{y}{(-15+16)}=\frac{1}{(4-3)}$$

$$\Rightarrow\quad\frac{x}{2}=\frac{y}{2}=\frac{1}{1}\qquad\Rightarrow\frac{x}{2}=\frac{1}{1}\text{ and }\frac{y}{1}=\frac{1}{1}$$

$\Rightarrow$ x = 2 and y = 1 **[Rest Try Yourself]**

2. **(i)** **For which values of a and b does the following pair of linear equations have an infinite number of solutions ?**

$$2x + 3y = 7$$
$$(a - b)x + (a + b)y = 3a + b - 2$$

(ii) **For which value of k will the following pair of linear equations have no solution ?**

$$3x + y = 1$$
$$(2k - 1)x + (k - 1)y = 2k + 1.$$

Sol. (i) $2x + 3y - 7 = 0$ …(i)

$(a - b)x + (a + b)y - (3a + b - 2) = 0$ …(ii)

For infinite number of solutions, we have

$$\frac{a-b}{2} = \frac{a+b}{3} = \frac{3a+b-2}{7}$$

For first and second, we have

$$\frac{a-b}{2} = \frac{a+b}{3}$$

or $3a - 3b = 2a + 2b$

or $a = 5b$ …(i)

From second and third, we have

$$\frac{a+b}{3} = \frac{3a+b-2}{7}$$

or $7a + 7b = 9a + 3b - 6$

or $4b = 2a - 6$

From (i) and (ii), eliminating a,

$$2b = 5b - 3 \Rightarrow b = 1$$

Substituting b = 1 in (i), we get a = 5

(ii) **[Try Yourself]**

3. **Solve following pair of linear equations by the substitution and cross-multiplication methods :**
8x + 5y = 9, 3x + 2y = 4

Sol. **[Try Yourself]**

4. **Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method .**

(i) **A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs. 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs. 1180 as hostel charges. Find the fixed charges and the cost of food per day.**

(ii) **A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.**

(iii) **Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer , then Yash would have scored 50 marks. How many questions were there in the test ?**

(iv) **Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If travel towards each other, they meet in 1 hour. What are the speeds of the two cars ?**

(v) **The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and breadth is increased by 3 units . If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangles.**

Sol. (i) Try Yourself

(ii) Try Yourself

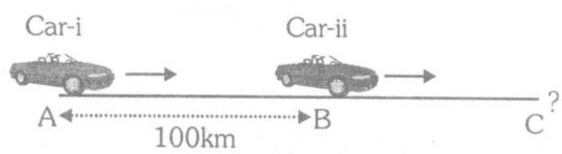
Hint (iii) number of right answers = x. Number of wrong answers = y

Then, $3x - y = 40$ and $4x - 2y = 50$

Hint (iv) Speed of car i = x km/hr

Speed of car ii = y km/hr

First case :



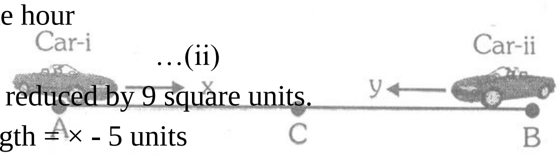
Two cars meet at C after 5 hrs.

AC – BC = AB

$\Rightarrow$ $5x - 5y = 100$ …(i)

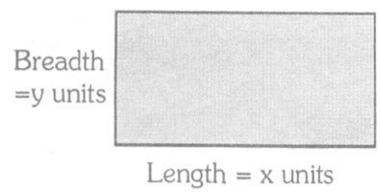
Second case:

Two cars meet at C after one hour



$x + y = 100$ …(ii)

Hint (v) In first case, area is reduced by 9 square units.



When length = x - 5 units

and breadth = y + 3 units

$\Rightarrow$ $xy - (x - 5) \times (y - 3) = 9$ …(i)

In second case area increases by 67 sq. units when length = x + 3 and breadth = y + 2.

$\Rightarrow$ $(x + 3) \times (y + 2) - xy = 67$ …(ii)

EXERCISE : 3 . 6

1. Solve the following pairs of equations by reducing them to a pair of linear equations :

(i) $\frac{1}{2x}+\frac{1}{3y}=2, \ \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$ **(ii)** $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2, \ \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14, \ \frac{3}{x}-4y=23$ **(iv)** $\frac{5}{(x-1)}+\frac{1}{(y-2)}=2, \ \frac{6}{(x-1)}-\frac{3}{(y-2)}=1$

(v) $\frac{7x-2y}{xy}=5, \ \frac{8x+7y}{xy}=15$ **(iv)** $6x+3y=6xy, \ 2x+4y=5xy$

(vii) $\frac{10}{(x+y)}+\frac{2}{(x-y)}=4, \ \frac{15}{(x+y)}-\frac{5}{(x-y)}=-2$

(viii) $\frac{1}{(3x+y)}+\frac{1}{(3x-y)}=\frac{3}{4}, \ \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Sol. (i) $\frac{1}{2x}+\frac{1}{3y}=2, \ \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$

We get $\frac{1}{2}u+\frac{1}{3}v=2,\ \frac{1}{3}u+\frac{1}{2}v=\frac{13}{6}$

Multiplying by 6 on both sides, we get

$\Rightarrow$ 3u + 2v = 12 …(i)

2u + 3v = 13 …(ii)

Multiplying (i) by 3 and (ii) by 2, then subtracting later from first, we get

3(3u + 2v) – 2(2u + 3v) = 3 × 12 – 2 × 13

$\Rightarrow$ 9u – 4u = 36 – 26 $\Rightarrow$ u = 2

Then substituting u =2 in (i), we get

6 + 2v = 12 $\Rightarrow$ v = 3

Now, u = 2 and v = 3

$\Rightarrow$ $\frac{1}{x}=2$ and $\frac{1}{y}=3$ $\Rightarrow x=\frac{1}{2}$ and $y=\frac{1}{3}$

(ii) [Hint : Put $\frac{1}{\sqrt{x}}=u$ & $\frac{1}{\sqrt{y}}=v$],

(iii) Try Yourself

(iv) [Hint : Put $\frac{1}{x-1}=u$ and $\frac{1}{y-2}=v$ to get :

5u + v = 2 and 6u – 3v = 1]

(v) [Hint : $\frac{7x-2y}{xy}=5,\ \frac{8x+7y}{xy}=15$

$\Rightarrow$ $\frac{7x}{xy}-\frac{2y}{xy}=5,\ \frac{8x}{xy}+\frac{7y}{xy}=15$

$\Rightarrow$ $\frac{7}{y}-\frac{2}{x}=5,\ \frac{8}{y}+\frac{7}{x}=15$

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$, we get

7v – 2u = 5, 8v + 7u = 15

[Rest Try Yourself]

2. Formulate the following problems as a pair of linear equations, and hence find their solutions :

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current .

(ii) **2 woman and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.**

(iii) **Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by has, she takes 10 minutes longer. Find the speed of the train and the bus separately .**

Sol. (i) [Hint Speed of Ritu in still water = x km/hr

Speed of current = y km/hr

Then speed downstream = (x + y) km/hr

Speed upstream = (x – y) km/hr

$$\frac{20}{x+y} = 2 \text{ and } \frac{4}{x-y} = 2$$

$\Rightarrow$ x + y = 10 ...(i)

x – y = 2 ...(ii)

Hint (ii) Let 1 woman finish the work in x days and let 1 man finish the work in y days.

Work of 1 woman in 1 day $= \frac{1}{x}$

Work of 1 man in 1 day $= \frac{1}{y}$

Work of 2 woman and 5 men in one day $= \frac{2}{x} + \frac{5}{y} = \frac{5x+2y}{xy}$

The number of days required for complete work $= \frac{xy}{5x+2y}$

We are given that $\frac{xy}{5x+2y} = 4$ ...(i)

Similarly, in second case $\frac{xy}{6x+3y} = 3$ ...(ii)]

(iii) **[Try Yourself]**

EXERCISE – 1 (FOR SCHOOL / BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

Choose The Correct One

1. If A : Homogeneous system of linear equations is always consistent. R : x = 0, y = 0 ix always a solution of the homogeneous system of equations with unknowns x and y, then which of the following statement is true ?

(A) A is true and R is the correct explanation of A

(B) A is false and R is not a correct explanation of A

(C) A is true and R is false

(D) A is false and R true

2. If the pair of linear equations x – y = 1, x + ky = 5 has a unique solution x = 2, y = 1, then value of k is –

(A) -2 (B) 3 (C)-3 (D) 4

3. The pair of linear equations 2x + ky – 3 = 0, 6x + $\frac{2}{3}$ y + 7 = 0 has a unique solution if –

(A) $k = \frac{2}{3}$ (B) $k \neq \frac{2}{3}$

(C) $k = \frac{2}{9}$ (D) $k \neq \frac{2}{9}$

4. The pair of linear equations 2kx + 5y = 7, 6x – 5y = 11 has a unique solution if –

(A) $k \neq -3$ (B) $k \neq 3$

(C) $k \neq 5$ (D) $k \neq -5$

5. The pair of equations 3x + 4y = k, 9x + 12y = 6 has infinitely many solutions if –

(A) k = 2 (B) k = 6

(C) k $\neq$ 6 (D) k = 3

6. The pair of linear equations 2x + 5y = k, kx + 15y = 18 has infinitely many solution if –

(A) k = 3 (B) k = 6 (C) k = 9 (D) k = 18

7. The pair of linear equations 3x + 5y = 3, 6x + ky = 8 do not have any solution if –

(A) k = 5 (B) k = 10 (C) k $\neq$ 10 (D) k $\neq$ 5

8. The pair of linear equations 3x + 7y = k, 12x + 2ky = 4k + 1 do not have any solution if

(A) k = 7 (B) k = 14 (C) k = 21 (D) k = 28

9. The pair of linear equations 7x – 3y = 4, 3x + $\frac{k}{7}$ y = 4 is consistent only when –

(A) k = 9 (B) k = -9 (C) k $\neq$ - 9 (D) k $\neq$ 7

10. The pair of linear equations kx + 4y = 5, 3x + 2y = 5 is consistent only when –

(A) k $\neq$ 6 (B) k = 6 (C) k $\neq$ 3 (D) k = 3

11. The pair of linear equations 7x + ky = k, 14x + 2y = k + 1 has infinitely many solution if –

(A) k = 1 (B) k $\neq$ 1 (C) k = 2 (D) k = 4

12. The pair of linear equations 13x + ky = k, 39x + 6y = k + 4 has infinitely many solutions if -

(A) k = 1 (B) k = 2 (C) k = 4 (D) k = 6

13. The pair of linear equations x + y = 3, 2x + 5y = 12 has a unique solution x = x_1, y – y_1 then value of x_1 is –

(A) 1 (B) 2 (C) -1 (D) -2

14. The pair of linear equations 3x – 5y + 1 = 0, 2x – y + 3 = 0 has a unique solution x = x_1, y = y_1 then y_1 =

(A) 1 (B) -1 (C) -2 (D) – 4

15. The pair of linear equations x + 2y = 5, 7x + 3y = 13 has a unique solution –

(A) x = 1, y = 2 (B) x = 2, y = 1

(C) x = 3, y = 1 (D) x = 1, y = 3

16. The pair of linear equations x + 2y = 5, 3x + 12y = 10 has –

(A) Unique solution

(B) No solution

(C) More than two solution

(D) Infinitely many solutions

17. If the sum of the ages of a father and his son in years is 65 and twice the difference of their ages in years is 50, then the age of father is –

(A) 45 years (B) 40 years (C) 50 years (D) 55 years

18. A fraction becomes $\frac{4}{5}$ when 1 is added to each of the numerator and denominator. However, if we subtract 5 from each then it becomes $\frac{1}{2}$. The fraction is –

(A) $\frac{5}{8}$ (B) $\frac{5}{6}$ (C) $\frac{7}{9}$ (D) $\frac{13}{16}$

19. Three chairs and two tables cost Rs. 1850 Five chairs and three tables cost Rs. 1850. Then the total cost of one chair and table is –

(A) Rs. 800 (B) Rs. 850 (C) Rs. 900 (D) Rs.950

20. Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son. The present age of the man is –

(A) 28 years (B) 30 years (C) 32 years (D) 34 years

OBJECTIVE							ANSWER KEY							EXERCISE - 1
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**A**	**B**	**D**	**A**	**A**	**B**	**B**	**B**	**C**	**A**	**A**	**B**	**A**	**B**	**A**
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	**A**	**A**	**C**	**B**	**B**										

EXERCISE – 2 (FOR SCHOOL / BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions

1. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find for whether the following pair of liner equations are consistent or inconsistent .

(i) x – 3y = 4 ; 3x + 2y = 1 (ii) $\frac{4}{3}$x + 2y = 8 ; 2x + 3y = 12

(iii) $4x + 6y = 7$; $12x + 18y = 21$ (iv) $x – 2y = 3$; $3x – 6y = 1$

2. On comparing the ratio $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines representing is following pair of linear equations intersect at a point, are parallel or coincident :

(a) (i) $2x – y = 3$; $4x – y = 5$ (ii) $x + 2y = 8$; $5x – 10y = 10$ (iii) $3x + 4y = - 2$; $12x +16y = - 8$

(b) (i) $6x + 3y = 18$; $2x + y = 6$ (ii) $x – 3y = 3$; $3x – 9y = 2$

(iii) $ax – by = c_1$; $bx + ay = c_2$, where $a \neq 0$, $b \neq 0$

3. For the linear equations given below, write anther linear equation in two variables, such that the geometrical representation of the pair so formed is -

(i) Intersecting (ii) Parallel lines (iii) Coincident lines

(a) $2x – 3y = +$ (b) $y = 2x + 3$

4. Find the value of k for which the given system of equations has a unique solution .

(a) $(k – 3) x + 3y = k$; $kx + ky = 12$ (b) $x – ky = 2$; $3x + 2y = - 5$

5. Find the value of k for which the given system of equations has no solution .

(a) $kx + 2y – 1 = 0$; $5x – 3y + 2 = 0$

(b) (i) $x + 2y = 3$; $5x + ky + 7 = 0$ (ii) $kx + 3y = k – 3$; $12x + ky = k$

6. (a) Find the value (s) of k for which the system of equations $kx – y = 2$ and $6x – 2y = 3$ has

(i) A unique solution (ii) No solution

(b)Find the value of k for which system $kx + 2y = 5$ and $3x + y = 1$ has

(i) A unique solution (ii) No solution

7. Find the value of k for which the given system of equations has an infinite number of solutions.

(a) $5x + 2y = 2k$ and $2(k + 1) x + ky = (3k + 4)$

(b) (i) $x + (k + 1) y = 5$ and $(k + 1) x + 9y = 8k – 1$

(ii) $10x + 5y – (k – 5) = 0$ and $20x + 10y – k = 0$

(c) $kx + 3y = k – 3$ and $12x + ky = k$

8. Find the value of a and b for which the given system of linear equation has an infinite number of solutions :

(a) $2x + 3y = 7$ and $(a – b) x + (a + b) y = 3a + b – 2$

(b) $(a + b) x – 2by = 5a + 2b + 1$ and $3x – y = 14$

(c) $(2a – 1) x + 3y – 5 = 0$ and $3x + (b – 1) y – 2 = 0$

Short Answer Type Questions

Based on graphical solution of system of equations :

Solve graphically each of the following pairs of equations (1–9) :

1. $x + y = 4$, $2x – 3y = 3$

2. $x + y = 3$, $2x + 5y – 12 = 0$

3. $\frac{4}{9}x + \frac{1}{3}y = 1$, $5x + 2y = 13$

4. $2x + 3y = 4$, $x – y + 3 = 0$

5. $x + y = 7$, $5x + 2y = 20$

6. $x + 4y = 0$, $2x + 8y = 0$

7. x + 2y = 3, 2x + 4y = 15

8. 3x + 2y = 3, 6x + 4y = 15

9. 2x + 3y – 5 = 0, 6x + 9y – 15 = 0

10. Check whether the pair of equations x + 3y = 6, and 2x – 3y = 12 is consistent. If so, solve graphically .

11. Show graphically that the pair of equations 2x – 3y + 7 = 0, 6x – 9y + 21 = 0 has infinitely many solutions.

12. Show graphically that the pair of equations 8x + 5y = 9, 16x + 10y = 27 has no solution.

13. Find whether the pair of equations 5x – 8y + 1 = 0, $3x - \frac{24}{5}y + \frac{3}{5} = 0$ has no solution, unique solution or infinitely many solutions.

14. Show graphically that the pair of equations 2x – 3y = 4, 3x – 2y = 1 has a unique solution.

15. Show graphically that the pair of equations 3x + 4y = 6, 6x + 8y = 12 represents coincident lines.

16. Determine by drawing graphs whether the following pair of equations has a unique solution or no :
2x – 3y = 6, 4x – 6y = 9. If yes, find the solution also.

17. Determine graphically whether the pair of linear equations 3x – 5y = - 1 , 2x – y = - 3 has a unique solution or not. If yes, find the solution also .

18. Solve graphically the pair of equations x + 3y = 6, and 3x – 5y = 18. Hence, find the value of K if 7x + 3y = K.

19. Solve graphically the pair of equations 2x – y = 1, x + 2y = 8. Also find the points where the lines meet the axis of y.

20. Solve graphically the following pair of linear equations :
2x + 3y – 12 = 0, 2x – y – 4 = 0. Also find the coordinates of the points where the lines meet the y-axis.

21. Solve the following pair of equations graphically : x + y = 4, 3x – 2y = 3
Shade the region bounded by the lines representing the above equations and x-axis.

22. Solve the following pair of linear equations graphically : 2x + y = 8, 3x – 2y = 12.
From the graph, read the points where the lines meet the x-axis.

23. Solve graphically the following pair of equations : x – y = 1, 2x + y = 8. Shade the area bounded by these lines and the y-axis.

24. On the same axes, draw the graph of each of the following equations :
2y – x = 8, 5y – x = 14, y – 2x = 1. Hence, obtain the vertices of the triangle so formed.

25. Solve graphically the pair of linear equations : 4x – 3y + 4 = 0, 4x + 3y – 20 = 0. Find the area of the region bounded by theses lines and x-axis,

Based on substitution method :
Solve the following equations by the substitution method : (26-41)

26. 3x + 11y = 13, 8x + 13y = 2

27. x + 2y = 1.6, 2x + y = 1.4

28. 11x – 8y = 27, 3x+5y = - 7

29. 0.04x + 0.02y = 5, 0.5x – 0.4y = 30

30. 5x + 8y = - 1, 6y – x = 4y – 7

31. 12x – 16y = 20, 8x + 6y = 30

32. 8x – 5y + 40 = 0, 7x – 2y = 0

33. $\frac{1}{2}$(9x +10y) = 23, $\frac{5x}{4}$ - 2y = 3

34. $\sqrt{2}x + \sqrt{3}y = 0, \ \sqrt{3}x - \sqrt{8}y = 0$

35. $\frac{(3x-y)}{5} = 2y - 1, \ \frac{3x}{8} - \frac{y}{4} = \frac{1}{2}$

36. 3x + 15 = 4y, 3y + 17 = 2 + 3x

37. x + 6y = 2x – 16, 3x – 2y = 24

38. x = 3y – 19, y = 3x – 23

39. 5x + 2y = 14, x + 3y = 8

40. x + y = 27, $\frac{3}{4}x + \frac{2}{3}y = 19$

41. $\frac{x+11}{7} + 2y = 10, \quad 3x = 8 + \frac{y+7}{11}$

42. Solve 2x – y = 12 and x + 3y + 1 = 0 and hence find the value of m for which y = mx + 3.

43. Solve 4x – 3y +17 = 0 and 5x + y + = 0 and hence find the value of n for which y = nx – 1.

Based on substitution method :

Solve the following pairs of lines equations by elimination method : (44-52)

44. (a) x + y = 5 and 2x – 3y = 4

(b) (i) 2x + 3y = 8 and 4x + 6y = 7

(ii) 11x +15y + 23 = 0 and 7x – 2y – 20 = 0

(c) 78x + 91y = 39 and 65x + 117y = 42

45. (a) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$ (b) $\frac{x}{3} + \frac{y}{4} = 11$ and $\frac{5x}{6} - \frac{y}{3} + 7 = 0$

46. (a) ax – by = $a^2 + b^2$ and x + y = 2a (b) $\frac{bx}{a} - \frac{ay}{b} + a + b = 0$ and $bx - ay + 2ab = 0$

47. ax + by – 2a + 3b = 0 and bx – ay – 3a – 2b = 0

48. (a) 2(ax – by) + (a + 4b) = 0 and 2(bx + ay) + (b – 4a) = 0

(b) (bx + ay) = 0 and (a + b) x + (a – b) y = $a^2 + b^2$

49. (a) (a + c) x – (a – c) y = 2ab and (a + b) x – (a – b) y = 2ab

(b) (a + 2b) x + (2a – b) y = 2 and (a – 2b) x + (2a + b) y = 3

50. (a) $\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{5}x + \sqrt{2}y = 0$

(b) $\sqrt{7}x + \sqrt{11}y = 0$ and $\sqrt{3}x - \sqrt{5}y = 0$

51. 0.5x + 0.7y = 0.74 and 0.3x + 0.5y = 0.5

52. (a) 23x – 29y = 98 and 29x – 23y =110

(b) (i) 217x + 131y = 913 and 131x + 127y = 827

(ii) 23x + 37y = 32 and 37x + 23y = 88

(c) (i) 65x – 33y = 97 and 33x – 65y = 1

(ii) 47x + 31y = 63 and 31x + 47y = 15

(iii) 99x + 101y = 499 and 101x + 99y = 501

Based on cross-multiplication method :

Solve each of the following pairs of equations by cross multiplication rule : (53-62)

53. x – 2y = 10, 4x + y = 13

54. 5x + 3y = 35, 2x + 4y = 28

55. 4x – 3y + 1 = 0, 2x – 5y + 11 = 0

56. 3x + 4y = 27, 5x – 3y = 16

57. 2x + 3y = 46, 3x + 5y = 74

58. 2x – y + 4 = 0, x + y – 1 = 0

59. $\frac{x}{2} - \frac{y}{3} + 4 = 0, \quad \frac{x}{2} - \frac{5y}{3} + 12 = 0$

60. ax + by + a = 0, bx + ay + b = 0

61. $\frac{x}{a} + \frac{y}{b} = 0, \quad \frac{x}{a} - \frac{y}{b} = 4$

62. x + y = a + b, ax – by = $a^2 - b^2$

Based on equations reducible to linear equations :
Solve for x and y : (63-82)

63. $\frac{2}{x}+\frac{3}{y}=2;\ \frac{1}{x}-\frac{1}{2y}=\frac{1}{3}$

64. $\frac{4}{x}+\frac{7}{y}=29;\ \frac{3}{y}+\frac{1}{x}=11$

65. $\frac{1}{3x}-\frac{1}{7y}=\frac{2}{3};\ \frac{1}{2x}-\frac{1}{3y}=\frac{1}{6}$

66. $\frac{1}{5x}+\frac{9}{y}=4;\ \frac{3}{x}+\frac{27}{y}=24$

67. $\frac{11}{2x}-\frac{9}{2y}=-\frac{23}{2};\ \frac{3}{4x}+\frac{7}{15y}=\frac{23}{6}$

68. x + y = 2xy ; x – y = xy

69. 4x + 3y = 8xy ; 6x + 5y = 13xy

70. 6x + 5y = 8xy ; 8x + 3y = 7xy

71. $\frac{x-y}{xy}=9;\ \frac{x+y}{xy}=5$

72. 9 + 25xy = 53x ; 27 – 4xy = x

73. $\frac{16}{x+3}+\frac{3}{y-2}=5;\ \frac{8}{x+3}-\frac{1}{y-2}=0$

74. $\frac{13}{x+y}-\frac{56}{x-y}=21;\ \frac{11}{x+y}-\frac{23}{x-y}=14\frac{2}{7}$

75. $\frac{24}{2x+y}-\frac{13}{3x+2y}=2;\ \frac{26}{3x+2y}+\frac{8}{2x+y}=3$

76. $\frac{29}{x-1}-\frac{81}{y+1}=-26;\ \frac{19}{y+1}-\frac{4}{x-1}=\frac{15}{2}$

77. $\frac{2}{x-1}+\frac{y-2}{4}=2;\ \frac{3}{2(x-1)}+\frac{2(y-2)}{5}=\frac{47}{20}$

78. $\frac{x-4}{x-3}=\frac{y+4}{y+7};\ \frac{x+5}{x+2}=\frac{y-1}{y-2}$

79. $\frac{3x-2}{3y+7}=\frac{5x-1}{5y+16};\ \frac{3x-15}{x-9}=\frac{6y-5}{2y+3}$

80. $\frac{x+y+3}{x-y-3}=\frac{-3}{2};\ \frac{x-y-3}{x-y-3}=-2$

81. $\frac{x+y-1}{x-y+1}=7;\ \frac{y-x+1}{x-y+1}=35$

82. $\frac{x+2y=1}{2x-y+1}=2;\ \frac{3x-y+1}{x-y+3}=5$

SUBJECTIVE **ANSWER KEY** **EXERCISE -2 (X) -CBSE**

- **Very Short Answer Type Questions**

1. (i) consistent (ii) consistent (iii) consistent (iv) inconsistent

2. (a) (i) intersect at point (ii) parallel (iii) coincident (b) (i) coincident (ii) parallel (iii) intersect at a point

3. (a) (i) 3x + 5y – 7 = 0 (ii) 4x – 6y – 8 = 0 (iii) 6x – 9y = 18

(b) (i) 2x + 3y – 4 = 0 (ii) 4x – 2y + 8 = 0 (iii) 8x – 4y + 12 = 0 (many such examples may be given)

4. (a) $k \neq 6$ (b) $k \neq \frac{-2}{3}$ **5.** (a) $k = \frac{-10}{3}$ (b) (i) k = 10 (ii) k = - 6 **6.** (a) (i) $k \neq 3$ (ii) k = 3 (b)(i) $k \neq 6$(ii) k = 6

7. (a) k = 4 (b) (i) k = 2 (ii) k = 10 (c) k = 6 **8.** (a) a = 5, b = 1 (b) a = 5, b = 1 (c) a = $\frac{17}{4}$, b = $\frac{11}{5}$

- **Short Answer Type Questions**

1. x = 3, y = 1 **2.** x = 1, y = 2 **3.** x = 3, y = - 1 **4.** x = - 1, y = 2 **5.** x = 2, y = 5 **6.** Infinite number of solutions
7. No solution **8.** No solution **9.** Infinite number of solutions **10.** Yes ; x = 6, y = 0 **13.** Infinitely many solutions
16. No **17.** Yes ; x = - 2, y = - 1 **18.** x = 6, y = 0 ; K = 42 **19.** x = 2, y = 3 ; (0, - 1), (0, 4)
20. x = 3, y = 2, (0, 4), (0, - 4) **21.** x = 1, y = 3
22. x = 4, y = 0 ; The two lines meet at the x-axis at a common point (4, 0). **23.** x = 3, y = 2 **24.** (2, 5), (-4, 2), (1, 3)
25. 12 sq units. **26.** x = -3, y = 2 **27.** x = 0.4, y = 0.6 **28.** x = 1, y = - 2 **29.** x = 100, y = 50
30. x = 3, y = - 2 **31.** x = 3, y = 1 **32.** x = $\frac{80}{19}$, y = $\frac{80}{19}$ **33.** x = 4, y = 1 **34.** x = y = 0 **35.** x = 2, y = 1
36. x = 35, y = 30 **37.** x = 7, y = - 3/2 **38.** x = 11, y = 10 **39.** x = 2, y = 2 **40.** x = 12, y = 15
41. x = 3, y = 4 **42.** x = 5, y = - 2 , - 1 **43.** x = - 2, y = 3, - 2
44. (a) x = $\frac{19}{5}$, y = $\frac{6}{5}$ (b) (i) No solution (ii) x = 2, y = - 3 (c) x = $\frac{3}{13}$,y = $\frac{3}{13}$ **45.** (a) x = 2, y = -3 (b) x = 6, y = 36
46. (a) x = (a + b), y = (a – b) (b) x = - a, y = b **47.** x = 2, y = - 3 **48.** (a) x = $\frac{-1}{2}$, y = 2, (b) x = a, y = - b
49. (a) x = b, y = - b (b) x = $\frac{5b-2a}{10ab}$, y = $\frac{a+10b}{10ab}$ **50.** (a) x = 0, y = 0 (b) x = 0, y = 0 **51.** x = 0.5, y = 0.7
52. (a) x = 3, y = - 1 (b) (i) x = 3, y = 2 (ii) x = 3, y = - 1 (c) (i) x = 2, y = 1 (ii) x = 2, y = - 1 (iii) x = 3, y = 2
53. x = 4, y = - 3 **54.** x = 4, y = 5 **55.** x = 2, y = 3 **56.** x = 5, **57.** x = 8, y = 10 **58.** x = - 1, y = 2
59. x = - 4, y = 6 **60.** x = - 1, y = 0 **61.** x = 2a, y = - 2b **62.** x = a, y = b **63.** x = 2, y = 3 64. x = $\frac{1}{2}$,y = $\frac{1}{3}$
65. x = $\frac{1}{5}$, y = $\frac{1}{7}$ **66.** x = $\frac{1}{5}$, y = 3 67. x = $\frac{1}{2}$, y = $\frac{1}{5}$ **68.** x = 2, = $\frac{2}{3}$ **69.** x = $\frac{1}{2}$, y = 2 **70.** x = 1, y = 2
71. x = - $\frac{1}{2}$, y = $\frac{1}{7}$ **72.** x = 3, y = 2 **73.** x = 5, y = 3 **74.** x = - 3, y = 4 **75.** x = 3, y = 2 **76.** x = 3, y = 1
77. x = 3, y = 6 **78.** x = 7, y = 5 **79.** x = 3, y = 2 **80.** x = 3, y = 2 **81.** x = $\frac{2}{9}$, y = $\frac{7}{6}$ **82.** x = 13, y = 10

EXERCISE – 3 (FOR SCHOOL / BOARD EXAMS)

APPLICATIONS TO WORD PROBLEMS

Based On Articles And Their Costs :

Q.1 4 charier and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs. 1750. Find the cost of a chair and a table separately .

Q.2 37 pens and 53 pencils together cost Rs. 320, while 53 pens and 37 pencils together cost Rs 40. Find the cost of a en and that of a pencil.

Q.3 4 tables and 3 chairs together cost Rs 2250 and 3 tables and 4 chairs cost Rs 1950. Find the cost of 2 chairs and 1 table.

Q.4 A and B each have certain number of oranges. A says to A says to B. “if you give me 10 of your oranges, I will have twice the number of oranges left with you. “B relies,” if you give me 10 of your oranges, If will have the same number of oranges as left with you.” Find the number of oranges with A and B separately.

Q.5 A and B each have a certain number of mangoes. A says to B, " if you give 30 of your mangoes, I will have twice as many as left with you." B replies, "if you give me 10, I will have thrice as many as left with you." How many mangoes does each have ?

Q.6 One says, " give me a hundred , friend ! I shall then become twice as rich as you, " The other replies," If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their respective capital ?

Q.7 Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Q.8 A man has only 20 paisa coins and 25 paisa coins in his purse. If he has 50 coins in all totaling Rs 11.25, how many cons of each kind does he have :

Q.9 A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Based on numbers :

Q.10 Sum of two numbers is 35 and their difference is 13. Find the numbers .

Q.11. The sum of two number is 8. If their sum is 4 times their difference. Find the number.

Q.12. The sum of two numbers is 1000 and the difference between their squares is 256000. Find the numbers .

Q.13 In a two digit number, the unit's digit is twice the ten's digit . If 27 is added to the number. the digits interchange their paces. Find the number.

Q.14. In a two digit number, the ten's digit is three times the unit's digit . When the number is decreased by 54, the digits are reversed. Find the number.

Q.15 The sum of the digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number .

Q.16 The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digit is 18. Find the number.

Q.17. The sum of a two digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of its digits in the first number. Find the first number .

Q.18 The sum of a two digit number and the number formed by interchanging its digits is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

Q.19 The sum of a two digit number and the number obtained by reversing the order of its digits is 121, and the two digits differ by 3. Find the number.

Q.20 A two digit number is 3 more than 4 times the sum of digits. If 18 is added to the number, the digits are reversed. Find the number.

Based On Fractions :

Q.21 A fraction becomes $\frac{4}{5}$, if 1 is added to both numerator and denominator. If however, 5 is subtracted from both numerator and denominator, the fraction becomes $\frac{1}{2}$. What is the fraction ?

Q.22 A fraction is such that if the numerator is multiplied by 3 and the denominator is reduced by 3, we get $\frac{18}{11}$. But , if the numerator is increased by 8 and the denominator is doubled, we get $\frac{2}{5}$. Find the fraction ?

Q.23 The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator . Determine the fraction.

Q.24 The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Q.25 The sum of the numerator and denominator of a fraction is 4 more than twice the numerator . If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction .

Q.26. The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator by 1, the numerator becomes half the denominator. Determine the fraction.

Based On ages :

Q.27. If twice the son's age in years is added to the father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son .

Q.28. A father is three times as old as his son. After twelve years, his age will be twice as that of his son then. Find their present ages.

Q.29. I am three times as old as my son. Five years later, I shall be two and a half times as old as my son. How old am I and how old is my son ?

Q.30 Ten years ago, father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be. Find their present ages.

Q.31 Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.

Q.32 The present age of a father is three years more than three times the age of the son. Three years hence, father's age will be 10 years more than twice the age the son. Determine their present ages.

Q.33 A and B are friends and their ages differ by 2 years. A's father D is twice as old as A and B is twice as old as his sister C. The age of D and C differ by 40 years. Find the ages of A and B.

Q.34 A is elder to B by 2 years. A's father F is twice as old as A and B is twice as old his sister S. If the ages of the father and sister differ by 40 years, find the age of A.

Q.35 Father's age is three times the sum of ages of his two children . After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Based On Time, Distance And Speed :

Q.36 Points A and B are 90 km. apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction, they meet in 9 hours and if they go in opposite directions, they meet in 9/7 hours. Find their speeds.

Q.37 Points A and B are 70 km. apart on a highway. A car starts from A and another from B simultaneously. If they ravel in the same direction, they meet in 7 hours but if they travel towards each other they meet in one hour. Find the speeds of the two cars.

Q.38 Points A and B are 80 km. apart from each other on a highway. A car starts from A and another from B at the same time. If they move in the same direction, they meet in 8 hours and if move in opposite directions, they meet in one hour and twenty minutes. Find the speeds of the two cars.

Q.39 Rahul travels 600 km to his home partly by train and partly by car. He takes 8 hours if he travels 120 km by train and rest by car. He takes 20 minutes longer if he travels 200 km by train and the rest by car. Find the speed of the train and the car.

Q.40 A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But , if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

Q.41 A boat covers 32 m upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of he boat in still water and that of the stream.

Q.42 A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current .

Q.43 The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours. It can go 40 km upstream and 55 km downstream. Determine the speed of stream and that of the boat in still water.

Q.44 A boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6^{1/2}$ hrs. Find the speed of the boat in still water and also speed of the stream.

Q.45 X takes 3 hours more than Y to walk 30 km, But, if X doubles his pace, he is ahead of Y by $1^{1/2}$ hours. Find their speed of walking.

Q.46 While covering a distance of 30 km. Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed, he would take 1 hour less than Amit. Find their speeds of walking .

Q.47 A man walks a certain distance with certain speed. If he walks $\frac{1}{2}$ km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. Find the distance covered by the man and his original rate of walking.

Q.48 A train covered a certain distances at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/h, it would have taken 6 hours more than the scheduled time. Find the length of the journey .

Based on geometrical applications :

Q.49 In a $\Delta ABC, \angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Q.50 Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (2x-1)^0, \angle B = (y+5)^0, \angle C = (2y+15)^0$ and $\angle D = (4x-7)^0$.

Q.51 In a $\Delta ABC, \angle A = x^0, \angle B = 3x^0$ and $\angle C = y^0$. If 3y – 5x = 30, prove that the triangle is right angled.

Q.52 The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units. Find the length and breadth of the rectangle .

Q.53 If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle .

Q.54 In a rectangle, if the length is increased by 3 meters and breadth is decreased by 4 metres, the area of the rectangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres. Find the dimensions of the rectangle .

Miscellaneous problems :

Q.55 A man starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was Rs.1500 after 4 years of service and Rs. 1800 after 10 years of service, what was his starting salary and what is the annual increment ?

Q.56 A railway half ticket costs half the full fare and the reservation charge is the same o half ticket as on full ticket. One reserved first class ticket from Mumbai to Ahmadabad costs Rs 216 and one full and one half reserved first class tickets cost Rs. 327. What is the basic first class full fare and what is the reservation charge ?

Q.57 Meena went to a bank to withdraw Rs. 2000. She asked the cashier to given her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received.

Q.58 Yash scored 40 marks in test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test ?

Q.59 The incomes of X and Y are in the ratio of 8 : 7 and their expenditures are in the ratio 19 : 16. If each saves Rs. 1250, find their incomes.

Q.60 The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them saves Rs. 200 per month, find their monthly incomes.

Q.61 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

Q.62 2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it ?

Q.63 2 women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman along to finish the embroidery, and that taken by 1 man alone.

Q.64 Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row, there would be 3 rows more. Find the number of students in the class.

Q.65 The students of a class are made to stand in rows. If 3 students are extra in raw, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

SUBJECTIVE **ANSWER KEY** **EXERCISE -3(X)-SBSE**

1. Cost of a chair = Rs. 15, Cost of a table = Rs. 500 **2.** Cost of a pen = Rs. 6.50, Cost of a pencil = Rs. 1.50

3. Rs. 150 **4.** A : 70 oranges, B : 50 oranges **5.** A : 34 mangoes, B : 62 mangoes

6. Rs. 40 , Rs. 170 **7.** Number of pens = 13, Number of pencils = 27 **8.** 25 coins of each kind.
9. Rs. 15, Rs. 3 **10.** 24, 11 **11.** 5, 3 **12.** 628, 372 **13.** 36 **14.** 93 **15.** 78 **16.** 53
17. 64 **18.** 48 **19.** 47 or 74 **20.** 35 **21.** $\frac{7}{9}$ **22.** $\frac{12}{25}$ **23.** $\frac{7}{18}$ **24.** $\frac{3}{7}$ **25.** $\frac{5}{9}$
26. $\frac{4}{7}$ **27.** Father's age = 40 years, Son's age = 15 years. **28.** Father's age = 36 years, Son's age = 12 years
29. My present age is 45 years and my son's present age is 15 fears.
30. Father's age = 34 years. Son's age = 12 years. **31.** Father's age = 40 years, Son's age = 10 years.
32. Father's age = 33 years, Son's age = 10 years.
33. A's age $= 27\frac{1}{3}$ years, B's age $= 29\frac{1}{3}$ years or A's age = 26 years, B's age = 24 years.
34. 26 years **35.** Father's age = 45 years **36.** 40 km/h & 30 km/hr
37. 40 km/h & 30 km/hr **38.** 35 km/h & 25 km/hr
39. Speed of train = 6 km/h & Speed of car = 80 km/hr **40.** Speed of train = 100 km/h & Speed of car = 80km/hr
41. Speed of boat = 10 km/h & Speed of steam = 2 km/hr **42.** Speed of sailor = 10 km/h & Speed of current =2km/hr
43. Speed of boat = 8 km/h & Speed of stream = 3 km/hr **44.** Speed of boat = 10 km/h & Speed of stream = 4 km/hr
45. X's speed $= \frac{10}{3}$ km/hr, Y's speed = 5 km/hr **46.** Ajeet's speed = 5 km/h & Amit's speed = 7.5 km/hr
47. Distance = 36 km, original speed = 4 km/hr **48.** 720 km
49. $\angle A = 20^0, \angle B = 40^0, \angle C = 120^0$ **50.** $\angle A = 65^0, \angle B = 55^0, \angle C = 115^0, \angle D = 15^0$
52. Length = 17 units breadth = 9 units **53.** 253 Sq. units
54. Length = 28 m, Bredth = 19 m
55. Starting salary = Rs. 1300, Annual increment = Rs. 50 **56.** Fare = Rs. 21, Reservation charge = Rs. 6
57. 10, 15 **58.** 20 **59.** X;s income = Rs. 6000, Y's income = Rs. 5250 **60.** Rs. 1800, Rs. 1400
61. Man : 140 days, Boy : 280 days. **62.** Man : 15 days, Boy : 60 days. **63.** Woman : 36 days, Man : 18 days
68. 60 **65.** 36

EXERCISE -4 (FOR SCHOOL / BOARD EXAMS)

PREVIOUS YEARS BOARD (SBSE) EMERSIONS

Short Answer Type – I

1. Find the value of k for which the following system of linear equations has infinite number of solutions
x + (k + 1) y = 5 ; (k + 1) x + 9y = 8k – 1 **[AI-2003]**

2. Find the value of k so that the system of linear equations will have infinite number of solutions :
x + (k + 2) y = 4 ; (2k – 1) x + 25y = 6k + 2 **[foreign-2003]**

3. Solve the following system of linear equations :
2(ax – by) + (a + 4b) = 0, 2(bx + ay) + (b – 4a) = 0.

OR

Two years ago, a father was five times as old as his son. Two years later , his age will be 8 more than three times the age of the son. Find the present ages of father and son. **[Delhi-2003]**

4. Solve the following system of linear equations : 6(ax + by) = 3a + 2b ; 6(bx – ay) = 3b – 2a .

OR

The sum of the digits of a two digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number **[AI-2004]**

5. Solve the following system of linear equations : 3(bx + ay) = a - +b, 3(ax – by) = - (6a + b).

OR

If 1 is added to each of numerator and denominator of a fraction, it becomes 2/3. However, if 1 is subtracted form each of numerator and denominator it becomes 3/5. Find the fraction. **[Foreign-2003]**

6. solve for x and y : $\frac{4}{x} + 3y = 14, \ \frac{3}{x} - 4y = 23$

OR

Solve for x and y : $\frac{b}{a}x+\frac{b}{b}y=a^2+b^2,\; x+y=2ab.$ **[Delhi-2004C]**

7. If (x – 4) is a factor of $x^3 + ax^2 + 2bx – 24$ and a – b = 8, find the values of a and b. **[Delhi-2004C]**

8. If (x + 3) is a factor of $x^3 + ax^2 – bx + 6$ and a + b = 7, find the values of a and b **[Delhi-2004C]**

9. If (x + 2) is a factor of $x^3 + ax^2 + 4bx + 12$ and a + b = - 4, find the values of a and b. **[Delhi-2004C]**

10. Solve for x and y : $\frac{2}{x}+\frac{3}{y}=13,\; \frac{5}{x}-\frac{4}{y}=-2, x, y\neq 0$

OR

Solve for x and y : ax + by – a + b = 0, bx – ay – a – b = 0 **[AI-2004C]**

11. If (x – 2) is a factor of x3+ax2+bx+18 and a – b = 7, find a and b. **[AI-2004C]**

12. Solve the following system of linear equations : ax + by = a – b, bx – ay = a + b. **[Delhi-2004C]**

13. Solve for x and y : $\frac{x}{a}+\frac{y}{b}=2,\; ax-by=a^2-b^2$

OR

A two digit number is four times the sum of its digits and twice the product of the digits. Find the number. **[AI-2005]**

14. Solve for x and y : $\frac{x}{a}-\frac{y}{b}=a-b,\; ax+by=a^3+b^3.$

OR

A number consisting of two digit, is equal to 7 times the sum of its digits. When 27 is subtracted from the number , the digit interchange places. Find the number . **[Foreign-2005]**

15. Solve for x and y : $\frac{2a}{x}+\frac{3b}{y}+1=0;\; \frac{3a}{x}-\frac{b}{y}-4=0$ **[Delhi -2005]**

16. Solve for x and y : $\frac{3a}{x}-\frac{2b}{y}+=0;\; \frac{a}{x}+\frac{3b}{y}-2=0$ **[AI-2005C]**

17. Solve for x and y ; 47x + 31y = 63, 31x + 47 = 15

OR

Solve for x and y : $\frac{ax}{b}-\frac{by}{a}=a+b; ax-by=2ab$ **[Delhi -2006]**

18. Solve the system of equations :

$\frac{bx}{a}-\frac{ay}{b}+a+b=0$ and $bx-ay+2ab=0$

OR

The sum of ht digits of a two digit number is 12. the number obtained by interchanging the two digits exceeds the given number by 18. Find the number **[AI-2006]**

19. Solve the system of equations for x ; $\frac{b^2x}{a}-\frac{a^2y}{b}=ab(a+b)$ and $b^2x-a^2y=2a^2b^2$

OR

A man sold a table and a chair together for Rs. 850 at a loss of 10% on the table and a gain of 10% on the chair. By selling them together for Rs. 950, he would have made a gain of 10% on the table and loss of 10% on the chair. Find the cost price of each . **[Foreign-2006]**

20. Solve the following equations for x and y : $mx – ny = m^2 + n^2$, x + y = 2m

OR

Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 2650 km by train and 240 km taxi he takes 6 minutes longer. Find the speed of the train and that of the taxi **[Delhi-2006C]**

21. Solve the following equations for x and y : $\frac{a^2}{x}-\frac{b^2}{y}=0\,;\; \frac{a^2b}{x}+\frac{b^2a}{y}=a+bx, y\neq 0.$

OR

The sum of the numerator and the denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction. **[AI-2006C]**

22. Solve for x and y : $x + \frac{6}{y} = 6,\ 3x - \frac{8}{y} = 5$

OR

Solve for x and y : $\frac{x+1}{2} + \frac{y-1}{3} = 8\ ;\ \frac{x-1}{3} + \frac{y+1}{2} = 9$ **[Delhi -2007]**

23. Solve for x and y : 8x – 9y = 6xy ; 10x + 6y = 19xy

OR

Solve for x and y : $4x + \frac{y}{3} = \frac{8}{3};\ \frac{x}{2} + \frac{3y}{4} = -\frac{5}{2}$ **[AI-2007]**

24. Find the value of k so that the following system of equations has no solution :
3x – y = 5 ; 6x – 2y – k = 0 **[Delhi-2008]**

25. Find the value of k so that the following system of equations has infinite solutions :
3x – y – 5 = 0 ; 6x – 2y + k = 0 **[Delhi-2008]**

26. Find the value (s) of k for which the pair of linear equations kx + 3y = k – 2 and 12x + ky = k has no solution **[Delhi-2008]**

27. Find the number of solutions of the following pair of linear equations :
x + 2y – 8 = 0 ; 2x + 4y = 16 **[AI-2009]**

28. Write whether the following pair of linear equations is consistent or not :
x + y = 14 ; x – y = 4 **[Foreign-2009]**

29. Without drawing the graph find out whether the lines representing the following pair of liner equations intersect at a point, are parallel or coincident : 9x – 10y = 21 ; $\frac{3}{2}x - \frac{5}{3}y = \frac{7}{2}$ **[Foreign-2009]**

30. Without drawing the graph find out whether the lines representing the following pair of linear equations intersect at a point , are parallel or coincident : 48x – 7y = 24 ; $\frac{9}{5}x - \frac{7}{10}y = \frac{9}{10}$ **[Foreign-2009]**

31. Without drawing the graph, find out whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident : 5x + 3y – 6 = ; $\frac{9}{5}$x + 3y = 6 **[Foreign-2009]**

SHORT ANSWER TYPE –II

1. Solve the following system of linear equations graphically : 2x – 3y = 1, 3x – 4y = 1 Does the point (3, 2) lie on any of the lies ? Write its equation **[Delhi-2003]**

2. Solve for x and y : $\frac{4}{x}$ + 5y = 7, $\frac{3}{x}$ + 4y = 5

OR

Father's age is three times the sum of ages of his two children . After 5 years his age will be twice the sum of age of two children. Find the age of father . **[Delhi-2003]**

3. Solve the following system of liner equations graphically : 3x – 5y = 19, 3y – 7x + 1 = 0 Does the point (4, 9) lie on any of the lines ? Write its equations **[AI-2003]**

4. Solve the following system of linear equations graphically : 2x + y = 10, 4x – y = 8. Does the point (1, -4) lie on any of the lines ? Write its equation. **[Foreign-2003]**

5. The sum of numerator and denominator of a fraction is 8. is added to both the numerator and denominator the fraction becomes 3/4. Find the fraction. **[AI-2003]**

6. Solve the following system of equations : $\frac{a}{x} - \frac{b}{y} = 0, \frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2\ ;\ x, y \neq 0.$

OR

5 years hence the age of a father shall be three times the age of his son while 5 years earlier the age of the father was 7 times the age of his son. Find their present ages. **[Foreign-2003]**

7. Solve the following system of linear equations graphically : 4x – 5y – 20 = 0, 3x + 5y – 15 = 0. Determine the vertices of the triangle formed by the lines, representing the above equations, and the y-axis. **[Delhi-2004]**

8. Solve the following system of linear equations graphically : 5x – 6y + 30 = 0, 5x + 4y – 20 = 0. Also find the vertices of the triangle formed by the above two lines and x-axis. **[AI-2004]**

9. Solve the following system of linear equations graphically : 2x + y + 6 = 0, 3x – 2y – 12 = 0. Also find the vertices of the triangle formed by the lines representing the above equations and x-axis. **[Foreign-2004]**

10. Solve the following system of linear equations graphically : 2x + 3y = 4, 3x – y = - 5. Shade the region bounded by the above lines and the x-axis. **[Delhi-2004C]**

11. Solve the following system of linear equations graphically : 3x + y = 1 = 0, 2x – 3y + 8 = 0 Shade the region bounded by the lines and the x-axis. **[AI-2004C]**

12. The monthly incomes of A and B are in the ratio of 9 : 7 and their monthly expenditures are in the ratio of 4 : 3 If each saves Rs. 1600 per month, find the monthly incomes of each. **[AI-2004C]**

13. Solve the following system of equations graphically : x + 2y = 5, 2x – 3y = - 4. Also find the points where the lines meet the x-axis. **[Delhi-2005]**

14. Solve the following system of equations graphically : 2x – y = 4 ; 3y – x = 3. Find the points where the lines meet the y-axis. **[AI-2005]**

15. Solve the following system of equations graphically : 3x – y = 3, x – 2y = - 4. Shade the are of the region bounded by the lines and x-axis. **[Delhi-2005C]**

16. Draw the graphs of the equations : 4x – y – 8 = 0 and 2x – 3y + 6 = 0. Also determine the vertices of the triangle formed by the lines and x-axis. **[Delhi-2006]**

17. Draw the graphs of the following equations : 3x – 4y + 6 = 0; 3x + y – 9 = 0. Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis. **[AI-2006]**

18. Draw the graphs of the equations : 4x – 3y – 6 = 0 ; x + 3y – 9 = 0. Determine the co-ordinates of the vertices of the triangle formed by the lines and the y-axis. **[Foreign-2006]**

19. Solve the following system of linear equations graphically : 3x – 2y – 1 = 0 ; 2x – 3y + 6 = 0. Shade the region bounded by the lines and x-axis. **[Delhi-2006C]**

20. Solve the following system of equations graphically for x and y : 3x +2y = 12 ; 5x – 2y = 4. Find the co-ordinates of the points where the lines meet the y-axis. **[AI-2006C]**

21. Solve the following system of equations graphically . 2x + 3y = 8 ; x + 4y = 9. **[Delhi-2007]**

22. Solve the following system of linear equations graphically. 2x + 3y = 12 ; 2y – 1 = x **[AI-2007]**

23. Represent the following system of linear equations graphically. From the graph, find the points where the lines intersect y-axis. 3x + y – = 0 ; 2x – y – 5 = 0. **[Delhi-2008]**

24. Solve for x and y : (a – b) x + (a + b) y = $a^2 – 2ab – b^2$; (a + b) (x + y) = $a^2 + b^2$.

OR

Solve for x and y : 37x + 43y = 123 ; 43x + 37y = 117. **[AI-2008]**

25. Represent the following pair of equations graphically and write and co-ordinates of points where the lines intersect y-axis ; x + 3y = 16 ; 2x – 3y = 12. **[Foreign-2008]**

26. Solve the following pair of equations : $\frac{5}{x-1}+\frac{1}{y-2}=2;\frac{6}{x-1}-\frac{3}{y-2}=1$ **[Delhi-2009]**

27. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds. They meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars ? **[Delhi-2009]**

28. Solve the following pair of equations : $\frac{10}{x+y}+\frac{2}{x-y}=4;\frac{15}{x+y}-\frac{2}{x-y}=-2$ **[Delhi-2009]**

29. Solve for x and y : $\frac{ax}{b} - \frac{by}{a} = a + b$; $ax - by = 2ab$. **[AI -2009]**

SUBJECTIVE **ANSWER KEY** **EXERCISE-4 (X) CBSE**

- **Short Answer Type-I**

1. k = 2 **2.** k = 3 **3.** x = -1/2, y = 2 or 42yrs, 10yrs **4.** x = 1/2, y = 1/3 or 78 **5.** x = -2, y = 1/3 or 7/11
6. x = 1/5, y = -2 or x = ab, y = ab **7.** a = 1, b = - 7 **8.** a = 0, b = 7 **9.** a = -3, b = -1
10. x = 1/2, y = 1/3 or x = 1, y = - 1 **11.** a = -2, b = - 9 **12.** x = 1, y = - 1 **13.** x = a, y = b or 36
14. $x = a^2$, $y = b^2$ or 63 **15.** x = a, y = - b **16.** x = - a, y = b **17.** x = 2, y = -1 or x = b, y = - a
18. x = - a, y = b or 57 **19.** $x = a^2$, $y = -b^2$ or cast of table = Rs. 700, cost of chair = Rs 200
20. x = m + n, y = m – n or speed of train = 100 km/h. Speed of taxi = 80 km/h **21.** $x = a^2$, $y = b^2$ or $\frac{5}{7}$
22. $x = -\frac{14}{5}$, $y = \frac{1}{13}$ or x = 7, y = 13 **23.** x = 3/2, y = 2/3 or x = 1, y = - 4 **24.** $k \neq 10$ **25.** k = - 10 **26.** $k = \pm 6$
27. Infinite number of solutions **28.** Consistent **29.** Coincident lines **30.** Parallel **31.** Unique solution.

- **Short Answer Type-II**

1. (-1, -1) ; Yes ; 3x – 4y = 1 **2.** x = 1/3, y = - or 45 years **3.** (-2, -5) ; Yes ; 3y – 7x + 1 = 0 **4.** (3, 4) ; yes ; 4x – y = 8
5. 3/5 **6.** (a, b) or 40, 10 years **7.** (0, -4), (5, 0), (0, 3) **8.** (-6, 0), (0, 5), (4,0) **9.** (-3, 0), (0, -6), (4, 0)
12. A's = Rs. 14400, B's = Rs. 11200 **13.** (5, 0), (-2, 0) **14.** (0, -4), (0, 1) **16.** (-3, 0), (2, 0), (3, 4) **17.** (-2, 0), (2, 3), (3, 0)
18. (0, 3), (3, 2), (0, -2) **19.** x = 3, y = 4 **20.** x = 2, y = 3 **21.** x = 1, y = 2 **22.** x = 3, y = 2 **23.** (0, 5) and (0, -5)
24. x = a + b, $y = \frac{-2ab}{a+b}$ or x = 1, y = 2 **25.** (0, 2) and (0, -4) **26.** x = 4, y = 5 **27.** 60 km/h ; 40 km/h
28. x = 3 ; y = 2 **29.** x = b, y = - a

EXERCISE -5 (FOR OLYMPIADS)

Choose The Correct One

1. The number of solutions of the equation 2x + y = 40, where both x and y are positive integers and $x \leq y$ is :
(A) 7 (B) 13 (C) 14 (D) 18

2. A confused bank teller transposed the rupees and paise when he cashed a cherub for Mansi, giving her rupees instead of paisa and paise instead of rupees. After buying a toffee for 50 paise, Mansi noticed that she was left with exactly three times as much as the amount on the cheque. Which of the following is a valid statement about the cheque amount ?
(A) Over Rs. 4 but less than Rs. 5 (B) Over Rs. 13 but less than Rs. 14
(C) Over Rs. 7 but less than Rs. 8 (D) Over Rs. 18 but less than Rs. 19

3. John inherited $25000 and invested part of it in a money market account, part in municipal bounds, and part in a mutual fund. After one year, he received a total of $ 1620 in simple interest from the three investments. The money market paid 6% annually, the bonds paid 7% annually, and the mutual funds paid 8% annually. There was $ 6000 more invested in the bonds than the mutual funds. The amount John invested in each category are in the ratio :
(A) 15 : 8 : 2 (B) 11 : 13 : 1 (C) 2 : 2 : 1 (D) None of these

4. Which one of the following conditions must p,q and r satisfy so that the following system of linear simultaneous equations has at least one solution , such that $p + q + r \neq 0$?

$$x + 2y - 3z = p \; ; \; 2x + 6y - 11z = q \; ; \; x - 2y + 7z = r$$

(A) 5p – 2q – r = 0 (B) 5p + 2q + r = 0 (C) 5p + 2q – r = 0 (D) 5p – 2q + r = 0

5. If x and y are integers, then the equation 5x + 19y = 64 has :
(A) No solution for x < 300 and y < 0 (B) No solution for x > 250 and y > - 100
(C) A solution for 250 < x < 300 (D) A solution for – 59 < y < - 56

6. The number of solutions of the equation 2x + y = 40, where both x and y are positive integers and $x \leq y$ is :

(A) 7 (B) 13 (C) 14 (D) 18

7. Study the question and statements given below : Decide whether any information provided in the statement (s) is redundant and / or can be dispensed with, to answer it.
If 7 is added to numerator and denominator each of fraction a/b. will the new fraction be less than the original one ? (Assume both a and b to be positive)
Statement-I : a = 73, b = 103
Statement-II : The average of a and b is less than b.
Statement-III : a – 5 is greater than b – 5.
(A) II and either I or III (B) Only I or III (C) Any two of them (D) Any one of them

8. A cyclist drove 1 km, with the wind in his back , in 3 min and drove the same way back, against the wind in 4 min. If we assume that the cyclist always puts constant force on the pedals, how much time would it take to drive 1 km without wind ?
(A) $2\frac{1}{3}$ min. (B) $3\frac{3}{7}$ min. (C) $2\frac{3}{7}$ min. (D) $3\frac{7}{12}$ min.

9. A person buys 18 local tickets for Rs. 110. Each first class ticket costs Rs. 10 and each second class ticket costs Rs. 3. What will another lot of 18 tickets in which the number of first class and second class tickets are interchanged cost ?
(A) Rs. 112 (B) Rs. 118 (C) Rs. 121 (D) Rs. 124

10. Rajesh walks to and fro to a shopping mall. He spends 30 min. shopping. If he walks at a speed of 10 km/h, he returns to home at 19:00h. It he walks at 15 km/h. he returns at 18:30 h. How fast must he walk in order to return home at 18:15 h ?
(A) 17 km/h (B) 17.5 km/h (C) 18 km/h (D) 20 km/h

11. A single reservoir supplies the petrol to the whole city, while the reservoir is fed by a single pipeline filling the reservoir with the stream of uniform volume. When the reservoir is full and if 40000 liters of petrol is used daily, the supply fails in 90 days. If 32000 liters of petrol is used daily, the supply fails in 60 days. How much petrol can be used daily without the supply ever failing ?
(A) 64000 litres (B) 56000 litres (C) 78000 litres (D) 60000 litres

12. Two horses start trotting towards each other, one from A to B and another from B to A . They cross each other after one hour and the first horse reaches B, 5/6 hours before the second hoarse reaches A. If the distance between A and B is 50 km. What is the speed of the slower hours ?
(A) 30 km/h (B) 15 km/h (C) 25 km/h (D) 20 km/h

13. A man row downstream at 12 km/h and upstream at 8 km/h. What is the speed of man in still water ?
(A) 12 km/h (B) 10 km/h (C) 8 km/h (D) 9 km/h

14. A motor boat takes 12 hours to go downstream and it takes 24 hours to return the same distance. What is the time taken by boat in still water ?
(A) 15 h (B) 16 h (C) 8 h (D) 20 h

15. Equation $xy^2 + xy^2 = 2xy$, $x, y \neq 0$ is
(A) Linear (B) Quadratic (C) Cubic (D) Not an equation

16. Sum of two integers is 88. If the greater is divided by the smaller, the quotient is 5 and the remainder is 10. the greater integer is :
(A) 13 (B) 75 (C) 65 (D) 23

17. The length of the sides of a triangle are $3x+2y, 4x+\frac{4}{3}y$ and $3(x+1)+\frac{3}{2}(y-1)$. If the triangle is equilateral , then its side is
(A) 8 (B) 10 (C) 12 (D) 16

18. The largest angle of a triangle is twice the sum of the other two. The smaller angle is one fourth of the largest. The largest angle is :
(A) 90^0 (B) 60^0 (C) 120^0 (D) None of these

19. In town, $\frac{2}{3}$ of men are married to $\frac{3}{7}$ of the women . In the town total population is more than 1000. If all marriages happen within the town. The smallest possible number of total population is (assume there are only adults in the town) :
(A) 1012 (B) 1035 (C) 1058 (D) None of these

20. The solution of the equations : $\frac{x}{4}=\frac{y}{3}=\frac{z}{2},\quad 7x+8y+5z=62$ is :

(A) (4, 3, 2) (B) (2, 3, 4) (C) (3, 4, 2) (D) (4, 2, 3)

21. If $\frac{1}{3}(x+y)2z=21,\ 3x-\frac{1}{2}(y+z)=65,\quad x+\frac{1}{2}(x+y-z)=38$, then its solution is :

(A) (24, 9, 5) (B) (2, 9, 5) (C) (4, 9, 5) (D) (5, 24, 9)

22. The solution of the equations : $\frac{xy}{y-x}=110,\ \frac{yz}{z-y}=132,\ \frac{zx}{z+x}=\frac{60}{11}$ is :

(A) (12, 11, 10) (B) (10, 11, 12) (C) (11, 10, 12) (D) (12, 10, 11)

23. Four men earn as much in a day as 7 women. 1 women earns as much as 2 boys. If 6 men, 10 women and 14 boys work together for 8 days to earn Rs. 2200, then what will be the earning of 8 men and 6 women working together is for 10 days ?

(A) Rs. 2000 (B) Rs. 1800 (C) Rs. 2400 (D) None of these

24. The point of intersection of the straight lines 2x – y + 3 = 0, 3x – 7y + 10 = 0 lies in :

(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant

25. A right-angled triangle is formed by the straight line : 4x + 3y = 12 with both the axis. Then length of perpendicular from the origin to the hypotenuse is :

(A) 3.5 units (B) 2.4 units (C) 4.2 units (D) None of these

OBJECTIVE — ANSWER KEY — EXERCISE - 5

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	A	A	C	B	C	B	D	D	B	D	B	B	A
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	B	C	C	A	A	A	B	A	B	B					

COMPETITION WINDOW

LINEAR IN EQUATIONS

Inequation : A statement involving variable (s) and the sign of inequality viz, $<, >, \leq$ or $\geq$ is called an inequation or an inequality.

An inequation may contain one or more variables. Also, is may be linear or quadratic or cubic etc.

E.g. (i) 3x – 2 < 0 (ii) 2x + 3y < 1 (iii) x2 – 5x + 4 ≤ 0

Linear Inequation In One Variable : Let a be a non-zero real number and x be a variable. Then inequations of the form ax + b < 0, ax + b ≤ 0, ax + b > 0 and ax + b ≥ 0 are known as linear equations in one variable x.

E.g. 9x – 15 > 0, 5x – 4 ≥ 0, 3x + 2 < 0, 2x – 3 ≤ 0

Solving Linear Inequation In One Variable : In the process of solving an inequation, we use mathematical simplifications which are governed by the following rules :

Rule-I : Same number may be added (or subtracted from) both sides of an inequation without changing the sign of inequality

Rule-II: Both sides of an inequation can be multiplied (or divided) by the same positive real number without changing the sign of inequality. However the sign of inequality is reversed when both sides of an inequation are multiplied (or divided) by a negative number.

Rule-III : Any tern of an inequation may be taken to the other side with it's sign changed without affecting the sign of inequality

Ex. Solve : 5x – 3 < 3x + 1, when (i) x is a real number (ii) x is an integer (iii) x is a natural number.

Sol. We have

5x – 3 < 3x + 1

⇒ 5x – 3x < 3x + 1 [Transposing 3x on LHS and – 3 on RHS]

$\Rightarrow \quad 2x < 4 \Rightarrow \frac{2x}{2} < \frac{4}{2} \Rightarrow x < 2$

(i) If $x \in R$, then $x < 2 \Rightarrow x \in (-\infty, 2)$

(ii) If $x \in Z$, then $x < 2 \Rightarrow x = 1, 0, -1, -2, -3, -4, \ldots\ldots\ldots..$

(iii) If $x \in N$, then $x < 2 \Rightarrow x = 1$

Ex. Solve ; $\frac{2x+4}{x-1} \geq 5$

Sol. We have,

$$\frac{2x+4}{x-1} \geq 5$$

$\Rightarrow \quad \frac{2x+4}{x-1} - 5 \geq 0 \Rightarrow \frac{-3x+9}{x-1} \geq 0$

$\Rightarrow \quad \frac{3x-9}{x-1} \leq 0$ [Multiplying both sides by – 1]

$\Rightarrow \quad \frac{x-3}{x-1} \leq 0$ [Dividing both sides by 3]

$\Rightarrow \quad 1 < x \leq 3 \Rightarrow x \in (1, \ 3)$

EXERCISE-6 (FOR IIT-JEE/AIEEE)

Choose The Correct One

1. Solve : $3x - 7 > x + 1, x \in R$:

(A) $(4, \infty)$ (B) $[4, \infty)$

(C) $(-\infty, \ 4]$ (D) $(-\infty, \ 4]$

2. Solve : $\frac{x}{5} < \frac{3x-2}{4} - \frac{5x-3}{5}, x \in R$:

(A) $(2/9, \ \infty)$ (B) $[2/9, \ \infty)$

(C) $(-\infty, \ \ 2.9)$ (D) $(-\infty, \ \ 2.9)$

3. Solve : $\frac{2(x-1)}{5} \leq \frac{3(2+x)}{7}, x \in R$:

(A) $(44, \ \infty)$ (B) $[44, \ \infty)$

(C) $[-44, \ \infty)$ (D) $(-44, \ \ \infty)$

4. Solve : $\frac{5x}{2} + \frac{3x}{4} \geq \frac{39}{4}, x \in R$:

(A) $(-3, \ \infty)$ (B) $(3, \ \infty)$

(C) $(-\infty, \ \ 3)$ (D) $[3, \ \ \infty)$

5. Solve : $\frac{x-1}{3} + 4 < \frac{x-5}{5} - 2, x \in R$:

(A) $(50, \ \infty)$ (B) $(-\infty, \ -50)$

(C) $[50, \ \ \infty)$ (D) None of these

6. Solve : $\dfrac{2x+3}{4}-3<\dfrac{x-4}{3}-2, x\in R$:

(A) $(13/2, \infty)$ (B) $(-\infty, -13/2)$
(C) $(-13/2, \infty)$ (D) $[13/2, \infty)$

7. Solve : $\dfrac{5-2x}{3}<\dfrac{x}{6}-5, x\in R$:

(A) $(8, \infty)$ (B) $[8, \infty)$ (C) $(-\infty, -8)$ (D) $(-\infty, 8)$

8. Solve : $\dfrac{4+2x}{3}\geq\dfrac{x}{2}-3, x\in R$:

(A) $(26, \infty)$ (B) $(-\infty, 26]$ (C) $[-26, \infty)$ (D) $(-\infty, -26)$

9. Solve : $\dfrac{2x+3}{5}-2<\dfrac{3(x-2)}{5}, x\in R$:

(A) $(-1, \infty)$ (B) $[1, \infty)$ (C) $(-\infty, -1)$ (D) $(-\infty, 1)$

10. Solve : $x-2\leq\dfrac{5x+8}{3}, x\in R$

(A) $[-7, \infty)$ (B) $(7, \infty)$ (C) $(-\infty, 7)$ (D) $(-\infty, 7)$

11. Solve : $\dfrac{6x-5}{4x+1}<0, x\in R$:

(A) $(-1/4, 5/6)$ (B) $[-1/4, 5/6]$ (C) $(-\infty, -1/4)$ (D) $(5/6, \infty)$

12. Solve : $\dfrac{2x-3}{3x-7}>0, x\in R$

(A) $[3/2, 7/3]$ (B) $(3/2, 7/3)$
(C) $(-\infty, 3/2)\cup(7/3, \infty)$ (D) None of these

13. Solve : $\dfrac{3}{x-2}<1, x\in R$:

(A) $(2, 5)$ (B) $[-\infty, 2)\cup(5, \infty)$
(C) $(-\infty, -2)\cup(5, \infty)$ (D) None of these

14. Solve : $\dfrac{1}{x-1}\leq 2, x\in R$:

(A) $(-1, 3/2]$ (B) $(-\infty, -1)\cup(3/2, \infty)$
(C) $(-1, 3/2)$ (D) $(-\infty, 1)\cup[3/2, \infty)$

15. Solve : $\dfrac{4x+3}{2x-5}<6, x\in R$

(A) $(5/2, 33/8)$ (B) $(-\infty, -5)\cup(4, \infty)$
(C) $(-5/2, 33/8)$ (D) $(-\infty, 5/2)\cup(33/8, \infty)$

16. Solve : $\dfrac{5x-6}{x+6}<1, x\in R$:

(A) $(-6, -3)$ (B) $(6, \infty)$ (C) $(-6, 3)$ (D) None of these

17. Solve : $\dfrac{5x+8}{4-x}<2, x\in R$:

(A) $[0,, 4)$ (B) $[4, \infty)$ (C) $(-\infty, 4)$ (D) $(-\infty, 0)\cup(4, \infty)$

18. Solve : $\frac{x-1}{x+3} > 2, x \in R$:

(A) $(7, 3)$ (B) $[7, 3]$ (C) $(-7, -3)$ (D) $[-7, -3]$

19. Solve : $\frac{7x-5}{8x+3} > 4, x \in R$:

(A) $(17/25, 3/8)$ (B) $(17/25, 3/8]$

(C) $(-17/25, -3/8)$ (D) $[17/25, 3/8]$

20. $\frac{2x-3}{3x-7} > 0, x \in R$

(A) $(-5, 5)$ (B) $[-5, 5]$

(C) $(-\infty, -5) \cup (5, \infty)$ (D) None of these

OBJECTIVE							ANSWER KEY							EXERCISE -6
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	**A**	**C**	**C**	**D**	**B**	**B**	**A**	**C**	**A**	**A**	**A**	**C**	**B**	**D**	**D**
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	**C**	**D**	**C**	**C**	**C**										

TRIGONOMETRY

★ INTRODUCTION

Trigonometry is the branch of Mathematics which deals with the measurement of angles and sides of a triangle.

The word Trigonometry is derived from three Greek roots : ‘trio’ meaning ‘thrice or Three’, ’gonia’ meaning an angle and ‘metron’ meaning measure. In fact, **Trigonometry is the study of relationship between the sides and the angles of a triangle.**

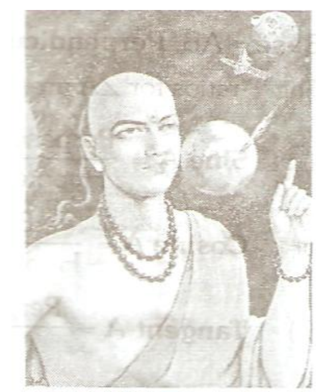
Trigonometry has its application in astronomy, geography, surveying, engineering and navigation etc. In the past, astronomers used it to find out the distance of stars and plants from the earth. Even now, the advanced technologies used in Engineering are based on trigonometric concepts.

In this chapter, we will define trigonometric ratios of angles in terms of ratios of sides of a right triangle. We will also define trigonometric ratios of angles of 0^0, 30^0, 45^0, 60^0, and 90^0. We shall also establish some identities involving these ratios.

★ HISTORICAL FACTS

Indian Mathematician has established keen interest in the study of Trigonometry since ages. They are known for their innovation in the use of size instead the use of choid. The most outstanding astronomer has been Aryabhatta.

Aryabhatta was born in 476 A.D. in Kerala. He studied in the university of Nalanda. In mathematics, Aryabhatta’s contribution are very valuable. He was the first mathematician to prepare tables of sines. His book ‘Aryabhatta’ deals with Geometry, Mensuration, Progressions, Square root, Cube root and Celestial sphere (spherical Trigonometry). This work, has won him recognition all over the world because of its logical and unambiguous presentation of astronomical observations.



ARYABHATTA (476 AD)

Aryabhatta was the pioneer to find the correct value of the constant π with respect to a circle $\left(\frac{Circumference}{Diameter} = \pi\right)$ up to four decimals as 3.1416. he found the approximate value of π and indirectly suggested that π is an irrational number. His observations and conclusions are very useful and relevant today.

Greek Mathematician Ptolemy, Father of Trigonometry proved the equation $\sin^2 A + \cos^2 A = 1$ using geometry involving a relationship between the chords of a circle. But ancient Indian used simple algebra to calculate sin A and cos A and proved this relation. Brahmagupta was the first to use algebra in trigonometry. Bhaskaracharya II (1114 A.D.) was very brilliant and most popular Mathematician. His work known as Siddhantasiromani is divided into four parts, one of which is Goladhyaya’s spherical trigonometry.

PTOLEMY (85 – 165 AD)

★ BASE, PERPENDICULAR AND HYPOTENUSE OF A RIGHT TRIANGLE

In ΔABC, if $\angle B = 90^0$, then :

(i) For $\angle A$, we have :

Base = AB, **Perpendicular** = BC and **Hypotenuse** = AC.

(ii) For $\angle$C, we have :

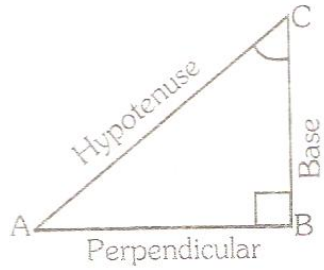
Base = BC, **Perpendicular** = AB and **Hypotenuse** = AC.

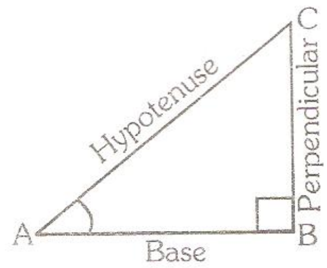
So, in a right angled triangle, for a given angle,

(i) The side opposite to the right angle is called **hypotenuse.**

(ii) The side opposite to the right angle is called **perpendicular.**

(iii) The third side (i.e., the side forming the given angle with the hypotenuse), is called **base.**





★ TRIGONOMETRICAL RARIOS (T-RATIOS) OF AN ANGLE

In ΔABC, let $\angle$B = 90^0 and let $\angle$A be acute.

For $\angle$A, we have :

Base = AB, **Perpendicular** = BC and **Hypotenuse** = AC.

The T-ratios for $\angle$A are defined as :

(i) **Sine A** = $\dfrac{Perpendicular(P)}{Hypotenuse(H)} = \dfrac{BC}{AC}$, written as **sin A.**

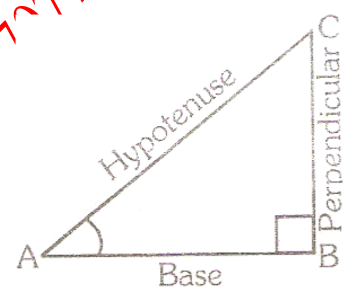
(ii) **Cosine A** = $\dfrac{Base(B)}{Hypotenuse(H)} = \dfrac{AB}{AC}$, written as **cos A.**

(iii) **Tangent A** = $\dfrac{Perpendicular(P)}{Base(B)} = \dfrac{BC}{AB}$, written as **tan A.**

(iv) **Cosecant A** = $\dfrac{Hypotenuse(H)}{Perpendicular(P)} = \dfrac{AC}{BC}$, written as **cosec A.**

(v) **Secant A** = $\dfrac{Hypotenuse(H)}{Base(B)} = \dfrac{AC}{AB}$, written as **sec A.**

(vi) **Cotangent A** = $\dfrac{Base(B)}{Perpendicular(P)} = \dfrac{AB}{BC}$, written as **cot A.**



Thus, there are six Trignometrical ratios based on the three sides of a right angled triangle.

Aid to Memory : The sine, cosine, and tangent ratios in a right triangle can be remembered by representing them as strings of letters, as in **SOH-CAH-TOA.**

Sine = **O**pposite $\div$ **H**ypotenuse

Cosine = **A**djacent $\div$ **H**ypotenuse

Tangent = **O**pposite $\div$ **A**djacent

The memorization of this mnemonic can be aided by expanding it into a phrase, such as " **S**ome **O**fficers **H**ave **C**urly **A**uburn **H**air **T**ill **O**ld **A**ge".

★ RECIPROCAL RELATIONS

Clearly, we have :

(i) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$	(ii) $\sec\theta = \dfrac{1}{\cos\theta}$	(iii) $\cot\theta = \dfrac{1}{\tan\theta}$

Thus, we have :

(i) $\sin\theta \ \text{cosec}\,\theta = 1$	(ii) $\cos\theta \ \sec\theta = 1$	(iii) $\tan\theta \ \cot\theta = 1$

★ QUOTAENT RELATIONS

Consider a right angled triangle in which for an acute angle θ, we have :

$$\sin\theta = \frac{Perpendicular}{Hypotenuse} = \frac{P}{H} \;:\; \cos\theta = \frac{Base}{Hypotenuse} = \frac{B}{H}$$

Now, $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{P}{H}}{\frac{B}{H}} = \dfrac{P}{H} \times \dfrac{H}{B} = \dfrac{P}{B}$ = **tan** θ (by def.)

and, $\dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{B}{H}}{\frac{P}{H}} = \dfrac{B}{H} \times \dfrac{H}{P} = \dfrac{B}{P}$ = **cos** θ (by def.)

Thus, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

★ POWER OF T-RATIOS

We denote :

(i) $(\sin\theta)^2$ by $\sin^2\theta$; (ii) $(\cos\theta)^2$ by $\cos^2\theta$; (iii) $(\sin\theta)^3$ by $\sin^3\theta$; (iv) $(\cos\theta)^3$ by $\sin^3\theta$;
and so on.

REMARK :

(i) The symbol sin A is used as an abbreviation for 'the sine of the angle A'. Sin A is not the product of 'sin' and A. 'sin' separated from A has no meaning. Similarly, cos A is not the product of 'cos' and A. similar interpretations follow for other trigonometric ratios also.

(ii) We may write sin2 A, cos2 A, etc., in place of (sin A)2, (cos A)2, etc., respectively. But cosec A = (sin A)-1 ≠ sin-1 A (it is called sine inverse A). sin-1 A has a different meaning, which will be discussed in higher classes. Similar conventions hold for the other trigonometric ratios as well.

(iii) Since the hypotenuse is the longest side in a right triangle, the value of sin A or cos A is always less than 1 (or, in particular, equal to 1).

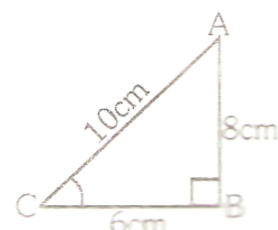
Ex.1 Using the information given in fig. write the values of all trigonometric ratios of angle C.

Sol. Using the definition of t-ratios,

$\text{SinC} = \dfrac{AB}{AC} = \dfrac{8}{10} = \dfrac{4}{5}$; $\quad \text{cosC} = \dfrac{BC}{AC} = \dfrac{6}{10} = \dfrac{3}{5}$

$\text{tanC} = \dfrac{AB}{BC} = \dfrac{6}{8} = \dfrac{4}{3}$; $\quad \text{cotC} = \dfrac{BC}{AB} = \dfrac{6}{8} = \dfrac{3}{4}$

$\text{tanC} = \dfrac{AC}{BC} = \dfrac{10}{6} = \dfrac{5}{3}$ and $\quad \text{cosC} = \dfrac{AC}{AB} = \dfrac{10}{8} = \dfrac{5}{4}$



Ex.2 In a right ΔABC, if $\angle$A is acute and tan A = $\dfrac{3}{4}$. find the remaining trigonometric ratios of $\angle$A.

Sol. Consider a ΔABC in which $\angle$B = 90^0

For $\angle$A, we have :

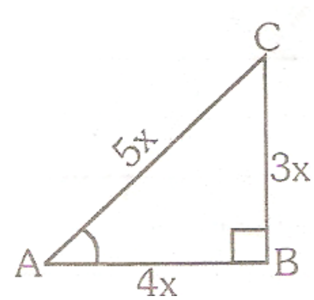
Base = AB, Perpendicular = BC and Hypotenuse = AC.

$\therefore \quad \tan A = \dfrac{Perpendicular}{Base} = \dfrac{3}{4}$

$\Rightarrow \quad \dfrac{BC}{AB} = \dfrac{3}{4}$

Let, BC = 3x units and AB = 4x units.

Then, $\quad AC = \sqrt{AB^2 + BC^2}$



$= \sqrt{(4x)^2 + (3x)^2}$

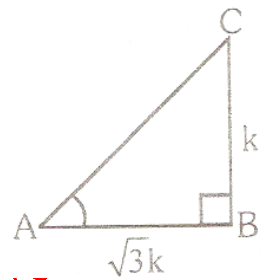
$= \sqrt{25x^2}$ = 5x units.

Ex.3 In a Δ ABC, right angled at B, if tan A = $\frac{1}{\sqrt{3}}$, find the value of

(i) sinA cosC + cosA sinC

(ii) cosA cosC – sinA sinC. **[NCERT]**

Sol. We know that



tanA = $\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

∴ BC : AB = 1 : $\sqrt{3}$

Let BC = k and AB = $\sqrt{3}$ k

Then, AC = $\sqrt{AB^2 + BC^2}$ …(Phythagoras theorem)

$= \sqrt{(\sqrt{3}k)^2 + (k)^2} = \sqrt{3k^2 + k^2}$

$= \sqrt{4k^2}$ = 2k

Now, sinA = $\frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

cosA = $\frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

sinC = $\frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

and cosC = $\frac{BC}{AC} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) sinA cosC + cosA sinC = $\frac{1}{2}.\frac{1}{2} + \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$

(ii) cosA cosC – sinA sinC = $\frac{\sqrt{3}}{2}\frac{1}{2} - \frac{1}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

Ex.4 If sinA = $\frac{1}{2}$, verify that 2 sinA cosA = $\frac{2\tan A}{1+\tan^2 A}$

Sol. We know that

SinA = $\frac{BC}{AC} = \frac{1}{2}$

Let BC = k and AC = 2k

∴ AB = $\sqrt{AC^2 - AB^2}$

$= \sqrt{(2k)^2 - k^2} = \sqrt{4k^2 - k^2} = \sqrt{3k^2} = \sqrt{3k}$

Now cosA = $\frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

and $\quad \cos A = \frac{BC}{AB} = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$

Now $\quad 2 \sin A \cos A = 2.\frac{1}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ ...(i)

and $\frac{2\tan A}{1+\tan^2 A} = \frac{2\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3}{4}}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$ ...(ii)

Ex.5 In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Find the value of sinP, cosP and tanP. **[NCERT]**

Sol. We are given

PR + QR = 25 cm

$\therefore$ PR = (25 – QR) cm

By Pythagoras theorem,

$PR^2 = QR^2 + PQ^2$

or $(25 - QR)^2 = QR^2 + 5^2$

or $625 + QR^2 - 50\,QR = QR^2 + 25$

or $50QR = 625 - 25 = 600$

$\therefore$ QR = 12 cm.

and PR = (25 – 12) cm = 13 cm

Now $\quad \sin P = \frac{QR}{PR} = \frac{12}{13}$

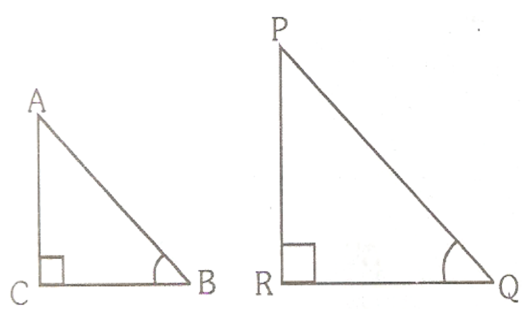
$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

and $\quad \tan P = \frac{QR}{PQ} = \frac{12}{5}$

Ex.6 If $\angle A$ and $\angle Q$ are acute angles such that sin B = sinQ, then prove that $\angle B = \angle Q$. [NCERT]

Sol. Consider two right ΔABC and ΔPQR such that sin B = sinQ.

We have,



$\sin B = \frac{AC}{AB}$ and, $\sin Q = \frac{PR}{PQ}$

$\because \quad \sin B = \sin Q$

$\Rightarrow \quad \frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow \quad \frac{AC}{PR} = \frac{AB}{PQ}$ k, (say) ...(i)

$\Rightarrow \quad AC = k\,PR$ and $AB = k\,PQ$ ...(ii)

Using Pythagoras theorem in triangles ABC and PQR, we have

$AB^2 = AC^2 + BC^2$ and $PQ^2 = PR^2 + QR^2$

$\Rightarrow \quad BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

$$\Rightarrow \quad \frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \frac{BC}{QR} = \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}} \qquad \text{[Using (ii)]}$$

$$\Rightarrow \quad \frac{BC}{QR} = \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k$$

From (i) and (ii), we have,

★ TRIGONOMETRICAL RATIO OF STANDARD ANGLES

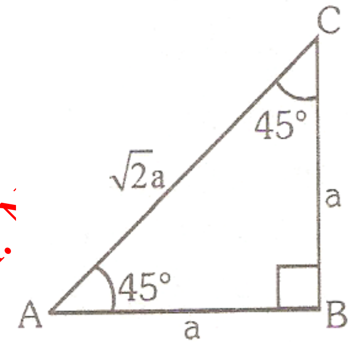
T-Ratios of 45^0

Consider a ΔABC in which $\angle B = 90^0$ and $\angle A = 45^0$

Then, clearly, $\angle C = 45^0$.

$\therefore \quad AB = BC = a$ (say).

$AC = \sqrt{AB^2 + CB^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a.$



$\therefore \quad$ **sin 45^0** $= \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$:

cos 45^0 $= \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$:

tan 45^0 $= \frac{BC}{AC} = \frac{a}{a} = 1$

$\therefore \quad$ **cosec 45^0** $= \frac{1}{\sin 45^0} = \sqrt{2}$; **sec 45^0** $= \frac{1}{\cos 45^0} = \sqrt{2}$; **cot 45^0** $= \frac{1}{\tan 45^0} = 1$

T-Ratios of 60^0 and 30^0

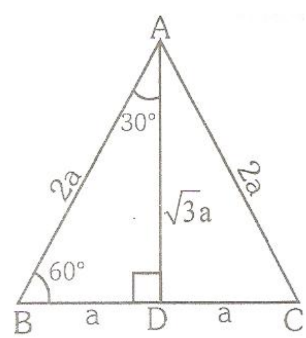
Draw an equilateral ΔABC with each side = 2a.

Then, $\angle A = \angle B = \angle C = 60^0$.

From A, draw $AD \perp BC$.

Then, BD = DC = a, $\angle BAD = 30^0$ and $\angle ADB = 90^0$.

Also, $AD = \sqrt{AB^2 - BD^2} = \sqrt{4a^2 - a^2} = \sqrt{3a^2} = \sqrt{3}a$.



T-Ratios of 60^0

In ΔADB we have : $\angle ADB = 90^0$ and $\angle ABD = 60^0$.

Base = BC = a, Perp. = AD = $\sqrt{3}a$ and Hyp. AB = 2a.

$\therefore \quad$ **sin 60^0** $= \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$:

cos 60^0 $= \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$:

tan 60^0 $= \frac{AD}{BD} = \frac{\sqrt{3}a}{a} = \sqrt{3}$

$\therefore$ **cosec 60⁰** $= \dfrac{1}{\sin 60^0} = \dfrac{2}{\sqrt{3}}$; **sec 60⁰** $= \dfrac{1}{\cos 60^0} = 2$; **cot 60⁰** $= \dfrac{1}{\tan 60^0} = \dfrac{1}{\sqrt{3}}$

T-Ratios of 30⁰

In ΔADB we have : $\angle$ADB = 90^0 and $\angle$ABD = 30^0.

$\therefore$ Base = AD = $\sqrt{3}a$, Perp. = BD = a and Hyp. AB = 2a.

$\therefore$ **sin 30⁰** $= \dfrac{BD}{AB} = \dfrac{a}{2a} = \dfrac{1}{2}$:

cos 30⁰ $= \dfrac{AD}{AB} = \dfrac{\sqrt{3}a}{2a} = \dfrac{\sqrt{3}}{2}$:

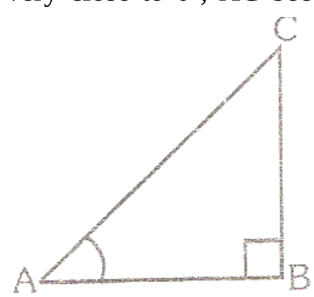
tan 30⁰ $= \dfrac{BD}{AD} = \dfrac{a}{\sqrt{3}a} = \dfrac{1}{\sqrt{3}}$

$\therefore$ **cosec 30⁰** $= \dfrac{1}{\sin 30^0} = 2$; **sec 30⁰** $= \dfrac{1}{\cos 30^0} = \dfrac{2}{\sqrt{3}}$; **cot 30⁰** $= \dfrac{1}{\tan 30^0} = \sqrt{3}$

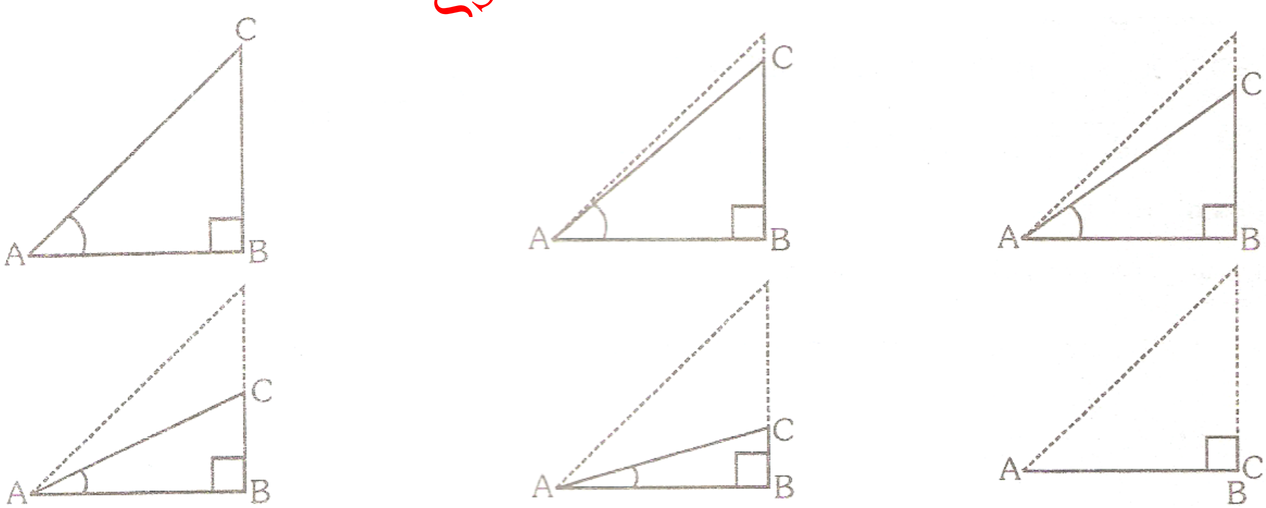
T-Ratios of 0⁰ and 90⁰

T-Ratios of 0⁰

We shall see what happens to the trigonometric ratios of angle A, if it is made smaller and smaller in the right triangle ABC (see figure), till it becomes zero. As $\angle$A gets smaller and smaller, the length of the side BC decreases. The point C gets closer to point B, and finally when $\angle$A becomes very close to 0^0, AC becomes almost the same as AB.



When $\angle$A is very close to 0^0, BC gets very close to 0 and so the value of sin A $= \dfrac{BC}{AC}$ is very close to O. Also when $\angle$A is very close to 0^0, AC is nearly same as AB and so the value of cos A $= \dfrac{AB}{AC}$ is very close to 1.



This helps us to see how we can define the values of sin A cos A when A = 0^0. We define :

sin 0⁰ = 0 and **cos 0⁰** = 1.

Using these, we have :

$$\textbf{tan } \mathbf{0^0} = \frac{\sin 0^0}{\cos 0^0} = 0,$$

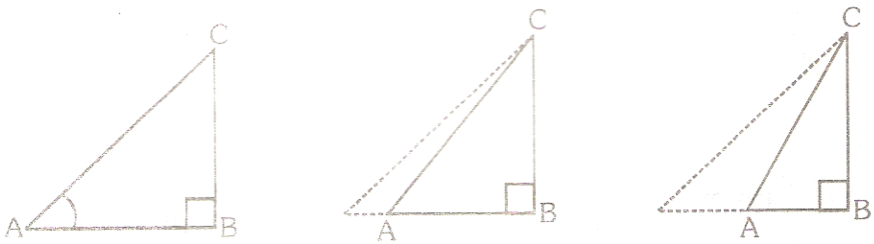
$$\textbf{cot } \mathbf{0^0} = \frac{1}{\tan 0^0} = \frac{1}{0} \qquad \text{(not defined)}$$

$$\textbf{sec } \mathbf{0^0} = \frac{1}{\cos 0^0} = 1$$

$$\text{and } \textbf{cosec } \mathbf{0^0} = \frac{1}{\sin 0^0} = \frac{1}{0} \qquad \text{(not defined)}$$

T-Ratios of 90^0

Now, we shall see what happens to the trigonometric ratios of $\angle A$ when it is made larger and larger in ΔABC till it becomes 90^0. As $\angle A$ gets larger and larger, $\angle C$ gets smaller and smaller. Therefore, as in the case above, the length of the side AB goes on decreasing. The point A gets closer to point B. Finally when $\angle A$ is very close 90^0, $\angle C$ becomes very close to 0^0 and the side AC almost coincides with side BC (see figure).



When $\angle C$ is very close to 0^0, $\angle A$ is very close 90^0, side AC is nearly the same as side BC, and so sin A is very close to 1. Also when $\angle A$ is very close to 90^0, $\angle C$ is very close 0^0, and the side AB is nearly zero, so cos A very close to 0. So, we define:

sin 90^0 = 1 and **cos 90^0** = 0.

Using these, we have :

$$\textbf{tan } \mathbf{90^0} = \frac{\sin 90^0}{\cos 90^0} = \frac{1}{0}, \qquad \text{(not defined)}$$

$$\textbf{cot } \mathbf{90^0} = \frac{\cos 90^0}{\sin 90^0} = \frac{1}{0} = 0$$

$$\textbf{cosec } \mathbf{90^0} = \frac{1}{\sin 90^0} = \frac{1}{1} = 1$$

$$\text{and } \quad \textbf{sec } \mathbf{90^0} = \frac{1}{\cos 90^0} = \frac{1}{0} \qquad \text{(not defined)}$$

table for T-Ratios of Standard Angles

Angle θ / Ratio	0^0	30^0	45^0	60^0	90^0
Sin θ	**0**	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	**1**
cos θ	**1**	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	**0**
tan θ	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\sqrt{3}$	**Not defined**

cot θ	**Not defined**	$\sqrt{3}$	**1**	$\frac{1}{\sqrt{3}}$	**0**
sec θ	**1**	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	**2**	**Not defined**
cosec θ	**Not defined**	**2**	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	**1**

REMARK:

(i) As θ increases from 0^0 to 90^0, sin θ increases from 0 to 1.

(ii) As θ increases from 0^0 to 90^0, cos θ decreases from 1 to 0.

(iii) As θ increases from 0^0 to 90^0, tan θ increases from 0 to ∞.

(iv) The maximum value of $\frac{1}{\sec\theta}, 0^0 \le \theta \le 90^0$ is one.

(v) As $\cos\theta$ decreases from 1^0 to0, θ increases from 0 to 90^0.

(vi) $\sin\theta$ and cos θ can not be greater than one numerically.

(vii) $\sec\theta$ and cosec θ can not be less than one numerically.

(viii) $\tan\theta$ and cot θ can have any value.

COMPETITION WINDOW

T-RATIOS OF SOME ANGLES LESS THAN 90^0

Angle θ / Ratio	15^0	18^0	$22\frac{1}{2}^0$	36^0
Sin θ	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$
cos θ	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$
tan θ	$2-\sqrt{3}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$

Ex.7 In ΔABC, right angled at B, BC = 5 cm, $\angle$BAC = 30^0, find the length of the sides AB and AC.

Sol. We are given

$\angle$BAC = 30^0, i.e., $\angle$A = 30^0

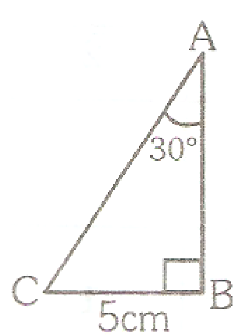
and BC = 5 cm

Now $\sin A = \frac{BC}{AC}$ or $\sin 30^0 = \frac{5}{AC}$

or $= \frac{5}{AC} = \frac{1}{2}$ ...[$\because \sin 30^0 = \frac{1}{2}$]

or AC = 2 × 5 or 10 cm

To find AB, we have,



$$\frac{AB}{AC} = \cos A$$

or $$\frac{AB}{10} = \cos 30^0$$

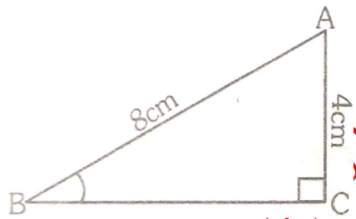
or $$\frac{AB}{AC} = \frac{\sqrt{3}}{2} \qquad \ldots[\because \cos 30^0 = \frac{\sqrt{3}}{2}]$$

$$\therefore \quad AB = \frac{\sqrt{3}}{2} \times 10 \text{ or } 5\sqrt{3} \text{ cm}$$

Hence, AB = $5\sqrt{3}$ cm and AC = 10 cm.

Ex.8 In ΔABC, right angled at C, if AC = 4 cm and AB = 8 cm. Find $\angle$A and $\angle$B.

Sol. We are given, AC = 4 cm and AB = 8 cm



Now $\sin B = \frac{AC}{AB} = \frac{4}{8} = \frac{1}{2}$

But we know that $\sin 30^0 = \frac{1}{2}$

$\therefore \quad B = 30^0$

Now $\angle A = 90^0 - \angle B \qquad \ldots[\because \angle A + \angle B = 90^0]$

$= 90^0 - 30^0 = 60^0$

Hence, $\angle A = 60^0$ and $\angle B = 30^0$.

Ex.9 Evaluate : $\dfrac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 - \cot 45^0}$ **[NCERT]**

Sol.
$$\frac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 - \cot 45^0} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{(3\sqrt{3} - 4)}{(3\sqrt{3} - 4)}$$

$$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16} = \frac{43 - 24\sqrt{3}}{11}$$

Ex.10 Find the value of θ in each of the following :

(i) $2 \sin 2\theta = \sqrt{3}$ (ii) $2 \cos 3\theta = 1$ (iii) $\sqrt{3} \tan 2\theta - 3 = 0$

Sol. (i) we have,

$$2 \sin 2\theta = \sqrt{3}$$

$$\Rightarrow \quad \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin 2\theta \ \sin 60^0 \Rightarrow 2\theta \Rightarrow 60^0 \Rightarrow \ \theta = 30^0$$

(ii) we have,

$$2 \cos 3\theta = 1$$

$$\Rightarrow \quad \cos 3\theta = \frac{1}{2}$$

$$\Rightarrow \quad \cos 3\theta \ \cos 60^0 \Rightarrow 3\theta \Rightarrow 60^0 \Rightarrow \ \theta = 20^0$$

(iii) we have,

$$\sqrt{3} \tan 2\theta - 3 = 0$$

$\Rightarrow \quad \sqrt{3}\ \tan 2\theta = 3$

$\Rightarrow \quad \tan 2\theta = \dfrac{3}{\sqrt{3}} = \sqrt{3}$

$\Rightarrow \quad \tan 2\theta\ \tan 60^0 \Rightarrow 2\theta \Rightarrow 60^0 \Rightarrow\ \theta = 30^0$

x^0	0’	6’	12’	18’	24’	30’	36’	42’	48’	54’	1’ 2’ 3’ 4’ 5 Mean Differences
43^0	0.6820								0.6921		8

COMPETION WINDOW

USING TRIGONOMETRIC TABLES

A Trigonometric Table consists of three parts :

(i) A column on the extreme left containing degrees from 0^0 to 89^0.

(ii) Ten column headed by 0’, 6’, 12’, 18’, 24’, 30’, 36’, 42’, 48’, and 54’,

(iii) Five column of mean differences, headed by 1’, 2’, 3’, 4’, and 5’, The mean differences is added in case of sines, tangents and secants. The mean difference is subtracted in case of cosines, cotangents and cosecants.

The method of finding T-ratios of given angles using trigonometric tables, will be clear from the following example :

Find the value of sin 43^0 52’.

We have, $43^0$52’ = $43^0$28’ + 4’

In the table natural sines, look at the numbers in the row against 43^0 and in the column headed 48’ as shown below.

From Table of Natural Sines :

Now, sin $43^0$48’ = 0.6921

Mean difference for 4’ = 0.0008 [To be added]

[See the number in the same row under 4’]

$\therefore$ sin 43^0 52’ = [0.6921 + 0 0.0008] = 0.6929

TO FIND THE ANGLE WHEN ITS T-RATIOS IS GIVEN

Find θ, when sin θ = 0.7114.

From the table, find the angle whose sine is just smaller than 0.7114.

We have sin θ = 0.7114
Sin 45^0 18’ = <u>0.7108</u>
Diff. = 0.0006

Mean difference of 6 corresponds to 3’.

$\therefore$ Required angle = (45^0 18’ + 3’) = $45^0$21’

Find θ, when cos θ = 0.5248

From the table, find the angle whose costing is just smaller than 0.5248

We have cos θ = 0.5248
cos 58^0 18’ = <u>0.5255</u>
Diff. = 0.0007

And 7 corresponds to 3'.

∴ Required angle = 58^0 18' + 3' = 58^0 21'

TRY OUT THE FOLLOWING

1. Using tables find the value of :
   (i) sin 83^0 12' (ii) cos 70^0 17' (iii) tan 24^0 14' (iv) cosec $(30.8)^0$ (v) sec 68^0 10' (vi) cot 39^0 15'
2. Using tables find the value of θ if :
   (i) sin θ = 0.42' (ii) cos θ = 0.8092 (iii) tan θ = 2.91 (iv) cosec θ = 2.8893 (v) sec θ = 1.2304
   (vi) cot θ = 0.1385

ANSWERS

1.	(i) 0.993.	(ii) 0.3373	(iii) 0.4536	(iv) 1.9530	(v) 2.6892	(vi) 1.2283
2.	(i) 24^0 50'	(ii) 35^0 59'	(iii) 71^0 2'	(iv) 20^0 15'	(v) 35^0 38'	(vi) 82^0 7'

★ T-RATIOS OF COMPLEMENTARY ANGLES

Complementary Angles

Two angles are said to be complementary, if their sum is 90^0.

Thus, θ^0 and $(90^0 - \theta)$ are complementary angles.

T-ratios of Complementary Angles

Consider Δ ABC in which ∠B = 90^0 and ∠A = θ^0.

∴ ∠C = $(90^0 - \theta)$.

Let AB = x. BC = y and AC = r.

When we consider the T-ratios of $(90^0 - \theta)$, then

Base = BC, **Perp.** = AB and **Hyp.** AC =

∴ $$\textbf{sin } (90^0 - \theta) = \frac{AB}{AC} = \frac{x}{r} = \textbf{cos } \theta.$$

$$\textbf{cos } (90^0 - \theta) = \frac{BC}{AC} = \frac{y}{r} = \textbf{sin } \theta.$$

$$\textbf{tan } (90^0 - \theta) = \frac{AB}{BC} = \frac{x}{y} = \textbf{cot } \theta.$$

∴ $$\textbf{cosec } (90^0 - \theta) = \frac{1}{\sin(90^0 - \theta)} = \frac{1}{\sin\theta} = \textbf{sec } \theta.$$

$$\textbf{sec } (90^0 - \theta) = \frac{1}{\cos(90^0 - \theta)} = \frac{1}{\sin\theta} = \textbf{cosec } \theta.$$

$$\textbf{cot } (90^0 - \theta) = \frac{1}{\tan(90^0 - \theta)} = \frac{1}{\cot\theta} = \textbf{tan } \theta.$$

(i) sin $(90^0 - \theta)$ = cos θ	**(ii) cos $(90^0 - \theta)$ = sin θ**	**(iii) tan $(90^0 - \theta)$ = cot θ**
(iv) cosec $(90^0 - \theta)$ = sec θ	**(v) sec $(90^0 - \theta)$ = cosec θ**	**(vi) cot $(90^0 - \theta)$ = tan θ**

Aid to memory:

Add co if that is not there

Remove co if that is there

Thus we have,

sine of $(90^0 - \theta)$ = cosine of $\theta \Rightarrow$ sine $(90^0 - \theta) = \cos\theta$

cosine of $(90^0 - \theta)$ = sine of $\theta \Rightarrow \cos(90^0 - \theta) = \sin\theta$
tangent of $(90^0 - \theta)$ = cotangent of $\theta \Rightarrow \tan(90^0 - \theta) = \cot\theta$
cotangent of $(90^0 - \theta)$ = tangent of $\theta \Rightarrow \cot(90^0 - \theta) = \tan\theta$
secant of $(90^0 - \theta)$ = cosecant of $\theta \Rightarrow \sec(90^0 - \theta) = \text{cosec}\,\theta$
cosecant of $(90^0 - \theta)$ = secant of $\theta \Rightarrow \text{cosec}(90^0 - \theta) = \sec\theta$

In other words :

sin (angle) = cos (complement) ; cos (angle) = sin (complement)
tan (angle) = cot (complement) ; cot (angle) = tan (complement)
sec (angle) = cosec (complement) ; cosec (angle) = sec (complement)

where complement = 90^0 – angle

Ex.11 Without using tables, evaluate :

(i) $\frac{\sin 53^0}{\cos 37^0}$ (ii) $\frac{\cos 49^0}{\sin 41^0}$ (iii) $\frac{\tan 66^0}{\cot 24^0}$

Sol. (i) $\frac{\sin 53^0}{\cos 37^0} = \frac{\sin(90^0 - 37^0)}{\cos 37^0} = \frac{\cos 37^0}{\cos 37^0} = 1$ [$\because \sin(90^0 - \theta)\cos\theta$]

(ii) $\frac{\cos 53^0}{\sin 37^0} = \frac{\cos(90^0 - 41^0)}{\sin 41^0} = \frac{\sin 41^0}{\sin 41^0} = 1$ [$\because \cos(90^0 - \theta)\sin\theta$]

(iii) $\frac{\tan 66^0}{\cot 24^0} = \frac{\tan(90^0 - 24^0)}{\cot 24^0} = \frac{\cot 24^0}{\cot 24^0} = 1$ [$\because \tan(90^0 - \theta)\cot\theta$]

REMARK :	(i)	The above example suggests that out of the two t-ratios, we convent one is term of the t-ratios of the complement.
	(ii)	For uniformity, we usually convert the angle greater than 45^0 in terms of its complement.

Ex.12 Without using tables, show that $(\cos 35^0 \cos 55^0 - \sin 35^0 \sin 55^0) = 0$.

Sol. LHS $= (\cos 35^0 \cos 55^0 - \sin 35^0 \sin 55^0)$
$= [(\cos 35^0 \cos 55^0 - \sin(90^0 - 55^0) \sin(90^0 - 35^0)]$
$= (\cos 35^0 \cos 55^0 - \cos 55^0 \cos 35^0) = 0$ = RHS.
[$\because \sin(90^0 - \theta)\cos\theta$ and $\cos(90^0 - \theta)\sin\theta$]

Ex.13 Express $(\sin 58^0 + \text{cosec}\, 85^0)$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

Sol. $(\sin 58^0 + \text{cosec}\, 85^0) = \sin(90^0 - 5^0)\ \text{cosec}(90^0 - 5^0) = (\cos 5^0 + \sec 5^0)$.

Ex.14 If $\tan 2A = \cot(A - 18^0)$, where 2A is an acute angle, find the value of A.

Sol. We are given,

$\tan 2A = \cot(A - 18^0)$

or $\cot(90^0 - 2A) = \cot(A - 18^0)$ …[$\because \cot(90^0 - 2A) = \tan 2A$]

$\therefore$ $90^0 - 2A = A - 18^0$

or $A + 2A = 90^0 - 18^0$

or $3A = 108^0$

$\therefore$ $A = 36^0$

Ex.15 Evaluate : $\frac{\sec 29^0}{\text{cosec}\,61^0} + 2\cot 8^0 \cot 17^0 \cot 45^0 \cot 73^0 \cot 82^0$.

Sol. $\frac{\sec 29^0}{\text{cosec}\,61^0} + 2\cot 8^0 \cot 17^0 \cot 45^0 \cot 73^0 \cot 82^0$.

$$= \frac{\sec 29^0}{\operatorname{cosec}(90^0 - 29^0)} + 2\cot 8^0 \cot 17^0 (1) \cot(90^0 - 17^0)\cot(90^0 - 8^0)$$

$$= \frac{\sec 29^0}{\sec 29^0} + 2\cot 8^0 \cot 17^0 \tan 17^0 \tan 8^0. \qquad [\because \cot(90^0 - \theta) = \tan\theta]$$

$$= 1 + 2\cot 8^0 \cot 17^0 . \frac{1}{\cot 17^0} . \frac{1}{\cot 8^0} \qquad \ldots\left(\because \tan\theta = \frac{1}{\cot\theta}\right)$$

Ex.16 For a triangle ABC, show that $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$, where A, B and C are interior angles of ΔABC.

Sol. We know that $\angle A + \angle B + \angle C = 180^0$

Thus we have, $B + C = 180^0 - A$

or $\left(\frac{B+C}{2}\right) = 90^0 - \frac{A}{2}$ or $\sin\left(\frac{B+C}{2}\right) = \sin\left(90^0 - \frac{A}{2}\right)$ or $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$.

★ T-IDENTITIES

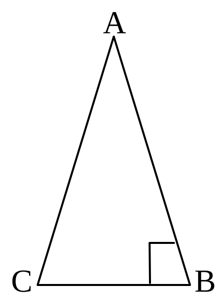
We know that an equation is called an identity when it is true for all value of the variables involved. Similarly, **an equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angle(s) the angle(s) involved.**

The three Fundamental Trigonometric Identities are –

(i) $\cos^2 A + \sin^2 A = 1$; $0^0 \le A \le 90^0$

(ii) $1 + \tan^2 A + \operatorname{cosec}^2 A = 1$; $0^0 \le A < 90^0$

(iii) $1 + \cot^2 A + \operatorname{cosec}^2 A = 1$; $0^0 < A \le 90^0$



Geometrical Proof :

Consider a ΔABC, right angled at B. Then we have :

$AB^2 + BC^2 = AC^2$ …(i) By Pythagoras theorem

(i) $\mathbf{\cos^2 A + \sin^2 A = 1 \,;\, 0^0 \le A \le 90^0}$

Dividing each term of (i) by AC^2, we get

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

i.e., $$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

i.e., $(\cos A)^2 + (\sin A)^2 = 1$

i.e., $\mathbf{\cos^2 A + \sin^2 A = 1}$ …(ii)

This is true for all A such that $\mathbf{0^0 \le A \le 90^0}$

So, this is a trigonometric identity.

(ii) $\mathbf{1 + \tan^2 A = \sec^2 A \,;\, 0^0 \le A < 90^0}$

Let us now divide (i) by AB^2. We get

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

or, $\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$

i.e., $\mathbf{1 + \tan^2 A = \sec^2 A}$ …(iii)

This equation is true for A = 0^0. Since tan A and sec A are not defined for A = 90^0, so (iii) is true for all A such that $\mathbf{0^0 \le A < 90^0}$

(iii) $\mathbf{1 + \cot^2 A = cosec^2 A : 0^0 < A < 90^0}$

Again, let us divide (i) by BC^2, we get

$$\Rightarrow \frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

$$\Rightarrow \left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

$\Rightarrow$ $\mathbf{1 + \cot^2 A = cosec^2 A}$ …(iii)

Since cosec A and cot A are not defined for A = 0^0, therefore (iv) is true for all A such that $\mathbf{: 0^0 < A < 90^0}$

Using the above trigonometric identities, we can express each trigonometric ratio in terms of the other trigonometric ratios, i.e., if any one of the ratios is known, we can also determine the value of other trigonometric ratios.

Fundamental Identities (Results)

$\sin^2\theta + \cos^2\theta = 1$	$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = cosec^2\theta$
$\sin^2\theta = 1 - \cos^2\theta$	$\sec^2\theta - \tan^2\theta = 1$	$cosec^2\theta - \cot^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$	$\tan^2\theta = \sec^2\theta - 1$	$\cot^2\theta = cosec^2\theta - 1$

To prove Trignometrical Identities

The following methods are to be followed :

Method-I : Table the more complicated side of the identity (L.H.S. or R.H.S. as the case may be) and by using suitable trigonometric and algebraic formula prove it equal to the other side.

Method-II : When neither side of the identity is in a simple form, simplify the L.H.S. and R.H.S. separately by using suitable formulae (by expressing all the T-ratios occurring in the identity in terms of the sine and cosine and show that the results are equal).

Method-III : If the identity to the proved is true, transposing so as to get similar terms on the same side, or cross-multiplication, and using suitable formulae, we get an identity which is true.

Ex.17 Express sin A, sec A and tan A in terms of cot A. **[NCERT]**

Sol. We know that

$$\sin A = \frac{1}{cosec A} = \frac{1}{\sqrt{cosec^2 A}} = \frac{1}{\sqrt{1+\cot^2 A}}$$

$$\sec A = \frac{1}{\cos A} = \frac{\frac{1}{\sin A}}{\frac{\cos A}{\sin A}}$$ …(Dividing num. and denom., by sin A)

$= \frac{cosec A}{\cot A} = \frac{\sqrt{1+\cot^2 A}}{\cot A}$ and tan A $= \frac{1}{\cot A}$

Ex.18 Prove $\sqrt{\sec^2\theta + cosec^2\theta} = \tan\theta + \cot\theta$

Sol. LHS $= \sqrt{\sec^2\theta + \csc^2\theta} = \sqrt{(1+\tan^2\theta)+(1+\cot^2\theta)} = \sqrt{\tan^2\theta+\cot^2\theta+2\tan\theta\cot\theta}$

$= \sqrt{\tan\theta+\cot\theta} = \tan\theta+\cot\theta$ = RHS

Hence, proved.

Ex.19 Prove $(\csc A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

Sol. LHS $= (\csc A-\sin A)(\sec A-\cos A) = \left(\dfrac{1}{\sin A}-\sin A\right)\left(\dfrac{1}{\cos A}-\cos A\right)$

$= \left(\dfrac{1-\sin^2 A}{\sin A}\dfrac{1-\cos^2 A}{\cos A}\right) = \dfrac{\cos^2 A}{\sin A}\dfrac{\sin^2 A}{\cos A}$ $[\because \sin^2 A+\cos^2 A=1]$

$= \sin A\cos A = \dfrac{\sin A\cos A}{\sin^2 A+\cos^2 A}$ $[\because \sin^2 A+\cos^2 A=1]$

$= \dfrac{\dfrac{\sin A\cos A}{\sin A\cos A}}{\dfrac{\sin^2 A}{\sin A\cos A}+\dfrac{\cos^2 A}{\sin A\cos A}}$ [Dividing the numerator and denominator by sin A cos A.]

$= \dfrac{1}{\tan A+\cot A}$ = RHS

Hence, proved.

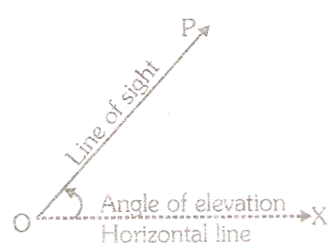
★ **APPLICATIONS OF TRIGONOMETRY**

Many times, we have to find the height and distances of many objects in real life. We use trigonometry to solve problems, such as finding the height of a tower, height of a flag mast, distance between two objects, where measuring directly is trouble, some and some times impossible. In those cases, we adopt indirect methods which involve solution of right triangles.

Thus Trigonometry is very useful in geography, astronomy and navigation. It helps us to prepare maps, determine the position of a landmass in relation to the longitudes and latitudes. Surveyors have made use of this knowledge since ages.

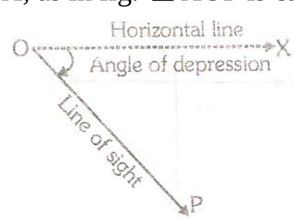
Angle of Elevation

The angle between the horizontal line drawn through the observer eye and line joining the eye to any object is called the angle of elevation of the object, if the object is at a higher level than the eye i.e., If a horizontal line OX is drawn through O, the eye of the observer, and P is an object in the vertical plane through OX, then if P is above OX, as in fig. $\angle XOP$ is called the angle of elevation or the altitude of P as seen from O.



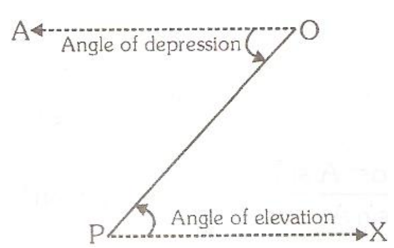
Angle of Depression

The angle between the horizontal line drawn through the observer eye and line joining the eye to any object is called the angle of depression of the object, if the object is at a higher level than the eye i.e., If a horizontal line OX is drawn through O, the eye of the observer, and P is an object in the vertical plane through OX, then if P is above OX, as in fig. $\angle XOP$ is called the angle of depression of P as seen from O.



REMARK :

1. The angle of elevation as well as angle of depression are measured with reference to horizontal line.
2. All objects such as towers, mountains etc. shall be considered as linear for mathematical convenience, throughout this section.



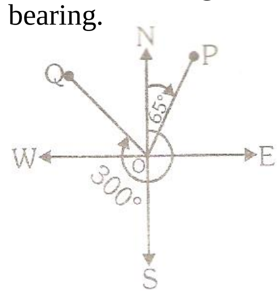
3. The height of the observer, is neglected, if it is not given in the problem.
4. Angle of depression of P as seen from O is equal to the angle of elevation of O, as seen from P.
   i.e., $\angle AOP = \angle OPX$.
5. To find one side a right angled triangle when another side and an acute angle are given, the hypotenuse also being regarded as a side.

   $\frac{\text{Re}quired}{Givenside}$ = a certain T-ratio of the given angle.
6. The angle of elevation increases as the object moves towards the right of the line of sight.
7. The angle of depression increases as the object moves towards the right of the line of sight.

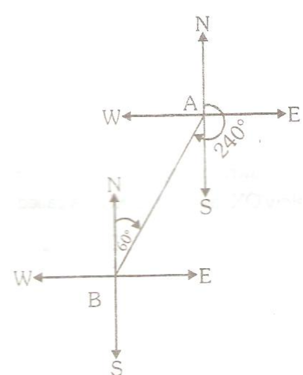
COMPETITION WINDOW

BEARING OF A POINT

The true bearing to a point is the angle measured in degrees in a clockwise direction from the north line. We will refer to the true bearing simply as the bearing.

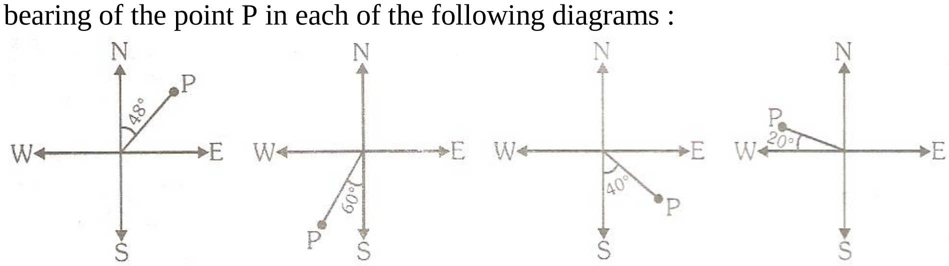


e.g. (i) the bearing of point P is 65^0 (ii) the bearing of point Q is 300^0
A bearing is used to represent the direction of one point relative to another point.
e.g., the bearing of A from B is 60^0. The bearing of B from A is 240^0.



TRY OUT THE FOLLOWING

State the bearing of the point P in each of the following diagrams :



(a) (b) (c) (d)

ANSWERS

(a) 48^0 (b) 240^0 (c) 140^0 (d) 290^0

Ex.20 An observer 1.5 m tall, is 28 5 m away from a tower 30 m high. Determine the angle of elevation of the top of the tower from his eye. **(NCERT)**

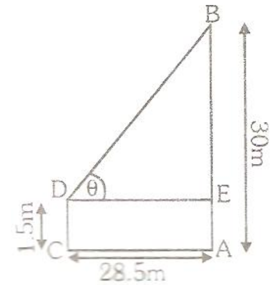
Sol. Let AB be the height of the tower, CD the height of the observer with his eye at the point D, AB = 30 m, CD = 1.5 m.

Through D, draw DE $\|$ CA than $\angle$BDE = θ where θ is the angle of elevation of the top of the tower from his eye.

AC = horizontal distance between the tower and the observer = 28.5 m

BE = AB – AE = (30 – 1.5) m = 28.5 m BDE is right triangle at E,

then $\frac{BE}{DE} = \tan\theta \Rightarrow \frac{28.5}{28.5} = \tan\theta \Rightarrow \tan\theta = 1$



$\Rightarrow \tan\theta = 1 = \tan 45^0 \Rightarrow \theta = 45^0$.

Required angle of elevation of the tower = θ = 45^0.

Ex.21 A vertical post casts a shadow 21 m long when the altitude of the sun is 30^0. Find :

(a) the height of the post.

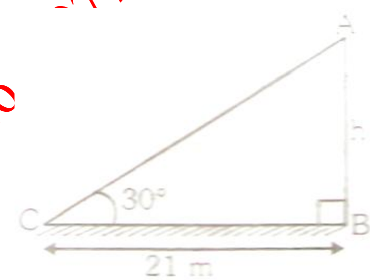
(b) the length of the shadow when the altitude of the sun is 60^0.

(c) the altitude of the sun when the length of the shadow is $7\sqrt{3}$ m.

Sol. Let AB be the vertical post and its shadow is 21 m when the altitude of the sun is 30^0.

(a) BC = 21 m, $\angle$ACB = 30^0, AB = h metres

ABC is rt. Δ, $\frac{AB}{BC} = \tan 30^0 \Rightarrow \frac{h}{21} = \frac{1}{\sqrt{3}}$



$\Rightarrow$ $h = \frac{21}{\sqrt{3}} = \frac{7\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = 7\sqrt{3}$

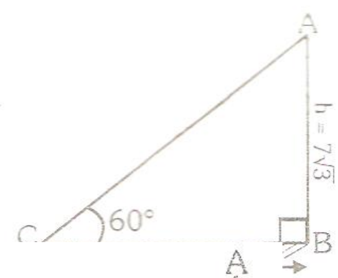
$\Rightarrow$ AB = h, Height of the pole = $7\sqrt{3}$

(b) In this case, we have,

$\angle$ACB = 60^0, BC = x m AB = $7\sqrt{3}$ m

ABC is rt. Δ, then :

$\frac{AB}{BC} = \tan 60^0 = \sqrt{3}$



$\Rightarrow$ $\frac{h}{x} = \sqrt{3} \Rightarrow \frac{7\sqrt{3}}{x} = \sqrt{3}$

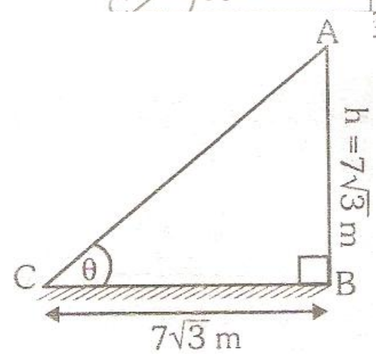
$\Rightarrow$ x = BC, Length of the shadow = 7 m.

(c) In this case :

AB = h = $7\sqrt{3}$

BC = The length of the shadow = $7\sqrt{3}$ m

when the altitude of the sun is θ



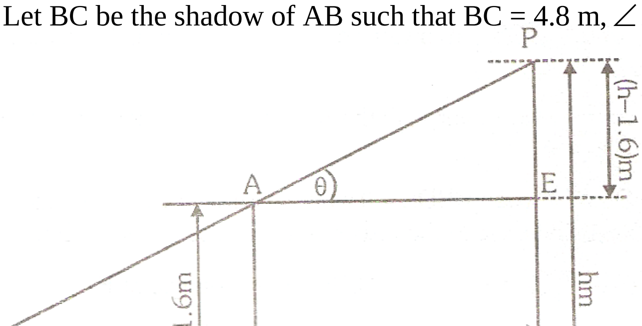
ABC is rt. Δ, then $\frac{AB}{BC} = \tan\theta = \frac{7\sqrt{3}}{7\sqrt{3}} = \tan\theta$

$\tan\theta$ = 1 = tan $45^0 \Rightarrow \theta$ 45^0

Altitude of the sun = θ = 45^0

Ex.22 A 1.6 m tall girl stands at a distance of 3.2 m from the lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using (i) trigonometric ratios (ii) property of similar triangles. **(NCERT)**

Sol. Let PQ be the position of the lamp-post whose height is h metres. i.e., PQ = h metres. AB be the position of the tall girl such that AB = 16. m. Let BC be the shadow of AB such that BC = 4.8 m, $\angle$ACB = $\angle$PAE = θ (corr. $\angle$s)



(i) In right ΔABC, $\frac{AB}{BC} = \tan\theta \Rightarrow \frac{1.6}{4.8} = \tan\theta \Rightarrow \tan\theta = \frac{1}{3}$

AB = EQ = 1.6 m. Also AE = BQ = 3.2 m

PE = PQ – EQ = (h – 1.6) m

In right ΔABC, $\frac{PE}{AE} = \tan\theta \Rightarrow \frac{(h-1.6)}{3.2} = \tan\theta = \frac{1}{3} \Rightarrow \frac{h-1.6}{3.2} = \frac{1}{3}$

3h – 4.8 = 3.2 $\Rightarrow$ 3h = 4.8 + 3.2 = 8 $\Rightarrow$ 3h = 8

The height of the lamp-post = $\frac{8}{3}$ m = $2\frac{2}{3}$ m

(ii) In two Δs ACB and PCQ, we have :

$\angle$ACB = $\angle$ACB = θ (common)

$\angle$ABC = $\angle$PQC = $90^0 \Rightarrow \Delta$ACB ~ ΔPAE

CQ = CB + BQ

= (4.8 + 3.2) m = 8 metres

(AA similar)

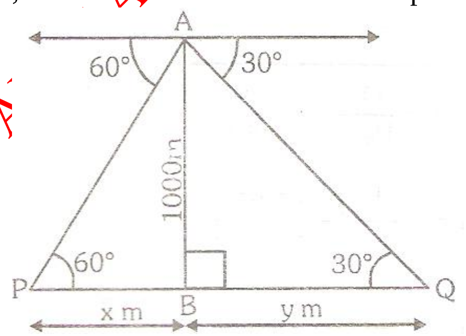
$$\frac{AC}{PC} = \frac{CB}{CQ} = \frac{AB}{PQ} \Rightarrow \frac{BC}{CQ} = \frac{AB}{PQ}$$

$$\frac{BC}{CQ} = \frac{AB}{PQ} \Rightarrow \frac{4.8}{8} = \frac{1.6}{h} \Rightarrow \frac{3}{8} = \frac{1}{h} \Rightarrow 3h = 8$$

Thus $3h = 8 \Rightarrow h = \frac{3}{8} m \Rightarrow h = 2\frac{2}{3}$ m

Required height of the lamp-post = PQ = h = $2\frac{2}{3}$ m

Ex.23 A captain of an airplane flying at an altitude of 1000 metres sights two ships as shown in the figure. If the angle of depressions is 60^0 and 30^0, find the distance between the ships.



Sol. Let A be the position of the captain of an airplane flying at the altitude of 1000 metres from the ground.

AB = the altitude of the airplane from the ground = 1000 m

P and Q be the position of two ships.

Let PB = x metres, and BQ = y metres.

Required : PQ = Distance between the ships = (x + y) metres.

ABP is rt. Δ at B

$$\frac{AB}{PB} = \tan 60^0$$

ABQ is rt. Δ at B

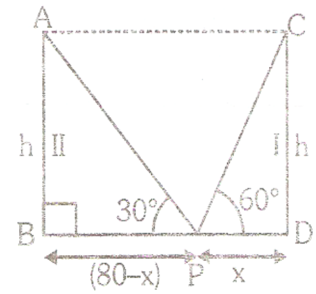
$$\frac{AB}{BQ} = \tan 60^0$$

$$\frac{1000}{x} = \sqrt{3} \Rightarrow x = \frac{1000}{\sqrt{3}} \qquad \frac{1000}{y} = \frac{1}{\sqrt{3}} \Rightarrow y = 1000\sqrt{3}$$

$$x = \frac{1000(1.732)}{3} = 577.3 \text{ m}$$

Required distance between the ships = (x + y) metres = (577.3 + 1732) m = 2309.3 m

Ex.24 Two poles of equal heights are standing opposite to each other on either side of a road, which is 80 metres wide. From a point between them on the road, the angles of elevation of their top are 30^0 and 60^0. Find the position of the point and also the height of the poles.



Sol. Let AB and CD be two poles of equal height standing opposite to each of them on either side of the road BD.

$\Rightarrow$ AB = CD = h metres.

Let P be the observation point on the road BD. The angles of elevation of their top are 30^0 and 60^0.

$\angle APB = 30^0$, $\angle CPD = 60^0$

The width of the road = BD = 80 m, let PD = x metres

Then BP = (80 - x) metres

Consider right Δ CDP, we have :

$$\frac{CD}{PD} \tan 60^0 \Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = \sqrt{3}x \qquad \ldots(i)$$

In right Δ ABP, we have :

$$\frac{AB}{BP} \tan 30^0 \Rightarrow \frac{h}{80-x} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{80-x}{\sqrt{3}} \qquad \ldots(ii)$$

From (i) and (ii), we get : $\left.\begin{array}{l} h = \sqrt{3}x \\ h = \dfrac{80-x}{\sqrt{3}} \end{array}\right\} \Rightarrow (80-x) = 3x \Rightarrow 4x = 80 \Rightarrow x = 20$

Height of each pole = AB = CD = $\sqrt{3}$. x = 20. $\sqrt{3}$ = 20 (1.732) = 34.64 metres.

Position of point P is 20 m from the first and 60 m from the second pole.

i.e., position of the point P is 20 m from either of the poles.

★ **SYNOPSIS**

1. In a right triangle ABC, with right angle B,

$\sin A = \frac{Perpendicular}{Hypotenuse}$	$\cos A = \frac{Hypotenuse}{Perpendicular}$
$\cos A = \frac{Base}{Hypotenuse}$	$\sec A = \frac{Hypotenuse}{Base}$
$\tan A = \frac{Perpendicular}{Base}$	$\cot A = \frac{Base}{Perpendicular}$

2. $\cos = \frac{1}{\sin A} : \sec A = \frac{1}{\cos A} : \tan A = \frac{1}{\cot A}$

3. The value of sinA or cosA never exceeds 1, whereas the value of secA or cosecA is always greater than or equal to 1.

4. $\sin(90^0 - A) = \cos A$ $\cot(90^0 - A) = \tan A$

$\cos(90^0 - A) = \sin A$ $\sec(90^0 - A) = cosec A$

$\tan(90^0 - A) = \cot A$ $cosec(90^0 - A) = \sec A$

5. $\sin^2 A + \cos^2 A = 1 : 1 + \tan^2 A = \sec^2 : 1 + \cot^2 A = cosec^2 A$

6. If one of the sides and any other part (either an acute angle or any side) of a right triangle is known, the remaining sides and angles of the triangle can be easily determined.

7. In a right triangle, the side opposite to 30^0 is half the side of the hypotenuse.

8. In a right triangle, the side opposite to 60^0 is $\frac{\sqrt{3}}{2}$ times the side of the hypotenuse.

9. (i) The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

(ii) The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level i.e., the case when we raise our head to look at the object.

(iii) The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level i.e., the case when we raise our head to look at the object.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. In ΔABC, $\angle B = 90^0$. If AB = 14 cm and AC = 50 cm then tan A equals :

(A) $\frac{24}{25}$ (B) $\frac{24}{7}$ (C) $\frac{7}{24}$ (D) $\frac{25}{24}$

2. If $\sin\theta = \frac{12}{13}$ then the value of the $\frac{2\cos\theta + 3\tan\theta}{\sin\theta + \tan\theta\sin\theta}$ is :

(A) $\frac{12}{5}$ (B) $\frac{5}{3}$ (C) $\frac{259}{102}$ (D) $\frac{259}{65}$

3. If $\sec\theta = \frac{\sqrt{p^2+q^2}}{q}$ then the value of the $\frac{p\sin\theta + q\cos\theta}{p\sin\theta + q\cos\theta}$ is :

(A) $\frac{p}{q}$ (B) $\frac{p^2}{q^2}$ (C) $\frac{p^2-q^2}{p^2+q^2}$ (D) $\frac{p^2+q^2}{p^2-q^2}$

4. If angle A is acute and cos A = $\frac{8}{17}$ then cot A is :

(A) $\frac{8}{15}$ (B) $\frac{17}{8}$ (C) $\frac{15}{8}$ (D) $\frac{17}{15}$

5. sec θ is equal to –

(A) $\frac{1}{\sqrt{1-\cos^2\theta}}$ (B) $\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$ (C) $\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$ (D) $\frac{\sqrt{cosec^2\theta - 1}}{cosec\theta}$

6. sin 30^0 + cos 60^0 equals :

(A) $\frac{1+\sqrt{3}}{2}$ (B) $\sqrt{3}$ (C) 1 (D) None of these

7. The value of $2\tan^2 60^0 - 4\cos^2 45^0 - 3\sec^2 30^0$ is :

(A) 0 (B) 1 (C) 12 (D) 8

8. The value of $\frac{3}{4}\tan^2 30^0 - 3\sin^2 60^0 + 3\,\text{cosec}^2 45^0$ is

(A) 1 (B) 8 (C) 0 (D) 12

9. $7\sin^2\theta + 3\cos^2\theta = 4$ then :

(A) $\tan\theta = \frac{1}{\sqrt{2}}$ (B) $\tan\theta = \frac{1}{2}$ (C) $\tan\theta = \frac{1}{3}$ (D) $\tan\theta = \frac{1}{\sqrt{3}}$

10. The solution of the trigonometric equation $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3, 0^0 < \theta < 90^0$:

(A) $\theta = 0^0$ (B) $\theta = 30^0$ (C) $\theta = 60^0$ (D) $\theta = 90^0$

11. If $\cot\theta + \cos\theta = p$ and $\cot\theta = q$, then the value of $p^2 - q^2$ is :

(A) $2\sqrt{pq}$ (B) $4\sqrt{pq}$ (C) 2 pq (D) 4 pq

12. The value of $\sin^2 15^0 + \sin 2\, 30^0 + \sin^2 45^0 + \sin^2 60^0 + \sin^2 75^0$ is :

(A) 1 (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) 3

13. The value of $\frac{\sin 29^0}{\cos 61^0} - \frac{\sin 61^0}{\cos 29^0}$ is :

(A) Zero (B) 1 (C) $\frac{61}{29}$ (D) $\frac{29}{61}$

14. The values of x and y which make the following solutions true are: $\cos x^0 = \sin 52^0$ and $\cos y^0 = \sin(y^0 + 10)$

(A) $x = 52^0, y = 30^0$ (B) $x = 38^0, y = 40^0$ (C) $x = 48^0, y = 52^0$ (D) $x = 40^0, y = 50^0$

15. If $\alpha + \beta = 90^0$ and $\alpha = 2\beta$ then $\cos^2\alpha + \sin^2\beta$ equal :

(A) 1 (B) Zero (C) $\frac{1}{2}$ (D) 2

16. A flagstaff 6 metres high throws shadow $2\sqrt{3}$ metres long on the ground. The angle of elevation is :

(A) 30^0 (B) 45^0 (C) 90^0 (D) 60^0

17. An observer $\sqrt{3}$ m tall is 3 m away from the pole $2\sqrt{3}$ m high. The angle of the top of elevation of the top from the pole is :

(A) 45^0 (B) 30^0 (C) 60^0 (D) 15^0

18. An observer 1.5 m tall is 28.5 m away from. a chimney. The angle of elevation of the top of the chimney from her eyes is 45^0. The height of the chimney is :

(A) 30 m (B) 27 m (C) 28.5 m (D) None of these

19. The angle of elevation of the top of a tower from a distance 100 m from its foot is 30^0. The height of the tower is :

(A) $1000\sqrt{3}$ m (B) $\frac{200}{\sqrt{3}}$ m (C) $5\sqrt{3}$ m (D) $\frac{100}{\sqrt{3}}$ m

20. A kite is flying at a height of 60 m above the ground. The sting attached to the kite is temporarily the to a point on the ground. The inclination of the string with the ground is 60^0. The length of the staring is :

(A) $40\sqrt{3}$ m (B) 30 m (C) $20\sqrt{3}$ m (D) $60\sqrt{3}$ m

21. A tree is broken by the wind. Its top struck the ground at an angle 30^0 at a distance of 30 m from its foot. The whole height of the tree is :

(A) $10\sqrt{3}$ m (B) $20\sqrt{3}$ m (C) $40\sqrt{3}$ m (D) $30\sqrt{3}$ m

22. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30^0 and 45^0 respectively. If the bridge is at a height of 3 m from the banks then the width of the river is :

(A) $3(\sqrt{3}-1)$ m (B) $3(\sqrt{3}+1)$ m (C) $(\sqrt{3}+3)$ m (D) $(\sqrt{3}-3)$ m

23. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is :

(A) $\sqrt{5}$ m (B) $\sqrt{13}$ m (C) 6 m (D) 2.25 m

24. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angles of elevation from his eyes to the top of the building increases from 30 to 60^0 as he walks towards the building. The distance he walked towards the building is :

(A) $19\sqrt{3}$ m (B) $57\sqrt{3}$ m (C) $38\sqrt{3}$ m (D) $18\sqrt{3}$ m

25. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30^0 and 60^0. if one strip is exactly behind the other on the same side of the light-house then the distance between the two ships is :

(A) $25\sqrt{3}$ m (B) $75\sqrt{3}$ m (C) $50\sqrt{3}$ m (D) None of these

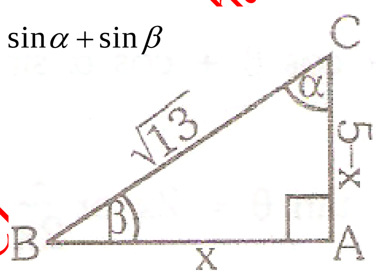
(OBJECTIVE)				ANSWER KEY					EXERCISE
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	B	C	C	A	B	C	A	C	D	C
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	B	C	A	B	C	D	B	A	D	A
Que.	**2**	**122**	**23**	**24**	**25**					
Ans.	D	B	C	A	C					

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. In the adjoining fig, determine : $\sin\alpha + \sin\beta$



2. If 5 tan θ = 4, find the value of $\dfrac{5\sin\theta - 3\cos\theta}{\sin\theta + \cos\theta}$:

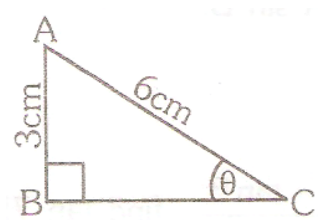
3. If A = 30^0. verify sin 2A = 2 sin A cos A :

4. Given that $\tan\theta = \dfrac{1}{\sqrt{5}}$, what is the value of $\dfrac{\operatorname{cos}ec^2 - \sec^2\theta}{\operatorname{cos}ec^2 + \sec^2\theta}$?

5. What is the maximum value of $\dfrac{1}{\sec\theta}$?

6. What is the value of θ if $\sin\theta = \cos\theta = \dfrac{\tan\theta}{\sqrt{2}}$

7. In the given fig ABC is right Δ at B such that AB = 3 cm and AC = 6 cm. Determine $\angle$ACB.



8. Evaluate : $\dfrac{2\tan 30^0}{1-\tan^2 30^0}$

9. Evaluate : $\dfrac{\cos\theta}{\sin(90^0-\theta)}+\dfrac{\sin\theta}{\cos(90^0-\theta)}$

10. Evaluate : $\sin 25^0 \cos 65^0 + \cos 25^0 \sin 65^0$

11. If tan A $=\dfrac{3}{4}$ and A + B = 90^0, then what is the value of cot B?

12. If tan A = cot B, prove that A + B = 90^0

13. Evaluate : $\dfrac{2\tan 80^0}{3\cot 10^0}$

14. The height of a tower is 10 m. Calculate the height of its shadow when sun's altitude is 45^0.

15. What is the angle of elevation of the sum when the length of the shadow of a pole is $\sqrt{3}$ times of the height of the pole ?

SHORT ANSWER TYPE QUESTIONS

1. If cos A = $\dfrac{3}{5}$. evaluate $\dfrac{5\sin A+3\sec A-3\tan A}{4\cot A+4\operatorname{cosec}A+5\cos A}$

2. Given $\cos\theta=\dfrac{q}{\sqrt{p^2+q^2}}$, find $\dfrac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}$

3. If $\sin\alpha=\dfrac{4}{5}$ and $\cos\beta=\dfrac{12}{13}$ find $\sin\alpha\cos\beta+\cos\alpha\sin\beta$

4. If $\sec\theta=x+\dfrac{1}{4x}$, prove that $\sec\theta+\tan\theta=2x$ or $\dfrac{1}{2x}$.

5. If $\dfrac{1+\cos\theta}{1-\cos\theta}=\dfrac{16}{9}$, find $\dfrac{1+\cot\theta}{1-\cot\theta}$.

6. If $\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}=\dfrac{36}{49}$, find $\dfrac{\operatorname{cosec}\theta-\sec\theta}{\operatorname{cosec}\theta+\sec\theta}$.

7. If 2 sin θ + cos θ = 2, find sin θ

8. If $\dfrac{1+\cos x}{1-\cos x}=7+4\sqrt{3}$. find the value of $\dfrac{1+\sin x}{1-\sin x}$.

9. (i) If sin (A + B) = 1 and cos (A – B) = 1, find A and B.

(ii) If tan (2A + B) = $\sqrt{3}$ and cot (3A – B) = $\sqrt{3}$, find A and B.

10. Find x if : (i) cos (5x – 40^0) = sin 30^0. (ii) cosec (x + 30^0) = cot 45^0.

11. If A = 60^0. B = 30^0, verify each of the following :

(i) cos (A - B) cos A cos B + sin A sin B.

(ii) cos (A + B) = $\dfrac{\cot A\cot B-1}{\cot A+\cot B}$

12. (i) Assume that tan (A + B) = $\dfrac{\tan A+\tan B}{1-\tan A\tan B}$, find tan 75^0 when A = 45^0, B = 30^0.

(ii) Assume cos (A - B) = cos A cos B + sin A sin B, find cos 15^0 when A = 45^0, B = 30^0.

13. Assume that sin (A + B) = cos A cos B + cos A sin B, if sin A = $\dfrac{1}{\sqrt{10}}$, sin B = $\dfrac{1}{\sqrt{5}}$. Then find angle (A + B)

14. Find the value of θ in each of the following if :

(i) 2 cos 3θ = 1 (ii) $2\sqrt{3}$ tanθ = 6 (iii) $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}=\dfrac{1}{2}$

15. Prove that : (i) sin 60^0 cos 30^0 + cos 60^0 sin 30^0 = sin 90^0 (ii) cos 90^0 = 4 $\cos^3$cos 30^0 – 3 cos 30^0

16. Evaluate the following :

(i) $\frac{3}{4}\cot^2 30^0 + 3\sin^2 60^0 - 2\operatorname{cosec}^2 60^0 - \frac{3}{4}\tan^2 30^0$

(ii) $\frac{1+\tan^2 30^0}{1-\tan^2 30^0} + \operatorname{cosec}^2 60^0 - \cos^2 45^0 + \sin^2 45^0 + \frac{1+\cot^2 60^0}{1-\cot^2 60^0}$

(iii) $\cos^2 30^0 \cos^2 45^0 + 4\sec^2 60^0 + \frac{1}{2}\cos^2 90^0 - 2\tan^2 60^0$

(iv) $4(\sin^4 30^0 + \cos^4 60^0) - 3(\cos^2 45^0 - \sin^2 90^0)$

17. Find the value of x in each of the following :

(i) $2x\tan^2 60^0 + 3x\sin^2 30^0 = \frac{27\cos^2 45^0}{4\sin^2 60^0}$.

(ii) $(x+1)(\sin^4 60^0 + \cos^4 30^0) - x(\tan^2 60^0 - \tan^2 45^0) + (x+2)\cos^2 45^0 = 1$.

(iii) $(x-4)\sin^2 60^0 + (x-5)\tan^2 30^0 - x\sin 45^0 \cos 45^0 = 0$.

(iv) $\tan x = \sin 45^0 \cos\ 45^0 + \sin 30^0$.

(v) $\sin 2x = \sin 60^0 \cos\ 30^0 - \cos 60^0 \sin 30^0$.

(vi) $\tan 3x = \sin 45^0 \cos\ 45^0 + \sin 30^0$.

18. If (sec x – 1) (sec x + 1) = 3 then find the value of x.

19. Prove : $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\operatorname{cosec}\theta - 2\sin\theta\cos\theta$

20. Prove : $(\tan\theta + \cot\theta + \sec)(\tan\theta + \cot\theta - \sec\theta) = \operatorname{cosec}^2\theta$.

21. Prove : $(\sec^2 A + \tan^2 A)(\operatorname{cosec}^2 A + \cot^2 A) = 1 + 2\sec^2 A\operatorname{cosec}^2 A$.

22. Prove : $\frac{1+\cot^2\theta}{1+\tan^2\theta} + \left(\frac{1+\cot\ \theta}{1+\tan\ \theta}\right)^2$

23. Prove : $\frac{\tan\ \theta}{(1+\tan^2\theta)^2} + \frac{\cot\ \theta}{(1+\cot^2\theta)} = \sin\theta\cos\theta$.

24. Prove : $(1-\cos\theta+\sin\theta)(1+\cos\theta+\sin\theta) = 2\sin\theta(1+\sin\theta)$

25. Prove : $\sin A(1+\tan A) + \cos A(1+\cot A) = \sec A + \operatorname{cosec} A$.

26. Prove : $\sec\theta(1+\sin\theta)(\sec\theta - \tan\theta) = 1$

27. Prove : $\sec\theta(1-\sin\theta)(\sec\theta + \tan\theta) = 1$

28. Prove : $\cos^2\theta(1+\tan^2\theta) + \sin^2\theta(1+\cot^2\theta) = 2$

29. Prove : $\frac{\cot\ \theta}{\cos^3\theta\operatorname{cosec}\theta + \sin\theta\cos\theta} = 1$

30. Prove : $(\sec\theta + \operatorname{cosec}\theta)(\sin\theta + \cos\theta) = \sec\theta\operatorname{cosec}\theta + 2$

31. Prove : $(\tan\theta + \sec\theta)^2 + (\tan\theta + \sec\theta)^2 = \frac{2(1+\sin^2\theta)}{\cos^2\theta}$

32. Prove : $\frac{(\sin\theta+\cos\theta)^2 - (\sin\theta-\cos\theta)^2}{(\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2} = 2\sin\theta\cos\theta$

33. Prove : $(1+\sin\theta+\cos\theta)^2 = 2(1+\sin\theta)(1+\cos\theta)$

34. Prove : $\sin^8\theta - cec^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

35. Prove : $\sec^6\theta - \tan^6\theta = 1 + 3\sec^2\theta\tan^2\theta$

36. Prove : $\sec^4\theta - \tan^4\theta = 2\sec^2 - 1$

37. Prove : $\dfrac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}-\dfrac{\cos^3\theta\sin^3\theta}{\cos\theta+\sin\theta}=2\sin\theta\cos\theta$

38. Prove : $\dfrac{\sin A+\cos A}{\sin A-\cos A}+\dfrac{\sin A-\cos A}{\sin A+\cos A}=\dfrac{2}{\sin^2 A-\cos^2 A}$

39. Prove : $\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta}=2\sec^2\theta$

40. Prove : $\dfrac{1}{\sec\theta-\tan\theta}+\dfrac{1}{\sec\theta+\tan\theta}=2\sec\theta$

41. Prove : $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}=2\sec\theta$

42. Prove : $\dfrac{1}{\mathrm{cosec}\theta+\cot\theta}-\dfrac{1}{\sin\theta}=\dfrac{1}{\sin\theta}-\dfrac{1}{\mathrm{cosec}\theta-\cot\theta}$

43. Prove : $\dfrac{1+\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{1+\cos\theta}=2\mathrm{cosec}\theta$

44. Prove : $\dfrac{\sin\theta\cos\theta}{\sin\theta-\cos\theta}+\dfrac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\dfrac{2}{\sin^2\theta-\cos^2\theta}=\dfrac{2\sec^2\theta}{\tan^2\theta-1}$

45. Prove : $\dfrac{\sin A-\sin B}{\cos A+\cos B}+\dfrac{\cos A-\cos B}{\sin\theta+\sin B}=0$

46. Prove : $\dfrac{1+\cot\theta}{1-\cot\theta}+\dfrac{1-\cot\theta}{1+\cot\theta}=\dfrac{2}{\sin^2\theta-\cos^2\theta}$

47. Prove : $\sqrt{\dfrac{\mathrm{cosec}\theta-1}{\mathrm{cosec}\theta+1}}+\sqrt{\dfrac{\mathrm{cosec}\theta+1}{\mathrm{cosec}\theta-1}}=2\sec\theta$

48. Prove : $\dfrac{\sec\theta+1-\tan\theta}{\sec\theta+1+\tan\theta}+\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=2\sec\theta$

49. Prove : $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}+=\mathrm{cosec}\theta-\cot\theta$

50. Prove : $\dfrac{1+\sin\theta}{1-\sin\theta}+=\dfrac{\cot^2\theta}{(\mathrm{cosec}\theta-1)^2}$

51. Prove : $\sqrt{\dfrac{\sec\theta-1}{\sec\theta+1}}+=\dfrac{\tan\theta}{\sec\theta+1}$

52. Prove : $\sqrt{\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}}+=\sec\theta-\tan\theta$

53. Prove : $\dfrac{1-\sin\theta}{1+\sin\theta}=1+2\tan^2\theta-2\tan\theta\sec\theta$

54. Prove : $\dfrac{\mathrm{cosec}\theta+\cot\theta}{\mathrm{cosec}\theta-\cot\theta}=1+2\cot^2\theta+2\mathrm{cosec}\theta\cot\theta$

55. Prove : $\sqrt{\dfrac{\mathrm{cosec}\theta-\cot\theta}{\mathrm{cosec}\theta+\cot\theta}}=\dfrac{1}{\mathrm{cosec}\theta+\cot\theta}$

56. Prove : $\sqrt{\dfrac{\sec\theta-1}{\sec\theta+1}}=\sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}}=2\mathrm{cosec}\theta$

57. Prove : $\sqrt{\frac{\operatorname{cosec}\theta-\cot\theta}{\operatorname{cosec}\theta+\cot\theta}}+\sqrt{\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}}=2\operatorname{cosec}\theta$

58. Prove : $\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\sec A+\tan A=\frac{1+\sin A}{\cos A}\sqrt{\frac{1+\sin A}{1-\sin A}}$

59. Prove : $\frac{1+\cos A-\sin A}{1+\cos A+\sin A}=\sec A-\tan A=\frac{1-\sin A}{\cos A}\sqrt{\frac{1-\sin A}{1+\sin A}}$

60. Prove : $\frac{\cos A-1-\sin A}{\cos A-1+\sin A}=\operatorname{cosec}A-\cot A=\frac{1+\cos A}{\sin A}\sqrt{\frac{1+\cos A}{1-\cos A}}$

61. What is the angle of elevation of a vertical flagstaff of height $100\sqrt{3}$ m from a point 100 m from its foot.

62. A ladder makes an angle of 60^0 with the floor and its lower end is 20 m from the wall. Find the length of the ladder.

63. The shadow of a building is 100 m long when the angle of elevation of the sun is 60^0. Find the height of the building.

64. A ladder 20 m long is placed against a vertical wall of height 10 metres. Find the distance between the foot of the ladder and the wall and also the inclination of the ladder to the horizontal.

65. What is the angle of elevation of the sun when the length of the shadow of the pole is $\frac{1}{\sqrt{3}}$ times the height of the pole ?

66. A flagstaff 6 metres high throws a shadow $2\sqrt{3}$ metres logn on the ground. Find the angle of elevation of the sun.

67. A tree $10(2+\sqrt{3})$ metres high is broken by the wind at a height $10\sqrt{3}$ metres from its root in shch a way that top struck the ground at certain angle and horizontal distance from the root of the tree to the point where the top meets the ground is 10 m. Find the angle of elevation made by the top of the tree with the ground.

68. A tree is broken at certain height and its upper part $9\sqrt{2}$ m long not completely separated meet the ground at an angle of 45^0. Find the height of the tree before it was broken and also find the distance from the root of the tree to the point where the top of the tree meets the ground.

LONG ANSWER TYPE QUESTIONS

1. The ladder resting against a vertical wall is inclined at an angle of 30^0 to the ground. The foot of the ladder is 7.5 m from the wall. Find the length of the ladder.

2. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30^0.

3. The length of a string between a kite and a point on the roof of a building 10 m high is 180 m. If the string makes an angle θ with the level ground such that $\tan\theta=\frac{4}{3}$, how high is the kite from the ground?

4. The angle of depression of a ship as seen from the top of 120 m high light house is 60^0. How far is the ship from the light house?

5. A boy 1.7 m tall, is 25 m away from a tower and observes the angle of elevation of the top of the tower to be 60^0. Find the height of the tower.

6. A man 1.8 m tall stands at a distance of 3.6 m from a lamp post and casts a shadow of 5.4 m on the ground. Find the height of the tower.

7. A straight highway leads to the foot of a tower of height 50 m. from the top of the tower, the angles of depression of two cars standing on the highway are 30^0 and 60^0. What is the distance between the two cars and how far is each car from the tower ?

8. Two points A and B are on opposite sides of a tower. The top of the tower makes an angle of 30^0 and 45^0 at A and B respectively. If the height of the tower is 40 metres, find the distance AB.

9. Two men on either side of a tower 60 metres high observe the angle of elevation of the top of the tower to be 45^0 and 60^0 respectively. Find the distance between the two men.

10. Two boats approach a light house in the middle of the sea from opposite directions. The angles of elevation of the top of the light house from two boats are α and β. If the distance between the two boats is x metres, prove that the height of the light house is

$$h = \frac{x}{\cot\alpha + \cot\beta}$$

(i) Find h if $\alpha = 60^0$, $\beta = 45^0$ and x = 250 m

(ii) Find h if $\alpha = 60^0$, $\beta = 30^0$ and x = 400 m

11. A boy standing on a horizontal plane finds a bird flying a distance of 100 m from him at an elevation of 30^0. A girl standing on the roof of 20 metres high building, finds the angle of elevation of the same bird to be 45^0. Both the boy and the girl are on opposite sides of the bird. Find the distance of the bird from the girl.

12. Two pillars of equal height stand on either side of a roadway which is 180 metres wide. The angle of elevation of the top of the pillars are 60^0 and 30^0 at a point on the roadway between the pillars. Find the height of the pillars and the position of the point.

13. Two lamp posts are 60 metres apart, and the height of the one is double that of the other. From the middle of the line joining their feet, an observer finds the angular elevation of their top to be complementary. Find the height of each lamp.

14. Two lamp posts are of equal height. A boy measured-the elevation of the top of each lamp-post from the mid-point of the line-segment joining the feet of lamp-post as 30^0. After walking 15 m towards one of them, he measured the elevation of its top at the point where he stands as 60^0. Determine the height of each lamp-post and the distance between them.

15. When the sun's altitude increases from 30^0 to 60^0, the length of the shadow of a tower decreases by 100 metres. Find the height of the tower.

16. The angle of elevation of the top of a tower from two points at distances a and b metres from the base and in the same straight lien with it are α and β respectively. Prove that the height of the tower is :

17. From the top of a church spire 96 m high, the angels of depression of two cars on a road, at the same level as the base of the spire and on the same side of it are θ and ϕ where $\tan\theta = \frac{1}{4}$ and $\tan\phi = \frac{1}{7}$ Calculate the distance between tow cars.

18. At a point on the level ground, the angle of the elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$. On waling 192 m towards the towards, the tangent of the angel is found to be $\frac{3}{4}$. Find the height of the tower.

19. AB is a straight road leading to C, the foot of a tower, A being at a distance of 120 m from C and B being 75 m nearer. It the angle of elevation of the tower at B be the double of the angle of elevation of the tower at A, find the height of the tower.

20. An aeroplane is observed at the same time by two anti-aircraft batteries distant 6000 m apart to be at elevation of 30^0 and 45^0 respectively. Assuming that the aeroplane is traveling directly towards the two batteries, find its height and its horizontal distance from the nearer battery.

21. AT.V tower stands vertically on a bank of canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60^0. From a point 20 m away from this point on the same bank, the angle of elevation of the top of the tower is 30^0. Find the height of the tower and the width of the canal.

22. A car is traveling on a straight road leading to a tower. From a point at a distance of 500 m from the tower as seen by the driver is 30^0. After driving towards the tower for 10 seconds, the angle of elevation of the top of the tower as seen by the driver is found to be 60^0. Find the speed of the car.

23. The height of a hill is 3300 metres. From a point P on the ground the angle of elevation of the top of the hill is 60^0. A balloon is moving with constant speed vertically upwards from P. After 5 minutes of its movement, a person sitting in it observes the angle of elevation of the top of the hill as 30^0. What is the speed of the balloon ?

24. A man in a boat rowing away from a light-house 100 m high, takes 2 minutes to change the angle of elevation of the top of the light-house from 60^0 to 45^0. Find speed of the boat.

25. From a point on the ground 40 m away from the foot of tower, the angle of elevation of the top of the tower is 30^0. The angle of elevation to the top of a water tank (on the top of the tower) is 45^0. Find
(i) The height of the tower (ii) The depth of the tank.

26. At a point on a level plane, a tower subtends an angle α and a man h metres high on its top an angle β. Prove that the height of the tower is : $\dfrac{h\tan\alpha}{\tan(\alpha+\beta)-\tan\alpha}$

27. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 12 metres. At a point on the plane, the angle of elevation of the bottom of the flagstaff is 45^0 and of the top of the flagstaff is 60^0. Determine the height of the tower:

28. The angles of elevation of the top and the bottom of a flagstaff fixed on a wall are 45^0 and 30^0 to a man standing on the other end of the road 20 metres wide. Find the height of the flagstaff and the height of the wall.

29. An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of the elevation of the two planes from the same point on the ground are 60^0 and 45^0 respectively. Find the vertical distance between the aeroplane at that instant.

30. An aeroplane when 6000 metres high passes vertically above another aeroplane at an instant when the angles of the elevation at the same observing point are 60^0 and 45^0 respectively. How many metres higher is the one than the other ?

31. Two aeroplane are observed to be in a vertical line. The angle of the upper plane is α and a that of the lower is β. If the height of the former be H metres, find the height of the latter plane if $\alpha = 60^0$, $\beta = 45^0$, H = 3500 m.

32. The angle of elevation of a Jet fighter from a point A on the ground is 60^0. After 10 seconds flight, the angle of the of elevation changes to 30^0. If the Jet is flying at a speed of 432 km/hour, find the height at which the jet is flying.

33. The angle of elevation of a Jet fighter from a point on the ground is 60^0. After 15 seconds flight, the angle of the of elevation changes to 30^0. If the Jet is flying at height of $1500\sqrt{3}$ m, find the speed of the Jet.

34. From the top of tower 60 metres high, the angle of depression of the top and bottom of a pole are observed to be 45^0 and 60^0 respectively. Find the height of the pole and distance of tower from the pole.

35. From the top of a building 60 metres high, the angle of depression of the top and bottom of a vertical lamp-post are observed to be 30^0 and 60^0 respectively. Find :
(i) The horizontal distance between the building and the lamp-post and
(ii) The difference between the height of the building and the lamp-post.

36. From the top of a cliff 200 metres high, the angles of depression of the top and bottom of a tower are observed to be 30^0 and 60^0. Find the height of the tower and calculate the distance between them.

37. A man on the deck of a ship is 12 m above water level. He observes that the angle of elevation, of the top of a cliff is 45^0 and the angle of depression of its base is 30^0. Calculate the distance of the cliff from the ship and the height of the cliff.

38. From a window (60 metres high above the ground) of a house in a street the angles of elevation and depression of the top and the foot of another house on opposite side of street are 60^0 and 45^0 respectively. Show that the height of the opposite house is 60 $(\sqrt{3}+1)$ metres.

39. A man on the deck of a ship, 16 m above water level, observes that the angle of elevation and depression respectively of the top and bottom of a cliff are 60^0 and 30^0. Calculate the distance of the cliff from the ship and the height of the cliff.

40. The angle of elevation of a cloud from a point 100 m above a lake is 30^0 and the angle of depression of its reflection in the lake is 60^0. Find the height of the cloud.

41. If the angle of elevation of a cloud from a point h metres above a lake be β, and the angle of depression of its reflection in the lake be α, prove that the height of the cloud is : $h\left(\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}\right)$

42. If the angle of elevation of a cloud from a point h metres above a lake be α, and the angle of depression of its reflection in the lake be β, prove that the distance (x) of the cloud from the point of observation is : $\frac{2h\sec\alpha}{\tan\beta-\tan\alpha}$. Find x if $\alpha = 30^0$, $\beta = 45^0$ and h = 250 m.

TRIGONOMETRY	**ANSWER KEY**	EXERCISE-2 (X)-CBSE

VERY SHORT ANSWER TYPE QUESTION :

1. $\frac{5}{\sqrt{3}}$ **2.** $\frac{5}{14}$ **4.** $\frac{2}{3}$ 5.1 **6.** 45^0 **7.** 30^0 **8.** $\sqrt{3}$ **9.** 2 **10.** 1 **11.** $\frac{4}{3}$ **13.** $\frac{2}{3}$ **14.** 10 m **15.** 30^0

SHORT ANSWER TYPE QUESTION :

1. $\frac{5}{11}$ **2.** $\frac{\sqrt{p^2+q^2}+q}{\sqrt{p^2+q^2}-q}$ **3.** $\frac{63}{65}$ **5.** $\frac{31}{17}$ **6.** $\frac{-71}{97}$ **7.** $1, \frac{3}{5}$ **8.** 3 **9. (i)** A = B 45^0 **(ii)** A = 18^0, B = 24^0

10. (i) 20^0 **(ii)** 60^0 **12. (i)** $2+\sqrt{3}$ **(ii)** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **13.** 45^0 **14. (i)** 20^0 **(ii)** 60^0 **(iii)** 30^0

16. (i) $\frac{10}{3}$ **(ii)** $\frac{16}{3}$ **(iii)** $\frac{83}{8}$ **(iv)** 2 **17. (i)** $\frac{2}{3}$, **(ii)** 3, **(iii)** 8, **(iv)** 45^0, **(v)** 15^0, **(vi)** 15^0 **18.** 60^0 **61.** 60^0 **62.** 40 m

63. 173.2 m **64.** 17.32 m, $\theta = 30^0$ **65.** 60^0 **66.** 60^0 **67.** 60^0 **68.** $9(\sqrt{2}+1)$ m , 9 m

LONG ANSWER TYPE QUESTION :

1. 8.66 m **2.** 10 m **3.** 154 m **4.** 69.28 m **5.** 45 m **6.** 3 m **7.** 86.5 m ; 57.67 m, 28.83 m **8.** 109.28 m **9.** 94.64 m
10. (i) 158.5 m **(ii)** 173.2 m **11.** 42.42 m **12.** 135 m from one end, h = 77.94 m
13. 21.21 m, 42.42 m **14.** Distance = 45m, height = 12.99 m **15.** 86.6 m **17.** 288 m **18.** 180 m **19.** 60 m
20. $3000(\sqrt{3}+1)$ m, $3000(\sqrt{3}-1)$ m, **21.** Height = 17.32 m, width = 10 m **22.** 120 km/hr **23.** 26.4 km/hr
24. 1.269 km/hr **25. (i)** 23.1 m **(ii)** 16.91 m **27.** 16.392 m **28.** 8.45 m, 11.55 m **29.** 1690.66 m, **30.** 2536 m
31. 2020.78 m **32.** 1039.2 m **33.** 720 km/hr **34.** h = 25.36 m, x = 34.64 m **35. (i)** 34.64 m, **(ii)** 20 m
36. Height = $133\frac{1}{3}$ m, Distance = 115.46 m **37.** Height = 32.784 m, Distance = 20.784 m
39. Height = 48 m, Distance = 27.71 m **40.** 200 m **42.** 1366 m

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD QUESTIONS

SHORT ANSWER TYPE 1

1. Without using tables, find the value of 14 sin 30^0 + 6 cos 60^0 = 5 tan 45^0. **[ICSE-2004]**

2. Prove that : $\frac{\mathrm{cosec}A}{\mathrm{cosec}A-1}+\frac{\mathrm{cosec}A}{\mathrm{cosec}A+1}=2+2\tan^2 A$ **[CBSE-AI-2004C]**

3. Evaluate : $\frac{\sec\theta.\mathrm{cosec}\theta(90^0-\theta)-\tan\theta\cot(90^0-\theta)+\sin^2 55^0+\sin^2 35^0}{\tan 10^0 \tan 20^0 \tan 60^0 \tan 70^0 \tan 80^0}$ **[CBSE-AI-2004C]**

4. Without using mathematical tables, find the value of x if $\cos x = \cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$. **[ICSE-2005]**

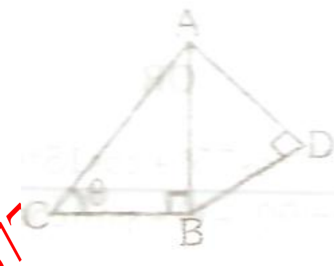
5. Without using trigonometric tables, evaluate : $\frac{2\tan 53^0}{\cot 37^0} - \frac{\cot 80^0}{\tan 10^0}$ **[ICSE-2006]**

6. Without using trigonometric tables, evaluate : $\frac{\sin 80^0}{\cos 10^0} + \sin 59^0 \sec 31^0$ **[ICSE-2007]**

7. Without using tables, evaluate : $\frac{\sin 25^0}{\sec 65^0} - \frac{\cos 25^0}{\operatorname{cosec} 65^0}$ **[ICSE-2008]**

8. Prove the $\frac{\sin A}{(1+\cos A)} = (\operatorname{cosec} A - \cot A)$
[ICSE-2008]

9. In the fig AD = 4 cm, BD = 3 cm and CB = 12 cm, find $\cot\theta$
[CBSE-Delhi-2008]



10. Without using the trigonometric tables, evaluate the following :

$$\frac{11\sin 70^0}{7\cos 20^0} - \frac{4}{7}\frac{\cos 53^0 \operatorname{cosec} 37^0}{7\tan 15^0 \tan 35^0 \tan 55^0 \tan 75^0}$$ **[CBSE-Delhi-2008]**

11. Without using the trigonometric tables, evaluate the following :

$$\frac{\sin 18^0}{\cos 72^0} + \sqrt{3}\,[\tan 10^0 \tan 30^0 \tan 40^0 \tan 50^0 \tan 80^0]$$ **[CBSE-Delhi-2008]**

12. If $\sin\theta = \cos\theta$, find the value of θ. **[CBSE-Al-2008]**

13. Without using the trigonometric tables, evaluate the following : $(\sin^2 25^0 + \sin^2 65^0) + \sqrt{3}(\tan 5^0 \tan 15^0 \tan 30^0 \tan 75^0 \tan 85^0)$. **[CBSE-Al-2008]**

14. Without using trigonometric tables, evaluate the following : $(\cos^2 25^0 + \cos^2 65^0) + \operatorname{cosec}\theta \sec(90-\theta) - \cot\theta \tan(90-\theta)$. **[CBSE-Al-2008]**

15. If $7\sin^2\theta + 3\cos^2\theta = 4$ show that $\tan\theta = \frac{1}{\sqrt{3}}$. **[CBSE-Al-2008]**

16. If $\tan A = \frac{5}{12}$, find the value of (sin A + cos A) sec A. **[CBSE-Foreign-2008]**

17. If sec 4A = cosec (A – 20^0), where 4A is an acute angle, find the value of A. **[CBSE-Foreign-2008]**

OR

In a ΔABC, right angled at C, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of sin A cos B + cos A sin B.

18. If cos A = $\frac{7}{25}$, find the value of tan A + cot A. **[CBSE-Foreign-2008]**

19. If $\sin\theta = \frac{1}{3}$, find the value of [2 cot2 θ + 2) **[CBSE-Delhi-2009]**

20. Simplify : $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$ **[CBSE-Delhi-2009]**

21. If $\sec^2\theta(1+\sin\theta)(1-\sin\theta) = k$, then find the value of k. **[CBSE-Al-2009]**

22. If $\cot\theta = \frac{15}{8}$, then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

OR

Find the value of tan 60^0 geometrically. **[CBSE-Al-2009]**

23. If sec A = $\frac{15}{7}$ and A + B = 90^0. find the value of cosec B. **[CBSE-Foreign-2009]**

24. Without using trigonometric tables, evaluate : $\frac{7\cos 70^0}{2\sin 20^0}+\frac{3}{2}\frac{\cos 55^0\,\mathrm{cos}ec 35^0}{2\tan 5^0\tan 25^0\tan 45^0\tan 85^0\tan 65^0}$ **[CBSE-Foreign-2009]**

SHORT ANSWER TYPE II

1. Evaluate : $\frac{\sec^2 54^0-\cot^2 36^0}{\mathrm{cos}ec^2 57^0-\tan^2 33^0}+2\sin^2 38^0\sec^2 52^0-\sin^2 45^0$ **[AI-2005]**

2. Prove that following : $(\tan A-\tan B)^2+(1+\tan A\tan B)^2=\sec^2 A\sec^2 B$. **[Foreign-2005]**

OR

Evaluate : $\sec^2 10^0-\cot^2 80^0+\frac{\sin 15^0\cos 75^0+\cos 15^0\sin 75^0}{\cos\theta\sin(90-\theta)+\sin\theta\cos(90-\theta)}$ **[Foreign-2005]**

3. Prove that: $\frac{\sec\theta+\tan\theta-1}{\tan\theta-\sec\theta+1}=\frac{\cos\theta}{1-\sin\theta}$.

OR

Without using trigonometric tables, evaluate the following :

$\frac{\cot(90-\theta).\sin\theta(90-\theta)}{\sin\theta}+\frac{\cot 40^0}{\tan 45^0}-(\cos^2 20^0+\cos^2 70^0)$. **[Delhi-2005C]**

4. Without using trigonometric tables, evaluate the following :

$\frac{\sec^2\theta-\cot^2(90-\theta)}{\mathrm{cos}ec^2 67^0-\tan^2 23^0}+(\sin^2 40^0+\sin^2 50^0)$. **[AI-2005C]**

5. Prove that: $(1+\tan A)^2+(1-\tan A)^2=2\sec^2 A$. **[ICSE-2005]**

6. Prove that: $\frac{\sin\theta\tan\theta}{1-\cos\theta}=1+\sec\theta$. **[ICSE-20056]**

7. Prove that: $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2\sec^2\theta}{\tan^2\theta-1}$. **[ICSE-20056]**

OR

Without using trigonometric tables : $\frac{\sec^2(90-\theta)-\cot^2\theta}{2(\sin^2 25^0+\sin^2 65^0)}+\frac{2\cos^2 60^0\tan^2 28^0\tan^2 62^0}{3(\sec^2 43^0-\cot^2 47^0)}$.

8. Prove that : $\frac{1}{\mathrm{cos}ec\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\mathrm{cos}ec\theta+\cot\theta}$ **[AI-2006]**

OR

Without using trigonometric tables : $\frac{\mathrm{cos}ec^2(90-\theta)-\tan^2\theta}{4(\cos^2 48^0+\cos^2 42^0)}+\frac{2\tan^2 30^0\sec^2 52^0\sin^2 38^0}{\mathrm{cos}ec^2 70^0-\tan^2 20^0}$.

9. Without using trigonometric tables : $\frac{\sin^2\theta+\sin^2(90-\theta)}{3(\sec^2 61^0-\cot^2 29^0)}+\frac{3\cot^2 30^0\sin^2 54^0\sec^2 36^0}{2(\mathrm{cos}ec^2 65^0-\tan^2 25^0)}$.

[Foreign-2006]

10. Without using trigonometric tables evaluate the following :

$\frac{\cos^2 20^0+\cos^2 70^0}{\sec^2 50^0-\cot^2 40^0}+2\,\mathrm{cos}ec^2 58^0-2\cot 58^0\tan 32^0-4\tan 13^0\tan 37^0\tan 45^0\tan 53^0\tan 77^0$

OR

Prove that : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2\,\mathrm{cos}ec\theta$ **[Delhi-2006C]**

11. Without using trigonometric tables evaluate the following :

(i) $\frac{\sec 39^0}{\mathrm{cos}ec 51^0}+\frac{2}{\sqrt{3}}\tan 17^0-\tan 38^0\tan 60^0\tan 52^0\tan 73^0-3(\sin^2 31^0+\sin^2 59^0)$ **[AI-2006C]**

(ii) $\dfrac{3\cos 55^0}{7\sin 35^0} - \dfrac{4(\cos 70^0.\text{cosec}20^0)}{7(\tan 5^0.\tan 25^0.\tan 45^0.\tan 65^0.\tan 85)}$ **[Delhi-2007]**

(iii) $\tan 7^0.\tan 23^0.\tan 60^0.\tan 67^0.\tan 83^0 + \dfrac{\cot 54^0}{\tan 36^0} + \sin 20^0.\sec 70^0 - 2$ **[AI-2007]**

12. Prove that : $\dfrac{\sin A - 1}{\sin A + 1} = \dfrac{1 - \cos A}{1 + \cos A}$ **[ICSE-2007]**

13. Prove that : $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \dfrac{\text{cosec}A - 1}{\text{cosec}A + 1}$

OR

Prove that : (1 + cot A – cosec A) (1 + tan A + sec A) = 2 **[CBSE (Delhi)-2008]**

14. Prove that : $(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

OR

Prove that $\sin\theta\,(1 + \tan\theta) + \cos\theta\,(1 + \cot\theta) = \sec\theta + \text{cosec}\,\theta$ **[CBSE - AI-2008]**

15. Prove that : $(1 + \cot A + \tan A)(\sin A - \cos A) = \sin A \tan A - \cot A \cos tA$. **[CBSE-foreign-2008]**

OR

Without using trigonometric tables evaluate the following : $2\left[\dfrac{\cos 58^0}{\sin 32^0}\right] - \sqrt{3}\left[\dfrac{\cos 38^0\ \text{cosec}52^0}{\tan 15^0\ \tan 60^0\ \tan 75^0}\right]$.

16. Find the value of sin 30^0 geometrically.

OR

Without using trigonometric tables, evaluate : $\dfrac{\cos 58^0}{\sin 32^0} + \dfrac{\sin 22^0}{\cos 68^0} - \dfrac{\cos 38^0\ \text{cosec}52^0}{\tan 18^0\ \tan 35^0\ \tan 60^0\ \tan 72^0\ \tan 55^0}$

[CBSE-Delhi-2009]

17. Evaluate : $\dfrac{2}{3}\text{cosec}^2 58^0 - \dfrac{2}{3}\cot 58^0\ \tan 32^0 - \dfrac{5}{3}\tan 13^0\ \tan 37^0\ \tan 45^0\ \tan 53^0\ \tan 77^0$. **[CBSE-AI-2009]**

18. Prove that : $\sec^2\theta - \dfrac{\sin^2\theta + 2\sin^4\theta}{2\cos^4\theta + \cos^2\theta} = 1$ **[CBSE-foreign-2009]**

LONG ANSWER TYPE

1. On a horizontal plane there is a vertical tower with a pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60^0 and 30^0 respectively. Find the height of the tower and flag pole mounted on it.

OR

From a building 60 metres high the angles of depression of the top and bottom of lamp-post are 30^0 and 60^0 respectively. Find the distance between lamp-post and building. Also find the difference of height between building and lamp-post. **[Delhi-2008]**

2. From the top of a cliff 92 cm high, the angle of depression of a buoy is 20^0. calculate to the nearest metre, the distance of the bucy from the foot of the cliff. **[ICSE-2005]**

3. The shadow of a vertical tower AB on level ground is increased by 10 m, when the altitude of the sun changes from 45^0 to 30^0. Find the height of the tower and give your answer correct to $\dfrac{1}{10}$ of a metre. **[ICSE-2006]**

4. The angle of depression of the top and the bottom of a building 50 metres high as observed from the top of a tower are 30^0 and 60^0 respectively. Find the height of the tower and also the horizontal distance between the building and the tower. **[Delhi-2006]**

OR

The angle of elevation of the top of a tower as observed from a point on the ground is $'\alpha'$ and on moving 'a' metres towards the tower, the angle of elevation is $'\beta'$ Prove that the height of the tower is $\dfrac{a\tan\alpha\tan\beta}{\tan\beta-\tan\alpha}$.

5. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly tower it. If it takes 12 minutes for the angle of depression to change from 30^0 to 45^0 how soon after this, will the car reach the tower? **[Al-2006C]**

6. A boy standing on a horizontal plane finds at a distance of 100 m from him at an elevation of 30^0. A girl standing on the roof of 20 metre high building, finds the angle of elevation of the same bird to be 45^0. Both the boy and the girl are on opposite sides of the bird. Find distance of bird from the girl. **[Delhi-2007]**

7. Statue 1.46 m tall, stands on the top of the pedestal . From a point on the ground, the angle of elevation of the top of the statue is 60^0 and from the same point, the angle of elevation of the top of the pedestal is 45^0. Find the height of the pedestal. (use $\sqrt{3}=1.73$) **[CBSE-Delhi-2008]**

8. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60^0. When he moves 40 m away from the bank, he finds the angle of elevation to be 30^0. Find the height of the tree and the width of the river. (use $\sqrt{3}$ = 1.732) **[CBSE-Delhi-2008]**

9. The angle of elevation of a jet fighter from a point A on the ground is 60^0. After a fight of 15 seconds, the angle of elevation changes to 30^0. If the jet is flying at a speed of 720 km/hour, find the constant height at which the jet is flying. (use $\sqrt{3}$ = 1.732) **[CBSE-Al-2008]**

10. The angle of elevation of an aeroplane from a point A on the ground is 60^0. After a flight of 30 seconds, the angel of elevation changed to 30^0. If the plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed in km/hour of the plane. **[CBSE-foreign-2008]**

11. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30^0, which is approaching the foot of the tower with a uniform speed. Six seconds later the angle of depression of the car is found to be 60^0. Find the time taken by the car to reach the foot of the tower from this point. **[CBSE-Delhi-2009]**

12. An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angels of elevation of the two planes from the same point on the ground are 30^0 and 60^0 respectively. Find the distance between the planes at that instant **[CBSE-Al-2009]**

13. A man is standing on the deck of a ship which is 25m above water level. He observes the angle of elevation of the top of a lighthouse as 60^0 and the angle of depression of the base of the light house as 45+0. Calculate the height of the lighthouse. **[CBSE-foreign-2009]**

TRIGONOMETRY	**ANSWER KEY**	EXERCISE-2 (X)-CBSE

SHORT ANSWER TYPE QUESTION-I

1 5 3. $\dfrac{2}{\sqrt{3}}$ **4.** 30^0 **5.** 1 **6.** 2 **7.** 1 **9.** $\dfrac{12}{5}$ **10.** 1 **11.** 2 **12.** 45^0 **13.** 2 **14.** 2 **16.** $\dfrac{17}{12}$ **17.** 22^0 or 1 **18.** $\dfrac{625}{168}$

19. 18 **20.** 1 **21.** 1 **22.** $\dfrac{625}{64}$ or $\sqrt{3}$ **23.** $\dfrac{15}{7}$ **24.** 5

SHORT ANSWER TYPE QUESTION-II

1. $\frac{5}{2}$ **2.** or 2 **3.** or 1 **4.** 2 **7.** or $\frac{2}{3}$ **8.** or $\left[\frac{-5}{12}\right]$ **9.** $-\frac{25}{6}$ **10.** 1 **11.** (i) 0 (ii) $\frac{-1}{7}$ (iii) $\sqrt{3}-1$ **15.** or 1

16. $\frac{1}{2}$ or $\frac{2\sqrt{3}-1}{\sqrt{3}}$ **17.** – 1

LONG ANSWER TYPE QUESTION-II

1. 15.588 m, 5.196 m or 34.64 m and 20 m **2.** 253 m **3.** 13.66 m **4.** 75 m and 43.3 m **5.** 16 minutes 23 seconds **6.** $30\sqrt{2}$ **7.** 2 m **8.** 34.64 m and 20 m **9.** 2598 m **10.** 864 km/hr **11.** 3 seconds **12.** 2083.33 m **13.** 68.25 m

EXERCISE – 4 (FOR OLYMPIADS)

CHOOSE THE CORRECT ONE

1. If $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2-b^2$ and $\frac{ax\sin\theta}{\cos^2\theta}-\frac{by\cos\theta}{\sin^2\theta}=0$ then $(ax)^{2/3}+(by)^{2/3}$ is equal to :

(A) $(a^2-b^2)^{2/3}$ (B) $(a^2+b^2)^{2/3}$ (C) $(a-b)^{2/3}$ (D) None of these

2. The sides of a right angled triangle form a geometric progression, find the cosines of the acute angles. (If a, b, c are in G.P. $\Rightarrow b^2 = ac$) :

(A) $\frac{\sqrt{5}-1}{2}$ and $\sqrt{\frac{\sqrt{5}+1}{2}}$ (B) $\frac{\sqrt{5}+1}{2}$ and $\sqrt{\frac{\sqrt{5}+1}{2}}$

(C) $\frac{\sqrt{5}-1}{2}$ and $\sqrt{\frac{\sqrt{5}-1}{2}}$ (D) None of these

3. If $y=\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$, then $\frac{1-\cos\alpha+\sin\alpha}{1+\sin}$ is equal to :

(A) 1+ y (B) 1 – y (C) $\frac{1}{y}$ (D) None of these

4. $\cot 36^0 \cot 72^0$ is equal to :

(A) $\frac{1}{5}$ (B) $\frac{1}{\sqrt{5}}$ (C) 1 (D) None of these

5. The value of $\cos^2 15^0 - \cos^2 30^0 + \cos^2 45^0 - \cos^2 60^0 + \cos^2 75^0$ is :

(A) 2 (B) 0 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

6. If θ $x=\sin^2\theta\cos\theta$ and $y=\cos^2\theta\sin\theta$, then :

(A) $(x^2y)^{2/3}+(xy^2)^{2/3}=1$ (B) $\left[\frac{x^2}{y}\right]^{2/3}+\left[\frac{y^2}{x}\right]^{2/3}=1$

(C) $x^2 + y^2 = x^2y^2$ (D) None of these

7. If $x=\sec\theta-\tan\theta$ and $y=\operatorname{cosec}\theta+\cot\theta$, then $xy+1$ is equal to :

(A) x + y (B) x – y (C) 2x + y (D) y – x

8. If $5\sin\theta=3$, then $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$ is equal to :

(A) $\frac{1}{4}$ (B) 4 (C) 2 (D) None of these

9. The value of the expression $1-\frac{\sin^2 y}{1+\cos y}+\frac{1+\cos y}{\sin y}-\frac{\sin y}{1-\cos y}$ is equal to :

(A) cos y (B) 1 (C) 0 (D) sin y

10. If $\sec\theta = x + \frac{1}{4x}, x \in R, x \neq 0$, then the value of $\sec\theta + \tan\theta$ is :

(A) 2x (B) $\frac{1}{2x}$ (C) 2x or $\frac{1}{2x}$ (D) None of these

11. If $\tan\theta = \frac{p}{q}$, then the value of $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$ is :

(A) $\frac{p^2 - q^2}{p^2 + q^2}$ (B) $\frac{p^2 + q^2}{p^2 - q^2}$ (C) 0 (D) None of these

12. If m = $\tan\theta + \sin\theta$ and $n = \tan\theta - \sin\theta$, then $(m^2 - n^2)^2$ is equal to :

(A) mn (B) 4 mn (C) 16 mn (D) $4\sqrt{mn}$

13. If $x = \cos\theta + b\sin\theta$ and $y = a\sin\theta + \cos\theta$ then $a^2 + b^2$ is equal to :

(A) $x^2 - y^2$ (B) $x^2 + y^2$ (C) (x+ y) (D) None of these

14. If $\cos\theta + \frac{y}{b}\sin\theta + 1 = 0$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta - 1 = 0$ then $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is equal to :

(A) 2 (B) 0 (C) – 2 (D) 1

15. ABC is a triangle, right angled at A. If the length of hypotenuse is $2\sqrt{2}$ times the length of perpendicular from A on the hypotenuse, the other angles of the triangle are :

(A) 22.5^0, 67.5^0 (B) 30^0, 60^0 (C) 45^0, 45^0 (D) None of these

16. If sin A + cos A = m and $\sin^3 A + \cos^3 A$ = n, then :

(A) $m^3 + 3m + 2n = 0$ (B) $m^3 - 3m + 2n = 0$
(C) $n^3 - 3n + 2m = 0$ (D) $m^3 - 3m + n = 0$

17. If $\sin^2\theta + 3\cos\theta - 2 = 0$, then $\cos^3\theta + \sec^3\theta$ is equal to :

(A) 18 (B) 9 (C) 4 (D) $\frac{1}{4}$

18. If $\sin\alpha + \cos\alpha = a$, then $\sin^6\alpha + \cos^6\alpha$ is equal to :

(A) $1 + \frac{3}{4}(a^2 - 1)^2$ (B) $1 - \frac{3}{4}(a^2 - 1)^2$ (C) $\frac{3 + 4(a^2 - 1)^2}{4}$ (D) $\frac{3 - 3(a^2 - 1)^2}{4}$

19. The quadratic equation whose roots are sin 18^0 and cos 36^0 is :

(A) $4x^2 + 2\sqrt{5}x + 1 = 0$ (B) $4x^2 - 2\sqrt{5}x - 1 = 0$
(C) $x^2 + 2\sqrt{5}x + 1 = 0$ (D) $4x^2 - 2\sqrt{5}x + 1 = 0$

20. If $\cos\theta + \sec\theta = 2$, then the value of $\cos^2\theta + \sec^2\theta$ is :

(A) 1 (B) 2 (C) 4 (D) None of these

21. If $\sin(A - B) = \cos(A + B) = \frac{1}{2}$, then the values of A and B lying between 0^0 and 90^0 are respectively:

(A) 30^0 and 60^0 (B) 60^0 and 30^0 (C) 45^0 15^0 (D) None of these

22. If $0 \leq x \leq \frac{\pi}{2}$ and $81^{\sin 2x} + 81^{\cos 2x} = 30$, then x is equal to :

(A) $\frac{\pi}{3}$ or $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ or 0 (C) $\frac{\pi}{2}$ or $\frac{\pi}{4}$ (D) None of these

23. If $m^2 + m'^2 + 2mn'\cos\theta = 1, n^2 + n'^2 + 2nn'\cos\theta = 1$, and $mn + m'n' + (mn' + m'n)\cos\theta = 0$, then $m^2 + n^2$ is equal to :

(A) $\sin^2\theta$ (B) $\text{cosec}^2\theta$ (C) $\cos^2\theta$ (D) None of these

24. If $\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos} = q$, then tan A is equal to :

(A) $\pm\frac{p}{q}\sqrt{\frac{q^2 - 1}{1 - p^2}}$ (B) $\pm\sqrt{\frac{q^2 - 1}{1 - p^2}}$ (C) $\pm\frac{p}{q}\sqrt{\frac{q^2 - 1}{1 - p^2}}$ (D) None of these

25. If $T_n = \sin^n\theta + \cos^n\theta$, then $\frac{T_3 - T_5}{T_1}$ is equal to :

(A) $\frac{T_5 - T_7}{T_3}$ (B) $\frac{T_3 - T_5}{T_7}$ (C) $\frac{T_9 - T_6}{T_4}$ (D) $\frac{T_6 - T_9}{T_4}$

26. The number of values of θ which lie between 0 and $\frac{\pi}{2}$ and satisfy the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$ is :

(A) 1 (B) 2 (C) 3 (D) None of these

27. The greatest angle of a cyclic quadrilateral is 3 times least. The circular measure of the least angle is :

(A) 60^0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) None of these

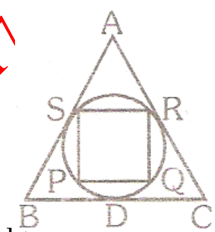
28. A circle is inscribed in an equilateral triangle of sides a, the area of any square inscribed in the circle is :

(A) $6a^2$

(B) $3a^2$

(C) $\frac{a^2}{6}$

(D) $\frac{a^2}{3}$



29. If sin x + $\sin^2$x = 1, then the value of $\cos^{12}$x + $3\cos^8$x + $\cos^6$x + $2\cos^4$x + $\cos^2$x – 2 is equal to :

(A) 0 (B) 1 (C) 2 (D) $\sin^2$x

30. The angles of elevation of the top of a TV tower from three points A, B and C in a straight line (in the horizontal plane) through the foot of tower are α, 2α and 3α respectively. If AB = a, the height of tower is :

(A) a tanα (B) a sinα (C) a sin 2α (D) a sin 3α

31. The expression $\text{cosec}^2A\cot^2A - \sec^2A\tan^2A - (\cot^2A - \tan^2A)(\sec^2A\,\text{cosec}^2A - 1)$ is equal to :

(A) 0 (B) 1 (C) –1 (D) None of these

32. $(1 + \tan\alpha\tan\beta)^2 + (\tan\alpha - \tan\beta)^2$ is equal to :

(A) $\cos^2\alpha\cos^2\beta$ (B) $\tan^2\alpha\tan^2\beta$ (C) $\tan^2\alpha + \tan^2\beta$ (D) $\sec^2\alpha\sec^2\beta$

33. From the top of a light house, 60 m high with its base at the sea level, the angle of depression of a boat is 15^0. The distance of the boat from the foot of the light house is :

(A) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)60$ m (B) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ m (C) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)60$ m (D) None of these

34. The angles of elevation of the top of a tower as observed from the bottom and top of a building of height 60 m are 60^0 and 45^0 respectively. The distance of the base of the tower from the base of the building is :

(A) $30(\sqrt{3}-1)$ m (B) $30(3+\sqrt{3})$ m (C) $30(3-\sqrt{3})$ m (D) $30(\sqrt{3}+1)$ m

35. $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$ is equal to :

(A) 0 (B) 1 (C) – 1 (D) None of these

36. If 0 < x < $\frac{\pi}{2}$, then the largest angle of a triangle whose sides are 1, sin x, cos x is :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2} - x$ (D) x

37. ABC is right angled at C, then tan A + tan B =

(A) $\frac{a^2}{bc}$ (B) $\frac{c^2}{ab}$ (C) $\frac{b^2}{ac} - x$ (D) a + b

38. A rectangle with an area of 9 square metre is inscribed in a triangle ABC having AB = 8 m, BC = 6 m and $\angle ABC = 90^0$. The dimensions of the rectangle (in metres) are :

(A) $2, \frac{9}{2}$ or $6, \frac{3}{2}$ (B) 1 , 9 or 3, 3 (C) 2, 4.5 (D) 4, 2.25

39. From the top of a light house, the angles of depression of two stations on opposite sides of it at distance 'a' apart are α and β. The height of the light house is :

(A) $\dfrac{a}{\cot\alpha\cot\beta}$ (B) $\dfrac{a}{\cot\alpha+\cot\beta}$ (C) $\dfrac{a\cot\alpha\cot\beta}{\cot\alpha+\cot\beta}$ (D) $\dfrac{a\tan\alpha\tan\beta}{\cot\alpha+\cot\beta}$

40. The value of the expression tan 1^0 tan 2^0 tan 3^0......tan 89^0 is equal to :

(A) 0 (B) Not defined (C) 1 (D) ∞

41. If $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$ then $\cos\theta_1+\cos\theta_2+\cos\theta_3$ is equal to :

(A) 3 (B) 2 (C) 1 (D) 0

42. If sinx + $\sin^2$x = 1, then $\cos^8$x + 2$\cos^6$x + $\cos^4$x is equal to :

(A) 0 (B) – 1 (C) 2 (D) 1

43. Which of the following is not possible ?

(A) $\sin\theta=\dfrac{5}{7}$ (B) $\cos\theta=\dfrac{1+t^2}{1-t^2}, t\neq 0$ (C) $\tan\theta=100$ (D) $\sec\theta=\dfrac{5}{2}$

44. $\cot\theta=2\sin\theta\cos\theta(0\le\theta\le 90^0)$ if θ equals :

(A) 45^0 and 90^0 (B) 45^0 and 60^0 (C) 45^0 only (D) 90^0 only

45. In a triangle ABC right angled at C, tan A and tan B satisfy the equation :

(A) $abx^2 - (a^2 + b^2)x - ab = 0$ (B) $abx^2 - c^2x + ab = 0$

(C) $c^2x^2 - abx + c^2 = 0$ (D) $ax^2 - bx + a = 0$

46. The area of the circle and the area of a regular polygon of n sides and of perimeter equal to that of the circle are in the ratio of :

(A) $\tan\left(\dfrac{\pi}{n}\right):\dfrac{\pi}{n}$ (B) $\cos\left(\dfrac{\pi}{n}\right).\dfrac{\pi}{n}$ (C) $\sin\left(\dfrac{\pi}{n}\right):\dfrac{\pi}{n}$ (D) $\cot\left(\dfrac{\pi}{n}\right):\dfrac{\pi}{n}$

47. If $\tan\theta+\sec\theta=\sqrt{3},\ 0<\theta<\dfrac{\pi}{2}$, then θ is equal to :

(A) $\dfrac{\pi}{3}$ (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{4}$ (D) None of these

48. A tower subtends an angle α at a point 'A' in the plane of it's base and the angle of depression of the foot of the tower at a height b just above A is B. Then the height of the tower is :

(A) $b\tan\alpha\cot\beta$ (B) $b\cot\alpha\tan\beta$ (C) $b\tan\alpha\tan\beta$ (D) $b\cot\alpha\cot\beta$

49. If sinx + $\sin^2$ x = 1, then $\cos^2$x + $\cos^4$x is equal to :

(A) 1 (B) –1 (C) 2 (D) 0

50. The angle of elevation of a tower from a point A due south of it is x and from a point b due to east of A is y. if AB = ℓ, the height h of the tower is :

(A) $\dfrac{\ell}{\sqrt{\cot^2 y-\cot^2 x}}$ (B) $\dfrac{\ell}{\sqrt{\tan^2 y-\tan^2 x}}$ (C) $\ell\sqrt{\cot^2 y-\cot^2 x}$ (D) $\ell\sqrt{\tan^2 y-\tan^2 x}$

OBJECTIVE **ANSWER KEY** EXERCISE -4

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	D	B	D	B	D	B	A	C	A	C	B	A	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	A	B	D	C	C	A	B	A	A	D	B	C	D	C
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	A	D	A	D	B	A	A	A	B	C	D	D	B	A	B
Que.	46	47	48	49	50										

Ans.	A	B	A	A	A	

COMPETITION WINDOW

LAW OF SINES

We use the sine rule for non-right angled triangles to find the lengths and angles. In trigonometry, the law of sines (also known as the sines law, sine formula, or sine rule) is an equation relating the lengths of the sides of an arbitrary triangle to the sines of it's angle. According to the law.

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Where a, b and c are the lengths of the sides of a triangle, and A, B and C are the opposite angles. To use the sine rule, choose an appropriate pair, depending on what you know in the triangle

e.g., $\frac{a}{\sin A}=\frac{b}{\sin B}$ or $\frac{a}{\sin a}=\frac{c}{\sin C}$ or $\frac{b}{\sin B}=\frac{c}{\sin C}$

If you are finding an angle, you can invert the formulae.

e.g., $\frac{\sin A}{a}=\frac{\sin B}{b}$ or $\frac{\sin a}{a}=\frac{\sin C}{c}$ or $\frac{\sin B}{b}=\frac{\sin C}{c}$

E.g. Find the length of PQ in triangle PQR.

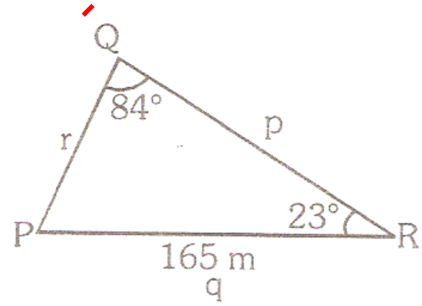
Use the sine rule

$$\frac{p}{\sin P}=\frac{q}{\sin Q}=\frac{r}{\sin R}$$

Use : $\frac{q}{\sin Q}=\frac{r}{\sin R}$

$$\Rightarrow \frac{165}{\sin 84^0}=\frac{r}{\sin 23^0}$$

$$\Rightarrow r=\frac{165\times\sin 23^0}{\sin 84^0}=64.8\text{ metres}$$

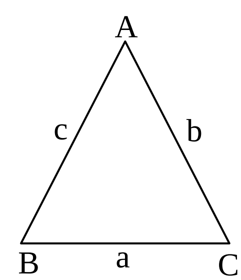


TRY OUT THE FOLLOWING

1. Find the length of BC in triangle ABC, if $\angle A=78^0, \angle C=18^0$ and AB = 26 cm. [Ans : 82.3 cm]
2. Find the length of AC in triangle ABC, if $\angle B=86^0, \angle A=74^0$ and AB = 35 cm. [Ans : 102.1 m]

RELATION TO THE CIRCUM RADIUS (R)

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R \quad \text{...(i)}$$



AREA OF A TRIANGLE

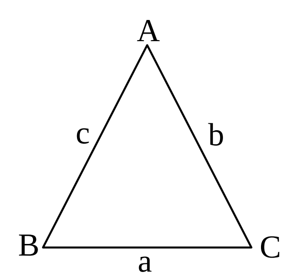
For any triangle ABC, the area is given by

$$\Delta=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin C \quad \text{...(ii)}$$

From (i) sin C = $\frac{c}{2R}$ ...(iii)

From (ii) $\Delta=\frac{1}{2}$ ab sin C ...(iv)

From (iii) and (iv) $R=\frac{abc}{4\Delta}$

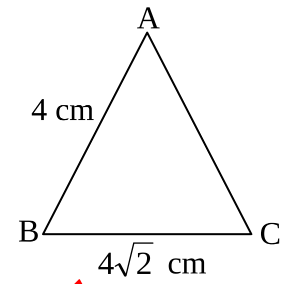


LAW OF COSINES

In any triangle ABC,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

To use the cosine rule, you need to know either two sides and the included angle or all three sides. e.g., Find the length of AC in ΔABC



use : $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$$\cos 45^0 = \frac{(4)^2 + (4\sqrt{2})^2 - AC^2}{2 \times 4 \times 4\sqrt{2}}$$

32 = 48 – AC^2

AC = 4 cm

EXERCISE – 5 (FOR IIT-JEE/AIEEE)

CHOOSE THE CORRECT ONE

Based On Sine Rule (Q.No. 1-20)

1. In ΔABC. AB = 30 cm and $\angle$C = 45^0. The length of the radius of circumcircle of ΔABC is :

(A) $15\sqrt{2}$ cm (B) $5\sqrt{2}$ cm (C) $15\sqrt{3}$ cm (D) $5\sqrt{3}$ cm

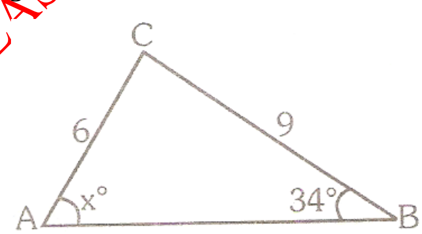
2. The radius of the circumcircle of ΔABC is $\dfrac{2\sqrt{3}}{3}$ cm. If BC = 2 cm, the size of angle A is :

(A) 30^0 (B) 60^0 (C) 90^0 (D) 45^0

3. In ΔABC, $\angle$A : $\angle$B = 1 : 3 : 8. If AB = 10 cm, the length of AC is : [Use : is $(180^0 - \theta) = \sin\theta$]

(A) $\dfrac{10\sqrt{6}}{3}$ cm (B) $\dfrac{10\sqrt{3}}{3}$ cm (C) $\dfrac{10\sqrt{3}}{6}$ cm (D) None of these

4. The measure of angle x in the triangle below is :



(A) 54^0 (B) 57.01^0 (C) 59^0 (D) None of these

5. In a circle of radius 7 cm, the are AB subtends an angle of 120^0 at the centre. The length of chord AB is :

(A) $7\sqrt{3}$ cm (B) $3\sqrt{2}$ cm (C) $5\sqrt{3}$ cm (D) $2\sqrt{3}$ cm

6. In a triangle ABC, a = 6, b = 12 and B = 60^0. The value of sin A is ;

(A) $\dfrac{\sqrt{3}}{4}$ cm (B) $\dfrac{1}{\sqrt{3}}$ cm (C) $\dfrac{1}{2}$ cm (D) None of these

7. In ΔABC. a = 2, b = 3 and $\sin A = \dfrac{2}{3}$, then $\angle$B is equal to :

(A) 30^0 (B) 60^0 (C) 90^0 (D) 120^0

8. In ΔABC. a = 4, c = 12 and $\angle$C = 60^0, then the value of sin A is :

(A) $\frac{1}{2\sqrt{3}}$ cm (B) $\frac{-1}{2\sqrt{3}}$ cm (C) $\frac{\sqrt{2}}{3}$ cm (D) $\frac{\sqrt{3}}{2}$

9. In an isosceles triangle ABC, the base AB = 12 cm and the angle at the top is 30^0. D is a point on the side BC such that $\angle$CAD : $\angle$DAB = 1 : 4. The length of the radius of circumcircle of ΔABC is :
(A) $3\sqrt{2}$ cm (B) $5\sqrt{2}$ cm (C) $6\sqrt{2}$ cm (D) $10\sqrt{2}$ cm

10. The base of an isosceles triangle is 10 cm, and the angle at the base is 2a. the length of the angle bisector of one of the base angles is : [Use : $\sin(180^0-\theta)=\sin\theta$]
(A) 10 sin 2a cos 2a (B) $\frac{10\sin 2a}{\sin 3a}$ (C) $\frac{10\sin 3a}{\sin 2a}$ (D) 1 0sin 4a

11. In the circumference with radius 50 cm is inscribed a quadrilateral. Two of its angles are 45^0 and 120^0. The length of diagonals is :
(A) $25\sqrt{2}$ cm; $25\sqrt{3}$ cm (B) $10\sqrt{2}$ cm; $10\sqrt{3}$ cm
(C) $50\sqrt{2}$ cm; $50\sqrt{3}$ cm (D) None of these

12. In ΔABC, $\angle$A = 45^0, $\angle$B = 30^0. M is a point on the side AB. The radius of the circumcircle of ΔAMC is R. The radius of the circumcircle of ΔMBC is :
(A) 2R cm (B) $R\sqrt{2}$ cm (C) $\frac{R}{\sqrt{2}}$ cm (D) None of these

13. The angle of a triangle are as 5 : 5 : 2, the ratio of the greatest side to the least side is :
(A) 2 + $\sqrt{3}$: 1 (B) 2 + $\sqrt{3}$: 2 – $\sqrt{3}$ (C) $\sqrt{3}$ – 1 : $\sqrt{3}$ + 1 (D) None of these

14. The perimeter of an acute angled triangle ABC is 6 times the arithmetic mean of the sines of its angles. If the side b is 2, the angle B is :
(A) 30^0 (B) 60^0 (C) 90^0 (D) None of these

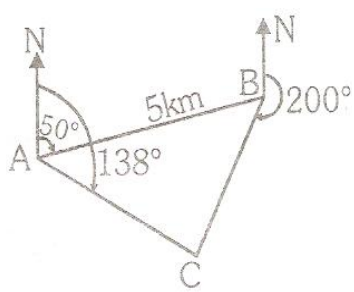
15. If the angles of a triangle be in the ratio 1 : 4 : 5, then the ratio of it's greatest side to the smallest side is :
(A) 5 : 1 (B) $(\sqrt{5}+1):1$ (C) $1:(\sqrt{5}-1)$ (D) None of these

16. In a ΔABC, if a sin A = b sin B, then the triangle is :
(A) Right angled (B) Equilateral (C) Right angled isosceles (D) Isosceles

17. Points D, E are taken on the side BC of a triangle ABC such that BD = DE = EC. If $\angle$BAD = x, $\angle$DAE = y, $\angle$EAC = z, then the value of $\frac{\sin(x+y)\sin(y+z)}{\sin x\sin z}$ is equal to :
(A) 1 (B) 2 (C) 4b (D) None of these

18. In a triangle ABC, A = 45^0, B = 75^0, then $a+\sqrt{2}$ c is equal to :
(A) 2b (B) b (C) 4b (D) $\frac{b}{2}$

19. A hiker starts her journey at point A. She notices a farm house at point C and works out its bearing is at 138^0. She then walks for 5 kilometers and stops at point B. At point B the hiker looks again at the farm house and calculates its bearing now to be 200^0. The distance AC and BC respectively are :



(A) 3.28 km, 6.55 km (B) 2.66 km, 5.83 km
(C) 2.83 km, 5.66 km (D) None of these

20. The angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the largest side to the perimeter is (Use : sin $(180^0-\theta)=\sin\theta$)

(A) $1 : (1+\sqrt{3})$ (B) $2 : 3$ (C) $\sqrt{3} : (2+\sqrt{3})$ (D) $1 : (2+\sqrt{3})$

Based On Cosine Rule (Q. No. 21-37)

21. In ΔABC, AB = 5 cm, AC = 6 cm, $\angle A = 60^0$. The length of the side BC is :
(A) $\sqrt{31}$ cm (B) $\sqrt{29}$ cm (C) 31 cm (D) 29 cm

22. Which of the following options contains the sides of a right angled triangle ?
(A) 13, 14, 15 (B) 12, 35, 37 (C) 13, 15, 24 (D) None of these

23. The size of $\angle C$ of ΔABC, if a = $2\sqrt{3}$ cm, b = 3 cm, c = $\sqrt{3}$ cm is :
(A) 90^0 (B) 60^0 (C) 30^0 (D) None of these

24. The size of $\angle C$ of ΔABC, if a = 11 cm, b = 60 cm, c = 61 cm is :
(A) 90^0 (B) 60^0 (C) 30^0 (D) None of these

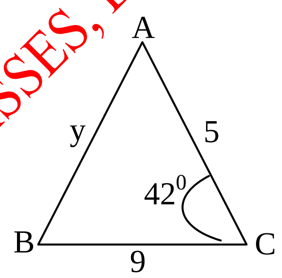
25. In ΔABC we have AC = 3 cm, BC = $\sqrt{5}$ cm, $\angle A = 45^0$. The length of the side AB is :
(A) $\sqrt{3}$ cm (B) $3\sqrt{3}$ cm (C) $\sqrt{2}$ cm or $2\sqrt{2}$ cm (D) $\sqrt{3}$ cm or $3\sqrt{3}$ cm

26. The length of a diagonal of a rectangle is 32 cm, and the angle between the diagonals is 135^0. The length of the sides of rectangle are :
(A) $4\sqrt{3-\sqrt{3}}$ cm and $4\sqrt{3+\sqrt{3}}$ cm (B) $16\sqrt{2-\sqrt{2}}$ cm and $16\sqrt{2+\sqrt{2}}$ cm
(C) 4 cm and 16 cm (D) None of these

27. The in centre of a right angled triangle is at distance $\sqrt{5}$ and $\sqrt{10}$ from the two ends of the hypotenuse. The length of the hypotenuse is :
(A) 5 cm (B) 10 cm (C) 15 cm (D) 7.5 cm

28. The in centre of ΔABC is at distance 7 and $3\sqrt{3}$ from the point A and B. If the angle at point C is 120^0, the length of the side AB is :
(A) $\sqrt{139}$ cm (B) $\sqrt{129}$ cm (C) $\sqrt{119}$ cm (D) None of these

29. Calculate the length y of the side in the triangle below :



(A) 5.25 (B) 4 (C) 6.25 (D) None of these

30. A ship sails from harbor and travels 25 km on a bearing of 300 before reaching a marker buoy. At this point the ship turns and follows a course on a bearing of 900 and travels for 32 km until it reaches an island. On the return journey, the ship is able to take the most direct route back to the harbor. The total distance travelled by the ship is :
(A) 105 km (B) 95 km (C) 112 km (D) 130 km

31. If the angles of a triangle ABC are in AP, then :
(A) $c^2 = a^2 + b^2 + ab$ (B) $a^2 + c^2 - ac = b^2$
(C) $c^2 = a^2 + b^2$ (D) None of these

32. If a = 4, b = 3 and A = 60^0, then c is a root of the equation :
(A) $x^2 - 3x - 7$ (B) $x^2 + 3x + 7 = 0$
(C) $x^2 - 3x + 7$ (D) $x^2 + 3x - 7 = 0$

33. If p1, p2, p3 are the altitudes of a triangle from the vertices A, B C and Δ, the area of the triangle, $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}$ $= \frac{ab(1+k)}{\Delta(a+b+c)}$, then k is equal to :

(A) cos C (B) cos A (C) cos B (D) None of these

34. In a ΔABC, 2ac $\sin\frac{A-B+C}{2}$ is equal to :

(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$
(C) $b^2 - c^2 - a^2$ (D) $c^2 - a^2 - b^2$

35. In a triangle the length of two larger sides are 10 and 9 respectively. If the angles are in A. P., then the third side can be : **[DCE-2001]**

(A) $5\pm\sqrt{6}$ (B) $5-\sqrt{6}$ (C) $3\sqrt{3}$ (D) 5

36. In a ΔABC if b = 20, c = 21 and sin A = $\frac{3}{5}$, then a = **[EAMCET-2003]**

(A) 12 (B) 13 (C) 14 (D) 15

37. In a ΔABC, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then cos C = **[Karnataka-CET-2003]**

(A) $\frac{5}{7}$ (B) $\frac{7}{5}$ (C) $\frac{16}{17}$ (D) $\frac{17}{36}$

Mixed Applications of Sine & Cosine Rule (Q.No. 38-41)

38. The sides of a triangle are $\sqrt{3}+1$ and $\sqrt{3}-1$ and the included angle is 60^0. The difference of the remaining angles is :

(A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

39. If two sides of a triangle and the included angle are given by a = $(1+\sqrt{3})$ cm, b = 2 cm, c = 60^0, the other two angles are :

(A) 90^0, 30^0 (B) 75^0, 45^0 (C) 60^0, 60^0 (D) None of these

40. In the previous Q., the third side is ;

(A) $\sqrt{6}$ cm (B) 6 cm (C) 9 cm (D) None of these

41. If $b^2 + c^2 = 3a^2$, then cot B + cot C – cot A =

(A) 1 (B) $\frac{ab}{4\Delta}$ (C) 0 (D) $\frac{ac}{4\Delta}$

Based On Area Of Triangle (Q.No. 42-46)

42. In a triangle ABC, B = 45^0, a = 2($\sqrt{3}+1$) and area of ΔABC = 6 + $2\sqrt{3}$ square units, then the side b is equal to

(A) $\frac{\sqrt{3}+1}{\sqrt{2}}$ (B) 4 (C) $\sqrt{2}(\sqrt{3}+1)$ (D) None of these

43. In any ΔABC, the expression $\frac{(a+b+c)(c+b-a)(c+a-b)(a+b-c)}{4b^2c^2}$ is equal to :

(A) $\cos^2$A (B) $\sin^2$A (C) 1 – cosA (D) 1 + cos A

44. In any ΔABC, the expression (a + b +c) (a + b – c) (b + c – a) (c + a – b) is equal to :

(A) 16Δ (B) $4\Delta^2$ (C) 4Δ (D) None of these

45. If x,y,z are perpendiculars drawn from the vertices of a triangle having sides a, b and c, then $\frac{bx}{c}+\frac{cy}{a}+\frac{az}{b}$ =

(A) $\frac{a^2+b^2+c^2}{2R}$ (B) $\frac{a^2+b^2+c^2}{R}$ (C) $\frac{a^2+b^2+c^2}{4R}$ (D) $\frac{2(a^2+b^2+c^2)}{R}$

46. In an equilateral triangle of each side $2\sqrt{3}$ cm, the radius of the circumcircle is :

(A) 2 cm (B) 1 cm (C) $\sqrt{3}$ cm (D) $2\sqrt{3}$ cm

47. A pole stands vertically inside a triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in ΔABC, the foot of the pole is at the : **[IIT-2001]**

(A) Centroid (B) Circumcentre (C) In centre (D) Orthocenter

48. A man from the top of a 100 m high tower sees a car moving towards the tower at an angle of depression of 30^0. After some time, the angle of depression becomes 60^0. the distance (in metres) traveled by the car during this time is : **[II'T-Screening-2001]**

(A) 100 $\sqrt{3}$ (B) $\frac{200\sqrt{3}}{3}$ (C) $\frac{100\sqrt{3}}{3}$ (D) $200\sqrt{3}$

49. The value of k for which $(\cos x + \sin x)^2$ + k in sin x cos x – 1 = 0 is an identity is : **[Kerala Engineering-2001]**

(A) – 1 (B) – 2 (C) 0 (D) 1

50. Which of the following pieces of does not uniquely determine an acute angled triangle ABC (R beign the radius of the circumcircle)?

(A) a, sinA, sinB (B) a, b, c (C) a, sinB, R (D) a, sinA, R.

51. The value of $\frac{1-\tan^2 15^0}{1+\tan^2 15^0} =$ **[II' Screening-2002]**

(A) a, sinA, sinB (B) a, b, c (C) a, sinB, R (D) a, sinA, R.

52. $\cos^2\frac{\pi}{12}+\cos^2\frac{\pi}{4}+\cos^2\frac{5\pi}{12}$ is equal to : **[Karnataka-CET-2002]**

(A) $\frac{2}{3+\sqrt{3}}$ (B) $\frac{2}{3}$ (C) $\frac{3+\sqrt{3}}{2}$ (D) $\frac{2}{3}$

53. If tanA + cotA = 4, then $\tan^4 A + \cot^4 A$ is equal to : **[Kerala Engineering-2002]**

(A) 110 (B) 191 (C) 80 (D) 194

54. If $\tan\theta+\sec\theta=e^x$, then $\cos\theta$ equals : **[AMU-2002]**

(A) $\frac{e^x+e^{-x}}{2}$ (B) $\frac{2}{e^x+e^{-x}}$ (C) $\frac{e^x-e^{-x}}{2}$ (D) $\frac{e^x-e^{-x}}{e^x+e^{-x}}$

55. In a ΔABC, if a2 + b2 + c2 – ab – bc – ca = 0, then $\sin^2 A + \sin^2 B + \sin^2 C =$ **[Karnataka-CET-2003]**

(A) $\frac{4}{9}$ (B) $\frac{9}{4}$ (C) $3\sqrt{3}$ (D) 1

56. **In a triangle ABC, medians AD and BE are drawn. If AD = 4,** $\angle DAB=\frac{\pi}{6}$ **and** $\angle ABE=\frac{\pi}{3}$, then the area of the triangle ABC is : **[AIEEE-2003]**

(A) $\frac{64}{3}$ (B) $\frac{8}{3}$ (C) $\frac{32}{3}$ (D) $\frac{32}{3\sqrt{3}}$

57. The upper $\left(\frac{3}{4}\right)$ the portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through it's foot and at a distance 40 m from the foot. A possible height of the vertical pole is:

[Hint : Use the formula $\tan(\theta+\alpha)=\frac{\tan\theta+\tan\alpha}{1-\tan\alpha+\tan\theta}$] **[AIEEE-2003]**

(A) 60 m (B) 20 m (C) 40 m (D) 80 m

58. If θ and ϕ are acute angles, $\sin\theta=\frac{1}{2}.\cos\phi=\frac{1}{3}$, then the value of θ ϕ lies in : **[IIT-Screening-2004]**

(A) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{3},\frac{2\pi}{3}\right)$ (C) $\left(\frac{2\pi}{3},\frac{5\pi}{6}\right)$ (D) $\left(\frac{5\pi}{6},\pi\right)$

59. The sides of a triangle are in the ratio 1 : $\sqrt{3}$: 2, the angles of the triangle are in the ratio :

(A) 1 : 3 : 5 (B) 2 : 3 : 4 **[IIT-Screening-2004]**

(C) 3 : 2 : 1 (D) 1 : 2 : 3

60. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60^0 and when he retires 40 metres away from the tree the angle of elevation becomes 30^0. The breadth of the river is : **[AIEEE-2004]**

(A) 20 m (B) 30 m (C) 40 m (D) 60 m

61. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 300 and tan 150, then the value of 2 + q – p is

(A) 1 (B) 2 **[AIEEE-2006]**

(C) 3 (D) 0

62. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60^0 at foot of the tower, and the angle of elevation of the top of the tower from A or B is 30^0. The height of the tower is : **[AIEEE-2007]**

(A) $\frac{2a}{\sqrt{3}}$ (B) $2a\sqrt{3}$ (C) $\frac{a}{\sqrt{3}}$ (D) $a\sqrt{3}$

63. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angles of elevation of the point A from a certain point C on the ground is 60^0. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45^0. then the height of the pole is :

(A) $\frac{7\sqrt{3}}{2(\sqrt{3}-1)}$ m (B) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m **[AIEEE-2008]**

(C) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m (D) $\frac{7\sqrt{3}}{2(\sqrt{3}+1)}$ m

OBJECTIVE						**ANSWER KEY**									EXERCISE -5
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	A	B	A	B	A	A	C	A	C	B	C	B	A	C	B
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	D	A	A	C	C	A	B	C	A	C	B	A	A	C	B
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**

Ans.	B	A	A	B	A	B	A	D	B	B	C	B	B	D	A
Que.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	A	B	B	B	D	C	D	D	B	B	D	C	B	D	A
Que.	**61**	**62**	**63**												
Ans.	C	C	B												

Sin x^0

Degree	0' $0^0.0$	6' $0^0.1$	12, $0^0.2$	18' $0^0.3$	24' $0^0.4$	30, $0^0.5$	36' $0^0.6$	42' $0^0.7$	48, $0^0.8$	54' $0^0.9$	MEAN DEFFERENCES				
											1	2	3	4	5
.0	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
.1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
.2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
.3	0523	0541	0558	0576	0593	0610	0628	1645	0663	0680	3	6	9	12	15
.4	0698	0715	0732	0750	0767	0785	0802	1819	2837	0854	3	6	9	12	15
.5	0872	0889	0906	0924	0941	0958	0976	1993	1011	1028	3	6	9	12	14
.6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
.7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
.8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
.9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
.10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
.11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
.12	2079	2096	2113	2300	2147	2164	2181	2198	2215	2233	3	6	9	11	14
.13	2250	2267	2284	2130	2317	2334	2351	2368	2385	2402	3	6	8	11	14
.14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
.15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
.16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
.17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
.18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
.19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
.20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
.21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
.22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
.23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
.24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
.25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
.26	4384	4399	4415	4431	4436	4462	4478	4493	4509	4524	3	5	8	10	13

.27	4540	4555	4571	4586	4602	4617	4433	4648	4664	4679	3	5	8	10	13
.28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
.29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
.30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	5	8	10	13
.31	5150	5165	5080	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
.32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
.33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
.34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
.35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
.36	5878	5892	5906	5920	5934	5948	5962	5976	5850	6004	2	5	7	9	12
.37	6018	6032	6046	6060	6074	6088	6101	6115	5990	6143	2	5	7	9	12
.38	6157	6170	6184	6198	6211	6225	6239	6252	6129	6280	2	5	7	9	12
.39	6293	6307	6320	6334	6347	6361	6374	6388	6266	6414	2	4	7	9	11
.40	6428	6441	6455	6468	6481	6494	6508	6521	6401	6547	2	4	7	9	11
.41	6561	6574	6587	6600	6613	6626	6639	6652	6534	6678	2	4	7	9	11
.42	6691	6704	6717	6730	6743	6756	6769	6782	6665	6807	2	4	6	9	11
.43	6820	6833	6845	6858	6871	6884	6896	6909	6794	6934	2	4	6	8	11
.44	6947	6959	6972	6984	6997	7009	7022	7034	6921	7059	2	4	6	8	10
.45	7071	7083	7096	7108	7120	7133	7145	7157	7046	7181	2	4	6	8	10

Sin x^0

Degree	0’ $0^0.0$	6’ $0^0.1$	12, $0^0.2$	18’ $0^0.3$	24’ $0^0.4$	30, $0^0.5$	36’ $0^0.6$	42’ $0^0.7$	48, $0^0.8$	54’ $0^0.9$	MEAN DEFFERENCES				
											1	2	3	4	5

Degree	0’ $0^{0}.0$	6’ $0^{0}.1$	12, $0^{0}.2$	18’ $0^{0}.3$	24’ $0^{0}.4$	30, $0^{0}.5$	36’ $0^{0}.6$	42’ $0^{0}.7$	48, $0^{0}.8$	54’ $0^{0}.9$	MEAN EFFERENCES 1	2	3	4	5
.46	7193	7206	7218	7230	7242	7256	7266	7278	7290	7302	2	4	6	8	10
.47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
.48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
.49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
.50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
.51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
.52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
.53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
.54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
.55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
.56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
.57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
.58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
.59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
.60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
.61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
.62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
.63	8910	8318	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
.64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
.65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
.66	9135	9193	9150	9157	9194	9171	9178	9184	9191	9198	1	2	3	5	6
.67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
.68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
.69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
.70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
.71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
.72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
.73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
.74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
.75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
.76	9403	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
.77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
.78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
.79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
.80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
.81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
.82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
.83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
.84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
.85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
.86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
.87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
.88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
.89	9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0

Degree	0’ $0^{0}.0$	6’ $0^{0}.1$	12, $0^{0}.2$	18’ $0^{0}.3$	24’ $0^{0}.4$	30, $0^{0}.5$	36’ $0^{0}.6$	42’ $0^{0}.7$	48, $0^{0}.8$	54’ $0^{0}.9$	MEAN EFFERENCES 1	2	3	4	5
.0	1.000	1.000	1.000	1.000	1.000	1.000	9999	9999	9999	9999	0	0	0	0	0
.1	.9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
.2	.9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
.3	.9986	9985	9984	9983	9982	9981	9980	9966	9978	9977	0	0	1	1	1
.4	.9976	9974	9973	9972	9971	9969	9968	9979	9965	9963	0	0	1	1	1
.5	.9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
.6	.9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
.7	.9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
.8	.9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
.9	.9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
.10	.9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
.11	.9816	9813	9810	9806	9805	9799	9796	9792	9789	9785	1	1	2	2	3
.12	.9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
.13	.9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
.14	.9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
.15	.9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
.16	.9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
.17	.9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
.18	.9511	9505	9500	9494	9489	9583	9478	9472	9466	9461	1	2	3	4	5
.19	.9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
.20	.9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
.21	.9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
.22	.9272	9265	5259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
.23	.9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
.24	.9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	3	5	6
.25	.9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
.26	.8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
.27	.8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
.28	.8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
.29	.8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
.30	.8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
.31	.8575	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
.32	.8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
.33	.8387	8377	8368	8358	8343	8339	8329	8320	8310	8300	2	3	5	6	8
.34	.8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
.35	.8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
.36	.8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
.37	.7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
.38	.7880	7859	7869	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
.39	.7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
.40	.7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
.41	.7547	7536	7424	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
.42	.7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
.43	.7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
.44	.7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10
.45	.7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
.46	.6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11

.47	.6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
.48	.6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
.49	.6428	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
.50	.6428	6414	6401	3688	6374	6361	6347	6334	6320	6307	2	4	7	9	11
.51	.9293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
.52	.6157	6124	6129	6115	6101	6088	6074	6060	6064	6032	2	5	7	9	12
.53	.6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
.54	.5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
.55	.5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
.56	.5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
.57	.5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
.58	.5299	5284	5270	5255	5240	5225	5210	5135	5180	5165	2	5	7	10	12
.59	.5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	12
.60	.5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
.61	.4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
.62	.4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
.63	.4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
.64	.4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
.65	.4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
.66	.4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
.67	.3907	3891	3875	3859	3843	3887	3811	3795	3778	3762	3	5	8	11	14
.68	.3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
.69	.3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
.70	.3420	3404	3387	3371	3335	3338	3322	3305	3289	3272	3	5	8	11	14
.71	.3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
.72	.3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
.73	.2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
.74	.2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
.75	.2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
.76	.2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
.77	.2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
.78	.2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
.79	.1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
.80	.1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
.81	.1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
.82	.1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
.83	.1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
.84	.1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
.85	.0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	14
.86	.0698	0680	0663	0645	0623	0610	0593	0576	0558	0541	3	6	9	12	15
.87	.0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
.88	.0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
.89	.0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15

tan x^0

Degree	0' $0^0.0$	6' $0^0.1$	12, $0^0.2$	18' $0^0.3$	24' $0^0.4$	30, $0^0.5$	36' $0^0.6$	42' $0^0.7$	48, $0^0.8$	54' $0^0.9$	MEAN DEFFERENCES				
											1	2	3	4	5
.0	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
.1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
.2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
.3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
.4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
.5	0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
.6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
.7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
.8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
.9	1584	1602	1620	1638	1635	1673	1691	1709	1727	1745	3	6	9	12	15
.10	1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
.11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
.12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
.13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
.14	2493	2512	2530	2549	2568	2586	2605	2623	6242	2661	3	6	9	12	16
.15	2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
.16	2867	2886	1905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
.17	3057	3076	3096	3115	3134	3253	3172	3191	3211	3230	3	6	10	13	16
.18	3249	3269	3288	3307	3327	3541	3365	3385	3404	3424	3	6	10	13	16
.19	3443	3463	3482	3502	3522	3739	3561	3581	3600	3620	3	7	10	13	16
.20	3640	3659	3679	3699	3719	4142	3759	3779	3799	3819	3	7	10	13	17
.21	3839	3859	3869	3899	3919	4348	3959	3979	4000	4020	3	7	10	13	17
.22	4040	4061	4081	4101	4122	4557	4163	4183	4204	4224	3	7	10	14	17
.23	4245	4265	4286	4307	4327	4770	4369	4390	4411	4431	3	7	10	14	17
.24	4452	4473	4494	4515	4536	4986	4578	4599	4621	4642	4	7	11	14	18
.25	4663	4684	4706	4727	4748	5206	4791	4813	4834	4856	4	7	11	14	18
.26	4877	4809	4921	4942	4962	5430	5008	5029	5051	5073	4	7	11	15	18
.27	5095	5117	5139	5161	5184	5658	5228	5250	5272	5295	4	7	11	15	18
.28	5317	5340	5362	5384	5407	5890	5452	5475	5498	5520	4	8	11	15	19
.29	5543	5566	5589	5612	5635	6128	5681	5704	5727	5750	4	8	12	15	19
.30	5774	5797	5820	5844	5867	6371	5914	5938	5961	5985	4	8	12	16	20
.31	6009	6032	6056	6080	6104	6619	6152	6176	6200	6224	4	8	12	16	20
.32	6249	6273	6297	6322	6346	6873	6395	6420	6445	6469	4	8	12	16	20
.33	6494	6519	6544	6569	6594	7133	6644	6669	6694	6720	4	8	13	17	21
.34	6745	6771	6796	6822	6874	7400	6899	6924	6950	6976	4	9	13	17	21
.35	7002	7028	7054	7080	7107	7673	7159	7186	7212	7239	4	9	13	18	22
.36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
.37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
.38	7513	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
.39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
.40	8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
.41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
.42	9004	9036	9067	9099	9131	9163	9195	9228	6260	9293	5	11	16	21	27
.43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
.44	9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29
.45	1.000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30

tan x^0

Degree	0' $0^0.0$	6' $0^0.1$	12, $0^0.2$	18' $0^0.3$	24' $0^0.4$	30, $0^0.5$	36' $0^0.6$	42' $0^0.7$	48, $0^0.8$	54' $0^0.9$	MEAN EFFERENCES 1	2	3	4	5
.46	1.0355	0392	0428	0464	0501	0838	0575	0612	0649	0686	6	12	18	25	31
.47	1.0724	0761	0299	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
.48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
.49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
.50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
.51	1.2349	2393	2437	2484	2527	2572	2617	2662	2708	2753	8	15	23	30	38
.52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
.53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
.54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
.55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
.56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
.57	1.5399	5458	4938	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
.58	1.6003	6066	5517	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
.59	1.6643	6709	6128	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
.60	1.7321	7391	6775	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
.61	1.8040	8115	7461	8265	8341	8418	8405	8572	8650	8728	13	26	38	51	64
.62	1.8807	8887	8190	9047	9128	9210	9262	9375	9458	9542	14	27	41	55	68
.63	1.9626	9711	8967	9883	9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
.64	2.0503	0594	9797	0778	0872	0965	1060	1155	1155	1348	16	31	47	63	78
.65	2.1445	1543	0686	1742	1842	1943	2045	2148	2148	2355	17	34	51	68	85
.66	2.2460	2566	1642	2781	2889	2998	3109	3220	3220	3445	18	37	55	73	92
.67	2.3559	3673	2673	3906	4023	4142	4262	4383	4383	4627	20	40	60	79	99
.68	2.4751	4876	3789	5129	5257	5386	5517	5649	5649	5916	22	43	65	87	108
.69	2.6051	6187	5002	6464	6605	6746	6889	7034	7034	7326	24	47	71	95	119
.70	2.7475	7625	6325	7929	8083	8239	8397	8556	8556	8878	26	52	78	104	133
.71	2.9042	9208	7776	9544	9714	9887	3.0061	3.0237	3.0237	3.0595	29	58	87	116	145
.72	3.0777	0961	9375	1334	1524	1716	1910	2106	2106	2506	32	64	96	129	161
.73	.02709	2914	1146	3332	3544	3759	3977	4197	4197	4646	36	72	108	144	180
.74	3.4874	5105	3122	5576	5816	6059	6305	6554	6554	7062	41	81	122	163	204
.75	3.7321	7583	5339	8118	8391	8667	8947	9232	9232	9812	46	93	139	186	232
.76	4.0108	0408	7848	1022	1335	1653	1976	2303	2303	2972	53	107	160	213	267
.77	4.3315	3662	0713	4374	4737	5107	5483	5864	5864	6646					
.78	4.7046	7453	4015	8288	8716	9152	9594	5.0045	5.0045	5.0970					
.79	5.1446	1923	7867	2924	3435	3955	4486	5026	5026	6140					
.80	5.6713	7297	2422	8502	9124	9758	6.0405	6.1066	6.1066	6.2432					
.81	6.3138	3859	7894	5350	6122	6912	7720	8548	8548	7.0264					
.82	7.1154	2066	4596	3962	4947	5958	6996	8062	8062	8.0285					
.83	8.1443	2636	3002	5126	6427	7769	9152	9.0579	9.0579	9.3572					
.84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.78	11.20					
.85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.30	13.95					
.86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.34	18.46					
.87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	24.90	27.27					
.88	28.64	30.14	31.82	33.69	35.80	38.19	40.62	44.07	44.07	52.08					
.89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	191.0	573.0					

Important Notes

SURFACE AREAS AND VOLUMES

★ INTRODUCTION

In this chapter, shall discuss problems on conversion of one of the solids like cuboid, cube, right circular cylinder, right circular cone and sphere in another.

In our day-to-day life we come across various solids which are combinations of two or more such solids, For example, a conical circus tent with cylindrical base is a combination of a right circular cylinder and a right circular cone, also an ice-cream cone is a combination of a cone and a hemi-sphere. We shall discuss problems on finding surface areas and volumes of such solids. We also come across solids which are a part of a cone. For example, a bucket , a glass tumbler, a friction clutch etc. these solids are known as frustums of a cone. In the end of the chapter, we shall discuss problems on surface area and volume of frustum of a cone.

★ UNITS OF MEASUREMENT OF AREA AND VOLUME

The inter-relationships between various units of measurement of length, area and volume are listed below for ready reference:

LENGTH

1 Centimetre (cm)	=	10 milimetre (mm)
1 Decimetre (dm)	=	10 centimetre
1 Metre (m)	=	10 dm = 100 cm = 1000mm
1 Decametre (dam)	=	10 m = 1000 cm
1 Hectometre (hm)	=	10 dam = 100 m
1 Kilometre (km)	=	1000 m = dam = 10 hm
1 Myriametre	=	10 kilocetre

AREA

1 cm^2 = 1 cm x 1 cm = 10 mm x 10 mm = 100 mm^2

1 dm^2 = 1 dm x 1 dm = 10 cm x 10 cm = 100 cm^2

1 m^2 = 1 m x 1 m = 10 dm x 10 dm = 100 dm^2

1 dam^2 = 1 dam x 1 dam = 10 m x 10 m = 100 m^2

1 hm^2 1 hectare = 1 hm x 1 hm = 100 m x 100 m = 10000 m^2 = 100 dm^2

1 km^2 = 1 km x 1km = 10 hm x 10 hm = 100 hm^2 or 100 hectare

VOLUME

1 cm^3 = 1 ml = 1 cm x 1 cm x 1cm = 10 mm x10 mm x10 mm = 1000 mm^3

1 litre = 1000 ml = 1000 cm^3

1 m^3 = 1 m x 1 m x1 m = 100 cm x100 cm x100 cm = 10^6 cm^3 = 1000 litre = 1 kilolitre

1dm^3 = 1000 cm^3

1 m^3 = 1000 dm^3

1 km^3 = 10^9 m^3

★ CUBOID

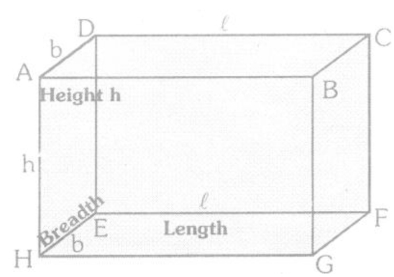
A rectangular solid bounded by six rectangular plane faces is called a cuboid. A match box, a tea-packet, a brick, a book, etc., are all examples of a cuboid.

A cuboid has 6 rectangular faces, 12 edges and 8 vertices.

The following are some definitions of terms related to a cuboid.

(i) The space enclosed by a cuboid is called its **volume.**

(ii) The line joining opposite comers of a cuboid is called its **diagonal.**
A cuboid has four diagonals.
A diagonal of a cuboid is the length of the longest rod that can be placed in the cuboid.



(iii) The sum of areas of all the six faces of a cuboid is known as its **total surface area.**

(iv) The four faces which meet the base of a cuboid are called the **lateral faces** of the cuboid.

(v) The sum of areas of the four walls of a cuboid is called its lateral **surface area.**

Formulae

For a cuboid of length = units, breadth = b units and height = h units, we have:

Sum of lengths of all edges = 4 $(\ell + b + h)$ units.
Diagonal of cuboid = $\sqrt{\ell^2 + b^2 + h^2}$ units.
Total Surface Area of cuboid = $2(\ell b + bh + \ell h)$ sq. units.
Lateral Surface Area of cuboid = [2 $(\ell + b) \times h$] sq. units.
Area of four walls of a room = [2 $(\ell + b) \times h$] sq. units.
Volume of cuboid = $(\ell \times b \times h)$ cubic units.

REMARK: For the calculation of surface area, volume etc. of a cuboid, the length, breadth and height must be expressed in the same units.

★ CUBE

A cuboid whose length, breadth and height are all squal

is called a cube .

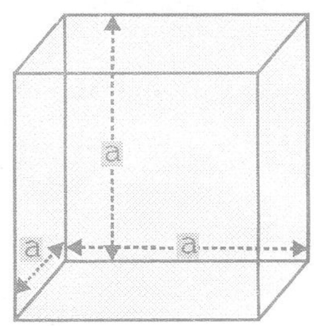
Ice-cubes, Sugar, Dice, etc. are all examples of a cube.

Each edge of a cube is called its side.

Formulae

For a cube of edge = a units, we have;

Sum of length of all edges = 12 a units.
Diagonal of cube = $(a\sqrt{3})$ units.
Total Surface Area of cube = $(6a^2)$ sq. units.
Lateral Surface Area of cube = $(4a^2)$ sq. units.
Volume of cube = a^3 cubic units.



Cube

★ CROSS SECTION

A cut which is made through a solid perpendicular to its length is called its cross section. If the cut has the same shape and size at every point of its length, then it is called **uniform cross-section.**

Volume of a solid with uniform cross section = (Area of its cross section) x (length).
Lateral Surface Area of a solid with uniform cross section
= (Perimeter of cross section) x (length).

Ex.1 The length, breadth and height of a rectangular solid are ratio 6 : 5 : 4. If the total surface area is 5328 cm^2, find the length, breadth and height of the solid.

Sol. Let length = (6x) cm, breadth = (5x) cm and height = (4x) cm.

Then, total surface area = $[2(6x \times 5x + 5x \times 4x + 4x \times 6x)]\ cm^2 = [2(30x^2 + 20x^2)]\ cm^2$ $= (148x^2)cm^2$.

$\therefore$ 148x2 = 5328 ⇨ $x^2 = 36$ ⇨ x = 6.

Hence, length = 36 cm, breadth = 30 cm, height = 24 cm.

Ex.2 **An open rectangular cistern is made of iron 2.5 cm thick. When measured from outside, it is 1 m 25 cm long. 1 m 5 cm broad and 90 cm deep.**

Find: (i) the capacity of the cistern in litres:
(ii) the volume of iron used;
(iii) the total surface area of the cistern.

Sol. External dimensions of the cistern are :
Length = 125 cm, Breadth = 105 cm and Depth = 90 cm.
Internal dimensions of the cistern are :
Length = 120 cm, Breadth = 100 cm and Depth = 87.5 cm.

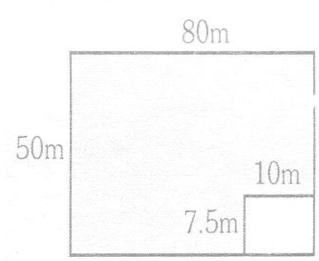
(i) Capacity = Internal volume = (120 x 100 x 87.5) $cm^3 = \left(\frac{120\times100\times87.5}{1000}\right)$ litres = 1050 litres.

(ii) Volume of iron = (External volume) – (Internal volume) = [(125 x 105 x 90) – (120 x 100 x 87.5)] cm^3
= (1181250 – 1050000) cm^3 = 131250 cm^3.

(iii) External area = (Area of 4 faces) + (Area of the base) = ([2(125 + 105) x 90] + (125 x 105)} cm^2.

= (41400 + 13125) cm^2 = 54525 cm^2.
Internal area = ([2(120 + 100) x 87.5] + (120 x 100)} cm^2 = (38500 + 12000) cm^2 = 50500 cm^2.
Area at the top = Area between outer and inner rectangles = [(125 x 105) – (120 x 100)] cm^2
= (13125 – 12000) cm^2 = 1125 cm^2.
Total surface area = (54525 + 50500 + 1125) cm^2 = 106150 cm^2.

Ex.3. **A field is 80 m long and 50 m broad. In one corner of the field, a pit which is 10 m long, 7.5 m broad and 8 m deep has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.**



Sol. Area of the field = (80 x 50) m^2 = 4000 m^2

Area of the pit = (10 x 7.5) m^2 = 15 m^2

Area over which the earth is spread out = (4000 – 75) m2 = 3925 m2

Volume of earth dug out = (10 x 7.5 8) m^3 = 600 m^3.

$\therefore$ Rise in level = $\left(\frac{volume}{Area}\right) = \left(\frac{600}{3925}\right)m = \left(\frac{600\times100}{3925}\right)cm = 15.3cm$

Ex.4. **A room is half as long again as it is broad. The cost of carpeting the room at Rs 18 per m^2 is Rs 972 and the cost of white-washing the four walls at Rs 6 per m^2 is Rs 1080. Find the dimensions of the room.**

Sol. Let breadth = (x) in. Then, length = $(\frac{3}{2}\times)$ m.

Let height of the room = y m.

Area of the floor = $\left(\frac{colt\ of\ carpeting}{Rate}\right) = \left(\frac{972}{18}\right) = 54\ m^2$

$\therefore$ $x\times\frac{3}{2}x = 54 \Rightarrow x^2 = \left(54\times\frac{2}{3}\right) = 36 \Rightarrow x = 6$.

So, breadth = 6 m and length = $\left(\frac{3}{2}\times6\right)$ m = 9 m.

Now, area of four walls = $\left(\frac{\text{cost of white} - \text{washing}}{Rate}\right) = \left(\frac{1080}{6}\right) m^2 = 180\, m^2$.

$\therefore \qquad 2(9+6)\times y = 180 \Rightarrow 30y = 180 \Rightarrow y = \left(\frac{180}{30}\right) = 6$.

Hence, length = 9 m, breadth = 6 m, height = m.

Ex.5. The water in a rectangular reservoir having a base 80 x m 60 m, is 6.5 m deep. In what time can the water be emptied by a pipe of which the cross section is a square of side 20 cm, if water runs through the pipe at the rate of 15 km/ hr ?

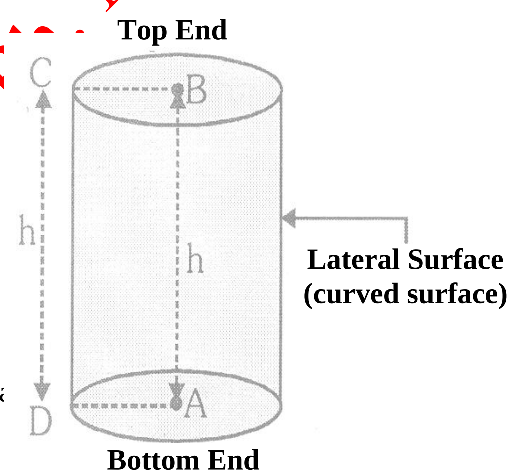
Sol. **V**olume of water in the reservoir = (80 x 60 x 6.5) m^3 = 31200 m^3.

Area of cross section of the pipe = $\left(\frac{20}{100}\times\frac{20}{100}\right) m^3 = \frac{1}{25}\, m^2$.

Volume of water emptied in 1 hr = $\left(\frac{1}{25}\times 15000\right) m^3 = 600\, m^3$.

Time taken to empty the reservoir = $\left(\frac{31200}{600}\right)$ **hrs = 52 hrs.**

RIGHT CIRCULAR CYLINDER



Solids like circular pillars, circular pencils, measuring jars, road rollers and gas cylinders, etc., are said to be in cylindrical shape.

In mathematical terms, **a right circular cylinder is a solid generated by the revolution of a rectangle about its sides.**

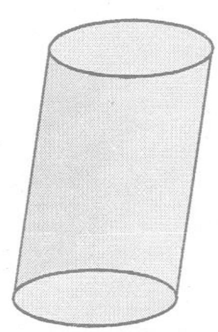
Let the rectangle ABCD revolve about its side AB, so as to describe a right circular cylinder as shown in the figure.

You must have observed that the cross-section of a right circular cylinder are circles congruent and parallel to each other.

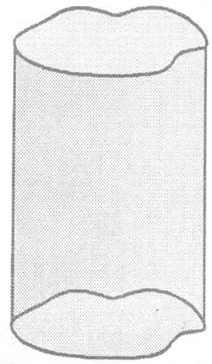
Cylinders Not Right Circular

There are two cases when the cylinder is not a right circular cylinder.

Case-I : In the following figure, we see a cylinder, which is certainly, but is not at right angles to the base, So we cannot say it is a right circular cylinder,



Case-II : In the following figure, we see a cylinder, with a non-circular base as the base is not circular. So we cannot say it is a right circular cylinder,



REMARK : Unless stated otherwise, here in this chapter the word cylinder would mean a right circular cylinder.

The following are definitions of some terms related to a right circular cylinder :

(i) The radius of any circular end is called the **radius** of the right circular cylinder.
Thus, in the above figure, AD as well as BC is a radius of the cylinder .

(ii) The line joining the centres of circular ends of the cylinder, is called the axis of the right circular cylinder. In the above figure, the line AB is the axis of the cylinder. Clearly, the axis is perpendicular to the circular ends.

REMARK : If the line joining the centres of circular ends of a cylinder is not perpendicular to the circular ends, then the cylinder is not a right circular cylinder.

(iii) The length of the axis of the cylinder is called the **height or length of the** cylinder.

(iv) The curved surface joining the two bases of a right circular cylinder is called its **lateral surface.**

Formulae

For a right circular cylinder of radius = r units & height = h units, we have :

Area of each circular end = πr^2 sq. units.

Curved (Lateral) Surface Area = $(2\pi rh)$ sq. units.

Total Surface Area = Curved Surface Area + Area of two circular ends.
= $(2\pi rh + 2\pi r^2)$ sq. units.
= $[2\pi r\,(h + r)]$ sq. units.

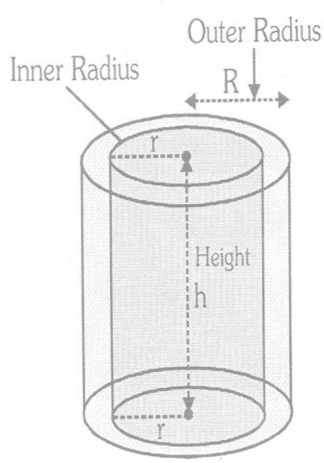
Volume of cylinder = $\pi r^2 h$ cubic units.

The above formulae are applicable to solid cylinders only.

Hollow Right Circular Cylinders

Solids like iron pipes, rubber, tubes, etc., are in the shape of hollow cylinder.

A solid bounded by two coaxial cylinders of the same height and different radii is called a hollow cylinder



Formulae

For a hollow cylinder of height h and with external and internal radii R and r respectively, we have :

Thickness of cylinder = $(R - r)$ units.
Area of a cross-section = $(\pi R^2 - \pi r^2)$ sq. units.
= $\pi(R^2 - r^2)$ sq. units.

Curved (Lateral) Surface Area = (External Curved Surface Area)
+ (Internal Curved Surface Area)
= (2πRh + 2πrh) sq. units = 2πh (R + r) sq. units.
Total Surface Area = (Curved Surface Area) + 2 (Area of Base Ring)
= [(2πRh + 2πrh) + ($\pi R^2 - \pi r^2$)] sq. units
= 2π(Rh + rh +v$R^2 - r^2$) sq. units
Volume of Material = π($R^2 - r^2$) h cubic units
Volume of Hollow region = πr^2h cubic units

Ex. 6 **2.2 cu dm of brass is to drawn into a cylindrical wire of diameter 0.50 cm. Find the length of the wire.**

Sol. Volume of brass = 2.2 cu dm = (2.2 x 10 x 10 x 10) cm^3 = 2200 cm^3. Let the required length of wire be x cm.

Then, its volume = $(\pi r^2 x) cm^3 = \left(\frac{22}{7} \times 0.25 \times 0.25 \times x\right) cm3$

$\therefore \quad \frac{22}{7} \times 0.25 \times 0.25 \times x = 2200$

$\Rightarrow \quad x = \left(2200 \times \frac{7}{22} \times \frac{1}{0.25 \times 0.25}\right) = 11200\, cm = 112\, m.$

Hence, the length of wire is 112 m.

Ex. 7 **A well 14 m diameter is dug 8 m deep. The earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. Find the height of the embankment.**

Sol. Volume of earth dug out from the well = $\pi r^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 8\right) m^3 = 1232\, m^3$.

Area of the embankment =π $(R^2 - r^2) = \frac{22}{7} \times \{(28)^2\} m^2 = \left(\frac{22}{7} \times 35 \times 21\right) m^2 = 2310\, m^2$**.**

Height of the embankment = $\frac{Volume\ of\ earth\ dug\ out}{Area\ of\ embankment} = \left(\frac{1232}{2310} \times 100\right) cm. = 53.3\, cm$**.**

Ex. 8 **The difference between the outside and inside surface of a cylinder metallic pipe 14 cm long is 44 cm^2. If the pipe is made of 99 cu cm of metal, find outer and inner radii of the pipe.**

Sol. Let, external radius = R cm and internal radius = r cm.

Then, outside surface = π Rh = $\left(2 \times \frac{22}{7} \times R \times 14\right) cm^2 = (88R)\, cm^2$.

Inside surface = 2 π rh = $\left(2 \times \frac{22}{7} \times r \times 14\right) cm^2 = (88r)\, cm^2$.

$\therefore \quad (88R - 88r) = 44 \Rightarrow (R - r) = \frac{44}{88} = \frac{1}{2} \Rightarrow (R - r) = \frac{1}{2}$

Internal volume = $\pi R^2 h = \left(\frac{22}{7} \times R^2 \times 14\right) cm^3 = (44R^2)\, cm^3$

$\therefore \quad (44R^2 - 44r^2) = 99 \Rightarrow (R^2 - r^2) = \frac{99}{44} \Rightarrow (R^2 - r^2) = \frac{9}{4}$

On dividing (ii) by (i), we get: (R + r) = $\left(\frac{9}{4} \times \frac{2}{1}\right) \Rightarrow (R + r) = \frac{9}{2}$

Solving (i) and (ii), we get, R = 2.5 and r = 2.

Hence, outer radius = 2.5 cm and inner radius = 2 cm.

Ex. 9 **A solid iron rectangular block of dimensions 4.4 m, 2.6 m and 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.**

Sol. Volume of iron = (440 x 260 x 100) cm^3.
Internal radius of the pipe = 30 cm.
External radius of the pipe = (30 + 5) cm = 35 cm.

Let the length of the pipe be h cm.
Volume of iron in the pipe = (External volume) – (Internal volume)

$$= [\pi \times (35)^2 \times h - \pi \times (30)^2 \times h] cm^3 = (\pi h)\{(35)^2 - (30)^2\} cm^3$$

$$= (65 \times 5)\pi h\, cm^3 = (325\pi h) cm^3.$$

$$\therefore \quad 325\pi h = 440 \times 260 \times 100 \quad \Rightarrow h = \left(\frac{440 \times 260 \times 100}{325} \times \frac{7}{22}\right) cm$$

$$\Rightarrow h = \left(\frac{11200}{100}\right) m = 112m.$$

Hence, the length of the pipe is 112 m.

Ex. 10 A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow in kilometers per hour.

Sol. Volume of water that flows per hour = (192.50 x 60) liters = (192.5 x 60 x 1000) cm^3.
Inner radius of the pipe = 3.5 cm.
Let the length of column of water that flows in 1 hour be h cm.

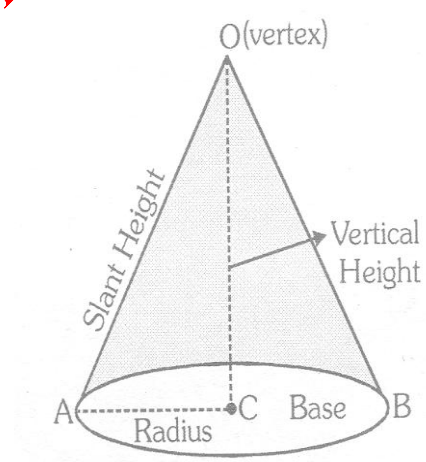
Then, $\frac{22}{7} \times 3.5 \times 3.5 \times h = 192.5 \times 60 \times 1000$

$$\Rightarrow h = \left(\frac{192.5 \times 60 \times 1000 \times 7}{3.5 \times 3.5 \times 22}\right) cm = 300000 cm = 3 \text{ km}$$

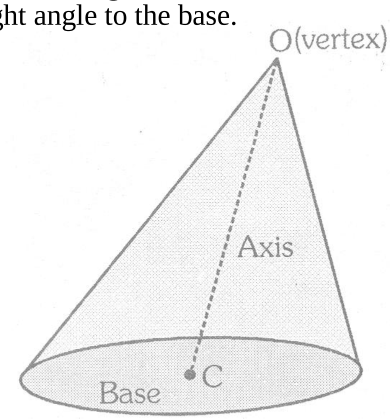
Hence, the rate of flow = 3 km per hour.

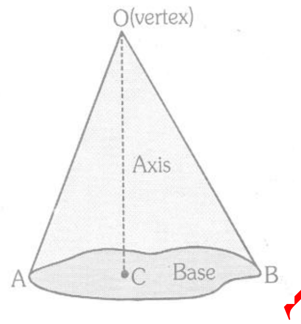
★ RIGHT CIRCULAR CONE

Solids like an ice-cream cone, a conical tent, a conical vessel, a clown's cap etc. are said to be in conical shape. In mathematical terms, **a right circular cone is a solid generated by revolving a right-angled triangle about one of the sides containing the right angle.**
Let a triangle AOC revolve about it's OC, so as to describe a right circular cone, as shown in the figure.



Cones Not Right circular
There are two cases when we cannot call a right circular cone.
Case-1 : The figure shown blown below is not a right circular cone because the line joining its vertex to the centre of its base is not at right angle to the base.





Case-II : The figure shown below is not a right circular cone because the base is not circular.

REMARK : Unless stated otherwise, by 'cone' in this chapter, we shall mean 'a right circular cone

The following are definitions of some terms related to right circular cone :

(i) The fixed point O is called the **vertex** of the cone.

(ii) The fixed line OC is called the **axis** of the cone.

(iii) A right circular cone has a plane end, which is in circular shape. This is called the **base** of the cone. The vertex of a right circular cone is farthest from its base.

(iv) The length of the line segment joining the vertex to the centre of the base is called the **height** of the cone. Length OC is the height of the cone.

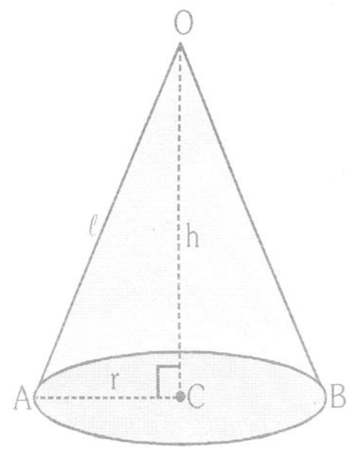
(v) The length of the line segment joining the vertex to any point on the circular edge of the base, is called the **slant height** of the cone.

(vi) The radius AC of the base circle the **radius** of the cone.

Relation Between Slant Height, Radius and Vertical Height.

Let us take a right circular cone with vertex at O, vertical height h, slant height ℓ and radius r. A is any point on the rim of the base of the cone and C is the centre of the base. Here, OC = h, AC = r and OA = ℓ. The cone is right circular and therefore, OC is at right angle to the base of the cone. So, we have OC $\perp$ CA, i.e., ΔOCA is right angled at C.

Then by Pythagoras theorem, we have :



$$\ell^2 = r^2 + h^2$$

Formulae

For a right circular cone of Radius = r, Height = h & Slant Height = ℓ, we have :

Area of the curved (lateral) surface = $(\pi r\ell)$ **sq. units.** = $\left(\pi r\sqrt{h^2+r^2}\right)$ **sq. units**

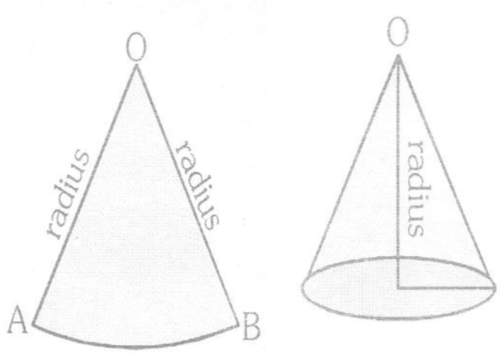
Total Surface Area of cone = (Curved surface Area + Area of Base)

= $(\pi r\ell + \pi r^2)$ **sq. units = π r** $(\ell + r)$ **sq. units.**

Volume of cone = $\left(\frac{1}{3}\pi r^2 h\right)$ **cubic units.**

Hollow Right Cricular Cone

Suppose a sector of a circle is folded to make the radii coincide, then we get a hollow right circular cone. In such a cone;



(i) Centre of the circle is vertex of the cone.

(ii) Radius of the circle is slant height of the cone.

(iii) Length of arc AB is the circumference of the base of the cone.

(iv) **Area of the sector is the curved surface area of the cone.**

Ex.11 **The total surface area of a right circular cone of slant height 13 cm us 90π cm^2.**

Calculate : (i) its radius in cm, (ii) its volume in cm^3, in terms of π.

Sol. Given : slant height, $\ell = 13$cm.
Let, radius = r cm and height = h cm.

(i) Total surface area = **π r** $(\pi + \mathrm{r}) = [\pi r(13 + r)]$ **cm^2.**

$\therefore$ $\pi r(13+r) = 90\pi \Rightarrow r^2 + 13r - 90 = 0 \Rightarrow (r+18)(r-5) = 0$

$\Rightarrow$ r = 5 [Neglecting r = - 18, as radius cannot be negative]

$\therefore$ Radius of the cone = 5 cm.

(ii) $h = \sqrt{\ell^2 - r^2} = \sqrt{(13)^2 - (5)^2}$

= Volume of the cone $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times 5 \times 5 \times 12\right) cm^3$

= 100π cm^3.

Ex.12 **A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find :**

(i) its radius, (ii) its slant height.

Sol. Height of cylindrical bucket, H = 32 cm.

Radius of cylindrical bucket, R = 18 cm.

Volume of sand = $\pi R^2 H = \left(\frac{22}{7} \times 18 \times 18 \times 32\right)$ cm^3.

(i) Height of conical heap, h = 24 cm.

Let the radius of the conical heap be r cm.

Then, volume of conical heap $= \frac{1}{3}\pi r^2 h \left(\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24\right)$ cm^3.

Now, Volume of conical heap = Volume of sand

$\Rightarrow$ $\left(\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24\right) = \left(\frac{22}{7} \times 18 \times 18 \times 32\right)$

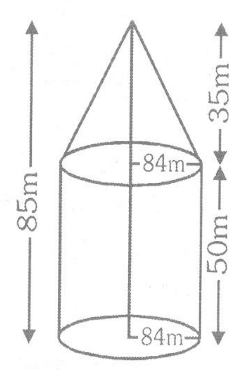
$\Rightarrow$ $r^2 = \left(\frac{18 \times 18 \times 32}{24}\right) = (18 \times 18 \times 4)$

$\Rightarrow$ $r = \sqrt{(18 \times 18 \times 4} = (18 \times 2)$ cm. = 36 cm.

$\therefore$ Radius of the heap = 36 cm.

(ii) Slant height, $\ell = \sqrt{h^2 + r^2} = \sqrt{(24)^3 + (36)^2} = \sqrt{1872} = 12\sqrt{13}$ cm.

Ex.13 **An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for folds and stitching.**



Given your answer to the nearest m^2.

Sol. Radius of the tent, $r = \left(\frac{168}{2}\right)$ m = 84 m.

Height of the tent = 85 m.
Height of the cylindrical part, H = 50 m.
Height of the conical part, h = (85 – 50) m = 35 m.

Slant height of the conical part, $\ell = \sqrt{h^2 + r^2} = \sqrt{(35)^2 + (84)^2} = \sqrt{8281}$ m = 91m.

Quantity of canvas required = Curved surface area of the tent
= Curved surface area of the cylindrical part
+ Curved surface area of the conical part
= $2\pi rH + \pi r\ell = \pi r(2H + \ell)$

$= \left[\frac{22}{7} \times 84(2 \times 50 + 91)\right] m^2 = (22 \times 12 \times 191)\ m^2 = 50424 m^2$.

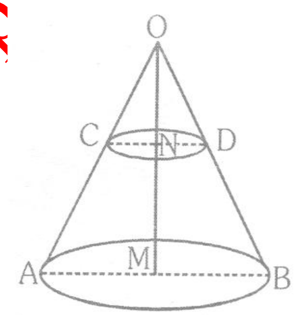
Area of canvas required for folds and stitching = (20% of 50424) m^2 = $\left(\frac{20}{100} \times 50424\right)$ m^2 = 10084.80 m^2.

∴ Total quantity of canvas required to make the tent
= (50424 + 10084.80) m^2 = 60508.80m^2 = 60509 m^2. (to the nearest m^2)

Ex.14 The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height, above the base is the section cut?

Sol. Let OAB be the given cone of height, H 30 cm and base radius R cm. Let this cone be cut by the plane CND to obtain the cone OCD with height h cm and base radius r cm.
Then, Δ OND ~ Δ OMB.

So, $\frac{ND}{MB} = \frac{ON}{OM} \Rightarrow \frac{r}{R} = \frac{h}{30}$ …(i)



Volume of cone OCD $= \frac{1}{27} \times \frac{1}{3}\pi R^2 \times 30$

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{1}{27} \times \frac{1}{3}\pi\ R^2 \times 30$

$\Rightarrow \left(\frac{r}{R}\right)^2 = \frac{10}{9h} \Rightarrow \left(\frac{h}{30}\right)^2 = \frac{10}{9h}$ [From (i)]

$\Rightarrow 9h^3 = 9000 \Rightarrow h^3 = 1000 \Rightarrow h = 10$.

∴ Height of the cone OCD = 10 cm.

Hence, the section is cut at the height of (30 – 10) cm, i.e., 20 cm from the base.

Ex.15 From a solid cylinder of height 30 cm and radius 7 cm, a conical cavity of height 24 cm and of base radius 7 cm is drilled out. Find the volume and the total surface of the remaining solid.

Sol. Radius, r = 7 cm.
Height of the cylinder, H = 30 cm.
Height of the cone, h = 24 cm.

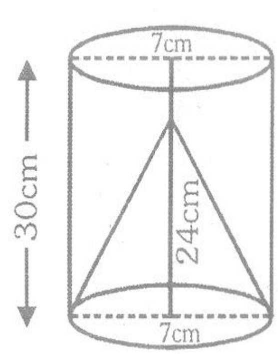
Slant height of the cone, $\ell = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (7)^2} = \sqrt{625} = 25$cm

(i) Volume of the remaining solid

= (Volume of the cylinder) – (Volume of the cone)

$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \pi r^2\left(H - \frac{h}{3}\right)$

$= \left[\frac{22}{7} \times 7 \times 7 \times \left(30 - \frac{24}{3}\right)\right] cm^3 = \left[\frac{22}{7} \times 7 \times 7 \times 22\right]$ cm^3



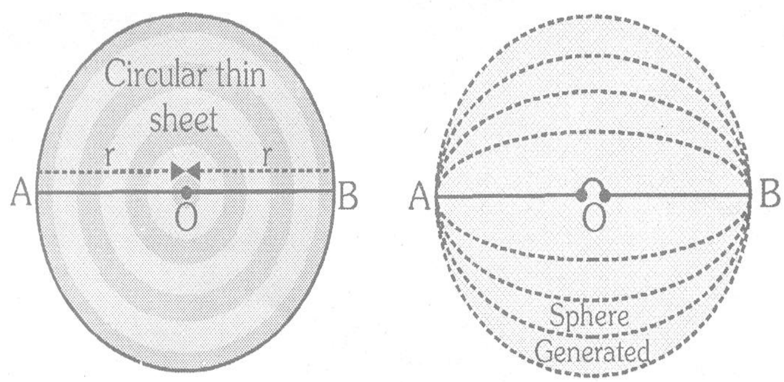
$= (22 \times 7 \times 22)\ cm^3 = 3388\, cm^3.$

(ii) Total surface area of the remaining solid
= Curved surface area of cylinder + Curved surface area of cone
+ Area of (upper) circular base of cylinder

$$= 2\pi rH + \pi r\ell + \pi r^2 = \pi r(2H + \ell + r) = \left[\frac{22}{7} \times 7 \times (60 + 25 + 7)\right] cm^2 = (22 \times 92)\, cm^2 = 2024 cm^2.$$

★ SPHERE

Objects like football, volleyball, throw-ball etc. are said to have the shape of a sphere.
In mathematical terms, **a sphere is a solid generated by revolving a circle about any of its diameters.**
Let a thin circular disc of card of card board with centre O and radius r revolve about its diameter AOB to describe a sphere as shown in figure.



Here, O is called the **centre of the sphere** and r is **radius of the sphere.** Also, the line segment AB is a **diameter of the sphere.**

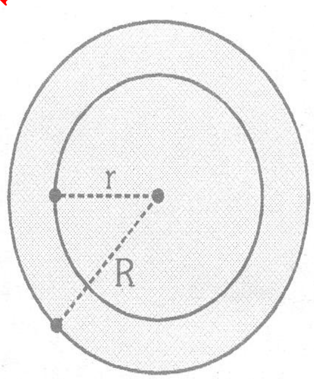
Formulae
For a solid sphere of radius = r, we have :

Surface area of the sphere = $(4\pi r^2)$ sq. units.

Volume of the sphere = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.

SPHERICAL SHELL

The solid enclosed between two concentric spheres to called a spherical shell.

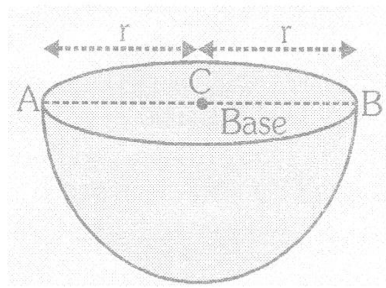


Formula
For a spherical shell with external radius = R and internal radius = r, we have :

Thickness of shell = (R – r) units.
Outer surface area = $4\pi R^2$ sq. units.
Inner surface area = $4\pi r^2$ sq. units.
Volume of material = $\frac{4}{3}\pi (R^3 - r^3)$ sq. units.

HEMISPHERE

When a plane through the centre of a sphere cuts it into two equal parts, then each part is called a hemisphere.
Form a solid sphere, the obtained hemisphere is also a solid and it has a base as shown in fig.



Formula

For a hemisphere of radius r, we have :

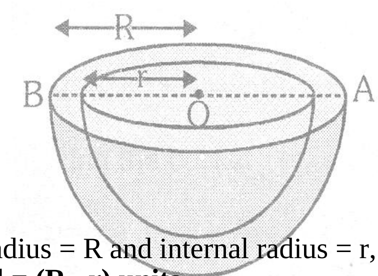
Curved surface area = 2 π r² sq. units.

Total Surface area = (2 π r² + π r²) = 3 π r² sq. units.

Volume $= \frac{2}{3}\pi r^3$ **cubic units.**

HEMISPHERICAL SHELL

The solid enclosed between two concentric hemispheres is called a hemispherical shell.



Formulae

For a hemispherical shell of external radius = R and internal radius = r, we have :

Thickness of the shell = (R –r) units.

Outer curved surface area = $(2\pi R^2)$ sq. units.

Inner curved surface area = $(2\pi r^2)$ sq. units.

Total surface area = $2\pi R^2 + 2\pi r^2 + \pi(R^3 - r^3) = \pi(3R^2 + r^2)$ sq. units.

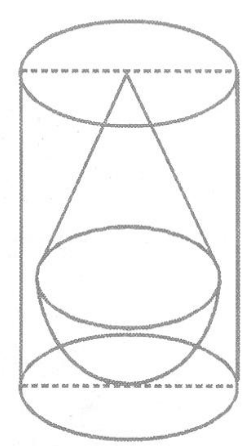
Ex.16 A solid consisting of a right circular cone, standing on a hemisphere, is placed upright, in a right circular cylinder, full of water, and touches the bottom. Find the volume of water left in the cylinder., having given that the radius of the cylinder is 3 cm and its height is 6 cm: the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to the nearest cm³ (Take π = 22/7)

Sol. Radius of the cylinder = 3 cm and its height = 6 cm.

Volume of water in the cylinder, when full $= \left[\pi \times (3)^2 \times 6\right] cm^3 = (54\pi)\ cm^3$.

Volume of solid consisting of cone hemisphere = (Volume of hemi-sphere) + (Volume of cone)

$$= \left[\frac{2}{3}\pi \times (2)^3 + \frac{1}{3}\pi \times (2)^2 \times 4\right] cm^3 = \left(\frac{32\pi}{3}\right) cm^3.$$



Volume of water displaced from cylinder

= Volume of solid consisting of cone and hemisphere

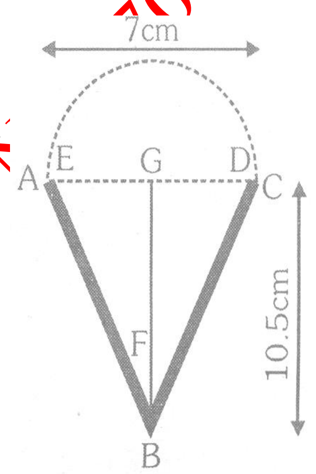
$$=\left(\frac{32\pi}{3}\right)cm^3$$

Volume of water left in the cylinder after placing the solid into it

$$\left(54\pi-\frac{32}{3}\right)cm^3=\left(\frac{130\pi}{3}\right)cm^3=\left(\frac{130}{3}\times\frac{22}{7}\right)cm^3=136.19cm^3$$

Hence, the volume of water left in the cylinder to the nearest cm^3 is 136 cm^3.

Ex.17 **The given figure shows the cross-section of an ice-cream cone consisting of a cone surmounted by a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 10.5 cm. The outer shell ABCDEF is shaded and is not filled with ice-cream. AE = DC = 0.5 cm, AB ‖ EF and BC ‖ FD. Caclculate:**
(i) the volume of the ice-cream in the cone (the unshaded poration including the hemisphere) in cm^3;
(ii) the volume of the outer shell (the shaded portion) in cm^3. Give your answer to the nearest cm^3.



Sol. Radius of hemisphere, R= AG = 3.5 cm.

External radius of conical shell, R = AG = 3.5 cm.
Internal radius of conical shell, r = EG = (AG – AE) = (3.5 – 0.5) cm = 3 cm
Now, ΔΔ BG ~ Δ EFG.

$$\therefore \quad \frac{FG}{BG}=\frac{EG}{AG}\Rightarrow\frac{FG}{1.05}=\frac{3}{3.5}\Rightarrow FG=9\,\text{cm.}$$

So, internal height of conical shell, h = FG = 9 cm.

(i) Volume of ice-cream

= Volume of hemisphere + Internal volume of conical shell

$$=\frac{2}{3}\pi R^3+\frac{1}{3}\pi r^2h=\frac{1}{3}\pi(2R^3+r^2h)$$

$$=\left[\frac{1}{3}\times\frac{22}{7}\times\{2\times(3.5)^3+(3)^2\times 9\}\right]cm^3=\left[\frac{1}{3}\times\frac{22}{7}\times\left(\frac{343}{4}+81\right)\right]\text{cm}^3$$

$$=\left(\frac{1}{3}\times\frac{22}{7}\times\frac{667}{4}\right)cm^3=\left(\frac{7337}{42}\right)cm^3=174.69cm^3=cm^3.\ \text{(to the nearest cm}^3\text{)}$$

(i) Volume of the shell = External volume – Internal volume

$$=\frac{1}{3}\pi R^2H-\frac{1}{3}\pi r^2h=\frac{1}{3}\pi(R^2H-r^2h)$$

$$=\frac{1}{3}\pi\left[(3.5)^2\times 10.5-(3)^2\times 9\right]cm^3=\frac{1}{3}\pi\left[\left(\frac{7}{2}\right)^2\times\left(\frac{21}{2}\right)-(9\times 9)\right]\text{cm}^3$$

$$=\left[\frac{1}{3}\times\frac{22}{7}\times\left(\frac{1029}{8}-81\right)\right]cm^3=\left(\frac{1}{3}\times\frac{22}{7}\times\frac{381}{8}\right)cm^3\left(\frac{1397}{28}\right)cm^3=49.89cm^3$$

= 50 cm^3 (to the nearest cm^3)

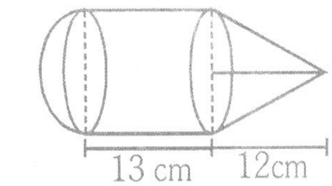
Ex.18 **A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical part are the same as that of the cylindrical part. Calculate the surface area of the height of the conical part is 12 cm.**

Sol. The toy is in the shape shown below :

Radius of the hemispherical part = 5 cm,

∴ Curved surface area of the Hemispherical part

$2\pi r^2 = [2\pi \times (5)^2\ cm^2 = (50\pi)\ cm^2$.



Cylindrical part has radius = 5 cm and height = 13 cm.

∴ Curved surface area of the cylindrical part =

$\pi rh = (2\pi \times 5 \times 13) cm^2 = (130\pi) cm^2$.

Conical part has radius = 5 cm and height = 12 cm.

∴ Its slant height $\sqrt{5^2 + (12)^2} = \sqrt{25+144} = \sqrt{169} = 13cm.$

∴ Curved surface area of the conical part = $\pi r \ell$

$= (\pi \times 5 \times 13) cm^2 = (65\pi) cm^2$

Hence, the surface area of the toy = (50π + 130 π + 65 π) cm^2 = (245 π) cm^2.

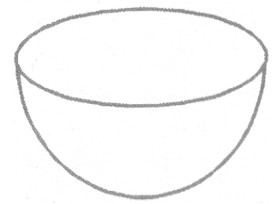
$= \left(245 \times \frac{22}{7}\right) cm^2 = 770 cm^2$.

Also, volume of the toy $= \left(\frac{2}{3}\pi r^3 + \pi r^2 h + \frac{1}{3}\pi r^2 H\right) cm^3 = \left(\frac{250\pi}{3} + 325\pi + 100\pi\right) cm^3 = \left(\frac{1525}{3}\right) cm^3$

Ex.19 The outer and inner diameters of a hemispherical bowl are 17 cm and 15 cm respectively. Find cost of polishing it all over at 25 paise per cm^2. (Take π = 22/7).

Sol. Outer radius $= \frac{17}{2}$ cm, Inner radius $= \frac{15}{2}$ cm.

Area of outer surface = 2 πR^2 $= \left[2\pi \times \left(\frac{17}{2}\right)^2\right] cm^2 = \left(\frac{289\pi}{2}\right) cm^2$.



Area of inner surface = 2 π r^2 $= \left[2\pi \times \left(\frac{15}{2}\right)^2\right] cm^2 = \left(\frac{225\pi}{2}\right) cm^2$.

Area of the ring at the top = π($R^2 - r^2$) = π [$(8.5)^2$ - $(7.5)^2$] cm^2 = (16 π) cm^2.

∴ Total area to be polished $= \left(\frac{289\pi}{2} + \frac{225\pi}{2} + 16\pi\right) cm^2$.

$= (273\pi) cm^2 = \left(273 \times \frac{22}{7}\right) cm^2 = 858 cm^2$.

∴ Cost of polishing the bowl = Rs $\left(\frac{858 \times 25}{100}\right)$ = Rs. 214. 50.

Ex.20 A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. What fraction of water overflows?

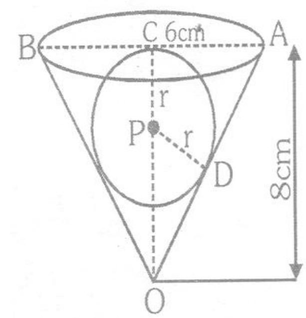
Sol. Radius of the conical vessel, R = AC = 6 cm.

Height of the conical vessel, H = OC = 8 cm.

Let the radius of the sphere be r.

Then, PC = PD = 6 cm.

[∵ lengths of two tangents from an external point to a circle are equal]



OA = $\sqrt{OC^2 + AC^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm.

OD = (OA – AD) = (10 – 6)

OP = (OC – PC) =(8 – R).
In right angled Δ ODP, we have :

$$OP^2 = OD^2 + PD^2$$

$$\Rightarrow \quad (8 - R)^2 = 4^2 + r^2 \Rightarrow 64 - 16r + r^2 = 16 + r^2$$

$$\Rightarrow \quad 16r = 48 \Rightarrow r = \frac{48}{16} = 3.$$

Volume of water overflown = volume of sphere $= \frac{4}{3}\pi r^3 = \left[\frac{4}{3}\pi \times (3)^3\right] cm^3 = (36\pi)\, cm^3$.

Volume of water in the cone before immersing the sphere

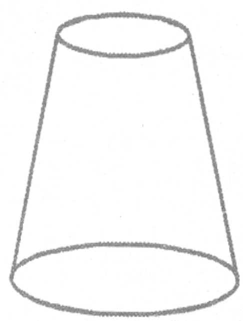
$= \text{Volume of cone} = \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times (6)^2 \times 8\right) cm^3 = (96\pi)\, cm^3$.

$\therefore$ Fraction of water overflown $= \frac{\text{Volume of water overflown}}{\text{Original volume of water}} = \frac{(36\pi)}{96\pi} = \frac{3}{8}$

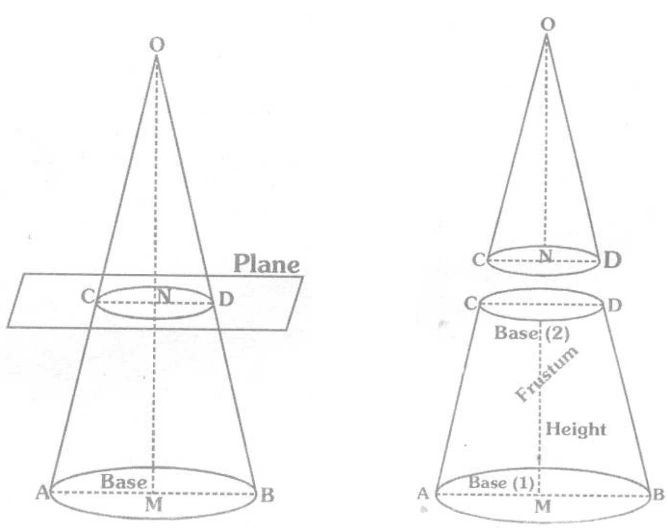
★ **FRUSTUM**

FRUSTUM OF A RIGHT CIRCULAR CONE

In our day-to-day life we come across a number of solids of the shape as shown in the figure. For example, a bucket or a glass tumbler. We observe that this type of solid is a part of a right circular cone and is obtained when the cone is cut by, a plane parallel to the base of the cone.



If right circular cone is cut off by a plane parallel to its base, the portion of the cone between the plane and the base of the cone is called a frustum of the cone.
We can see this process from the figures given below:
The lower portion in figure is the frustum of the cone. It has two parallel flat circular bases, mark as Base (1) and Base (2). A curved surface joins the two bases.



(i) (ii)

The line segment MN joining the centres of the two bases is called the height of the frustum. Diameter CD of Base (2) is parallel to diameter AB of base (1). Each of the line segments AC and BD is called the slant height of the frustum. We observe from the figures (i) and (ii) that,

1. **Height of the frustum = (the height of the cone OAB) – (the height of the cone OCD)**
2. **Slant height of the frustum = (the height of the cone OAB) – (the height of the cone)**

⏩ Volume of a Frustum of a Right Circular Cone

Let h be the height ; r_1 and r_2 be the radii of the two bases ($r_1 > r_2$) of frustrum of a right circular cone. The frustum is made from the complete cone OAB by cutting off the conical part OCD. Let h_1 be the height of the cone OAB and h_2 be the height of the cone OCD.

Here, $h_2 = h_1 - h$.

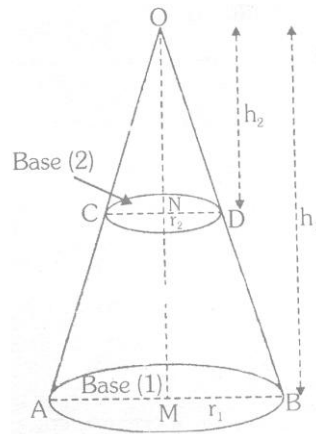
Since right angled triangles OND and OMB are similar, therefore, we have:

$$\frac{h_2}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{h_1 - h}{h_1} = \frac{r_2}{r_1} \qquad \Rightarrow \quad 1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1} \qquad \Rightarrow \quad h_1 = \frac{hr_1}{r_1 - r_2}$$

$$\text{and} \quad h_2 = h_1 - h = \frac{hr_1}{r_1 - r_2} - h \Rightarrow \qquad h_2 = \frac{hr_2}{r_1 - r_2}$$



Volume V of the frustum of cone = Volume of the cone OAB – volume of the cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2 \qquad = \frac{1}{3}\pi r_1^2 \times \frac{hr_1}{(r_1 - r_2)} - \frac{1}{3}\pi r_2^2 \times \frac{hr_2}{(r_1 - r_2)}$$

$$= \frac{1}{3}\pi h \left\{ \frac{r_1^3 - r_1^3}{r_1 - r_2} \right\} = \frac{1}{3}\pi h \{ r_1^2 + r_1 r_2 + r_2^2 \}$$

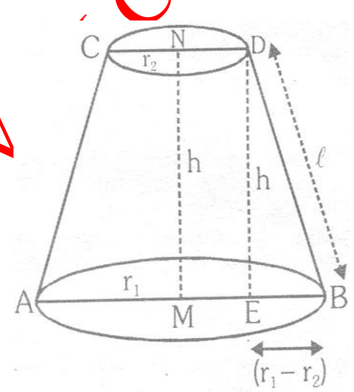
$$\therefore \qquad V = \frac{1}{3}\pi h \{ r_1^2 + r_1 r_2 + r_2^2 \}$$

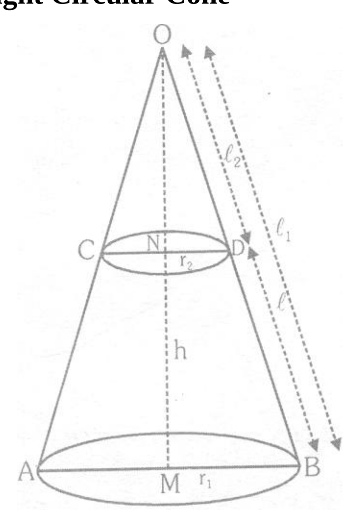
Note : Volume $V = \frac{1}{3}\pi h \{ r_1^2 + r_1 r_2 + r_2^2 \}$

$$= \frac{h}{3}(\pi r_1^2 + \pi r_2^2 + \pi r_1 r_2) = \frac{h}{3}\{\pi r_1^2 + \pi r_2^2 + \sqrt{(\pi r_1^2)(\pi r_2^2)}\}$$

$$= \frac{h}{3}\{(area of\ base) + (area of\ base 2) + \sqrt{(area of\ base)(area of\ base 2)}\}$$

⏩ Curbed Surface Area of a Frustum of a Right Circular Cone





Let h be the height, ℓ be the slant height and r_1, r_2 be the radii of the bases where $r_1 > r_2$.

In figure (i), we observe EB = $r_1 - r_2$

Aad $\qquad \ell^2 = h^2 + (r_1 - r_2)^2$

$\therefore \qquad \ell = \sqrt{h^2 + (r_1 - r_2)^2}$

In figure (ii, we have OAB as the complete cone from which cone OCD is cut off to make the frustum ABDC.

Let ℓ be the slant height of the cone OAB and ℓ_2 be the slant height of the cone OCD.

Since, Δ OMB are similar,

$$\frac{\ell_2}{\ell_1} = \frac{r_2}{r_1} \quad \Rightarrow \quad \frac{\ell_1 - \ell}{\ell_1} = \frac{r_2}{r_1} \quad \Rightarrow \quad \ell_1 = \frac{\ell r_1}{r_1 - r_2}$$

Now, $\ell_2 = \ell_1 - \ell = \dfrac{\ell r_1}{r_1 - r_2} - \ell \quad \Rightarrow \quad \ell_2 = \dfrac{\ell r_2}{r_1 - r_2}$

Curved surface area of frustum ABCD

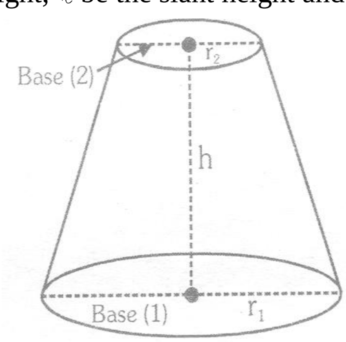
= (Curved surface area of cone OAB) – (Curved surface area of cone OCD)

$$= \pi r_1 \ell_1 - \pi r_2 \ell_2 = \pi r_1 \times \frac{\ell r_1}{(r_1 - r_2)} - \pi r_2 \times \frac{\ell r_2}{(r - r_2)} = \pi \ell \left\{ \frac{r_1^2 - r_2^2}{r_1 - r_2} \right\}$$

Therefore, curved surface area of frustum = $\pi \ell (r_1 + r_2)$.

Total surface Area of a Frustum of a solid Right Circular Cone

Let h be the height, ℓ be the slant height and r_1, r_2 the radii of the bases where $r_1 > r_2$ as shown in figure.



Total surface area of this frustum

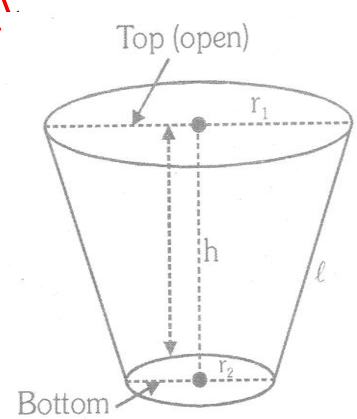
= Curved surface area + Area of Base 1 + Area of Base 2

= $\pi \ell (r_1 + r_2) + \pi r_2^2$

Area of the Metal Sheet Used To Make a Bucket

A bucket is in the shape of a frustum of a right circular hollow cone.

Let h be the depth, ℓ be the slant height, r_1 be the radius of the top and r_2 be the radius of the bottom as shown in figure



The area of the metal sheet used for making the bucket

= $\qquad$ Outer (or inner) curved surface area + Area of bottom

= $\qquad \pi \ell (r_1 + r_2) + \pi r_2^2$

Ex.21 A bucket of height 16 cm and made up of metal sheet is in the form of frustum of a right circular cone with radii of its lower and upper ends as 3 cm and 15 cm respectively. Calculate:

(i) the height of the cone of which the bucket is a part.

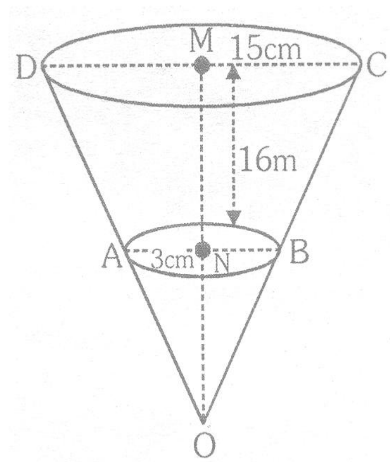
(ii) the volume of water which can be filled in the bucket.

(iii) the slant height of the bucket.

(iv) the area of the metal sheet required to make the bucket.

Sol. Let ABCD be the bucket which is frustum of a cone with vertex 0 (as shown in figure). Let ON = x cm

$\Delta OAB - \Delta OMC$



$\therefore \quad \frac{x}{16+x} = \frac{3}{15} \qquad \left\{\because \frac{ON}{OM} = \frac{NB}{MC}\right\}$

$\Rightarrow \quad \frac{x}{16+x} = \frac{1}{5} \qquad \Rightarrow \quad 5x = 16 + x1$

$\Rightarrow \quad 4x = 16 \qquad \Rightarrow \quad$ x = 4

$\therefore$ ON = 4 cm and OM = 4 + 16 = 20 cm

$\therefore$ the height of the cone = 20 cm

volume of the bucket $= \frac{1}{3}\pi(15)^2 \times 20 - \frac{1}{3}\pi(3)^2 \times 4cm^3$

{i.e., Volume of the large cone – Volume of the small cone}

$= \frac{1}{3}\pi[225 \times 20 - 36]cm^3$

$= \pi[75 \times 20 - 12]cm^3$

$= 1488\pi\, cm^3$

Slant height of cone of radius 15 cm

$= \sqrt{(15)^2 + (20)^2}cm = \sqrt{625}\ cm = 25cm$

Slant height of cone of radius 3 cm

$= \sqrt{(4)^2 + (3)^2}cm = 5cm$

$\therefore$ Slant height of bucket $= (25-5)cm = 20cm, i.e, \ell = 20cm$

$\therefore$ The area of the metal sheet $= \pi\ell(R+r) + \pi r^2$

$= \pi \times 20 \times (15+3) + \pi(3)^2\, cm^2 = 360\pi + 9\pi\, cm^2$

$= 369\pi\, cm^2$

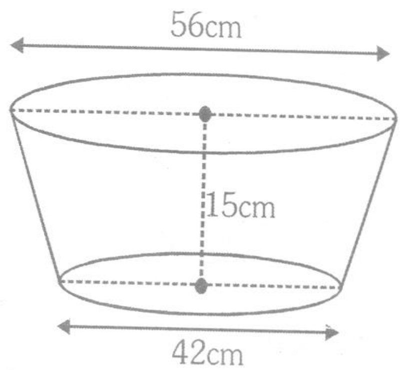
Note. The area of the metal sheet used = C.S. of larger cone – C.S. of smaller cone + Area of the base of the bucket

$= [\pi \times 25 \times 15 - \pi \times 5 \times 3 + \pi \times (3)^2]cm^2 = [375\pi - 15\pi + 9\pi]cm^2$

$= 369\pi\ cm^2$

Ex.22 A bucket is in the form of a frustum of a cone, depth is 15 cm and the diameters of the top and the bottom are 56 cm and 42 cm respectively. Find how many liters of water can the bucket hold ? (Take $\pi = 22/7$)

Sol. R = 28 cm
r = 21 cm



h = 15 cm

Capacity of the bucket $= \frac{1}{3}\pi h\{R^2 + r^2 + R\,r\}$

$= \frac{1}{3} \times 22 \times 15 \times \{(28)^2 + (21)^2 + (28)(21)\} cm^3$

$= \frac{22}{7} \times 5 \times \{784 + 441 + 588\} cm^3$

$= \frac{22}{7} \times 5 \times 1813 cm^3 \quad = 22 \times 5 \times 259 cm^3$

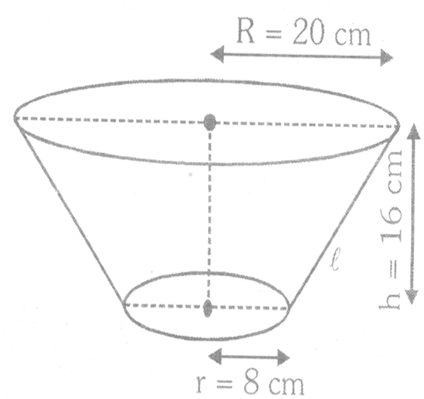
$= 28490 \text{ cm}^3 \quad = \frac{28490}{1000}$ liters

= 28.49 liters

Ex.23 A container made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container at the rate of Rs. 15 per liter and the cost of the metal sheet used, if it costs Rs. 5 per 100 cm^2. (Take π 3.14)

Sol. R = 20 cm, r = 8 cm, h = 16 cm

$\ell = \sqrt{h^2 + (R - r)^2} = \sqrt{256 + 144} cm = 20 cm$



Volume of contamer $= \frac{1}{3}\pi h\{R^2 + r^2 + Rr\}$

$= \frac{1}{3} \times (3.14) \times 16\{400 + 64 + 160\} cm^3$

$= 3.14 \times \frac{16}{3}\{624\} cm^3$

$= 3.14 \times 16 \times 208 cm^3$

$= 10449.92 cm^3$

Therefore, the quantity of milk in the container $= \frac{10449.92}{1000}$ liters = 10.45 liters

Colt of milk at the rate of Rs. 15 per liter = Rs. $\{10.45 \times 15\}$ = Rs. 156.75

Surface area of the metal sheet used to make the container

$= \pi\ell(R + r) + \pi r^2 = \pi\{\ell(R + r) + r^2\}$

$= (3.14) \times \{20 \times 28 + 64\} cm^2$

$= (3.14) \times 624 cm^2 = 1959.36 cm^2$

Therefore, the cost of the metal sheet at rate of Rs. 5 per 100 cm^2

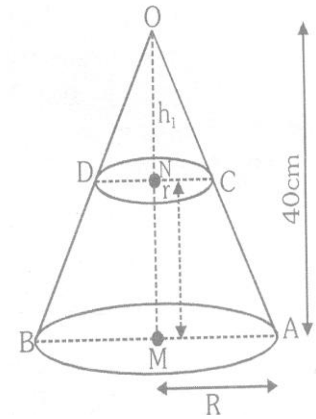
$= Rs. \frac{1959.36 \times 5}{100} = Rs.\, 97.97$ approx.

Ex.24 The height of a cone is 40 cm. A small cone is cut off at the top by a plane parallel to the bases. It the volume of the small cone be $\frac{1}{64}$ of the volume of the given cone, at what height above the base is the section made

Sol. Let R be the radius of the given cone, r the radius of the small cone, h be the height of the frustum and h_1 be the height of the small cone.

In figure 13.49, ΔONC and ΔOMA are similar (ΔONC - ΔOMA)

$\therefore \quad \frac{ON}{OM} = \frac{NC}{MA} \quad \Rightarrow \quad \frac{h_1}{40} = \frac{r}{R}$



$\Rightarrow \quad h_1 = \left(\frac{r}{R}\right)40 \quad \ldots(i)$

We are given that $\frac{Volume\ of\ small\ cone}{Volume\ of\ given\ cone} = \frac{1}{64}$

$$\Rightarrow \quad \frac{\frac{1}{3}\pi r^2 \times h_1}{\frac{1}{3}\pi R^2 \times 40} = \frac{1}{64}$$

$$\Rightarrow \quad \frac{r_2}{R_2} \times \frac{1}{40} \times \left\{\left(\frac{r}{R}\right)40\right\} = \frac{1}{64} \quad \text{(By 1)}$$

$$\Rightarrow \quad \left(\frac{r}{R}\right)^3 = \frac{1}{64} = \left(\frac{1}{4}\right)^3 \Rightarrow \frac{r}{R} = \frac{1}{4} \quad \ldots(2)$$

From (i) and (ii) $h_1 = \frac{1}{4} \times 40 = 10cm$

Therefore, $h = 40 - h_1 = (40 - 10)$ cm

$\Rightarrow$ h = 30 cm

Ex.25 The radius of the base of a right circular cone is r. It is cut by a plane parallel to the base a height h from the base. The slant height of the frustum is $\sqrt{h^2 + \frac{4}{9}r^2}$. Show that volume of the frustum is $\frac{13}{27}\pi r^2 h$.

Sol. In figure 13.50, $\ell = \sqrt{h^2 + \frac{4}{9}r^2}$ is the slant height of frustum of the given cone having base radius r. O is the centre of the base and O' is the centre of the top of the frustum.

OO' = h (given)

AOB and COD are diameters of the lower and upper faces of the frustum. Draw DE $\perp$ OB. Let O' P = x

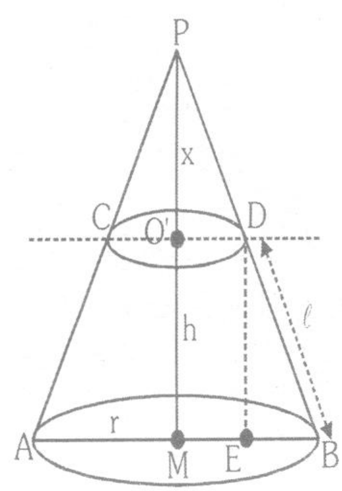
In right angled Δ DEB,

$DB^2 = BE^2 + DE^2$

$\Rightarrow \quad \ell^2 = BE + h^2 \quad (\because DE = OO' = h)$

$\Rightarrow \quad h^2 + \frac{4}{9}r^2 = h^2 + BE^2 \quad \Rightarrow \quad BE^2 = \frac{4}{9}r^2$

$\Rightarrow \quad BE = \frac{2}{3}r \quad \Rightarrow \quad OE = r - \frac{2}{3}r = \frac{1}{3}r$



$\Rightarrow \quad O'D = \frac{1}{3}r$ is the radius of the top face of the frustum.

Now, $\Delta PO'D \sim \Delta POB$

$\Rightarrow \quad \frac{PO'}{O'D} = \frac{PO}{PB} \qquad \Rightarrow \quad \frac{x}{\frac{1}{3}r} = \frac{h+x}{r}$

$\Rightarrow \quad 3x = h + x \qquad \Rightarrow \quad x = \frac{1}{2}h.$

Volume of the frustum = Volume of the cone PAB – Volume of the cone PCD

$$= \frac{1}{3}\pi \times (r^2) \times OP - \frac{1}{3}\pi \times \left(\frac{1}{3}r\right)^2 \times O'P$$

$$= \frac{1}{3}\pi r^2 \times (h+x) - \frac{1}{27}\pi r^2 \times x$$

$$= \frac{1}{3}\pi r^2 \times \left(h + \frac{1}{2}h\right) - \frac{1}{27}\pi r^2 \times \frac{1}{2}\ h$$

$$= \left(\frac{1}{2}\pi r^2 - \frac{1}{54}\pi r^2\right)h = \frac{26}{54}\pi r^2 h = \frac{13}{27}\pi r^2 h$$

Hence, the required volume is $\frac{13}{27}\pi r^2 h$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

1. If the volume of a cube is 1728 cm^3, the length of its edge is equal to
(a) 12 cm (b) 14 cm (c) 16 cm (d) 24 cm

2. Two cubes each of 10 cm edge are joined end to. The surface area of the resulting cuboid is
(a) 1200 cm^2 (b) 1000 cm^2 (c) 800 cm^2 (d) 1400 cm^2

3. A rectangular sheet of paper 44 cm $\times$ 18 cm is rolled along its length and a cylinder is formed. The volume the cylinder so formed is equal to $(Take\,\pi = \frac{22}{7})$
(a) 2772 cm^3 (b) 2505 cm^3 (c) 2460 cm^3 (d) 2672 cm^3

4. If the radius and height of a cylinder are in ratio 5 : 7 and its volume is 550 cm^3, then its radius is equal to $(Take\pi = \frac{22}{7})$

(a) 6 cm (b) 7 cm (c) 5 cm (d) 10 cm

5. If the curved surface area of a solid right circular cylinder of height h and radius r is one-third of its total surface area, then

(a) $h = \frac{1}{3}r$ (b) $h = \frac{1}{3}r$ (c) $h = r$ (d) $h = 2r$

6. A hollow cylindrical pipe is 21 cm long. If its outer and inner diameters are 10 cm and 6 cm respectively, then the volume of the metal used in making the pipe is $(Take\pi = \frac{22}{7})$

(a) 1048 cm^3 (b) 1056 cm^3 (c) 1060 cm^3 (d) 1064 cm^3

7. If the radius and slant height of a cone are in the ratio 4 : 7 and is curved surface area is 792 cm^2, then its radius is $(Take\pi = \frac{22}{7})$

(a) 10 cm (b) 8 cm (c) 12 cm (d) 9 cm

8. If the radius of the base and the height of a right circular cone are respectively 21 cm and 28 cm, then the curved surface area of the cone is $(Take\pi = \frac{22}{7})$

(a) 3696 cm^2 (b) 2310 cm^2 (c) 2550 cm^2 (d) 2410 cm^2

9. A conical tent with base-radius 7 m and height 24 m is made from 5 m wide canvas. The length of the canvas used is $(Take\pi = \frac{22}{7})$

(a) 100 m (b) 105 m (c) 110 m (d) 115 m

10. The total surface area of a solid hemisphere of radius 3.5 m is covered with canvas at the rate of Rs. 20 per m^2. The total cost to cover the hemisphere is $(Take\pi = \frac{22}{7})$

(a) Rs. 2210 (b) Rs. 2310 (c) Rs. 2320 (d) Rs. 2420

11. If the volume of a vessel in the form of a right circular cylinder is 448π cm^3 and its height is 7 cm, then the curved surface area of the cylinder is

(a) 224π cm^2 (b) 212π cm^2 (c) 112π cm^2 (d) none of these

12. If the curved surface area of a right circular cone is 12320 cm^2 and its base-radius is 56 cm, then its height is $(Take\pi = \frac{22}{7})$

(a) 42 cm (b) 36 cm (c) 48 cm (d) 50 cm

13. If a solid metallic sphere of radius 8 cm is melted and recasted into spherical solid balls of radius 1 cm, then n =

(a) 500 (b) 510 (c) 512 (d) 516

14. If the diameter of a metallic sphere is 6 cm, it melted and a wire of diameter 0.2 cm is drawn, then the length of the wire made shall be

(a) 24 m (b) 28 m (c) 32 m (d) 36 m

15. If n coins each of diameter 1.5 cm and thickness 0.2 cm are melted and a right circular cylinder of height 10 cm and diameter 4.5 cm is made, then n =

(a) 336 (b) 450 (c) 512 (d) 545

16. A tent is in the form of a cylinder of diameter 8 m and height 2 m, surmounted by a cone of equal base and height 3 m. The canvas used for making the tent is equal to

(a) 36π m^2 (b) 28π m^2 (c) 24π m^2 (d) 32π m^2

17. A toy is in the form of a cone mounted on a hemisphere with same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. The surface area of the toy is
(a) 36π cm^2 (b) 33π cm^2 (c) 35π cm^2 (d) 24π cm^2

18. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm. The volume of the frustum is
(a) 3328π cm^3 (b) 3228π cm^3 (c) 3240π cm^3 (d) 3340π cm^3

19. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm has lateral surface area equal to
(a) 540π cm^2 (b) 580π cm^2 (c) 560π cm^2 (d) 680π cm^2

20. A solid metal cone with base-radius 12 cm and height 24 cm, is melted to form solid spherical balls, each of diameters 6 cm. The number of such balls made is
(a) 32 (b) 36 (c) 48 (d) none of these

OBJECTIVE				ANSWER						EXERCISE-4
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	A	B	A	A	B	B	C	B	C	B
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	C	A	C	D	B	A	B	A	C	A

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CONVERSION OF SOUPS

1. Two cubes of the side 10 cm are joined end to end. Find the surface area of the resulting rectangular shaped solid

2. Three cubes each of side 4 cm are joined end to end. Find the surface area of the resulting rectangular cuboid.

3. The six cube marked I, II, III, IV, V, VI each of side 3 cm are placed as shown in fig. It takes the shape of a cuboid. Find the surface area of the cuboid.

I	II	II
I	V	V

4. A rectangular solid metallic cuboid 18 cm × 15 cm × 4.5 cm is melted and recast into solid cubes each of side 3 cm. How many solid cubes can be made ?

5. A rectangular solid metallic cuboid 32 cm × 27 cm × 15 cm is melted and recast into solid cubes each of side 6 cm. How many solid cubes can be made from the metal.

6. Two rectangular solid metallic cuboid 12 cm × 10 cm × 5 cm and 12 cm × 5 cm × 4 cm are melted together and recast into solid cubes each of side 2 cm. How many solid cubes can be made from the metal.

7. Three rectangular solid metallic cuboid 20 cm × 10 cm × 5 cm, 15 cm × 10 cm × 4 cm and 15 cm × 12 cm × 5 cm are melted together and recast into solid cubes each of side 2 cm. How many solid cubes can be made from the metal.

8. The side of a metallic cube 35 cm. The cube is melted and recast into 1000 equal solid dice. Determine the side of the dice.

9. Two solid metallic cube sides 40 cm and 30 cm are melted together into 160 equal solid cubical dice. Determine the side of the dice.

10. Three solid metallic cubes 60 cm, 50 cm and 30 cm are melted together and recast into 875 equal solid cubical dice. Determine the side of the dice.

11. The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform circular cross-section. If the length of the wire is 36 m, find the radius of its cross-section.

12. The diameter of a metallic sphere is 18 cm. The sphere is melted and drawn into a wire having diameter of the cross-section as 0-4 cm. Find the length of the wire.

13. How many balls, each of radius 0.5 cm, can be made from a solid sphere of metal of radius 10 cm by melting the sphere ?

14. A spherical ball of lead 5 cm in diameter is melted and recast into three spherical balls. The diameters of two of these balls are 2 cm and $2\,(1405)^{1/3}$ cm. Find the diameter of the third ball.

15. How many bullets, can be made out of a solid cube of lead whose edge measures 44 cm cm and diameter of each bullet being 4 cm.

16. How many spherical lead shots each 4.2 cm in dimater can be obtained from a rectangular solid (cuboid)of lead with dimensions 66 cm, 42 cm, 21 cm.(Take $\pi = 22/7$)

17. H**ow** many spherical balls each of 5 cm in diameter can be cast from a rectangular block of metal 11× dm ×10 dm ×5 dm ? (1 dm = 10 cm)

18. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 32 m of uniform thickness (diameter). Find the thickness of the wire.

19. 56 circular plates, each of radius 5 cm and thickness 0.25 cm, are placed one above another to form a solid right circular cylinder. Find the curved surface and the volume of the cylinder so formed.

20. The diameter of a metallic sphere is 4.2 cm. It is melted and recast into a right circular cone of height 8.4 cm, Find the radius of the base of the cone.

21. A right circular metallic cone of height 20 cm and radius of base 5 cm is melted and recast into a sphere. Find the radius of the sphere.

22. A right circular cone of height 81 cm and radius of base 16 cm is melted and recast into a right circular cylinder of height 48 cm. Find the radius of the base of the cylinder.

23. A spherical shell of lead, whose external diameter is 24 cm, is melted and recast into a right circular cylinder, whose height is 12 cm and diameter 16 cm. Determine the internal diameter of the shell.

24. The internal and external radii of a metallic spherical shell are 4 cm and 8 cm, respectively. It is melted and recast into a solid right circular cylinder of height $9\frac{1}{3}$ cm. Find the diameter of the base of the cylinder.

25. A right circular cone is of height 3.6 cm and radius of its base 1.6 cm. It is melted and recast into a right circular cone with radius of its base 1.2 cm. Find the height of the cone so formed.

26. A solid metallic right circular cylinder 1.8 m high with diameter of its base 2 m is melted and recast into a right circular cone with base of diameter 3 m. Find the height of the cone.

27. Find the umber of coins, 1.5 cm in diameter and 0.2 cm thickness, to be melted to form a right circular cylinder o f height 10 cm and diameter 4.5 cm.

28. A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylinder vessel with internal radius 10 cm. Find the height to which the water rises.

29. A well, whose diameter is 4 m, has been dug 16 m deep and the earth dug out is used to form an embankment 8 m wide around it. Find the height of the embankment.

30. A well, whose diameter is 3.5 m, has been dug 16 m deep and the earth dug out is used to form a platform 27.5 m by 7 m just near the site of the well. Find the height of the platform. (Take $\pi = 22/7$)

31. The base radius and height of a right circular solid cone are 12 cm and 24 cm respectively. It is melted and recast into spheres of diameter 6 cm each. Find the number of spheres so formed.

32. The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. It is melted and recast into a solid cone of base diameter 14 cm. Find the number of spheres so formed.

33. A solid metallic sphere of diameter 28 cm is melted and recasted into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find the number of cones so formed.

34. A conical flask is full of water. The flask has base-radius 3 cm and height 15 cm. The water is poured into a cylindrical glass tube of uniform inner radius 1.5 cm, placed vertically and closed at the lower end. Find the height of water in the glass tube.

35. Find the depth of a cylindrical tank of radius 10.5 m, if its capacity is equal to that of a rectangular tank of size 15 m $\times$ 11 m $\times$ m $\times$ 10.5 m. (Take $\pi = 22/7$)

36. A rectangular tank 28 m ling and 22 m wide is required to receive entire water from a full cylindrical tank of internal diameter 28 m and depth 4 m. Find the least height of the tank that will serve the purpose(Take $\pi = 22/7$)

37. A conical flask is full of water. The flask has base-radius a and height 2 a. The water is poured into a cylindrical flask of base-radius $\frac{2a}{3}$ Find the height of water in the cylindrical flask.

38. A sphere of diameter 2 a is dropped into a cylindrical vessel partly filled with water. The diameter of the base of the vessel is $\frac{8a}{3}$ If the sphere is completely submerged, by how much will the level of water rise ?

39. The rain water from a roof 44 m $\times$ 20 m drains into a cylindrical vessel having diameter 2 m and height 2.8 m. If the vessel is justfull, find the rainfall in cm.

40. An agricultural field is in the form of a rectangle of length 20 m and width 14 m. A 10 m deep well of diameter 7 m is dug in a corner of the field and the earth taken out of the well is spread evenly over the remaining part of the field. Find the rise in its level. (Take $\pi = 22/7$)

41. 600 persons took dip in a rectangular tank which is 60 m long and 40 m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m^3 ?

42. The largest sphere is curved cut of a cube whose edge is of length ℓ units. Find the volume of the sphere

43. The largest right circular cone is curved out of a cube whose edge is o length p units. Find the volume of the cone.

44. Two solid right circular cones have same height. The radii of their bases are 4 cm and 3 cm. They are melted and recast into a right circular cylinder of same height. Find the radius of the base of the cylinder. $\left(Take \frac{1}{\sqrt{3}} = .577\right)$

45. Water is being pumped out through a circular pipe whose diameter is p cm. If the flow of water is 14 p cm per second, how many litres of water are being pumped cut in one hour? $(Take\ \pi = 22/7)$

46. Water flow out through a circular pipe, whose internal diameter is $1\frac{1}{3}$ cm, at the rate of 0.63 m per second into a cylinder tank, the radius of whose base is 0.2 m. By how much will the level of water rise in one hour.

47. Water in a canal 4 m wide and 1.5 m deep is flowing with velocity 12 km per hour. How much area will it irrigate in 30 minutes, if 9 cm of standing water is required for irrigation?

48. Water flow at the rate of 15 m per minute through a cylindrical pipe having its diameter 1.2 cm. How much time will it tank to fill a conical vessel whose diameter of base is 40 cm and depth 81 cm ?

49. A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to half-empty the tank, if the tank is 3 metres in diameter $(Take\ \pi = 22/7)$

50. A conical tank is full of water. Its base-radius is 1.75 m and height 2.25 m. It is connected with a pipe which empties it at the rate of 7 litres per second. How much time will it take to empty the tank completely ? $(Take\ \pi = 22/7)$

51. Two solid metallic right circular cones have same height h. The radii of their bases are r_1 and r_2. The two cones ae melted together and recast into a right circular cylinder of height h. Show that radius of the base of the cylinder is $\sqrt{\frac{1}{3}(r_1^2 + r_2^2)}$

52. The radii of the bases of two right circular solid metallic cones of same height h are r_1 and r_2. The cones are melted together and recast into a solid sphere of radius R. Show that $h = 4\left(\frac{R^3}{r_1^2 + r_2^2}\right)$

53. The radii of the solid metallic spheres are r_1 and r_2. The sphere are melted together and recast in a solid cone of height ($r_1 + r_2$). Show that the radius of the cone is $2 \times \sqrt{r_1^2 + r_2^2 + r_1 r_2}$

54. The radii of a solid metallic sphere is r. A solid metallic cone of height h has base radius r. The two are melted together and recast into a solid right circular cone with base radius r. Prove that the height of the resulting cone is 4 r + h.

55. A solid metallic right circular cylinder and a solid metallic right circular cone are giver. The cylinder and cone both have same height h and same base radii r. The two solids are melted together and recast into a solid cylinder of radius $\frac{1}{2}$ r. Prove that the height of the cylinder is $\frac{16}{3}$ h.

SURFACE AREAS & VOLUMES OF COMBINATIONS OF SOLIDS

1. A solid is in the form of a cone mounted on a right circular cylinder both having same radii of their bases. Base of the cone is placed on the top base of the cylinder. If the radius of the base and height of the cone be 4 cm and 7 cm respectively and the height of the cylindrical part of the solid is 3.5 cm, find the volume of the solid.

2. A solid is in the form of a right circular mounted on a solid hemisphere of radius 14 cm. The radius of the base of the cylindrical part is 14 cm and the vertical height of the complete solid is 28 cm. Find :

(i) The volume of the solid

(ii) The surface area of the solid

(iii) Cost of painting the solid at the rate of Rs. 0.80 cm^2.

3. A solid is in the form of a cone of vertical height 9 cm mounted on the top base of a right circular cylinder of height 40 cm. The radius of the base of the cone and that of the cylinder are both equal to 7 cm. Find the weight of the solid if 1 cm^3 of the solid weight 4 gm.

4. A solid is in the form of a right circular cone mounted on a solid hemisphere with same radius is made from a piece of metal. The radius of the hemisphere is $\frac{1}{3}$ of the vertical height of the conical part. If the radius of the base of the cone is r, prove that the volume of the piece of metal is $\frac{5}{3}\pi r^3$

5. A solid wooden toy is in the shape of a right circular cone mounted on a solid hemisphere with same radius. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy. $(Take\ \pi = 22/7)$

6. A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and diameter of the base is 4 cm. If a right circular cylinder circumscribes the solid, find how much more space it will cover. $(Take\ \pi = 3.14)$

7. A cylindrical tub of radius 5 cm and height 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed completely into the tub. If the radius of the hemisphere is 3.5 cm and the height of the conical part is 5 cm, find the volume of water left in the tub. $(Take\ \pi = 22/7)$

8. A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The ice-cream is to be distributed to 10 children in equal cones with hemisphere tops. If the height the height of the conical portion is 4 times the radius of its base, find the radius of the ice-cream cone.

9. A right circular cylinder having diameter 18 cm and height 20 cm is full of ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

10. A circus tent has cylindrical shape surmounted by a conical reef. The radius of the cylindrical base is 40 m. The heights of the cylindrical and conical portions are 6.3 m and 4.2 m, respectively. Find the volume of the tent. $(Take\ \pi = 22/7)$

11. A circus tent is cylindrical up to a height of 3 m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53 m, find the total cast of the canvas used to make the tent when the cost per square metre of the canvas is Rs. 10. $(Take\ \pi = 22/7)$

12. A tent of height 11 m is in the form of a right circular cylinder with diameter of base 30 m and height 3 m, surmounted by a right circular cone of the same base. Find the cost of the canvas of the tent at the rate of Rs. 25 per m^2. $(Take\ \pi = 22/7)$

13. A tent is in the form of a cylinder of diameter 15 m and height 2.4 m, surmounted by a cone of equal base and height 4 m. Find the capacity of the tent and the cost of the canvas at Rs. 50 per square metre. $(Take\ \pi = 22/7)$

14. A iron pillar has some part in the form of a right circular and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if cone cubic cm of iron weight 7.8 gracs.

15. The interior of a building is in the form of a right circular of diameter 4.2 m and height 4 m, surmounted by a cone. The vertical height of the cone is 2.1 m. Find the outer surface area and volume of the building. $(Take\ \pi = 22/7)$

16. The in interior of a building is in the form of a right circular of diameter 4.3 m and height 3.8 m, surmounted by a cone whose vertical angel is right angle. Find the area of the surface and the volume of the building. $(Take\ \pi = 22/7)$

17. A vessel is in the form of a hemispherical bowl, surmounted by a hollow cylinder. The diameter of the hemisphere is 12 cm and the total height of the vessel is 16 cm. Find the capacity of the vessel. $(Take\ \pi = 22/7)$ Also find the internal surface area of the vessel by taking $\pi = 3.14$.

18. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid. $(Take\ \pi = 22/7)$

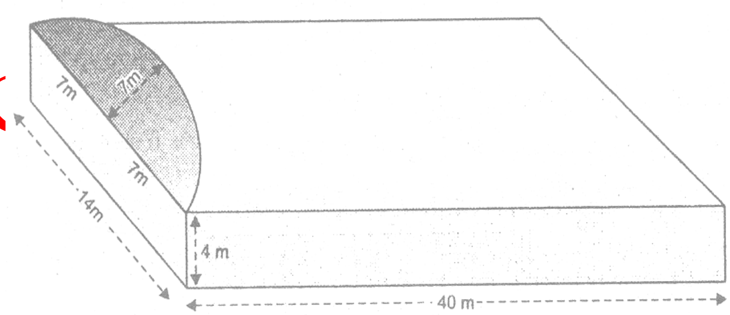
19. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm and the diameter of the hemisphere ends is 36 cm, find the cost of polishing the surface of the solid at the rate of 10 paise per cm^2. $(Take\ \pi = 22/7)$

20. A solid toy is in the form of a right circular cylinder with a hemispherical shape at cone end and a cone at the other end. Their common diameter is 4.2 cm and the heights of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the solid. $(Take\ \pi = 22/7)$

21. A petrol tank is a cylinder of base diameter 28 cm and length 24 cm fitted with conical ends each of axis-length 9 cm. Determine the capacity of the tank.

22. Form a solid right circular cylinder with height h and radius of the base r, a right circular cone of the same height And same base is removed. Find the volume of the remaining solid.

23. A right circular cone with sides 12 cm and 16 cm is revolved around its hypotenuse. Find the volume of the double cone so formed.



24. A godown building as shown in figure is made in the form of a cuboidal base with dimensions 40 m × 14 m ×4 m, surmounted by a half cylindrical curved roof having same length as that of the base. The diameter of the cylinder is 14 m. Find the volume of the building and its total outer surface area.

25. Find the mass of a 3.5 m long lead pipe, if the external diameter of the pipe is 2.4 cm, thickness of the metal is 2 mm and mass of 1 cm^3 of lead is 11.4 g. $(Take\ \pi = 22/7)$

26. A cylindrical vessel of diameter 16 cm and height h cm is fixed symmetrically inside a similar vessel of diameter 20 cm and height h cm. The total space between the two vessels is filled with cork dust. How many cubic centimeters of cork dust is used.

27. The interior of a building is in the form of cylinder of radius 4 m and height 3.5 m, surmounted by a cone of vertical angle 90^0. Find the surface area of the interior of the building (excluding the flooring area of the building). Also find the cost of painting the interior of the building at the rate of Rs. 5 per m^2. Use $\pi = 22/7$ $and \sqrt{2} = 1.414$

28. A solid is in the form of a cone of vertical height h mounted on a right circular cylinder of height 2 h and both having same radii of their bases. Base of the cone is placed on the top base of the cylinder. If V cube units be the volume of the solid, prove that the radius of the cylinder is $\sqrt{\frac{3V}{7\pi h}}$.

29. A solid is in the form of a cone of vertical height h mounted on the top base of a right circular cylinder of height $\frac{1}{3}h$. The circumference of the base of the cone and that of the cylinder are both equal to C. If V be the volume of the solid, prove that $C = 4\sqrt{\frac{3\pi V}{7h}}$.

30. A conical vessel of radius 12 cm and depth 16 cm is completely filled with water. A sphere is lowered into the water and its size is such that than when it touches inner curved surface of the vessel, it is just immersed upto the topmost point of the sphere. How much water over flows out of the vessel out of the total volume V cubic nuits.

FRUSTUM OF A RIGHT CIRCULAR CONE

1. A bucket of height 3 cm and made up of metal sheet is in the form of frustum of a right circular cone with radii of its lower and upper ends as 6 cm and 10 cm respectively. Calculate:

(i) the height of the cone of which the bucket is a part.
(ii) the volume of water which can be filled in the bucket.
(iii) the slant height of the bucket.
(iv) the area of the metal sheet required to make the bucket.

2. The radii of the circular ends of a frustum of a right circular cone are 5 cm and 8 cm and its lateral height (slant height) is 5 cm. Find the volume of the frustum. $(Take\ \pi = 22/7)$

3. The radii of the circular ends of a bucket frustum of a right circular cone are 14 cm an d2 cm and is thickness is 9 cm. Find the lateral surface of the frustum. $(Take\ \pi = 22/7)$

4. If the radii of the circular ends of a bucket 24 cm high are 5 cm and 15 cm respectively, find the inner surface area of the bucket (i.e., the area of the metal sheet required to make the bucket) $(Take\ \pi = 3.14)$

5. A bucket is in the form of a cone, its depth is 30 cm and the diameters of the top and the bottom are 42 cm and 14 cm respectively. Find how many litres of water can the bucket hold ? $(Take\ \pi = 22/7)$

6. A container made up of a metal sheet is in the form of a frustum of a cone of height 12 cm with radii of its lower and upper ends as 3 cm and 12 cm respectively. Find the cost of metal sheet used, if it costs Rs. 4 per 100 cm^2. $(Take\ \pi = 22/7)$

7. A vessel is in the form of a frustum of a cone of height 21 cm with radii of its lower and upper ends as 8 cm and 18 cm respectively. Find the cost of milk which can completely fill the vessel at the rate of Rs. 10 per litre.

8. The perimeters of the ends of a frustum are 48 cm and 36 cm. If the height of the frustum be 11 cm, fid the volume of the frustum. $(Take\ \pi = 22/7)$

9. The slant height of the frustum of a cone is 4 cm. If the perimeters of its circular bases be 18 cm and 6 cm, find the curved surface area of the frustum and also find the colt of painting its total surface at the rate of Rs. 12.50 per 100 cm^2.

10. The height of a cone is 30 cm. A frustum is cut off from this cone by a plane parallel to the base of the cone. If the volume of the frustum is $\frac{19}{27}$ of the volume of the cone, find the height of the frustum.

11. The height of a cone is 10 cm. The cone is divided into two parts by drawing a plane through the midpoint of the axis of the cone, parallel to the base. Compare the volume of the two parts.

12. A hollow cone is cut by a plane parallel to the base and upper part is removed. If the curved surface of the remainder is $\frac{15}{16}$ of the curved surface of the whole cone, find the ratio of the line-segments into which the cone's altitude is divided by the plane.

13. A right circular cone is cut by a plane parallel to the base of the cone and the upper portion is removed. If the curved surface of the frustum is $\frac{8}{9}$ of the curved surface of the whole given cone, prove that the height of the frustum is $\frac{2}{3}$ of the height of the whole cone.

14. The altitude of a right circular cone is trisected by two parallel planes, drawn parallel to the base of the cone. The cone is cut into three parts. The topmost part is a right circular cone, the middle one and last one at the bottom are two frustums. If V_1 be the volume of the small cone, V_2 be the volume of the middle portion frustum and V_3 be the volume of the frustum made at the bottom, prove the $V_1 : V_2 : V_3 = 1 : 7 : 19$.

15. A right circular cone is divided by a plane parallel to its base into a small cone of volume V_1 at the top and a frustum of volume V_2 as second part at the bottom. If $V_1 : V_2 = 1 : 3$, find the ratio of the height of the altitude of small cone and that of the frustum.

SURFACE AREAS AND VOLUMES | **ANSWER KEY** | EXERCISE-2 (X)-CBSE

• Conversion Of Solids

1. 1000 cm^2 **2.** 224 cm^2 **3.** 198 cm^2 **4.** 45 **5.** 60 **6.** 105 **7.** 20 **8.** 3.5 cm **9.** 2.5 cm
10. 2 cm. **11.** 0.1 cm **12.** 243 m **13.** 8000 **14.** 1 cm **15.** 2541 **16.** 1500 **17.** 8400 **18.** 0.05 cm
19. 1100 cm^3 **20.** 2.1 cm **21.** 5 cm **22.** 12. cm **23.** 8 $(18)^{1/3}$cm **24.** 16 cm **25.** 6.4 cm
26. 2.4 cm **27.** 450 **28.** 2 cm **29.** 75 cm **30.** 80 cm **31.** 32 **32.** 4 cm **33.** 672 **34.** 20 cm
35. 5 m **36.** 4 m **37.** $\frac{3}{2}$ a **38.** $\frac{3a}{4}$ **39.** 1 cm **40.** 1.6 cm approax. **41.** 1cm **42.** $\frac{\pi\ell^3}{6}$
43. $\frac{\pi p^3}{12}$ **44.** 2.885 cm **45.** 39.6 p^3 litres **46.** 2.52 m **47.** 400000 m^2 **48.** 20 min.
49. 16.5 min. **50.** $17\frac{3}{16}$ m.

• Surface Areas And Volumes Of Combinations Of Solids

1. $293\frac{1}{3} cm^3$ **2.** (i) $14373\frac{1}{3} cm^3$ (ii) 3080 cm^2 (iii) Rs. 2464 **3.** 26.488 kg **5.** 266.11 cm3 **6.** 25.12 cm3
7. 616 cm^3 **8.** 3 cm **9.** 30 **10.** 48720 m^3 **11**. Rs. 97350 **12.** Rs. 27082.5 **13.** 660 m3 ; Rs. 15675
14. 395. 4 kg approx. **15.** 72. 4 m^2 ; 65. 142 m^3 **16.** 71.83 m^2, **17.** 1584 cm^3 ; 602.88 cm^2
18. 641 66 cm^3 ; 418 cm^2 **19.** Rs. 1221.94 **20.** 218.064 cm^3 **21.** 18480 cm^3 **22.** $\frac{2}{3}\pi r^2 h$
23. 798 .816 cm^3 **24.** 4320 m^3 ; 1096 m^2 **25.** 50518 kg **26.** $36\pi h cm3$ **27.** Rs. 159.104 m^2 ; Rs. 795.52
28. $\frac{3}{8} V$

• Frustrum Of A Right Circular Cone

1. 7.5 cm, 196 π cm^3, 5 cm 116 π cm^2. **2.** 540.57 cm^3 **3.** 753.6 cm^2 **4.** 1711.30 cm^2 **5.** 20.02 liters
6. Rs. 47.52 **7.** Rs. 117.04 **8.** 1554 cm3 **9.** Rs. 9.58 **10.** 10 cm **11.** 1 : 7 **12.** 1 : 3 **15.** 1 : $(4^{1/3} – 1)$

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PEREVIOUS YEARS BOARD (CBSE) QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. The surface area of a sphere is 616 cm^2. Find its radius. **[Foreign – 2008]**

2. A cylinder and a cone area of same base radius and of same height. Find the ratio of the volume of cylinder to that of the cone. **[Delhi – 2009]**

3. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm. write the height of the frustum. **[Al – 2010]**

SHORT ANSWER TYPE QUESTIONS

1. A solid metallic sphere of diameter 21 cm is melted and recasted into a number of smaller cones, each of diameter 7 cm and height 3 cm. Find number of cones so formed. **[Delhi – 2004]**

2. A solid metallic sphere of diameter 28 cm is melted and recasted into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find number of cones so formed. **[Delhi – 2004]**

3. A hemispherical bowl of internal diameter 30 cm contains some liquid. This liquid is to be filled into cylindrical shaped bottles each of diameter 5 cm and height 6 cm. Find the number of bottles necessary to empty the bowl. **[Al – 2004]**

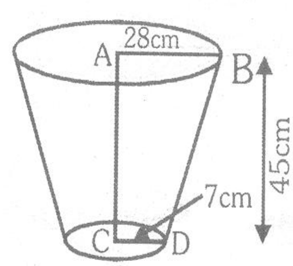
4. Solid spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 18 cm and the water rises by 40 cm, find the number of solid spheres dropped in the water. **[Foreign – 2004]**

5. A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy. [use $\pi = 22/7$] **[Delhi – 2007]**

6. A toy is in the form of a cone mounted on a hemisphere with same radius. The diameter of the base of the conical portion is 7 cm and total height of the toy is 14.5 cm. Find the volume of the toy. [use $\pi = 22/7$] **[Al – 2007]**

LONG ANSWER TYPE QUESTOINS

1. If the radii of the circular ends of a bucket, 45 cm high are 28 cm and 7 cm (as shown in given fig.), find the capacity of the bucket. **[Al – 2004]**



2. A hollow cone is cut by a plane parallel to the base and the upper is removed. If the curved surface of the remainder is $8/7^{th}$ of the curved surface of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane.

OR

If the radii of the ends of a bucket, 45 cm high, are 28 cm and 7 cm, find its capacity and surface area. **[Delhi – 2004C]**

3. A well, of diameter 3m, is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4m, to form an embankment. Find the height of the embankment. [use $\pi = 22/7$] **[Al – 2004C]**

4. If the radii of the ends of a bucket, 45 cm high are 28 cm and 7 cm, determine the capacity and total surface area of the bucket. **[Al – 2005]**

5. The rain water form a roof 22 m × 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the vessel is just full, find the rainfall in cm. **[Delhi – 2006]**

6. Water flows at the rate of 10 m per minute through a pipe having its diameter as 5 mm? How much time will it take to fill a conical vessel whose diameter of base is 40 cm and depth 24 cm? **[Foreign – 2006]**

7. A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

OR

A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter of the sphere. **[Delhi – 2007]**

8. A hemispherical bowl of internal diameter 36 cm is full of some liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. Find the number of bottles needed to empty the bowl.

OR

Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm/second into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will be level of water rise in the tank in half an hour. **[Al – 2007]**

9. A gulab jamun, when ready for eating, contains sugar syrup of about 30% of its volume. Find approximately how much syrup would be found in 45 such gulab jamuns, each shaped like a cylindrical with two hemispherical ends, if the complete length of each of them is 5 cm and it's diameter is 2.8 cm.

OR

A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. This ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream. **[Delhi – 2008]**

10. A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with diameters of it's lower and upper ends as 16 cm and 40 cm respectively. Find the volume of the bucket. Also find the cost of the bucket if the cost of metal sheet used is Rs. 20 per 100 cm^2 [use $\pi = 3.14$]

OR

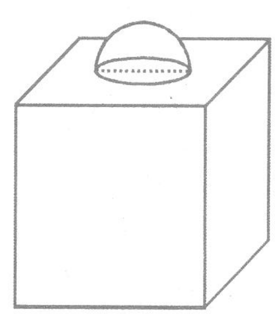
A farmer connects a pipe of internal diameter 20 cm from a canal in to a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 6 km/hr, in how much time will be tank be fillrd? **[Delhi – 2008]**

11. A tent consists of a frustum of a cone, surmounted by a cone. If the diameter of the upper and lower circular ends of the frustum be 14 m and 26 m respectively, the height of the frustum be 8 m and the slant height of the surmounted conical portion be 12 m, find the area of canvas required to make the tent. (Assume that the radii of the upper circular end of the frustum and the base of surmounted conical portion are equal). **[Al – 2008]**

12. If the radii of the circular ends of a conical bucket, which is 16 cm high, are 20 cm and 8 cm, find the capacity and total surface area of the bucket. [use $\pi = 22/7$] **[Foreign – 2008]**

13. Form a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid cored to two places of decimals. Also, find the total surface of the remaining solid. [Take $\pi = 3.1416$] **[Delhi – 2009]**

14. In figure, a decorative block which is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere, fixed on the top, has a diameter of 4.2 cm. Find the total surface area of the block. [Take $\pi = 22/7$] **[Al – 2009]**



15. A spherical copper shell, of external diameter 18 cm is melted and recast into a solid cone of base radius 14 cm and height $4\frac{3}{7}$ cm. Find the inner diameter of the shell.

OR

A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of metal sheet used in making it. [use π = 22 / 7]

16. The rain-water collected on the roof of a building, of dimensions 22 m × 20 m, is drained into a cylindrical vessel having base diameter 2m and height 3.5 m. If the vessel is full up to the brim, find the height of rain-water on the roof. [use π = 22 / 7] **[AI – 2010]**

- **VERY SHORT ANSWER TYPE QUESTIONS**

1. 7 cm **2.** 3 : 1 **3.** 3 cm

- **SHORT ANSWER TYPE QUESTIONS**

1. 126 **2.** 672 **3.** 60 **5.** 858 cm^2 **6.** 231 cm^3

- **ANSWER TYPE QUESTIONS**

1. 48510 cm^2 **2.** 1 : 2 or 48510 cm^3, 5616.38 cm^2 **3.** 1.125 m 4. 48510 cm^3, 8079.5 cm^2 **5.** 2.5 cm
6. 51.2 min 7. 18 cm or 6 cm **8.** 72 or 90 cm **9.** 338.184 cm^3 or 10 cones **10.** Rs. 391.87 or 50 minutes

11. (284π) m^2 **12.** $\frac{73216}{7} cm^3, \frac{13728}{7} cm^2$ **13.** 150.79 cm^3, 259.55 cm^2 **14.** 163.86 cm^2
15. 16 cm or cm ; 2160.32 cm^2 **16.** 2.5 cm

EXERCISE-4 FOR OLYMPIADS

1. One cubic metre piece of copper is melted and recast in to a square cross-section bar, 36 m long. An exact cube is cut off from this. If cubic metre of copper cost Rs. 108, then the cost of this cube is :
(A) 50 paisa (B) 75 paisa (C) One paisa (D) 1.50 paisa

2. If the surface areas of two spheres are in the ratio 4 : 9, then the ratio of their volume is :
(A) 8 : 25 (B) 8 : 26 (C) 8 : 27 (D) 8 : 28

3. In a shower 10 cm of rain fall. The volume of water that falls on 1.5 hectares of ground is :
(A) 1500 m^3 (B) 1400 m^3 (C) 1200 m^3 (D) 1000 m^3

4. The radius of base and the volume of a right circular cone are doubled. The ratio of the length of the larger cone to that of the smaller cone is :
(A) 1 : 4 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1

5. A cone and a hemisphere have equal base diameter and equal volume. The ratio of their heights is :
(A) 3 : 1 (B) 2 : 1 (C) 1 : 2 (D) 1 : 3

6. If the lateral surface of a right circular cone is 2 times its base, then the semi-vertical angle of the cone must be :
(A) 15^0 (B) 30^0 (C) 45^0 (D) 60^0

7. The slant height of a conical tent made of canvas is $\frac{14}{3}$ m. The radius of tent is 2.5 m. The width of the canvas is 1.25 tube. If the height of the tube is 15 cm, then the diameter of the tube (in Rs.) is :

(A) 726 (B) 950 (C) 960 (D) 968

8. A hemispherical basin 150 cm in diameter holds water one hundred and twenty times as much a cylindrical m. If the rate of canvas per metre is Rs. 33, then the total cost of the canvas required for the tube (in cm) is :

(A) 23 (B) 24 (C) 25 (D) 26

9. A river 3 m deep and 60 m wide is flowing at the rate of 2.4 km/h. The amount of water running into the sea per minute is :

(A) 6000 m^3 (B) 6400 m^3 (C) 6800 m^3 (D) 7200 m^3

10. If a solid right circular cylinder is made of iron is heated to increase its radius and height by 1 % each, then the volume of the solid is increased by :

(A) 1.0 % (B) 3.03 % (C) 2.02 % (D) 1.2 %

11. If the right circular cone is separated into three solids of volumes V_1, V_2, and V_3 by two planes which are parallel to the base and trisects the altitude, then $V_1 : V_2 : V_3$ is :

(A) 1 : 2 : 3 (B) 1 : 4 : 6 (C) 1 : 6 : 9 (D) 1 : 7 : 19

12. Water flows at the rate of 10 m per minute from a cylindrical pipe 5 mm in diameter. A conical vessel whose diameter is 40 cm and depth 24 cm is filled. The time taken to fill the conical vessel is :

(A) 50 min (B) 50 min 12 sec. (C) 51 min 12 sec. (D) 51 min 15 sec.

13. A cylinder circumscribes a sphere. The ratio of their volume is :

(A) 1 : 2 (B) 3 : 2 (C) 4 : 3 (D) 5 : 6

14. If form a circular sheet of paper of radius 15 cm, a sector of 144^0 is removed and the remaining is used to make a conical surface, then the angle at the vertex will be :

(A) $\sin^{-1}\left(\frac{3}{10}\right)$ (B) $\sin^{-1}\left(\frac{6}{5}\right)$ (C) $2\sin^{-1}\left(\frac{3}{5}\right)$ (D) $2\sin^{-1}\left(\frac{4}{5}\right)$

15. A right circular cone of radius 4 cm and slant height 5 cm is curved out from a cylindrical piece of wood of same radius and height 5 cm. The surface area of the remaining wood is:

(A) 84 π (B) 70 π (C) 76 π (D) 50 π

16. If h, s, V be the height, curved surface area and volume of a cone respectively, then $(3\pi Vh^3 + 9V^2 - s^2h^2)$ is

(A) 0 (B) π (C) $\frac{V}{sh}$ (D) $\frac{36}{V}$

17. If cone is cut into two parts by a horizontal plane passing through the mid point of its axis, the ratio of the volume of the upper part and the frustum is :

(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 7

18. A cone, a hemisphere and a cylinder stand on equal bases of radius R and have equal heights H. Their whole surfaces are in the ratio:

(A) $(\sqrt{3}+1) : 3 : 4$ (B) $(\sqrt{2}+1) : 7 : 8$ (C) $(\sqrt{2}+1) : 3 : 4$ (D) None of these

19. If a sphere is placed inside a right circular cylinder so as to touch the top, base and the lateral surface of the cylinder. If the radius of the sphere is R, the volume of the cylinder is :

(A) $2\pi R^3$ (B) $8\pi R^3$ (C) $\frac{4}{3}\pi R^3$ (D) None of these

20. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of one end and the other end as its base. The volumes of the cylinder, hemisphere and the cone are respectively in the ratio of :

(A) $3 : \sqrt{3} : 2$ (B) 3 : 2 : 8 (C) 1 : 2 : 3 (D) 2 : 3 : 1

21. A hollow sphere of outer diameter 24 cm its cut into two equal hemisphere. The total surface area of one of the hemisphere is $1436\frac{2}{7}cm^2$. Each one of the hemisphere is filled with water. What is the volume of water that can be filled in each of the hemisphere?

(A) $3358\frac{2}{3}cm^3$ (B) $3528\frac{2}{3}cm^3$ (C) $2359\frac{2}{3}cm^3$ (D) $9335\frac{2}{3}cm^3$

22. A big cube of side 8 cm is formed by rearranging together 64 small but identical cubes each of side 2 cm. Further, if the corner cubes in the topmost layer of the big cube are removed, what is the change in total surface area of the big cube ?
(A) 16 cm^2, decreases (B) 48 cm^2, decreases
(C) 32 cm^2, decreases (D) Remains the same as previously

23. A large solid sphere of diameter 15 m is melted and recast into several small spheres of diameter 3 m. What is the percentage increase in the surface area of the smaller sphere over that of the large sphere?
(A) 200 % (B) 400 % (C) 500 % (D) Can't be determined

24. A cone is made of a sector with a radius of 14 cm and an angle of 60^0. What is total surface area of the cone ?
(A) 119.78 m^2 (B) 191.87 m^2 (C) 196.5 m^2 (D) None of these

25. If a cube of maximum possible volume is cut off from a solid sphere of diameter d, then the volume of the remaining (waste) material of the sphere would be equal to :

(A) $\frac{d^3}{3}\left(\pi - \frac{d}{2}\right)$ (B) $\frac{d^3}{3}\left(\frac{\pi}{2} - \frac{1}{\sqrt{3}}\right)$ (C) $\frac{d^3}{4}\left(\sqrt{2} - \pi\right)$ (D) None of these

26. A piece of paper is in the form of a right angle triangle in which the ratio of base and perpendicular is 3 : 4 and hypotenuse is 20 cm. What is the volume of the biggest cone that can be formed by taking right angle vertex of the paper as the vertex of the cone?
(A) 45.8 m^3 (B) 56.1 m^3 (C) 61.5 m^3 (D) 48 m^3

27. In a particular country the value of diamond is directly proportional to the surface area (exposed) of the diamond. For thieves steel a cubical diamond piece and then divide equally in four parts. What is the maximum percentage increase in the value of diamond after cutting it ?
(A) 50 % (B) 66.66 % (C) 100 % (D) None of these

28. In a bullet the gun powder is to be filled up inside the metallic enclosure. The metallic enclosure is made up of a cylindrical base and conical top with the base of radius 5 cm. The ratio of height of cylinder and cone is 3 : 2. A cylindrical hole is drilled through the metal solid with height two-third the height of metal solid. What should be the radius of the hole, so that the volume of the hole (in which gun powder is to be filled up) is one third the volume of metal solid after drilling ?

(A) $\sqrt{\frac{88}{5}}$ cm **(B)** $\sqrt{\frac{55}{8}}$ cm **(C)** $\frac{55}{8}$ cm **(D)** 33π cm

29. A cubical cake is cut into several smaller cubes by dividing each edge in 7 equal parts. The cake is cut from the top along the two diagonals forming four prisms. Some of them get cut and rest remained in the cubical shape. A complete cubical (smaller) cake was given to adults and the cut off part of a smaller cake is given to a child get the cake?
(A) 343 (B) 448 (C) 367 (D) 456

30. In a factory there are two identical solid blocks of iron. When the first block is melted and recast into spheres of equal radii Y, then 14 cc of iron was left. The volumes of the solid blocks and all the spheres are in integers. What is the volume (in cm^3) of each of the large sphere of radius '2r'?
(A) 176 (B) 12π (C) 192 (D) Data insufficient

31. Initially the diameter of a balloon is 28 cm. It can explode when the diameter becomes 5/2 times of the initial diameter. Air is blown at 156 cc/s. It is knows that the shape of balloon always remains spherical. In how many seconds the balloon will explode?
(A) 1078s (B) 1368s (C) 1087s (D) None of these

32. The radius of a cone is $\sqrt{2}$ times the height of the cone. A cube of maximum possible volume is cut from the same cone. What is the ratio of the volume of the cone to the volume of the cube?

(A) $2\sqrt{3}$ ft (B) $(2+\sqrt{3})$ ft (C) $(3+\sqrt{2})$ ft (D) $(2+2\sqrt{3})$ ft

34. A blacksmith has a rectangular sheet of iron. He has to cut out 7 circular discs from this sheet. What is the minimum possible width of the iron sheet if the radius of each disc is 1 ft?

(A) $\frac{1}{11}$ (B) $\frac{2}{17}$ (C) $\frac{3}{22}$ (D) None of these

35. Barun needs an open box of capacity 864 m^3. Actually where he lives, the rates of paints are soaring high so he wants to minimize the surface area of the box keeping the capacity of the box same as required. What is the base area and height of such a box?

(A) 36 m2, 24 m (B) 216 m2, 4 m (C) 144 m2, 6 m (D) None of these

36. There are two cylindrical containers of equal capacity and equal dimensions. If the radius of one of the containers a increased by 12 ft and the height of another container is increased by 12 ft, then the capacity of both the containers is equally increased by K cubic ft. If the actual heights of the container be 4 ft, then find the increased volume of each of the container :

(A) 1680 π cu ft (B) 2304 π cu ft (C) 1480 π cu ft (D) Can’t be determined

OBJECTIVE **ANSWER KEY** EXERCISE-4

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	A	B	B	B	D	C	D	B	D	C	B	C	C
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	A	D	C	A	B	A	D	B	A	B	B	C	B	B	A
Que.	**31**	**32**	**33**	**34**	**35**	**36**									
Ans.	A	B	B	A	C	B									

STATISTICS

★ INTRODUCTION

In class IX, we have studied about the presentation of given data in the form of ungrouped us well as grouped frequency distributions. We have also studied how to represent the statistical data in the form of various graphs such as bar graphs, histograms and frequency polygons. In addition, we have studied the measure of central tendencies such as mean, median and mode of ungrouped data.

In this chapter, we shall discuss about mean, median and mode of grouped data. We shall also discuss the concept of cumulative frequency, cumulative frequency distribution and cumulative frequency curve (ogive).

★ MEAN OF UNGROUPED DATA

We know that the mean of observations is the sum of the values of all the observations divided by the total number of observations i.e., if $x_1, x_2, x_3, \ldots\ldots, x_n$ are n observations, then

mean, $\bar{x} = \dfrac{x_1 + x_2 + x_3 + \ldots.. + x_n}{n}$ or $\bar{x} = \dfrac{\sum_{1=1}^{n} x_1}{n}$, where $\sum_{1=1}^{n} x_1$ denotes the sum $x_1 + x_2$ $x_3 + \ldots\ldots + x_n$.

● Direct method ● Short-cut method or Assumed-mean method ● Step-deviation method.

★ MEAN OF GROUPED DATA

● Direct method

If $x_1, x_2, x_3, \ldots\ldots, x_n$ are n observations with respective frequencies $f_1, f_2, f_3, \ldots\ldots.., f_n$ then mean, $(\bar{x})$ defined by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \ldots.. + f_nx_n}{f_1 + f_2 + f_3 + \ldots. + f_n} \text{ or } \bar{x} = \frac{\sum_{1=1}^{n} f_1x_1}{\sum_{1=1}^{n} f_1}, \text{ where } \sum_{1=1}^{n} f_1 = f_1 + f_2 + f_3 + \ldots.. + f_n .$$

To find mean of grouped Data

The following steps should be followed in finding the arithmetic mean of grouped data by direct method.

STEP-1: Find the class mark (x_i) of each class using, $x_i = \dfrac{lower\,\text{limit} + Upper\,\text{limit}}{2}$

STEP-2: Calculate f_ix_i for each i

STEP-3: Use the formula : mean, $\bar{x} = \dfrac{\sum_{1=1}^{n} f_1x_1}{\sum_{1=1}^{n} f_1}$,

● SHORTCUT METHOD OR ASSUMED MEAN METHOD

In this case, to calculate the mean, we follow the following steps :

STEP-1: Find the class mark (x_i) of each class using

$$x_i = \frac{lower\,\text{limit} + Upper\,\text{limit}}{2}$$

STEP-2: Choose a suitable value of x_i in the middle as the assumed mean and denote it by ‘a’.

STEP-3: Find $d_i = x_i - a$ for each i

STEP-4: Find $f_i \times d_i$ for each i

STEP-5: Find n = $\sum f_1$

STEP-6: Calculate the mean, $(\bar{x})$ by using the formula $\bar{x} = a + \dfrac{\sum f_id_i}{N}$.

● STEP-DEVIATION METHOD

Sometimes, the values of x and f are so large that the calculation of mean by assumed mean method becomes quite inconvenient. In this case, we follow the following steps:

STEP-1: Find the class mark (x_i) of each class using, $x_i = \dfrac{lower\,\text{limit} + Upper\,\text{limit}}{2}$

STEP-2: Choose a suitable value of x_i in the middle as the assumed mean and denote it by 'a'.

STEP-3: Find h = (upper limit – lower limit) for each class.

STEP-4: Find $u_i = \dfrac{x_i - a}{h}$ for each class.

STEP-5: Find $f_i u_i$ for each i.

STEP-6: Calculate, the mean by using the formula $\bar{x} = a + \left\{\dfrac{\sum f_i \times u_i}{N}\right\} \times h$, where $N = \sum f_i$

Ex.1 Find the mean of the following data:

Class Interval	0-8	8-16	16-24	24-32	32-40
Frequency	6	7	10	8	9

Sol. We may prepare the table as given below :

Class Interval	**Frequency (f_i)**	**Class mark (x_i)**	**$f_i x_i$**
0-8	6	4	24
8-16	7	12	84
16-24	10	20	200
24-32	8	28	224
32-40	9	36	324
	$\sum f_i = 40$		$\sum f_i x_i = 856$

$$\therefore \quad \text{Mean, } \bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{856}{40} = 21.4$$

Ex.2 The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs. 18. Find the missing frequency f.

Daily pocket allowance (in Rs.)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of children	7	6	9	13	f	5	4

Sol. We may prepare the table as given below :

Daily pocket Allowance	**Number of Children (f_i)**	**Class mark (x_i)**	**$f_i x_i$**
11-13	7	12	84
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	20f
21-23	5	22	110
23-25	4	24	96
	$\sum f_i = 44 + f$		$\sum f_i x_i = 752 + 20f$

$$\therefore \quad \text{Mean, } \bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{752 + 20f}{44 + f}$$

Given, mean = 18

$\therefore \quad 18 = \dfrac{752+20f}{44+f} \Rightarrow 792+18f = 752+20f \Rightarrow f = 20$

Ex.3 Find the missing frequencies f1 and f2 in the table given below, it is being given that the mean of the given frequency distribution is 50.

Class	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

Sol. We may prepare the table as given below :

Class	**Number of (f_i)**	**Class mark (x_i)**	**$f_i x_i$**
0-20	17	10	170
20-40	f_1	30	$30f_1$
40-60	32	50	1600
60-80	f_2	70	$70f_2$
80-100	19	90	1710
	$\sum f_i = 68 + f_1 + f_2$		$\sum f_i x_i = 3480 + 30f_1 + 70f_2$

$\therefore \quad$ Mean, $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum f_i} = \dfrac{3480+30f_1+70f_2}{68+f_1+f_2}$

Given, mean = 50

$\therefore \quad 50 = \dfrac{3480+30f_1+70f_2}{68+f_1+f_2} \Rightarrow 3400+50f_1 = 50f_2 = 3480+30f_1+70f_2$

$\Rightarrow \quad 20f_1 - 20f_2 = 80 \Rightarrow f_1 - f_2 = 4 \quad$ ...(i)

And $\sum f_i = 68 + f_1 + f_2$

$\therefore \quad 120 = 68 + f_1 + f_2 \quad [\because \sum f_1 = 120]$

$\Rightarrow \quad f_1 + f_2 = 52 \quad$ ...(ii)

Adding (1) and (2), we get $2f_1 = 56 \Rightarrow f_1 = 28 \quad \therefore \quad f_2 = 24$

Hence, following missing frequencies f1 and f2 are 28 and 24 respectively.

Ex.4 The following table gives the marks scored by 100 students in a class test :

Mark	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	12	28	27	20	17	6

Sol. We may prepare the table with assumed mean, a = 35 as given below :

Mrks	**No.of students (f_i)**	**Class mark (x_i)**	**$d_i = x_i - a$ = xi – 35**	**$f_i d_i$**
0-10	12	5	– 30	– 360
10-20	18	15	– 20	– 360
20-30	27	25	– 10	– 270
30-40	20	30 = a	0	0
40-50	17	45	10	170
50-60	6	55	20	120
	N = 100			$\sum f_i d_i = -700$

$\therefore$ Mean, $\bar{x} = a + \frac{\sum f_i d_i}{N} = 35 + \frac{(-700)}{100} = 35 - 7 = 28$

Ex.5 Thirty women were examined in a hospital by a doctor and the number of heart beats per minute, were recorded and summarized as follows. Find the mean heart beats per minute for these women, by using assumed.

No. of heart beats per minute	65-68	68-71	71-74	74-77	77-80	80-83	83-86
Frequency	2	4	3	8	7	4	2

Sol. We may prepare the table with assumed mean, a = 35 as given below :

No. of heart beats per minute	**No.of women (f_i)**	**Class mark (x_i)**	**$d_i = x_i - a$ = xi – 75.5**	**$f_i d_i$**
65-68	2	66.5	– 9	– 18
68-71	4	69.5	– 6	– 24
71-74	3	72.5	**– 3**	– 9
74-77	8	75.5 = a	0	21
77-80	7	78.5	3	24
80-83	4	81.5	6	18
83-86	2	84.5	9	
	N = 30			$\sum f_i d_i = 12$

$\therefore$ Mean, $\bar{x} = a + \frac{\sum f_i d_i}{N} = 75.5 + \frac{12}{30} = 75.5 + \frac{2}{5} = 75.9$

Ex.6 Find the mean of the following distribution by step-deviation method :

Class	50-70	70-90	90-110	110-130	130-150	150-170
Frequency	18	12	13	27	8	22

Sol. We may prepare the table with assumed mean a = 120 as given below :

Class	**Frequency (f_i)**	**Class mark (x_i)**	$u_i = \frac{x_i - a}{h} = \frac{x_i - 120}{20}$	**$f_i u_i$**
50-70	18	60	– 3	– 54
70-90	12	80	– 2	– 24
90-110	13	100	– 1	– 13
110-130	27	120 = a	0	0
130-150	8	140	1	8
150-170	22	160	2	44
	N = 100			$\sum f_i u_i = -39$

$\therefore$ Mean, $\bar{x} = a + \frac{\sum f_i u_i}{N} \times h = 120 + \frac{(-39)\times 20}{100} = 20 - \frac{39}{5} = \frac{561}{5} = 112.2$

Ex.7 Find the mean marks from the following data :

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80	Below 90	Below 100
No. of Students	5	9	17	29	45	60	70	78	83	z85

Sol. We may prepare the table as given below :

Marks	**No. of students**	**Class Interval**	**f_i**	**Class mark (x_i)**	**$f_i x_i$**
Below 10	5	0-10	5	5	25
Below 20	9	10-20	4	15	60
Below 30	17	20-30	9	25	225
Below 40	29	30-40	12	35	420
Below 50	45	40-50	16	45	720
Below 60	60	50-60	15	55	825
Below 70	70	60-70	10	65	650
Below 80	78	70-80	8	75	600
Below 90	83	80-90	5	85	425
Below 100	85	90-100	2	95	190
			N = 85		$\sum f_i x_i = 4140$

$\therefore$ Mean, $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{4140}{85} = 48.41$

Ex.8 Find the mean marks of students from the adjoining frequency distribution table.

Marks	**No. of Students**
Above 0	80
Above 10	77
Above 20	72
Above 30	65
Above 40	55
Above 50	43
Above 60	23
Above 70	16
Above 80	10
Above 90	8
Above 100	0

Sol. We may prepare the table as given below :

Marks	No. of students	Class Interval	f_i	Class mark (x_i)	f_ix_i
Above 0	80	0-10	3	5	15
Above 10	77	10-20	5	15	75
Above 20	72	20-30	7	25	175
Above 30	65	30-40	10	35	350
Above 40	55	40-50	12	45	540
Above 50	43	50-60	20	55	1100
Above 60	23	60-70	7	65	455
Above 70	16	70-80	6	75	450
Above 80	10	80-90	2	85	170
Above 90	8	90-100	8	95	760
Above 100	0	100-110	0	105	0
			N = 80		$\sum f_i x_i = 4090$

$\therefore$ Mean, $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{4090}{80} = 51.125 = 51.1$ (approax)

Ex.9 Find the arithmetic mean of the following frequency distribution.

Class	25-29	30-24	35-39	40-44	45-49	50-54	55-59
Frequency	14	22	16	6	5	3	4

Sol. The given series is in inclusive form. We may prepare the table in exclusive form with assumed mean a = 42 as given below :

Class	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - a = x_i - 75.5$	$f_i u_i$
24.5-29.5	14	27	– 15	– 210
29.5-34.5	22	32	– 10	– 220
34.5-39.5	16	37	– 5	– 80
39.5-44.5	6	42 = a	0	0
44.5-49.5	5	47	5	25
49.5-54.5	3	52	10	30
54.5-59.5	4	57	15	60
	N = 70			$\sum f_i d_i = -395$

$\therefore$ Mean, $\bar{x} = a\frac{\sum f_i d_i}{N} = 42 + \frac{(-395)}{70} = \frac{2940-395}{70} = \frac{2545}{70} 36.36$ (approx)

★ MEDIAN OF A GROUPED DATA

MEDIAN : It is a measure of central tendency which gives the value of the middle most observation in the data. In a grouped data, it is not possible to find the middle observation by looking at the cumulative frequencies as the middle observation will be some value in a class interval. It is, therefore, necessary to find the value inside a class that divides the whole distribution into two halves.

MEDIAN CLASS : The class whose cumulative frequency is greater than $\frac{N}{2}$ is called the median class.

To calculate the median of a grouped data, we follow the following steps :

STEP-1: Prepare the cumulative frequency table corresponding to the given frequency distribution and obtain $N = \sum f_i$.

STEP-2: Find $\frac{N}{2}$

STEP-3: Look at the cumulative frequency just greater than $\frac{N}{2}$ and find the corresponding class (Median class).

STEP-4: Use the formula Median, $M = \ell + \left\{\frac{\frac{N}{2} - C}{f}\right\} \times h$

Where ℓ = Lower limit of median class.
F = Frequency of the median class.
C = Cumulative frequency of the class preceding the median class.
h = Size of the median class.
$N = \sum f_i$

Ex.10. Find the median of the following frequency distribution :

Marks	0-10	10-20	20-30	30-40	40-50	Total
No. of Students	8	20	36	24	12	100

Sol. At first we prepare a cumulative frequency distribution table as given below :

Marks	Number of students (f_i)	Cumulative frequency
0-10	8	8
10-20	20	28
20-30	36	64
30-40	24	88
40-50	12	100
	N = 100	

Here, N = 100

$\therefore \quad \frac{N}{2} = 50$

The cumulative frequency just greater than 50 is 64 and the corresponding class is 20-30.
So, the median class is 20-30.

$\therefore \quad \ell = 20$, N = 100, C = 28, f = 36 and h = 10

Therefore, median $= \ell + \left\{\frac{\frac{N}{2} - C}{f}\right\} \times h$

$$= 20 + \left(\frac{50-28}{36}\right) \times 10 = 20 + \frac{22 \times 10}{36} = 20 + \frac{55}{9} = \frac{180+55}{9} = \frac{235}{9} = 36.1$$

Ex.11 A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60

No. of policy Holders	2	6	24	45	78	89	92	98	100

Sol. From the given table we can find the frequency and cumulative frequency as given below :

Age (in years)	**Number of students (f_i)**	**Cumulative frequency**
15-20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100
	N = 100	

Here, N = 100

$\therefore \quad \frac{N}{2} = 50$

The cumulative frequency just greater than 50 is 78 and the corresponding class is 35-40.
So, the median class is 65-40.

$\therefore \quad \ell = 20$, N = 100, C = 45, f = 33 and h = 5

Therefore, median $= \ell + \left\{ \frac{\frac{N}{2} - C}{f} \right\} \times h$

$$= 35 + \left(\frac{50 - 45}{33} \right) \times 5 = 35 + \frac{5 \times 5}{33} = \frac{1155 + 25}{33} = \frac{1180}{33} = 35.76$$

Hence, the median age is 35.76 years.

Ex.12 The length of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table. Find the median length of the leaves.

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No. of leaves	3	5	9	12	5	4	2

Sol. The given series is in inclusive form. We may prepare the table in exclusive form and prepare the cumulative frequency table as given below :

Length (in mm)	**Number of leaves (f_i)**	**Cumulative frequency**
117.5-126.5	3	3
126.5-135.5	5	8
135.5-144.5	9	17
144.5-153.5	12	29
153.5-162.5	5	34
162.5-171.5	4	38
171.5-180.5	2	40
	N = 40	

Here, N = 40

$\therefore \quad \frac{N}{2} = 20$

The cumulative frequency just greater than 20 is 29 and the corresponding class is 144.5-153.5
So, the median class is 144.5-153.5

$\therefore \quad \ell = 144.5$, N = 40, C = 17, f = 12 and h = 9

Therefore, median $= \ell + \left\{ \dfrac{\frac{N}{2} - C}{f} \right\} \times h$

$$= 144.5 + \frac{(20-17)}{12} \times 9 = 144.5 + \frac{3 \times 9}{12} = 144.5 + 2.25 = 146.75$$

Hence, median length of leaves is 146.75 mm.

Ex.13 Calculate the missing frequency 'a' from the following distribution, it is being given that the median of the distribution is 24.

Age (in mm)	0-10	10-20	20-30	30-40	40-50
No. of persons	5	25	a	18	7

Sol. At first we prepare a cumulative frequency distribution table as given below :

Age (in years)	0-10	10-20	20-30	30-40	40-50	Total
No. of persons (f_i)	5	25	a	18	7	55+a
Cumulative frequency	5	30	30+a	48+a	55+a	

Since the median is 24, therefore, the median class will be 20-30.
Hence, $\ell = 20$, N = 55+a, C = 30, f = a and h = 10

Therefore, median $= \ell + \left\{ \dfrac{\frac{N}{2} - C}{f} \right\} \times h$

$$\Rightarrow \quad 24 = 20 + \left(\frac{\frac{55+a}{2} - 30}{a} \right) \times 10$$

$$\Rightarrow \quad 24 = 20 + \frac{(a-5)}{2a} \times 10$$

$$\Rightarrow \quad 4 = \frac{(a-5)}{2a} \times 5$$

$\Rightarrow \quad$ 4a = 5a – 25 $\Rightarrow$ a = 25

Hence, the value of missing frequency a is 25.

Ex.14 The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class Interval	Frequency (f_i)
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4
	N = 100

Sol. At first we prepare a cumulative frequency distribution table as given below :

Class Interval	**frequency (f_i)**	**Cumulative frequency**
0-100	2	2
100-200	5	7
200-300	x	7+x
300-400	12	19+x
400-500	17	36+x
500-600	20	56+x
600-700	y	56+x+y
700-800	9	65+x+y
800-900	7	72+x+y
900-1000	4	76+x+y
	N = 100	

We have N = 100

$\therefore 76 + x + y = 100 \Rightarrow x + y = 24$ …(i)

Since the median is 525, so, the median class is 500 – 600

$\therefore \ell = 500$, N = 100, C = 36 + x, f = 20 and h = 100

Therefore, median $= \ell + \left\{\dfrac{\dfrac{N}{2} - C}{f}\right\} \times h$

$\Rightarrow \quad 525 = 500 + \left(\dfrac{50 - 36 - x}{20}\right) \times 100 \Rightarrow 25 = (14 - x) \times 5$

$\Rightarrow \quad 5 = 14 - x \Rightarrow x = 9$

Also, putting x = 9 in (1), we get 9 + y = 24 $\Rightarrow$ y = 15

Hence, the values of x and y are 9 and 15 respectively.

★ **MODE OF A GROUPED DATA**

MODE : Mode is that value among the observations which occurs most often i.e., the value of the observation having the maximum frequency.

In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequency.

MODAL CLASS : The class of a frequency distribution having maximum frequency is called modal class of a frequency distribution .

The mode is a value inside the modal class and is calculated by using the formula.

$$\text{Mode} = \ell + \left\{\frac{f_1 f_0}{2f_1 - f_0 - f_2}\right\} \times h$$

Where ℓ = Lower limit of the modal class.

h = Size of class interval.

f_1 = Frequency of modal class.

f_0 = Frequency of the class preceding the modal class

f_2 = Frequency of the class succeeding the modal class

Ex15 The following data gives the information on the observed lifetimes (in hours) of 225 electrical components :

Lifetimes (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Sol. Here the class 60-80 has maximum frequency, so it is the modal class.

$\therefore \quad \ell = 60, h = 20, f_1 = 61, f_0 = 52$ and $f_2 = 38$

$$\text{Therefore, mode} = \ell + \left\{\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right\} \times h$$

$$= 60 + \left(\frac{61-52}{2\times61-52-38}\right) \times 20 = 60 + \frac{9}{20} \times 20 = 60 + 5.625 = 65.625$$

Hence, the modal lifetimes of the components is 65.625 hours.

Ex.16 Given below is the frequency distribution of the heights of players in a school.

Heights (in cm)	160-162	136-165	166-168	169-171	172-174
No. of students	15	118	142	127	18

Find the average height of maximum number of students.

Sol. The given series is in inclusive form. We prepare the table in exculsive form, as given below :

Heights (in cm)	159.5-162.5	162.5-165.5	165.5-168.5	168.5-171.5	171.5-174.5
No. of students	15	118	142	127	18

We have to find the mode of the data.

Here, the class 165.5-168.5 has maximum frequency, so it is the modal class.

Ex.17 The mode of the following series is 36. Find the missing frequency f in it.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-71
Frequency	8	10	f	16	12	6	7

Sol. Since the mode is 36, so the modal class will be 30-40

$\therefore \quad \ell = 30, h = 10, f_1 = 16, f_0 = f$ and $f_2 = 12$

Therefore, mode = $\ell + \left\{\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right\} \times h$

$\Rightarrow \quad 66 = 30 + \left(\dfrac{61 - f}{2 \times 16 - f - 12}\right) \times 10 \Rightarrow 6 = \dfrac{(16 - f)}{(20 - f)} \times 10$

$\Rightarrow \quad 120 - 6f = 160 - 10f \Rightarrow 4f = 40 \Rightarrow f = 10$

Hence, the value of the missing frequency f is 10.

★ GRAPHICAL. REPRESENTATION OF CUMULATIVE FREQUENCY DISTRIBUTION

● CUMULATIVE FREQUENCY POLYGON CURVE (OGIVE)

Cumulative frequency is of two types and corresponding to these, the ogive is also of two types.

- **LESS THAN SERIES**
- **MORE THAN SERIES**

● **LESS THAN SERIES** To construct a cumulative frequency polygon and an ogive, we follow these steps :

STEP-1 : Mark the upper class limit along x-axis and the corresponding cumulative frequencies along y-axis.

STEP-2 : Plot these points successively by line segments. We get a polygon, called cumulative frequency polygon.

STEP-3 : Plot these points successively by smooth curves, we get a curve called cumulative frequency or an ogive.

★ APPLICATION OF AN OGIVE

Ogive can be used to find the median of a frequency distribution. To find the median, we follow these steps.

METHOD –I

STEP-1 : Draw anyone of the two types of frequencies curves on the graph paper.

STEP-2 : Compute $\frac{N}{2}(N = \sum f_i)$ and mark the corresponding points on the y-axis.

STEP-3 : Draw a line parallel to x-axis from the point marked in step 2, cutting the cumulative frequency curve at a point P.

METHOD –II

STEP-1 : Draw less than type and more than type cumulative frequency curves on the graph paper.

STEP-2 : Mark the point of intersecting (P) of the two curves draw2n in step 1.

STEP-3 : Draw perpendicular PM from P on the x-axis. The x-coordinate of point M gives the median .

Ex.18 The following distribution gives the daily income of 50 workers of a factory.

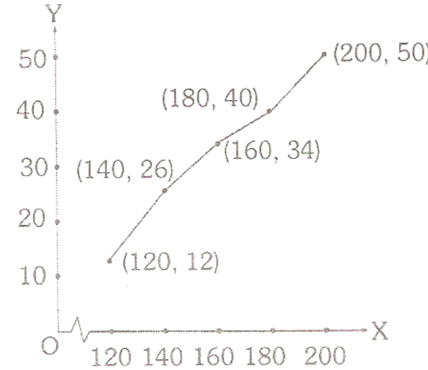
Daily income (in Rs.)	100-120	120-140	140-160	160-180	180-200
No. of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Sol. From the given table, we prepare a less than type cumulative frequency distribution table, as given below :

Income less than (in Rs)	120	140	160	180	200
Cumulative frequency	12	26	34	40	50

Join these points by a freehand curve to get an ogive of 'less than' type.



Ex.19 The following table gives production yield per hectare of wheat of 100 farms of a village.

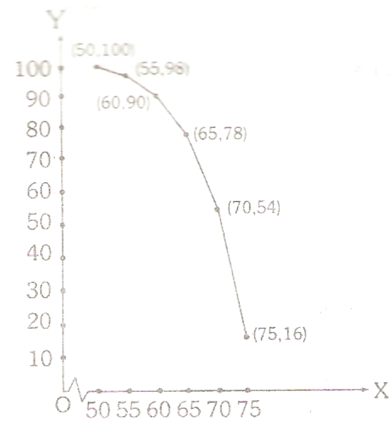
Production yieid (in kg/ha)	50-55	55-60	60-65	65-70	70-75	75-80
No. of farms	2	8	12	24	38	16

Change the distribution to more than type distribution and draw its ogive.

Sol. From the given table, we may prepare more than type cumulative frequency distribution table, as given below :

Production more than (in kg/ha)	50	55	60	65	70	75
Cumulative frequency	100	98	90	78	54	16

Now, plot the points (50, 100), (55,98), (60,90), (65,78), (70,54) and (75,16).
Join these points by a freehand curve to get an ogive of 'more than' type.



Ex.20 The annual profits earned by 30 shops of a shopping complex in a locality gives rise to the following distribution

Profit (in lakhs Rs.)	No. of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Draw both ogives for the data above. Hence, obtain the median profit.

Sol. We have a more than type cumulative frequency distribution table. We may also prepare a less than type cumulative frequency distribution table from the given data, as given below :

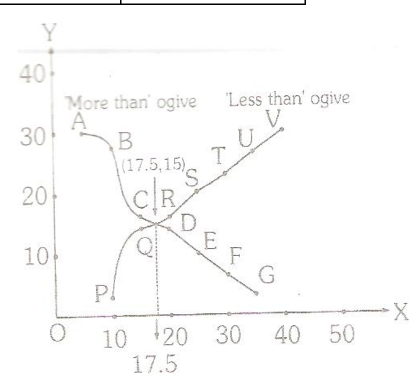
'More than' type

Profit more than (Rs. in lakhs)	No. of shops
5	30
10	28
15	16
20	14
25	10
30	7
35	3

'Less than' type

Profit less than (Rs. in lakhs)	No. of shops
10	2
15	14
20	11
25	20
30	23
35	27
40	30

Now, plot the points A(5,30), B(10,28), C(15,16), D(20,14), E(25,10), F(30,7) and G(35,3) for the more than type cumulative frequency and the points P(10,2), Q(15,14), R(20,16), S(25,20), T(30,23), U(35,27) and V(40,30) for the less then type cumulative frequency table.



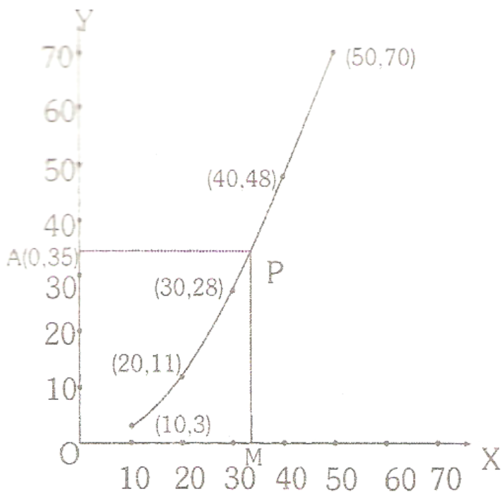
Join these points by a freehand to get ogives for 'more than' type and 'less than' type.
The tow ogives intersect each other at point (17.5, 15).
Hence, the median profit is Rs. 17.5 lakhs.

Ex.21 The following data gives the information on marks of 70 students in a periodical test :

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50
No. of students	3	11	28	48	70

Draw a cumulative frequency curve for the above data and find the median.

Sol. We have a less than cumulative frequency table. We mark the upper class limits along the x-axis and the corresponding cumulative frequency (no. of students) along the y-axis. Now, plot the points (10,3), (20,11), (30,28), (40,48) and (50,70). Join these points by a freehand curve to get an ogive of 'less than' type.



Here, N = 70

$\therefore \frac{N}{2} = 35$

Take a point A(0,35) on the y-axis and draw AP $\parallel$ x-axis, meeting the curve at P.

Draw PM $\perp$ x-axis, intersecting the x-axis, at M.

Then, OM = 33.

Hence, the median marks is 33.

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. Which of the following is a measure of central tendency ?

(A) Frequency (B) Cumulative frequency

(C) Mean (D) Class limit

2. Class mark of a class is obtained by using –

(A) Class mark (B) $\frac{1}{2}$ [upper limit – lower limit]

(C) $\frac{1}{2}$ [upper limit + lower limit] (D) $\frac{1}{2}$ [upper limit + lower limit] – 1

3. The value of $\sum_{i=1}^{n} x_i$ is –

(A) $\frac{\bar{x}}{2}$ (B) 2 $\bar{x}$ (C) n $\bar{x}$ (D) $\frac{\bar{x}}{n}$

4. The mean of the following data $1^2, 2^2, 3^2, \ldots\ldots n^2$ is –

(A) $\frac{(n+1)(2n+1)}{6}$ (B) $\frac{n(n-1)(2n+1)}{6}$ (C) $\frac{n(n+1)(2n-1)}{6}$ (D) $\frac{n(n-1)(2n-1)}{6}$

5. The mean of following distribution is –

x_i	10	12	15	25
f_i	2	3	7	8

(A) 18.50 (B) 18.50 (C) 18.15 (D) 18.25

6. The mean of following data is 18.75 then the value of p is –

x_i	10	15	p	25	30
f_i	5	10	7	8	2

(A) 21 (B) 20.6 (C) 20 (D) 22

7. To find mean, we use the formula.

(A) $\sum_{i=1}^{n} f_i x_i$ (B) $N\sum_{i=1}^{n} f_i x_i$ (C) $\frac{1}{N}\sum_{i=1}^{n} f_i x_i$ (D) $\sum_{i=1}^{n}\left(\frac{f_i x_i}{N}\right)$

8. Which of the following can not be determined graphically –

(A) Mean (B) Median (C) Mode (D) Standard deviation

9. If the median of the following data is 40 then the value of p is –

Class	0- 10	10-30	30-60	60-80	80-90
Frequency	5	15	30	p	2

(A) 7 (B) 8 (C) 9 (D) 7.6

10. Which of the following is true?

(A) Mode = 2median – Mean (B) Mode = 3median + 2Mean

(C) Mode = 3median – 2Mean (D) None of these

11. Mode is –

(A) Most frequent value (B) Least frequent value

(C) Middle most value (D) None of these

12. Which of the following is true –

(A) Mode = 2median + Mean (B) Median = Mode + $\frac{3}{2}$ [Mean – Median]

(C) Mean = Mode + $\frac{3}{2}$ [Median – Mode] (D) Median = Mode + $\frac{3}{2}$ [Mean +Median]

13. In the formula for mode of a grouped data, mode = $\ell + \left\{\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right\} \times h$, where symbols have their usual meaning f_0 represents :

(A) Frequency of modal class
(B) Frequency of median class
(C) Frequency of the class preceding the modal class
(D) Frequency of class succeeding the modal class

14. Median of a given frequency distribution is found with the help of a –
(A) Bar graph (B) Ogive (C) Histogram (D) None of these

15. The measure of central tendency which is given by the x-coordinate of the point of intersection of the 'more than' ogive and 'less than' ogive is –
(A) Mean (B) Median (C) Mode (D) None of these

OBJECTIVE ANSWER KEY EXERCISE

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	C	C	C	A	B	C	C	A	B	C
Que.	**11**	**12**	**13**	**14**	**15**					
Ans.	A	C	C	B	B					

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

(A) MEAN OF A GROUPED DATA

1. Find the mean of the following data :

(a)

Class Interval	0-6	6-12	12-18	18-24	24-30
Frequency	6	8	10	9	7

(b)

Number of Plant	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of house	1	2	1	5	6	2	3

2. Find the mean of the following distribution :

(a)

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	3	5	9	5	3

(b) (i)

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	12	16	6	7	9

(ii)

Class Interval	100-120	120-140	140-160	160-180	180-200
Frequency	12	14	8	6	10

(iii)

Class Interval	0-100	100-200	200-300	300-400	400-500
Frequency	6	9	15	12	8

3. (a) The arithmetic mean of the following frequency distribution is 25.25. Determine the value of p :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	7	8	p	15	4

(b) The arithmetic mean of the following frequency distribution is 47. Determine the value of p :

Class Interval	0-20	20-40	40-60	60-80	80-100
Frequency	8	15	20	p	5

4. Find the value of f, the missing frequency, if the mean of the following distribution is 67.

Class Interval	25-35	35-45	45-55	55-65	65-75	75-85	85-98
Frequency	10	6	4	f	4	12	26

5. (a) Find the missing frequencies f_1 and f_2 if the frequency distribution is 62.8 and the sum of all frequency is 50

Class	0-20	20-40	40-60	60-80	80-100	100-120	Total
Frequency	5	f_1	10	f_2	7	8	50

(b) Find the missing frequencies f_1 and f_2 in the following data if the mean is $166\frac{9}{26}$ and the sum of the observation is 52.

Class	140-150	150-160	160-170	170-180	180-190	190-200
Frequency	7	f_1	20	f_2	7	8

(c) The mean of following frequency table is 53. But the frequency f_1 and f_2 in the classes 20-40 and 60-80 are missing

Age (in years)	0-20	20-40	40-60	60-80	80-100	Total
No. of people	15	f_1	21	f_2	17	100

6. (a) Find the mean of the following data, by using the assumed mean method.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	7	8	12	13	10

(b)

Marks	0-100	100-200	200-300	300-400	400-500	500-600
No. of students	2	8	12	20	5	3

7. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

No. of days	0-6	6-10	10-14	14-20	20-28	28-38	38-40
No. of students	11	10	7	4	4	3	1

8. (a)

Class	4-8	8-12	12-16	16-20	20-24	24-28	28-32	32-36
No. of students	2	12	15	25	18	12	13	3

Find the arithmetic mean of the following frequency distribution by using step deviation method :

(b) The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45-55	55-65	65-75	75-85	85-95
No. of cities	3	10	11	8	3

(c) The distribution show the number of wickets taken by bowlers in one day cricket matches. Find the mean number o

No. of wickets	20-60	60-100	100-150	150-250	250-350	350-450
No. of bowlers	7	5	16	12	2	3

9. (a) The following table gives the distribution of expenditures of different families on education. Find the mean expenditure on education of a family.

Expenditure (in Rs.)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
No. of families	24	10	33	28	30	22	16	7

(b) (i) To find the concentriation of SO_2 in the air (in per million), the data was collected for 30 localities in a certain city and is presented below :

Concentration of SO_2 (in ppm.)	0.00-0.04	0.04-0.08	0.08-0.12	0.12-0.16	0.16-0.020	0.20-0.24
Frequency	4	9	9	2	4	2

Find the mean concentration of SO_2 in the air.

(ii) The following table shows that the daily expenditure on food of 25 house holds in a localities. Find the mean daily expenditure on food by a suitable method.

Daily expenditure (in Rs.)	100-150	150-200	200-250	250-300	300-350
No. of house holds	4	5	12	2	2

10. (a) Find the mean marks from the following data :

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of students	4	10	18	28	40	70

(b) Compute the mean for the following data :

Marks	Less than 10	Less than 30	Less than 50	Less than 70	Less than 90	Less than 110	Less than 130	Less than 150
No. of students	0	10	25	43	65	87	96	100

11. (a) Find the average marks of student from the following data :

Marks	No. of Students	Marks	No. of Students
Above 0	80	Above 60	23
Above 10	77	Above 70	16
Above 20	72	Above 80	10
Above 30	65	Above 90	8
Above 40	55	Above 100	0
Above 50	43		

(b) Find the mean wage of the following data :

Wages (in Rs.)	No. of Workers
0 and above	120
20 and above	108
40 and above	90
60 and above	75
80 and above	50
100 and above	24
120 and above	9
140 and above	0

12. (a) In a retail market, fruit vendors selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

No. of mangoes	50-52	53-55	56-58	59-61	62-64
No. of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a pocket box.

(b) The following data shows that the age distribution of patients of malaria in a village during a particular month. Find the average age of the patients.

Age (in years)	5-14	15-24	25-34	35-44	45-54	55-64
No. of cases	6	11	21	23	14	5

(B) MEDIAN OF A GROUPED DATA

1. Find the median for the following frequency distribution :

(a)

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	9	14	2	19	10

(b) (i)

Class Interval	25-35	35-45	45-55	55-65	65-75
Frequency	20	25	5	7	4

(ii)

Class Interval	0-8	8-16	16-24	24-32	32-40	40-48
Frequency	8	10	16	24	15	7

(c) 100 surnames were randomly picket up from a local telephone directly and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows :

No. of letters	1-4	4-7	7-10	10-13	13-16	16-19
No. of Surnames	6	30	40	16	4	4

Find the median number of letters in the surnames. Find the mean number of letters in the surnames.

2. (a) Find the median from the following data :

Class groups	110-120	120-130	130-140	140-150	150-160	160-170	170-180	180-190	190-200
Frequency	6	25	48	72	116	60	38	22	3

(b) (i) The following distribution gives the weights of 30 students of a class. Find the median weight of the student

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
No. of students	2	3	8	6	6	3	2

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

(ii) Find the median of the following frequency distribution :

Marks	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency	2	5	9	12	17	20	15	9	7	4

(c) The following table gives the distribution of the life time of 400 neon lamps :

Life Time (in hours)	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
No. of lamps	14	56	60	86	74	62	48

Find the median life time of a lamp.

3. (a) A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 years.

Age in years	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
No. of policy holders	2	6	24	45	78	89	92	98	100

(b) A survey regarding the heights (in cm) of 51 girls of class X of a school was conducted and the data obtained follows :

Heights (in cm)	Less than 140	Less than 145	Less than 150	Less than 155	Less than 160	Less than 165
No. of girls	4	11	29	40	46	51

Find the median height.

4. (a) The following table gives the marks obtained by 50 students in a class test :

Marks	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
No. of Students	2	3	6	7	14	12	4	2

Find the median.

(b) The following table gives the population of males in different age groups :

Age group (in years)	5-14	15-24	25-34	35-44	45-54	55-64	65-74
No. of males	447	307	279	220	157	91	39

Find their median age.

5. (a) The following table gives the distribution of IQ of 100 students. Find the median IQ.

IQ	75-84	85-94	95-104	105-114	115-124	125-134	135-144
Frequency	8	11	26	31	18	4	2

(b) The length of 70 leaves of a plant are measured correct to the nearest millimeter and the data obtained is represented in the following table :

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No. of leave	10	8	13	22	7	6	4

Find the median length of the leaves.

6. Calculate the missing frequency f from the following distribution, it being given that the median of the distribution is 24.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	25	f	18	7

7.

Variable	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	f_1	65	f_2	25	18

(a) If the median of the following frequency distribution is 28.5, find the missing frequencies.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	f_1	20	15	f_2	5	60

(b) If the median of the following frequency distribution is 32.5, find the values of f_1 and f_2.

Class interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	7	10	x	13	y	10	14	9

(c) (i) An incomplete distribution is given below :
If median value is 46 and the total number of items is 230.
(α) Find the missing frequencies f_1 and f_2.
(β) Find the arithmetic mean (AM) of the completed distribution.
(ii) The median of the following data is 20.75 Find the missing frequencies x and y, if the total frequency is 100

(C) MEDIAN OF A GROUPED DATA

1. (a) Calculate the mode for the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

(b) A student noted the number of cars passing through spot on a road for 100 periods each of 3 minutes and summarized it in the table given below. Find the mode of the data .

No. of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

2. (a) The given distribution shows the number of runs scored by some top batsmen of the world in one day international cricket matches :

Runs Scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000	10000-11000
No. of batsman	4	18	9	7	6	3	1	1

Find the mode of the data.

(b) (i) The following tables gives the ages of the patients admitted in a hospital during a year.

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
No. of patients	6	11	21	23	14	5

Find the mode and the mean of the data

(ii) The following data gives the distribution of total monthly house hold expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure

Expenditure (in Rs.)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
No. of families	24	40	33	28	30	22	16	7

(c) (i) The following distribution gives the state-wise teacher student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret two measures.

No. of students per teacher	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
No. of state/U.T.	3	8	9	10	3	0	0	2

(ii) The following table shows the marks obtained by 100 students of Class X in school during a particular academic session. Find the mode of this distribution

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80
No. of students	7	21	34	46	66	77	92	100

3. (a) Compute the mode of the following data :

Class Interval	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
Frequency	3	8	13	18	28	20	13	8	6	4

(b) Compute the mode of the following data :

Score	80-90	90-100	100-110	110-120	120-130	130-140	140-150
No. of pupil	18	27	48	39	12	6	16

4. Calculate the mode of the following data :

Wages (In Rs.)	51-56	57-62	63-68	69-74	75-80	81-86	87-92
No. of workers	12	24	40	30	18	8	20

5. The mode of the following data is 85.7 Find the missing frequency in it.

Size	45-55	55-65	650-75	75-85	85-95	95-105	105-115
Frequency	7	12	17	f	32	6	10

(C) GRAPHICA REPRESENTATION OF CUMULATIVE FREQUENCY DISTRIBUTION

1. The following distribution gives the mark obtained by 102 students of class X.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	9	10	25	50	5	3

Convert the above distribution to a less than type cumulative frequency distribution and draw its ogive.

2. The following table gives the distribution of IQ of 60 pupils of class X in a school.

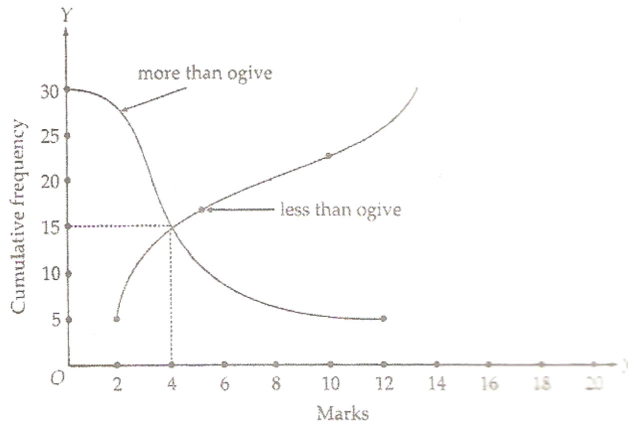
IQ	60-70	70-80	80-90	90-100	1005-110	110-120	120-130
No. of pupils	2	3	5	16	14	13	7

Convert the above distribution to a more than type cumulative frequency distribution and draw its ogive.

3. (a) The following table gives the height of trees :

Height	Less than 140	Less than 145	Less than 150	Less than 155	Less than 160	Less than 165
No. of trees	4	11	29	40	46	50

(b) What is the value of the median of the data using the graph in the given figure, of less than ogive and more than ogive?

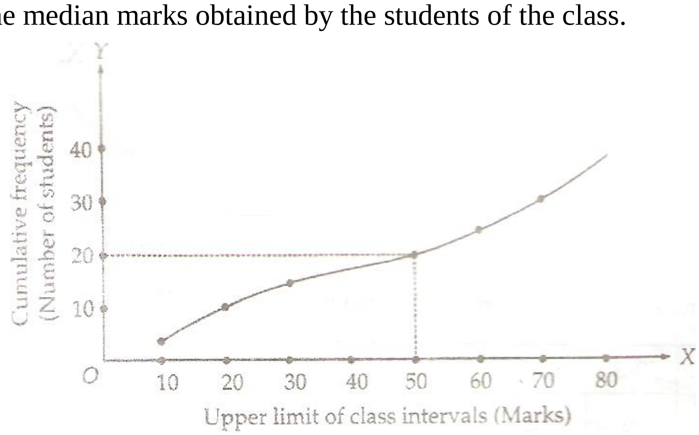


Draw both ogives for the data above. Hence, obtain the median of the data.

4. (a) Following is the age distribution of a group of students. Draw a cumulative frequency curve for the data and find the median.

Age in years	**No. of students**
Less than 5	36
Less than 6	78
Less than 7	136
Less than 8	190
Less than 9	258
Less than 10	342
Less than 11	438
Less than 12	520
Less than 13	586
Less than 14	634
Less than 15	684
Less than 16	700

(b) A student draws a cumulative frequency curve for the marks obtained by 40 students of a class as shown below. Find the median marks obtained by the students of the class.



5. The table given below shows the frequency distribution of the scores obtained by 200 candidates in a MCA entrance examination.

Score	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of students	30	15	45	20	25	40	10	15

Draw cumulative curve of more than type and hence find median.

(A) MEAN OF A GROUPED DATA :

1. (a) 15.45, (b) 8.1 **2.** (a) 25, (b) (i) 22 (ii) 145.20 (iii) 264 **3.** (a) 6, (b) 12 **4.** 23.71
5. (a) 8, 12, (b) $f_1 = 7$, $f_2 = 10$, (c) $f_1 = 18$, $f_2 = 29$, **6.** (a) 27.2 (b) 304 **7.** 12.48 days
8. (a) 19.92 (b) 69.43%, (c) On an average the number of wickets taken by bowers in one day cricket is 152.89.

9. (a) 2823.53 (b) (i) 0.099 ppm (ii) Rs.211 **10.** (a) $40\frac{5}{7}$ marks (b) 74.80

11. (a) 51.1 (b) (i) 69.34 **12.** (a) 57.19 (b) 34.87 years

(B) MEDIAN OF A GROUPED DATA :

1. (a) 35 (b) (i) 39.2 (ii) 26 (c) Median = 8.05, Mean = 8.32
2. (a) 153.8 (b) (i) 56.67 kg (ii) 532.5 (c) 3406.98 hours
3. (a) 35.76 years (b) 149.03 cm **4.** (a) 33 (b) 25.07 years
5. (a) 106.1 (b) 146.14 m **6.** 25
7. (a) 8, 7 (b) 3, 6 (c) (i) (α) 34 & 46 (β) 45.87 (ii) x = 17, y = 20

(C) MODE OF A GROUPED DATA :

1. (a) 46.67 (b) 44.7 cars
2. (a) 4608.7 runs
(b) (i) mode = 36.8, mean = 35.37 years, (ii) Rs. 1847.83, Rs. 2662.5
(c) (i) mode = 30.6, mean = 29.2 Most states U. T., have a student teacher ratio of 30.6 and on an average, this ratio is 29.2
(ii) 44.7
3. (a) 23.28 (b) 107 4. 66.2 5. 30 (approx.)

(D) GRAPHICAL REPRESENTATION OF CUMULATIVE FREQUENCY DISTRIBUTION :

3. (a) Median = 148.9 (b) Median = 4
4. (a) Median = 10 (b) Median marks = 50
5. Median = 375

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. Which measure of central tendency is given by the x-coordinate of the point of intersection of the "more than ogive" and "less than ogive"? **Delhi-2008**
2. Find the median class of the following data : **AI-2008**

Marks Obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	10	12	22	30	18

3. Write the median class of the following distribution : **Delhi-2009**

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

4. What is the lower limit of the modal class of the following frequency distribution? **Foreing-2009**

Age (in years)	0-10	10-20	20-30	30-40	40-50	50-60
Number of patients	16	13	6	11	27	18

SHORT ANSWER TYPE QUESTIONS

1. The mean of the following frequency distribution is 57.6 and the sum of observations is 50. Find the missing frequencies f_1 and f_2 : **Al-2004**

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	7	f_1	12	f_2	8	5

2. The following table gives the distribution of expenditure of different families on education. Find the mean expenditure on education of a family : **Delhi-2004C**

Expenditure (in Rs.)	Number of families
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

3. Find the mean of the following distribution : **Delhi-2005**

Class	4-8	8-12	12-16	16-20	20-24	24-28	28-32	32-36
Number of students	2	12	15	25	18	12	13	3

4. If the mean of the following data is 18.75 find the value of p : **Al-2005**

x_i	10	15	p	25	30
f_i	5	10	7	8	2

5. The Arithmetic Mean of the following frequency distribution is 50. Find the value of p : **Delhi-2006**

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	p	32	24	19

6. If the mean of the following is 50, find the value of f_1 : **Delhi-2006**

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	28	32	f_1	19

7. The mean of the following frequency distribution is 62.8. Find the missing frequency x. **Delhi-2007**

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	x	12	7	8

LONG ANSWER TYPE QUESTIONS

1. A survery regarding the heights (in cm) of 50 girls of class x of a school was conducted and the following data was obtained : **Delhi-2008**

Height in cm	120-130	130-140	140-150	150-160	160-170	Total
Number of girls	2	8	12	20	8	50

Find the mean, median and mode of the above data.

2. Find the mean, mode and median of the following data. **Al-2008**

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	10	18	30	20	12	5

3. Find the mean, median and mode of the following data. **Foreign-2008**

Class	Frequency
0-50	2
50-100	3
100-150	5
150-200	6
200-250	5
250-300	3
300-350	1

4. The following table gives the daily income of 50 workers of a factory : **Delhi-2009**

Daily income (in Rs.)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Find the mean, mode and median of the above data.

5. During the medical check-up of 35 students of a class their weights were recorded as follows : **Al-2009**

Weight (in kg)	Number of students
38-40	3
40-42	2
42-44	4
44-46	5
46-48	14
48-50	4
50-52	3

Draw a less than type and a more than type ogive from the given data. Hence obtain the median weight from the graph.

6. Find the mode, median and mean for the following data : **Foreign-2009**

Marks obtained	Number of students
25-35	7
35-45	31
45-50	33
50-55	17
55-65	11
65-75	1

VERY SHORT ANSWER TYPE QUESTION

1. Median **2.** 30-40 **3.** 17.5 and 45 **4.** 30-40 5. 40

SHORT ANSWER TYPE QUESTION

1. $f_1 = 8$, $f_2 = 10$ **2.** Rs. 2662.5 **3.** 19.92 **4.** p = 20 **5.** p = 28 **6.** $f_1 = 24$ **7.** 10

LONG ANSWER TYPE QUESTION

1. mean = 150.25 ; Median = 151.5 ; Mode = 154.
2. mean = 35.76 ; Median = 35.66 ; Mode = 35.44
3. mean = 59.9 ; Median = 61.6 ; Mode = 65.
4. mean = 145.20 ; Median = 138.57 ; Mode = 125
5. 42.2 kg
6. mean = 49.7 ; Median = 48.5 ; Mode = 46.1